

PROBABILITY 2 : Conditional Probability

Combinators

Previously we talked about Permutations which focused on the number of ways we can arrange a set of elements.

Variations with Repetition

Variations deal with the number of ways we can pick and arrange elements of a set. There are 2 variation types. Variations with and without repetition. We can calculate the number of variations for a combination lock with 10 total elements (0 - 9) and 3 possible total positions by taking 10 to the power of 3. It is going to take quite some time to try 1,000 possible values.

Variations without Repetition

What if you weren't able to use the same values twice? Let's say we have 3 spaces left in our Pokemon deck and 5 possible Pokemon card options. How many possible variations could we have if we only use a card once?

The formula for the number of variations without repetition is $n! / (n-p)!$ where n is the number of potential elements and p represents the number of open positions. If $n = 5$ and $p = 3$ we see that we have 60 possible variations of Pokemon cards.

Combinations without Repetition

Permutations are used when you care about order, but if you don't care about order and just want to know how many ways there are to select a certain number of items then calculate combinations.

Let's say we want to buy 3 shirts out of 5 options. We obviously don't care in which order the cashier processes the payments. The formula for finding the possible combinations is $n! / p! * (n - p)!$ where n is the number of shirts and p represents the number of shirts I can buy.

We see if $n = 5$ and $p = 3$ that there is a total of 10 possible variations of t-shirts.

Combinations with Repetition

But, what if we want to calculate the total possible number of combinations with repetitions? The formula for that is $(n + p - 1)! / p! * (n - 1)!$. If $n = 5$ and $p = 3$ then we see we now have 35 possible variations. This makes sense because there are now way more options.

Buying More Leads to Fewer Combinations

One interesting thing to note however is that if we actually buy more shirts the number of combinations actually decreases. We can see that here with an increase of p from 3 to 4. If we change those values in our combinations without repetition formula that our variations fall from 10 to 5 variations.

How Do We Decide Which to Use?

You should use Permutations when you have both an equal number of elements and positions. Use Variations when you have more elements than positions. And, when you only care about which elements made it into a position use combinations.

What are the Odds of Rolling an 11?

To mix up this tutorial I want to revisit what we covered last time as a review. I'll also cover some new concepts.

Previously we covered the basics of probabilities. This time we'll calculate the probability of rolling a 3 if we are using 2 dice.

Last time we found out that the probability of rolling an 11 was 0.06. But, that is before either die is thrown. What if we roll a 5 with the 1st die? What is the probability now?

Conditional Probability

Conditional probability deals with the probability of rolling an 11 given you roll a 5. The notation is $P(11 | 5)$, where $|$ stands for the word Given. Now we deal with the probability of getting a 6 and its probability is $1/6$ or .167.

The formula for conditional probability is $P(A | B) = P(A \text{ AND } B) / P(B)$. And, we see here that given B is 5 that the probability of getting an 11 is $1/6$.

Independent & Dependent Events

Dice rolls are however what we call Independent Events. The roll of 1 has no effect on the other. However as soon as the value of 1 die is known it does affect the chance that the sum will be 11. So in this case they are dependent.

Probability Based on Multiple Dice Throws

Now we will calculate the probability of rolling a 1 with 4 dice throws. To find this answer I'll calculate the odds of Not rolling a 1. That is found by taking $5/6$ to the power of 4 which represents the number of throws. The answer is .482. We can then subtract .482 from 1 to get a probability of .518.

That's it for now. In the next video we'll cover Bayesian Inference.