Randomness and Probability

Logistics

- Midterm on Thursday
- All assignments have been returned to you.
  - If you haven’t gotten something back, check with me.
- Office hours tomorrow, post on Piazza.

Probability Review

- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (100%)
  - Chance of event that is certain

If an event has a chance of 70%, then the chance that it doesn’t happen is:

$$100\% - 70\% = 30\%$$

$$1 - 0.7 = 0.3$$

Experiments & Events

- Experiment: toss a die
- Event:
  - A: observe a number greater than 2
  - B: observe an odd number
  - C: observe a 6
  - D: observe a 3

- Mutual Exclusivity: if one event occurs, the other cannot
  - Which events above are mutually exclusive?

- Simple events vs compound events
  - Which events above are compound? Which are simple?

(demo - simulating coin toss)

Probability of an Event

Assuming all outcomes are equally likely, the chance of an event A happening is:

$$P(A) = \frac{\text{\# of outcomes that make A happen}}{\text{total \# of outcomes}}$$

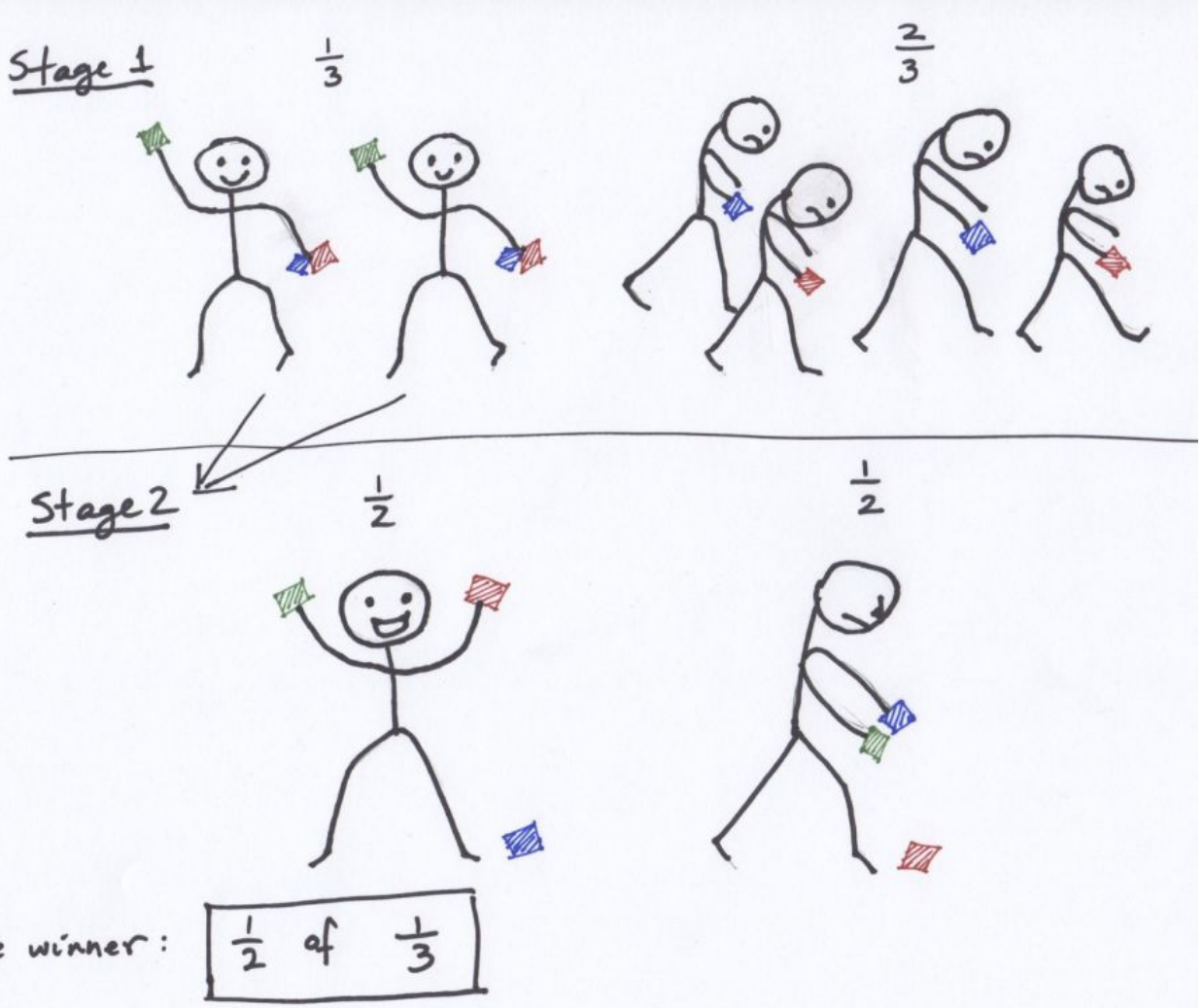
Multiplication Rule

Chance that two events *A* and *B* both happening:

= P(*A* happens) x P(*B* happens given that *A* has happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

Example -- red, green, blue cards


Stage 1
1/3
2/3
Stage 2
1/2
1/2
The winner:
1/2 of 1/3

Example: At Least One Head

- In 3 tosses?

- In 10 tosses?

Example: At Least One Head

- In 3 tosses?
  - Any outcome *except* TTT
  - P(TTT) = (½) X (½) X (½) = ⅛
  - P(at least one head) = 1 - P(TTT) = ⅞ = 87.5%

- In 10 tosses?

Example: At Least One Head

- In 3 tosses?
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- In 10 tosses?
  - 1 - (½)**10

Addition Rule

If event *A* can happen in *exactly* one of two ways:

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

Sampling

- Deterministic sample:
  - Sampling scheme that doesn’t involve chance

- Probability sample:
  - Before the sample in drawn, know the selection probability of every group in the population
  - Not all individuals have an equal chance of being selected

Demo - movie sampling

Sample of Convenience

- Example: sample consists of whoever walks by

- Just because you think you're sampling "at random" doesn't mean you are

- If you can't figure out ahead of time:
  - What's the population
  - What's the chance of selection for each group in the population

  Then you don't have a random sample.

Probability Distribution

- Random quantity with various possible values

- Probability distribution:
  - All the possible values of the quantity
  - The probability of each of those values

- In some cases, the probability distribution can be worked out mathematically without ever having to generate or simulate the random quantity

Demo -- die rolls

Empirical Distribution

- Based on observations

- Observations can be from repetitions of an experiment

- Emperical Distribution:
  - All observed values
  - The proportion of counts of each value

Demo - die rolls

Law of Averages

If a chance experiment is repeated many times,

Independently and under the same conditions,

The the proportion of times that an event occurs

Gets closer to the theoretical probability of the event

As you increase the number of rolls of a die, the proportion of times you see the face w/ five spots gets closer and closer to ⅙.

Large Random Samples

If the sample size is large,

Then the empirical distribution of a uniform random sample

Resembles the distribution of the population,

With high probability.