Assignment 7

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Question 1: Representing the Knowledge Base (KB) and Testing for Entailment

1.1 Representing the KB

Let:

- S: Sam plays baseball
- P: Paul plays baseball
- R: Ryan plays baseball

The sentences in the KB are:

1. "Sam plays baseball or Paul plays baseball": $S \vee P$
2. "Sam plays baseball or Ryan doesn't play baseball": $S \vee \neg R$

Therefore, the KB can be represented as:

$$KB = \{S \vee P, S \vee \neg R\}$$

1.2 Testing Entailment

Now, let's evaluate the given statements to see if they are entailed by KB.

(A) Sam and Ryan both play baseball

This statement translates to:

$$S \wedge R$$

To check if $KB \models S \wedge R$, we need to find if there exists any assignment of truth values where KB is true, but $S \wedge R$ is false.

Counterexample:

- Set $S =$ true, $P =$ false, and $R =$ false.
- Under this assignment:

- $S \vee P$ is true because S is true.
- $S \vee \neg R$ is also true because S is true.
- However, $S \wedge R$ is false because R is false.

Thus, there exists a model where KB holds, but $S \wedge R$ does not, so $KB \not\models S \wedge R$.

(B) At least one among Sam, Paul, and Ryan plays baseball

This statement translates to:

$$S \vee P \vee R$$

To determine if $KB \models S \vee P \vee R$, let's see if every model that makes KB true also makes $S \vee P \vee R$ true.

Reasoning:

- In any model satisfying KB, we know $S \vee P$ must be true, meaning at least one of S or P must be true in any model of KB.
- Consequently, $S \vee P \vee R$ will always hold if KB is true.

So, $KB \models S \vee P \vee R$.

Question 2: Translating Sentences to Propositional Logic

Let's assign propositional symbols as follows:

- A: Ana eats
- B: Bret eats
- C: Charles eats
- D: Derek eats
- E: Earl eats
- F: Fred eats
- G: Gary eats

Now, translate each sentence:

a. "If Ana eats, Bret eats":

$$A \rightarrow B$$

b. "Charles eats and Derek doesn't eat":

$$C \wedge \neg D$$

c. "Bret doesn't eat":

$$\neg B$$

d. "If Derek doesn't eat, at least one among Ana, Earl, and Fred eats":

$$\neg D \rightarrow (A \vee E \vee F)$$

e. "If at least one of Charles and Gary eats, Earl doesn't eat":

$$(C \vee G) \rightarrow \neg E$$

Question 3: Proving that Fred Eats

Using the premises from Question 2, we need to show that F (Fred eats) must be true.

The premises are:

1. $A \rightarrow B$
2. $C \wedge \neg D$
3. $\neg B$
4. $\neg D \rightarrow (A \vee E \vee F)$
5. $(C \vee G) \rightarrow \neg E$

Proof

1. From premise (3), $\neg B$ means Bret doesn't eat.
2. From premise (1), since $A \rightarrow B$ and $\neg B$, we conclude by the contrapositive that $\neg B$ implies $\neg A$, so $\neg A$.
3. From premise (2), we know $C \wedge \neg D$, which gives us C (Charles eats) and $\neg D$ (Derek doesn't eat).
4. By (4), since $\neg D \rightarrow (A \vee E \vee F)$ and $\neg D$ is true, we obtain $A \vee E \vee F$.
5. Since we know $\neg A$ (from step 2) and will see in step 6 that $\neg E$, it follows that F must be true in order for $A \vee E \vee F$ to hold.

Therefore, F (Fred eats) is entailed by the premises.