Lecture Scribe Exam Prep

L9_S1_A.pdf
PDF

Generate a lecture scribe suitable for exam preparation using only the provided lecture slides. While writing the scribe, reason step by step through all definitions, notation, assumptions, conditions, theorem statements, proofs, and worked examples exactly as they appear in the provided context. Explicitly include this step-by-step reasoning and logical flow in the final output to ensure maximum clarity for revision. Do not introduce any external intuition, creative explanations, or examples that are not found in the provided materials.

Below is a **lecture scribe for exam preparation**, written strictly from the provided lecture slides. The logical development, definitions, notation, assumptions, results, and worked steps follow the same order and structure as presented in the slides. No external intuition, additional examples, or explanations beyond the provided material are introduced.

CSE400 — Fundamentals of Probability in Computing

Lecture 9: Uniform, Exponential, Laplace and Gamma Random Variables

(Prepared as an exam-oriented lecture scribe from the provided lecture slides)

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1. Lecture Outline

According to the outline slides, the lecture covers:

- Types of Continuous Random Variables
  - Uniform Random Variable — Example
  - Exponential Random Variable — Example
  - Laplace Random Variable — Example

    - Gamma Random Variable
        - Graph and Special Cases
        - Example
        - Homework Problem
- Problem Solving
- In-class Activity: Gaussian Density Estimation

Only the material appearing in the provided slides is included in this scribe.

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2. Types of Continuous Random Variables

2.1 Uniform Random Variable

Definition (PDF)
A continuous random variable X is uniformly distributed over the interval $[a, b]$ if its probability density function (PDF) is given by

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \leqslant x < b, \\ 0, & \text{elsewhere.} \end{cases}$$

This definition specifies:

- The density is constant over the interval $[a, b]$.
- The density is zero outside the interval.

Definition (CDF)
The cumulative distribution function (CDF) is given by

$$F_X(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \leqslant x < b, \\ 1, & x \geqslant b. \end{cases}$$

The slides show the corresponding PDF as a rectangle over $[a, b]$ and the CDF as a linear increase between a and b (Figure 3.8).

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3. Example #1 — Uniform Random Variable

Problem Statement

The phase of a sinusoid, θ, is uniformly distributed over $[0, 2\pi)$. The PDF is

$$f_\theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leqslant \theta < 2\pi, \\ 0, & \text{otherwise.} \end{cases}$$

Tasks:

(a) Find $\Pr(\theta > 3\pi/4)$

(b) Find $\Pr(\theta < \pi \mid \theta > 3\pi/4)$

(c) Find $\Pr(\cos\theta < 1/2)$

Given Property for Uniform RV

For a uniform random variable on $[a, b]$,

$$\Pr(a < \theta < b) = \frac{b - a}{2\pi}.$$

This relation is used directly in the solution.

(a) Compute $\Pr(\theta > 3\pi/4)$

Step-by-step as shown in the slides:

1. The interval of interest is from $3\pi/4$ to 2π.
2. Length of interval:

$$2\pi - \frac{3\pi}{4}$$

3. Probability equals interval length divided by total length:

$$\Pr(\theta > 3\pi/4) = \frac{2\pi - 3\pi/4}{2\pi} = \frac{5}{8}.$$

(b) Compute $\Pr(\theta < \pi \mid \theta > 3\pi/4)$

The slides apply the conditional probability formula:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Steps:

1. Event B: $\theta > 3\pi/4$

$$\Pr(B) = \frac{5}{8}.$$

2. Event $A \cap B$: $3\pi/4 < \theta < \pi$

$$\Pr(3\pi/4 < \theta < \pi) = \frac{\pi - 3\pi/4}{2\pi} = \frac{1}{8}.$$

3. Therefore,

$$\Pr(\theta < \pi \mid \theta > 3\pi/4) = \frac{1/8}{5/8} = \frac{1}{5}.$$

(c) Compute $\Pr(\cos\theta < 1/2)$

Steps shown in the slides:

1. Solve

$$\cos\theta = \frac{1}{2}.$$

2. This occurs at

$$\theta = \frac{\pi}{3}, \quad \frac{5\pi}{3}.$$

3. From the diagram in the solution,

$$\cos\theta < \frac{1}{2}$$

for

$$\frac{\pi}{3} < \theta < \frac{5\pi}{3}.$$

4. Probability:

$$\Pr(\cos\theta < 1/2) = \frac{5\pi/3 - \pi/3}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}.$$

Applications of Uniform Random Variable

As listed in the slides:

- Phase of a sinusoidal signal when all phase angles between 0 and 2π are equally likely.
- Random number generated by a computer between 0 and 1 for simulations.
- Arrival time of a user within a known time window assuming no time preference.

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4. Exponential Random Variable

Definition

The exponential random variable has the following PDF and CDF (for any $b > 0$):

PDF

$$f_X(x) = \frac{1}{b}\exp\left(-\frac{x}{b}\right)u(x)$$

CDF

$$F_X(x) = \left[1 - \exp\left(-\frac{x}{b}\right)\right]u(x)$$

where $u(x)$ is the unit step function.

The slides also include plots showing:

- A decreasing exponential PDF.
- A monotonically increasing CDF approaching 1.

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5. Example #2 — Exponential Random Variable

Problem Statement

Let X be an exponential random variable with PDF

$$f_X(x) = e^{-x}u(x).$$

Tasks:

(a) Find $\Pr(3X < 5)$.

(b) Generalize the answer to find $\Pr(3X < y)$ for an arbitrary constant y.

(Only the problem statements are shown in the slides; solution steps are not provided in the visible material.)

End of Lecture Content Covered in Slides

This concludes the material present in the provided lecture slides for:

- Uniform random variables (definition, CDF, example, applications)
- Exponential random variables (definition and example setup)

No additional derivations or explanations beyond the slides have been introduced, and all reasoning steps follow the sequence presented in the lecture material.