1) Ket - Notation:- $|-\rangle$

Define as a two dimensional vector who 0th location has one in it.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{array}{l} -0\text{th} \\ -1\text{th} \end{array}$$

Similarly:-

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{array}{l} -0\text{th} \\ -1\text{th} \end{array}$$

Example:-

as it is 5 in decimal so it 5 th has 1 in it

$|101\rangle$ = $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ → 5th location will has 1 as 101 in decimal is 5

↳ $2^3 = 8$ Dimensional

Example:- $|11\rangle$ = $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ — 3th location will has 1 as 11 in decimal is 3

↳ $2^2 = 4$ Dimensional

2 Dimension:- $\{|0\rangle, |1\rangle\}$ Standard Basis

$$\begin{pmatrix} 7 \\ 3+5i \end{pmatrix} = 7|0\rangle + (3+5i)|1\rangle$$

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4D $\begin{pmatrix} 7 \\ 0 \\ i+3 \\ 0 \end{pmatrix} = 7|00\rangle + i+3|10\rangle$

Basis : $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

2 Bra-Notation $\langle - |$:-

is a two dimensional vector whose zero location has one in it.

$\langle 0| [1, 0]$, $\langle 1| = [0, 1]$

$|\psi\rangle^{+} = \langle\psi|$, $\langle\psi|^{+} = |\psi\rangle$

Conjugate transpose (+) :-

Compose of two operation the

+ = Conjugate (*) & Transpose (T)

Example :-

$|\psi\rangle = (3+5i)|0\rangle + 7|1\rangle$

$\langle\psi| = |\psi\rangle^{+} = (3-5i)\langle 0| + 7\langle 1|$

change the sign ~~of~~ of iota term

$|\psi\rangle = \begin{pmatrix} 3+5i \\ 7 \end{pmatrix}$, $\langle\psi| = [3-5i \;\; 7]$

4D $[3 \;\; 0 \;\; i \;\; 7]$

$= 3\langle 00| + i\langle 10| + 7\langle 11|$

3 Ket-Ket Notation :- $(\cancel{\langle + \langle} + |-\rangle \; |-\rangle)$:-

gives us the tensor product of two column vector creating a column vector of high dimension.

Tensor Product

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} 1 \cdot B & 0 \cdot B \\ 0 \cdot B & 2 \cdot B \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 4 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & 8 & 14 \end{pmatrix}$$

$|\psi\rangle \otimes |\phi\rangle = |\psi\rangle|\phi\rangle = |\psi\phi\rangle$] $\rightarrow$ similar way to write it

Example:-

$|\psi\rangle = i|0\rangle + 7|1\rangle$

$|\phi\rangle = |00\rangle + 3|10\rangle + 7|11\rangle$

$|\psi\phi\rangle = i|000\rangle + 3i|010\rangle + 7i|011\rangle$
$+ 7|100\rangle + 21|110\rangle + 49|111\rangle$

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in term of vector:-

$$|\psi\rangle = \begin{bmatrix} i \\ 7 \end{bmatrix}, \quad |\phi\rangle = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 7 \end{bmatrix}$$

$$|\psi\phi\rangle = \begin{array}{l} \begin{bmatrix} i \\ 0 \\ 3i \\ 7i \\ 7 \\ 0 \\ 21 \\ 49 \end{bmatrix} \begin{array}{l} -0 \\ \\ \\ \\ \\ \\ \\ -7 \end{array} \end{array}$$

4 Bra-Bra Notation $(\langle -| \langle -|)$:-

gives us the tensor product of two row vector of higher dimension.

Ex
$$\langle\psi| = 3\langle 0| + 7\langle 1|$$
$$\langle\phi| = \langle 0| + i\langle 1|$$

$$\langle\psi\phi| = 3\langle 00| + 3i\langle 01| + 7\langle 10| + 7i\langle 11|$$

in terms of vector:-

$$\langle\psi| = [3 \ \ 7) \qquad \langle\phi| = [0, i]$$

$$\langle\psi\phi| = \begin{bmatrix} 3 \\ 3i \\ 7 \\ 7i \end{bmatrix} \begin{array}{l} -0th \\ \\ \\ -4th \end{array}$$

5// Ket Bra Notation (|-x-|) :-

$|\alpha\rangle\langle\beta|$ → uncool to write it that way

So we write is as → $|\alpha \times \beta|$

" my result will alway be a matrix and no matter this operation will always be valid."

(many rows but one column) (many column but one row)

as we know $|\alpha\rangle$ represent column vector and $\langle\beta|$ represent row vector.

$m\times 1$ $1\times m$

$|\alpha \times \beta|$

$m\times 1$ $1\times m$

$= m\times n$

Example:-

$|00 \times 10|$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ex

$|\alpha\rangle = 3|0\rangle + i|1\rangle$

$\cancel{\langle\beta| = |00\rangle + 2|10\rangle}$ $|\beta\rangle = |00\rangle + 2|10\rangle + 7|11\rangle$

To Find

$|\alpha\rangle\langle\beta|$

$|\beta\rangle^{\dagger} = \langle\beta| = \langle 00| + 2\langle 10| + 7\langle 11|$

$|\alpha\rangle\langle\beta| = \left(3|0\rangle + i|1\rangle\right)\left(\langle 00| + 2\langle 10| + 7\langle 11|\right)$

$= 3|0\rangle\langle 00| + 6|0\rangle\langle 10| + 21|0\rangle\langle 11|$
$+ i|1\rangle\langle 00| + 2i|1\rangle\langle 10| + 7i|1\rangle\langle 11|$

Matrix:-

= 2 rows

= 4 columns

$$\begin{array}{cc} \begin{bmatrix} 3 & 0 & 6 & 21 \\ i & 0 & 2i & 7i \end{bmatrix} & \begin{array}{l} \text{0th row} \\ \text{1th row} \end{array} \\ \begin{array}{cccc} \text{0 colum} & \text{1th colum} & \text{2th} & \text{3th colum} \end{array} & \end{array}$$

Example (Ket-Bra)

$$A = \begin{array}{c} \\ 0 \text{ row} \\ \\ \\ 3 \text{ row} \end{array} \begin{bmatrix} 0 & 1 \\ 3 & i \\ 7 & 0 \\ 0 & 13 \end{bmatrix}$$

$$\begin{array}{cc} 0 & 1 \\ c & c \end{array}$$

$$= 1|00\rangle\langle 1| + 3|01\rangle\langle 0| + i|01\rangle\langle 1| + 7|10\rangle\langle 0| + 13|11\rangle\langle 1|$$

6) Bra-Ket Notation:-

$$\langle \alpha | \beta \rangle$$

$1 \times n$ $n \times m$ $m \times 1$ → 1×1 (inner product of two vector) → number.

condition $n = m$ needs to fulfil otherwise we cann't multiple.

Example:-

$$\langle 0|0\rangle = [1 \;\; 0]\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= 1\times 1 + 0\times 1 = 1$$

∴ If the inner product of ~~of~~ a vector ~~is equal~~ with itself is equals to one then it implies that its magnitude will also be equal to 1. because magnitude or Norm of any vector say Ket is equal to square root of Inner product of Ket Alpha with itself norm

$|\alpha\rangle$

is

$\sqrt{\langle\alpha|\alpha\rangle}$

If $\sqrt{\langle\alpha|\alpha\rangle} = 1$ (then it will be called unit vector.

Orthogonal vector:-

When $\langle\alpha|\beta\rangle = 0$

$\langle 0|1\rangle = [1, 0] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1\times0 + 0\times1 = 0$

Example:-

$|\alpha\rangle = i|0\rangle + 7|1\rangle$

$|\beta\rangle = 3|0\rangle + |1\rangle$

$\langle\alpha|\beta\rangle = ?$

1st check the dimension, as in this the dimensions are same

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Sol//

conjugate Transpose

$\langle\alpha| = |\alpha\rangle^{\dagger} = -i\langle 0| + 7\langle 1|$

$\langle\alpha|\beta\rangle = (-i\langle 0| + 7\langle 1|)(3|0\rangle + |1\rangle)$

$= -3i\underbrace{\langle 0|0\rangle}_{=1 \text{ unit vector}} - i\underbrace{\langle 0|1\rangle}_{0 \text{ orthogonal vectors}} + 21\underbrace{\langle 1|0\rangle}_{0} + 7\underbrace{\langle 1|1\rangle}_{1 \text{ unit}}$

$= \boxed{-3i+7}$ → inner product

For:-

Real number:- $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle$

Complex number:- $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^{*}$

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Lec #2 Qubits & Measurements

Classical Computer:-

- Basic unit of data on classical computer is called bit.
- A bit can store either a value of 0 or a value of 1.
- In quantum computer we have quantum bits. In a quantum computer, we have qubit.

Qubit states:-

① Pure state:-

In pure state qubit behave exactly like classical bit so it can have the value of 0 or 1.

0 or 1

② Super Position:-

0 and 1

0 → probability p

1 → probability 1-p

Mathematical Representation:-

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

α, β are amplitude of qubits.

$(\alpha, \beta \in \mathbb{C})$:- α, β are complex number that can also take negative values

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prob of 0 $= |\alpha|^2$

prob of 1 $= |\beta|^2$

$|\alpha|^2 = \alpha^* \alpha$

$|\beta|^2 = \beta^* \beta$

Normalization Constraint:-

• We know sum of probability is always equal to 1 that is why

$|\alpha|^2 + |\beta|^2 = 1$ this is called normalization constraint

$$|\psi\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \leftarrow \text{unit vector.}$$

It implies the magnitude or norm of this vector is equal to 1

$$\| |\psi\rangle \| = 1$$

or

$$\langle \psi | \psi \rangle = 1$$

Exampler (Single Qubit)

$$|\psi\rangle = \frac{-4i}{5}|0\rangle + \frac{3}{5}|1\rangle$$

ⓐ Is $|\psi\rangle$ a valid qubit?

ⓑ What is the probability of $|\psi\rangle$ equal to zero, and prob of 1

Part a-

✓ $\left.\begin{array}{l} \bullet\ |\alpha|^2 + |\beta|^2 = 1 \\ \bullet\ \||\psi\rangle\| = 1 \\ \bullet\ \langle\psi|\psi\rangle = 1 \end{array}\right\}$ method we can use

$$\underbrace{\left|\frac{-4}{5}i\right|^2}_{\alpha} + \underbrace{\left|\frac{3}{5}\right|^2}_{\beta} = 1$$

$$|\alpha|^2 = \alpha^* \alpha$$

$$= \left(\frac{4}{5}i \times \frac{-4}{5}i\right)$$

$$|\beta|^2 = \left(\frac{3}{5} \times \frac{3}{5}\right)$$

$$= \frac{16}{25} + \frac{9}{25} = 1$$

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Part (b):-

$$\text{Prob of } 0 = \left|-\frac{4}{5}i\right|^2$$

$$= \frac{16}{25} = 0.64$$

$$\text{Prob of } 1 = 1 - 0.64 = 0.36$$

or

$$= \left|\frac{3}{5}\right|^2 = 0.36$$

(Multiple Qubit):

- Tensor Product

$$|\psi\rangle = -\frac{4}{5} i|0\rangle + \frac{3}{5}|1\rangle$$

$$|\phi\rangle = \frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}$$

$$|\psi\phi\rangle = \left(-\frac{4}{5} i|0\rangle + \frac{3}{5}|1\rangle\right)\left(\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}\right)$$

$$\underset{(\alpha)}{=\frac{-4i}{5\sqrt{2}}}|00\rangle + \underset{(\beta)}{\frac{-4i}{5\sqrt{2}}}|01\rangle + \underset{(\gamma)}{\frac{3}{5\sqrt{2}}}|10\rangle + \underset{(\delta)}{\frac{3}{5\sqrt{2}}}|11\rangle$$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

Prob of 00

$$|\alpha|^2 = \left|\frac{-4}{5\sqrt{2}} i\right|^2$$

$$= \left(\frac{4i}{5\sqrt{2}} \times \frac{-4i}{5\sqrt{2}}\right) = \frac{16}{50} = 0.32$$

Prob of 01

$$|\beta|^2 = \left|\frac{-4}{5\sqrt{2}} i\right|^2$$

$$= \frac{16}{50} = 0.32$$

Prob 10

$$|\gamma|^2 = \left|\frac{3}{5\sqrt{2}}\right|^2$$

$$= \frac{9}{50} = 0.18$$

Prob 11

$$|\delta|^2 = \left|\frac{3}{5\sqrt{2}}\right| = \frac{9}{50} = 0.18$$

$0.32 + 0.32 + 0.18 + 0.18 = 1$

Operation on Qubit:-

• Measure Qubit. • Transform using quantum gates.

Full Partial

Full measurement:-

It implies that given n-qubits register you measure all the n-qubits.

Partial measurement:-

It implies that given n-qubit register you measure you subset of these n-qubits

Rules:-

1) Superposition $\xrightarrow{m}$ pure state

2) Normalization constraint must always uphold.

Ex 1)

$|\psi\rangle = \frac{1}{2}|00\rangle - \frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Full measurement:-

Measurement	Probability	Resultant state
00	$\lvert\frac{1}{2}\rvert^2 = \frac{1}{4} = 0.25$	$\lvert\psi\rangle = \lvert 00\rangle$
10	$\lvert-\frac{i}{2}\rvert^2 = \frac{1}{4} = 0.25$	$\lvert\psi\rangle = \lvert 10\rangle$
11	$\lvert\frac{1}{\sqrt{2}}\rvert^2 = \frac{1}{2} = 0.5$	$\lvert\psi\rangle = \lvert 11\rangle$

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$\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $\left(\frac{-i}{2} + \frac{1}{\sqrt{2}}\right) \times \frac{\sqrt{4}}{\sqrt{3}}$

$\frac{2}{\sqrt{2} \times \sqrt{3}}$

Partial measurement:-

Measurement	Probability	Resultant state
Ist qubit = 1	$\left\|\frac{-i}{2}\right\|^2 + \left\|\frac{1}{\sqrt{2}}\right\|^2$ $= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$	$\|\psi\rangle = \frac{\frac{-i}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle}{\left\|\frac{-i}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle\right\|}$ (not normalized so we divide it with the norm) → it will the the square of probability $= \frac{\frac{-i}{2}\|10\rangle + \frac{1}{\sqrt{2}}\|11\rangle}{\sqrt{3/4}}$ $= \frac{-i}{\sqrt{3}}\|10\rangle + \frac{\sqrt{2}}{\sqrt{3}}\|11\rangle$
Ist qubit = 0	$\left\|\frac{1}{2}\right\|^2 + \left\|\frac{-i}{2}\right\|^2$ $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$	$\|\psi\rangle = \frac{\frac{1}{2}\|00\rangle - \frac{i}{2}\|10\rangle}{1/\sqrt{2}}$ $= \frac{1}{\sqrt{2}}\|00\rangle - \frac{i}{\sqrt{2}}\|10\rangle$

Example:-

$$\frac{1}{\sqrt{5}}|0000\rangle - \sqrt{\frac{2}{5}}|0100\rangle + \sqrt{\frac{1}{5}}|1111\rangle + \sqrt{\frac{1}{5}}|0110\rangle$$

what is the prob and resultant state if 1st and 4th qubit are 0.

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$$\text{Prob} = \left|\frac{1}{\sqrt{5}}\right|^2 + \left|-\sqrt{\frac{2}{5}}\right|^2 + \left|\frac{1}{\sqrt{5}}\right|^2 = \frac{4}{5}$$

$$\text{Resultant state} = \frac{|\psi\rangle + \frac{1}{\sqrt{5}}|0000\rangle - \sqrt{\frac{2}{5}}|0100\rangle + \frac{1}{\sqrt{5}}|0110\rangle}{2/\sqrt{5}}$$

$= \frac{1}{2}|0000\rangle - \frac{1}{\sqrt{2}}|0100\rangle + \frac{1}{2}|0110\rangle$

Measuring in Orthonormal Basis

1) Standard Basis:-

Is a set of linearly independent vector where each vector has exactly one 1 in it where rest of the elements of that vector are all zeros.

2D $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ ✓

$= \{ |0\rangle, |1\rangle \}$ ✓

3D// $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

4D// : $\{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \}$ ✓

2) Orthonormal Basis:-

Composed of two different words orthogonal and normalized. That basis must has vector which are orthogonal and as well normalized.

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1) Normalized "unit vector"

2) Orthonormal " $\perp$ "

Example:

$\{|+\rangle, |-\rangle\}$ $\quad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

unit vector

$\| |+\rangle \| = \sqrt{\langle + | + \rangle} = 1$

$= \sqrt{\left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)}$

$= \frac{1}{2}\left[\underbrace{\langle 0|0\rangle}_{1} + \underbrace{\langle 0|1\rangle}_{0} + \underbrace{\langle 1|0\rangle}_{0} + \underbrace{\langle 1|1\rangle}_{1}\right]$

$= \frac{1}{2} \times 2 = 1$

$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)^2$

$\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$

$\frac{1 + 0 + 0 + 1}{2}$

$\frac{2}{2}$

unit vector

$\| |-\rangle \| = \sqrt{\langle - | - \rangle} = 1$

$\sqrt{\left(\frac{\langle 0| - \langle 1|}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)}$

$= \sqrt{\frac{1}{2}\left[\underbrace{\langle 0|0\rangle}_{1} - \underbrace{\langle 0|1\rangle}_{0} - \underbrace{\langle 1|0\rangle}_{0} + \underbrace{\langle 1|1\rangle}_{1}\right]} = \sqrt{\frac{1}{2} \times 2} = \sqrt{1} = 1$

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orthogonal

If they are orthogonal there inner product will be zero

$$\langle +|-\rangle = \left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left[\underbrace{\langle 0|0\rangle}_{1} - \underbrace{\langle 0|1\rangle}_{0} + \underbrace{\langle 1|0\rangle}_{0} - \underbrace{\langle 1|1\rangle}_{1}\right]$$

$$= \frac{1}{2} \times 0 = 0$$

3) **Formula :-**

To write any vector in the orthonormal basis of your liking.

$$\{|\alpha\rangle, |\beta\rangle\}$$

$$|\psi\rangle = \underbrace{\langle \psi|\alpha\rangle}_{\text{amplitude/number}} |\alpha\rangle + \underbrace{\langle \psi|\beta\rangle}_{\text{amplitude/number}} |\beta\rangle$$

probability of measuring $|\alpha\rangle = |\langle \psi|\alpha\rangle|^2$

probability of measuring $|\beta\rangle = |\langle \psi|\beta\rangle|^2$

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Exampler

$|\psi\rangle = |0\rangle$

Standard basis

probability m $0 = |1|^2 = 1$

probability m $1 = 0$

Hadamard Basis:-

$\{|+\rangle, |-\rangle\}$

prob of m $|+\rangle = |\langle\psi|+\rangle|^2$

$$\langle\psi|+\rangle = \left(\langle 0|\right)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{\langle 0|0\rangle + \langle 0|1\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times 1 = \frac{1}{\sqrt{2}}$$

$$|\langle\psi|+\rangle|^2 = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} = 0.5$$

prob of m $|-\rangle = |\langle\psi|-\rangle|^2$

$$= \left(\langle 0|\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\left(\frac{\langle 0|0\rangle}{1} - \frac{\langle 0|1\rangle}{0}\right) = \frac{1}{\sqrt{2}}$$

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$|\langle\psi|-\rangle|^2 = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

Exampler

$|\psi\rangle = \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)|+\rangle + \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}\right)|-\rangle$

Sol// Hadamard Basis

prob of $|+\rangle = \left|\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)\right|^2 = \frac{1}{6} + \frac{1}{3} - \frac{2}{\sqrt{6}\sqrt{3}} = \frac{1}{6} + \frac{1}{3} - \frac{\sqrt{2}}{3}$

$= \frac{1+2-2\sqrt{2}}{6} = \frac{3-2\sqrt{2}}{6}$

prob of $|-\rangle = 1 - \frac{3-2\sqrt{2}}{6} = \frac{3+2\sqrt{2}}{6}$

Standard Basis = $\{|0\rangle, |1\rangle\}$

prob of (M) $|0\rangle = |\langle\psi|0\rangle|^2$

$\langle\psi|0\rangle = \left[\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)\langle+| + \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}\right)\langle-|\right]\left[|0\rangle\right]$

$= \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)\langle+|0\rangle + \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}\right)\langle-|0\rangle$

$= \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\langle+|0\rangle = \left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)(|0\rangle)$

$= \frac{\overset{1}{\langle 0|0\rangle} + \overset{0}{\langle 1|0\rangle}}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

$\langle-|0\rangle \left(\frac{\langle 0| - \langle 1|}{\sqrt{2}}\right)(|0\rangle)$

$= \frac{1}{\sqrt{2}}$

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$= \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{6}} \cancel{- \frac{1}{\sqrt{3}}} + \frac{1}{\sqrt{6}} \cancel{+ \frac{1}{\sqrt{3}}} \right] = \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{6}} = \frac{\sqrt{4}}{\sqrt{12}} = \frac{1}{\sqrt{3}}$

$|\langle \psi | 0 \rangle|^2 = \frac{1}{3}$

now

Prob (m) of $|1\rangle = |\langle \psi | 1 \rangle|^2$

$\langle \psi | 1 \rangle = \left[\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) \langle + | + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) \langle - | \right] \left[|1\rangle \right]$

$\langle 0 | 1 \rangle = \left\{ \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) \langle + | 1 \rangle + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \right) \langle - | 1 \rangle \right.$

$\langle + | 1 \rangle = \frac{\langle 0 | + \langle 1 |}{\sqrt{2}} \left(|1\rangle \right)$

$= \frac{\langle 0 | 1 \rangle + \langle 1 | 1 \rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\langle - | 1 \rangle = \frac{\langle 0 | - \langle 1 |}{\sqrt{2}} \left(|1\rangle \right)$

$= -\frac{1}{\sqrt{2}}$

$\cancel{\langle \psi | 1 \rangle}$ $|\langle \psi | 1 \rangle|^2 = \frac{2}{3}$

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Quantum Gate

$$U|\alpha\rangle = |\beta\rangle$$

unitary matrix — qubit(s) — transform qubit(s)

The operation U must be special which preserve the sum of probability that means this operation U must preserve the norm a vector and that why we know that there only single type of matrix that preserve norm of a vector is called unitary matrix.

- $U^{\dagger}U = UU^{\dagger} = I$ (identity)

  $U^{\dagger} = U^{-1}$

- $H^{\dagger} = H$ (Hermission matrix)

Example:-

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y^{\dagger} = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}$$

$Y^{\dagger} = Y$ $\rightarrow$ Hermission matrix

$$Y^{\dagger}Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} -i \times i & 0 \\ 0 & -i \times i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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Gate :-

Pauli-x: $-\boxed{X}-$ Pauli-x (single qubit as input and single qubit as output) like all quantum gates it is reversable and it is reverse of itself it is because this gate is both (unitary and Hermission) all the gate who are **U & H** are inverse of themselve.

Matrix:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Example:

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$X(\alpha|0\rangle + \beta|1\rangle) = \alpha X|0\rangle + \beta X|1\rangle$

$= \alpha|1\rangle + \beta|0\rangle$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0\times\alpha + 1\times\beta \\ 1\times\alpha + 0\times\beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \beta|0\rangle + \alpha|1\rangle$$

Pauli-z: $-\boxed{Z}-$ is also a single qubit gate so it takes a single cubit as input and produces a single qubit output. and it is also **U & H** gate that means it is inverse of itself. so if we apply two pauli-z gate in sequence they will cancel each other effect and my input will become equal to my output.

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Matrix:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Example:

$$\boxed{\begin{aligned} Z|0\rangle &= |0\rangle \\ Z|1\rangle &= -|1\rangle \end{aligned}}$$

Superposition:-

$$Z(\alpha|0\rangle + \beta|1\rangle) = \alpha|0\rangle - \beta|1\rangle$$

Hadamard Gate:

—[H]— it is a single qubit gate which is also U & H.

Matrix:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Example:

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$

$$H|+\rangle = |0\rangle$$

$$H|-\rangle = |1\rangle$$

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Rotation Gate:

$-\boxed{R_\theta}-$ this is unitary gate but not Hermission

U but not H.

Matrix:

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

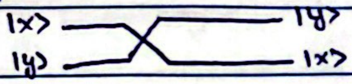
Example:

$$R_\theta \neq R^{+}_{\theta}$$

$$R^{+}_{\theta} \cdot R_\theta = 1$$

Two Qubit gates:

Swap gate:



U & H

Matrix:

swap

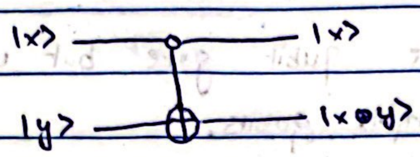
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Example:

$$\text{Swap}\,|01\rangle = |10\rangle \rightarrow \text{test} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

CNOT:

U & H



Matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Example:

(control bit if it is 1 then the other will be flip)

$CNOT|10\rangle = |11\rangle$

$CNOT|01\rangle = |01\rangle$

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Hadamard (Single Qubit)

Hadamard gate is a single qubit gate but we can expend it and apply it on multiple qubits.

$$\text{matrix of } H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

because it is a two by two matrix so we can aply this matrix on single qubit.

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \rightarrow \text{complex number}$$

$$H|\psi\rangle = |\psi\rangle'$$

example:-

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

Equel suppersposition

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1+0 \\ 1+0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left[\underset{|0\rangle}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} + \underset{|1\rangle}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right]$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right] = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$\therefore$ Hadmard gate is both unitary and hermitian.

unitary gate :-

$$HH^{+} = \text{Identity}$$

Hermition :-

$$H = H^{+}$$

Combining both:- we can write:-

$$H^{-1} = H$$

example:-

$$H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{H|0\rangle}{\sqrt{2}} - \frac{H|1\rangle}{\sqrt{2}}$$

$$= \frac{\frac{|0\rangle + |1\rangle}{\sqrt{2}}}{\sqrt{2}} - \frac{\frac{|0\rangle - |1\rangle}{\sqrt{2}}}{\sqrt{2}}$$

$$= \frac{\cancel{|0\rangle} + |1\rangle - \cancel{|0\rangle} + |1\rangle}{2}$$

$$= \frac{\cancel{2}|1\rangle}{\cancel{2}} = |1\rangle$$

Hadamard (Multiple Qubit)

examples:- $\xrightarrow{\text{Tensil product}}$

$H^{\otimes 2} |00\rangle$

1 method (matrix)

$H^{\otimes 2} |00\rangle$

$H^{*2} = H \otimes H$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}}_{H^{\otimes 2}} \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{|00\rangle}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

2nd method: single qubit

often used method:-

$$H^{\otimes 2} |00\rangle = H^{\otimes 2} [|0\rangle |0\rangle]$$

$$= H|0\rangle \; H|0\rangle$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \; \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$= \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$$

Hadmard (For n qubit)

$H|0\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $H|1\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$

$$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{x\cdot y} |y\rangle \longrightarrow$$

Derived:

$$H|x\rangle = \frac{|0\rangle + (-1)^x |1\rangle}{\sqrt{2}}, \quad x\in\{0,1\}$$

$$H|x\rangle = \frac{1}{\sqrt{2}} \sum_{y\in\{0,1\}} (-1)^{x\cdot y} |y\rangle \quad \text{— (1)}$$

or

$$H^{\otimes n}|0^n\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=1}^{n-1} |y\rangle$$

↓

Derive:

$H^{\otimes 2}$

Controlled NOT

Unitary $CNOT \times CNOT^{*} = I$ $\therefore CNOT^{+} = CNOT^{-1}$

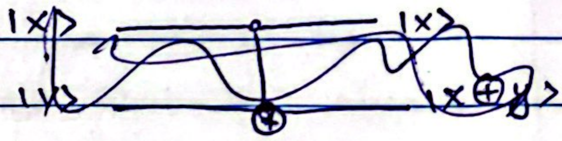
Hermiton $CNOT = CNOT^{+}$

$CNOT^{-1} = CNOT$

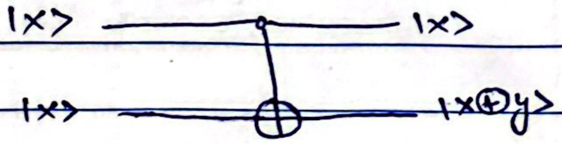
Swap

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{4\times4}$$

First qubit is represented as a control bit and if the first qubit is 1 then it flips the second qubit, if the first qubit is 0 then the second qubit remein the same,



$CNOT\ |00\rangle = |00\rangle$
$CNOT\ |10\rangle = |11\rangle$
$CNOT\ |01\rangle = |01\rangle$
$CNOT\ |11\rangle = |10\rangle$



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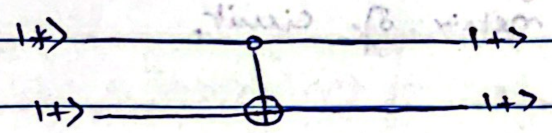
example:-

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{|11\rangle} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |10\rangle$$

example

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle$$



$CNOT|++\rangle = |++\rangle$

$CNOT|-+\rangle = |-+\rangle$

$CNOT|--\rangle = |+-\rangle$ → verify

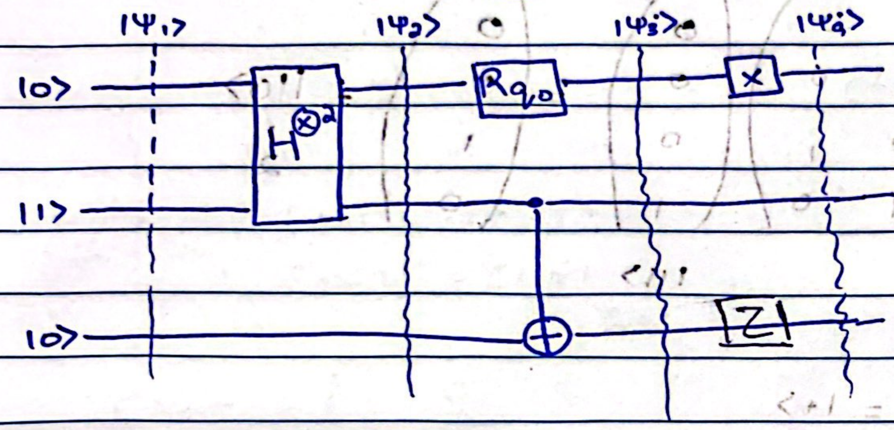
$CNOT|+-\rangle = |--\rangle$

$$|--\rangle = \frac{|0\rangle - \sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{0} - \sqrt{1}}{\sqrt{2}}$$

$$= \frac{|00\rangle - |01\rangle - |10\rangle + |11\rangle}{2}$$

$$= \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

Quantum Circuit



a) make reverse circuit

b) write output of the circuit.

c) write complete circuit as unitary matrix.

d) write reverse circuit as unitary matrix.

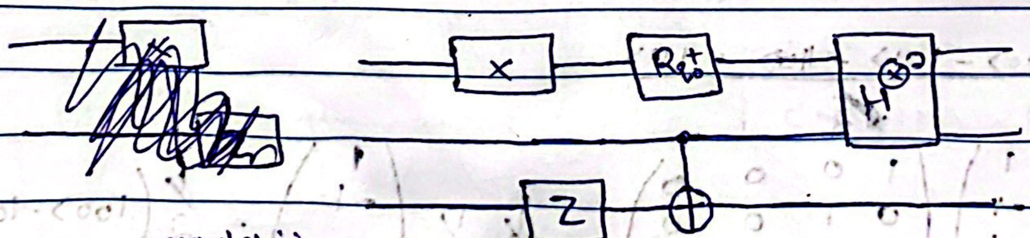
e) complete output using unitary matrix of circuit.

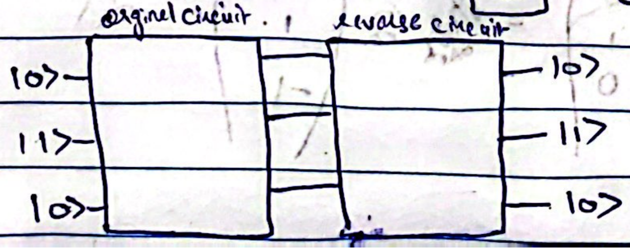
a) To do:

1) Conjugate transpose of each gate

$U^{-1} = U^{+}$

$H^{+} = H$





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b)

$|\psi_1\rangle = |0\rangle|1\rangle|0\rangle$

$|\psi_2\rangle = H^{\otimes 2}(|0\rangle|1\rangle)|0\rangle$

$= \frac{|0\rangle+|1\rangle}{\sqrt{2}} \frac{|0\rangle-|1\rangle}{\sqrt{2}} |0\rangle$

$= \frac{|00\rangle-|01\rangle+|10\rangle-|11\rangle}{2} |0\rangle$

$= \frac{|000\rangle-|010\rangle+|100\rangle-|110\rangle}{2}$

$\theta = 90$

$|\psi_3\rangle = R_{90}|\psi_2\rangle$: $R_\theta \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$= \frac{|100\rangle-|110\rangle-|000\rangle+|010\rangle}{2}$ $= \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$

= CNOT gate check second bit

$|\psi_3\rangle = \frac{|100\rangle-|111\rangle-|000\rangle+|011\rangle}{2}$ $+|000\rangle + |011\rangle - |100\rangle - |111\rangle$

$|\psi_4\rangle$ = Pauli-x : flip 1 to 0 and 0 to 1

Pauli-z : change sign of qubit if qubit is $|1\rangle$

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$$|\psi_4\rangle = \frac{|000\rangle + |011\rangle - |100\rangle \mp |111\rangle}{2}$$

(c)

$$Stage_1 = H \otimes H \otimes I_{8\times 8}$$

$$Stage_2 = R_{q_0} \otimes CNOT$$

$$Stage_3 = X \otimes I \otimes Z_{8\times 8}$$

$\cancel{S_1 \times S_2 \times S_3}$ $\quad S_1 \times S_2 \times S_3$ ✗

$\cancel{S_3 \times}$ $\quad S_3 \times S_2 \times S_1$ ✓

$$(S_3 (S_2 (S_1 |010\rangle)))$$

S_1 =

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_2 = R_{90} \otimes CNOT$$

$$= \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{pmatrix}$$

$$S_3 = X \otimes I \otimes Z$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_3 \times S_2 \times S_1$$

e) $(S_3 \times S_2 \times S_1)(|010\rangle)$

$(S_3(S_2(S_3($

Entanglement

entanglement can be describe using two different equivalent definitions.

① Given a state of multiple entangled qubit, one cannot express individual qubit sepeartly.

Example :-

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \neq (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$$

So they are entangled.

Seperable

$$\frac{|01\rangle + |00\rangle}{\sqrt{2}} = |0\rangle \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

② Given a state of multiple entangled qubits, measuring any qubit individually reveals an other qubits.

Example :- (Entanglement)

$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$ | 1st qubit (m) = 1/0 → 2nd qubit will also be 1

2nd qubit (m) = 1/0 → 1st qubit will also be 1

Separable qubit

$\frac{|01\rangle + |00\rangle}{\sqrt{2}}$ | 1st qubit $|0\rangle$

2nd qubit : $|0\rangle$ —— $\frac{1}{2}$

$|1\rangle$ prob $\frac{1}{2}$

Bell state:- (EPR states)

We have 4 entangled qubit describe as bell state.

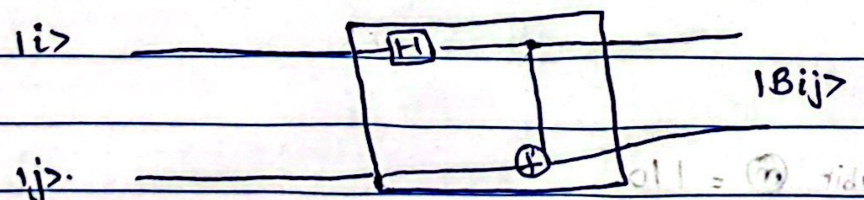
① $|\Phi^+\rangle = |B_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

② $|\Phi^-\rangle = |B_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$

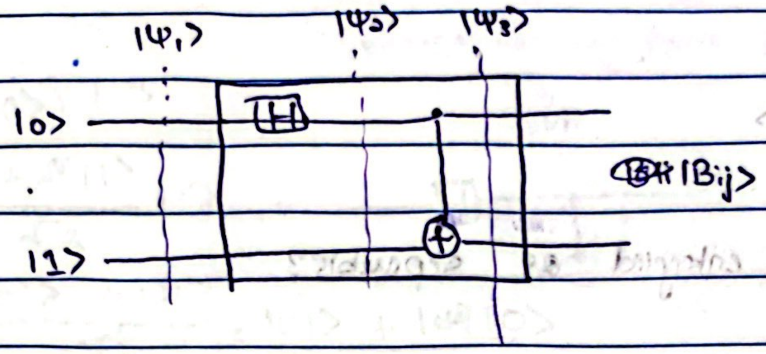
③ $|\Psi^+\rangle = |B_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

④ $|\Psi^-\rangle = |B_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

quantum circuit that create the bell state.



(3)



$|\psi_1\rangle = |0\rangle\,|1\rangle = |01\rangle$

$|\psi_2\rangle = ((|0\rangle)H)|1\rangle$

$= \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}\,|1\rangle$

$= \dfrac{|01\rangle + |11\rangle}{\sqrt{2}}$

$|\psi_3\rangle = CNOT$

$= \dfrac{|01\rangle + |10\rangle}{\sqrt{2}} = |B_{01}\rangle$

Given :-

$$|\omega\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Q :- Find $|\omega\rangle$ is entangled or separable?

Proof :- Proof by contradiction.

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$$

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} = (\alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle)$$

~~$\delta\beta$~~ β and δ cannot be equal to zero $\beta \neq 0$ $\delta \neq 0$ so this equation cannot exist this is ~~contradic~~ contradiction L.H.S $\neq$ R.H.S

Thus $|\omega\rangle$ is entangled.

By checking right hand side : as 00 and 11 are not present at right hand side

$\alpha\gamma = 0$

$\alpha\delta = \frac{1}{\sqrt{2}}$

$\beta\gamma = -\frac{1}{\sqrt{2}}$

$\beta\delta = 0$ — either $\beta = 0$ $\delta = 0$

β cannot be equal to 0 otherwise (3) will be zero

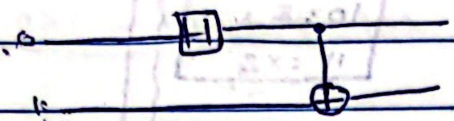
$\beta \neq 0$

$\delta \neq 0$

How to create Bell state
(Einstein - Podolsky - Rosen pair)
$\downarrow$ EPR

$(H|0\rangle)|1\rangle$

$\frac{|0\rangle + |1\rangle}{\sqrt{2}} |1\rangle$

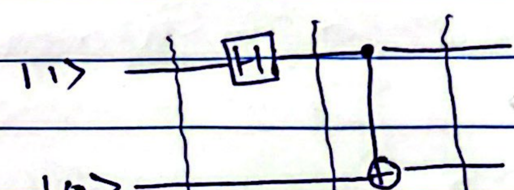


$\frac{|01\rangle + |11\rangle}{\sqrt{2}} = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

General Formula:

$$|xy\rangle \longrightarrow \frac{|0y\rangle + (-1)^x |1\bar{y}\rangle}{\sqrt{2}}$$

Input	Bell state
$\|00\rangle$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}} = \|\Phi^+\rangle$
$\|01\rangle$	$\frac{\|01\rangle + \|10\rangle}{\sqrt{2}} \cdot \|\Psi^+\rangle$
$\|10\rangle$	$\frac{\|00\rangle + (-1)^1 \|11\rangle}{\sqrt{2}} = \frac{\|00\rangle - \|11\rangle}{\sqrt{2}} \quad \|\Phi^-\rangle$
$\|11\rangle$	$\frac{\|01\rangle - \|10\rangle}{\sqrt{2}} \quad \|\Psi^-\rangle$



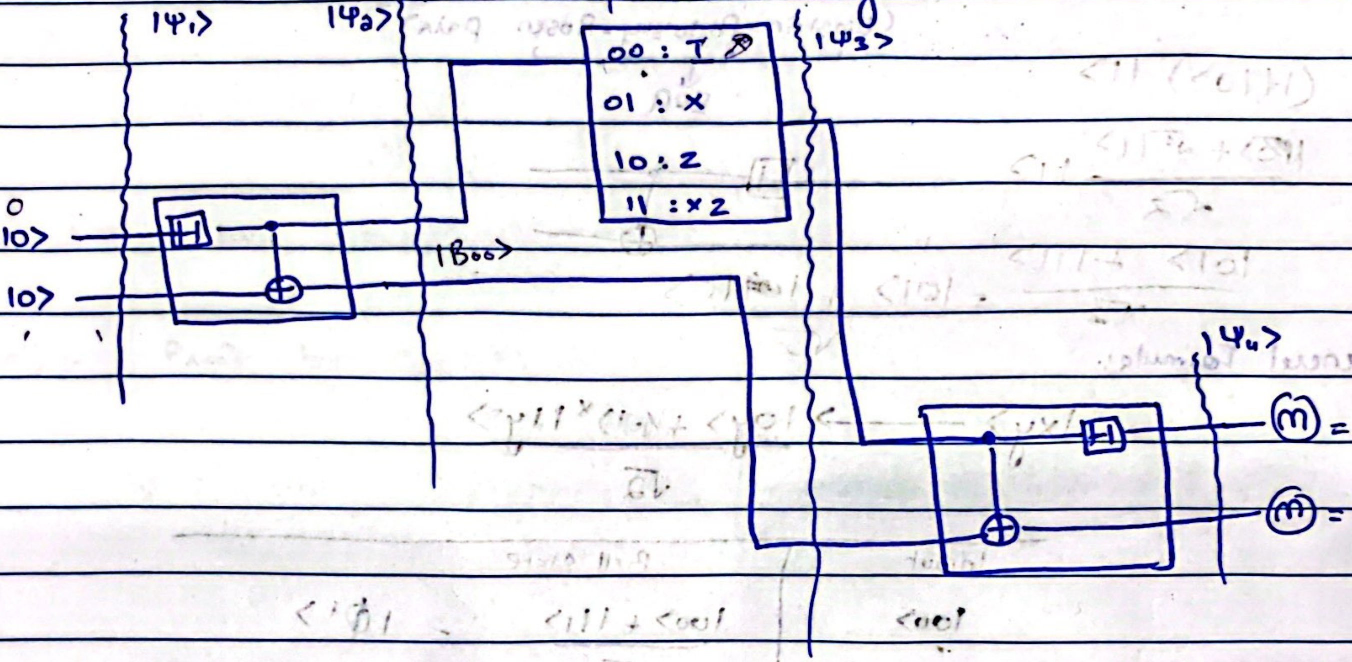
$|\alpha_1\rangle = |10\rangle$

$|\alpha_2\rangle = H|1\rangle |0\rangle$

$= \frac{|00\rangle - |10\rangle}{\sqrt{2}}$

$|\alpha_3\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$

Superdense Coding



$|\psi_1\rangle = 00$

$|\psi_2\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = |\Phi^+\rangle$

00 $\quad |\psi_3\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Pauli X
01 $\quad |\psi_3\rangle = \frac{|10\rangle + |01\rangle}{\sqrt{2}}$

Pauli Z
10 $\quad |\psi_3\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$

XZ
11 $\quad |\psi_3\rangle = \cancel{\frac{-|10\rangle + |01\rangle}{\sqrt{2}}} \quad X(Z|\psi_2\rangle) = \frac{|10\rangle - |01\rangle}{\sqrt{2}}$

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$H(|0\rangle + |1\rangle)$

00 $|\psi_4\rangle \overset{CNOT}{=} \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

$= \frac{|00\rangle + |10\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} |0\rangle + \frac{|0\rangle - |1\rangle}{2} |0\rangle$

$= \frac{|00\rangle + \cancel{|10\rangle} + |00\rangle - \cancel{|10\rangle}}{2}$

$= \frac{\cancel{2} \cdot |00\rangle}{\cancel{2}} = |00\rangle$

01 $|\psi_4\rangle \overset{CNOT}{=} \frac{|10\rangle + |01\rangle}{\sqrt{2}}$

$= \frac{|11\rangle + |01\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} |1\rangle + \frac{|0\rangle + |1\rangle}{\sqrt{2}} |1\rangle$

$= \cancel{|01\rangle + |01\rangle - |00\rangle - |11\rangle}$

$= \frac{|01\rangle - \cancel{|11\rangle} + |01\rangle + \cancel{|11\rangle}}{2}$

$= \frac{\cancel{2}|01\rangle}{\cancel{2}} = |01\rangle$

So on for 10 and 11

Quantum Teleportation

NY (New York) $|B_{AB}\rangle$

Mars

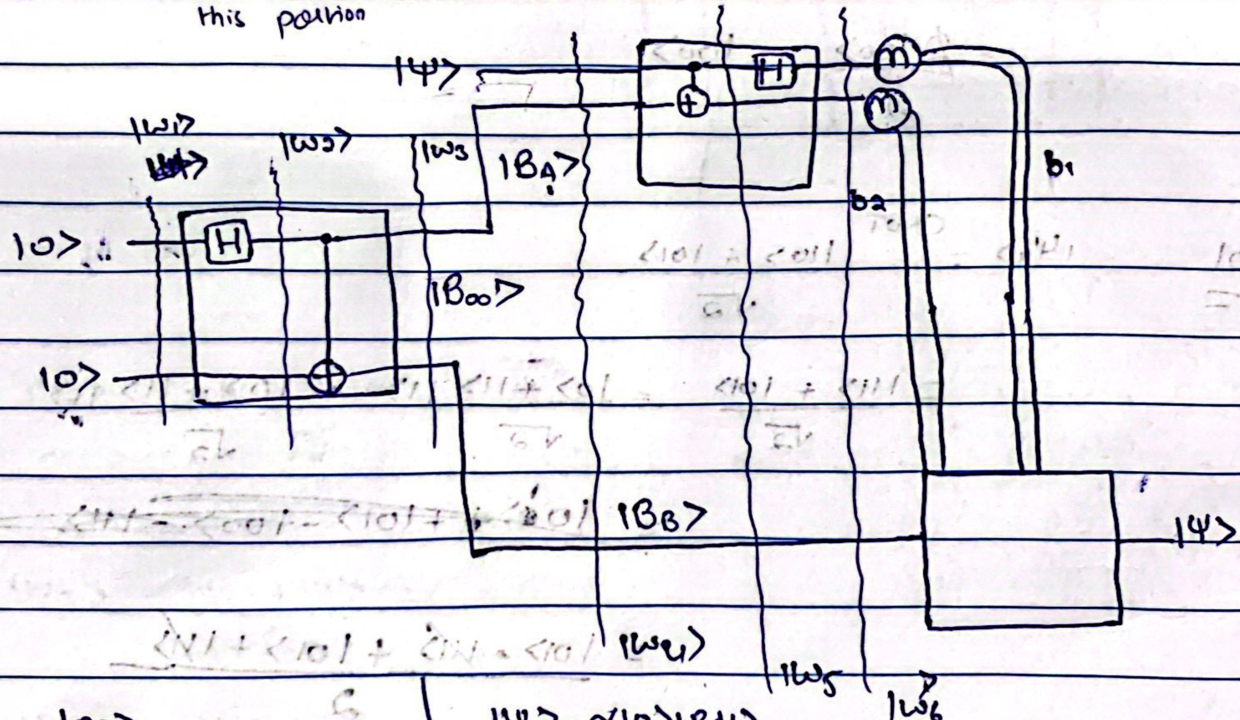
Alice $|B_A\rangle$

$|B_{00}\rangle$

Bob

↳ $|B_B\rangle$

Bob $|B_B\rangle \longrightarrow$ moved to mars and take this portion



$|w_1\rangle = |00\rangle$

$|w_2\rangle = (H|0\rangle)|0\rangle$

$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} (|0\rangle)$

$= \frac{|00\rangle + |10\rangle}{\sqrt{2}}$

$|w_3 = \frac{|00\rangle + |11\rangle}{\sqrt{2}}\ |B_{00}\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|w_4\rangle = |\psi\rangle|B_{00}\rangle = (\alpha|0\rangle + \beta|1\rangle)\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$

$= \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle}{\sqrt{2}}$

$\cancel{|w_5\rangle = \frac{\alpha|000\rangle + \alpha|010\rangle + \beta|110\rangle + \beta|110\rangle}{\sqrt{2}}}$

$|w_5\rangle = \frac{\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle}{\sqrt{2}}$

$$\left|\tfrac{1}{2}\right|^2$$

$$|w_0\rangle = \frac{1}{\sqrt{2}}\left[\alpha\frac{(|0\rangle+|1\rangle)}{\sqrt{2}}|00\rangle + \alpha\frac{(|0\rangle+|1\rangle)}{\sqrt{2}}|11\rangle + \beta\frac{(|0\rangle-|1\rangle)}{\sqrt{2}}|10\rangle + \beta\frac{(|0\rangle-|0\rangle)}{\sqrt{2}}|01\rangle\right]$$

$$= \frac{1}{2}\left[\alpha|000\rangle+\alpha|100\rangle+\alpha|011\rangle+\alpha|111\rangle+\beta|010\rangle-\beta|110\rangle+\beta|001\rangle-\beta|1\right.$$

$$|w_0\rangle = \frac{1}{2}\left[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle) + |01\rangle(\alpha|1\rangle+\beta|0\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)\right]$$

Measure qubit

m	Prob	Resultant state	operation
00	$1/4$	$\alpha|0\rangle+\beta|1\rangle$	no operation
10	$1/4$	$\alpha|0\rangle-\beta|1\rangle$	Pauli-Z
01	$1/4$	$\alpha|1\rangle+\beta|0\rangle$	Pauli-X
11	$1/4$	$\alpha|1\rangle-\beta|0\rangle$	Pauli $X(Z(|\psi\rangle)$

$$|\psi\rangle = \alpha|0\rangle+\beta|1\rangle$$

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