PROBABILITY 4 : Conditional Probability, Additive / Multiplication Rule

In this video we start to have fun by calculating increasingly more complex probabilities. I'll be using the exercise data I used in the previous tutorial since you should already be comfortable with it.

For the rest of the video M stands for Men, W for Women, E for a Person that Exercises and D for a Person that doesn't Exercise in the formulas I use.

	NO EXERCISE	DID EXERCISE	TOTAL
MEN	78	22	100
WOMEN	83	17	100
TOTAL	161	39	200

Conditional Probability Formula

The Conditional Probability Formula, which you can see on the right, allows us to pick a randomly classified element and then based on it calculate the probability that that element fits in another classification. An example will make that make sense.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Let's say I randomly pick a person that exercises.
What is the probability that that person is a woman? A = Event that the Woman Exercises, B = Event that the Person Exercises.

I find my answer by taking the Probability of the Union of Women that Exercise and Women and divide it by the Probability that the person is a Woman. This works out to 17/200 / 39/200 = .436 or 43.6%. So if I pick a person that exercises there is a 43.6% chance it is a woman. How cool is that!

Additive Rule

Now what if I wanted to find the Probability that a randomly chosen person is either a Woman, Exerciser, or a Woman that Exercises? We can use the Additive Rule to figure that out. Here is its formula.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Now previously we found that if 2 probabilities are exclusive, to find the probability that either will occur we add their probabilities P(A or B) = P(A) + P(B).

When 2 events aren't exclusive, we can have Women that also Exercise, we have to subtract them from the result. Hence we subtract the intersection of Women and Exercisers.

So to answer our question we find P(A) = 100/200 which represents the 100 Women in our sample. We find that P(B) = 39/100 which represents all people that exercise. When we add them we get 139/200. Then we find $P(A \cap B)$ = 17/200, which represents Women Exercisers.

When we subtract Women that Exercise we get a result of .61, or 61% of people in our sample will either be Women, Exercisers, or Women that Exercise.

Multiplication Rule

The Multiplication Rule finds the Probability that 2 events will occur. We want to calculate the probability that if we pick 2 random people from our sample that we will get an Exerciser and then a Woman Exerciser. Here is our formula.

$$P(A \cap B) = P(A|B) \times P(B)$$

Here P(A) represents all women and P(B) represents All Exercisers. P(A|B) represents all Women Exercisers and equals 17/200 = .085. P(B) is all Exercisers and equals 39/100 = .195. If we multiply them together we get .0165.

That means there is a 1.65% probability that if you pick 2 random people that you'll get a Woman and a Woman Exerciser.

That is it for now. In the next video I'll cover Bayes' Theorem and more.