



Contents lists available at ScienceDirect

Engineering Science and Technology, an International Journal

journal homepage: www.elsevier.com/locate/jestch



Full Length Article

The application of ant colony optimization in the solution of 3D traveling salesman problem on a sphere



Hüseyin Eldem ^{a,*}, Erkan Ülker ^b

^a Karamanoğlu Mehmetbey University, Computer Technologies Department, Karaman, Turkey

^b Selçuk University, Computer Engineering Department, Campus, Konya, Turkey

ARTICLE INFO

Article history:

Received 7 July 2017

Revised 18 August 2017

Accepted 23 August 2017

Available online 14 September 2017

Keywords:

Ant colony optimization

Metaheuristic

Spherical geometry

Max-Min Ant System

nonEuclidean TSP

ABSTRACT

Traveling Salesman Problem (TSP) is a problem in combinatorial optimization that should be solved by a salesperson who has to travel all cities at the minimum cost (minimum route) and return to the starting city (node). Today, to resolve the minimum cost of this problem, many optimization algorithms have been used. The major ones are these metaheuristic algorithms. In this study, one of the metaheuristic methods, Ant Colony Optimization (ACO) method (Max-Min Ant System – MMAS), was used to solve the Non-Euclidean TSP, which consisted of sets of different count points coincidentally located on the surface of a sphere. In this study seven point sets were used which have different point count. The performance of the MMAS method solving Non-Euclidean TSP problem was demonstrated by different experiments. Also, the results produced by ACO are compared with Discrete Cuckoo Search Algorithm (DCS) and Genetic Algorithm (GA) that are in the literature. The experiments for TSP on a sphere, show that ACO's average results were better than the GA's average results and also best results of ACO successful than the DCS.

© 2017 Karabuk University. Publishing services by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

Traveling salesman problem (TSP) is the problem of a salesman who needs to visit all the cities in the schedule and return to the starting point by spending less. One of the parameters such as path, time, cost and path in TSP, can be optimized. TSP is called also as a Hamiltonian path problem that is used in computer science for data modeling. The TSP's, evaluated in discrete and combinatorial problems has been comprehensively studied in the field of similar graph theory problems. TSP is considered in two categories as symmetric and asymmetric. In the symmetric TSP, always, the distance between x. city and y. city is equal, i.e., $d_{xy} = d_{yx}$. In the asymmetric TSP, the distance matrix between cities may not be the equal for all cities.

In order to solve TSP, many methods have been developed. They are divided into two groups as heuristics and exact methods in terms of obtaining the optimal results. Branch-and-bound, branch-and-cut and iterative improvement are the exact solution methods for TSP [4,23]. Various heuristic algorithms, based on

Simulated Annealing [21,12], Genetic Algorithms (GA) [35,16,18,38,26]), Tabu Search [14,15,25], Artificial Neural Networks [19,22,30,28] and Ant Colony Systems [2,3,5,13,7,8,33,32,1] have been developed which make the closest possible solutions to the best solutions at a reasonable time. In the meantime, to solve TSP, 2-opt, 3-opt and 4-opt local search algorithms were also used [20]. Some researchers to make optimum results of TSP, have studied hybrid evolution algorithms [24,39,27,34,25]. Some TSP applications were executed on the basic 3D geometric figures like spheres and cuboids [36,37,31,29,10,9,11]. An algorithm was proposed by making the solution of TSP with GA on a cuboid [36] and a sphere [37]. In [31], the particle swarm optimization algorithm (PSO) was proposed by making the solution of TSP on cuboid. An algorithm was proposed by making the solution of spherical TSP with Cuckoo Search algorithms on a sphere [29]. And also, algorithms were proposed by making the solution of spherical TSP and cuboid TSP with ACO and PSO on a sphere and cuboid [9].

One of the metaheuristic algorithms, Ant colony optimization (ACO), used to solve discrete optimization problems, was proposed by Marco Dorigo in 1992 as a PhD thesis [5]. ACO is a metaheuristic computational algorithm technique. ACO was used to solve graph problems by investigating possible paths on the graphs. ACO is inspired by the behavior of ants that provides to find shortest distance between their nest and food resource by means of pheromone.

* Corresponding author.

E-mail addresses: heldem@kmu.edu.tr (H. Eldem), eulker@selcuk.edu.tr (E. Ülker).

Peer review under responsibility of Karabuk University.

Ants choose shortest way while searching food resources rapidly in progress of time. Various *TSP* applications have been successfully solved with *ACO* techniques.

MAX-MIN Ant System (MMAS) that is an improvement over the *Ant System (AS)* proposed by Stützle and Hoos [32]. *MMAS* differs from *AS* at pheromone update. In *AS*, when complete the tours, each of ants updates their pheromone trials. But in *MMAS* just the best ant updates the pheromone trials and pheromone level is bounded between minimum-maximum limit.

In this study, *TSP* was solved for the points on a sphere by *ACO* algorithm (*MMAS*). To our knowledge, so far there is no study solving *TSP* by this technique in 3D. For the available *TSPs*, the coordinates of the points and the distances between them are known. Since all the points are present on a sphere and passage from one point to the other is carried out from the sphere surface, this problem is different from the existing *TSPs*. The study covers the definition of points on a sphere, finding the distances between the points and adaptation of the problem to the *ACO*.

2. The basic of a sphere

A sphere is a 3D object made up of points that are at the same distance from a given point in space. Every point (with coordinates of x, y, z) distributed at an equal distance r from the center is located on the sphere surface. In other words, a sphere is obtained by turning of an arc, drawn at a same distance from the origin with coordinates of $x-y$, around the z -axis. The relation between the x, y, z coordinates and the radius of a sphere is formulated by the Eq. (1):

$$r = \sqrt{x^2 + y^2 + z^2} \quad (1)$$

The radius of a sphere, r is the distance from the center (point A) to the points on the sphere (B, C, D and E) and shown in Fig. 1. Every point on the sphere has coordinates of x, y, z and these values always satisfy the Eq. (1).

When a problem is considered on a sphere, the first example that comes to mind is the geometric similarity of the Earth to a sphere. The circle passing throughout the sphere center and bounded by a sphere is big circle called equator of the Earth. This circle becomes important when the minimum distance between two points, i.e., geodesic, on a sphere along the lower cross-section is considered. The curves are called as geodesics on any surface of sphere that minimize the distances between their points [37].

2.1. Mathematical notation of points on a sphere

Euclidean curves have a single dimension. These curves can be defined by a single parameter called u along a 3D curve. In other words, in terms of parameter u points out the Cartesian coordinates. Any point on a curve can be represented by a point vector function according to the given reference Cartesian coordinates [17]:

$$\mathbf{P}(u) = (x(u), y(u), z(u)) \quad (2)$$

Generally, coordinate equations can be set up in a way that where parameter u is described between 0 and 1. As an example, a circle on the xy -plane centered at the origin is defined in a parametric form given below [17]:

$$x(u) = r \cos(2\pi u) \quad y(u) = r \sin(2\pi u) \quad z(u) = 0, \quad 0 \leq u \leq 1 \quad (3)$$

Circles and circular curves can also be defined in other parametric forms. Sloping Euclidean surfaces are two-dimensional varieties described by parameters u and v . A coordinate position on a surface can be represented by a parameterized vector function with u and v parameters for the coordinate values of x, y and z [17].

$$\mathbf{P}(u, v) = (x(u, v), y(u, v), z(u, v)) \quad (4)$$

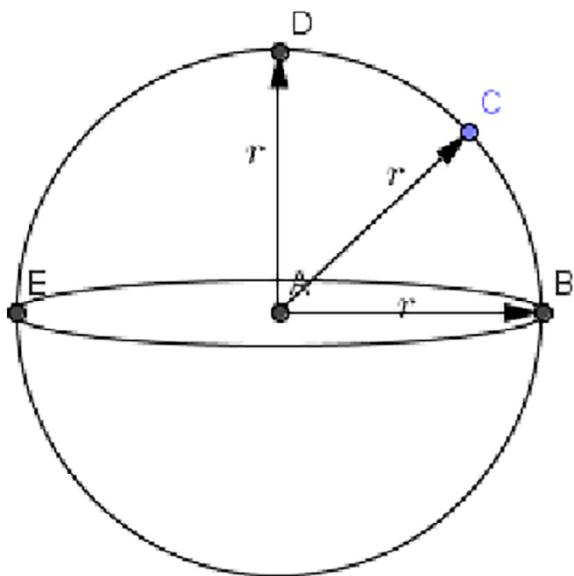
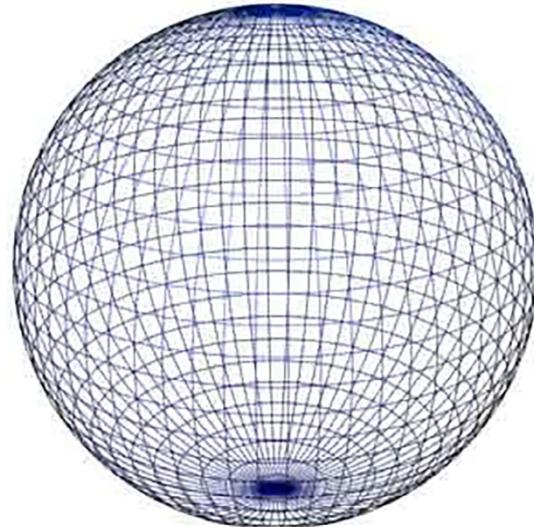


Fig. 1. Spherical surface and the radius.

Each of Cartesian coordinate values is a function of surface parameters u and v , which change between 0 and 1. The coordinates of a spherical surface centered at the origin with a radius r can be defined by the Eq. (5) [17]:

$$\begin{aligned} x(u, v) &= r \cos(2\pi u) \sin(\pi v) \\ y(u, v) &= r \sin(2\pi u) \sin(\pi v) \\ z(u, v) &= r \cos(\pi v) \end{aligned} \quad (5)$$

where, the parameters u and v define the constant lines of latitude and constant lines of longitudes on the surface, respectively [17]. To illustrate, x, y, z coordinates for the different values of parameters u and v were calculated according to the Eq. (5) and are given in Table 1. Note that r was taken as 1 and upon the increase in value of r , the values of x, y, z should also be increased in the same manner.

2.2. Finding the shortest distance between all pairs of points on the surface of unit sphere

On a spherical surface, minimum distance between two points (P_1, P_2) is along the arc of a great circle (Fig. 2). So, in radians, the

Table 1

Coordinates on a spherical surface for different values of parameters u and v .

u	v	x	y	z
0	0	0	0	1
0	0.5	1	0	6.123233e-17
0	1	1.224646e-16	0	-1
0.5	0	0	0	1
0.5	0.5	-1	1.224646e-16	6.123233e-17
0.5	1	-1.224646e-16	1.499759e-32	-1
1	0	0	0	1
1	0.5	1	-2.449293e-16	6.123233e-17
1	1	1.224646e-16	-2.999519e-32	-1

value of angle θ (θ) can be used between two vectors $\vec{V1}$ and $\vec{V2}$. The scalar product of two vectors is [37]:

$$\vec{V1} \cdot \vec{V2} = |\vec{V1}| |\vec{V2}| \cos\theta \quad (6)$$

where θ is a small angle between the direction of two vectors.

$$\vec{V1} \cdot \vec{V2} = P_{1X}P_{2X} + P_{1Y}P_{2Y} + P_{1Z}P_{2Z} \quad (7)$$

The size of vectors $\vec{V1}$ and $\vec{V2}$ are 1 for points on the surface of unit sphere. So shortest distance is [37]:

$$\theta = \arccos(\vec{V1} \cdot \vec{V2}) \quad (8)$$

The problem is *nonEuclidean TSP* and different from *Euclidean TSP*. Because on the *Euclidean TSP*, the shortest distance between two points (P_i, P_j) is calculated by using Euclidean distance which is a straight line instead of arc length [37]. The points distance matrix on a sphere is the same with symmetric *TSP* ($d(P_i, P_j) = d(P_j, P_i)$).

3. On the unit sphere, solution of TSP by using ACO (MMAS)

The three-dimensional TSP to be applied to the surface of the sphere differs from the normal two-dimensional TSP. In two-dimensions, the ant moves only in one plane. But in a three-dimensional TSP, salesperson (agent, ant, etc.) could only travel between two points through the surface (not through inside of sphere). In this study, the TSP points are on the surface of the sphere.

Similarly to standard *TSP*, the problem to be solved can be described as the detection of the shortest tour distance for an robot

to travel all points (N points in total with known coordinates and stored distance matrix) located on a surface of a sphere and return to the original point. In this study, it is aimed to solve the described problem by the *MMAS* method that improved pheromone update according to *ACO*.

According to Eq. (8), the solution of the problem is equal to standard *TSP*, after calculation of the distances between each pair of points. After this step, the solution of the problem can be examined by each method to solve the *TSP* described in the literature survey of the introduction section. In this article, solutions for a specific number of randomly generated points were obtained for each iteration by using *MMAS*.

General structure of the MMAS:

```
set initial pheromone level for all edges;
place ants to random cities on the problem;
for each iteration do:
```

```
    According to the probability function, move each ant to next
    city
```

```
    for each ant with a complete tour do:
```

```
        if ant's tour length is best of tours
```

```
            calculate the pheromone on each edge of best ant's tour
```

```
                if new pheromone level >  $\tau_{max}$ 
```

```
                    set pheromone level to  $\tau_{max}$ 
```

```
                else if pheromone level <  $\tau_{min}$ 
```

```
                    set pheromone level to  $\tau_{min}$ 
```

```
                apply pheromone update;
```

```
                if (iteration best tour is shorter than the global solution)
```

```
                    update global solution to iteration best
```

```
                end
```

```
            until all ants have completed its solution
```

```
end
```

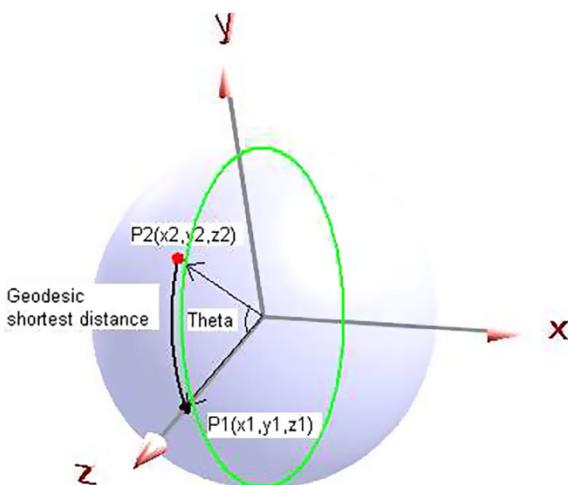


Fig. 2. Geodesic: shortest distance between two points on a spherical surface [37].

According to this general structure, first, the initial values of the parameters of the *ACO* algorithm are adjusted. Thus, the initial pheromone data of each corner between points are adjusted and written into the pheromone matrix. The distance matrix, where the distance of each point to all other points is given by Eq. (8), is obtained. Initially, in *ACO* algorithm, each agent ant, helping for solution, is located on nodes (cities) randomly. In each iteration, ants select the next city that they will travel according to Eq. (9).

$$p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta}{\sum [\tau_{ik}(t)]^\alpha \cdot [\eta_{ik}]^\beta} & \text{if } j \in \text{allowed}_k \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

At a time, t ; $\tau_{ij}(t)$ is the density of the mark left at the arc. η_{ij} , i.e., visibility, is multiplicative reciprocal of d_{ij} . d_{ij} is the distance between i and j points on a sphere. α and β are two parameters control the importance of the pheromone according to its visibility. After completing the tours of the each ants, i.e., and finding the solution of each ants, the evaporation of the pheromone and the updating density process are performed respectively. With evaporation, it

Table 2

Calculated average Spherical TSP tour lengths with GA [37] and ACO (MMAS) for $N = 100, 150, 200, 250, 300, 350$ and 400 points on the surface of unit sphere.

Evolution number	Number of points													
	100		150		200		250		300		350		400	
	GA	ACO	GA	ACO	GA	ACO	GA	ACO	GA	ACO	GA	ACO	GA	ACO
10	90.1194	24.810	157.6443	29.861	227.2337	37.717	299.1799	42.686	374.1841	46.633	441.3127	50.870	532.6288	54.706
20	72.0467	24.745	132.9872	29.796	187.7657	37.564	265.2366	42.262	333.5504	46.588	393.6967	50.508	472.2036	54.325
30	53.0306	24.676	100.9841	29.762	162.8873	37.420	224.2341	42.150	288.0024	46.450	353.1749	50.456	439.496	54.260
40	42.6922	24.647	82.2588	29.623	141.7211	37.461	200.0556	42.024	259.6622	46.432	323.2888	50.350	393.7457	54.037
50	37.1942	24.401	70.5737	29.615	115.1155	37.657	165.5674	42.155	226.1183	46.371	291.1789	50.234	354.375	54.136

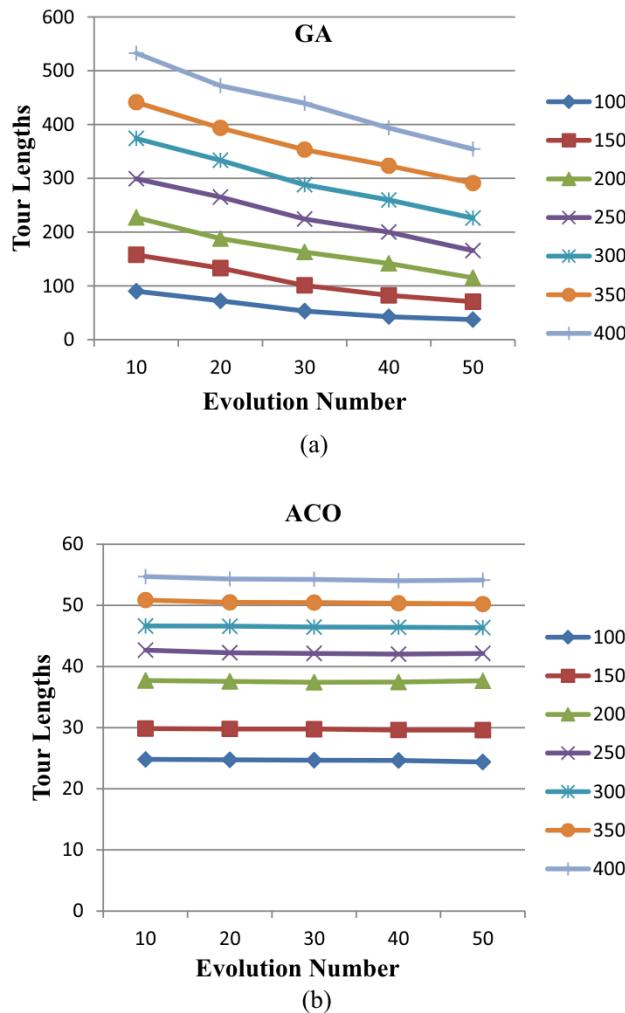


Fig. 3. Average tour lengths for different amount points on the surface of unit sphere founded by GA [37] and ACO solutions.

may be possible to forget the wrong solutions and give opportunity to the new tours by preventing pheromone accumulation. In the

Table 3

Calculated average Spherical TSP tour computation times with ACO (MMAS) for $N = 100, 150, 200, 250, 300, 350$, and 400 points on the surface of unit sphere.

Evolution Number	Number of Points						
	100	150	200	250	300	350	400
Time (sec)	Time (sec)	Time (sec)	Time (sec)	Time (sec)	Time (sec)	Time (sec)	Time (sec)
10	5.79	9.45	12.68	16.81	25.47	27.78	38.20
20	18.52	30.01	39.43	50.93	52.67	55.78	64.21
30	27.43	42.45	58.44	75.70	78.73	83.65	102.73
40	24.62	43.48	60.16	76.27	87.78	108.93	127.01
50	28.07	45.75	63.12	83.90	108.64	139.08	167.39

meantime, the best solution provided by ants is a solution that will provide global best displacement and is the best end result when iterations are completed.

4. Experimental results

On this experiment, MMAS is tested for $N = 100, 150, 200, 250, 300, 350$, and 400 points for the unit sphere. For each value of N , MMAS algorithm for TSP on unit sphere were repeated 100 times. Instead of using a predefined set of points to generalize the results in a unit sphere, a new set of random points was created for each trial. Like Dorigo et al.'s paper, the default value of α and β parameters was 1 and 5 respectively. And also pheromone trail evaporation parameter ρ was set to 0.5 [6]. In this paper, in case study #1, results of our method ACO (MMAS) was compared with the GA method given in [37]. In case study #2, results obtained using ACO (MMAS) algorithm was compared with the results of novel Discrete Cuckoo Search Algorithm (DCS) [29] having better performance than GA based method. The results were obtained by using Matlab R2010a software.

Üğur et al. [37] calculated the results of TSP on a sphere with GA and for different GA generation sizes (10, 20, 30, 40, and 50 generation) and for $N = 100, 150, 200, 250, 300, 350$, and 400 points. For each generation, Üğur et al. [37] fixed GA population size as 100. The mutations of individuals in a population observed in every generation can be called evolution. The total evolution equals the population size multiplied by the number of generation.

For the TSP's solutions in the literature, generally, the number of ants should be equal to the number of cities for optimum results. In this study, spherical TSP solution was applied considering this equality. To make a fair comparison with the GA [37] results in the literature and to achieve an equal number of evolution for MMAS, the number of tours for each generation is determined by using Eq. (10):

$$\text{Number of Tours}_{\text{MMAS}} = \frac{\text{The Size of Generation}_{\text{GA}} \times \text{The Size of Population}_{\text{GA}}}{\text{Number of Ants}_{\text{MMAS}}} \quad (10)$$

For the optimum length of the tours, the results were obtained for different evolution numbers, i.e., 10, 20, 30, 40 and 50 evolutions. For all simulations, constants are $\alpha = 1$, $\beta = 5$ and $\rho = 0.50$ (the coefficient of evaporation). And also the number of ants were

Table 4

Calculated best Spherical TSP tour lengths with DCS [29] and ACO (MMAS) for $N = 100, 150, 200, 250, 300, 350$ and 400 points on the surface of unit sphere.

Evolution Number	Number of Points													
	100		150		200		250		300		350		400	
	DCS	ACO	DCS	ACO	DCS	ACO	DCS	ACO	DCS	ACO	DCS	ACO	DCS	ACO
10	25,5412	22,7628	31,1714	28,8980	36,5940	34,5537	41,1404	40,4088	45,6111	43,9647	45,6336	44,1669	52,2950	51,7597
20	25,4120	22,6715	31,1952	28,7744	36,4697	34,4814	40,9506	39,1512	45,3540	43,7251	45,3357	44,1524	51,9555	50,7783
30	25,3846	22,3405	30,9975	28,6522	36,4538	34,4579	40,8280	39,0770	45,2773	43,7411	45,2284	44,1019	51,8304	50,6808
40	25,3409	22,3147	30,9768	28,0550	36,3856	34,2200	40,7622	38,9580	45,2367	42,7122	45,1079	44,0692	51,7429	50,6607
50	25,3341	22,2944	30,9735	27,9894	36,3792	34,1734	40,7557	38,7441	45,2005	42,3198	45,0918	43,9462	51,6725	49,5383

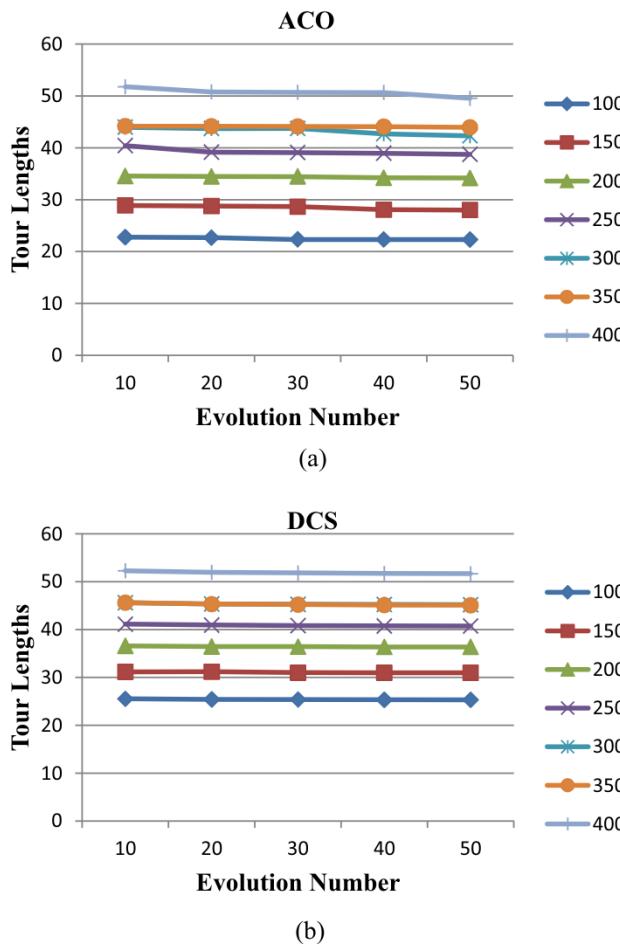


Fig. 4. Best tour lengths for different amount points on the surface of unit sphere founded by DCS [29] and ACO solutions.

equal to that of points (cities) number for all experiments. The calculated average tour distances by proposed ACO (MMAS) approach were compared with GA's average tour distances as shown in Table 2 and Fig. 3. Meanwhile, average calculation times are shown in Table 3. The values were obtained for ways at a unit surface of the sphere.

If Table 2 is examined, when evolution number for 150 points is taken as 50, the tour length obtained with ACO is 29.615 and with GA is 70.5737. For example, when evolution number is 50 for 250 and 400 points, GA's tour lengths are 165.5674 and 354.375, ACO's tour lengths are 42.155 and 54.136 respectively. When GA and MMAS results were compared, it is observed that the results of the MMAS algorithm were much more successful than GA for the spherical TSP, considering the other columns in Table 2.

In literature [29], Discrete Cuckoo Search Algorithm (DCS) is also tested with the same generation size and number of points

as like in Uğur et al. [37] and results are given. But, in Uğur et al. [29], only best results of DCS are compared with GA. In this paper, results that indicated in Table 2 found by the proposed ACO are average results of iterations. In case study #2, the calculated best tour distances by ACO were compared with the DCS's best tour distances as shown in Table 4. In Table 4, when evolution number for 300 points is taken as 40, the tour length obtained with ACO is 42.7122 and with DCS is 45.2367. In all evaluation numbers for 100 points ACO results better than DCS. For each other set of points (150, 200, 250, 300, 350 and 400), the ACO method is more successful than the DCS method for each evolution number. When Fig. 4 and Table 4 are examined, it is seen that the tour lengths are within the range of [22, 52] for ACO (MMAS) and [25, 53] for DCS. When the results of DCS and those of ACO were compared, see another columns of Table 4, it was observed that the results of ACO algorithm were successful than DCS for sphericalTSP.

The optimum route determined by SphereTSP for $N = 100, 250$ and 400 points is shown in Fig. 5 and Fig. 6. For Fig. 5 and Fig. 6 all points and route are viewed simultaneously with a transparent mode and solid view, respectively.

5. Conclusion

Spherical geometry shows differences from Euclidean geometry. In planar geometry, the shortest distance is given by a straight line, while in spherical geometry is formed by large circles. That is, distance between two points is traveled through a curved line on the surface of the sphere in place of a straight line. Where, the angular distance is in consideration. The contribution of this study that it suggests a ACO (MMAS) algorithm giving reliable results for the solution of spherical TSP.

ACO (MMAS) has been successful in the spherical TSP, which can be used effectively with optimal results in the existing planar TSP solution. When the results of the proposed method and those of spherical TSP application through GA given by Uğur et al. [37] are compared, it can be seen that the ACO is more successful than GA for spherical TSP. When the best results for the spherical TSP of the proposed ACO method and DCS method given by Ouyang et al. [29] are compared, ACO is more successful than DCS in all best results of spherical TSP.

In the future studies, as a suggestion, other heuristic methods used in the TSP solution, such as Particle Swarm Optimization (PSO), can be tested for spherical TSP solution. Meanwhile, spherical TSP problems can be studied by hybrid utilization of heuristic methods. It can be predicted that with the increasing algorithm evolution and iteration, the optimum results obtained for unit sphere can be improved further. By changing the value of ACO constants, much better optimum results can be obtained.

The application of TSP for spherical conditions and the proposed method are important for the planning of the motions on the surface of the world. For vehicles traveling to different points on the surface of the world due to variety of reasons such as transportation, this method can be used to optimize the cost-time problem.

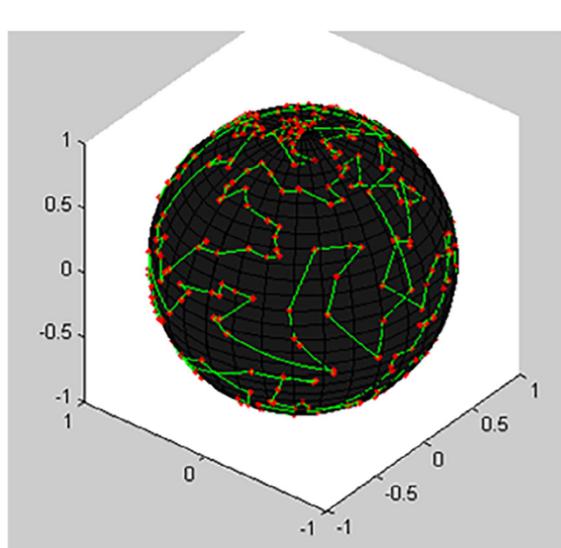
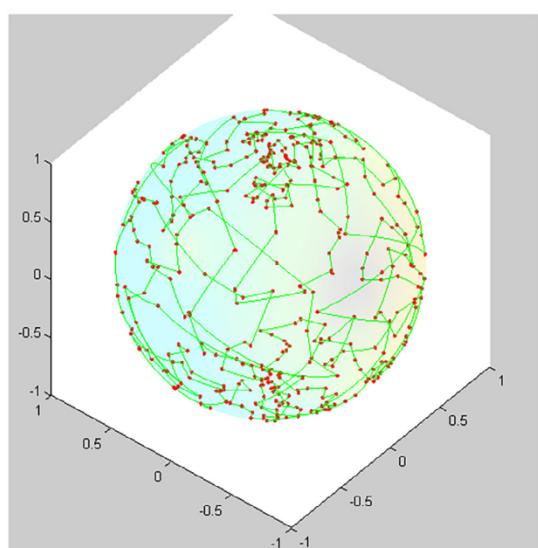
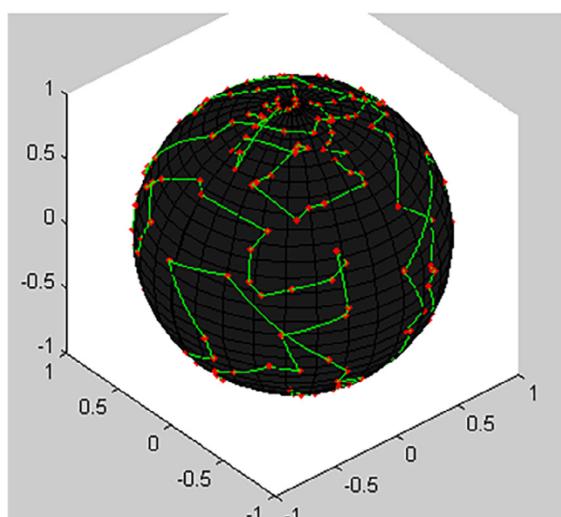
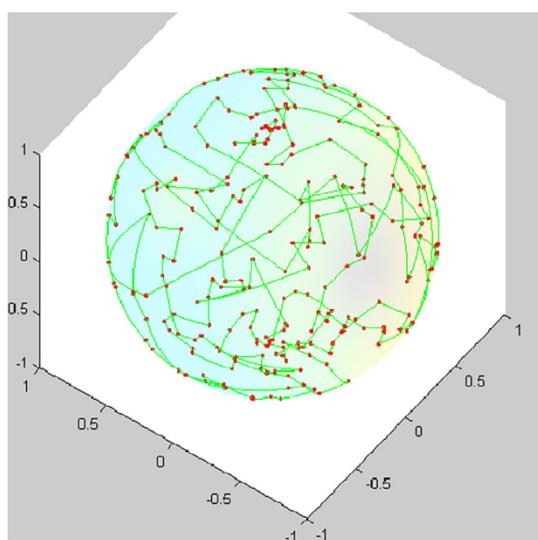
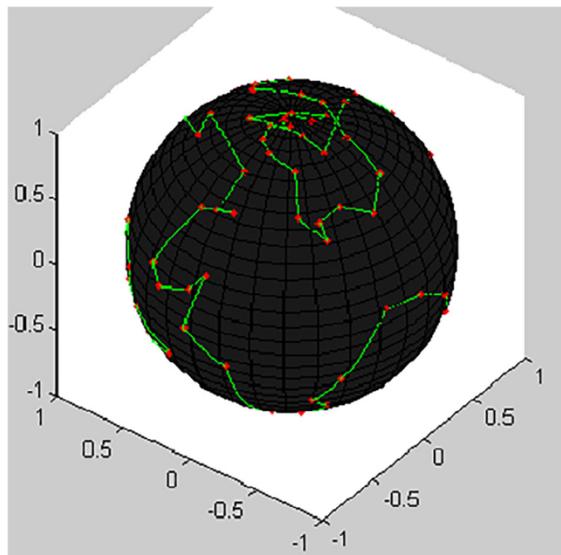
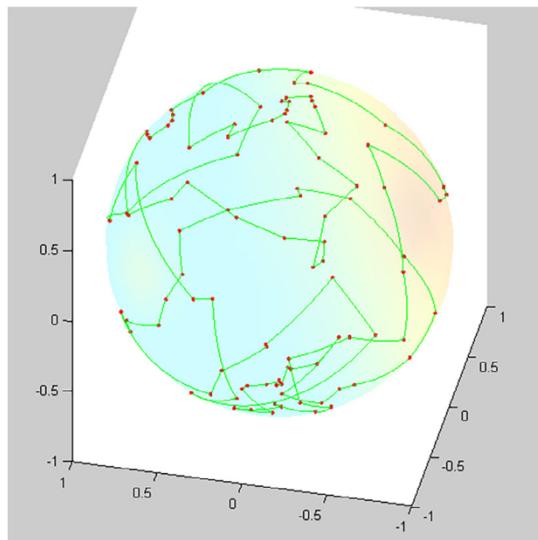


Fig. 5. The transparent view of the shortest routes obtained for 100, 250 and 400 points randomly placed on the sphere.

Fig. 6. The solid view of the shortest routes obtained for 100, 250 and 400 points randomly placed on the sphere.

Proposed method is beneficial to understand the behavior of the ant (agent) present on each spherical object in the real world. Meanwhile the use of ACO, metaheuristic methods and hybrid approaches for optimization problems of 3D shapes including sphere could be the source of inspiration for different studies.

References

- [1] S. Chen, C. Chien, Parallelized genetic ant colony systems for solving the traveling salesman problem, *Expert Syst. Appl.* 38 (2011) 3873–3883.
- [2] A. Colorni, M. Dorigo, V. Maniezzo, *Distributed Optimization by Ant Colonies*, Elsevier Publishing, Amsterdam, 1991, pp. 134–142.
- [3] A. Colorni, M. Dorigo, V. Maniezzo, An investigation of some properties of an ant algorithm, North-Holland, Amsterdam, 1992, pp. 509–520.
- [4] G. Dantzig, R. Fulkerson, S. Johnson, Solution of a Large-Scale Traveling Salesman Problem, *J. Oper. Res. Soc.* 2 (1954) 393–410.
- [5] M. Dorigo, Optimization, Learning and Natural Algorithms, Politecnico di Milano, Italy, 1992. Ph.D. thesis.
- [6] M. Dorigo, V. Maniezzo, A. Colorni, Ant system: optimization by a colony of cooperating agents, *IEEE Trans. Syst. Man Cybern.-Part B Cybern.* 26–1 (1996) 29–41.
- [7] M. Dorigo, L.M. Gambardella, Ant colony system: a cooperative learning approach to the traveling salesman problem, *IEEE Trans. Evolut. Comput.* 1 (1) (1997) 53–66.
- [8] M. Dorigo, L.M. Gambardella, Ant colonies for the travelling salesman problem, *BioSystems* 43 (1997) 73–81.
- [9] H. Eldem, E. Ülker, The Application of Particle Swarm Optimization In The Solution Of 3D Traveling Salesman Problem On A Sphere. Akademik Bilişim'14 – XVI. Akademik Bilişim Konferansı Bildirileri, 2014, pp. 461–469 (in Turkish Language).
- [10] H. Eldem, E. Ülker, Application of ant colony optimization for the solution of 3 dimensional cuboid structures, *J. Comput. Commun.* 2 (2014) 99–107.
- [11] H. Eldem, E. Ülker, Optimizing The Tour On 3D Cuboid Structures With Particle Swarm Optimization Method. 3rd International Symposium on Innovative Technologies in Engineering and Science. Universidad Politecnica de Valencia, 2014, pp. 1607–1617.
- [12] A.E. Ezugwu, A.O. Adewumi, M.E. Frincu, Simulated annealing based symbiotic organisms search optimization algorithm for traveling salesman problem, *Expert Syst. Appl.* 77 (2017) 189–210.
- [13] L.M. Gambardella, M. Dorigo, Solving Symmetric and Asymmetric TSPs by Ant Colonies. Proceedings of the Int. Conf. on Evolutionary Computation. Nagoya, Japan, 1996, pp. 622–627.
- [14] F. Glover, Tabu search – Part I, *ORSA J. Comput.* 1 (3) (1989) 190–206.
- [15] F. Glover, Tabu search – Part II, *ORSA J. Comput.* 2 (1) (1990) 4–32.
- [16] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, 1989. Reading.
- [17] D. Hearn, M.P. Baker, *Computer Graphics C Version*, second ed., Prentice Hall, 1996.
- [18] J.H. Holland, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, 1975.
- [19] J.J. Hopfield, D.W. Tank, Neural computation of decisions in optimization problems, *Biol. Cybern.* 52 (1985) 141–152.
- [20] D.S. Johnson, L.A. McGeoch, The traveling salesman problem: A case study in local optimization, in: E.H.L. Aarts, J.K. Lenstra (Eds.), *Local Search in Combinatorial Optimization*, John Wiley & Sons, New York, 1997. 215–310.
- [21] S. Kirkpatrick, C.D. Gelatt, M.P. Vecchi, Optimization by simulated annealing, *Science* 220 (1983) 671–680.
- [22] T. Kohonen, *Self-Organizing Maps*, Springer, Berlin, 1995.
- [23] G. Laporte, The vehicle routing problem: an overview of exact and approximate algorithms, *Eur. J. Oper. Res.* 59 (1992) 345–358.
- [24] Z.J. Lee, A hybrid algorithm applied to travelling salesman problem, *Networking, Sensing and Control, Proceedings of the IEEE International Conference*, 2004, pp. 237–242.
- [25] Y. Lin, Z. Bian, X. Liu, Developing a dynamic neighborhood structure for an adaptive hybrid simulated annealing – tabu search algorithm to solve the symmetrical traveling salesman problem, *Appl. Soft Comput.* 49 (2016) 937–952.
- [26] S. Maity, A. Roy, M. Maiti, An imprecise multi-objective genetic algorithm for uncertain constrained multi-objective solid travelling salesman problem, *Expert Syst. Appl.* 46 (2016) 196–223.
- [27] Y. Marinakis, A. Migdalas, P.M. Pardalos, A hybrid genetic-GRASP algorithm using lagrangean relaxation for the traveling salesman problem, *J. Comb. Optim.* 10 (4) (2005) 311–326.
- [28] T.A.S. Masutti, L.N. Castro, Neuro-immune approach to solve routing problems, *Neurocomputing* 72 (2009) 2189–2197.
- [29] X. Ouyang, Y. Zhou, Q. Luo, H. Chen, A novel discrete cuckoo search algorithm for spherical traveling salesman problem, *Appl. Math. Inf. Sci.* 7 (2) (2013) 777–784.
- [30] K. Shinozawa, T. Uchiyama, K. Shimohara, An approach for solving dynamic TSPs using neural networks Neural Networks, *Proceedings of the IEEE International Joint Conference*. 3 (1991) 2450–2454.
- [31] S. Shoubao, C. Xibin, Jumping PSO with expanding neighborhood search for TSP on a cuboid, *Chin. J. Electron.* 22 (1) (2013) 202–208.
- [32] T. Stützle, H. Hoos, MAX-MIN ant system, *Future Gener. Comput. Syst.* 16 (8) (2000) 889–914.
- [33] T. Stützle, H. Hoos, Improvements on the ant system: introducing the MAX-MIN ant system, *Artif. Neural Networks Genet. Algorithms* (1998) 245–249.
- [34] C. Tsai, C. Tseng, A new hybrid heuristic approach for solving large traveling salesman problem, *Inf. Sci.* 166 (2004) 67–81.
- [35] Y. Tsujimura, M. Gen, Entropy-based genetic algorithm for solving TSP, *Knowledge-Based Intell. Electron. Syst.* 2 (1998) 285–290.
- [36] A. Uğur, Path planning on a cuboid using genetic algorithms, *Inf. Sci.* 178 (2008) 3275–3287.
- [37] A. Uğur, S. Korukoglu, A. Caliskan, M. Cinsdikici, A. Alp, Genetic algorithm based solution for Tsp on a sphere, *Math. Comput. Appl.* 14 (3) (2009) 219–228.
- [38] Y. Wang, The hybrid genetic algorithm with two local optimization strategies for traveling salesman problem, *Comput. Ind. Eng.* 70 (2014) 124–133.
- [39] C.M. White, G.G. Yen, A hybrid evolutionary algorithm for traveling salesman problem, *Congress on Evolutionary Computation (CEC2004)*. 2, 2004, 1473–1478.