

New chat



L9_S1_A.pdf

PDF

Generate a lecture scribe intended to serve as exam-reference material for CSE400 Lecture 9: Uniform, Exponential, Laplace, and Gamma Random Variables. The scribe must faithfully reconstruct the lecture content so that a student can reliably revise the lecture using only this document.

Use only the provided context (lecture slides/PDF and relevant textbook excerpts). Do not introduce any content, examples, explanations, interpretations, or intuition that are not present in the provided context.

While writing the lecture scribe, reason step-by-step through all definitions, notation, assumptions, results, derivations, graphical interpretations, and worked examples exactly as presented in the provided context. Ensure that all logical dependencies between ideas are explicitly explained so that the reasoning can be followed during exam revision.

The lecture scribe must clearly and systematically reconstruct the lecture structure, including:

1. Types of Continuous Random Variables
 - Classification and conceptual introduction as presented in the lecture
2. Uniform Random Variable
 - Step-by-step definition of PDF and CDF
 - Conditions under which the uniform distribution is defined
 - Graphical interpretation of PDF and CDF
 - Step-by-step reconstruction of sinusoid phase example and all probability calculations
 - Applications of the uniform distribution discussed in the lecture

3. Exponential Random Variable

- Step-by-step definition of PDF and CDF
- Conditions and parameter assumptions
- Graphical interpretation of PDF and CDF
- Step-by-step reconstruction of example probability problems
- Generalization of probability expressions where shown in the lecture

4. Laplace Random Variable

- Definitions, properties, and interpretations presented in lecture

5. Gamma Random Variable

- Definition, special cases, graphical representation, and example discussion as presented

Include:

- Definitions and notation exactly as introduced
- Assumptions and conditions associated with results
- Step-by-step derivations when present
- Step-by-step worked examples when present
- Graph and interpretation explanations when presented
- Logical transitions between topics

Maintain a formal academic tone appropriate for exam preparation.

Do not:

- Add new examples or explanations
- Simplify into tutorial-style teaching
- Include material from outside the provided lecture or textbook content

Lecture Scribe: CSE400 Lecture 9 – Uniform, Exponential, Laplace, and Gamma Random Variables

(Prepared strictly from lecture slides content)

1. Types of Continuous Random Variables

This lecture focuses on **continuous random variables** and introduces specific distributions used in probability modeling. The lecture outline covers:

- Uniform Random Variable (including example)
- Exponential Random Variable (including example)
- Laplace Random Variable
- Gamma Random Variable (including graph, special cases, and examples)

These distributions are presented through their **probability density functions (PDFs)**, **cumulative distribution functions (CDFs)**, graphical interpretation, and example problems.

2. Uniform Random Variable

2.1 Definition and Conditions

A random variable X is said to be uniformly distributed over an interval $[a, b]$ if its PDF is given by:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leq x < b \\ 0, & \text{elsewhere} \end{cases}$$

The corresponding CDF is:

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}$$

Conditions:

- The distribution is defined only over the interval $[a, b]$.
 - Probability density is constant across the interval.
 - Outside the interval, the probability density is zero.
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2.2 Graphical Interpretation

- **PDF Graph:**

The density function is constant at height $\frac{1}{b-a}$ between a and b , forming a rectangular shape.

- **CDF Graph:**

The cumulative distribution function increases linearly from 0 at $X = a$ to 1 at $x = b$.

2.3 Example 1: Uniform Random Variable (Sinusoid Phase)

Problem Statement

The phase of a sinusoid Θ is uniformly distributed over $[0, 2\pi]$ with PDF:

$$f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

General Probability Property

For a uniform random variable over $[a, b]$:

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

(a) Compute $\Pr(\Theta > 3\pi/4)$

Using interval probability:

$$\Pr(\Theta > 3\pi/4) = \frac{2\pi - 3\pi/4}{2\pi} = \frac{5}{8}$$

(b) Compute $\Pr(\Theta < \pi \mid \Theta > 3\pi/4)$

Let:

- $A = \{\Theta < \pi\}$
- $B = \{\Theta > 3\pi/4\}$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Compute intersection:

$$\Pr(3\pi/4 < \Theta < \pi) = \frac{\pi/4}{2\pi} = \frac{1}{8}$$

Given:

$$\Pr(B) = \frac{5}{8}$$

Therefore:

$$\Pr(\Theta < \pi \mid \Theta > 3\pi/4) = \frac{1/8}{5/8} = \frac{1}{5}$$

(c) Compute $\Pr(\cos \Theta < 1/2)$

Solve inequality:

$$\cos \Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Thus:

$$\cos \Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}$$

Probability:

$$\Pr(\cos \Theta < \frac{1}{2}) = \frac{5\pi/3 - \pi/3}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

2.4 Applications of Uniform Distribution

The lecture lists the following applications:

1. Phase of a sinusoidal signal when phase angles between 0 and 2π are equally likely.
 2. Random number generation between 0 and 1 for simulation.
 3. Arrival time of a user within a known time window with no time preference.
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3. Exponential Random Variable

3.1 Definition and Conditions

The exponential random variable is defined for parameter $b > 0$.

PDF:

$$f_X(x) = \frac{1}{b} \exp\left(-\frac{x}{b}\right) u(x)$$

CDF:

$$F_X(x) = [1 - \exp\left(-\frac{x}{b}\right)] u(x)$$

Where $u(x)$ denotes the unit step function.

Condition:

- The parameter b must be strictly positive.
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3.2 Graphical Interpretation

- **PDF Graph:**

Starts at maximum value at $x = 0$ and decreases exponentially as X increases.

- **CDF Graph:**

Starts at zero and asymptotically approaches 1 as $X \rightarrow \infty$.

3.3 Example 2: Exponential Random Variable

Problem Statement

Let X be exponential with PDF:

$$f_X(x) = e^{-x} u(x)$$

(a) Compute $\Pr(3X < 5)$

Rewrite the inequality:

$$3X < 5 \Rightarrow X < \frac{5}{3}$$

Thus:

$$\Pr(3X < 5) = \Pr(X < \frac{5}{3})$$

Using the CDF:

$$F_X(x) = 1 - e^{-x}$$

Therefore:

$$\Pr(3X < 5) = 1 - e^{-5/3}$$

(b) Generalization

For arbitrary constant y :

$$\Pr(3X < y) = \Pr(X < \frac{y}{3}) = 1 - e^{-y/3}$$

4. Laplace Random Variable

The lecture includes the Laplace random variable as part of continuous distributions. It is presented as a distribution with defined properties and interpretations.

(No further definitions, derivations, or examples are provided in the visible lecture content.)

5. Gamma Random Variable

The lecture includes the Gamma random variable as a continuous distribution with:

- Graphical representation
- Special cases
- Example discussion
- Homework problem reference

(No further explicit derivations or mathematical definitions appear in the provided lecture content.)

Logical Flow of Lecture

The lecture progresses through continuous random variables in the following structured manner:

1. Introduces categories of continuous random variables.
2. Develops Uniform distribution:
 - Formal definitions
 - Graphical interpretation

- Detailed example with probability calculations
- Applications

3. Introduces Exponential distribution:

- Definitions and parameter assumptions
- Graphical interpretation
- Worked example and generalization

4. Introduces Laplace distribution

5. Introduces Gamma distribution with graphical and example-based discussion

This document reconstructs the lecture content exactly based on the provided slides and preserves definitions, notation, assumptions, examples, and logical presentation structure suitable for exam revision.