. Let S = {2, 4, {3, 4}, 5, {6}, 8} be a set.

(a) Mark true or false for each of the following:

i. $\{2\} \in S$ True

ii. $\{2\} \subset S$ False

iii. $\{6\} \in S$ True

iv. $\{6\} \subset S$ True

v. $\{\{3, 4\}\} \subset S$

(b) Give a partition of S.

S1 = {2} S2 = {3, 4} S3 = {5} S4 = {6} S5 = {8}

2. Let f be a function from the set A to the set B.

(a) Say that the size of A is 6 and the size of B is 10. Is it possible to for f to be onto? Is it possible

for f to be one-to-one? Why or why not?

A and B can be one-to-one if every element of A maps to a unique element in B. A and B can't be onto because onto is for every B there is an A such that F(A) = B. Since there are more elements in B than A this criteria can't be satisfied.

(b) Say that the size of A is 8 and the size of B is 5. Is it possible to for f to be onto? Is it possible

for f to be one-to-one? Why or why not?

It is possible for A and B to be onto because there are more elements in A than B therefore each element in B can be mapped in A. However, A and B can't be one to one because there are more elements in A in B therefore each element of A cannot have a unique mapping.

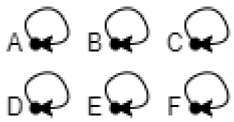
(c) If we know that f is a bijection, what can we assume about the size of A and the size of B?

If we know it's a bijection, then A and B have the same size because in order to be both onto and one-to-one the cardinality of A and B must be equal.

3. Let X = {a, b, c, d, e, f } be a set. Create a binary relation on X that is reflexive and antisymmetric.

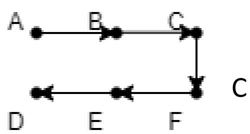
Represent your relation as a matrix and a digraph.

Reflexive



$$\begin{array}{c|c} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Anti-Symmetric



$$\begin{array}{c|c} & \begin{matrix} A & B & C & D & E & F \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4. Let Z = {a, b, c, d, e, f, g, h, i, j} be a set, and define a relation on Z as b > a if and only if b is after a in the English alphabet. The English alphabet, in order, is included below for your reference.

Find the minimal and maximal elements of the poset (Z, >).

The minimal element is j and the maximal element is a.

5. Determine if the following relation is an equivalence relation: x and y are integers and xRy if $x-y = 3m$ for some integer m. Justify your reasoning.

In order for x and y to be an equivalence relation, it must be reflexive, symmetric, and transitive. The relation x and y is $(x - y) = 3m$. Therefore, $(x - y)$ is divisible by 3, or $(x - y) \% 3 = 0$.

Is it reflexive? In order to be reflexive, every element $a \in A$, we have aRa. So, we will check if (x, x) belongs to the relation. $x - x = 0$ and 0 is divisible by 3 which means for every value of x, (x,x) will be present in the relation. Thus, the relation is reflexive.

Is it symmetric? To be symmetric, (x,y) and (y,x) must both belong to the relation. (x,y) is $x-y=3m$.

$(x - y) = 3m$

$-(y - x) = 3m$

$(y - x) = -3m$

$(y - x)$ is divisible by 3 therefore (y,x) belongs to the relation and the relation is symmetric.

Is it transitive? In order to be transitive if (x,y) and (y,z) belong to the relation then (x,z) must also belong to the relation. Let us assume (x,y) belongs to the relation such that $x - y = 3a$ and (y,z) belongs to the relation such that $y - z = 3b$. If we add them together we get:

$(x - y) + (y - z) = 3a + 3b$

$(x - z) = 3(a + b)$

$(x - z) = 3k, k = a + b$

$(x - z)$ is divisible by 3, therefore (x, z) is part of the relation. Thus, the relation is transitive.

Since the relation is reflexive, symmetric and transitive than it is an equivalence relation.