Assignment 8

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1. **Explain how first-order logic overcomes shortcomings of propositional logic**

   First-order logic (FOL) overcomes the limitations of propositional logic by enabling representation of relationships between objects and general statements about these objects. Unlike propositional logic, which deals only with simple, fixed statements that cannot express individual objects, properties, or relations, FOL introduces quantifiers and variables, allowing it to:

   - Express generalizations with **quantifiers** (e.g., "for all" or "there exists"), making it possible to formulate rules about groups or classes of objects.
   - Capture relationships between objects using **predicates** (e.g., loves(Yiheng, Beth)).
   - Represent statements about **individual entities** with **variables** and functions, unlike propositional logic, which cannot specify individuals.
   - Model complex sentences and logical inferences about specific objects or classes of objects, enhancing its expressive power over propositional logic.

2. **Express universal quantification in terms of existential quantification**

   Universal quantification ($\forall x : P(x)$) can be expressed in terms of existential quantification by using negation:

   $$\forall x\, P(x) \equiv \neg \exists x\, \neg P(x)$$

   This means that stating "$P(x)$ is true for all x" is equivalent to saying "there does not exist an x for which $P(x)$ is not true."

3. **Translate the following into first-order logic**

   (a) **Some students took CS411 in Spring 2020.**

   $$\exists x\, (\text{Student}(x) \land \text{Took}(x, \text{CS411, Spring2020}))$$

   (b) **Some students wear a hoodie with UIC logo on it.**

   $$\exists x\, (\text{Student}(x) \land \text{Wears}(x, \text{Hoodie}) \land \text{HasLogo}(x, \text{UIC}))$$

   (c) **Something that glitters is not always gold, whereas gold always glitters.**

   $$\exists x\, (\text{Glitters}(x) \land \neg \text{Gold}(x)) \land \forall y\, (\text{Gold}(y) \rightarrow \text{Glitters}(y))$$

(d) **No one can win with everyone all the time.**

$$\neg\exists x\,\forall y\,\forall t\,(\text{Person}(x) \wedge \text{Person}(y) \wedge \text{Wins}(x, y, t))$$

(e) **All CS courses are difficult, except two.**

$$\exists x, y\,(\text{CS}(x) \wedge \text{CS}(y) \wedge x \neq y \wedge \neg\text{Difficult}(x) \wedge \neg\text{Difficult}(y)) \wedge \forall z\,((\text{CS}(z) \wedge z \neq x \wedge z \neq y) \rightarrow \text{Difficult}(z))$$