Parsian Note Book

Parsian Note Book

Subject:

Date:

1

$$AA^T = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 & 4 \\ -4 & 8 & -8 \\ 4 & -8 & 8 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 2-\lambda & -4 & 4 \\ -4 & 8-\lambda & -8 \\ 4 & -8 & 8-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{cases} \lambda = 0 \\ \lambda = 18 \end{cases} \Rightarrow \left\{ k_1 = \begin{bmatrix} 1/2 \\ -1 \\ 1 \end{bmatrix} \quad k_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad k_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right.$$

$$\Downarrow$$

$$\sigma = 3\sqrt{2}$$

$$\Sigma = \begin{bmatrix} 3\sqrt{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1/3 & 2\sqrt{5}/5 & -2\sqrt{5}/15 \\ -2/3 & \sqrt{5}/5 & 4\sqrt{5}/15 \\ 2/3 & 0 & \sqrt{5}/3 \end{bmatrix}$$

$$\Rightarrow V = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

3

الف)

$$D = P^{-1}AP \Rightarrow A = PDP^{-1}$$

$$\Rightarrow A^m = (PDP^{-1})^m = \underbrace{(PD\cancel{P^{-1}})(\cancel{P}D\cancel{P^{-1}})(\cancel{P}D\cancel{P^{-1}})\cdots(\cancel{P}DP^{-1})}_{m}$$

$$\Rightarrow A^m = PD^mP^{-1}$$

ب)

$$AA = \begin{pmatrix} -5 & 14 & 14 \\ 18 & 7 & -18 \\ -39 & 23 & 48 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} -47 & 74 & 74 \\ 122 & 3 & -122 \\ -233 & 135 & 260 \end{pmatrix}$$

$$A^5 = A^2A^3 = \begin{pmatrix} -1319 & 1362 & 1362 \\ 4202 & -1077 & -4202 \\ -6545 & 3663 & 6788 \end{pmatrix}$$

4

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{4\times 3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}_{3\times 4}$$

$$= \begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}_{4\times 4}$$

$$\det\left((A^TA) - \lambda I\right) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} 2-\lambda & 2 & 0 & 1 \\ 2 & 2-\lambda & 0 & 1 \\ 0 & 0 & -\lambda & 0 \\ 1 & 1 & 0 & 2-\lambda \end{bmatrix} \right)$$

$$= -\lambda^2(-\lambda^2 + 6\lambda - 6) = 0 \Rightarrow \begin{cases} \lambda = 0 \\ \lambda = 3 - \sqrt{3} \\ \lambda = 3 + \sqrt{3} \end{cases}$$

$$\boxed{\sigma = \begin{cases} \sqrt{3+\sqrt{3}} \\ \sqrt{3-\sqrt{3}} \end{cases}}$$

$$AX = B \Rightarrow (LU)X = B \Rightarrow LY = B$$
$$Y = UX$$

$$\begin{array}{ccc} A & X & B \\ \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = & \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} L & U & A \\ \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} & \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = & \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \end{array}$$

$$\Rightarrow \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & u_{12}l_{21} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} u_{11} = 1 & u_{12} = 1 & u_{13} = 1 \\ l_{21} = 3 & u_{22} = -2 & u_{23} = -1 \\ l_{31} = 1 & l_{32} = 1 & u_{33} = 1 \end{cases}$$

$$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = 1 \\ y_3 = 3 \end{cases}$$

$$UX = Y \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \Rightarrow \boxed{\begin{cases} x_3 = 3 \\ x_2 = -2 \\ x_1 = -1 \end{cases}}$$