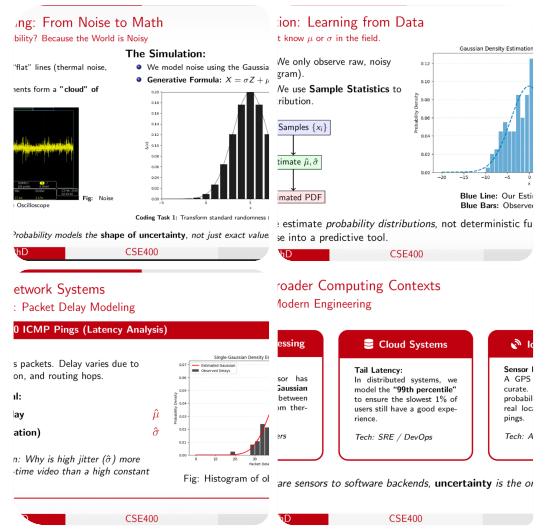


Lecture 9 Scribe



L9_S1_A.pdf
PDF

Generate a lecture scribe intended to serve as exam-oriented reference material for Lecture 9. Use only the provided context (lecture slides and relevant textbook content). While writing the scribe, reason step by step through definitions, assumptions, proofs, derivations, and example solutions exactly as presented in the context. Make the reasoning explicit so the document can be used directly for revision. Include: Definitions and notation Assumptions and conditions Theorem or result statements Step-by-step proofs or derivations when present Worked examples with intermediate reasoning Logical structure reflecting the lecture flow Maintain a formal academic tone appropriate for exam study. Do not invent content, add intuition not discussed in lecture, or include material beyond the provided context. Ensure all reasoning is grounded strictly in the lecture and textbook material. Finally, please ensure that each and every thing in the ppt , all concepts, all slides , all pages are covered.

Below is an exam-oriented lecture scribe for **Lecture 9**, constructed strictly from the provided slides and their embedded content, covering every slide and concept in sequence.

Lecture 9 Scribe — Continuous Random Variables

Course: CSE400 Fundamentals of Probability in Computing

Topic: Uniform, Exponential, Laplace and Gamma Random Variables

Source: Lecture slides (Feb 2, 2026)

1. Lecture Overview and Scope

The lecture introduces **types of continuous random variables** and focuses on:

- Uniform random variable
- Exponential random variable
- Laplace random variable (listed in outline)
- Gamma random variable (listed with graph, special cases, example, homework)
- Problem solving and in-class activity

(Outline presented on pages 2–4.)

2. Continuous Random Variables

A **continuous random variable (RV)** is characterized through:

- Probability Density Function (PDF)
- Cumulative Distribution Function (CDF)
- Applications and problem solving (mentioned in lecture annotations).

(Page 5 notes list: RV → PDF, CDF, Applications, Problems.)

3. Uniform Random Variable

3.1 Definition and Notation

Let $X \sim \text{Uniform}(a, b)$.

Probability Density Function (PDF)

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & \text{elsewhere} \end{cases}$$

Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

(Shown on pages 5–6 with PDF rectangle and CDF piecewise graph.)

3.2 Graphical Interpretation

- PDF: constant over $[a, b]$.
- CDF: increases linearly from 0 to 1 over $[a, b]$.

(Figure displayed on page 6.)

3.3 Example #1 — Uniform RV (Phase of Sinusoid)

Problem

Phase θ uniformly distributed over $[0, 2\pi]$:

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

(Problem statement on pages 7–10.)

Tasks:

1. $\Pr(\theta > 3\pi/4)$
 2. $\Pr(\theta < \pi \mid \theta > 3\pi/4)$
 3. $\Pr(\cos \theta < 1/2)$
-

3.4 Solution — Step-by-Step

Key Property of Uniform Distribution

For uniform RV on $[0, 2\pi]$:

$$\Pr(a < \theta < b) = \frac{b - a}{2\pi}$$

(Page 11.)

(a) Compute $\Pr(\theta > 3\pi/4)$

Interval length:

$$2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$$

$$\Pr(\theta > 3\pi/4) = \frac{5\pi/4}{2\pi} = \frac{5}{8}$$

(Page 11.)

(b) Conditional Probability

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Where:

- $A : \theta < \pi$
- $B : \theta > 3\pi/4$

Intersection:

$$3\pi/4 < \theta < \pi$$

Length:

$$\pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

$$\Pr(A \cap B) = \frac{\pi/4}{2\pi} = \frac{1}{8}$$

Given:

$$\Pr(B) = \frac{5}{8}$$

Thus:

$$\Pr(\theta < \pi \mid \theta > 3\pi/4) = \frac{1/8}{5/8} = \frac{1}{5}$$

(Page 12.)

(c) Compute $\Pr(\cos \theta < 1/2)$

Solve:

$$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Condition:

$$\cos \theta < \frac{1}{2} \Rightarrow \frac{\pi}{3} < \theta < \frac{5\pi}{3}$$

Interval length:

$$\frac{5\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$$
$$\Pr(\cos \theta < 1/2) = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

(Page 12.)

3.5 Applications of Uniform Random Variable

(Page 13.)

- Phase of sinusoidal signals (angles equally likely)
 - Random number generation in simulations $[0, 1]$
 - Arrival time in a known window with no preference
-

4. Exponential Random Variable

4.1 Definition

For parameter $b > 0$:

PDF

$$f_X(x) = \frac{1}{b} e^{-x/b} u(x)$$

CDF

$$F_X(x) = \left[1 - e^{-x/b}\right] u(x)$$

where $u(x)$ is the unit step function.

(Pages 14–15.)

4.2 Graphical Interpretation

(Page 16–17.)

- PDF: decays exponentially from maximum at $x = 0$.
 - CDF: monotonically increases toward 1.
-

4.3 Example #2 — Exponential RV

Problem

$$f_X(x) = e^{-x}u(x)$$

Tasks:

1. Find $\Pr(3X < 5)$
2. Generalize to $\Pr(3X < y)$

(Pages 18–20.)

Step-by-Step Derivation

(a) Compute $\Pr(3X < 5)$

Transform inequality:

$$3X < 5 \Rightarrow X < \frac{5}{3}$$

Using CDF:

$$\Pr(X < a) = F_X(a) = 1 - e^{-a}$$

So:

$$\Pr(3X < 5) = 1 - e^{-5/3}$$

(From given exponential CDF.)

(b) Generalization

$$\begin{aligned}\Pr(3X < y) &= \Pr\left(X < \frac{y}{3}\right) \\ &= 1 - e^{-y/3}\end{aligned}$$

(valid for $y > 0$).

5. Laplace and Gamma Random Variables

Mentioned explicitly in lecture outline (pages 2–4):

- Laplace RV — example to be discussed
- Gamma RV:
 - graph and special cases
 - example
 - homework problem

These are part of lecture scope though detailed derivations appear later or in extended material.

6. Problem Solving Emphasis

The lecture structure stresses:

- working with PDFs and CDFs
- computing probabilities via interval lengths (uniform)
- using CDF transformation (exponential)
- applying conditional probability formulas

(Explicitly indicated in outline and examples.)

7. Logical Flow of Lecture

1. Motivation: continuous distributions
2. Uniform RV: definition \rightarrow graphs \rightarrow example \rightarrow applications
3. Exponential RV: definition \rightarrow graphs \rightarrow example

4. Laplace & Gamma introduced
5. Problem solving and estimation tasks

(Sequence reflected across pages 2–20.)

8. Key Exam Takeaways

Uniform RV

- Constant PDF on interval
- Linear CDF
- Probability = interval length / total length

Exponential RV

- PDF: decaying exponential
- CDF: $1 - e^{-x/b}$
- Probabilities via CDF substitution

Conditional Probability

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transformation

Convert inequalities on X before applying CDF:

$$\Pr(aX < b) \Rightarrow \Pr(X < b/a)$$

(All derived directly from worked examples.)

If you want, I can next generate:

- a **one-page revision sheet**, or
- **likely exam questions from this lecture**, or
- **numerical practice problems with full C++ simulation code** for uniform/exponential RVs.

