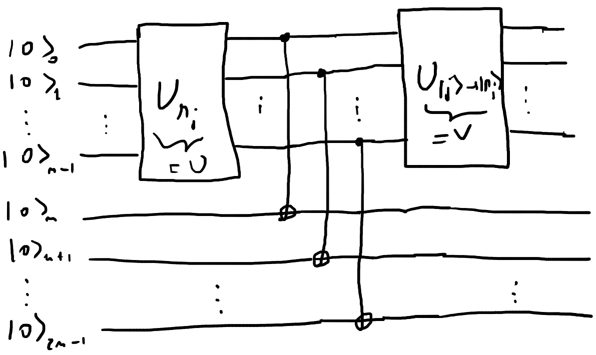

$$\rho = \sum_{j=0}^{d-1} n_j |n_j\rangle\langle n_j| \quad , \quad d = 2^m$$

$$|\psi_{real}\rangle = U_{n_j}|0\rangle^{\otimes m} = \sum_{j=0}^{d-1} \sqrt{n_j}\, |j\rangle$$
$$= \sum_{j_0,\dots,j_{m-1}=0}^{1} \sqrt{n_{j_0,\dots,j_{m-1}}}\, |j_0 \cdots j_{m-1}\rangle$$

$$|\psi_{purificado}\rangle = CNOT^{\otimes m}\, |\psi_{real}\rangle$$
$$= \sum_{j_0,\dots,j_{m-1}=0}^{1} \sqrt{n_{j_0,\dots,j_{m-1}}}\; CNOT|j_0\rangle_0|0\rangle_m\, CNOT|j_1\rangle_1|0\rangle_{m+1} \cdots CNOT|j_{m-1}\rangle_{m-1}|0\rangle_{2m-1}$$
$$= \sum_{j_0,\dots,j_{m-1}=0}^{1} \sqrt{n_{j_0,\dots,j_{m-1}}}\; |j_0\rangle_0|j_0\rangle_m\, |j_1\rangle_1|j_1\rangle_{m+1} \cdots |j_{m-1}\rangle_{m-1}|j_{m-1}\rangle_{2m-1}$$
$$= \sum_{j_0,\dots,j_{m-1}=0}^{1} \sqrt{n_{j_0\cdots j_{m-1}}}\; |j_0 j_1 \cdots j_{m-1}\rangle_{0\cdots m-1}\, |j_0 j_1 \cdots j_{m-1}\rangle_{m,\dots,2m-1}$$
$$= \sum_{j=0}^{d-1} \sqrt{n_j}\; |j\rangle_{0\cdots m-1}\, |j\rangle_{m\cdots 2m-1}$$

$$Tr_{m,\dots,2m-1}\left(|\psi_{pur.}\rangle\langle\psi_{pur.}|\right) = \sum_{j=0}^{d-1} n_j |j\rangle\langle j|_{0\cdots m-1} = \rho'$$

$$U_{|j\rangle\to|n_j\rangle} = \Big[\, |n_0\rangle \;\; |n_1\rangle \;\cdots\; |n_{d-1}\rangle \,\Big] = V \;\Rightarrow\; V|j\rangle = |n_j\rangle$$

Aplica V

$$V\otimes I\, |\psi_{purificado}\rangle = \sum_{j=0}^{d-1} \sqrt{n_j}\; V|j\rangle_{0\cdots m-1} \otimes |j\rangle_{m\cdots 2m-1}$$
$$= \sum_{j=0}^{d-1} \sqrt{n_j}\; |n_j\rangle_{0\cdots m-1} \otimes |j\rangle_{m\cdots 2m-1}$$
$$= |\psi_{final}\rangle$$

$$Tr_{m\cdots 2m-1}\left(|\psi_{final}\rangle\langle\psi_{final}|\right) = \sum_{j=0}^{d-1} n_j |n_j\rangle\langle n_j|_{0\cdots m-1} = \rho$$

Algoritmo

$$\rho \to \text{lapack}: \; W = \begin{bmatrix} n_0 \\ \vdots \\ n_{d-1} \end{bmatrix}, \quad N = \Big[\, |n_0\rangle \cdots |n_{d-1}\rangle \,\Big]$$

W ↓ state prep. qiskit U

N ↓ unitary compilation qiskit V

$$\xrightarrow{U,V} \; |0\rangle^{\otimes 2m} \to U \to \text{CNOTS} \to V \to \text{tomo}$$