2ª Lista de Cálculo de Várias Variáveis (Derivadas parciais)

1 – Determine as derivadas parciais de primeira ordem da função:

a) $f(x,y) = 3x - 2y^4$ b) $f(x,y) = x^5 + 3x^2y^3 + 3xy^4$ c) $z = x\,e^{3y}$ d) $f(x,t) = \sqrt{x}\,\ln t$

e) $z = (2x+3y)^{10}$ f) $z = \operatorname{tg} xy$ g) $f(x,y) = \frac{x-y}{x+y}$ h) $f(x,y) = x^y$ i) $w = \operatorname{sen}\alpha + \cos\beta$

j) $w = \frac{e^v}{u+v^2}$ k) $f(r,s) = r\,\ln(r^2+s^2)$ l) $f(x,t) = \arctan(x\sqrt{t})$ m) $u = t\,e^{\frac{w}{t}}$

n) $f(x,y,z) = zx - 5x^2y^3z^4$ o) $f(x,y,z) = x\operatorname{sen}(x-y)$ p) $w = \ln(x+2y+3z)$ q) $w = z\,e^{xyz}$

r) $u = \frac{xy}{\operatorname{sen}(yz)}$ s) $u = x^{\frac{y}{z}}$ t) $f(x,y,z,t) = xyz^2\operatorname{tg}(yt)$ u) $f(x,y,z,t) = \frac{x\,y^2}{t+2z}$

v) $u = \sqrt{(x_1)^2 + (x_2)^2 + \cdots + (x_n)^2}$ w) $u = \operatorname{sen}(x_1 + 2x_2 + \cdots + nx_n)$

2 – Determine as derivadas parciais indicadas: (obs: $f_x = \frac{\partial f}{\partial x}$ e $f_y = \frac{\partial f}{\partial y}$)

a) $f(x,y) = \ln\left(x + \sqrt{x^2+y^2}\right)$; $f_x(3,4)$ b) $f(x,y) = \arctan\left(\frac{x}{y}\right)$; $f_x(2,3)$

c) $f(x,y,z) = \frac{y}{x+y+z}$; $f_y(2,1,-1)$ d) $f(x,y,z) = \sqrt{\operatorname{sen}^2 x + \operatorname{sen}^2 y + \operatorname{sen}^2 z}$; $f_z\left(0,0\frac{\pi}{4}\right)$

3 – Determine todas as derivadas parciais de segunda ordem:

a) $f(x,y) = x^3y^5 + 3x^4y$ b) $f(x,y) = \operatorname{sen}^2(mx+ny)$ com $m \in \mathcal{R}$ e $n \in \mathcal{R}$

c) $w = \sqrt{u^2+v^2}$ d) $v = \frac{xy}{x-y}$ e) $z = \operatorname{arc\,tg}\frac{x+y}{1-xy}$ f) $v = e^{xe^y}$

4 – *Teorema de Clairaut*: "Suponha que f esteja definida em uma região R que contenha o ponto (a,b). Se as funções f_{xy} e f_{yx} forem ambas contínuas em R, então $f_{xy}(a,b) = f_{yx}(a,b)$."

Verifique que a conclusão do Teorema de Clairaut é válida, isto é, que $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$:

a) $u = x\operatorname{sen}(x+2y)$ b) $u = x^4y^2 - 2xy^5$ c) $u = \ln\sqrt{x^2+y^2}$ d) $u = xye^y$

5 – Determine as derivadas parciais indicadas:

a) $f(x,y) = 3xy^4 + x^3y^2$; f_{xyz} , f_{yyy} b) $f(x,t) = x^2e^{-ct}$; f_{ttt} , f_{txx} (com $c \in \mathcal{R}$)

c) $f(x,y,z) = \cos(4x+3y+2z)$; f_{xyz} , f_{yzz} d) $f(r,s,t) = r\ln(rs^2t^3)$; f_{rss} , f_{rst}

e) $u = e^{r\emptyset}\operatorname{sen}\emptyset$; $\frac{\partial^3 u}{\partial r^2\partial\emptyset}$ f) $z = u\sqrt{v-w}$; $\frac{\partial^3 z}{\partial u\partial v\partial w}$ g) $w = \frac{x}{y+2z}$; $\frac{\partial^3 w}{\partial z\partial y\partial x}$, $\frac{\partial^3 w}{\partial x^2\partial y}$

h) $u = x^ay^bz^c$; $\frac{\partial^6 u}{\partial x\partial^2 y\partial^3 z}$ (com $a \in \mathcal{R}$, $b \in \mathcal{R}$ e $c \in \mathcal{R}$)

6 – Use a derivação implícita para determinar a $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$:

a) $x^2+y^2+z^2 = 3xyz$ b) $yz = \ln(x+z)$ c) $x - z = \operatorname{arc\,tg}(yz)$ d) $\operatorname{sen}(xyz) = x+2y+3z$

7 – Determine $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$:

a) $z = f(x) + g(y)$ b) $z = f(x+y)$ c) $z = f(x)\,g(y)$ d) $z = f(xy)$ e) $z = f\left(\frac{x}{y}\right)$