# Advanced Digital Communications (EQ2410)
**Period 3, 2016**

**Homework Project 2**
Due: Monday, March. 4, 2016, 12:00 M. Xiao

## Instructions

This is the second homework project in the Advanced Digital Communications course, period 3, 2016. The purpose of this project is to predict the performance of LDPC codes and to verify the predictions with the help of simulations.

Solve the following problems as a group. You are free to organize the group work as you like. However, make sure that everybody understands what was done.

Part 1, Solutions should be handed in via **email** (`mingx@kth.se`)

1. A short report which contains the solutions and answers to the problems 1 and 2 as well as the plots for the result,
2. All matlab scripts which were used to generate your results.

**Problem 1:** The matlab script `main.m` contains the implementation of a simple LDPC coded transmission system. Explain

(a) why transmission of the all-zero codeword (mapped to the all-one codeword after BPSK) is sufficient to evaluate the performance of the code,

(b) which channel model is implemented,

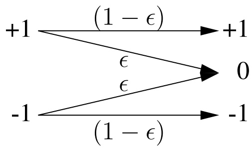
(c) which decoder is implemented,

(d) to which component decoders the decoders `decoder_1` and `decoder_2` correspond,

(e) which degree distributions are given by the distributions `Distr_1A`, `Distr_1B`, `Distr_2A`, and `Distr_2A`, and

(f) which problems can occur when generating the check matrix.
(Hint: Verify how the check matrix was constructed in `main.m` , and consider a regular length 1000 (3,7) LDPC code. How many edges and how many check nodes do you get?).

**Problem 2:** In the following, we focus on belief-propagation decoding for the binary erasure channel (BEC) with erasure probability $\epsilon$:




(a) Change the channel model in the implementation to the BEC, and implement the belief-propagation decoder.

Note that the bits at the output of the channel are either erased or correct and that therefore the messages exchanged by the component decoders can be represented either by the correct bits ($-1$ or $+1$) or by the erasure symbol 0.

(b) Explain why it is sufficient in the BEC case to track the erasure probability for the messages which are exchanged by the component decoders in order to perform density evolution.

(c) Let in the following $p(l)$ and $q(l)$ denote the erasure probabilities at the output for the variable-node decoder and the check-node decoder, respectively, at iteration $l$.

1. For a variable node with degree $d_v$, show that
$$p(l) = \epsilon \cdot q(l-1)^{d_v - 1}.$$
2. For a check node with degree $d_c$, show that
$$q(l) = 1 - (1 - p(l))^{d_c - 1}.$$

(d) Verify your implementation in the following way:

1. Choose Gallagher's (3,6) LDPC code, and run computer simulations for $\epsilon \in \{0.1, 0.15, \ldots, 0.8\}$ and a block length of $n = 10000$ code symbols. Set the maximum number of iterations to 20.
Plot the erasure probability after decoding over $\epsilon$, and check for which value of $\epsilon$ the decoder starts to converge.
2. Let in the following $f(.)$ and $g(.)$ denote the functions $p(l) = f(q(l-1))$ and $q(l) = g(p(l))$.
Plot the function $f(q)$ and the inverse function $g^{-1}(q)$ for $q \in [0, 1]$ for $\epsilon \in \{0.1, 0.15, \ldots, 0.8\}$ into one diagram.
3. Explain why convergence is only possible if $g^{-1}(q) > f(q)$. Determine the decoding threshold $\epsilon_T$ for the (3,6) LDPC code, and compare it to the result of your simulation.

The following problems have to be solved and handed in individually by each student. The solutions have to be handed in in hand-written format[1] together with print-outs of the result plots. Please attach as well print-outs of the matlab scripts which were used to generate your results.

**Problem 3:** Consider LDPC decoding for the binary symmetric channel (BSC) with Gallagher's Algorithm A. Consider a modified decoding algorithm where the variable-node decoder is replaced by the majority-voting decoder specified as follows:

$$v_i = \begin{cases} +1, & \text{if } |S_{+1}^{(i)}| \geq |S_{-1}^{(i)}|, \\ -1, & \text{else,} \end{cases}$$

[1] If necessary a scanned version can be sent by email.

with the sets

$$\begin{aligned} S_{+1}^{(i)} &= \{i : i \in \{0, \dots, i-1, i+1, \dots, d_v\} \text{ and } u_i = +1\} \quad \text{and} \\ S_{-1}^{(i)} &= \{i : i \in \{0, \dots, i-1, i+1, \dots, d_v\} \text{ and } u_i = -1\}, \end{aligned}$$

the channel output $u_0 = y$, and the messages $u_i$ ($i \in \{1, \dots, d_v\}$) provided by the check nodes (i.e., the decoder output is $v_i = +1$ if the number of incoming messages $u_i$ with $u_i = +1$ is bigger than the number of incoming messages $u_i$ with $u_i = -1$).

(a) Implement the modified decoder and compare the performance for a block length $n = 10000$ by running computer simulations.

**Problem 4:** Based on the results from the group part, try to design a good regular LDPC code with $d_c \geq 3$ for the BEC for the decoding thresholds $\epsilon_T$ given in the following table.

| | $\epsilon_T$ |
|---|---|
| Student A | 0.2 |
| Student B | 0.3 |
| Student C | 0.4 |

Try to maximize the code rate $R$ under the constraint that the decoder converges for $\epsilon \leq \epsilon_T$. Explain how you have optimized your code and show simulation results which verify that your design fulfils the requirements.

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## Plagiarism

Cooperation between groups is forbidden. If groups hand in obviously identical solutions or parts of solutions, all members of the respective groups will get zero points for Part 1.