Advanced Digital Communications (EQ2410)
Period 3, 2014/15

Homework Project 1

Due: Monday, Feb. 16, 2014, 12:00 M. XIAO

Instructions

This is the first homework project in the Advanced Digital Communications course, period 3, 2014/15. The purpose of this project is to use analytical derivations to verify your implementation of simple equalization techniques.

You can group together to finish the project. Each group can have maximum 2 students. The contribution of each student within a group should be clarified in the report.
Please hand in via **email** (`mingx@kth.se`).
Each problem has 0.5 points

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1. your answers to Problem 1 and 2 and the BER plots for Problem 3 in one PDF document;
2. one copy of your matlab implementation for Problem 3.

Problem 1: The matlab script `main.m` contains parts of an implementation of a digital communications system. In order to get familiar with the code, answer the following questions:

(a) Identify the signals `Signal_1`, `Signal_2`, `Signal_3`, `Signal_4`, `Signal_5`, and `Signal_6` as well as the vectors `Vec_1` and `Vec_2`.

Give the mathematical definition for a each signal/vector.

(b) Which purpose do the filters `Filter_1`, `Filter_2`, `Filter_3`, `Filter_4`, and `Filter_5` have?

(c) Explain why the convolutional products in `main.m` are multiplied with the variable `delta_t`. Explain furthermore why we have a factor 1/`delta_t` when we generate `Signal_1`.

(Hint: What happens in the limit when `delta_t` tends to zero?)

Problem 2: The MMSE solution to the linear equalization problem is given as $\boldsymbol{c}_{MMSE} = \boldsymbol{R}^{-1}\boldsymbol{p}$. In preparation for the implementation show that

$$\boldsymbol{R} = P_s\boldsymbol{U}\boldsymbol{U}^H + \boldsymbol{C}_w \quad \text{and} \quad \boldsymbol{p} = P_s\boldsymbol{U}\boldsymbol{e} = P_s\boldsymbol{u}_0,$$

with $P_s = \mathrm{E}[\|b[n]\|^2]$ and with $\boldsymbol{C}_w$ denoting the covariance matrix of the noise vector $\boldsymbol{w}[n]$ in the geometric model.

Problem 3:

(a) Implement the following equalization methods:

- the linear ZF equalizer,
- the linear MMSE equalizer,
- the ZF decision feedback equalizer, and
- the MMSE decision feedback equalizer.

(b) For the parameters given in `main.m` , measure the average bit error probability, and plot it in logarithmic scale (e.g., by using `semilogy`) over E_b/N_0 in dB. Use a block length of 500 symbols, and average the BER over a sufficiently large number of transmissions.

Verify your results by comparing with Figure 5.14 in Madhow's book (for $m = 1$). Note that the linear ZF equalizer does not exist for $m = 1$. You can however verify your implementation in this case for $m = 2$.

Hand in plots of your results for both $m = 1$ and $m = 2$.

The solutions to Problem 4 have to be handed in in hand-written format[1] `main.m` .

We are considering the following system parameters; note that in the list specifying the groups it is as well indicated which student is student A, B, etc.

- Binary transmission: $b[n] \in \{-1, +1\}$, with $\Pr(b[n] = -1) = \Pr(b[n] = +1) = 0.5$
- Real-valued Gaussian noise with variance σ_n^2: $n(t) \sim N(0, \sigma_n^2)$
- Symbol duration: $T = 2$
- Transmit filter: $g_T(t) = I_{[0,2]}$
- Channel filter: $g_C(t) = a_1 \cdot \delta_0(t-1) + a_2 \cdot \delta_0(t-2)$
- Receive filter: $g_R(t) = I_{[0,1]}$
- Sampling interval: $T_s = T/m$ with $m = 2$
- Length of the observation interval: $L = 3$

	a_1	a_2
Student A	1	-2
Student B	1	-3
Student C	1	-4
Student D	1	-5

Problem 4:

(a) For the parameters specified above show analytically that the noise samples $w[k]$ in the sampled output of the receive filter are uncorrelated and have variance σ_n^2.

(b) Give the discrete-time impulse response $f[k]$ and the matrix $\boldsymbol{U}$ of the geometric model for your realization of a_1, a_2 (see table above).

(c) Show for the parameters specified above that the error probability at the output of a linear equalizer $\boldsymbol{c}$ is given as

$$P_e = \frac{1}{4} \sum_{b[n-1] \in \{\pm 1\}} \sum_{b[n+1] \in \{\pm 1\}} Q\left(\frac{\boldsymbol{c}^H \boldsymbol{U} \cdot [b[n-1], b[n], b[n+1]]^T}{\sqrt{\boldsymbol{c}^H \boldsymbol{C}_w \boldsymbol{c}}} \right)$$

(d) Let $\boldsymbol{c}_{FF}$ be the zero-forcing solution for the DFE designed for the system above. Note that for the parameters above only one previous decision $\hat{b}[n-1]$ is fed back.

Give an analytical expression for the error probability P_e of the ZF-DFE.

Hint: Derive first $\Pr(Z_{DFE}[n] < 0 | b[n] = 1, b[n-1], \hat{b}[n-1])$, average it over all realizations of $b[n-1], \hat{b}[n-1]$, and exploit that $\Pr(b[n-1] = \hat{b}[n-1]) = P_e$.

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[1] If necessary a scanned version can be sent by email.

Plagiarism

Part 1 Cooperation between groups is forbidden. If groups hand in obviously identical solutions or parts of solutions, all members of the respective groups will get zero points for Part 1.

Part 2 Discussions between students are allowed. However, from the solutions, it should become clear that the material was prepared individually. If two groups hand in obviously identical solutions or parts of solutions, they will get zero points for Part 2.