Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

Specimen

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

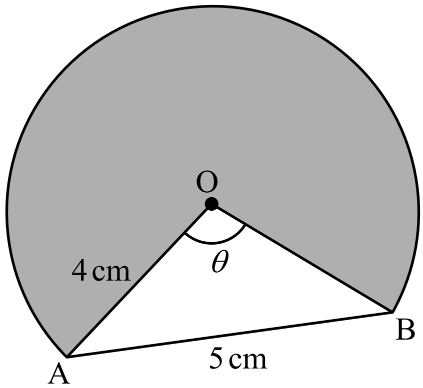
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\mathrm{cm}$.



Chord AB has a length of $5\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

2. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

3. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "six" is $\frac{7}{10}$.

The die is tossed five times. Find the probability of obtaining

(a) at most three "sixes". [3]

(b) the third "six" on the fifth toss. [3]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

4. [Maximum mark: 7]

The following table below shows the marks scored by seven students on two different mathematics tests.

Test 1 (x)	15	23	25	30	34	34	40
Test 2 (y)	20	26	27	32	35	37	35

Let L_1 be the regression line of x on y. The equation of the line L_1 can be written in the form $x = ay + b$.

(a) Find the value of a and the value of b. [2]

Let L_2 be the regression line of y on x. The lines L_1 and L_2 pass through the same point with coordinates (p, q).

(b) Find the value of p and the value of q. [3]

(c) Jennifer was absent for the first test but scored 29 marks on the second test. Use an appropriate regression equation to estimate Jennifer's mark on the first test. [2]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

5. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

6. [Maximum mark: 6]

In a city, the number of passengers, X, who ride in a taxi has the following probability distribution.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.60	0.30	0.03	0.05	0.02

After the opening of a new highway that charges a toll, a taxi company introduces a charge for passengers who use the highway. The charge is $\$2.40$ per taxi plus $\$1.20$ per passenger. Let T represent the amount, in dollars, that is charged by the taxi company per ride.

(a) Find $\mathrm{E}(T)$. [4]

(b) Given that $\mathrm{Var}(X) = 0.8419$, find $\mathrm{Var}(T)$. [2]

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

7. [Maximum mark: 5]

Two ships, A and B, are observed from an origin O. Relative to O, their position vectors at time t hours after midday are given by

$$\boldsymbol{r}_{\mathrm{A}} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\boldsymbol{r}_{\mathrm{B}} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 12 \end{pmatrix}$$

where distances are measured in kilometres.

Find the minimum distance between the two ships.

8. [Maximum mark: 7]

The complex numbers w and z satisfy the equations

$$\frac{w}{z} = 2\mathrm{i}$$
$$z^{*} - 3w = 5 + 5\mathrm{i}.$$

Find w and z in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

9. [Maximum mark: 5]

Consider the graphs of $y = \dfrac{x^2}{x-3}$ and $y = m(x+3)$, $m \in \mathbb{R}$.

Find the set of values for m such that the two graphs have no intersection points.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

The length, $X\,\text{mm}$, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\text{P}(24.15 < X < 25)$. [2]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a seashell selected at random has a length greater than $26\,\text{mm}$. [5]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than $26\,\text{mm}$.

(c) Find $\text{E}(Y)$. [3]

(d) Find the probability that exactly three of these seashells have a length greater than $26\,\text{mm}$. [2]

A seashell selected at random has a length less than $26\,\text{mm}$.

(e) Find the probability that its length is between $24.15\,\text{mm}$ and $25\,\text{mm}$. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

A large tank initially contains pure water. Water containing salt begins to flow into the tank The solution is kept uniform by stirring and leaves the tank through an outlet at its base. Let x grams represent the amount of salt in the tank and let t minutes represent the time since the salt water began flowing into the tank.

The rate of change of the amount of salt in the tank, $\frac{dx}{dt}$, is described by the differential equation $\frac{dx}{dt} = 10e^{-\frac{t}{4}} - \frac{x}{t+1}$.

(a) Show that $t + 1$ is an integrating factor for this differential equation. [2]

(b) Hence, by solving this differential equation, show that $x(t) = \frac{200 - 40e^{-\frac{t}{4}}(t+5)}{t+1}$. [8]

(c) Sketch the graph of x versus t for $0 \le t \le 60$ and hence find the maximum amount of salt in the tank and the value of t at which this occurs. [5]

(d) Find the value of t at which the amount of salt in the tank is decreasing most rapidly. [2]

The rate of change of the amount of salt leaving the tank is equal to $\frac{x}{t+1}$.

(e) Find the amount of salt that left the tank during the first 60 minutes. [4]

12. [Maximum mark: 19]

(a) Show that $\cot 2\theta = \frac{1 - \tan^2\theta}{2\tan\theta}$. [1]

(b) Verify that $x = \tan\theta$ and $x = -\cot\theta$ satisfy the equation $x^2 + (2\cot 2\theta)x - 1 = 0$. [7]

(c) Hence, or otherwise, show that the exact value of $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. [5]

(d) Using the results from parts (b) and (c) find the exact value of $\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$.
Give your answer in the form $a + b\sqrt{3}$ where $a, b \in \mathbb{Z}$. [6]