Lesson Plan

Teacher: Abigail Gogan	**Course:** Analysis & Approaches	**Date:** N/A
Subject Group: Mathematics	**Topic:** Functions	**Grade:** 11

Content 2.2:

- Functions, domain, range, graph
- Function notation and seeing a function as a mathematical model
- Inverse functions, $f^{-1}(x)$ notation, and understanding them as the reverse/undoing of a function
- Inverse function as a reflection of a function over the line y=x
- Mention command terms *plot*, *label*

Lesson Objective: introduce function notation, identifying domain and range, reading graphs and graphing functions, finding the inverse of a function, understanding the relationship of an inverse to its original function

Required Materials: worksheet, desmos.com (unless class has access to graphing calculators?), geogebra.com

DAY 1

Warmup / Hook (10 Minutes):

- Display on board: y = *f(input) = output in terms of input = output in simple terms*
- Give silly example like *make word plural function: y = f(word) = plural version of word* and have students "calculate" *f(question)*, *f(tooth)*, *f(deer)* and ask for answers/explain why they are *questions*, *teeth*, *deer*
- Note that the placeholder *word* functions the same way as a variable like *x* would

Introduction / Opener (10 Minutes): *[Review - Activate - Recap]*

- Hand out daily problems that were marked last day
- Anonymous peer assessment of last day's daily problem. Hand out the papers randomly and have students hand them back after the assessment.

Body (20 Minutes):

- Introduce numerical example like *double number function: y = f(number) = number *2* and have them calculate *f(3)*, *f(5)*, *f(0.5)* and ask for answers/review why they are *6*, *10*, *1*
- Make concise statement about how a function in its general form tells us something to do with a number. A function with a variable input will usually output some altered version of that number.
- Have them try to calculate *y = f(x)* using the *double number function*. Answer should be *y = f(x) = 2x*, which is just the general form of the function because we never specified what number *x* was a placeholder for
- Have them now calculate *y = f(4)* using the function we just defined.

- If we define a new function to be $g(x) = 3x + 2$:
    - describe this function in words, like how we described the double number function
    - have them calculate $y = g(4)$
- walk through how to check whether a function contains a certain coordinate:
    - given general form and coordinates, for example $f(x) = (½)x + 5$, $(4, 8)$ and $(10, 10)$
    - Plug the x value into the function and solve: $f(4) = (½)4 + 5 = 2 + 5 = 7 \neq y$ so **no**
    - If output matches y value, then coordinate **is** in function, if different then **it is not**
    - Next: $f(10) = (½)10 + 5 = 5 + 5 = 10 = y$ so **yes**

Closure (10 mins):

Example of functions being used as a mathematical model:

- Video: Making a polynomial regression for a line of music
- Video: Rush E in Desmos
- ask class: what is the *x-axis* representing? (time) The *y-axis*? (pitch)
- Video: Never gonna give you up in Desmos

Assessment (10 mins): *(how will you know that the lesson objective has been achieved?)*

Daily problem: Students write their names on the paper with invisible ink!

- define a function that contains the points $(x, y) = (2, 6)$ **and** $(x, y) = (3, 10)$, ie. a function that satisfies $(2, 6) \in f(x)$ **and** $(3, 10) \in f(x)$

Reflection:

DAY 2

Warmup / Hook (10 Minutes):

- What is the difference between $3*0$ and $3/0$?
- What about $\sqrt{2}$ and $\sqrt{(-2)}$?
- Think-pair-share with neighbour, eventually share answers as a class
- Explain direct association with $3/0$ not being allowed and $x=0$ not being allowed as input in $f(x) = 3/x$ (“allowed” = definable)
- Defining $g(x) = x^2$, what value of x gives us $g(x) = -2$?
- Think-pair-share with neighbour
- Similarly to before, it is impossible to find a (real) input $g(?)$ that would give us the output $g(?) = -2$ meaning it is an undefined output

Introduction / Opener (5 Minutes): *[Review - Activate - Recap]*

- Hand out daily problems that were marked last day
- Anonymous peer assessment of last day’s daily problem. Hand out the papers randomly and have students hand them back after the assessment.
- Students plug the two coordinate pairs into the function and verify whether they both satisfy the equality

Body (30 Minutes):

- Back to intro example, we identified that:
  - $x = 0$ is not a valid input for the function $f(x) = 3/x$ but all other inputs are definable
  - This actually tells us what is called **domain**: “x (input) can be anything except for 0”
  - $y = g(?) = -2$ is not a valid output of the function $g(x) = x^2$
  - using desmos.com we can observe that no negative numbers are valid outputs of this function. this tells us about what is called **range**: “y (output) can be any non-negative number”
- The way I remember which one goes with which variable is that we always say “domain and range” and we always write coordinates as *(x, y)*. can also imagine *f(domain) = range*
- Bracket notation:
  - exclusive () works like < and >
  - *D: (3, 4)* is equivalent to $3 < x < 4$
  - inclusive [] works like ≤ and ≥
  - *R: [5, 6]* is equivalent to $5 \leq y \leq 6$
- have class try examples *R: [0, ∞)* and $6.9 < x \leq 7.5$
- on desmos.com, graph $A: \sqrt{(x^2 + (y - 1)^2)} = 2$ and $B: y = sinx - 3$ have students define domain and range for both
  - answer: *(A) D: [-2, 2], R: [-1, 3]* and *(B) D: (-∞, ∞), R: [-4, -2]*

15 min lecture ⬆, 15 min activity ⬇

- have students gather in groups of 3-4. Using geogebra.com to graph functions quickly, each group must find a function with one of the following domain and range pairs:
  - D: (-∞, ∞), R: [0, ∞)
  - D: [0, ∞), R: [0, ∞)
  - D: [1, 3], R: [-5, -3]
  - D: (-∞, ∞), R: [-3, 3]
- some function ideas to help groups get started (display on the board after 5 mins in groups):
  - $y = x^2$, $y = \sqrt{x}$, $y = |x|$, $y = sinx$, $y^2+x^2=1$, $y = -x$, $y = 1/x$, $y = lnx$, $y = 3^x$
- Have students write their answers on the board (to be graphed) and explain their process for deciding on their functions. possible answers:
  - $y = |x|$
  - $y = \sqrt{x}$
  - $(x - 2)^2 + (1/4)y^2 = 1$
  - $y = 3sinx$

Closure (5 Minutes):

- If needed, fix any of the functions that didn’t quite satisfy the given domain and range
- Mention things like public domain (ideas that anyone can use without copyright, input of ideas into other art), field goal range (distance of a football kicker, output of kick)

Assessment (10 minutes): *(how will you know that the lesson objective has been achieved?)*

Daily problem: Students write their names on the paper with invisible ink!

- what is the range of the function $y = f(x) = 4$? try graphing it on scrap paper/the back of your paper if you are stuck
- are the following statements the same? explain using bracket notation:
  - x is any non-negative number

- x is any positive number

Reflection:

DAY 3

Warmup / Hook (10 Minutes):

Have students do this independently and then compare with a neighbour afterwards:

- Define a function which takes in a number and outputs that number plus 1
  - (answer) $f(x) = x + 1$
- Display on the board: what is the opposite of adding 1 to a number?
  - (answer) *subtracting 1 from a number*
- Define a function which takes in a number and outputs the opposite of adding 1 to that number
  - (answer) $g(x) = x - 1$; note the importance of choosing a new letter so that we don't get the functions confused with one another
- Have students compare answers with a neighbour

Introduction / Opener (5 Minutes): *[Review - Activate - Recap]*

- Hand out daily problems that were marked last day
- Anonymous peer assessment of last day's daily problem. Hand out the papers randomly and have students hand them back after the assessment.
- answers are *R: [4, 4]* and *no, because of the 0 in the middle; D: [0, ∞) versus D: (0, ∞)*

Body (30 Minutes):

- Go through examples of finding inverse of a function using the switching variables trick
  - $y = f(x) = x + 2$, switch to $x = y + 2$, rearrange to get inverse $y = x - 2$
  - $y = g(x) = x/4$, switch to $x = y/4$, rearrange to get inverse $y = 4x$
- model inverse function notation: $f^{-1}(x) = x - 2$, $g^{-1}(x) = 4x$
- note that worksheet contains terms *plot* and *label*
  - define *plotting a point* as calculating a coordinate pair that exists in the function and accurately placing it on the graph
  - define *labelling* as writing on your graph the following: title, x-axis, y-axis, horizontal axis title, vertical axis title, units/scale and sketch out an example on board
- inductive definition of function inverse (worksheet) (**20 mins**)

Closure (5-10 Minutes):

- Ask for groups' final observations/similarities
- If groups are not getting there, ask for any special observations about the diagonal line $y=x$ (draw it on the board for reference)
- Reiterate that though it doesn't work for 100% of functions, usually the inverse of a function just looks like the regular function if it were mirrored across the $y = x$ line
- if students need more explanation, draw an example like $f(x) = 3x - 4$ and $f^{-1}(x) = (⅓)(x + 4)$

Assessment (5-10 minutes): *(how will you know that the lesson objective has been achieved?)*

Daily problem: Students write their names on the paper with invisible ink!

- if we try to find the inverse of an inverse function (ie. the inverse of the inverse of a function), what do we get? Explain your reasoning using examples of both written functions and graphs.

Reflection: