UCI MCSB Bootcamp Dry (Mathematical/Computational)

Project: Discrete logistic growth

Suppose a rabbit colony has a population *x*(*n*) at month *n*, where *x* is measured in thousands. If the population were growing in an unbounded environment, the population obeys

*x*(*n* + 1) = *x*(*n*) + *r · x*(*n*) (1)

where *r* is the per-capita growth rate. Suppose if instead the population is in a bounded environment (like an island), growth is limited, and the population obeys

*x*(*n* + 1) = *x*(*n*) + *r* r1 *− x*(*n*)) *x*(*n*) (2) where *K* is a parameter we refer to as the carrying capacity.

*K*

Suppose *r* = 0*.*1 and *K* = 0*.*6.

1. According to your intuition, what population sizes are *steady states*, meaning that if the population had that value at time *n* = 0, then it would remain at that value?
2. Sketch your intuition for the population *x*(*t*) from a starting population *x*(1) = 0*.*2.

Write code to solve the dynamical system, and answer the following questions:

1. Suppose *r* = 0*.*1 and *K* = 0*.*6. Generate time series of the populations for a few starting populations *x*(1). Does it match your intuition?
2. Suppose *r* = 2*.*1 and *K* = 0*.*6. Generate time series of the populations for a few starting populations *x*(1).

In a discrete-time dynamical system, if the population cycles between two values, the solution is called a two-cycle. Cycling between *N* values is called an *N* -cycle.

1. Check that at *r* = 2*.*5 and *K* = 0*.*6 there is a 4-cycle.
2. (Optional) Can you find a value of *r, K* and *x*(1) that gives a 3-cycle?
3. In this part, we will do a parameter sweep for 0 *< r <* 3*.*0, with fixed *K* = 0*.*6. The goal is to generate a diagram where the horizontal axis is the parameter value *r*. On the vertical axis, if there is a stable steady state, plot the steady-state population. If there is an *N* -cycle, plot the *N* values of *x* that it cycles through. 1
   * Hint: One way to plot the steady state or the *N* -cycle is to simulate the system until *n*max, and plot the last half values of *x*(*n*). You need to choose *n*max large enough so that the dynamics have settled into their steady state (or steady cycle) by *n*max*/*2.
   * Hint: How many *r* values should you explore?

1This type of behavior in a dynamical system is called *chaos*! This particular type of chaos is called period-doubling chaos.