

Exam-oriented Lecture Scribe

Tutorial_1_Solution.pdf

PDF

Generate a lecture scribe suitable for exam preparation using only the provided solution pdf. While writing the scribe, reason step by step through all definitions, notation, assumptions, conditions, theorem statements, proofs, and worked examples exactly as they appear in the provided context. Explicitly include this step-by-step reasoning and logical flow in the final output to ensure maximum clarity for revision. Do not write the whole solution, just the required important formulae and theories. Do not introduce any external intuition, creative explanations, or examples that are not found in the provided materials.

Here is the **exam-oriented lecture scribe** constructed strictly from the provided solution PDF. Only key formulae, definitions, conditions, and derived results are included, with the logical flow preserved exactly as in the solutions.

CSE 400: Fundamentals of Probability in Computing

Tutorial-1: Important Formulae and Theories

Q1. Division into Unlabeled Groups (Multinomial Counting)

(a) Total number of ways

Given:

- 20 distinct dishes
- Divided into 4 groups
- Each group contains 5 dishes
- Order within groups does not matter
- Order of groups does not matter

Using the multinomial formula:

$$\text{Total ways} = \frac{20!}{(5!)^4 4!}$$

(b) Probability all platters contain same cuisine

Since:

- Each cuisine has exactly 5 dishes

- Only one arrangement satisfies condition
- Platters are unlabeled

$$P = \frac{1}{\frac{20!}{(5!)^4 4!}} = \frac{(5!)^4 4!}{20!}$$

Q2. Uniform Distribution (Waiting Time)

Arrival uniformly distributed over 30 minutes.

$$P = \frac{\text{Favorable time}}{30}$$

(a) Waiting < 5 minutes

Total favorable time = 10 minutes

$$P(\text{wait} < 5) = \frac{10}{30} = \frac{1}{3}$$

(b) Waiting > 10 minutes

Total favorable time = 10 minutes

$$P(\text{wait} > 10) = \frac{10}{30} = \frac{1}{3}$$

Q3. Conditional Probability & De Morgan's Law

Let:

- L : arrives late
- E : leaves early

Given:

$$P(L) = 0.15, \quad P(E) = 0.25, \quad P(L \cap E) = 0.08$$

Compute:

$$P(E^c) = 1 - P(E) = 0.75$$

$$P(L \cup E) = P(L) + P(E) - P(L \cap E) = 0.32$$

Using De Morgan's law:

$$P(L^c \cap E^c) = 1 - P(L \cup E) = 0.68$$

Conditional probability:

$$P(L^c|E^c) = \frac{P(L^c \cap E^c)}{P(E^c)} = \frac{0.68}{0.75} \approx 0.907$$

Q4. Poisson Distribution ($\lambda = 0.2$)

Poisson PMF:

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

(a) $P(X = 0)$

$$P(X = 0) = e^{-0.2} = 0.8187$$

(b) $P(X \geq 2)$

Using complement:

$$P(X \geq 2) = 1 - P(0) - P(1)$$

$$P(1) = 0.2e^{-0.2} = 0.1637$$

$$P(X \geq 2) = 0.0176 \approx 0.0175$$

Q5. Poisson Distribution ($\lambda = 3.5$)

$$X \sim \text{Poisson}(3.5)$$

$$P(X = k) = \frac{e^{-3.5} 3.5^k}{k!}$$

(a) $P(X \geq 2)$

$$P(X \geq 2) = 1 - P(0) - P(1)$$

$$P(0) = 0.0302$$

$$P(1) = 0.1057$$

$$P(X \geq 2) = 0.8641$$

(b) $P(X \leq 1)$

$$P(X \leq 1) = P(0) + P(1) = 0.1359$$

Q6. Poisson Distribution ($\lambda = 3$)

$$X \sim \text{Poisson}(3)$$

(a) $P(X \geq 3)$

$$P(X \geq 3) = 1 - P(0) - P(1) - P(2)$$

$$P(0) = 0.0498$$

$$P(1) = 0.1494$$

$$P(2) = 0.2240$$

$$P(X \geq 3) = 0.5768$$

(b) Conditional Probability

$$P(X \geq 3 | X \geq 1) = \frac{P(X \geq 3)}{P(X \geq 1)}$$

$$P(X \geq 1) = 1 - P(0) = 0.9502$$

$$P(X \geq 3 | X \geq 1) \approx 0.607$$

Q7. Gaussian Distribution & Q-Function

Given PDF:

$$f_X(x) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x+3)^2}{8}\right)$$

Comparing with:

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

Identify:

$$m = -3, \quad \sigma = 2$$

Gaussian relations:

$$F_X(x) = \Phi\left(\frac{x-m}{\sigma}\right)$$

$$P(X > x) = Q\left(\frac{x-m}{\sigma}\right)$$

Results:

$$P(X \leq 0) = 1 - Q\left(\frac{3}{2}\right)$$

$$P(X > 4) = Q\left(\frac{7}{2}\right)$$

$$P(|X + 3| < 2) = 1 - 2Q(1)$$

$$P(|X - 2| > 1) = 1 - Q(2) + Q(3)$$

Q8. Binomial Model (Jury Problem)

Correct decision probability:

If defendant guilty:

$$\sum_{i=8}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

If innocent:

$$\sum_{i=5}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

If $\alpha = P(\text{guilty})$:

$$\alpha \sum_{i=8}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i} + (1-\alpha) \sum_{i=5}^{12} \binom{12}{i} \theta^i (1-\theta)^{12-i}$$

Q9. Poisson PMF from Normalization

Given:

$$p(i) = \frac{c\lambda^i}{i!}$$

Since:

$$\sum_{i=0}^{\infty} p(i) = 1$$

Using:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

Thus:

$$ce^\lambda = 1$$

$$c = e^{-\lambda}$$

Hence:

$$P(X = 0) = e^{-\lambda}$$

$$P(X > 2) = 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2 e^{-\lambda}}{2}$$

CDF of Discrete Random Variable

$$F(a) = \sum_{x \leq a} p(x)$$

For discrete ordered values $x_1 < x_2 < \dots$:

- F is a step function
 - Jump at x_i equals $p(x_i)$
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Q10. Binomial Reliability Model

Let number of functioning components be Binomial (n, p) .

(a) 5-component vs 3-component

5-component effective probability:

$$10p^3(1-p)^2 + 5p^4(1-p) + p^5$$

3-component effective probability:

$$3p^2(1-p) + p^3$$

Inequality reduces to:

$$3(p-1)^2(2p-1) > 0$$

$$p > \frac{1}{2}$$

(b) General Case

Difference:

$$P_{2k+1} - P_{2k-1} = \binom{2k-1}{k} p^k (1-p)^k (2p-1)$$

Since binomial coefficient and powers are positive:

$$> 0 \iff p > \frac{1}{2}$$

End of Exam-Revision Scribe

Only essential formulae, distributions, identities, and final derived results have been included exactly as in the provided material.

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