PROBABILITY 6 : Multiplication Rule

In this video I will review the Multiplication Rule. With the Multiplication Rule you can find the probability of multiple separate events.

Multiplication Rule

The formula for the Multiplication Rule is $P(A \text{ and } B) = P(A) \times P(B)$ or $P(A \cap B) = P(A) \times P(B)$. Remember from the last video that intersection or $\cap$ is the same as and.

I'll show you how to find probabilities using 1st Independent Events and then Dependent Events.

Probability of 2 Tails

When calculating the probability of flipping a coin twice and getting 2 tails we use the probability of flipping a tail. If you flip a coin once you have a 50% chance of getting a tail. So if we use our formula we have :

$P(A \cap B) = .5 \times .5 = .25$ or 25% chance of flipping a tail twice.

This is an independent event because each time we flip a coin, previous flips have no effect.

Probability of 4 Tails

To find the probability of more than one flip we just keep multiplying for each flip. So to find the probability of flipping 4 tails in a row we use :

$P(A \cap B) = .5 \times .5 \times .5 \times .5 = .0625$ or 6.25% chance of flipping 4 tails in a row.

Other Independent Event Probabilities

Some people think that Independent Events are always 50% probabilities, but that isn't true. Lionel Messi scores 24.9% of the goals he attempts. Each time he attempts a goal however the probability that he will score does not change.

However if you were going to calculate the probability that he will make 2 goals in a row the odds decrease as we see here :

$P(A \cap B) = .249 \times .249 = .062 = 6.2\%$ Probability of Making 2 Shots in a Row

Find Probability with Dependent Events

Probabilities related to drawing cards that are not replaced would be an example of a dependent event. The reason this is true is because every time you draw a card we decrease the total number of cards in the deck.

Probability of Drawing a King and a Queen

Here we will calculate the probability of drawing both a King and Queen from 2 cards dealt.

Probability with Replacement

If we replace the card dealt each time both probabilities are the same :

P(A and B) = 4/52 x 4/52 = 0.0059 or .59%

Probability without Replacement

If we don't replace the card dealt each time probabilities change :

P(A and B) = 4/52 x 4/51 = 0.006 or .60%

Probability of Drawing 4 of a Kind

Now we'll get more complicated by finding the probability of drawing 4 of a kind from a 5 card draw without replacement. We start by not caring what the 1st card is :

P(4OK) = 52/52

We continue by trying to trying to match the 1st card drawn. We now have both 1 fewer card and 1 fewer matching card :

P(4OK) = 52/52 x 3/51

Again we decrement both the denominator and numerator 2 more times :

P(4OK) = 52/52 x 3/51 x 2/50 x 1/49

We don't care what the final card is :

P(4OK) = 52/52 x 3/51 x 2/50 x 1/49 x 48/48

Now we have to think about what if we drew cards in a different order like this for example if we are matching for 8 : 8, 7, 8, 8, 8 or :

P(4OK) = 52/52 x 51/51 x 3/50 x 2/49 x 1/48

Well 1st we would get rid of the 1 values and then we will multiply by a Combinatoric Number. I'll cover it in more detail later, but basically we are calculating for how many ways we have to pick an element. Here we have 5 for 5 cards. This provides are target probability :

P(4OK) = 3/51 x 2/50 x 1/49 x 5/1 = 30/124950 = 1/4165 = .02% Probability of Drawing 4 of a Kind out of a 5 Card Draw.

That is all for now. In the next video I'll cover Bayes' Theorem which finds probabilities using previously occurring event data.