Lecture 9 Scribe — Continuous Random Variables

Course: CSE400 Fundamentals of Probability in Computing

Topic: Uniform, Exponential, Laplace and Gamma Random Variables
Source: Lecture slides (Feb 2, 2026)

1 Lecture Overview and Scope

The lecture introduces types of continuous random variables and focuses on:

- Uniform random variable
- Exponential random variable
- Laplace random variable (listed in outline)
- Gamma random variable (listed with graph, special cases, example, homework)
- Problem solving and in-class activity

2 Continuous Random Variables

A continuous random variable (RV) is characterized through:

- Probability Density Function (PDF)
- Cumulative Distribution Function (CDF)
- Applications and problem solving (mentioned in lecture annotations)

3 Uniform Random Variable

3.1 Definition and Notation

Let $X \sim \text{Uniform}(a, b)$.

Probability Density Function (PDF)

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x < b \\ 0, & \text{elsewhere} \end{cases}$$

Cumulative Distribution Function (CDF)

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x < b \\ 1, & x \ge b \end{cases}$$

3.2 Graphical Interpretation

PDF: constant over $[a, b]$.

CDF: increases linearly from 0 to 1 over $[a, b]$.

3.3 Example #1 — Uniform RV (Phase of Sinusoid)

Problem

Phase uniformly distributed over $\theta \in [0, 2\pi]$.

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

Tasks:

1. $Pr(\theta > 3\pi/4)$
2. $Pr(\theta < \pi \mid \theta > 3\pi/4)$
3. $Pr(\cos\theta < 1/2)$

3.4 Solution — Step-by-Step

Key Property of Uniform Distribution

For uniform RV on $[a, b]$:

$$Pr(a < \theta < b) = \frac{b-a}{2\pi}$$

(a) Compute $Pr(\theta > 3\pi/4)$

Interval length:

$$2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$$

$$Pr(\theta > 3\pi/4) = \frac{5\pi/4}{2\pi} = \frac{5}{8}$$

(b) Conditional Probability

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Where:

$$A : \theta < \pi, \quad B : \theta > 3\pi/4$$

Intersection:

$$3\pi/4 < \theta < \pi$$

Length:

$$\pi - \frac{3\pi}{4} = \frac{\pi}{4}$$

$$Pr(A \cap B) = \frac{\pi/4}{2\pi} = \frac{1}{8}$$

$$Pr(B) = \frac{5}{8}$$

$$Pr(\theta < \pi \mid \theta > 3\pi/4) = \frac{1/8}{5/8} = \frac{1}{5}$$

(c) Compute $Pr(\cos\theta < 1/2)$
Solve:

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Condition:

$$\frac{\pi}{3} < \theta < \frac{5\pi}{3}$$

Interval length:

$$\frac{5\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$$

$$Pr(\cos\theta < 1/2) = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

3.5 Applications of Uniform Random Variable

- Phase of sinusoidal signals (angles equally likely)
- Random number generation in simulations
- Arrival time in a known window with no preference

4 Exponential Random Variable

4.1 Definition

For parameter $b > 0$:
PDF

$$f_X(x) = \frac{1}{b} e^{-x/b} u(x)$$

CDF

$$F_X(x) = [1 - e^{-x/b}]u(x)$$

where $u(x)$ is the unit step function.

4.2 Graphical Interpretation

PDF: decays exponentially from maximum at $x = 0$.
CDF: monotonically increases toward 1.

4.3 Example #2 — Exponential RV

Problem
Tasks:

1. Find $Pr(3X < 5)$
2. Generalize to $Pr(3X < y)$

Step-by-Step Derivation
(a) Compute $Pr(3X < 5)$
Transform inequality:

$$3X < 5 \Rightarrow X < \frac{5}{3}$$

Using CDF:

$$Pr(X < a) = F_X(a) = 1 - e^{-a}$$

So:

$$Pr(3X < 5) = 1 - e^{-5/3}$$

(b) Generalization
(valid for $y > 0$)

$$Pr(3X < y) = Pr\left(X < \frac{y}{3}\right) = 1 - e^{-y/3}$$

5 Laplace and Gamma Random Variables

Mentioned explicitly in lecture outline:

- Laplace RV — example to be discussed
- Gamma RV:
  - graph and special cases
  - example
  - homework problem

These are part of lecture scope though detailed derivations appear later or in extended material.

6 Problem Solving Emphasis

The lecture structure stresses:

- working with PDFs and CDFs
- computing probabilities via interval lengths (uniform)
- using CDF transformation (exponential)
- applying conditional probability formulas

7 Logical Flow of Lecture

1. Motivation: continuous distributions
2. Uniform RV: definition → graphs → example → applications
3. Exponential RV: definition → graphs → example
4. Laplace & Gamma introduced
5. Problem solving and estimation tasks

8 Key Exam Takeaways

Uniform RV

- Constant PDF on interval
- Linear CDF
- Probability = interval length / total length

Exponential RV

- PDF: decaying exponential
- CDF: $1 - e^{-x/b}$
- Probabilities via CDF substitution

Conditional Probability Transformation

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Convert inequalities before applying CDF:

$$Pr(aX < b) \Rightarrow Pr\left(X < \frac{b}{a}\right)$$