# Stat 159 HW03

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## Abstract

In this paper, we analyze a dataset containing continuous values for Sales, Newspaper, Radio, and TV. We attempt to establish a linear relationship regressing Radio, Newspaper and TV on Sales through, providing some visualization of the data and results of the model. This paper is supported by a similar analysis in Chapter 3 of An Introduction to Statistical Learning, by Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani.

## Introduction

We are typically concerned with two types of analysis: prediction and inference. In the case of prediction, we are interested in converting a vector of predictors into a target variable through some model: our primary concern is to accurately predict what the target variable will be given predictor values that we haven't seen before. In inference, we are more interested in providing interpretable results about the relationship between two or more quantities, where we may hope that there is a causal relationship.

In this analysis, we are focused on inference: we wish to understand how spending on different types of media advertising impacts sales. If there is a strong positive relationship between Sales and one of the media types, it may benefit the bottom line to increase or maintain spending in that area; if there is no relationship or possibly a negative correlation, we may consider decreasing the budget.

# Data

We are analyzing a set of 200 observations across 4 variables: the target, Sales, and three predictors, TV, Radio, and Newspaper. Sales represents number of sales of a particular product in thousands, while TV, Radio, and Newspaper represent dollar amounts of the advertising budget for each form of media respectively.

We can do a preliminary visual exploration of the variables to get a sense of their distributions - see page 2 and 3.

We then proceed to modeling the relationship between the variables.

# Methodology

A linear model is almost never *true*: if there is a relationship between two variables, it is almost invariably more complicated than linear. However, linear models are surprisingly useful for prediction, and are very easy to interpret. They're also quite easy to compute - these three properties have made them popular in analysis for a long time.

A multiple variable linear model (in this case, 3 variables without interactions) is often written as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

Where Y is the target variable and the  $X_i$  are the predictor or input variables.

For this analysis, our model is

$$Sales = \beta_0 + \beta_1 TV + \beta_2 Radio + \beta_3 Newspaper$$

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Figure 1: The distribution of sales across markets looks approximately normal.

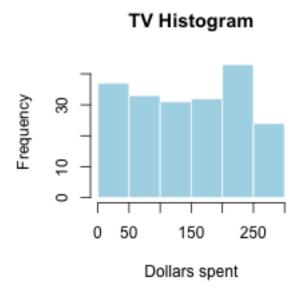


Figure 2: Spending on TV advertising looks fairly uniform.

# Radio Histogram 8 0 0 10 20 30 40 50 Dollars spent

Figure 3: Spending on Radio advertising is skewed lower.

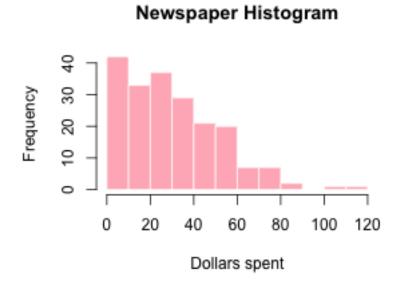


Figure 4: Spending on Newspaper advertising skews extremely low.

 $\hat{\beta}_0$  is simply an intercept which translates the line of best fit. The  $\hat{\beta}_i$  are the values we are particularly interested in. A positive  $\hat{\beta}_i$  indicates a positive correlation: more of  $X_i$  spending is associated with higher Sales. We are interested in how large  $\hat{\beta}_i$  might be, as this indicates the strength of the relation. However, we must also look at its *significance* to understand whether a relationship actually exists - just because we find a nonzero  $\hat{\beta}_i$  does not imply that there is a true relationship. We use the t-test to evaluate how likely it is that the true  $\beta_i$  is actually zero, and that our estimate  $\hat{\beta}_i$ 's distance from zero is due to chance.

## Results

First we do some exploration: we can perform a scatterplot to get a visual impression of their relationships, and we will plot our linear model on top to see how accurate our model seems.

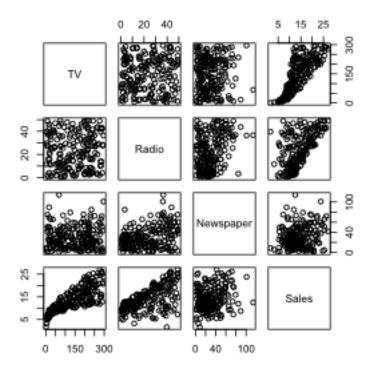


Figure 5: Scatterplot matrix of all variables, with regression line

This matrix can help us get a sense of whether a relationship exists between predictor and target, and can help us identify collinearity between variables, which can create problems in linear regression. We can also look at the quantitative correlations to help us.

Table 1: Correlation Matrix between all variable

|               | $\mathrm{TV}$ | Radio   | Newspaper | Sales  |
|---------------|---------------|---------|-----------|--------|
| $\mathbf{TV}$ | 1             | 0.05481 | 0.05665   | 0.7822 |
| Radio         | 0.05481       | 1       | 0.3541    | 0.5762 |
| Newspaper     | 0.05665       | 0.3541  | 1         | 0.2283 |
| Sales         | 0.7822        | 0.5762  | 0.2283    | 1      |

Fortunately, the relationship between TV and Radio seems very random- there does not seem to be any relationship. But Neswpaper seems somewhat correlated with Radio.

TV and Radio seem to have strong relationships with Sales. We take a closer look at the relationship between Sales and the three target variables, this time including the regression lines for one-variable models.

# Regression of TV on Sales

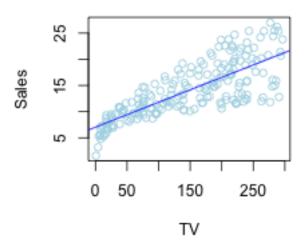


Figure 6: We can see that the relationship is positive, but clearly non-linear. The data is heteroskedastic and looks more like a log function than a linear one. However, our linear model may still be useful for interpretation.

# Regression of Radio on Sales

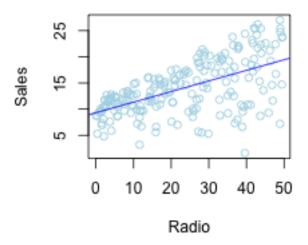


Figure 7: This relationship is also positive but seems nonlinear, and the spread of the data is larger.

# Regression of Newspaper on Sale

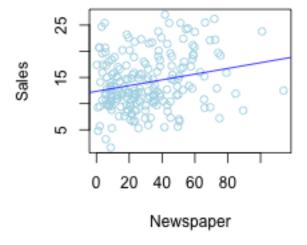


Figure 8: Here there does not seem to be much of of a relationship. Newspaper seems unlikely to be a useful predictor in our model.

Then we compute the linear model with  $lm(Sales \sim .)$  to perform inference:

Table 2: Summary of Linear Regression Model of TV on Sales

|             | Estimate  | Std. Error | t value | $\Pr(> t )$ |
|-------------|-----------|------------|---------|-------------|
| ${f TV}$    | 0.04576   | 0.001395   | 32.81   | 1.51e-81    |
| Radio       | 0.1885    | 0.008611   | 21.89   | 1.505e-54   |
| Newspaper   | -0.001037 | 0.005871   | -0.1767 | 0.8599      |
| (Intercept) | 2.939     | 0.3119     | 9.422   | 1.267 e-17  |

We first consider  $\beta_1$  and its signficance. While the coefficient seems small, at 0.05, we see that the t-statistic is quite large, making it very significant. In comparison,  $\beta_2$  is larger, but its t-value is smaller (although still quite signficant). It's important to keep in mind that the value of a coefficient is highly dependent on the units of the variables and how they are scaled, so we are most interested in whether the  $\beta_i$ 's are positive and how significant they are. We see that  $\beta_3$ , representing the linear relationship between Newspaper and Sales, is slightly negative but quite insignificant at a p-value of 0.86. It seems that we may want to exclude Newspaper as a predictor for Sales, as it seems generally uncorrelated with sales, and somewhat correlated with TV and Radio.

We compute some other goodness-of-fit statistics to evaluate how far off our model is.

Table 3: Goodness of Fit Statistics

| Statistic               | Value  |
|-------------------------|--------|
| Residual Sum of Squares | 556.8  |
| R-squared               | 0.8972 |
| Total Sum of Squares    | 5417   |
| F-statistic             | 570.3  |
| Residual Standard Error | 1.686  |

We will also plot some visuals that can help us understand our model's fit.

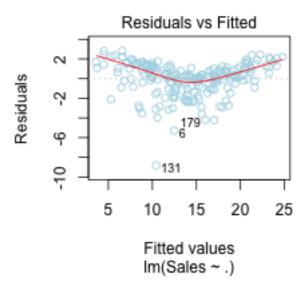


Figure 9: The residual plot confirms that the true relationship is not linear- when a linear model is accurate, there should be no trend in the residuals. Here there is a clear parabolic pattern indicating a nonlinear relationship between the predictors and target.

# Conclusions

From our analysis, we can see that our model fits the data fairly well, even if the true relationship is not linear. Our R-squared value is fairly good, and this linear model may provide some value. If we wished to further explore the data, we would probably exclude Newspaper as a predictor and perhaps evaluate whether there is any significant interation between the two remaining predictors, TV and Radio.

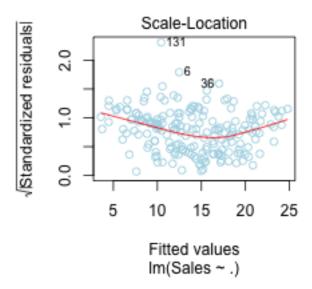


Figure 10: A Scale-Location plot is helpful for identifying whether data is heteroskedastic. Surprisingly, the spread seems fairly random and unrelated to the fitted values. There is a slight trend, but generally heteroskedasiticity does not seem to be a large problem in our model.

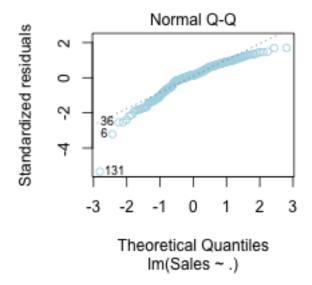


Figure 11: Here we evaluate whether the data seems normally distributed, which is an assumption in a linear regression model. The data is not perfectly normal, especially in the tails. However, the data is reasonably close to a normal distribution, and this may explain why our goodness-of-fit statistics are fairly strong.