

## Discrete Graded Homework 2, Abby Miller, amm0257

1.  $H(x)$ : new house       $J(x)$ : get a job       $C(x)$ : new car.

Argument form:

$$J(x) \rightarrow (H(x) \wedge C(x))$$

$$\neg H(x) \underline{\hspace{2cm}}$$

$$\therefore \neg J(x)$$

Proof:

1.  $J(x) \rightarrow (H(x) \wedge C(x))$  Premise
2.  $\neg J(x) \vee (H(x) \wedge C(x))$  Conditional Identity
3.  $(\neg J(x) \vee H(x)) \wedge (\neg J(x) \vee C(x))$  Distributive Law
4.  $\neg J(x) \vee H(x)$  Simplification
5.  $\neg H(x)$  Premise
6.  $\neg J(x)$  Disjunctive syllogism 4, 5

2.  $P(x)$ : Practice hard  $B(x)$ : Play badly

Argument form:

$$\forall x (P(x) \vee B(x))$$

$$\exists x (\neg P(x)) \underline{\hspace{2cm}}$$

$$\therefore \exists x B(x)$$

Proof:

1.  $\forall x (P(x) \vee B(x))$  Premise
2.  $a$  is a particular element Element definition
3.  $P(a) \vee B(a)$  1, 2 Universal instantiation
4.  $\exists x (\neg P(x))$  Premise
5.  $\neg P(a)$  4, 2 Existential instantiation
6.  $B(a)$  5, 3 Disjunctive syllogism
7.  $\exists x B(x)$  6 Existential generalization

3. The proof is incorrect because it only proves when  $n = 10$ . It is too specific.  
The proof must be more general and universal.

Proof by contraposition:

Assume there is an integer  $k$  such that  $n = 2k + 1$

$$n^2 = (2k + 1)^2$$

$$= 4k^2 + 4k + 1$$

$$= 2(2k^2 + 2k) + 1$$

$$= 2(j) + 1 \text{ where } j = 2k^2 + 2k$$

Since  $j$  is an integer,  $2j + 1$  is odd whenever  $j$  is odd.

$j$  is odd whenever  $n$  is odd.  $n^2$  is odd when  $n$  is odd.

Therefore,  $n^2$  is even when  $n$  is even.

4. The proof is wrong because no explanation is given for why  $x-y$  is even.  
They are defined but not used.

Proof:

Assume  $x$  and  $y$  are two odd integers such that  $x = 2k + 1$  and  $y = 2j + 1$  for some integer  $k$  and  $j$

$$x - y = (2k+1) - (2j + 1)$$

$$= 2k + 1 - 2j - 1$$

$$= 2k - 2j$$

$$= 2(k - j)$$

$$x - y = 2(h) \text{ where } h \text{ is an integer } = k-j$$

2 times an integer is always even, so  $x-y$  is always even.

Therefore, the difference of two odd numbers is always even.

5. Proof by cases:

Assume  $y = 0$ .

Case 1:  $x$  is a negative integer

$$|-x - 0| = |0 - -x|$$

$$|-x| = |x|$$

When  $x < 0$ ,  $|x| = -x$

$$-x = -x$$

True

Case 2:  $x$  is zero

$$|0 - 0| = |0 - 0|$$

$$|0| = |0|$$

When  $x = 0$ ,  $|x| = x$

$$0 = 0$$

True

Case 3:  $x$  is positive

$$|x - 0| = |0 - x|$$

$$|x| = |-x|$$

When  $x > 0$ ,  $|x| = x$

$$x = x$$

True

There exists a  $y$ ,  $y = 0$  where all  $x$   $|x - y| = |y - x|$ . Therefore the theorem is true.