School of Engineering and Applied Science (SEAS), Ahmedabad University

CSE 400: Fundamentals of Probability in Computing

Lecture 9: Continuous Random Variables

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1. Introduction to Continuous Random Variables

A **continuous random variable** is a random variable that can take infinitely many values within a given interval. Unlike discrete random variables, the probability that a continuous random variable takes any exact value is zero.

Instead of probability mass functions (PMFs), continuous random variables are described using **probability density functions (PDFs)**.

A function $f_X(x)$ is a valid PDF if:

- $f_X(x) \geq 0$ for all x
- $\displaystyle\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

The probability that X lies in an interval $[a, b]$ is:

$$P(a \leq X \leq b) = \int_a^b f_X(x)\,dx$$

2. Uniform Random Variable

A continuous random variable X is said to have a **uniform distribution** over the interval $[a, b]$ if its PDF is:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

This distribution models situations where all values in an interval are equally likely.

The mean and variance of a uniform random variable are:

$$\mathbb{E}[X] = \frac{a+b}{2}, \qquad \mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

3. Exponential Random Variable

An **exponential random variable** is commonly used to model waiting times. Its PDF is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where $\lambda > 0$ is the rate parameter.

The exponential distribution has the **memoryless property**:

$$P(X > s + t \mid X > s) = P(X > t)$$

The mean and variance are:

$$\mathbb{E}[X] = \frac{1}{\lambda}, \qquad \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

4. Laplace Random Variable

The **Laplace distribution** is symmetric about its mean and is defined by:

$$f_X(x) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}}, \quad -\infty < x < \infty$$

where μ is the location parameter and $b > 0$ is the scale parameter.

The Laplace distribution is often used in modeling noise and error distributions.

Its mean and variance are:

$$\mathbb{E}[X] = \mu, \qquad \mathrm{Var}(X) = 2b^2$$

5. Gamma Random Variable

A **Gamma distribution** generalizes the exponential distribution. Its PDF is:

$$f_X(x) = \begin{cases} \dfrac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the rate parameter.

The Gamma function is defined as:

$$\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$$

The mean and variance are:

$$\mathbb{E}[X] = \frac{k}{\lambda}, \qquad \mathrm{Var}(X) = \frac{k}{\lambda^2}$$

6. Gaussian (Normal) Distribution

The **Gaussian distribution** is one of the most important distributions in probability and statistics. Its PDF is:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Here, μ is the mean and σ^2 is the variance.

The Gaussian distribution is symmetric about its mean and satisfies:

$$\mathbb{E}[X] = \mu, \qquad \mathrm{Var}(X) = \sigma^2$$

7. Gaussian Density Estimation

Gaussian density estimation is used to model unknown data distributions. Given data points $x_1, x_2, \ldots, x_n$, the parameters are estimated as:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2$$

This technique is widely used in machine learning, pattern recognition, and statistical inference.

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