ECE 6350

Introduction to Object Oriented Concepts Using F90/95

D. R. Wilton

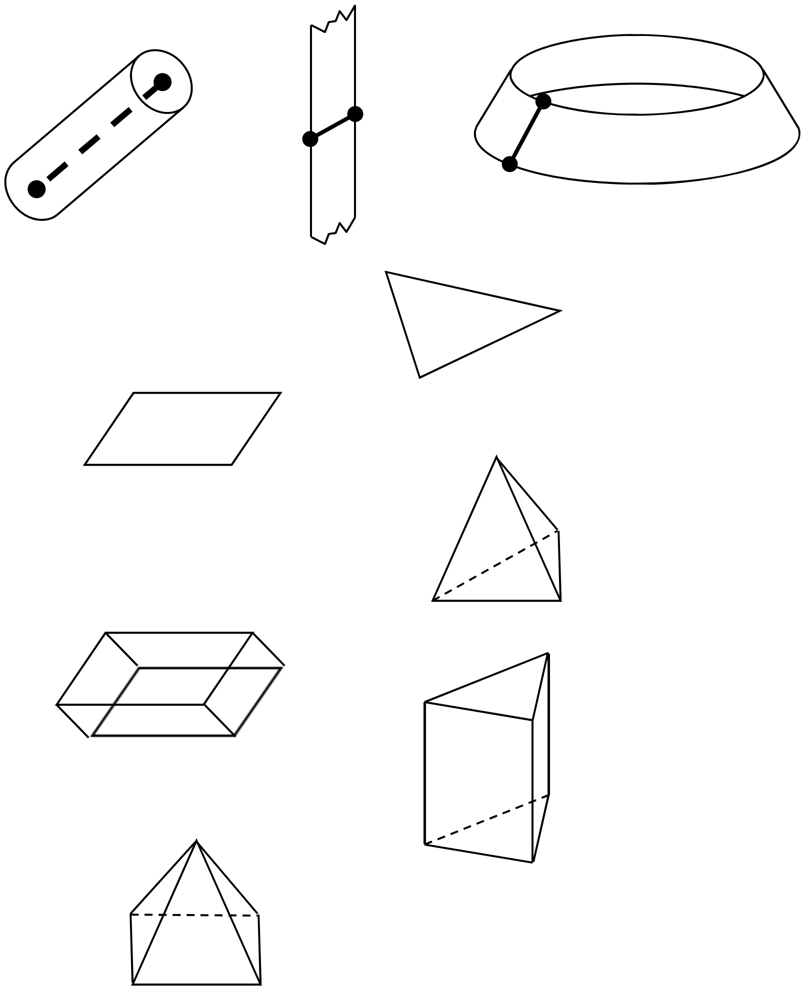
University of Houston

Objects Needed for an Object-Oriented CEM Computer Code (EMPACK)

- **Elements**
- **Basis/Testing functions**
- **Green's functions**
- **(Discretized) Operators**
- **Element matrices/vectors**
- **System matrices/vectors**
- **Quadrature rules**
- **Excitations**
- **Solvers**

Elements

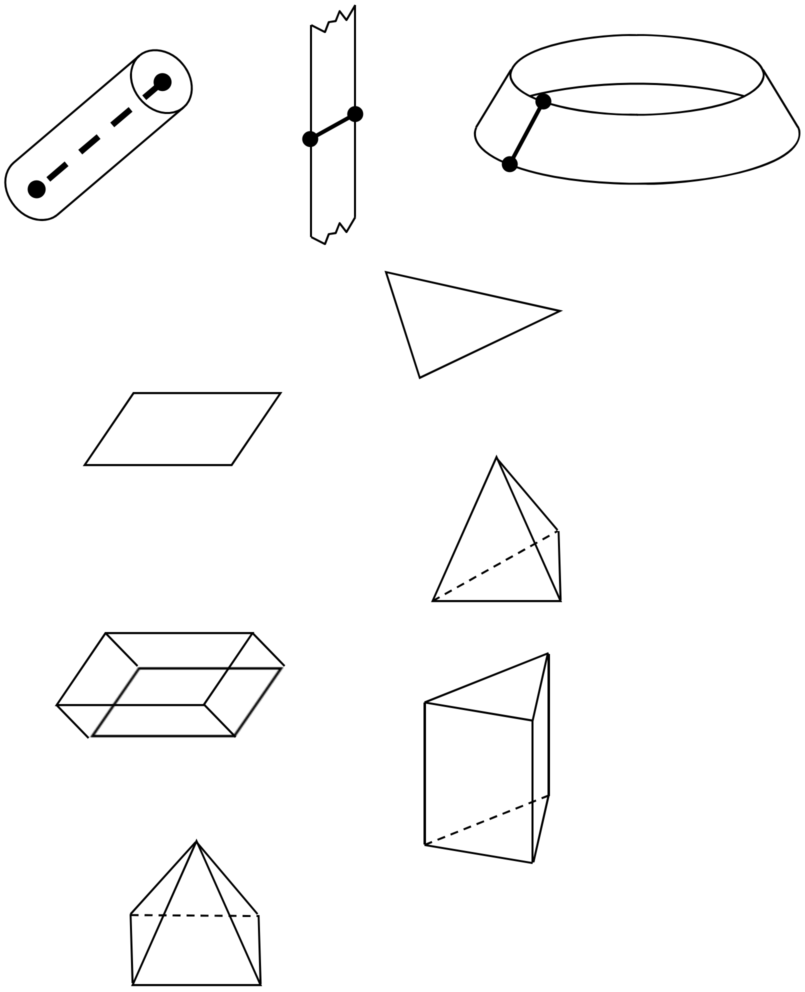
- **Line segments**
- **Triangles**
- **Rectangles**
- **Tetrahedrons**
- **Bricks**
- **Wedges**
- **Pyramids**



Elements (cont'd)

For each element type, we also need

- Geometry order (i.e., linear or curvilinear)
- Node list
- DoF list
- Unknown types (J,M)
- Basis/testing type (scalar, div-, or curl-conforming)
- Basis order, etc.



Basis/Testing Functions

	Basis	Derivative(s)
Scalar, PWC :	$\Pi_n(\mathbf{r})$	$--$
Scalar, PWL :	$\Lambda_n(\mathbf{r})$	$\dfrac{d\Lambda_n(\mathbf{r})}{d\ell}, \nabla\Lambda_n(\mathbf{r})$
Vector PWC :	$\Pi_n(\mathbf{r})$	$--$
Vector PWL :	$\Lambda_n(\mathbf{r}), \Omega_n(\mathbf{r})$	$\nabla\cdot\Lambda_n(\mathbf{r}), \nabla\times\Omega_n(\mathbf{r})$

Green's Functions

Static 2D : $\dfrac{1}{2\pi}\ln\dfrac{1}{D}$

Static 3D : $\dfrac{1}{4\pi R}$

Dynamic 2D : $H_0^{(2)}(kD)$

Dynamic 3D : $\dfrac{e^{-jkR}}{4\pi R}$

Layered media : $\mathcal{G}^A,\ K^\Phi, P_z,$ etc.

Wire, waveguide, cavity, periodic, etc....

$$D=\sqrt{(x-x')^2+(y-y')^2}$$
$$=\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|$$

$$R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$
$$=\left|\mathbf{r}-\mathbf{r}'\right|$$

Discretized System Operators

TX Line : $j\omega\mu < \Lambda_m, \Lambda_n > \; + \; \dfrac{1}{j\omega\varepsilon} < \dfrac{d\Lambda_m}{d\ell}, \dfrac{d\Lambda_n}{d\ell} >$

FEM Helmholtz : $j\omega\mu < \Lambda_m ; \Lambda_n > \; + \; \dfrac{1}{j\omega\varepsilon} < \nabla \times \Lambda_m, \nabla \times \Lambda_n >$

Static Potential IE : $\dfrac{1}{\varepsilon} < \Pi_m, G, \Pi_n >$

TM EFIE : $j\omega\mu < \Lambda_m ; G, \Lambda_n >$

TE, 3D EFIE : $j\omega\mu < \Lambda_m ; G, \Lambda_n > \; + \; \dfrac{1}{j\omega\varepsilon} < \nabla \cdot \Lambda_m, G, \nabla \cdot \Lambda_n >$

Etc. ...

Discretized Element Operators

TX Line : $$\sigma_i^e \sigma_j^e \left[j\omega\mu < \Lambda_i^e, \Lambda_j^e > \; + \; \frac{1}{j\omega\varepsilon} < \frac{d\Lambda_i^e}{d\ell}, \frac{d\Lambda_j^e}{d\ell} > \right]$$

FEM Helmholtz : $$\sigma_i^e \sigma_j^e \left[j\omega\mu < \Lambda_i^e; \Lambda_j^e > \; + \; \frac{1}{j\omega\varepsilon} < \nabla \times \Lambda_i^e, \nabla \times \Lambda^e_{jn} > \right]$$

Static Potential IE : $$\frac{1}{\varepsilon} < \Pi_m, G, \Pi_n >$$

TM EFIE : $$j\omega\mu < \Lambda_i^e; G, \Lambda_j^f > \qquad \left(\Lambda_i^e = \Lambda_i^e \hat{\mathbf{z}} \right)$$

TE, 3D EFIE : $$\sigma_i^e \sigma_j^f \left[j\omega\mu < \Lambda_i^e; G, \Lambda_j^f > \; + \; \frac{1}{j\omega\varepsilon} < \nabla \cdot \Lambda_i^e, G, \nabla \cdot \Lambda_j^f > \right]$$

MFIE, CFIE, PMCHWT, etc. ...

Element and System Matrices/Vectors

	Global Matrix/Vector		Element Matrix/Vector
EFIE, TX Line :	Z_{mn}, V_m	$\Leftrightarrow$	Z_{ij}^{ef}, V_i^e
FEM Helmholtz :	Y_{mn}, I_m	$\Leftrightarrow$	Y_{ij}^{e}, I_i^e
Static 3D :	S_{mn}, V_m	$\Leftrightarrow$	S_{ij}^{ef}, V_i^e

Quadrature Rules of Various Orders

- Gauss Legendre
- Gauss triangle
- MRWlog ("Log-Lin")
- Singularity specific

$$\int_{\mathcal{D}} f(\mathbf{r})\, d\mathcal{D} \approx \sum_{k=1}^{K} w_k\, f\left(\mathbf{r}(\xi^{(k)})\right)$$

Quadrature Rule:

$$\left\{ w_k, \xi^{(k)} \right\}_{k=1}^{K}$$

Excitations

- Plane waves:

$$\mathbf{E}^i = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}}, \ \mathbf{H}^i = \frac{\mathbf{k}\times\mathbf{E}_0}{k\eta} e^{-j\mathbf{k}\cdot\mathbf{r}}$$

- Voltage sources:

$$\mathbf{E}^i = V_0 \delta(\ell - \ell_0)\hat{\ell}$$

- Pt. sources, $G(\mathbf{r},\mathbf{r}') = \dfrac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$:

$$\mathbf{E}^i = -j\omega\mu\left(\mathcal{I} + \frac{\nabla\nabla}{k^2}\right)\cdot I_0 G(\mathbf{r},\mathbf{r}')d\ell,$$

$$\mathbf{H}^i = I_0 \nabla G(\mathbf{r},\mathbf{r}') \times d\ell$$

- Line sources,

$$G(\boldsymbol{\rho},\boldsymbol{\rho}') = \frac{H_0^{(2)}(|\boldsymbol{\rho}-\boldsymbol{\rho}'|)}{4j}:$$

TM case:

$$\mathbf{E}^i = -j\omega\mu I_0 G(\boldsymbol{\rho},\boldsymbol{\rho}')\hat{\mathbf{z}},$$

$$\mathbf{H}^i = I_0 \nabla G(\boldsymbol{\rho}-\boldsymbol{\rho}') \times \hat{\mathbf{z}}$$

TE case:

$$\mathbf{E}^i = -j\omega\mu\left(\mathcal{I} + \frac{\nabla\nabla}{k^2}\right)\cdot I_0 G(\boldsymbol{\rho},\boldsymbol{\rho}')d\ell,$$

$$\mathbf{H}^i = I_0 \nabla G(\boldsymbol{\rho},\boldsymbol{\rho}') \times d\ell$$

Solvers

- Direct (Gauss elimination)
- Iterative (QMR, BiConGStab)
- Eigenvalue

Examples of Other Objects

- **Constants (**$\pi, \; j = \sqrt{-1}, \; \mu_0, \; \varepsilon_0, \; \hat{\mathbf{x}},$ **etc.)**
- **Special Functions (**$H_0^{(2)}(x), \;\; \mathrm{Arcsinh}(x),$ **etc.)**
- **Utilities (open files, etc.)**
- **Vectors, dyads (w / their algebra defined)**

The End