Scribe Questions 4-6: System Modelling and Analysis

Group 3

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Scribe Question 4: Model-Implementation Alignment

The theoretical model developed in Milestone 2 assumes an M/M/1 queueing system for packet delay analysis [1]. The alignment between the mathematical model and its implementation is described below.

1. Arrival Process

We assume packet arrivals follow a Poisson process with rate λ. Therefore:

$$N(t) \sim Poisson(\lambda t)$$

and inter-arrival times are exponentially distributed:

$$A \sim Exponential(\lambda), \quad E[A] = \frac{1}{\lambda}$$

In implementation, arrivals are generated using exponential random sampling consistent with this assumption.

2. Service Process

Service times are modeled as:

$$S \sim Exponential(\mu), \quad E[S] = \frac{1}{\mu}$$

The implementation generates service durations using exponential sampling with parameter μ.

3. Stability Condition

Traffic intensity is defined as:

$$\rho = \frac{\lambda}{\mu}$$

The system is stable only when:

$$\lambda < \mu$$

This condition is enforced in implementation to prevent unbounded queue growth.

4. Performance Metrics

The theoretical expectations:

$$E[N] = \frac{\rho}{1 - \rho}$$

$$E[D] = \frac{1}{\mu - \lambda}$$

$$E[N] = \lambda E[D] \quad (Little's Law)$$

Simulation outputs are validated against these closed-form expressions to ensure correctness of implementation.

5. Tail Probability

Exact tail probability of delay:

$$P(D > t) = e^{-(\mu-\lambda)t}$$

When exact distribution is unknown, Markov's inequality [2] is used:

$$P(D \geq t) \leq \frac{E[D]}{t}$$

Thus, the implementation aligns directly with the theoretical probabilistic model.

Scribe Question 5: Cross-Milestone Consistency and Change

The project evolved across milestones while maintaining theoretical consistency.

1. From Deterministic to Probabilistic Modeling

Earlier milestone discussions focused on defining system components. Milestone 2 introduced random variables:

$$A = Inter - arrivaltime$$

$$S = Servicetime$$

$$D = Totaldelay$$

$$N = Numberinsystem$$

This ensured mathematical rigor while keeping the system interpretation unchanged.

2. Introduction of Concentration Bounds

To handle uncertainty in delay analysis and to study worst-case behavior, we introduced probabilistic bounds beyond the exact exponential formula.

Markov Inequality

For any non-negative random variable X,

$$P(X \geq t) \leq \frac{E[X]}{t}$$

Applying to packet delay D:

$$P(D \geq t) \leq \frac{E[D]}{t}$$

Interpretation:

- Requires only the mean $E[D]$.
- Does not require knowledge of the full distribution.
- Decays linearly as $1/t$.
- Provides a worst-case (loose) bound.

Example: If $E[D] = 1$ and $t = 3$,

$$P(D \geq 3) \leq \frac{1}{3} \approx 0.33$$

Exact value (from exponential model):

$$P(D > 3) = e^{-3} \approx 0.05$$

Thus, Markov inequality is safe but pessimistic.

Chernoff Bound

For any non-negative random variable X and any $s > 0$,

$$P(X \geq t) \leq \inf_{s>0} \frac{E[e^{sX}]}{e^{st}}$$

Applying to Packet Delay:
In the M/M/1 system,

$$D \sim Exponential(\mu - \lambda)$$

Moment generating function (MGF):

$$M_D(s) = E[e^{sD}] = \frac{\mu - \lambda}{\mu - \lambda - s}, \quad 0 < s < \mu - \lambda$$

Using Chernoff inequality:

$$P(D \geq t) \leq \inf_{0<s<\mu-\lambda} \frac{\mu - \lambda}{\mu - \lambda - s} e^{-st}$$

Optimizing over s gives the tight bound:

$$P(D \geq t) \leq e^{-(\mu-\lambda)t}$$

Interpretation:

- Produces exponentially decaying bounds.
- Much tighter than Markov inequality.
- Matches the exact exponential tail of delay.
- Directly connects reliability to $(\mu - \lambda)$ (spare capacity).

3. Consistency with Little's Law

Across milestones, Little's Law remained valid:

$$L = \lambda E[D]$$

All derived numerical examples (e.g., $\lambda = 2$, $\mu = 3$) satisfy:

$$\rho = \frac{2}{3}, \quad E[N] = 2$$

This confirms cross-milestone logical consistency.

4. Key Change

The major shift was from:

- Exact distribution-based delay formulas

to

- Worst-case probabilistic bounds using concentration inequalities.

However, the underlying system assumptions remained unchanged.

Scribe Question 6: Open Issues and Responsibility Attribution

1. Open Issues

Despite strong theoretical modeling, several open issues remain:

- The M/M/1 assumption may not reflect real-world bursty traffic.
- Exponential service time assumption may not match router hardware behavior.
- Markov's inequality provides loose bounds.
- Chebyshev's inequality requires variance estimation.
- Real-world networks may involve multiple routers (M/M/1 may be insufficient).

2. Model Limitations

The analysis assumes:

- Independence of arrivals.
- Memoryless property.
- Single server queue.

These assumptions simplify analysis but reduce realism.

3. Responsibility Attribution

Team responsibilities can be categorized as:

- **Modeling Responsibility:** Defining λ, μ, and ensuring $\lambda < \mu$.

- **Mathematical Validation:** Verifying formulas such as:

$$E[D] = \frac{1}{\mu - \lambda}$$

- **Implementation Validation:** Matching simulation outputs with theoretical expectations.
- **Risk Analysis:** Applying concentration inequalities for worst-case delay bounds.

4. Future Work

- Extend to M/M/c or M/G/1 models.
- Empirical validation using real network traces.
- Tighter tail bounds using Chernoff inequalities.

References

[1] Milestone 2: Probabilistic Modelling and Performance Analysis of Packet Delay in a Single Router Network.

[2] Alex Tsun, *Probability & Statistics with Applications to Computing*, Chapter 6: Concentration Inequalities.