

CA7

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بخش اول

تمرین اول - الف -

$$V_{in} = R i(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

$$\frac{d}{dt} V_{in} = \frac{i(t)}{C} + R \frac{di(t)}{dt} + L \frac{d^2 i(t)}{dt^2}$$

ب -

Laplace $\rightarrow s V_{in} = s^2 I(s) + s R I(s) + \frac{1}{C} I(s)$

$$\Rightarrow I(s) = \frac{s V_{in}}{s^2 + R s + \frac{1}{C}}$$

ج -

$$V_C(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau \xrightarrow{L} V_C(s) = \frac{1}{C s} I(s)$$

$$\Rightarrow I(s) = C s V_C(s)$$

$$\Rightarrow V_C(s) = \frac{1}{C s} \times \frac{s V_{in}(s)}{s^2 + R s + \frac{1}{C}} = \frac{V_{in}(s)}{s^2 + R s + 1}$$

$$\Rightarrow Y(s) = \frac{X(s)}{s^2 + R s + 1}$$

د -

$$Y(s) = X(s) \times \frac{3}{s^2 + s + 3} = X(s) \left(\frac{\frac{3}{2}}{s+1} - \frac{\frac{3}{2}}{s+2} \right)$$

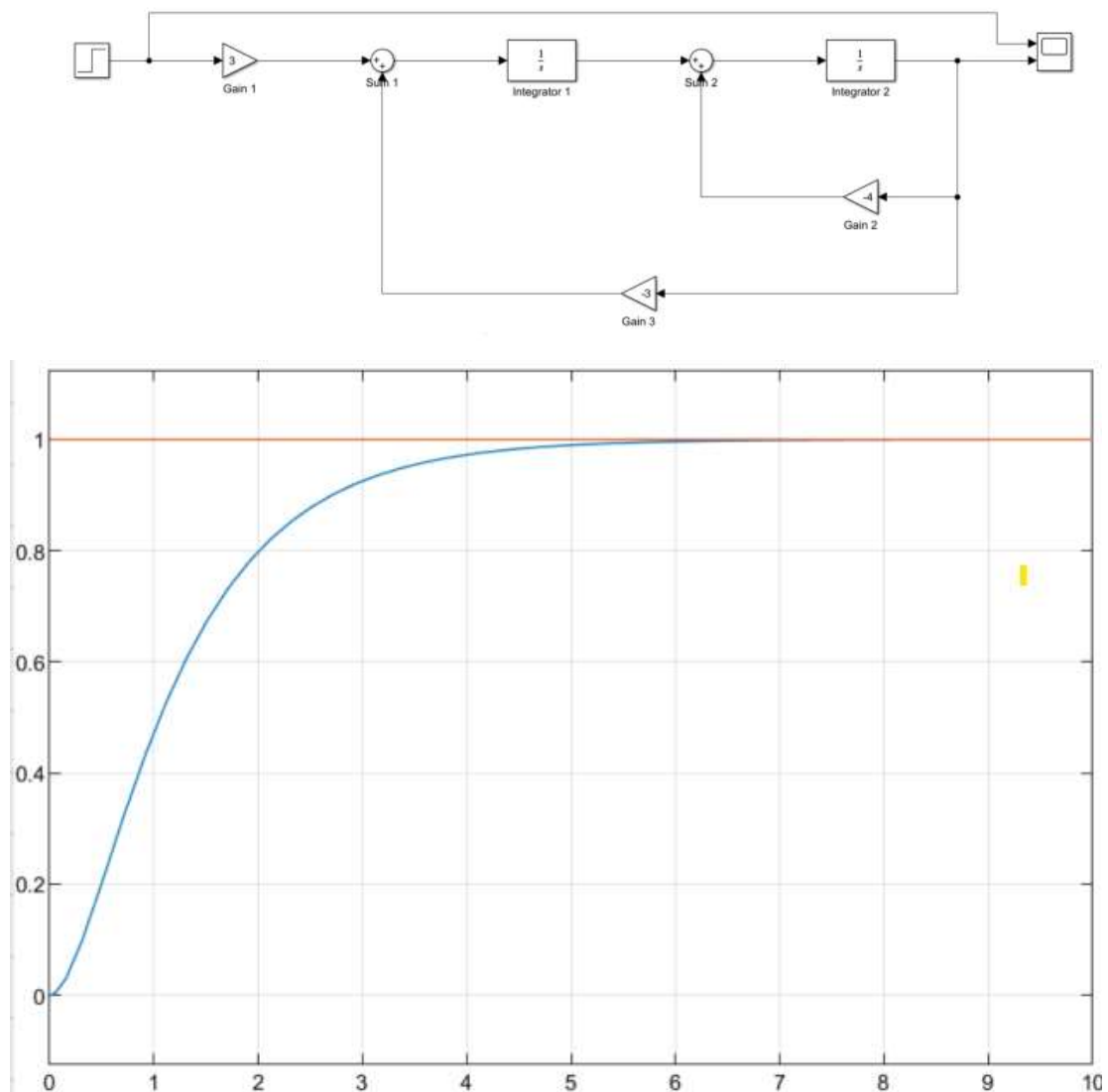
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input: $u(t) \Rightarrow X(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{3}{s^3 + 4s^2 + 3s}$

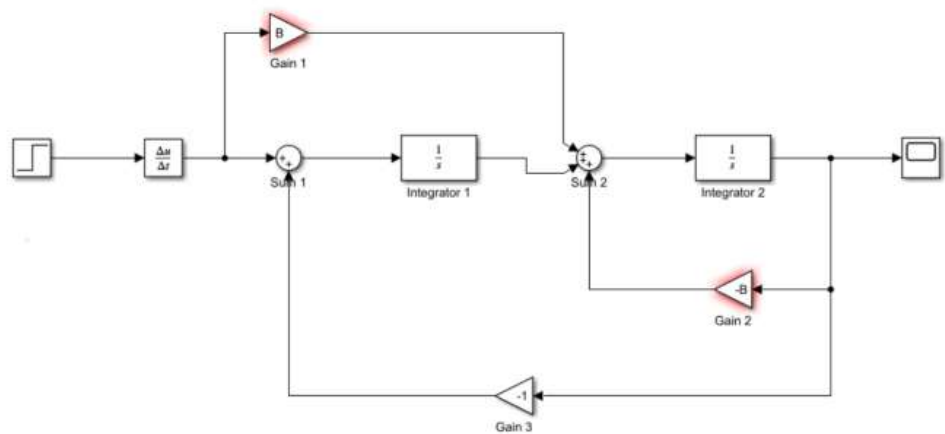
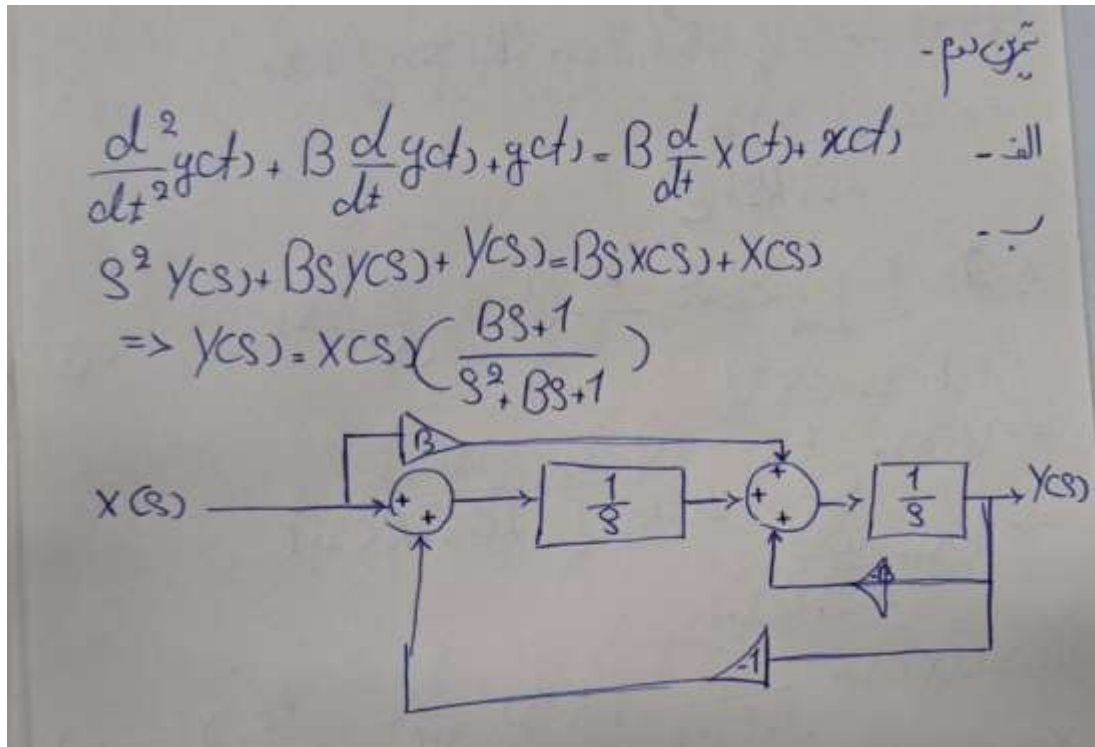
$\xrightarrow{L^{-1}} y(t) = u(t) - \frac{3}{2}e^{-t} + \frac{1}{2}e^{-3t}$

مسئله با Simulation حل می شود

(۵)

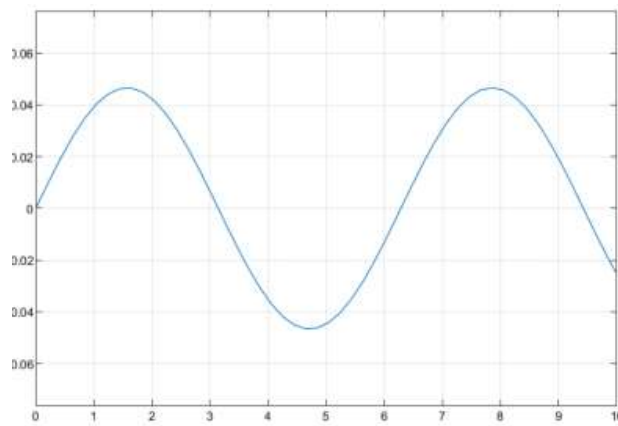
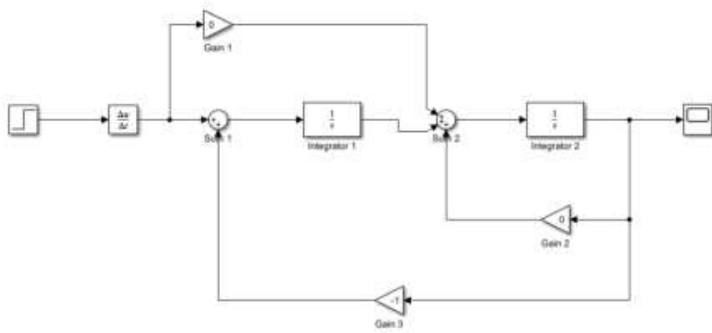


تمرین دوم

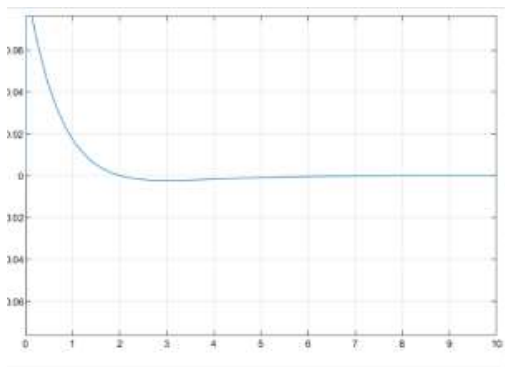


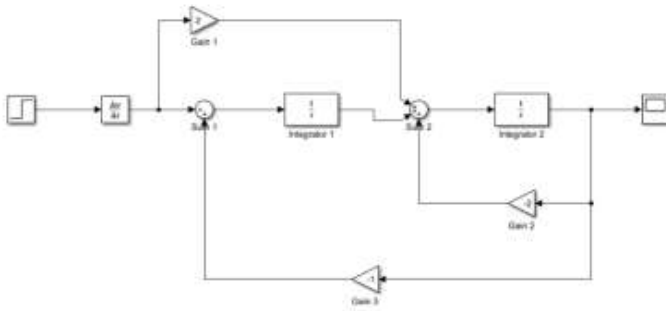
$$x(t) = \delta(t) \Rightarrow Y(s) = \frac{Bs+1}{s^2 + Bs+1} \xrightarrow{B=0} y(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 1} \right\} = \sin(t)$$

ج -



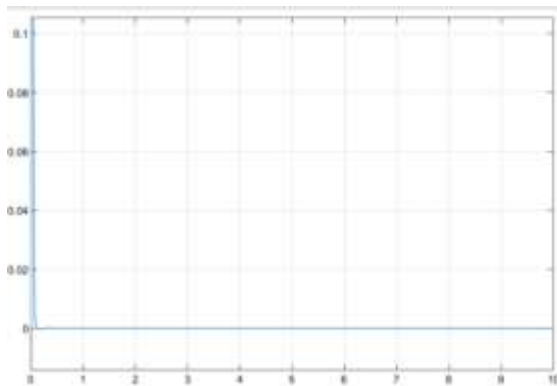
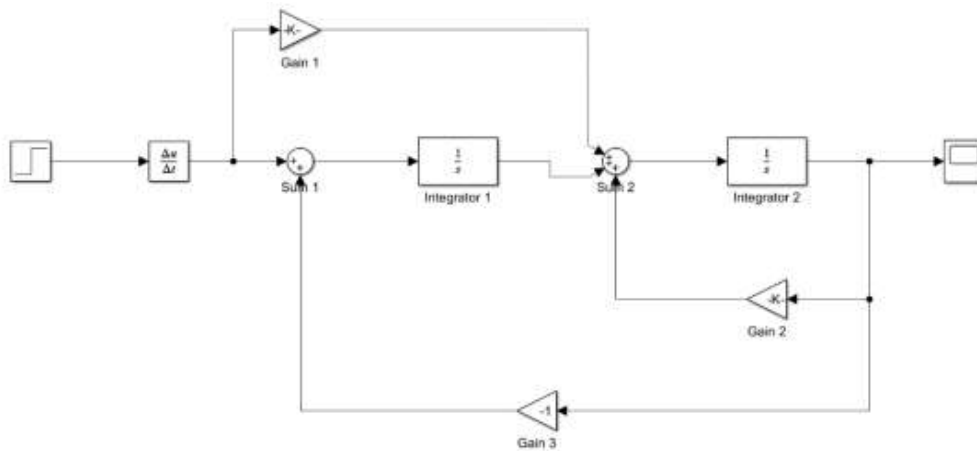
$$\begin{aligned}
 s^2 + Bs + 1 &= 0 & \Delta &= B^2 - 4 \Rightarrow |B| \geq 2 \\
 B &= 2 \\
 x(t) = \delta(t) & \} \Rightarrow y(t) = \mathcal{L}^{-1} \left\{ \frac{2s+1}{s^2+2s+1} \right\} = 2e^{-t} - e^{-t}
 \end{aligned}$$





$B = 100$
 $x(t) = \delta(t)$

$$y(t) = \mathcal{L}^{-1} \left\{ \frac{100s+1}{s^2+100s+1} \right\} = 100e^{-50t} \cosh(7\sqrt{51}t) - \frac{4999}{7\sqrt{51}} e^{-50t} \cosh(7\sqrt{51}t)$$



(۵)

باید تا جای ممکن قطب ها نزدیک به هم باشند. اگر قطب حقیقی نداشته باشیم به حالت پایداری نمی رسیم. $B = +2, -2$ بهترین حالت برای سیستم هستند.

تمرین سوم)

تمرین سوم - الف -

$$\frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2y(t) = 5u(t)$$
$$s^2 Y(s) + 3sY(s) + 2Y(s) = \frac{5}{s}$$
$$\Rightarrow Y(s) = \frac{5}{s(s^2 + 3s + 2)} \quad \xrightarrow{\mathcal{L}^{-1}} y(t) = \left(\frac{5}{2} e^{-2t} - \frac{5}{2} e^{-t} + \frac{5}{2} \right) u(t)$$

باید شرایط اولیه را در نظر بگیریم:

$$s^2 Y(s) - sy(0) - y'(0) + 3sY(s) - 3y(0) + 2Y(s) = 0$$
$$\Rightarrow Y(s) = \frac{s+2}{(s+2)(s+1)} \quad \xrightarrow{\mathcal{L}^{-1}} y(t) = (-2e^{-2t} + 3e^{-t}) u(t)$$

```
p3.m x +
1  clc;
2  clear all;
3  syms y(t)
4  dy = diff(y);
5  equation = diff(y,t,2) + 3*diff(y,t,1) + 2*y == 5*heaviside(t);
6  cond1 = y(0) == 1;
7  cond2 = dy(0) == 1;
8  conditions = [cond1 cond2];
9  ySol(t) = dsolve(equation,conditions);
10 ySol = simplify(ySol);
11 fprintf('The answer of the differential equation:\n%s\nis\ny(t) = %s\n', equation, ySol);
12 fplot(ySol, [0, 10]);
13
```

Command Window

New to MATLAB? See resources for [Getting Started](#).

The answer of the differential equation:
 $2*y(1) + 3*subs(diff(y(t), t), t, 1) + subs(diff(y(t), t, t), t, 1) == 5$
 is
 $y(t) = (exp(-2)*(10*exp(2) - 8*exp(1) + 2))/4$

