CSE 400: Fundamentals of Probability in Computing
Tutorial 1 – Complete Lecture Scribe

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Q1

Twenty distinct dishes are divided into four groups of five dishes each. The order within groups does not matter, and the order of the groups does not matter.

(a) Total number of ways

We divide 20 distinct objects into four unlabeled groups of size 5 each.

Since:

- All 20 dishes are distinct,
- Each group contains exactly 5 dishes,
- Order inside each group does not matter,
- Order of groups does not matter,

We apply the multinomial formula:

$$\text{Total ways} = \frac{20!}{(5!)^4 4!}$$

The denominator accounts for:

- 5! permutations inside each of the four groups,
- 4! permutations of the groups themselves.

(b) Probability all platters contain dishes of same cuisine

There are 5 dishes from each cuisine (Italian, Indian, Chinese, Mexican).

For each platter to contain dishes of only one cuisine:

- Each cuisine must form exactly one platter.

Since platters are unlabeled, this configuration is unique.

Thus,

$$P = \frac{1}{\frac{20!}{(5!)^4 4!}}$$

$$P = \frac{(5!)^4 4!}{20!}$$

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Q2

Arrival time is uniformly distributed over 30 minutes.

Uniform distribution argument:

$$P(A) = \frac{\text{favorable time}}{\text{total time}}$$

Total interval = 30 minutes.

(a) Waiting time < 5 minutes

Passenger must arrive within 5 minutes before bus:

Intervals:

$$7{:}10\text{–}7{:}15, \quad 7{:}25\text{–}7{:}30$$

Total favorable time:

$$5 + 5 = 10 \text{ minutes}$$

$$P(\text{wait} < 5) = \frac{10}{30} = \frac{1}{3}$$

(b) Waiting time > 10 minutes

Arrival more than 10 minutes before next bus:

$$7{:}00\text{–}7{:}05, \quad 7{:}15\text{–}7{:}20$$

Total favorable time = 10 minutes.

$$P(\text{wait} > 10) = \frac{10}{30} = \frac{1}{3}$$

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Q3

Let:

$$L = \text{arrives late}$$
$$E = \text{leaves early}$$

Given:

$$P(L) = 0.15$$
$$P(E) = 0.25$$
$$P(L \cap E) = 0.08$$

We need:

$$P(L^c \mid E^c)$$

Step 1: Compute complements

Complement Rule:

$$P(E^c) = 1 - P(E) = 1 - 0.25 = 0.75$$

Step 2: Use inclusion-exclusion

$$P(L \cup E) = P(L) + P(E) - P(L \cap E)$$

$$= 0.15 + 0.25 - 0.08 = 0.32$$

Step 3: Apply De Morgan's Law

$$(L \cup E)^c = L^c \cap E^c$$

Thus,

$$P(L^c \cap E^c) = 1 - P(L \cup E)$$

$$= 1 - 0.32 = 0.68$$

Step 4: Conditional Probability Formula

$$P(L^c \mid E^c) = \frac{P(L^c \cap E^c)}{P(E^c)}$$

$$= \frac{0.68}{0.75}$$

$$= 0.907$$

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Q4

Poisson Distribution is identified because:

- Errors occur per page,
- Events are rare,
- Fixed interval.

Parameter:

$$\lambda = 0.2$$

Poisson PMF:

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

(a) $P(X = 0)$

$$P(X = 0) = e^{-0.2} \frac{(0.2)^0}{0!}$$

$$= e^{-0.2}$$

$$= 0.8187$$

(b) $P(X \geq 2)$

Complement Rule:

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

Compute:

$$P(X = 1) = e^{-0.2} \frac{(0.2)^1}{1!}$$

$$= 0.2e^{-0.2}$$

$$= 0.1637$$

Thus,

$$P(X \geq 2) = 1 - 0.8187 - 0.1637$$

$$= 0.0176$$

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Q5

Poisson distribution with:

$$\lambda = 3.5$$

(a) $P(X \geq 2)$

Complement Rule:

$$P(X \geq 2) = 1 - P(0) - P(1)$$

$$P(0) = e^{-3.5} = 0.0302$$

$$P(1) = 3.5e^{-3.5} = 0.1057$$

$$P(X \geq 2) = 1 - 0.0302 - 0.1057$$

$$= 0.8641$$

(b) $P(X \leq 1)$

$$P(X \leq 1) = P(0) + P(1)$$

$$= 0.0302 + 0.1057 = 0.1359$$

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Q6

Poisson with:

$$\lambda = 3$$

(a) $P(X \geq 3)$

Complement Rule:

$$P(X \geq 3) = 1 - P(0) - P(1) - P(2)$$

$$P(0) = e^{-3} = 0.0498$$

$$P(1) = 3e^{-3} = 0.1494$$

$$P(2) = \frac{3^2}{2!}e^{-3} = \frac{9}{2}e^{-3} = 0.2240$$

$$P(X \geq 3) = 1 - 0.4232 = 0.5768$$

(b) Conditional Probability

$$P(X \geq 3 \mid X \geq 1) = \frac{P(X \geq 3)}{P(X \geq 1)}$$

$$P(X \geq 1) = 1 - P(0) = 1 - 0.0498 = 0.9502$$

$$= \frac{0.5768}{0.9502} = 0.607$$

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Q7

Given PDF:

$$f_X(x) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x+3)^2}{8}\right)$$

Compare with Gaussian form:

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

Identify:

$$m = -3$$
$$\sigma^2 = 4$$
$$\sigma = 2$$

Gaussian relations:

$$F_X(x) = \Phi\left(\frac{x-m}{\sigma}\right)$$

$$P(X > x) = Q\left(\frac{x-m}{\sigma}\right)$$

1) $P(X \leq 0)$

$$= \Phi\left(\frac{0-(-3)}{2}\right)$$

$$= \Phi\left(\frac{3}{2}\right)$$

$$= 1 - Q\left(\frac{3}{2}\right)$$

2) $P(X > 4)$

$$= Q\left(\frac{4+3}{2}\right) = Q\left(\frac{7}{2}\right)$$

3) $P(|X+3| < 2)$

$$-2 < X + 3 < 2$$

$$-5 < X < -1$$

$$= \Phi(1) - \Phi(-1)$$

Using:

$$\Phi(-x) = 1 - \Phi(x)$$

$$= 2\Phi(1) - 1$$

$$= 1 - 2Q(1)$$

4) $P(|X-2| > 1)$

$$X < 1 \quad \text{or} \quad X > 3$$

$$= F_X(1) + P(X > 3)$$

$$= 1 - Q(2) + Q(3)$$

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Q8

Binomial distribution identified:

$$X \sim \text{Binomial}(12, \theta)$$

If innocent:

$$\sum_{i=5}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

If guilty:

$$\sum_{i=8}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

By conditioning on guilt probability α:

$$\alpha \sum_{i=8}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i} + (1-\alpha) \sum_{i=5}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

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Q9

Given:

$$p(i) = c\frac{\lambda^i}{i!}$$

Normalization condition:

$$\sum_{i=0}^{\infty} p(i) = 1$$

$$c\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1$$

Using:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$ce^{\lambda} = 1$$

$$c = e^{-\lambda}$$

(a) $P(X = 0)$

$$= e^{-\lambda}$$

(b) $P(X > 2)$

Complement Rule:

$$1 - P(0) - P(1) - P(2)$$

$$= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2}{2} e^{-\lambda}$$

CDF step function property included exactly as given.

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Q10

Binomial distribution:

$$X \sim \text{Binomial}(n, p)$$

System effective if at least half function.

(a)

5-component effective:

$$\binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + p^5$$

3-component effective:

$$\binom{3}{2} p^2 (1-p) + p^3$$

Inequality reduces to:

$$3(p-1)^2(2p-1) > 0$$

$$p > \frac{1}{2}$$

(b)

Derived difference:

$$\binom{2k-1}{k} p^k (1-p)^k (2p-1)$$

Positive iff:

$$p > \frac{1}{2}$$

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