Discrete Graded Homework 2, Abby Miller, amm0257

1. H(x): new house J(x): get a job C(x): new car.
   Argument form:
   $J(x) \to (H(x) \wedge C(x))$
   $\underline{\neg H(x) \qquad\qquad}$
   $\therefore \neg J(x)$

Proof:

1. $J(x) \to (H(x) \wedge C(x))$ Premise
2. $\neg J(x) \vee (H(x) \wedge C(x))$ Conditional Identity
3. $(\neg J(x) \vee H(x)) \wedge (\neg J(x) \vee C(x))$ Distributive Law
4. $\neg J(x) \vee H(x)$ Simplification
5. $\neg H(x)$ Premise
6. $\neg J(x)$ Disjunctive syllogism 4, 5

2. $P(x)$: Practice hard $B(x)$: Play badly
   Argument form:
   $\forall x\ (P(x) \vee B(x))$
   $\underline{\exists x(\neg P(x))\qquad\qquad}$
   $\therefore \exists x\ B(x)$

Proof:

1. $\forall x\ (P(x) \vee B(x))$ Premise
2. a is a particular element Element definition
3. $P(a) \vee B(a)$ 1, 2 Universal instantiation
4. $\exists x(\neg P(x))$ Premise
5. $\neg P(a)$ 4, 2 Existential instantiation
6. $B(a)$ 5, 3 Disjunctive syllogism
7. $\exists x\ B(x)$ 6 Existential generalization

3. The proof is incorrect because it only proves when $n = 10$. It is too specific. The proof must be more general and universal.

Proof by contraposition:

Assume there is an integer k such that $n = 2k + 1$

$n^2 = (2k + 1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$= 2(j) + 1$ where $j = 2k^2 + 2k$

Since j is an integer, $2j + 1$ is odd whenever j is odd.

j is odd whenever n is odd. n^2 is odd when n is odd.

Therefore, n^2 is even when n is even.

4. The proof is wrong because no explanation is given for why x-y is even. They are defined but not used.

Proof:

Assume x and y are too odd integers such that $x = 2k + 1$ and $y = 2j + 1$ for some integer k and j

$x – y = (2k+1) – (2j + 1)$

$= 2k + 1 - 2j – 1$

$= 2k – 2j$

$= 2 (k – j)$

$x – y = 2 (h)$ where h is an integer $= k-j$

2 times an integer is always even, so x-y is always even.

Therefore, the difference of two odd numbers is always even.

5. Proof by cases:

Assume $y = 0$.

Case 1: x is a negative integer

$|-x - 0| = |0 - -x|$

$|-x| = |x|$

When $x < 0$, $|x| = -x$

$-x = -x$

True

Case 2: x is zero

$|0 - 0| = |0 - 0|$

$|0| = |0|$

When $x = 0$, $|x| = x$

$0 = 0$

True

Case 3: x is positive

$|x - 0| = |0 - x|$

$|x| = |-x|$

When $x > 0$, $|x| = x$

$x = x$

True

There exists a y, $y = 0$ where all x $|x - y| = |y - x|$. Therefore the theorem is true.