Statistics, Simulation And Hypothesis Testing

Logistics

- Midterm makeups due now
- Conditional probability worksheet due Thursday

Diagnostic Problem from Last Week

Will post an update online -- short answer is the slides were actually OK

Statistics

Why sample?

Probability

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Probability

Statistics

Why sample?

Probability

Statistics

Sampling

Estimation

Statistical Inference:

Making conclusions based on data in random samples

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Example:

Use the data to guess the value of an unknown number

Estimation

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Create an **estimate** of the unknown quantity

Estimation

Statistical Inference:

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Example:

Use the data to guess the value of an unknown number

fixed

Create an **estimate** of the unknown quantity

Estimation

Statistical Inference:

Making conclusions based on data in random samples

Example:

Use the data to guess the value of an unknown number

fixed

depends on the random sample

Create an **estimate** of the unknown quantity

Terminology

Parameter

A number associated with the population

Terminology

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Statistic

A number calculated from the sample

Terminology

Parameter

A number associated with the population

Statistic

A number calculated from the sample

A statistic can be used as an **estimate** of a parameter

(Demo)

Estimation

How many enemy planes?

Assumptions

- Planes have serial numbers 1, 2, 3, …, N.
- We don’t know N.
- We would like to estimate N based on the serial numbers of the planes that we see.

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The main assumption

- The serial numbers of the planes that we see are a uniform random sample drawn with replacement from 1, 2, 3, …, N.

Discussion question

If you saw these serial numbers, what would be your estimate of N?

```
170   271   285   290   48
235    24    90   291   19
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One idea: 291. Just go with the largest one.

The largest number observed

- Is it likely to be close to N?
  - How likely?
  - How close?

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Option 1. We could try to calculate the probabilities and draw a probability histogram.

Option 2. We could simulate and draw an empirical histogram.

(Demo)

Verdict on the estimate

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Average of the serial numbers observed ~ N/2

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New estimate: 2 times the average

(Demo)

Bias & Variance

Bias

- **Biased estimate:** On average across all possible samples, the estimate is either too high or too low.

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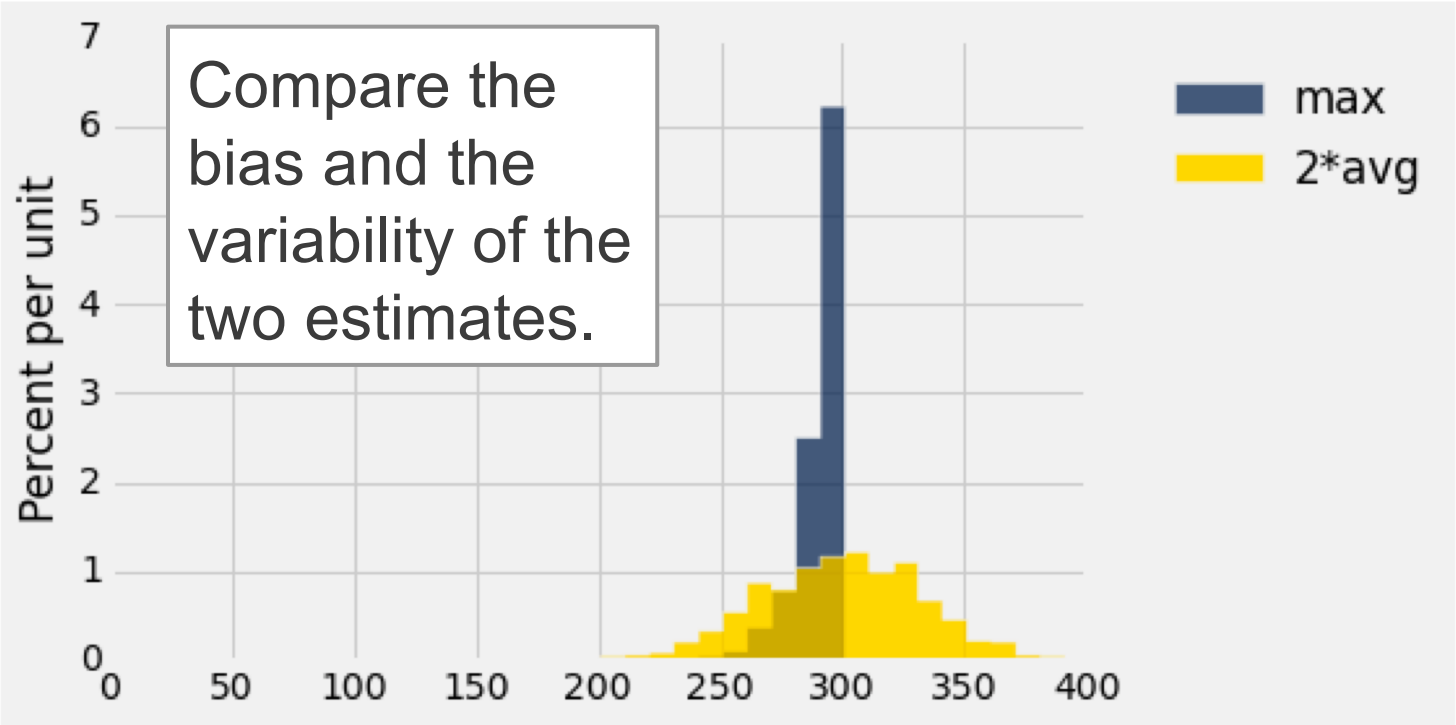
Discussion question



Bias-variance trade-off

- The **max** has low variability, but it is biased.
- **2*average** has little bias, but it is highly variable.
- Life is tough.

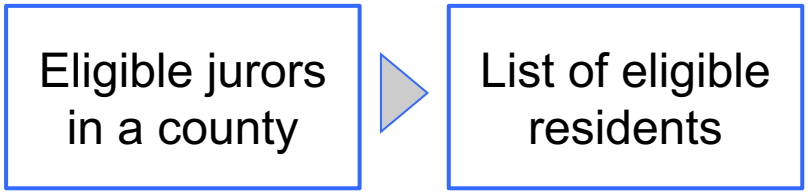
(Demo)

Comparing Samples to Distributions

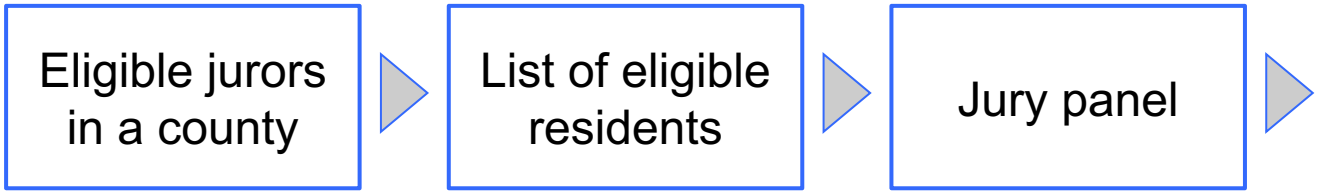
Jury Panels

Eligible jurors in a county

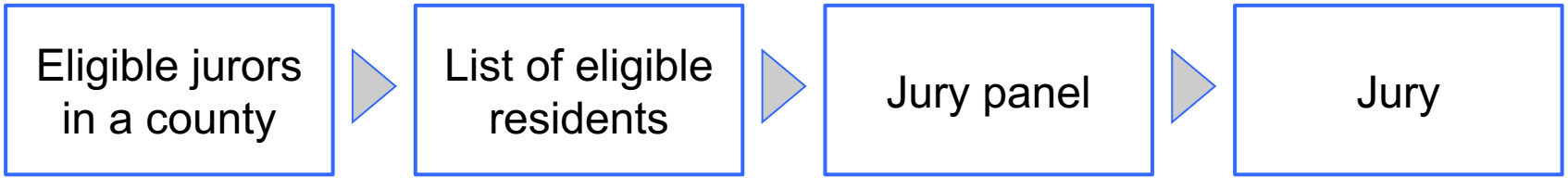
Jury Panels



Jury Panels


Eligible jurors in a county
List of eligible residents
Jury panel

Jury Panels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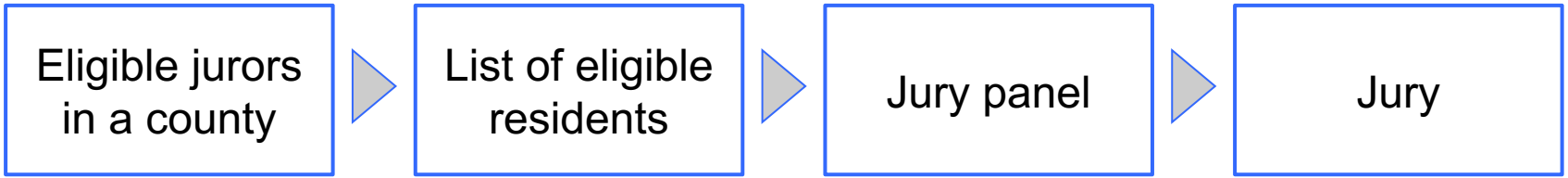

Section 197 of California's Code of Civil Procedure: All persons selected for jury service shall be selected at random, from a source or sources inclusive of a representative cross section of the population of the area served by the court.

Sixth Amendment to the US Constitution: … the accused shall enjoy the right to a speedy and public trial, by an impartial jury of the state and district wherein the crime shall have been committed.

Robert Swain v. Alabama

1965 Supreme Court case about jury selection

- In Talladega, Alabama, 26% of residents were black
- In Swain's jury panel, 8 of 100 panelists were black
- All 8 were struck from the jury by the prosecution (using peremptory challenges)

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Ruling: "The overall percentage disparity has been small and reflects no studied attempt to include or exclude a specified number of [black men]."

Is the actual jury panel likely?

(Demo)

Hypothesis Testing

Testing a Hypothesis

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- What the test statistic might be if the null hypothesis were true
- Approximate the sampling distribution by an empirical distribution

Jury Hypothesis Test

Step 1: The Hypotheses

- *Null hypothesis:* The panel of jurors was selected at random from the eligible juror population. Any differences between the ethnicity distributions of panel and population are due to randomness.
- *Alternative hypothesis*: The panel was not selected at random.

Step 2: The Test Statistic

- TVD between a panel and the population distribution

Step 3: The Sampling Distribution of the Test Statistic

- Simulate drawing panels from population. See if the actual panel TVD is likely.