CSE400 – Fundamentals of Probability in Computing
Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

Dhaval Patel, PhD

February 2, 2026

Lecture Outline

- Types of Continuous Random Variables
  - Uniform Random Variable: Example
  - Exponential Random Variable: Example
  - Laplace Random Variable: Example
  - Gamma Random Variable
    - Graph and Special Cases
    - Example
    - Homework Problem
- Problem Solving

1 Types of Continuous Random Variables

1.1 Uniform Random Variable

Let X be a continuous random variable uniformly distributed on the interval $[a, b]$.

Probability Density Function (PDF)

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x < b \\ 0, & \text{elsewhere} \end{cases}$$

Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x < b \\ 1, & x \ge b \end{cases}$$

Graphical Representation

- PDF is constant with height $\frac{1}{b-a}$ over $[a, b]$
- CDF increases linearly from 0 to 1 over $[a, b]$

2 Example 1: Uniform Random Variable

Problem Statement

The phase of a sinusoid, Θ, is uniformly distributed over $[0, 2\pi)$. The PDF is given by:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $\Pr(\Theta > \frac{3\pi}{4})$

For a uniform random variable on $[0, 2\pi)$:

$$\Pr(a < \Theta < b) = \frac{b-a}{2\pi}$$

$$\Pr\left(\Theta > \frac{3\pi}{4}\right) = \frac{2\pi - \frac{3\pi}{4}}{2\pi} = \frac{5}{8}$$

(b) Find $\Pr(\Theta < \pi \mid \Theta > \frac{3\pi}{4})$

Using conditional probability:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr\left(\frac{3\pi}{4} < \Theta < \pi\right) = \frac{\pi - \frac{3\pi}{4}}{2\pi} = \frac{1}{8}$$

$$\Pr(B) = \Pr\left(\Theta > \frac{3\pi}{4}\right) = \frac{5}{8}$$

$$\Pr\left(\Theta < \pi \mid \Theta > \frac{3\pi}{4}\right) = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5}$$

(c) Find $\Pr(\cos\Theta < \frac{1}{2})$

$$\cos\Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos\Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}$$

$$\Pr(\cos\Theta < \tfrac{1}{2}) = \frac{\frac{5\pi}{3} - \frac{\pi}{3}}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

3 Uniform Random Variable: Application Examples

- Phase of a sinusoidal signal when all phase angles between 0 and 2π are equally likely
- A random number generated by a computer between 0 and 1 for simulations
- Arrival time of a user within a known time window assuming no time preference

4 Exponential Random Variable

Definition

The exponential random variable has PDF and CDF given by (for any $b > 0$):

Probability Density Function

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x)$$

Cumulative Distribution Function

$$F_X(x) = \left[1 - \exp\left(-\frac{x}{b}\right)\right] u(x)$$

Graphical Representation

- PDF decreases exponentially
- CDF increases asymptotically to 1
- Example plots shown for $b = 2$

5 Example 2: Exponential Random Variable

Problem Statement

Let X be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x)$$

(a) Find $\Pr(3X < 5)$

$$\Pr(3X < 5) = \Pr\left(X < \frac{5}{3}\right)$$

Using the CDF:

$$F_X(x) = 1 - e^{-x}$$

$$\Pr\left(X < \frac{5}{3}\right) = 1 - e^{-5/3}$$

(b) Generalize to find $\Pr(3X < y)$

$$\Pr(3X < y) = \Pr\left(X < \frac{y}{3}\right)$$

$$= 1 - e^{-y/3}$$