Wind Energy Harvesting with a Kite

Antonin Bavoil[1], Jean-Baptiste Caillau[1], Lamberto Dell'Elce[2], Alain Nême[3] , Jean-Baptiste Leroux[3]

1: Université Côte d'Azur, CNRS, Inria, LJAD
2: Inria
3: ENSTA Bretagne, iRDL

FGS Conference on Optimization

Gijón, June 2024

Kite Electrical Energy Production (KEEP)

- KEEP is born as a follow up of Beyond the Sea®.
- Idea: generate (on land) electricity with a kite.
- 10x less material than a wind turbine, <u>soft.</u>
- Portable source of power in remote areas.

→ Islands, military.

https://www.dailymotion.com/video/x5fwyox

Kite Electrical Energy Production (KEEP)

- Approximately the same power as a wind turbine.
- Currently: 10% of the production used for control.
- Previous works:
  - U. Ahrens, M. Diehl, R. Schmehl. *Airborne Wind Energy.* Springer, 2013,
  - U. Fechner et al. *Dynamic Model of a Pumping Kite Power System*, Pergamon, 2015.

→ *Can we generate a good amount of electricity while requiring close to no control?*

https://www.iksurfmag.com/reviews/kites/north-kiteboarding-evo-6m-2013/

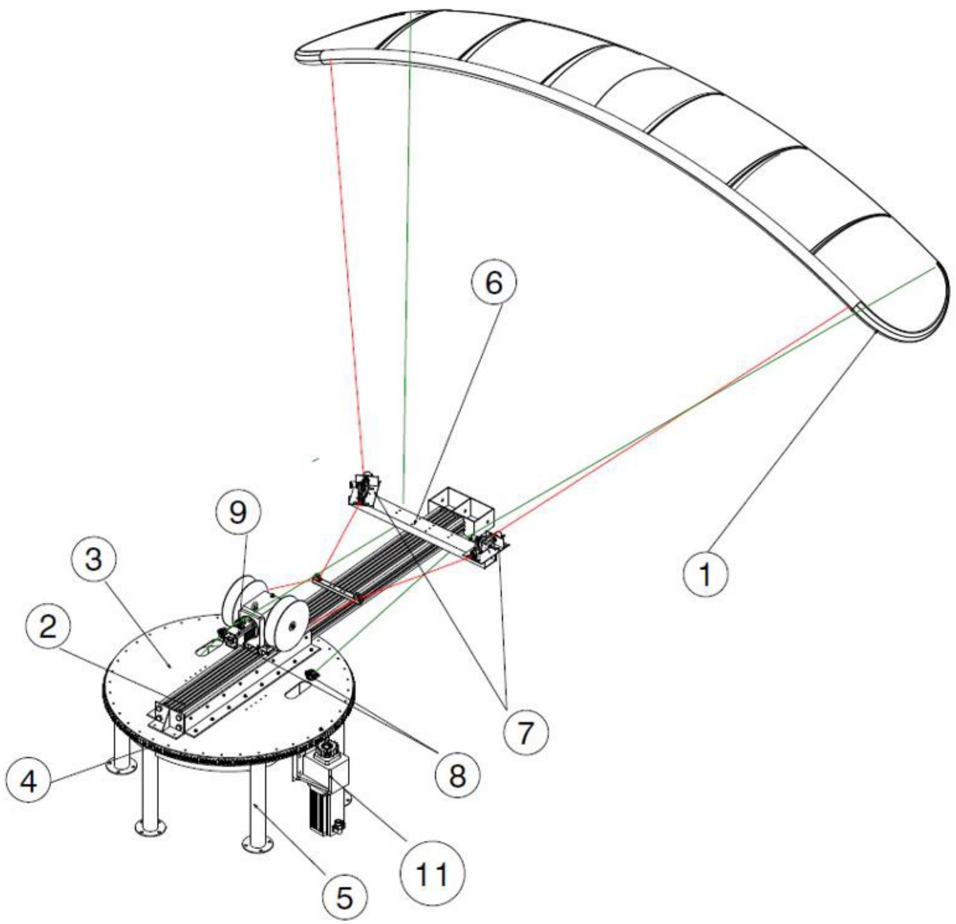
Wind Energy Harvesting with a Kite

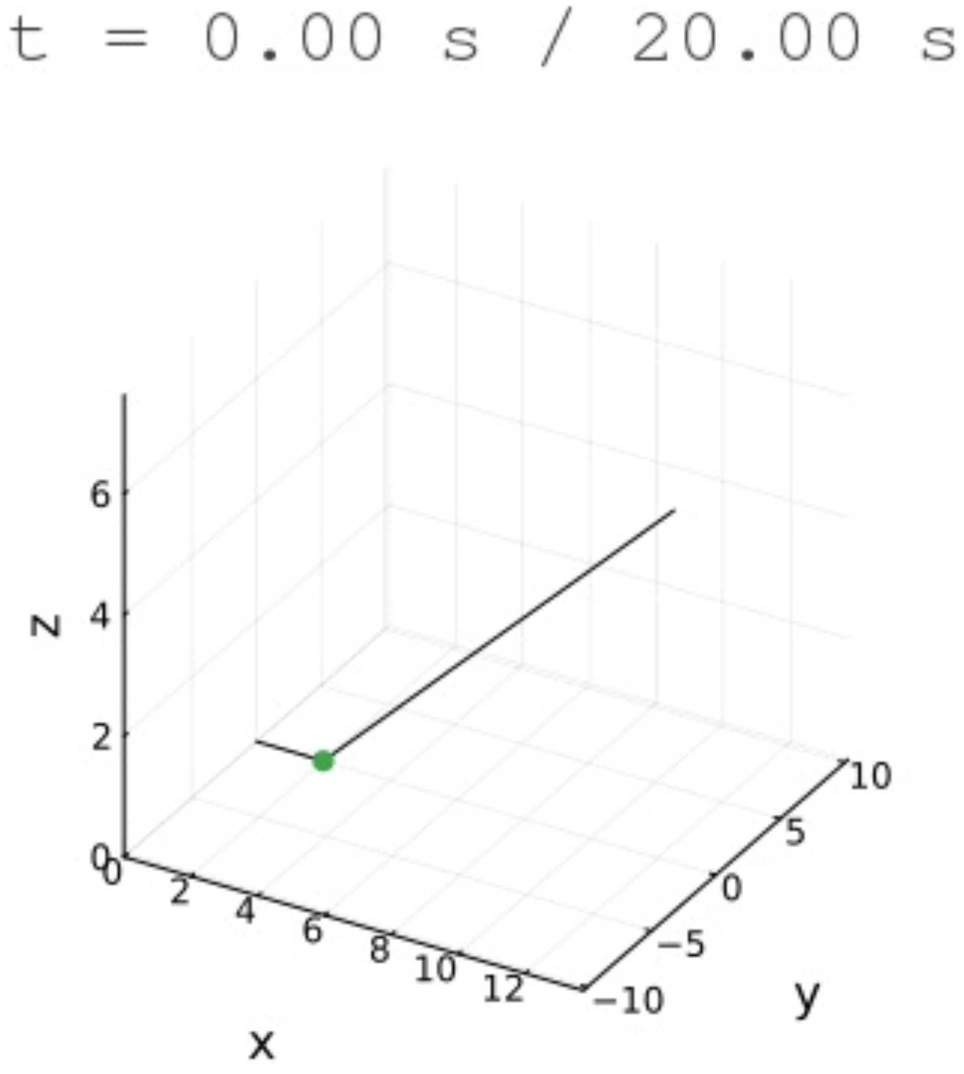
I. Modelling

II. Numerical Results

III. Optimization

Modelling





→ The kite does eights in the sky to swing the arm left to right as much as possible, without tangling the lines.

Modelling

Replacing the control with an algebraic constraint

- As a preliminary study, we replace the control with a geometric constraint.

→ The kite will stay on an 8-based cone centered on O.

- Spherical coordinates in the inertial frame $Oxyz$:

Set of (R, θ, φ) s.t.

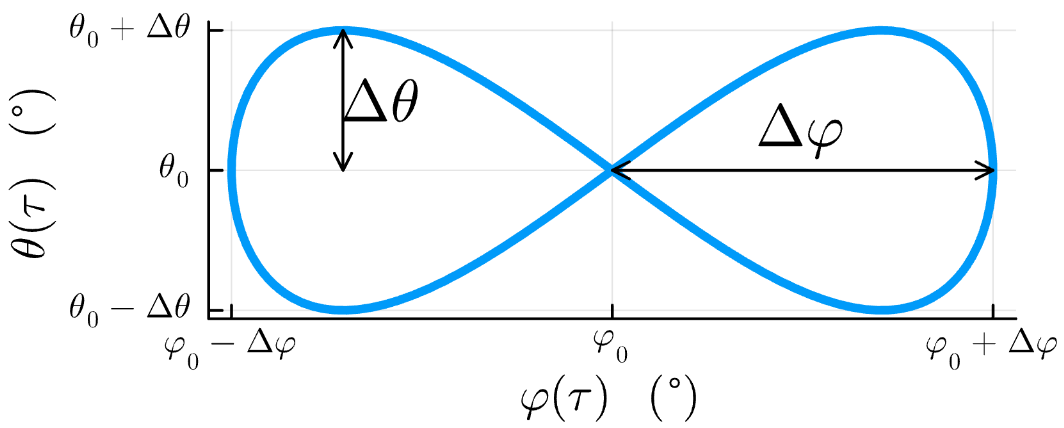
$$\theta = \theta_0 + \Delta\theta \sin(2\tau)$$

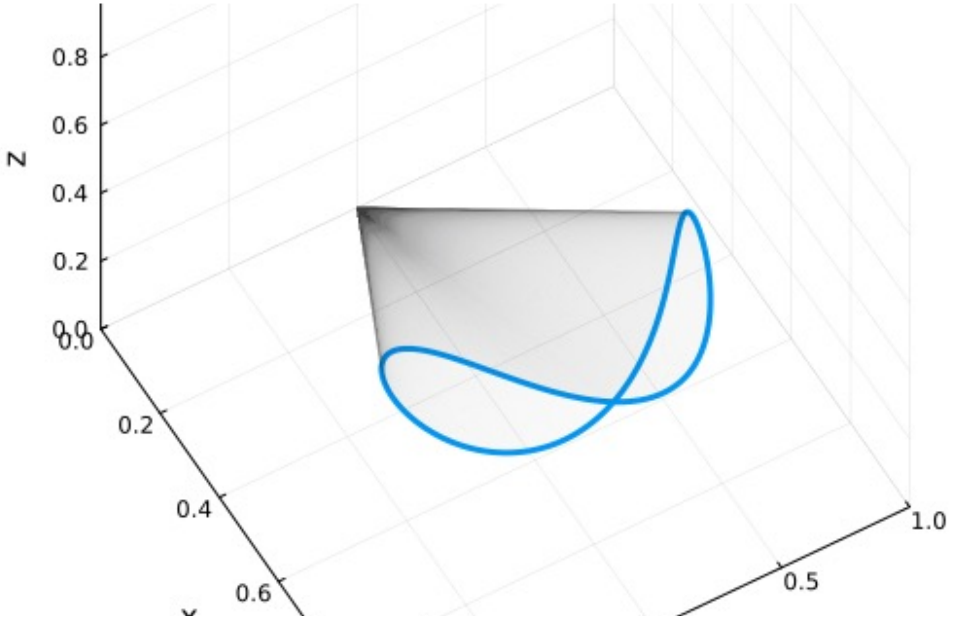
$$\varphi = \varphi_0 + \Delta\varphi \sin(\tau)$$

$$R = R(\alpha, \tau).$$

→ **Like a railway track guiding a train.**

Base of the cone

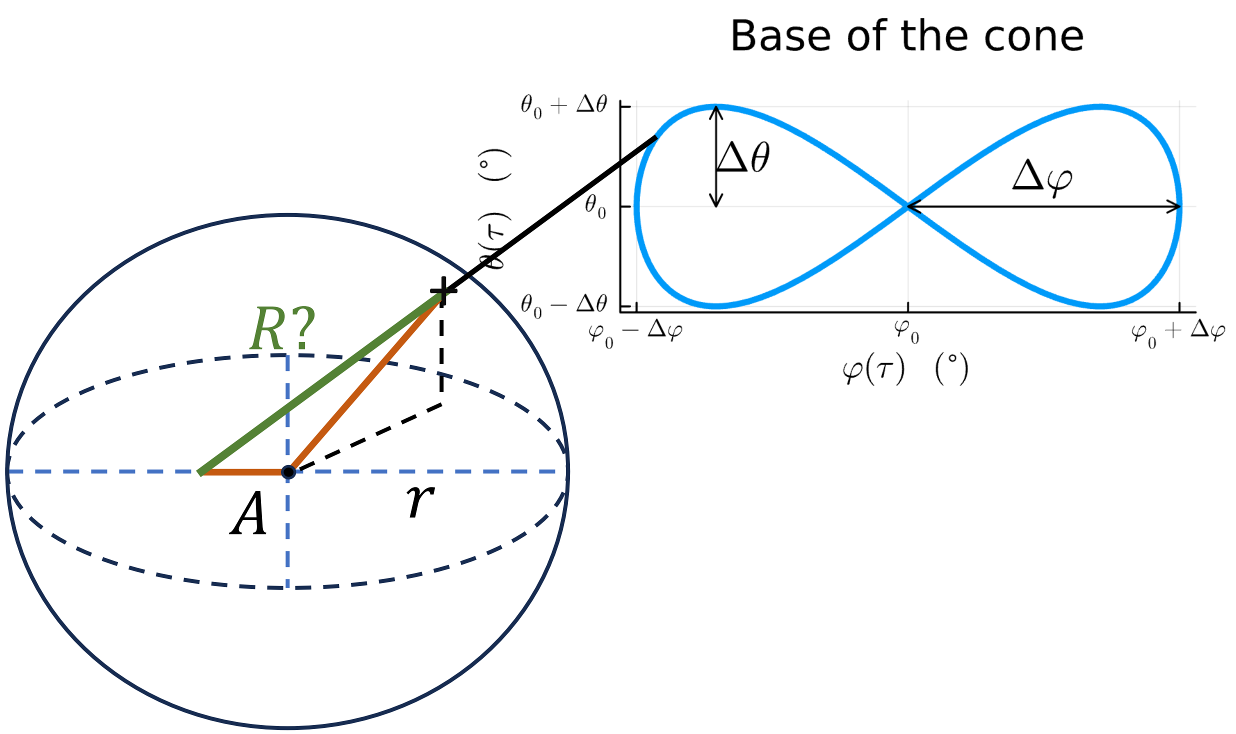




Modelling

Coordinates

- State of the system in the 3d space:
  - α: 1D position of the tip of the arm (on a circle),
  - (R, τ): 2D position of the kite (on the 8-based cone).



We can do better: get R through the intersection of a sphere (A, r) with a line:

$$R(\alpha, \tau) = \hat{\tau} \cdot \overrightarrow{\mathrm{OA}} + \sqrt{\left(\hat{\tau} \cdot \overrightarrow{\mathrm{OA}}\right)^2 + r^2 - \mathrm{OA}^2}.$$

→ (α, τ) is enough to determine the state of the system.

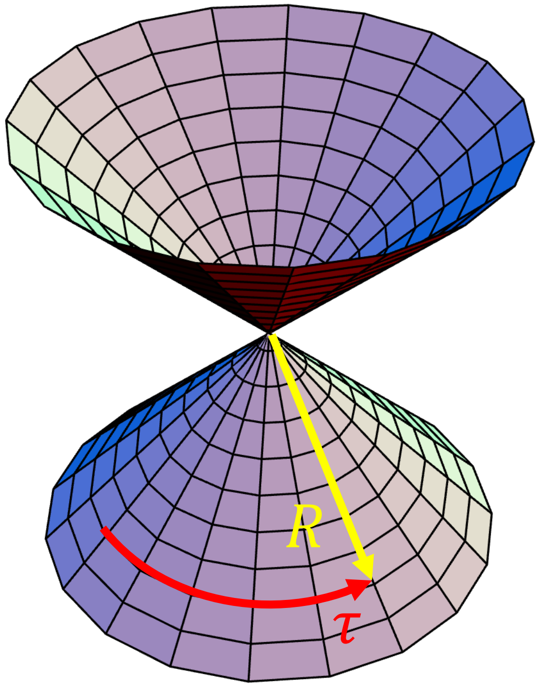
Modelling

Replacing the control with an algebraic constraint

- How? Add a fictitious force $\overrightarrow{F_{\text{cone}}}$ that would result from a control.
- No friction with the cone:

$$\overrightarrow{F_{\text{cone}}} \cdot \frac{\partial \overrightarrow{OK}}{\partial R} = 0 \text{ and } \overrightarrow{F_{\text{cone}}} \cdot \frac{\partial \overrightarrow{OK}}{\partial \tau} = 0.$$

Circle-based cone

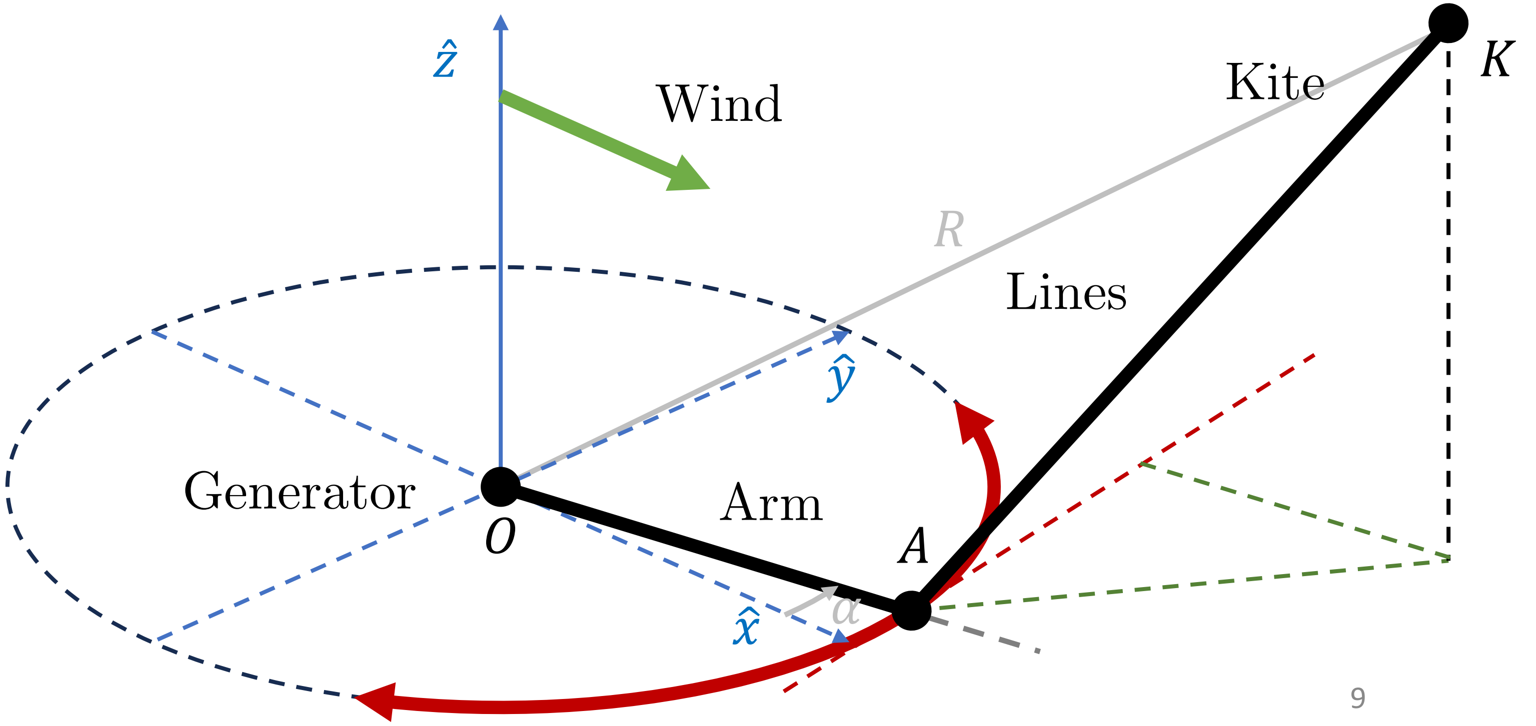


https://mathworld.wolfram.com/Cone.html

Modelling

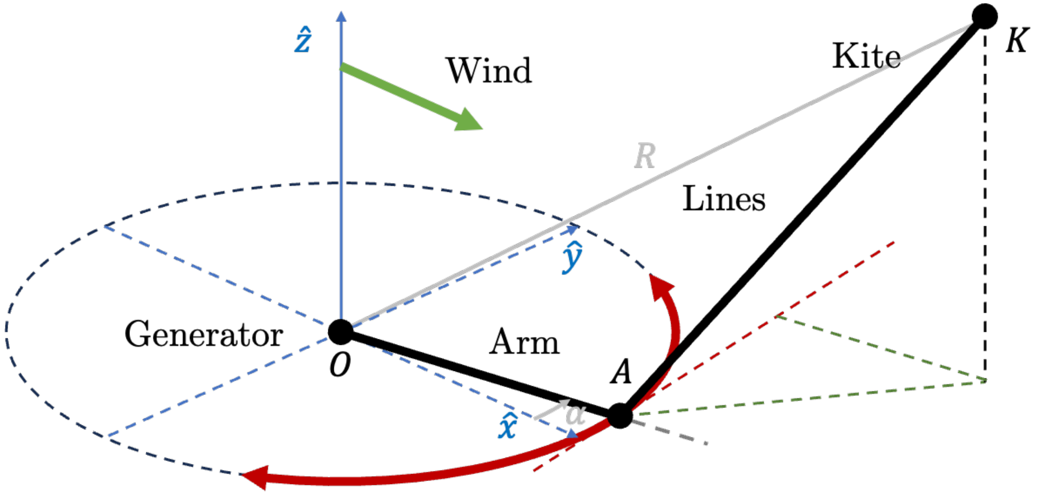
Compute the dynamics

- Dynamics: conservation of linear & angular momentum.
- All forces apply at point K (the kite):
  - Gravity,
  - Aerodynamical forces,
  - Line tension,
  - Fictitious cone force.



Modelling

Gravity

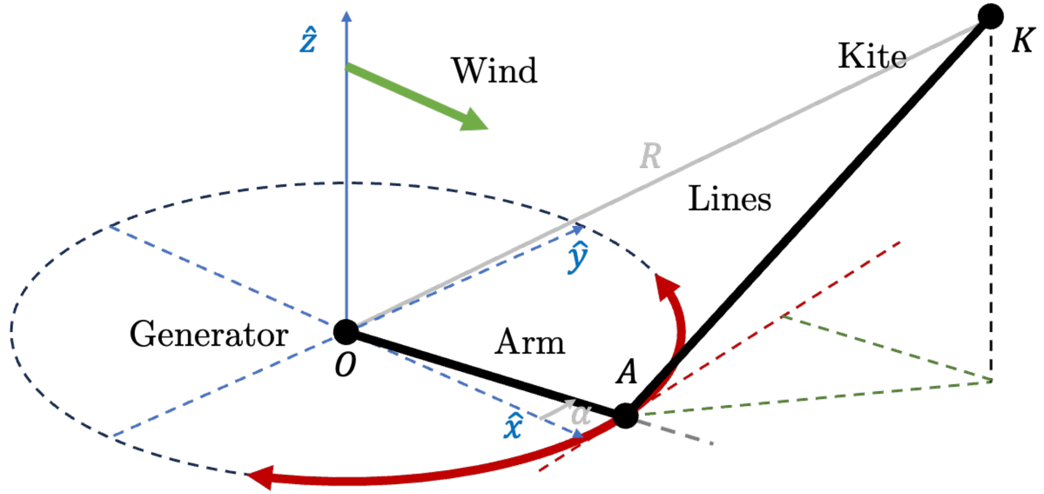


- Gravity force = weight of the kite + $\frac{1}{2}$ weight of the lines:

$$\overrightarrow{F_{\text{grav}}} = -g\left(m_{\text{kite}} + \frac{1}{2}m_{\text{lines}}\right)\hat{z}.$$

Modelling

Aerodynamical forces



- Apparent wind: $\overrightarrow{w_{\text{app}}} = \vec{v} - \vec{w}$ (velocity – wind).
- Aerodynamical force on the kite:

$$\overrightarrow{F_{\text{aero,kite}}} = -\frac{1}{2}\, S\, \rho_{\text{air}} \left\| \overrightarrow{w_{\text{app}}} \right\|^2 (C_L \hat{z}_w + C_D \hat{x}_w).$$

- Aerodynamical force on the lines, applied at point K:

$$\overrightarrow{F_{\text{aero,lines}}} = -\frac{1}{2} \frac{n b_l\, r\, d_l}{3} \rho_{\text{air}} \left\| \overrightarrow{w_{\text{app}}} - (\hat{r} \cdot \vec{v})\hat{r} \right\|^2 C_{Dl}\, \hat{e}_w.$$

$$\overrightarrow{F_{\text{aero}}} = \overrightarrow{F_{\text{aero,kite}}} + \overrightarrow{F_{\text{aero,lines}}}.$$

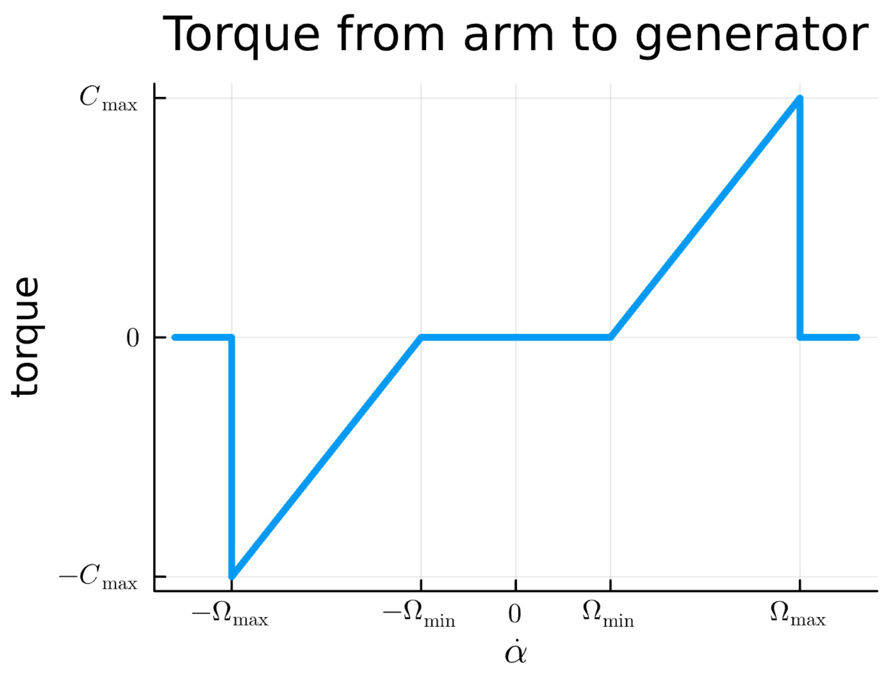
Modelling

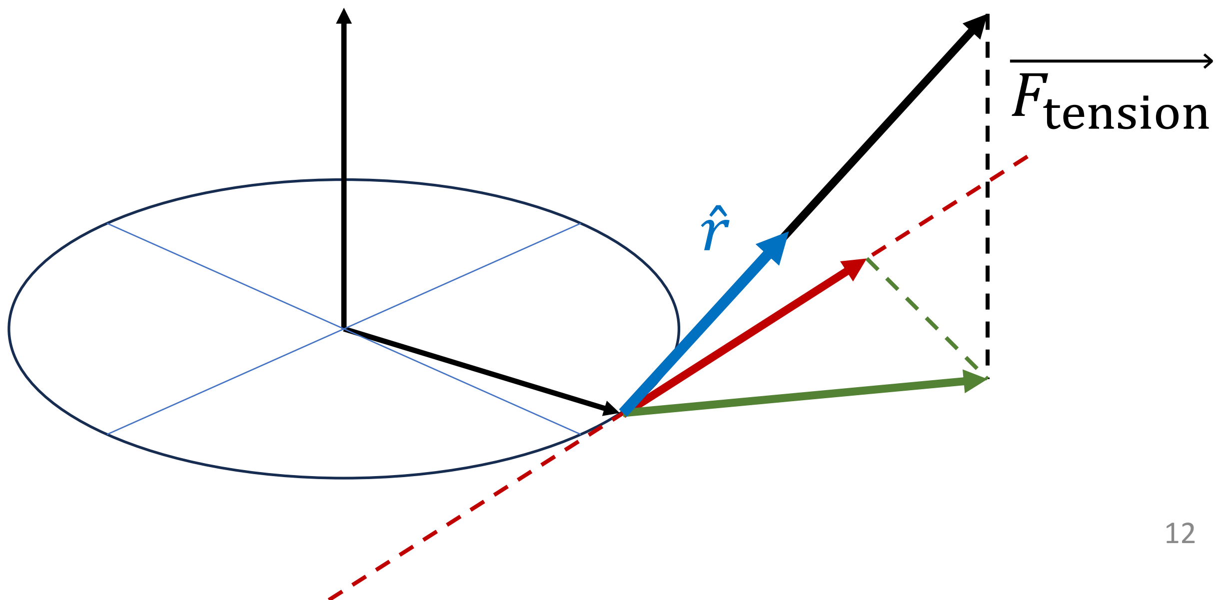
Line tension & conservation of angular momentum

- Conservation of angular momentum of the arm gives:

$$I_{\mathrm{eq}}\ddot{\alpha} = -\mathrm{torque}(\dot{\alpha}) + F_{\mathrm{tension}}\,\hat{r}\cdot\left(\hat{z}\times\overrightarrow{\mathrm{OA}}\right).$$

- Hence $\overrightarrow{F_{\mathrm{tension}}} = \frac{I_{\mathrm{eq}}\ddot{\alpha}-\mathrm{torque}(\dot{\alpha})}{\hat{r}\cdot(\hat{z}\times\overrightarrow{\mathrm{OA}})}\hat{r}$, where $\ddot{\alpha}$ is obtained by solving the dynamics cf. next slide.





Modelling

Computing the explicit dynamics

- Recall that $\overrightarrow{F_{\text{cone}}} \cdot \frac{\partial \overrightarrow{OK}}{\partial R} = 0$ and $\overrightarrow{F_{\text{cone}}} \cdot \frac{\partial \overrightarrow{OK}}{\partial \tau} = 0$.

- Conservation of linear momentum gives: $m \frac{d\vec{v}}{dt} = \overrightarrow{F_{\text{cone}}} + \overrightarrow{F_{\text{grav}}} + \overrightarrow{F_{\text{aero}}} + \overrightarrow{F_{\text{tension}}}$.

  $(\ddot{\alpha}, \ddot{\tau})$ $\quad$ $\ddot{\alpha}$

- Hence $\overrightarrow{F_{\text{cone}}} = m \frac{d\vec{v}}{dt} - \overrightarrow{F_{\text{grav}}} - \overrightarrow{F_{\text{aero}}} - \overrightarrow{F_{\text{tension}}}$. Linear in $\ddot{\mathbf{u}} = (\ddot{\alpha}, \ddot{\tau}) \in \mathbb{R}^2$.

- Solve $A(\mathbf{u})\,\ddot{\mathbf{u}} = b(\mathbf{u}, \dot{\mathbf{u}})$.

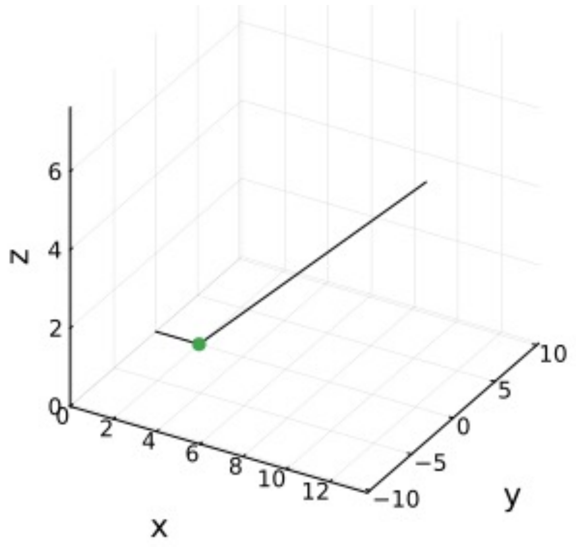
First Numerical Results

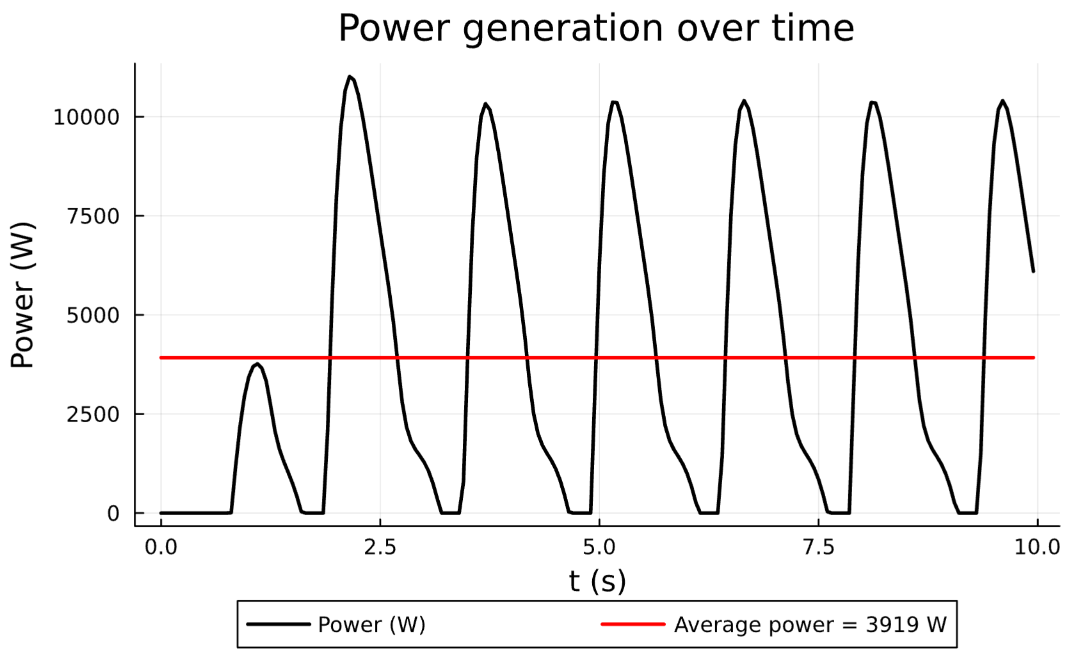
Solving the dynamics

- Assemble $A(\mathbf{u})$ and $b(\mathbf{u}, \dot{\mathbf{u}})$ using forward auto. diff. for exact, fast & maintainable derivatives:

```
julia> kite_speed = jvp(u -> OK(u, params), u, du)
```

- Runge-Kutta 5(4) (Tsitouras 2011):





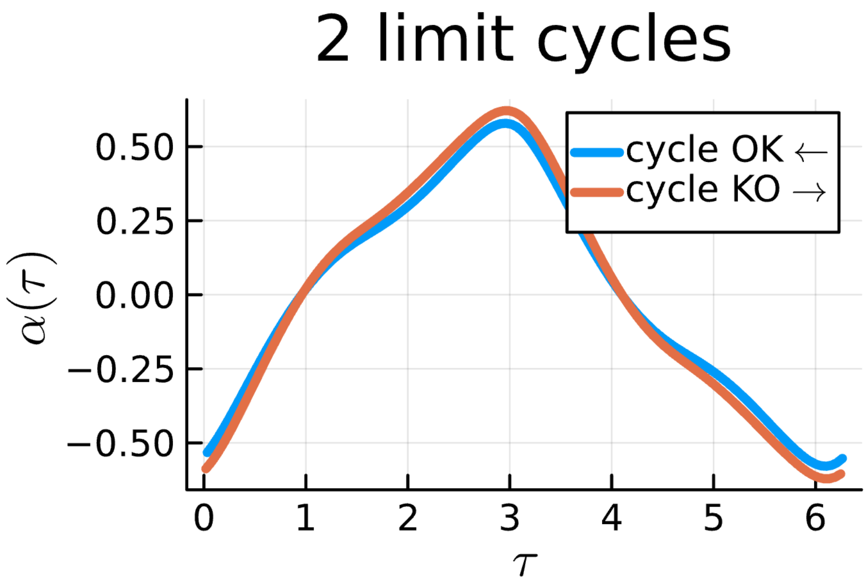
→ Cyclic behavior: toward a limit cycle...

First Numerical Results

Limit Cycle

- Define a Poincaré section at $\tau \equiv 0\ [2\pi]$ where the cycle begins (arbitrary).
- Dense ODE output & continuous callback allow us to save the state every time it crosses one of these hyperplanes:

Iteration	$\Delta t = t(\tau = 2\pi) - t(\tau = 0)$ (seconds)
1	2.7857519406370397
2	3.0933710067285087
3	3.0930984474625856
…	…
7	3.093098056921132
8	3.0930980569206064



→ 2 limit cycles, interested in the one with $\dot{\tau} < 0$ that does not tangle the lines.

Optimization

Problem: Optimize the Average Power over a limit cycle

- We define the following problem

$$\frac{1}{t_f}\int_0^{t_f} \underbrace{\dot{\alpha}(\mathrm{t})\ \mathrm{torque}(\dot{\alpha}(t))}_{P(t)}\ \mathrm{d}t \ \rightarrow \max$$

Over $\left((\alpha_0, \dot{\alpha_0}, \dot{\tau_0}, t_f), p\right) \in \mathbb{R}^4 \times \mathbb{R}^n$ with n the number of available parameters to optimize.

With constraints:

- $\alpha(t_f) = \alpha_0,\ \dot{\alpha}(t_f) = \dot{\alpha_0},\ \ \dot{\tau}(t_f) = \dot{\tau_0},\ \ \tau(t_f) = \tau_0 - 2\pi$
- Box constraints on the state.

With final state obtained by the ODE solver.

Optimization

Augmentations

- Augment ODE state:
  - $(\alpha, \tau, \dot{\alpha}, \dot{\tau}) \rightarrow (\alpha, \tau, \dot{\alpha}, \dot{\tau}, W)$ with W being the cumulated work done by the generator,
  - with its associated differential equation $\dot{W} = \dot{\alpha}\, \mathrm{torque}(\dot{\alpha})$.

- Augment optimization variables: $\left((\alpha_0, \mathrm{d}\alpha_0, \mathrm{d}\tau_0, t_f), p\right) \rightarrow \left((\alpha_0, \mathrm{d}\alpha_0, \mathrm{d}\tau_0, t_f, W_f), p\right)$

→ Maximize W_f / t_f over $\left((\alpha_0, \mathrm{d}\alpha_0, \mathrm{d}\tau_0, t_f, W_f), p\right) \in \mathbb{R}^5 \times \mathbb{R}^n$ with constraints:

- $\alpha(t_f) = \alpha_0, \dot{\alpha}(t_f) = \dot{\alpha_0}, \dot{\tau}(t_f) = \dot{\tau_0}, \tau(t_f) = \tau_0 - 2\pi$
- The augmented constraint $W_f = W(t_f)$
- Box constraints over the state

Optimization

Ongoing work

- Gradients and Hessians of the constraints computed with automatic differentiation
- Initialization on the limit cycle, where constraints are satisfied
- Use Interior Point OPTimizer (IPOPT)

→ Look for saturated box constraints

→ perform a sensitivity analysis around the optimal point

Conclusion

- Solve the ODE → find the limit cycle → optimize on the limit cycle, thanks to Julia tools
- Move forward to a controlled model, without the cone constraint
- Have an engineer build a prototype that would (in)validate our results through measurements

K. Desenclos, 2022, Production d'électricité par voile de kite.

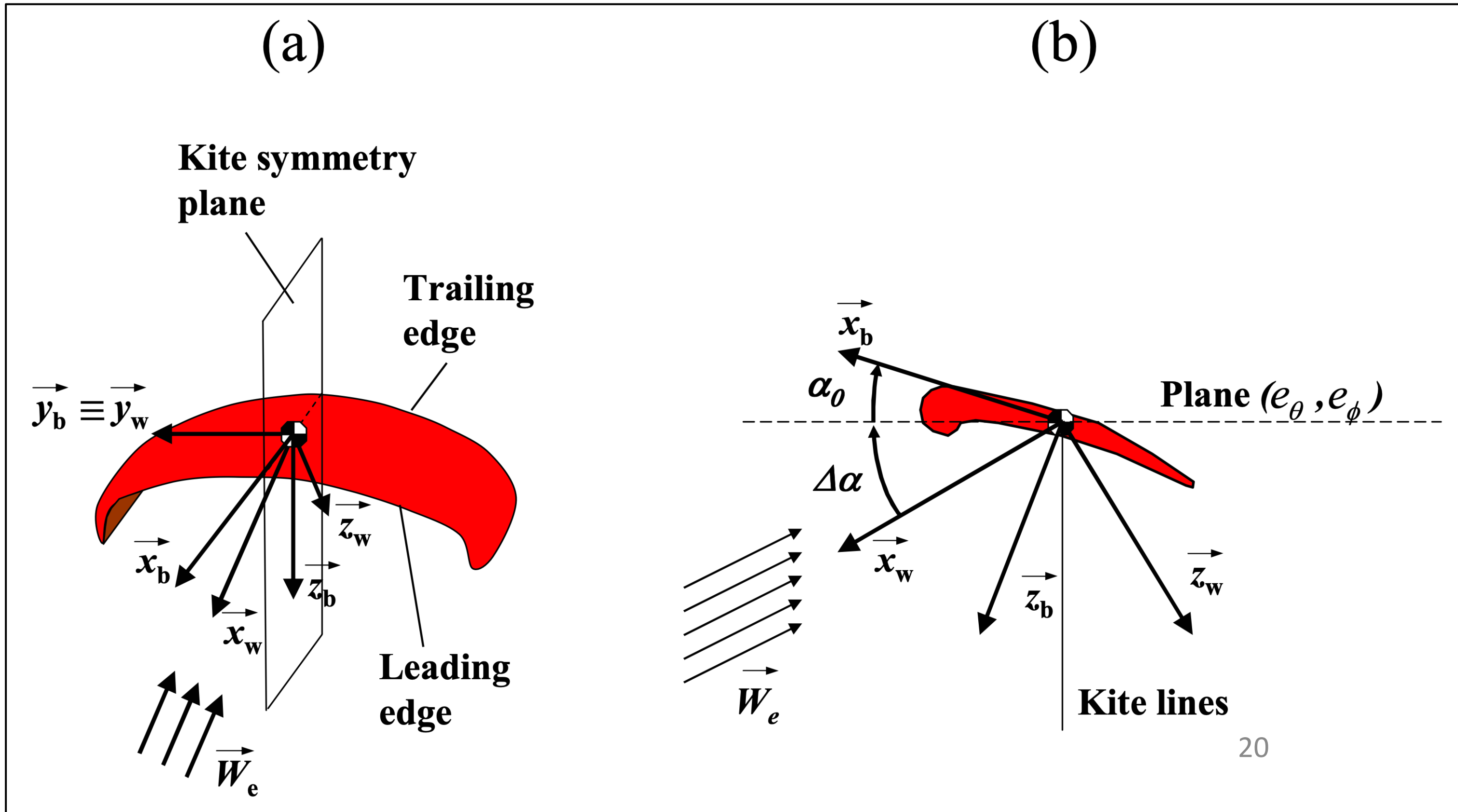
Annex 1: Basis vectors related to the local wind

- Apparent wind perpendicular to the lines:

$$\widehat{e_w} = \frac{\overrightarrow{w_{app}} - (\hat{r}\cdot\vec{v})\hat{r}}{\left\|\overrightarrow{w_{app}} - (\hat{r}\cdot\vec{v})\hat{r}\right\|}$$

- Basis of the apparent wind:

$$\widehat{x_w} = -\frac{\overrightarrow{w_{app}}}{\left\|\overrightarrow{w_{app}}\right\|}$$
$$\widehat{y_w} = \hat{r}\times\widehat{e_w}$$
$$\widehat{z_w} = \widehat{x_w}\times\widehat{y_w}$$



Annex 2: Forward Automatic Differentiation

- Define ε st. $\varepsilon^2 = 0$
- A dual number is $a + b\varepsilon$
- Define the base operations:
  - $(a + b\varepsilon) + (c + d\varepsilon) = (a + c) + (b + d)\varepsilon$
  - $(a + b\varepsilon)(c + d\varepsilon) = ac + (ad + bc)\varepsilon$
  - …
- sin, cos, exp, etc. are defined using $+$, $\times$, $/$, etc.
- $f(x + \varepsilon) = f(x) + \varepsilon f'(x)$

→ See ForwardDiff.jl

$$f(w_1, w_2) = \sin(w_1) + w_1 w_2$$