School of Engineering and Applied Science (SEAS), Ahmedabad University

CSE 400: Fundamentals of Probability in Computing

Lecture 5 Scribe

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Date of Submission:	February 7$^{\text{th}}$, 2026 (11:59 PM)

Objective

This scribe serves as an academic reference for exam preparation for **Lecture 5**. All definitions, derivations, and examples are strictly based on the lecture slides and are presented in a step-by-step manner suitable for a closed-notes examination.

1 Bayes' Theorem as a Weighted Average of Conditional Probabilities

1.1 Decomposition of an Event

Let A and B be two events. Any outcome that belongs to A must satisfy exactly one of the following:

- It lies in $A \cap B$
- It lies in $A \cap B^c$

Thus,

$$A = (A \cap B) \cup (A \cap B^c)$$

The events $A \cap B$ and $A \cap B^c$ are mutually exclusive.

1.2 Application of Axiom 3

By Axiom 3 (Additivity),

$$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c)$$

1.3 Use of Conditional Probability

Using the definition of conditional probability:

$$\Pr(A \cap B) = \Pr(A \mid B)\Pr(B)$$

$$\Pr(A \cap B^c) = \Pr(A \mid B^c)\Pr(B^c)$$

Hence,

$$\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\Pr(B^c)$$

1.4 Interpretation

The probability of A is a weighted average of:

$$\Pr(A \mid B) \quad \text{and} \quad \Pr(A \mid B^c)$$

with weights $\Pr(B)$ and $\Pr(B^c)$ respectively.

2 Example: Insurance Policyholder Problem

2.1 Problem Statement

An insurance company classifies policyholders as:

- Accident prone
- Not accident prone

Given:

$$\Pr(A \mid B) = 0.4, \quad \Pr(A \mid B^c) = 0.2, \quad \Pr(B) = 0.3$$

where:

- A: policyholder has an accident within one year
- B: policyholder is accident prone

2.2 Step-by-Step Solution

Using the law of total probability:

$$\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\Pr(B^c)$$

$$\Pr(A) = (0.4)(0.3) + (0.2)(0.7)$$

$$\Pr(A) = 0.12 + 0.14 = 0.26$$

2.3 Bayes' Theorem Application

Find $\Pr(B \mid A)$:

$$\Pr(B \mid A) = \frac{\Pr(A \mid B)\Pr(B)}{\Pr(A)}$$

$$\Pr(B \mid A) = \frac{(0.4)(0.3)}{0.26} = \frac{6}{13}$$

3 Law of Total Probability

3.1 Partition of the Sample Space

Let $B_1, B_2, \ldots, B_n$ be mutually exclusive and exhaustive events such that:

$$\Pr(B_i) > 0 \quad \forall i$$

3.2 Derivation

Since:

$$A = \bigcup_{i=1}^{n} (A \cap B_i)$$

and all intersections are mutually exclusive,

$$\Pr(A) = \sum_{i=1}^{n} \Pr(A \cap B_i)$$

Using conditional probability:

$$\Pr(A) = \sum_{i=1}^{n} \Pr(A \mid B_i) \Pr(B_i)$$

4 Bayes' Theorem

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i) \Pr(B_i)}{\sum_{j=1}^{n} \Pr(A \mid B_j) \Pr(B_j)}$$

- $\Pr(B_i)$: a priori probability
- $\Pr(B_i \mid A)$: a posteriori probability

5 Three-Card Problem

5.1 Card Definitions

- RR: card with two red sides
- BB: card with two black sides
- RB: card with one red and one black side

Let R be the event that the upper side is red.

5.2 Required Probability

Find $\Pr(RB \mid R)$.

5.3 Solution

$$\Pr(RB \mid R) = \frac{\Pr(R \mid RB) \Pr(RB)}{\Pr(R \mid RR) \Pr(RR) + \Pr(R \mid RB) \Pr(RB) + \Pr(R \mid BB) \Pr(BB)}$$

$$= \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1/6}{1/2} = \frac{1}{3}$$

6 Random Variables

A random variable is a real-valued function defined on the sample space.

6.1 Example

Let Y be the number of heads in three coin tosses.

$$Y \in \{0, 1, 2, 3\}$$

7 Probability Mass Function (PMF)

7.1 Definition

Let X be a discrete random variable with range:

$$R_X = \{x_1, x_2, x_3, \dots\}$$

The PMF is defined as:

$$p(x_i) = \Pr(X = x_i)$$

7.2 PMF Properties

1. $p(x_i) \geq 0$
2. $\displaystyle\sum_{i=1}^{\infty} p(x_i) = 1$

End of Lecture 5 Scribe