Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Thursday 4 May 2017 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2217 – 7203

13 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$.

2. [Maximum mark: 6]

Consider the complex numbers $z_1 = 1 + \sqrt{3}\,\mathrm{i}$, $z_2 = 1 + \mathrm{i}$ and $w = \dfrac{z_1}{z_2}$.

(a) By expressing z_1 and z_2 in modulus-argument form write down

(i) the modulus of w;

(ii) the argument of w. [4]

(b) Find the smallest positive integer value of n, such that w^n is a real number. [2]

3. [Maximum mark: 5]

Solve the equation $\sec^2 x + 2\tan x = 0$, $0 \le x \le 2\pi$.

4. [Maximum mark: 5]

Three girls and four boys are seated randomly on a straight bench. Find the probability that the girls sit together and the boys sit together.

5. [Maximum mark: 7]

ABCD is a parallelogram, where $\overrightarrow{AB} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\overrightarrow{AD} = 4\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$.

(a) Find the area of the parallelogram ABCD. [3]

(b) By using a suitable scalar product of two vectors, determine whether $\mathrm{A\hat{B}C}$ is acute or obtuse. [4]

6. [Maximum mark: 5]

Consider the graphs of $y = |x|$ and $y = -|x| + b$, where $b \in \mathbb{Z}^+$.

(a) Sketch the graphs on the same set of axes. [2]

(b) Given that the graphs enclose a region of area 18 square units, find the value of b. [3]

7. [Maximum mark: 7]

An arithmetic sequence $u_1, u_2, u_3 \ldots$ has $u_1 = 1$ and common difference $d \neq 0$. Given that u_2, u_3 and u_6 are the first three terms of a geometric sequence

(a) find the value of d. [4]

Given that $u_N = -15$

(b) determine the value of $\sum_{r=1}^{N} u_r$. [3]

8. [Maximum mark: 6]

Use the method of mathematical induction to prove that $4^n + 15n - 1$ is divisible by 9 for $n \in \mathbb{Z}^+$.

9. [Maximum mark: 5]

Find $\int \arcsin x \, \mathrm{d}x$.

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k. [4]

(b) By considering the graph of f write down

(i) the mean of X;

(ii) the median of X;

(iii) the mode of X. [3]

(c) (i) Show that $P(0 \le X \le 2) = \frac{1}{4}$.

(ii) Hence state the interquartile range of X. [6]

(d) Calculate $P(X \le 4 \mid X \ge 3)$. [2]

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11. [Maximum mark: 17]

(a) (i) Express $x^2 + 3x + 2$ in the form $(x+h)^2 + k$.

(ii) Factorize $x^2 + 3x + 2$. [2]

Consider the function $f(x) = \dfrac{1}{x^2 + 3x + 2}$, $x \in \mathbb{R}$, $x \neq -2$, $x \neq -1$.

(b) Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. [5]

(c) Show that $\dfrac{1}{x+1} - \dfrac{1}{x+2} = \dfrac{1}{x^2 + 3x + 2}$. [1]

(d) Hence find the value of p if $\int_0^1 f(x)\,\mathrm{d}x = \ln(p)$. [4]

(e) Sketch the graph of $y = f(|x|)$. [2]

(f) Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x-axis and the lines with equations $x = -1$ and $x = 1$. [3]

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12. [Maximum mark: 18]

Consider the polynomial $P(z) = z^5 - 10z^2 + 15z - 6$, $z \in \mathbb{C}$.

(a) Write down the sum and the product of the roots of $P(z) = 0$. [2]

(b) Show that $(z - 1)$ is a factor of $P(z)$. [2]

The polynomial can be written in the form $P(z) = (z - 1)^3 \left(z^2 + bz + c\right)$.

(c) Find the value of b and the value of c. [5]

(d) Hence find the complex roots of $P(z) = 0$. [3]

Consider the function $q(x) = x^5 - 10x^2 + 15x - 6$, $x \in \mathbb{R}$.

(e) (i) Show that the graph of $y = q(x)$ is concave up for $x > 1$.

(ii) Sketch the graph of $y = q(x)$ showing clearly any intercepts with the axes. [6]

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