Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: analysis and approaches
Higher level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

15 pages

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

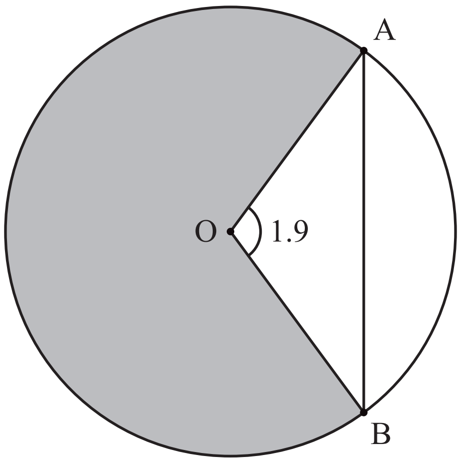
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $\text{A}\hat{\text{O}}\text{B} = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord $[\text{AB}]$. [3]

(b) Find the area of the shaded sector. [3]

2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

3. [Maximum mark: 6]

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

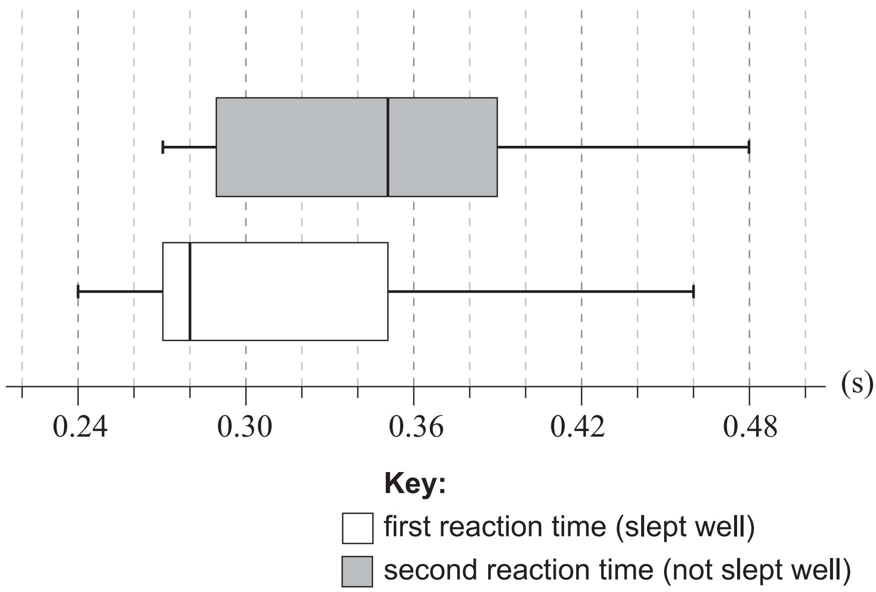
Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$.

4. [Maximum mark: 6]

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

(a) State the median reaction time after sleeping well. [1]

(b) Verify that the measurement of 0.46 seconds is not an outlier. [3]

(c) State why it appears that the mean reaction time is greater than the median reaction time. [1]

Now consider the two box and whisker diagrams.

(d) Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time. [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v = \dfrac{(t^2+1)\cos t}{4}$, $0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

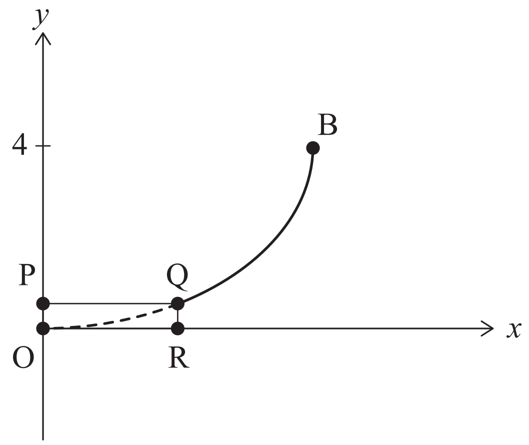
(b) Find the times when the particle's acceleration is $-1.9\,\mathrm{m\,s^{-2}}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

6. [Maximum mark: 5]

The following diagram shows the curve $\frac{x^2}{36}+\frac{(y-4)^2}{16}=1$, where $h \le y \le 4$.

diagram not to scale



The curve from point Q to point B is rotated $360°$ about the y-axis to form the interior surface of a bowl. The rectangle OPQR, of height h cm, is rotated $360°$ about the y-axis to form a solid base.

The bowl is assumed to have negligible thickness.

Given that the interior volume of the bowl is to be $285\,\mathrm{cm}^3$, determine the height of the base.

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7. [Maximum mark: 8]

Consider $\lim\limits_{x \to 0} \dfrac{\arctan(\cos x) - k}{x^2}$, where $k \in \mathbb{R}$.

(a) Show that a finite limit only exists for $k = \dfrac{\pi}{4}$. [2]

(b) Using l'Hôpital's rule, show algebraically that the value of the limit is $-\dfrac{1}{4}$. [6]

8. [Maximum mark: 7]

Rachel and Sophia are competing in a javelin-throwing competition.

The distances, R metres, thrown by Rachel can be modelled by a normal distribution with mean 56.5 and standard deviation 3.

The distances, S metres, thrown by Sophia can be modelled by a normal distribution with mean 57.5 and standard deviation 1.8.

In the first round of competition, each competitor must have five throws. To qualify for the next round of competition, a competitor must record at least one throw of 60 metres or greater in the first round.

Find the probability that only one of Rachel or Sophia qualifies for the next round of competition.

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9. [Maximum mark: 4]

Consider the set of six-digit positive integers that can be formed from the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9.

Find the total number of six-digit positive integers that can be formed such that

(a) the digits are distinct; [2]

(b) the digits are distinct and are in increasing order. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, $h_A\,\text{cm}$, at time t weeks can be modelled by the function $h_A(t) = \sin(2t+6) + 9t + 27$, where $0 \le t \le 9$.

The scientist found that the height of Plant B, $h_B\,\text{cm}$, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \le t \le 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $t > 6$, prove that Plant A was always taller than Plant B. [3]

(d) For $0 \le t \le 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

Two airplanes, A and B, have position vectors with respect to an origin O given respectively by

$$\boldsymbol{r}_A = \begin{pmatrix} 19 \\ -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$$

$$\boldsymbol{r}_B = \begin{pmatrix} 1 \\ 0 \\ 12 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$$

where t represents the time in minutes and $0 \le t \le 2.5$.

Entries in each column vector give the displacement east of O, the displacement north of O and the distance above sea level, all measured in kilometres.

(a) Find the three-figure bearing on which airplane B is travelling. [2]

(b) Show that airplane A travels at a greater speed than airplane B. [2]

(c) Find the acute angle between the two airplanes' lines of flight. Give your answer in degrees. [4]

The two airplanes' lines of flight cross at point P.

(d) (i) Find the coordinates of P.

(ii) Determine the length of time between the first airplane arriving at P and the second airplane arriving at P. [7]

Let $D(t)$ represent the distance between airplane A and airplane B for $0 \le t \le 2.5$.

(e) Find the minimum value of $D(t)$. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

The population, P, of a particular species of marsupial on a small remote island can be modelled by the logistic differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{N}\right)$$

where t is the time measured in years and k, N are positive constants.

The constant N represents the maximum population of this species of marsupial that the island can sustain indefinitely.

(a) In the context of the population model, interpret the meaning of $\frac{\mathrm{d}P}{\mathrm{d}t}$. [1]

(b) Show that $\frac{\mathrm{d}^2P}{\mathrm{d}t^2} = k^2P\left(1 - \frac{P}{N}\right)\left(1 - \frac{2P}{N}\right)$. [4]

(c) Hence show that the population of marsupials will increase at its maximum rate when $P = \frac{N}{2}$. Justify your answer. [5]

(d) Hence determine the maximum value of $\frac{\mathrm{d}P}{\mathrm{d}t}$ in terms of k and N. [2]

Let P_0 be the initial population of marsupials.

(e) By solving the logistic differential equation, show that its solution can be expressed in the form

$$kt = \ln\frac{P}{P_0}\left(\frac{N - P_0}{N - P}\right).$$ [7]

After 10 years, the population of marsupials is $3P_0$. It is known that $N = 4P_0$.

(f) Find the value of k for this population model. [2]

References: