Conditional Probability

Logistics

- Midterms:
    - Graded
    - Threw out question 5d — everyone got full credit, and anyone who got it right got bonus points
    - Grade sheets will be handed back at end of class, actual exams handed back next week
    - You have two options:
        - Take the grade you got and be done with it
        - Take your grade sheet and any problems you didn't get full credit on you can re-do at home with any resources you want
            - You'll get ½ of the points you missed for questions you get right
            - Due Tuesday
- Midterm course grades
    - Posted
    - Will be emailing out your snapshot of grades to date — make sure everything looks right and come to me if there are any issues

Marginal Probability: the probability of an event, irrespective of the outcomes of other random events

What is the probability of drawing a 7 of clubs?

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What is the probability of drawing a 7 of clubs?

1 / 52

Joint Probability: what is the probability of two simultaneous events?

What is the probability of that, given two decks of cards, one person draws a 7 of clubs and one person draws an ace of spades?

Joint Probability: what is the probability of two simultaneous events?

What is the probability of that, given two decks of cards, one person draws a 7 of clubs and one person draws an ace of spades?

P(draw 7 of clubs) * P(draw a 4 of hearts)

Joint Probability: what is the probability of two simultaneous events?

What is the probability of that, given two decks of cards, one person draws a 7 of clubs and one person draws an ace of spades?

P(draw 7 of clubs) * P(draw a 4 of hearts)

1/52 * 1/52

What is the probability of drawing a 7 of clubs, and then a 4 of hearts without replacement?

“given that”

P(draw 7 of clubs) * P(draw a 4 of hearts | 7 of clubs was already drawn)

What is the probability of drawing a 7 of clubs, and then a 4 of hearts without replacement?

"given that"

P(draw 7 of clubs) * P(draw a 4 of hearts | 7 of clubs was already drawn)

= 1/52 * 1/51

Conditional Probabilities

- Table shows a population by gender and political views
- What is P(Female)?
- What is P(Female and Liberal)?
- What is P(Moderate | Female)?

	Liberal	Moderate	Conservative	**Total**
Male	17	29	14	**60**
Female	30	24	23	**77**
Total	**47**	**53**	**37**	**137**

Conditional Probabilities

- Table shows a population by gender and political views
- What is P(Female)?

  77 / 137 = 0.562

- What is P(Female and Liberal)?
- What is P(Moderate | Female)?

	Liberal	Moderate	Conservative	**Total**
Male	17	29	14	**60**
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Total	**47**	**53**	**37**	**137**

Conditional Probabilities

- Table shows a population by gender and political views

- What is P(Female)?

  77 / 137 = 0.562

- What is P(Female and Liberal)?

  30 / 137 = 0.219

- What is P(Moderate | Female)?

	Liberal	Moderate	Conservative	**Total**
Male	17	29	14	**60**
Female	30	24	23	**77**
Total	**47**	**53**	**37**	**137**

Conditional Probabilities

- P(Moderate | Female) =

  24 / 77 = 0.311

- This is the **conditional probability**

- P(B|A) : the probability of event B, given that event A occurred

	Liberal	Moderate	Conservative	**Total**
Male	17	29	14	**60**
Female	30	24	23	**77**
Total	**47**	**53**	**37**	**137**

Conditional Probability

Formal Definition:

$$P(B \mid A) = \frac{P\ (A \text{ and } B)}{P(B)}$$

Example: $\frac{P(\text{Moderate and Female})}{P(\text{Female})}$

$= (24 / 137) / (77 / 137)$

$= 0.175 / 0.562 = 0.311$

	Liberal	Moderate	Conservative	**Total**
Male	17	29	14	**60**
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Revisiting the Multiplication Rule

Multiplication rule for ***independent*** events:

$$P(A \text{ and } B) = P(A) * P(B)$$

Independent: the occurence of one event has no effect on the probability of the occurrence of another event.

Example: 60% of AAA members made airline reservations last year.

Two members are selected at random.

What is the probability both made airline reservations in the last year?

General Multiplication Rule

When events are ***dependent***:

P(A and B) = P(A) * P(B | A)

For two events, A and B, the joint probability that both events will happen is found by multiplying the probability that event A will happen by the **conditional probability** of event B occurring.

General Multiplication Rule

$$P(A \text{ and } B) = P(A) * P(B \mid A)$$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

When events are independent

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(B \mid A) = \frac{P(A) * P(B)}{P(A)}$$

$$P(B \mid A) = P(B)$$

When events are independent

$$P(B \mid A) = P(B)$$

When events are ***not*** independent

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 1

In a recent study, it was found that the probability that a randomly selected student is a girl is .51.

The probability that a randomly selected student is a girl and plays sports is .1.

If a student is female, what is the probability that she plays sports?

Example 1

In a recent study, it was found that the probability that a randomly selected student is a girl is .51.

The probability that a randomly selected student is a girl and plays sports is .1.

If a student is female, what is the probability that she plays sports?

$$P(S \mid F) = \frac{P(S \text{ and } F)}{P(F).} = \frac{.1}{.51} = .1961$$

Example 2

The probability that a randomly selected student plays sports if (*given that*) they are male is .31.

The probability that a student is male is .49.

What is the probability that the student is male and plays sports?

Example 2

The probability that a randomly selected student plays sports if (*given that*) they are male is .31.

The probability that a student is male is .49.

What is the probability that the student is male and plays sports?

$$P(S \mid M) = \frac{P(S \text{ and } M)}{P(M)} \qquad .31 = \frac{x}{.49} \qquad x = .1519$$

When events are not independent

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

When events are not independent

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

THESE ARE NOT EQUIVALENT

P(B | A) != B(A | B)

But you can re-arrange them to find each other:

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

$$P(B \mid A) = \frac{P(A \mid B) * P(B)}{P(A)}$$

Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Bayes' Theorem in Diagnosis

Consider a human population that may or may not have cancer, and a medical test that returns positive or negative for detecting cancer.

If a randomly selected patient has the test and it comes back positive, what is the probability that the patient has cancer?

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85% of the people who have cancer and are tested will test positive.

Bayes' Theorem in Diagnosis

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If a randomly selected patient has the test and it comes back positive, what is the probability that the patient has cancer?

$$P(\text{Test} = \text{Positive} \mid \text{Cancer} = \text{True}) = 0.85$$

85% of the people who have cancer and are tested will test positive.

Does that mean there's an 85% chance a patient with a positive test has cancer?

Bayes' Theorem in Diagnosis

P(Test = Positive | Cancer = True) = 0.85

P(Test = Negative | Cancer = False) = 0.95

P(Cancer = True) = 0.0002 P(Cancer = False) = .9998

P(A | B) = P (B | A) * P(A) / P(B)

P(Cancer = True | Test = Positive) = P(Test = Positive | Cancer = True)

* P(Cancer = True) / P(Test = Positive)

P(Cancer = True | Test = Positive) = 0.85 * 0.0002 / P(Test = Positive)

Bayes’ Theorem in Diagnosis

$$P(\text{Cancer} = \text{True} \mid \text{Test} = \text{Positive}) = 0.85 * 0.0002 / P(\text{Test} = \text{Positive})$$

How do we find P(Test = Positive)?

$$P(\text{Test} = \text{Positive}) = P(\text{Test}=\text{Positive} \mid \text{Cancer} = \text{True}) * P(\text{Cancer} = \text{True}) +$$

$$P(\text{Test} = \text{Positive} \mid \text{Cancer} = \text{False}) * P(\text{Cancer} = \text{False})$$

$$= 0.85 * 0.0002 + 0.05 * 0.9998 = 0.05016$$

Bayes' Theorem in Diagnosis

P(Cancer = True | Test = Positive) = 0.85 * 0.0002 / P(Test = Positive)

P(Cancer = True | Test = Positive) = 0.85 * 0.0002 / 0.05016

P(Cancer = True | Test = Positive) = 0.003389

Bayes’ Theorem in Diagnosis

P(Cancer = True | Test = Positive) = 0.85 * 0.0002 / P(Test = Positive)

P(Cancer = True | Test = Positive) = 0.85 * 0.0002 / 0.05016

P(Cancer = True | Test = Positive) = 0.003389

So what does that mean?

If a patient is informed they have cancer with this test, there is only a 0.33% chance they have cancer!

Homework — Conditional Probability Worksheet

- Handing out in class today.

- Due Thursday, October 31