Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7205

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The acute angle between the vectors $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ and $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ is denoted by θ. Find $\cos\theta$.

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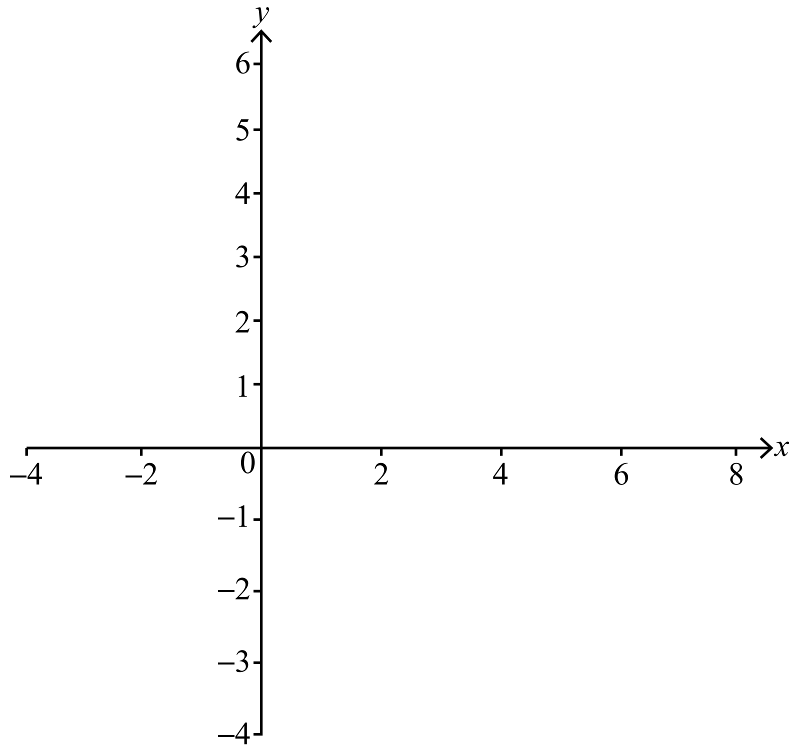
2. [Maximum mark: 7]

(a) Sketch the graphs of $y = \frac{x}{2} + 1$ and $y = |x - 2|$ on the following axes. [3]



(b) Solve the equation $\frac{x}{2} + 1 = |x - 2|$. [4]

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3. [Maximum mark: 6]

The discrete random variable X has the following probability distribution, where p is a constant.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0.5-p$	0.25	0.125	p^3

(a) Find the value of p. [2]

(b) (i) Find μ, the expected value of X.

(ii) Find $\mathrm{P}(X>\mu)$. [4]

4. [Maximum mark: 6]

Consider the curve $y = \frac{1}{1-x} + \frac{4}{x-4}$.

Find the x-coordinates of the points on the curve where the gradient is zero.

5. [Maximum mark: 7]

The geometric sequence $u_1, u_2, u_3, \ldots$ has common ratio r.
Consider the sequence $A = \{a_n = \log_2 |u_n| : n \in \mathbb{Z}^+\}$.

(a) Show that A is an arithmetic sequence, stating its common difference d in terms of r. [4]

A particular geometric sequence has $u_1 = 3$ and a sum to infinity of 4.

(b) Find the value of d. [3]

6. [Maximum mark: 7]

Consider the functions f, g, defined for $x \in \mathbb{R}$, given by $f(x) = \mathrm{e}^{-x}\sin x$ and $g(x) = \mathrm{e}^{-x}\cos x$.

(a) Find

(i) $f'(x)$;

(ii) $g'(x)$. [3]

(b) Hence, or otherwise, find $\int_0^{\pi} \mathrm{e}^{-x}\sin x\,\mathrm{d}x$. [4]

7. [Maximum mark: 6]

Consider the distinct complex numbers $z = a + \mathrm{i}b$, $w = c + \mathrm{i}d$, where $a, b, c, d \in \mathbb{R}$.

(a) Find the real part of $\dfrac{z + w}{z - w}$. [4]

(b) Find the value of the real part of $\dfrac{z + w}{z - w}$ when $|z| = |w|$. [2]

8. [Maximum mark: 7]

(a) Use the substitution $u = x^{\frac{1}{2}}$ to find $\int \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4]

(b) Hence find the value of $\frac{1}{2}\int_1^9 \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, expressing your answer in the form $\arctan q$, where $q \in \mathbb{Q}$. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 24]

The points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D have position vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$, relative to the origin O.
It is given that $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{DC}}$.

(a) (i) Explain why ABCD is a parallelogram.

(ii) Using vector algebra, show that $\overrightarrow{\mathrm{AD}} = \overrightarrow{\mathrm{BC}}$. [4]

The position vectors $\overrightarrow{\mathrm{OA}}$, $\overrightarrow{\mathrm{OB}}$, $\overrightarrow{\mathrm{OC}}$ and $\overrightarrow{\mathrm{OD}}$ are given by

$$\begin{aligned}
\boldsymbol{a} &= \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k} \\
\boldsymbol{b} &= 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k} \\
\boldsymbol{c} &= q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} \\
\boldsymbol{d} &= -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}
\end{aligned}$$

where p, q and r are constants.

(b) Show that $p = 1$, $q = 1$ and $r = 4$. [5]

(c) Find the area of the parallelogram ABCD. [4]

The point where the diagonals of ABCD intersect is denoted by M.

(d) Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

(e) Find the Cartesian equation of Π. [3]

The plane Π cuts the x, y and z axes at X, Y and Z respectively.

(f) (i) Find the coordinates of X, Y and Z.

(ii) Find YZ. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The function f is defined by $f(x) = \frac{ax + b}{cx + d}$, for $x \in \mathbb{R}, x \neq -\frac{d}{c}$.

(a) Find the inverse function f^{-1}, stating its domain. [5]

The function g is defined by $g(x) = \frac{2x - 3}{x - 2}, x \in \mathbb{R}, x \neq 2$.

(b) (i) Express $g(x)$ in the form $A + \frac{B}{x - 2}$ where A, B are constants.

(ii) Sketch the graph of $y = g(x)$. State the equations of any asymptotes and the coordinates of any intercepts with the axes. [5]

The function h is defined by $h(x) = \sqrt{x}$, for $x \geq 0$.

(c) State the domain and range of $h \circ g$. [4]

11. [Maximum mark: 12]

(a) Show that $\log_{r^2} x = \frac{1}{2} \log_r x$ where $r, x \in \mathbb{R}^+$. [2]

It is given that $\log_2 y + \log_4 x + \log_4 2x = 0$.

(b) Express y in terms of x. Give your answer in the form $y = px^q$, where p, q are constants. [5]

The region R, is bounded by the graph of the function found in part (b), the x-axis, and the lines $x = 1$ and $x = \alpha$ where $\alpha > 1$. The area of R is $\sqrt{2}$.

(c) Find the value of α. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.