

L5_A_S1.pdf
PDF

Generate a lecture scribe intended strictly as exam-oriented reference material for CSE400 – Fundamentals of Probability in Computing, Lecture 5. Use ONLY the provided context (Lecture 5 slides / PDF and relevant textbook content explicitly covered in this lecture). Do not introduce any new material, examples, interpretations, or intuition beyond what appears in the context. While writing the scribe, reason step by step through:

- Definitions and notation
- Assumptions and conditions
- Statements of results
- Proofs, derivations, and worked examples

Make the logical flow explicit exactly as presented in the lecture so that a student can reconstruct reasoning during exam revision. The scribe must include, where present in the context:

- Bayes' Theorem as a weighted average of conditional probabilities
- Learning-by-example problems and their step-by-step solutions
- Law of Total Probability and Bayes Formula, including assumptions
- Definitions and properties of random variables
- Discrete random variables and distributions
- Probability Mass Function (PMF) with properties
- Worked PMF examples exactly as shown

Maintain a formal academic tone suitable for a closed-notes exam. Do not invent content, do not simplify explanations, and do not add examples.

Give me Letex code based on this pdf Lec-5 slides, please add my Name = Chaudhari Ansh Nareshkumar and my emailId = ansh.c1@ahduni.edu.in.

Below is a **complete, exam-oriented LaTeX lecture scribe** for
CSE400 – Fundamentals of Probability in Computing, Lecture 5,

STRICTLY derived from the provided Lecture-5 slides/PDF only, with no added material, intuition, or examples.

The structure, notation, logical flow, derivations, and worked examples **follow the slides exactly**, so a student can reconstruct arguments in a **closed-notes exam**.

The content is based entirely on **Lecture-5 slides (pages 1–42)**  L5_A_S1 .

✓ How to use

- Copy everything below into a `.tex` file (e.g., `CSE400_Lecture5_Scribe.tex`)
- Compile with `pdflatex`

latex

```
\documentclass[12pt]{article}

\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb}
\usepackage{setspace}
\usepackage{hyperref}

\onehalfspacing

\begin{document}

\begin{center}
{\Large \textbf{CSE400 – Fundamentals of Probability in Computing}}\[4pt]
{\large \textbf{Lecture 5: Bayes’ Theorem, Random Variables, and Probability Mass}}

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\end{center}

\vspace{1em}
```

`\section{Bayes' Theorem}`

`\subsection{Weighted Average of Conditional Probabilities}`

Let A and B be events.

Event A can be expressed as:

`\[`

$$A = AB \cup AB^c$$

`\]`

because for an outcome to be in A , it must either be in both A and B , or in

The events AB and AB^c are mutually exclusive. Hence, by Axiom 3:

`\[`

$$\Pr(A) = \Pr(AB) + \Pr(AB^c)$$

`\]`

Using the definition of conditional probability:

`\[`

$$\Pr(AB) = \Pr(A \mid B)\Pr(B)$$

`\]`

`\[`

$$\Pr(AB^c) = \Pr(A \mid B^c)\Pr(B^c)$$

`\]`

Thus,

`\[`

$$\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)[1 - \Pr(B)]$$

`\]`

`\textbf{Conclusion:}`

The probability of event A is a **weighted average** of the conditional prob

`\subsection{Learning by Example: Example 3.1 (Part 1)}`

An insurance company classifies people as:

`\begin{itemize}`

`\item Accident prone`

`\item Not accident prone`

`\end{itemize}`

Given:

$$\begin{aligned} & \Pr(\text{Accident} \mid \text{Accident prone}) = 0.4 \\ & \Pr(\text{Accident} \mid \text{Not accident prone}) = 0.2 \\ & \Pr(\text{Accident prone}) = 0.3 \end{aligned}$$

Let:

$$\begin{aligned} A_1 &= \text{event that policyholder has an accident within a year} \\ A &= \text{event that policyholder is accident prone} \end{aligned}$$

Using conditioning:

$$\Pr(A_1) = \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^c)\Pr(A^c)$$

Substituting values:

$$\Pr(A_1) = (0.4)(0.3) + (0.2)(0.7) = 0.26$$

Learning by Example: Example 3.1 (Part 2)

Given that a policyholder has an accident, find the probability that they are acc

We compute:

$$\Pr(A \mid A_1) = \frac{\Pr(A \mid A_1)\Pr(A_1)}{\Pr(A_1)}$$

Using:

```
\[
\Pr(A \mid A_1) = \Pr(A)\Pr(A_1 \mid A)
\]
```

Thus,

```
\[
\Pr(A \mid A_1) = \frac{(\Pr(A)\Pr(A_1 \mid A))}{\Pr(A_1)} = \frac{6}{13}
\]
```

```
\section{Formal Introduction}
```

```
\subsection{Law of Total Probability}
```

Suppose B_1, B_2, \dots, B_n are mutually exclusive events such that:

```
\[
\bigcup_{i=1}^n B_i = B
\]
```

Since exactly one of the events must occur, we write:

```
\[
A = \bigcup_{i=1}^n AB_i
\]
```

Using mutual exclusivity:

```
\[
\Pr(A) = \sum_{i=1}^n \Pr(AB_i)
\]
```

Applying conditional probability:

```
\[
\Pr(A) = \sum_{i=1}^n \Pr(A \mid B_i)\Pr(B_i)
\]
```

This is known as the **Law of Total Probability**.

```
\subsection{Bayes Formula}
```

Using:

```
\[
\Pr(AB_i) = \Pr(B_i \mid A)\Pr(A)
\]
```

\]

We obtain:

\[

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^n \Pr(A \mid B_j)\Pr(B_j)}$$

\]

This is known as the **Bayes Formula** (Proposition 3.1).

Here:

\begin{itemize}

\item $\Pr(B_i)$ is the **a priori probability**

\item $\Pr(B_i \mid A)$ is the **posteriori probability**

\end{itemize}

\section{Bayes Formula: Example 3.2}

Three cards:

\begin{itemize}

\item RR (both sides red)

\item BB (both sides black)

\item RB (one red, one black)

\end{itemize}

Let:

\[

$R = \text{event that upturned side is red}$

\]

We want:

\[

$\Pr(RB \mid R)$

\]

Using Bayes formula:

\[

$$\Pr(RB \mid R) = \frac{\Pr(R \mid RB)\Pr(RB)}{\Pr(R)}$$

\]

Where:

```
\[
\Pr(R) = \Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)
\]
```

Substituting values:

```
\[
\Pr(RB \mid R) = \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1}{3}
\]
```

```
\section{Random Variables}
```

```
\subsection{Definition}
```

A **random variable** is a real-valued function defined on a sample space Ω

```
\[
X: \Omega \rightarrow \mathbb{R}
\]
```

It assigns a real number $X(\omega)$ to each outcome $\omega \in \Omega$.

In this lecture, we restrict attention to **discrete random variables**.

```
\subsection{Distribution of a Random Variable}
```

Let a be a value in the range of X .

The event:

```
\[
\{ \omega \in \Omega : X(\omega) = a \}
\]
```

is written as $X = a$.

The probability:

```
\[
\Pr(X = a)
\]
```

defines the distribution of X .

```
\subsection{Example: Tossing 3 Fair Coins}
```

Let Y be the number of heads.

```
\[
\Pr(Y=0)=\frac{1}{8}, \quad
\Pr(Y=1)=\frac{3}{8}, \quad
\Pr(Y=2)=\frac{3}{8}, \quad
\Pr(Y=3)=\frac{1}{8}
\]
```

Since Y must take one of these values:

```
\[
\sum_{i=0}^3 \Pr(Y=i) = 1
\]
```

```
\section{Probability Mass Function (PMF)}
```

```
\subsection{Definition}
```

A random variable that takes at most a countable number of values is called `\text`

Let X be a discrete random variable with range:

```
\[
R_X = \{x_1, x_2, x_3, \dots\}
\]
```

The function:

```
\[
p_X(x) = \Pr(X = x)
\]
```

is called the `\textbf{Probability Mass Function (PMF)}` of X .

Since X must take one of its values:

```
\[
\sum_x p_X(x) = 1
\]
```

```
\subsection{PMF Example: Two Independent Coin Tosses}
```

Let X be the number of heads.

```
\[
```



```

p_X(x) =
\begin{cases}
\frac{1}{4}, & x=0 \text{ or } x=2 \\
\frac{1}{2}, & x=1 \\
0, & \text{otherwise}
\end{cases}
\end{cases}
\]

```

```

\subsection{PMF Example}

```

Given:

```

\[
p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots
\]

```

Since:

```

\[
\sum_{i=0}^{\infty} p(i) = 1
\]

```

We obtain:

```

\[
c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1
\]

```

Using:

```

\[
e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}
\]

```

Thus:

```

\[
c = e^{-\lambda}
\]

```

Hence:

```

\[
\Pr(X=0) = e^{-\lambda}
\]
\]

```

```
\Pr(X>2) = 1 - \sum_{i=0}^2 e^{-\lambda}\frac{\lambda^i}{i!}
\]

\section*{End of Lecture 5}

\end{document}
```

If you want, I can also:

- Convert this into a **ready-to-submit PDF**
- Add **exam-margin notes**
- Format it exactly like **previous CSE400 scribes** you've submitted



Sources

