

$$\begin{aligned}
E &\rightarrow FE \\
E &\rightarrow GE \\
E &\rightarrow id \\
C &\rightarrow + \\
D &\rightarrow * \\
F &\rightarrow EC \\
G &\rightarrow ED
\end{aligned}$$

In the previous grammar, all the productions are in the form of non-terminal  $\rightarrow$  string of exactly two non-terminals or non-terminal  $\rightarrow$  single terminal. So, the grammar is in CNF.

**Example 6.40**

Convert the following grammar into CNF.

$$\begin{aligned}
S &\rightarrow abAB \\
A &\rightarrow bAB / \epsilon \\
B &\rightarrow bAa / \epsilon
\end{aligned}$$

**Solution:** In the grammar, there are two null productions  $A \rightarrow \epsilon$  and  $B \rightarrow \epsilon$ . First, these productions must be removed, and after that the grammar can be converted into CNF. After removing the null production  $A \rightarrow \epsilon$ , the modified grammar becomes

$$\begin{aligned}
S &\rightarrow abAB/abB \\
A &\rightarrow bAB/bB \\
B &\rightarrow Ba
\end{aligned}$$

After removing the null production  $B \rightarrow \epsilon$ , the modified grammar becomes

$$\begin{aligned}
S &\rightarrow abAB/abA/abB/ab \\
A &\rightarrow bAB/bA/bB/b \\
B &\rightarrow Ba/Aa/a
\end{aligned}$$

Now, this grammar can be converted into CNF. In the grammar, except  $A \rightarrow b$  and  $B \rightarrow a$ , all the productions are not in CNF. Let us take two non-terminals  $C_a$  and  $C_b$  which will replace  $a$  and  $b$ , respectively. So, two new productions  $C_a \rightarrow a$  and  $C_b \rightarrow b$  will be added to the grammar. After replacing and adding new productions, the modified grammar is

$$\begin{aligned}
S &\rightarrow CaC_bAB/C_aC_bA/C_aC_bB/C_aC_b \\
A &\rightarrow C_bAB/C_bA/C_bB/b \\
B &\rightarrow BC_a/AC_a/a \\
C_a &\rightarrow a \\
C_b &\rightarrow b
\end{aligned}$$

In this modified grammar,  $S \rightarrow CaC_bAB$ ,  $S \rightarrow CaC_bA$ ,  $S \rightarrow CaC_bB$ , and  $A \rightarrow C_bAB$  are not in CNF. Let us take two other non-terminals  $D$  and  $E$ , which will be replaced in the place of  $CaC_b$  and  $AB$ , respectively. So, two new productions  $D \rightarrow CaC_b$  and  $E \rightarrow AB$  will be added to the grammar. By replacing and adding two new productions, the modified grammar is

$$\begin{aligned}
S &\rightarrow DE/DA/DB/C_aC_b \\
A &\rightarrow C_bE/C_bA/C_bB/b \\
B &\rightarrow BC_a/AC_a/a
\end{aligned}$$