The goal of this mathematical model is to derive the needed volume of the compliance chamber to accommodate physiological compliances. In other words, **we want to know the total volume of air in the compliance chamber when there is only atmospheric pressure**.

Compliance (C) is defined as a change in volume for a given change in pressure. In the physiological context, dV and dP will always be the same sign, as an increase in vascular pressure leads to an increase in volume of the vessel and vice versa. In our compliance chamber it is the opposite, as an increase in pressure results in a reduction of the volume of air in the chamber. As such there is a negative sign in the equation.

$$[1]\ \ C = -\frac{\Delta V}{\Delta P}\ where\ \Delta V = V_2 - V_1\ and\ \Delta P = P_2 - P_1$$

Additionally, we want to assume that there will be isothermal compression and expansion, which yields the second equation.

$$[2]\ \ V_1 P_1 = V_2 P_2$$

Rearranging equation [1] to solve for V_2:

$$[3]\ \ V_2 = V_1 - C\Delta P$$

Substituting V_2 from equation [3] into equation [2] and solving for V_1:

$$V_1 = \frac{P_2}{P_1}(V_1 - C\Delta P)$$

$$V_1 = \frac{P_2}{P_1}V_1 - \frac{P_2}{P_1}C\Delta P$$

$$V_1\left(1 - \frac{P_2}{P_1}\right) = -\frac{P_2}{P_1}C\Delta P$$

$$V_1 = \frac{-\frac{P_2}{P_1}C\Delta P}{\left(1 - \frac{P_2}{P_1}\right)}$$

Multiply by $-P_1/-P_1$

$$V_1 = \frac{C\Delta P * P_2}{P_2 - P_1}$$

$$[4]\ \ V_1 = C * P_2$$

The limiting case would be the case of highest compliance, as the air would be compressing the most for a given change in pressure. For this we use the pulmonary compliance value of
$C = 2.6 * 10^{-8} \frac{\mathrm{m5}}{\mathrm{N}}$ [i] and
P_2 = peak pulmonary pressure + atmospheric pressure = 20 mmhg + 760 mmHg = 780 mmHg = 103991.16 Pa

Plug into equation [4] to get

$$V_1 = 2730\ mL$$

Therefore, we need approximately 2.7 L of air in the compliance chamber for it to behave similarly to vessels in terms of the amount of back pressure it will generate for a given stroke volume. This value doesn't need to be exact as we will also be able to control the pump and the fluidic resistor, but I believe this is a solid rationale for having the compliance chamber be in the realm of 2.7 L in size. Doing the same calculations with the minimum systemic compliance (systemic compliance is not always constant) yields a volume of approximately 1L. This would be accomplished by raising the water level in the compliance chamber until there is only ~1L of air.

Accounting for the fact that we will have fittings into and out of the compliance chamber which will always be submerged, the actual volume of the chamber will be closer to 3L.

i B. Knierbein, H. Reul, R. Eilers, M. Lange, R. Kaufmann, and G. Rau, "Compact mock loops of the systemic and pulmonary circulation for blood pump testing," *Int. J. Artif. Organs*, vol. 15, no. 1, pp. 40–48, 1992.