# Lecture 6:
# Back-testing statistical-arbitrage strategies

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# Simulation of trading Profit/Loss

$Q_{i,n}$ = investment in stock $i$ at the start of period $n$

$R_{i,n}$ = dividend-adjusted return of stock over period $n$

$r$ = Fed Funds rate or reference rate for cash

$r + \delta r$ = interest paid for cash on long stock

$r - \delta r$ = interest received for cash on short stock

$\varepsilon$ = market impact + clearing & commissions

$E_n$ = equity in the account at start of period $n$

Typically, we will assume $\varepsilon = 5$ bps $= 0.0005$,

and $\delta r = 0$, for simplicity

# Basic P/L equation

$$E_{n+1} = E_n + r\Delta t E_n + \sum_{i=1}^{N} Q_{i,n} R_{i,n} - r\Delta t \left( \sum_{i=1}^{N} Q_{i,n} \right)$$

$$- \delta r \Delta t \sum_{i=1}^{N} \left| Q_{i,n} \right| - \varepsilon \sum_{i=1}^{N} \left| Q_{i,n+1} - Q_{i,n} \right|$$

$$\Lambda = \frac{\sum_{i=1}^{N} \left| Q_{i,n} \right|}{E_n} = \text{leverage ratio}$$

$$\Lambda = \frac{\text{Long Market Value} + \left| \text{Short Market Value} \right|}{\text{Equity}}$$

# Examples of Leverage

Long-only: $\Lambda = \frac{L}{E}$

Long-only, Reg T: (margin acct) $L \le 2E \quad \therefore \quad \Lambda \le 2$

130-30 Investment funds $L = 1.3E, \ |S| = 0.3E \quad \therefore \quad \Lambda = 1.6$

Long-short $-Neutral, Reg T: $L + |S| \le 2E \quad \therefore \quad \Lambda \le 2$

:

Long-short, Equal target position in each stock $Q_i \le \pm \frac{\Lambda_{\max} E}{N} \quad \therefore \sum_i |Q_i| \le \Lambda_{\max} E$

# Sharpe Ratio

$$\mu = \frac{1}{\Delta t N_{\text{periods}}} \sum_{n=1}^{N_{\text{periods}}} \frac{E_n - E_{n-1}}{E_{n-1}}$$

Expected return over simulation period

$$\sigma^2 = \frac{1}{\Delta t N_{\text{periods}}} \sum_{n=1}^{N_{\text{periods}}} \left( \frac{E_n - E_{n-1}}{E_{n-1}} - \mu \Delta t \right)^2$$

Variance over simulation period

$$S = \frac{\mu - r}{\sigma}$$

Sharpe Ratio

The Sharpe ratio measures returns above the risk-free rate.

It is independent of the leverage of the strategy (dimensionless).

# Modeling the Evolution of Stock Residuals

$$\frac{dS_i(t)}{S_i(t)} = \sum_{k=1}^{N_{factors}} \beta_{ik} \frac{dP_k(t)}{P_k(t)} + \varepsilon_i(t)$$

Stock returns a sum of the market return and a residual process

$$\varepsilon_i(t) = \alpha_i dt + dX_i(t)$$

Residual= drift component (expected excess return above mkt.) + increment of a stationary process

$$dX_i(t) = \kappa_i(m_i - X_i(t))dt + \sigma_i dW_i(t)$$

Ornstein-Uhlenbeck AR-1 process

Daily sampling frequency
Statistical Estimation Window=3 months (~ 60 business days)

# Defactoring using ETFs

$$R_i = \beta_{ij} R_j^{ETF} + \varepsilon_i$$

Regress returns on sector ETF returns

$$\beta_{ij} = \begin{cases} \overline{\sigma_i}\,\overline{\rho_{ij}} / \overline{\sigma_j} \\ 0 \end{cases}$$

If stock I is in sector j

otherwise

In some cases, we construct ``synthetic'' ETFs (e.g., if the ETF did not exist in the past). These are taken to be Capitalization-Weighted

# Trading Signals

Introduce the **s-score** for each stock:

$$s_i(t) = \frac{X_i(t) - m_i}{\sigma_{eq,i}}$$

Example:

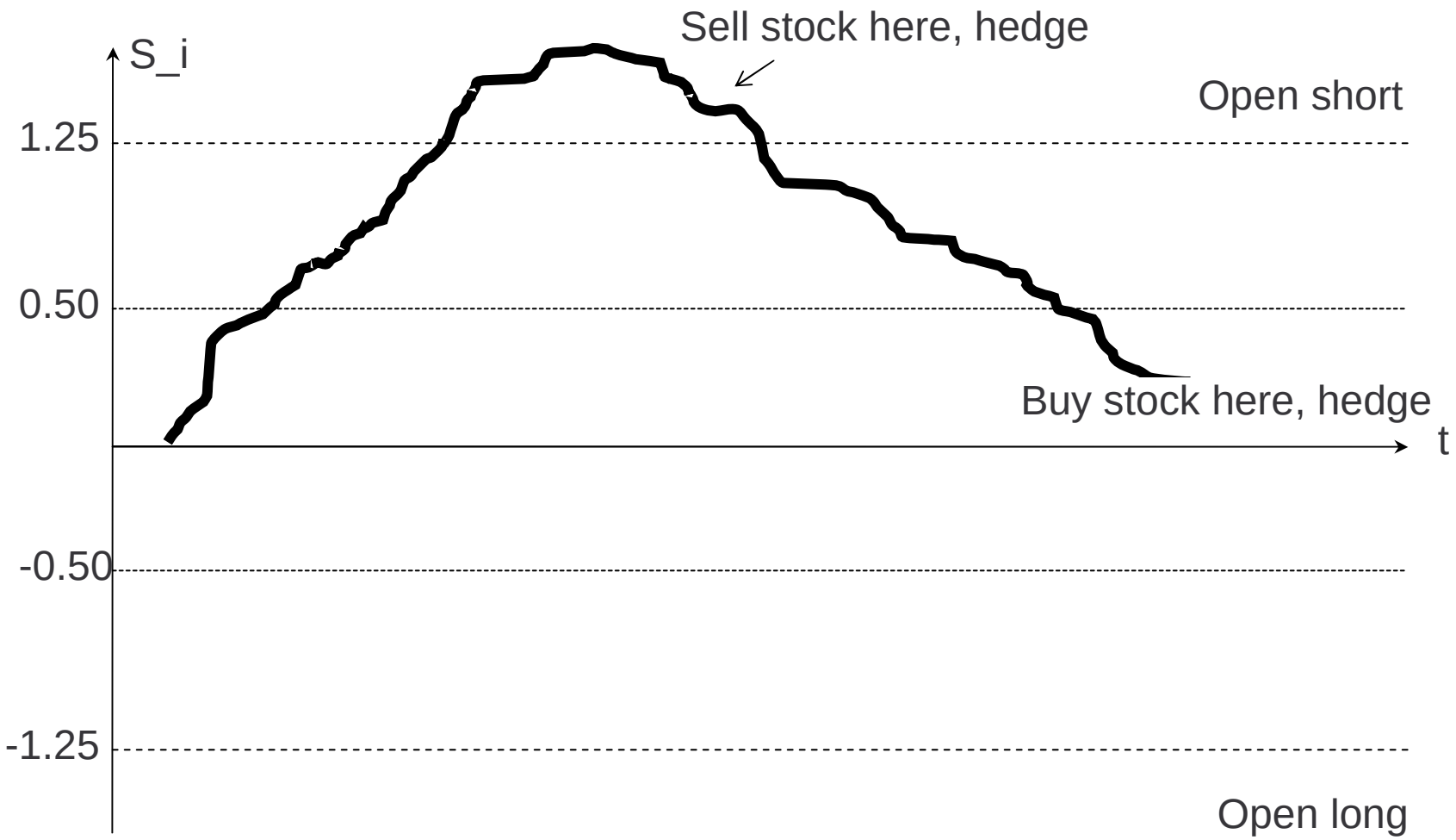
Open long position if $s_i < -1.25$

Open short position if $s_i > +1.25$

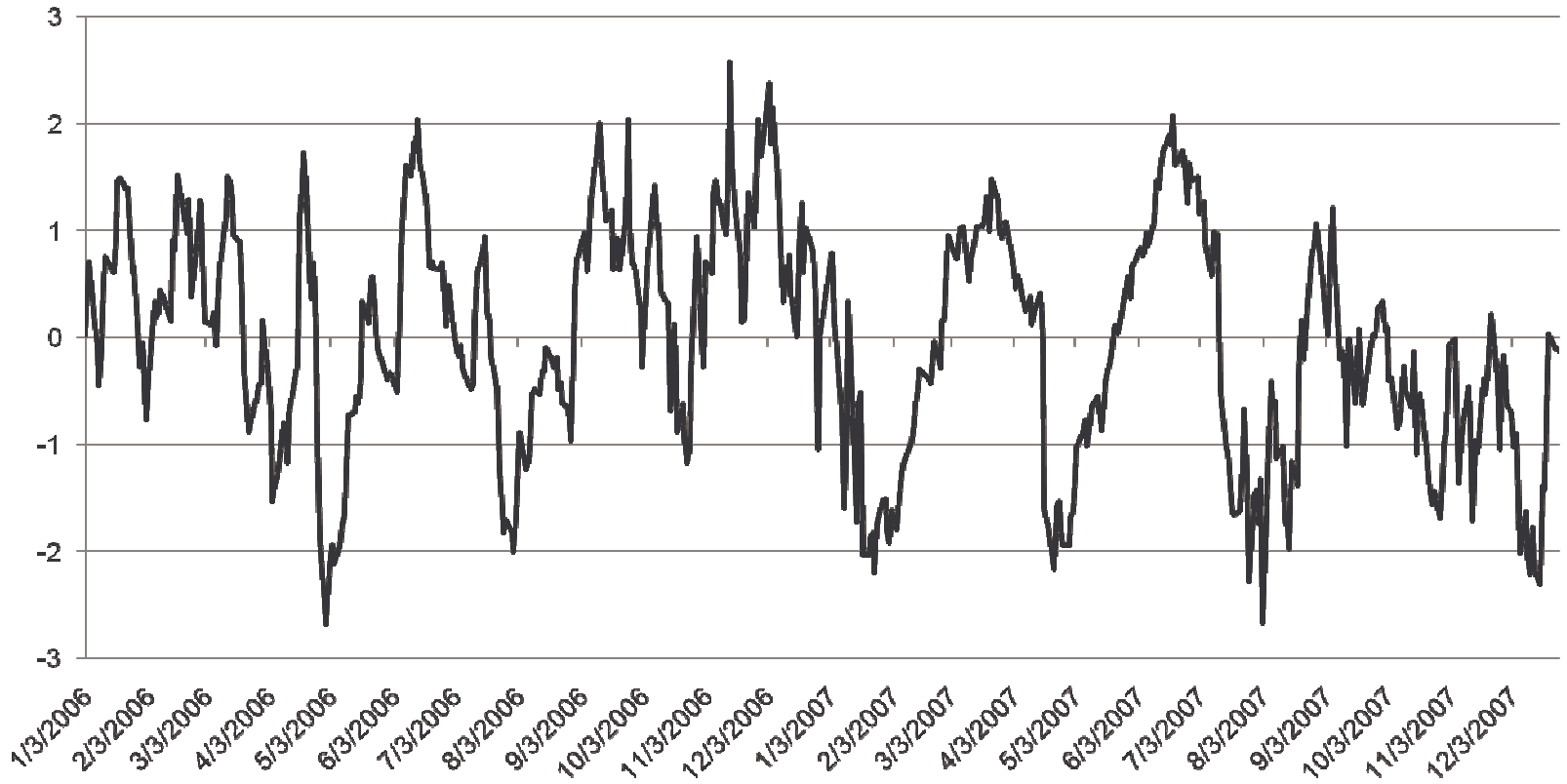
Close long position if $s_i > -0.50$

Close short position if $s_i < +0.50$

# The trading signal seen graphically



# S-score of JPM ( vs. XLF )



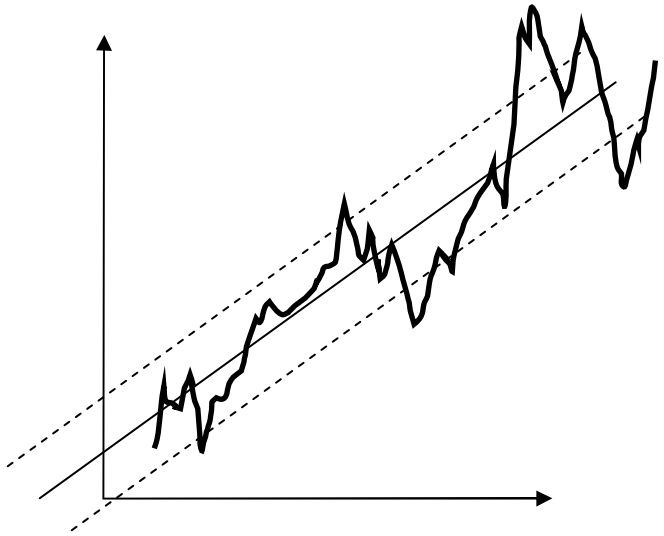
# Including the drift in signal-generation

$$\mu_i = \alpha_i + \frac{E\{dX_i(t) \mid X_i(t)\}}{dt}$$

$$= \alpha_i + \kappa_i (m_i - X_i(t))$$

$$= \alpha_i - \kappa_i \sigma_{eq,i} s_i$$

$$= -\kappa_i \sigma_{eq,i} \left( s_i - \frac{\alpha_i}{\kappa_i \sigma_{eq,i}} \right)$$



$$\therefore \qquad s_{\mathrm{mod},i} = s_i - \frac{\alpha_i}{\kappa_i \sigma_{eq,i}} = s_i - \frac{\alpha_i \tau_i}{\sigma_{eq,i}}$$

# Statistics on the Estimated OU Parameters

| ETF | Abs(Alpha) | Beta | Kappa | Reversion days | EquiVol | Abs(m) |
|---|---|---|---|---|---|---|
| **HHH** | 0.20% | 0.69 | 38 | 7 | 4% | 3.3% |
| **IYR** | 0.11% | 0.90 | 39 | 6 | 2% | 1.8% |
| **IYT** | 0.18% | 0.97 | 41 | 6 | 4% | 3.0% |
| **RKH** | 0.10% | 0.98 | 39 | 6 | 2% | 1.7% |
| **RTH** | 0.17% | 1.02 | 39 | 6 | 3% | 2.7% |
| **SMH** | 0.19% | 1.01 | 40 | 6 | 4% | 3.2% |
| **UTH** | 0.09% | 0.81 | 42 | 6 | 2% | 1.4% |
| **XLF** | 0.11% | 0.83 | 42 | 6 | 2% | 1.8% |
| **XLI** | 0.15% | 1.15 | 42 | 6 | 3% | 2.4% |
| **XLK** | 0.17% | 1.03 | 42 | 6 | 3% | 2.7% |
| **XLP** | 0.12% | 1.01 | 42 | 6 | 2% | 2.0% |
| **XLV** | 0.14% | 1.05 | 38 | 7 | 3% | 2.5% |
| **XLY** | 0.16% | 1.03 | 39 | 6 | 3% | 2.5% |
| **Total** | 0.15% | 0.96 | 40 | 6 | 3% | 2.4% |

Average over 2006-2007

Correction to the s-score due to alpha is of the order of 0.15 (small).

# Constructing market-neutral portfolios from signals

-- Large, diversified trading universe of U.S. equities (~ 1000 names)

-- Select within the trading universe those stocks that have a trading signal (large magnitude of s-score) and open trades

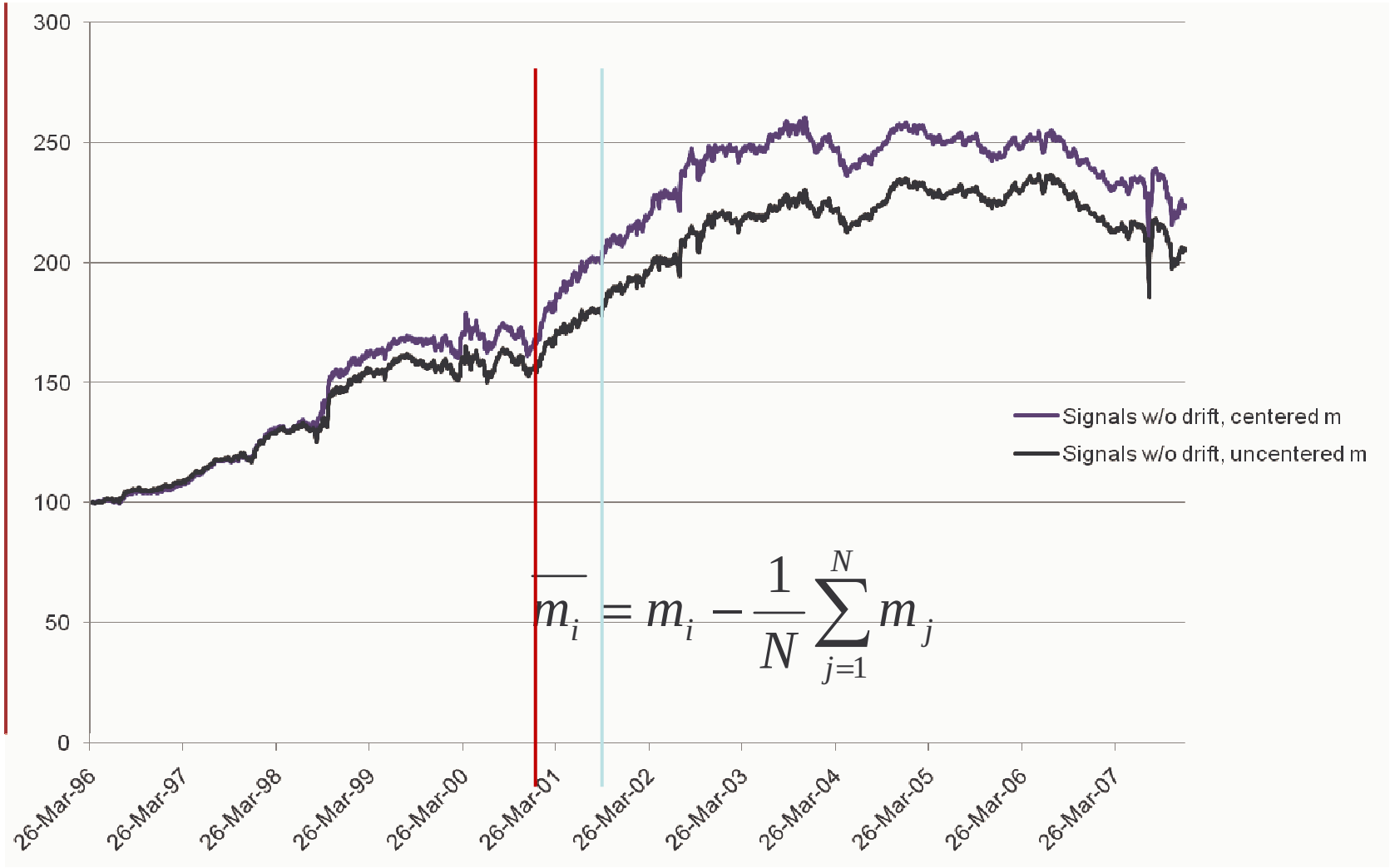
-- Monitor for closing trades through s-score as well

-- Keep all sectors beta-neutral by using ETFs to balance the portfolio and maintain sector-neutrality

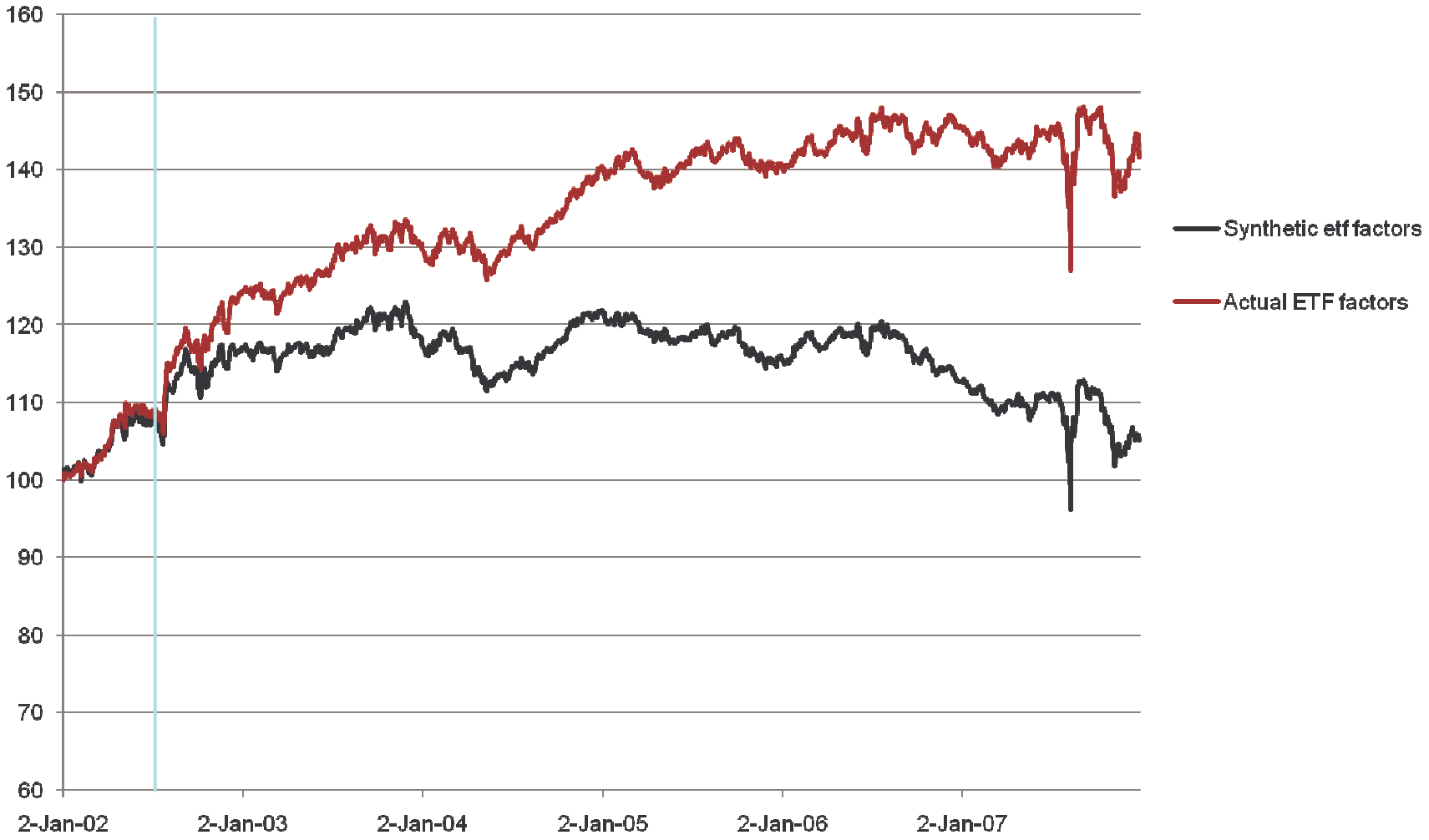
-- Leverage = 2+2 (i.e. $2 long, $2 short for $1 of capital)

-- Expected Volatility for this Leverage < 7% annualized (< 50 bps/day).

# Using synthetic ETFs as factors



# Signals based on actual ETFs versus synthetic ETFs



# Annualized Returns (synthetic ETFs)

| | HHH | IYR | IYT | OIH | RKH | RTH | SMH | UTH | XLE | XLF | XLI | XLK | XLP | XLV | XLY | Porfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1996** | 9% | 10% | -13% | 8% | 7% | -11% | 10% | 12% | 3% | 14% | 6% | 17% | -5% | 15% | 4% | 6% |
| **1997** | 1% | 14% | 0% | 25% | 16% | 19% | 46% | 10% | -8% | 23% | 8% | 16% | 7% | 19% | 35% | 15% |
| **1998** | 15% | -7% | -8% | 9% | 62% | 55% | 47% | 24% | 10% | 30% | 18% | 59% | 1% | -2% | 63% | 25% |
| **1999** | -31% | -13% | 22% | -13% | -13% | 8% | 37% | -14% | 10% | 27% | 18% | 41% | -21% | 2% | 13% | 5% |
| **2000** | -18% | 10% | 17% | -7% | 55% | 3% | -35% | 12% | 10% | 4% | -15% | 36% | 4% | -9% | -32% | 2% |
| **2001** | -32% | 35% | 12% | 7% | 72% | 42% | -40% | 10% | 17% | 1% | 45% | 68% | 12% | 37% | 84% | 25% |
| **2002** | 30% | 20% | -1% | 9% | 50% | 20% | -14% | -22% | -12% | 28% | 17% | 41% | 31% | 1% | 46% | 16% |
| **2003** | 9% | 1% | -6% | -3% | 46% | 26% | -27% | -11% | 1% | -5% | -11% | 38% | -11% | -19% | -22% | 0% |
| **2004** | 10% | 1% | 19% | 2% | 19% | -7% | 2% | -10% | 6% | 2% | 11% | 15% | 0% | -12% | 0% | 4% |
| **2005** | 1% | -29% | -4% | -5% | -1% | 4% | 6% | -18% | 0% | -9% | -1% | 10% | -15% | -8% | -9% | -5% |
| **2006** | -9% | -24% | -1% | -3% | 22% | -8% | -3% | 2% | -9% | -12% | 12% | 9% | -12% | -17% | 17% | -2% |
| **2007** | 27% | -46% | 16% | -19% | -32% | -27% | 3% | -19% | -11% | -25% | 19% | 0% | 0% | -12% | 31% | -6% |

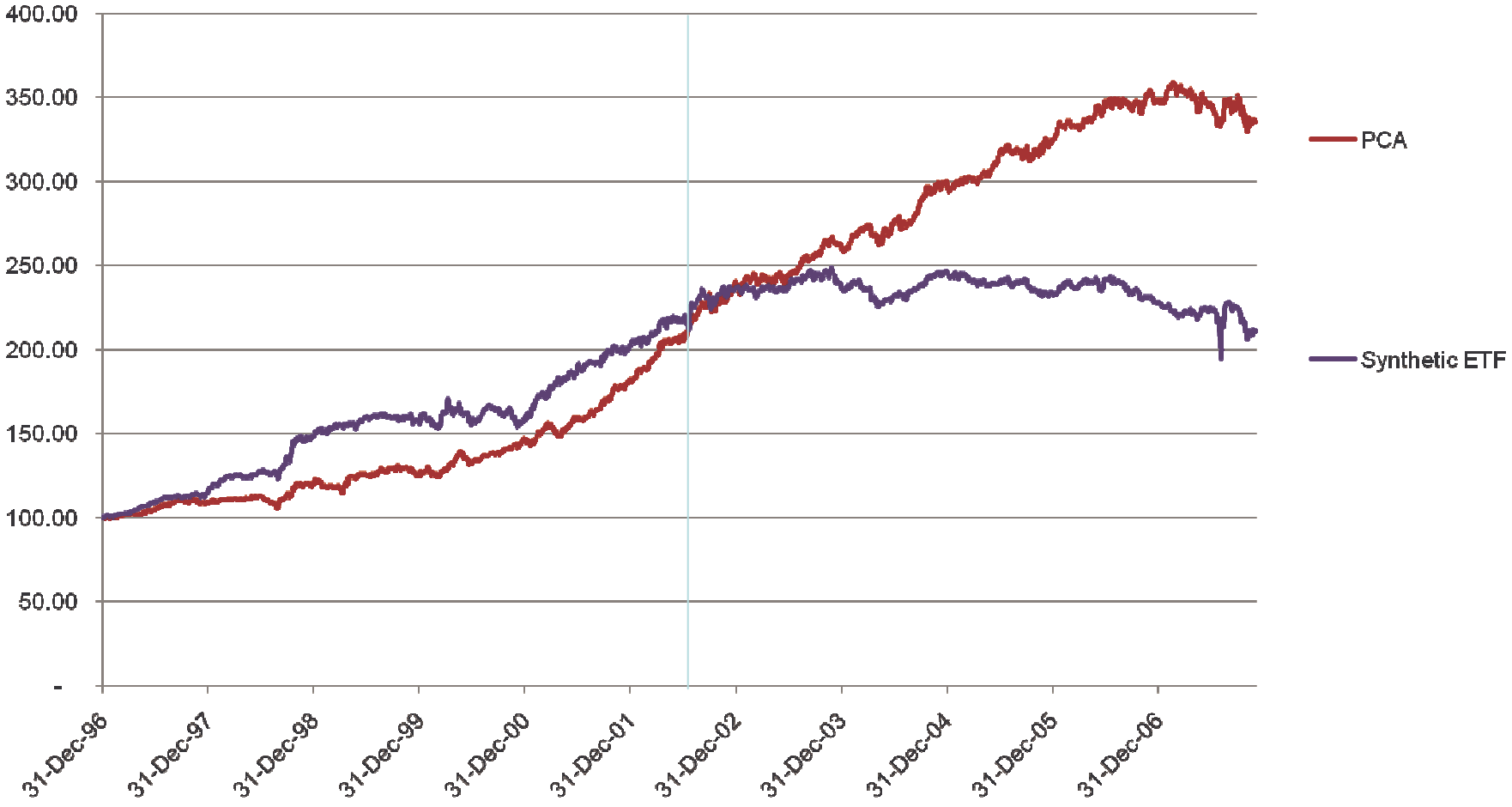
# Sharpe ratio: synthetic ETFs

| | HHH | IYR | IYT | OIH | RKH | RTH | SMH | UTH | XLE | XLF | XLI | XLK | XLP | XLV | XLY | Porfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1996** | 1.7 | 1.7 | (1.2) | 1.0 | 0.8 | (0.6) | 0.6 | 1.4 | 0.6 | 2.3 | 0.5 | 1.5 | (0.5) | 1.1 | 0.4 | 1.7 |
| **1997** | 0.1 | 1.5 | (0.0) | 2.5 | 1.2 | 1.1 | 2.2 | 1.1 | (1.0) | 2.3 | 0.6 | 1.1 | 0.4 | 1.5 | 2.9 | 3.6 |
| **1998** | 0.9 | (0.5) | (0.5) | 0.8 | 2.5 | 1.8 | 2.4 | 2.0 | 1.1 | 2.1 | 0.8 | 3.0 | 0.1 | (0.1) | 2.8 | 3.4 |
| **1999** | (1.0) | (1.3) | 1.5 | (1.3) | (0.7) | 0.3 | 1.2 | (1.2) | 1.4 | 1.9 | 1.1 | 1.9 | (1.1) | 0.1 | 0.6 | 0.8 |
| **2000** | (0.4) | 1.0 | 1.2 | (0.6) | 2.1 | 0.1 | (0.7) | 0.7 | 1.0 | 0.2 | (0.8) | 0.9 | 0.1 | (0.5) | (1.1) | 0.3 |
| **2001** | (0.9) | 2.8 | 0.7 | 0.6 | 2.7 | 1.5 | (0.9) | 0.6 | 1.6 | 0.1 | 1.9 | 1.9 | 0.6 | 1.4 | 3.3 | 2.9 |
| **2002** | 1.9 | 1.5 | (0.1) | 1.0 | 2.1 | 0.7 | (0.5) | (1.1) | (1.3) | 1.6 | 0.8 | 2.0 | 1.3 | 0.0 | 1.8 | 2.0 |
| **2003** | 0.5 | 0.0 | (0.4) | (0.4) | 2.6 | 1.3 | (1.3) | (0.9) | 0.1 | (0.4) | (0.8) | 2.5 | (0.6) | (1.0) | (1.1) | 0.1 |
| **2004** | 0.7 | 0.1 | 1.2 | 0.3 | 1.3 | (0.4) | 0.1 | (1.1) | 0.6 | 0.1 | 1.1 | 1.2 | (0.0) | (0.8) | (0.0) | 0.8 |
| **2005** | 0.1 | (2.1) | (0.3) | (0.8) | (0.1) | 0.2 | 0.5 | (2.1) | 0.0 | (0.8) | (0.1) | 1.0 | (1.1) | (0.6) | (0.5) | (1.3) |
| **2006** | (0.7) | (1.8) | (0.1) | (0.3) | 1.6 | (0.4) | (0.2) | 0.3 | (0.7) | (1.1) | 0.9 | 0.7 | (0.9) | (1.0) | 1.1 | (0.5) |
| **2007** | 2.1 | (2.1) | 0.6 | (1.4) | (1.1) | (0.9) | 0.1 | (1.1) | (0.8) | (1.0) | 1.0 | (0.0) | 0.0 | (0.6) | 1.1 | (0.5) |
| | 0.4 | 0.1 | 0.2 | 0.1 | 1.2 | 0.4 | 0.3 | (0.1) | 0.2 | 0.6 | 0.6 | 1.5 | (0.2) | (0.0) | 0.9 | 1.1 |

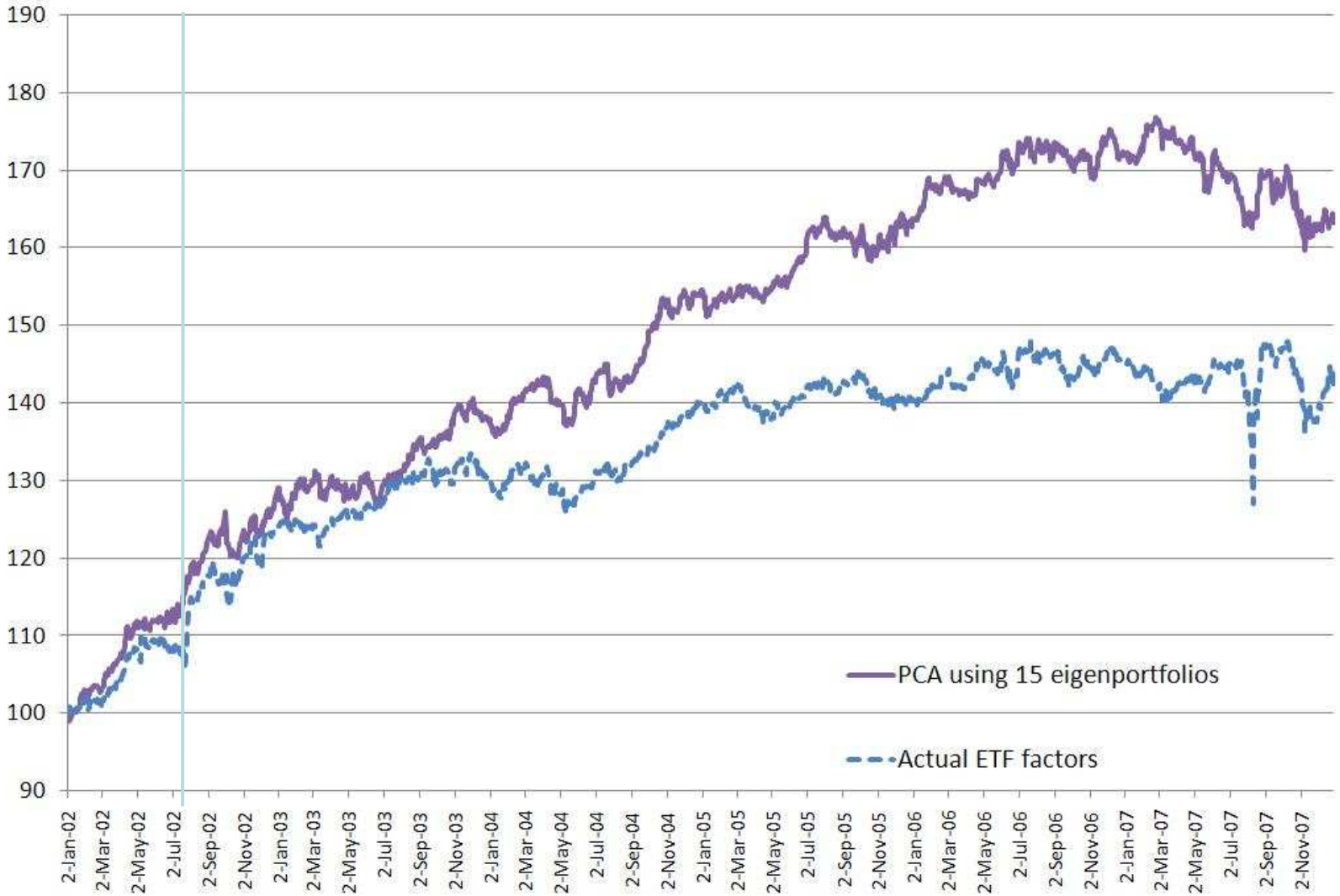
# Sharpe Ratio: Actual ETFs

| | HHH | IYR | IYT | OIH | RKH | RTH | SMH | UTH | XLE | XLF | XLI | XLK | XLP | XLV | XLY | Porfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2002** | 1.9 | 2.1 | 1.4 | 0.6 | 2.4 | 1.5 | (0.7) | (0.2) | (0.2) | 1.8 | 0.7 | 1.5 | 1.8 | (0.1) | 2.4 | 2.7 |
| **2003** | (0.2) | 0.8 | (0.3) | (0.5) | 1.4 | 1.1 | (1.0) | (0.1) | 0.5 | 0.6 | (0.6) | 2.6 | 0.3 | (0.4) | (0.4) | 0.8 |
| **2004** | 0.9 | 1.6 | (0.7) | 0.4 | 0.5 | 0.1 | 0.2 | (0.4) | 0.6 | 0.6 | 1.4 | 1.9 | 0.5 | (0.6) | 0.3 | 1.6 |
| **2005** | 0.3 | (1.5) | 0.8 | (0.6) | 0.3 | 0.5 | 0.5 | (1.1) | (0.1) | 0.9 | 0.6 | 1.3 | (0.7) | 0.2 | 0.0 | 0.1 |
| **2006** | (0.2) | (1.3) | 0.0 | (0.2) | 0.9 | (0.1) | 0.5 | 1.7 | (0.5) | (0.6) | 1.7 | 1.7 | (0.0) | (0.4) | 2.0 | 0.7 |
| **2007** | (0.4) | (0.3) | 0.0 | (1.3) | (1.2) | (0.7) | 0.9 | (0.7) | (1.0) | (0.6) | 1.1 | 0.6 | 0.4 | (0.5) | 1.3 | (0.2) |
| | 0.4 | 0.2 | 0.2 | (0.3) | 0.7 | 0.4 | 0.1 | (0.1) | (0.1) | 0.5 | 0.8 | 1.6 | 0.4 | (0.3) | 0.9 | 0.9 |

# Signals based on 15 PCA factors outperform synthetic ETFs



# PCA 15 Factors vs. ETFs (actual)



# 15 PCA factors: performance measures

| year | AnnRetPL | AnnVolPL | SharpePL |
|---|---|---|---|
| 1997 | 9% | 5% | 1.73 |
| 1998 | 11% | 6% | 1.71 |
| 1999 | 3% | 7% | 0.49 |
| 2000 | 16% | 7% | 2.44 |
| 2001 | 22% | 8% | 2.86 |
| 2002 | 28% | 7% | 3.73 |
| 2003 | 8% | 7% | 1.23 |
| 2004 | 14% | 5% | 2.56 |
| 2005 | 8% | 5% | 1.53 |
| 2006 | 7% | 5% | 1.42 |
| 2007 | -6% | 16% | (0.36) |

# Sensitivity to number of PCA factors

-- The separation between systematic and idiosyncratic components is a theoretical construct. **It is not observable!**

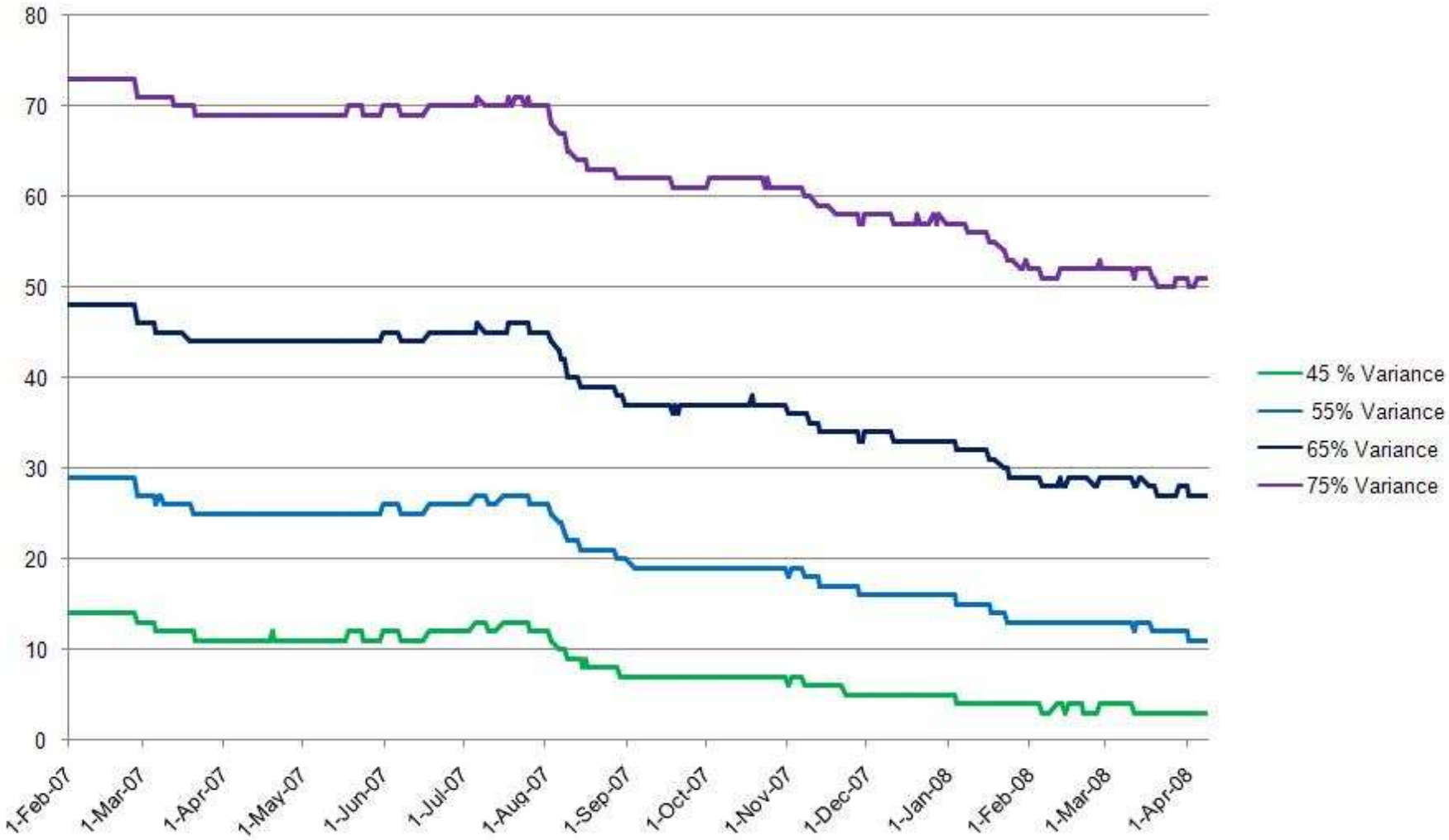
-- Tradeoff between ``systematic'' and ``idiosyncratic'' model of stock returns

-- **Too few factors** (think CAPM) lead to large variance of residuals and low sensitivity to size, industry, etc.

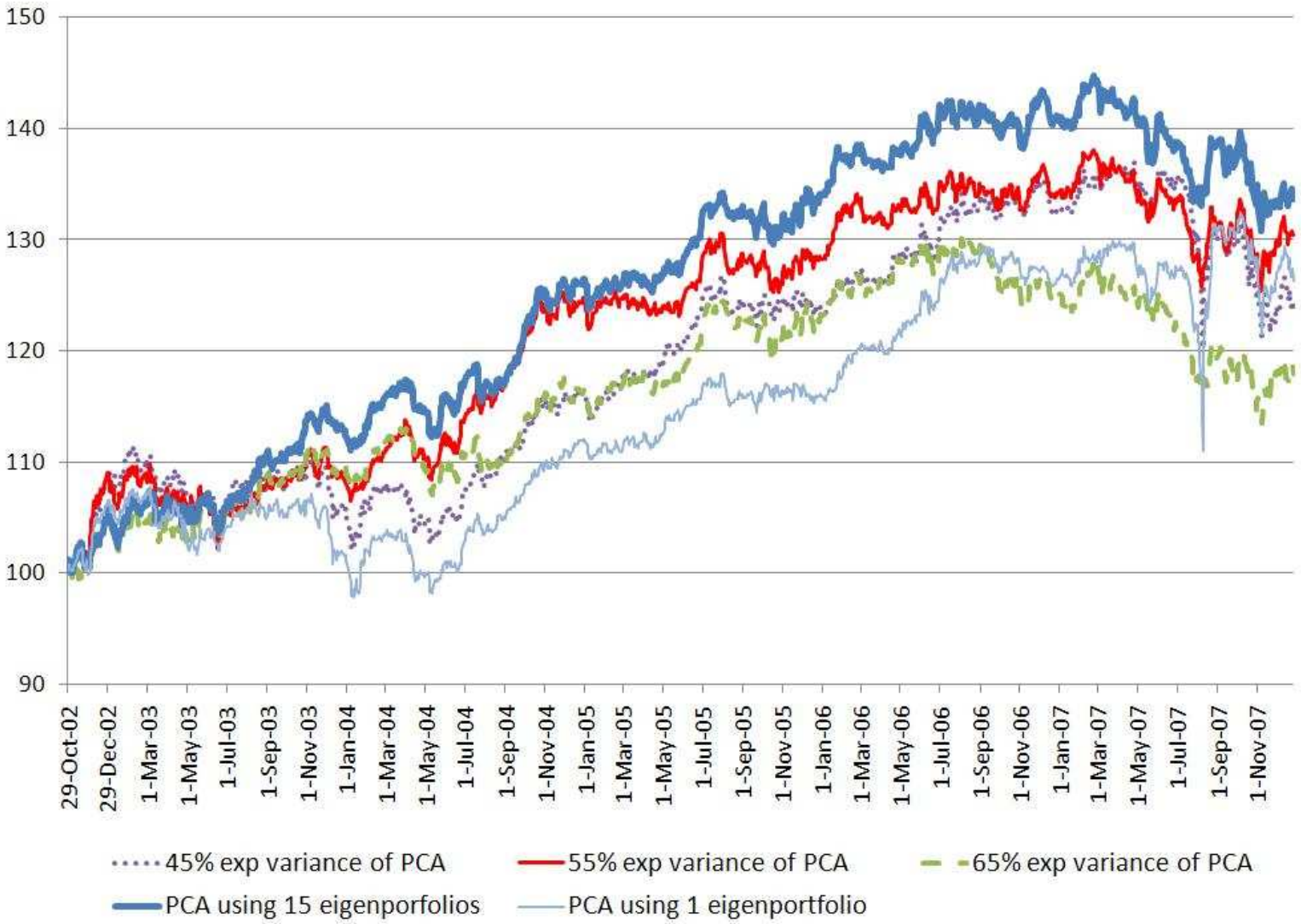
-- **Too many factors** lead to noise trading (negative P/L).

-- Interesting question: is the ``correct'' number of factors variable? (I.e. dependent on market conditions?) . Yes.

# Evolution of the Number of Factors for different levels of explained variance



# P/L for different truncation levels



# Sharpe Ratios for Variable PCA strategies (2007-08)

| | 1 Eigenportfolios | 15 Eigenportfolios | 45 % Exp Variance | 55% Exp Variance | 65% Exp Variance |
|---|---|---|---|---|---|
| **2003** | -0.7 | | -0.5 | | 0.4 |
| **2004** | 1.7 | | 1.7 | | 1.3 |
| **2005** | 0.8 | | 1.3 | | 1.0 |
| **2006** | 1.8 | | 1.3 | | 0.3 |
| **2007** | 0.0 | | -0.7 | | -0.9 |
| **Since Inception** | 0.7 | | 0.6 | | 0.4 |

# Trading Time vs. Actual Time

Statistics on equity returns can be done

-- in **actual time** (% change/day)

-- in **trading time** ( % change per share/day)

Trading time incorporates **volume information**.

In trading-time framework, mean-reverting signals (S-Scores) are

-- **weaker** when volume is heavy

-- **stronger** when volume is light

# Trading Time vs. Actual Time II

Using the daily trading volume, construct a residual process which measures the change in price **per share**

$$\varepsilon = \frac{\Delta S}{S} - \beta \frac{\Delta I}{I} \qquad \text{(usual residual)}$$

$$\bar{\varepsilon} = \frac{< \Delta V >}{\Delta V} \varepsilon, \qquad \Delta V = \text{daily volume} \qquad < \Delta V > = \text{average } \Delta V$$

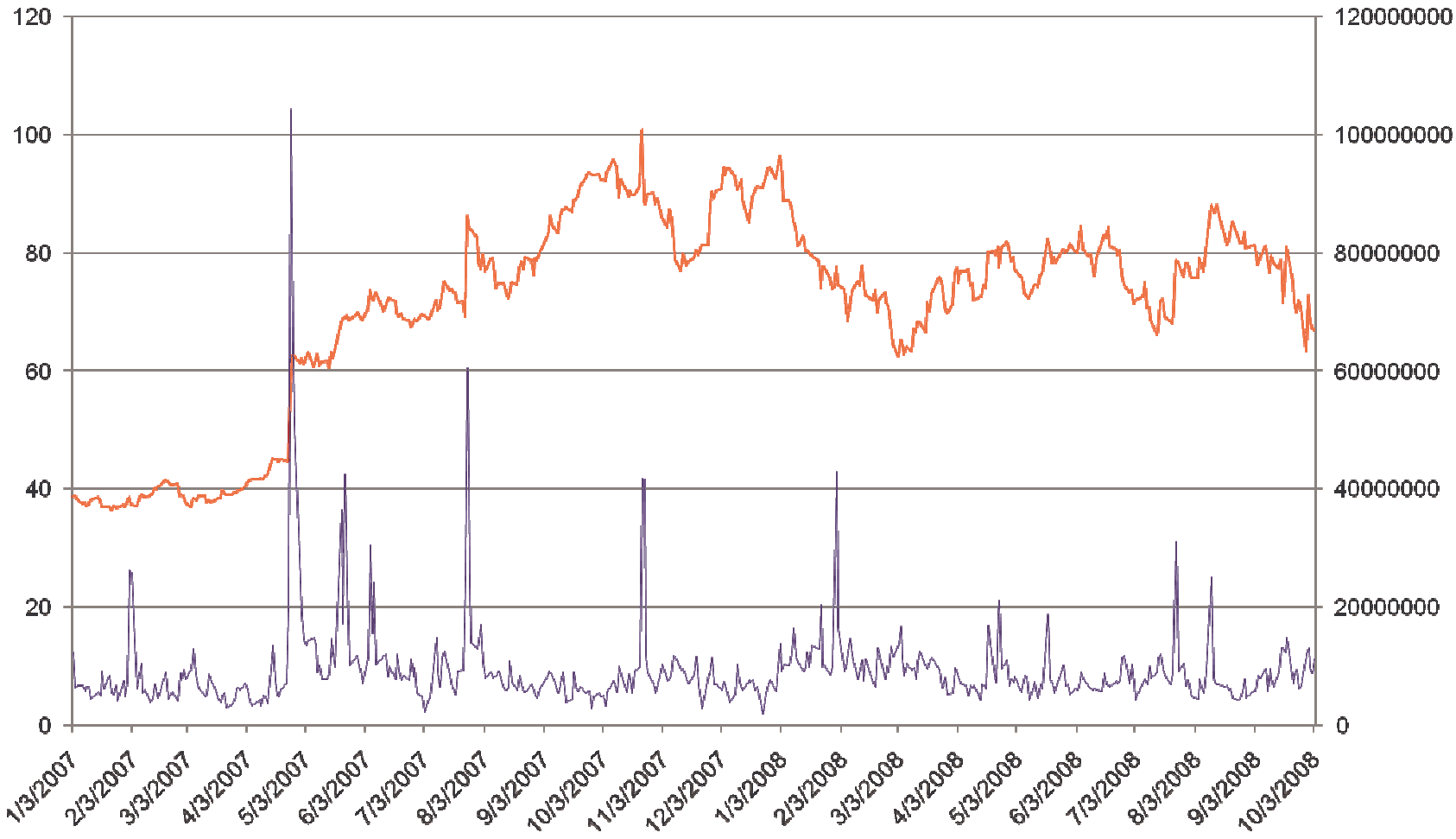
$$Y_t = \sum_{i=1}^{t} \bar{\varepsilon}_i$$

$$dY = \kappa(m - Y)dt + \sigma dW$$

Estimate AR-1 / OU process for the new process Y(t)

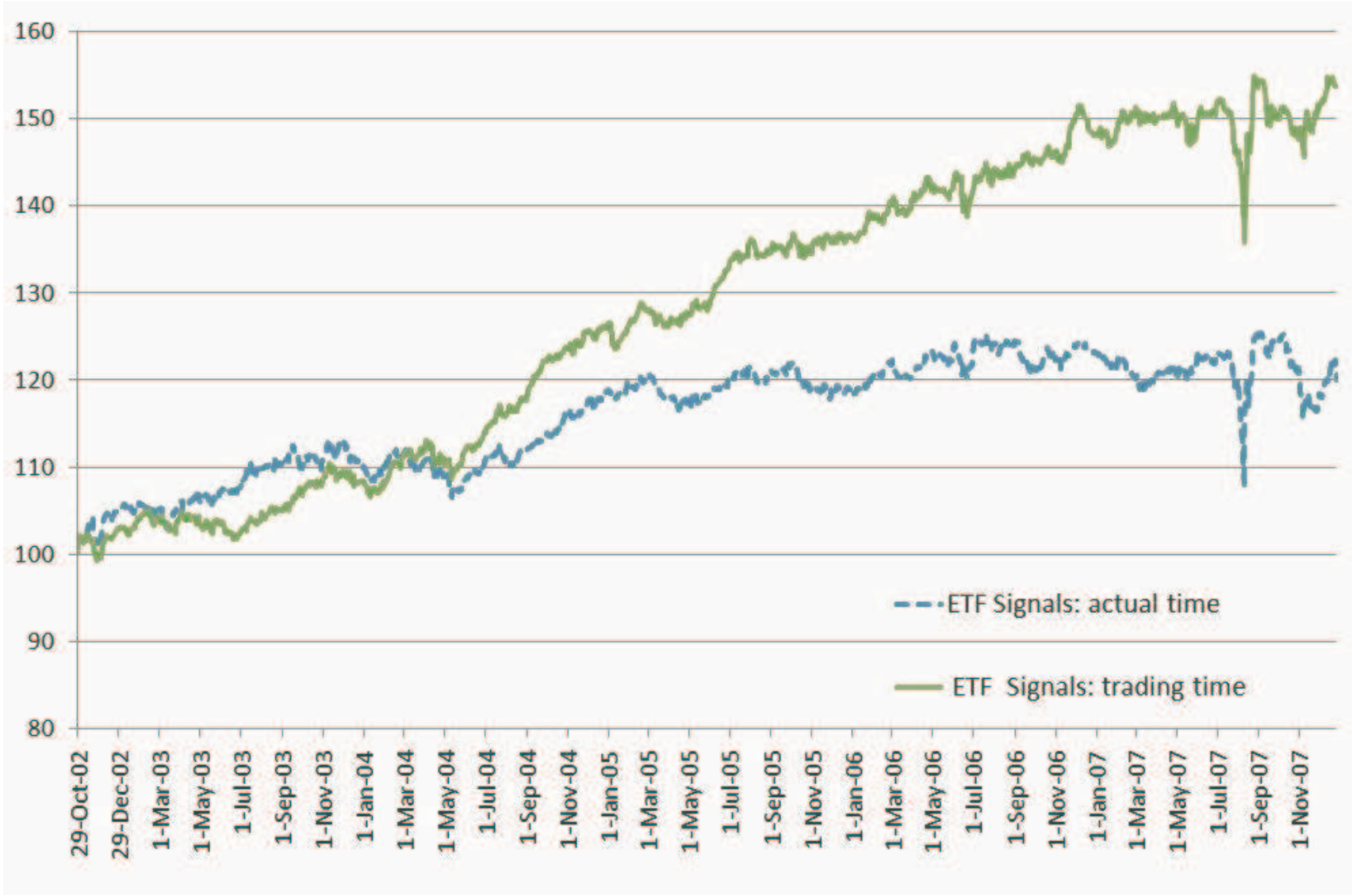
This makes deviations on unusually high-volume more likely, so the signal is weaker

# Amazon.com  Jan 2007-Oct 2008

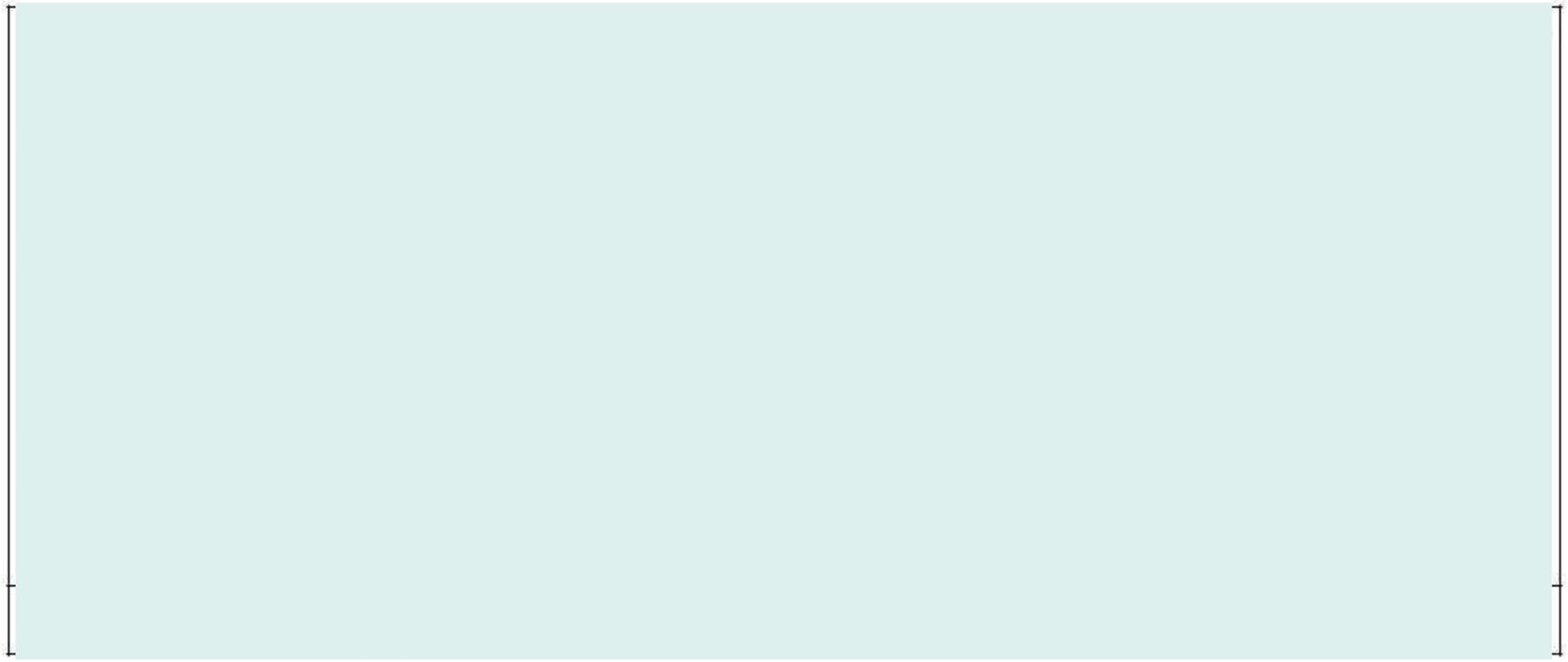



Short-covering after earnings surprise makes the stock run up!

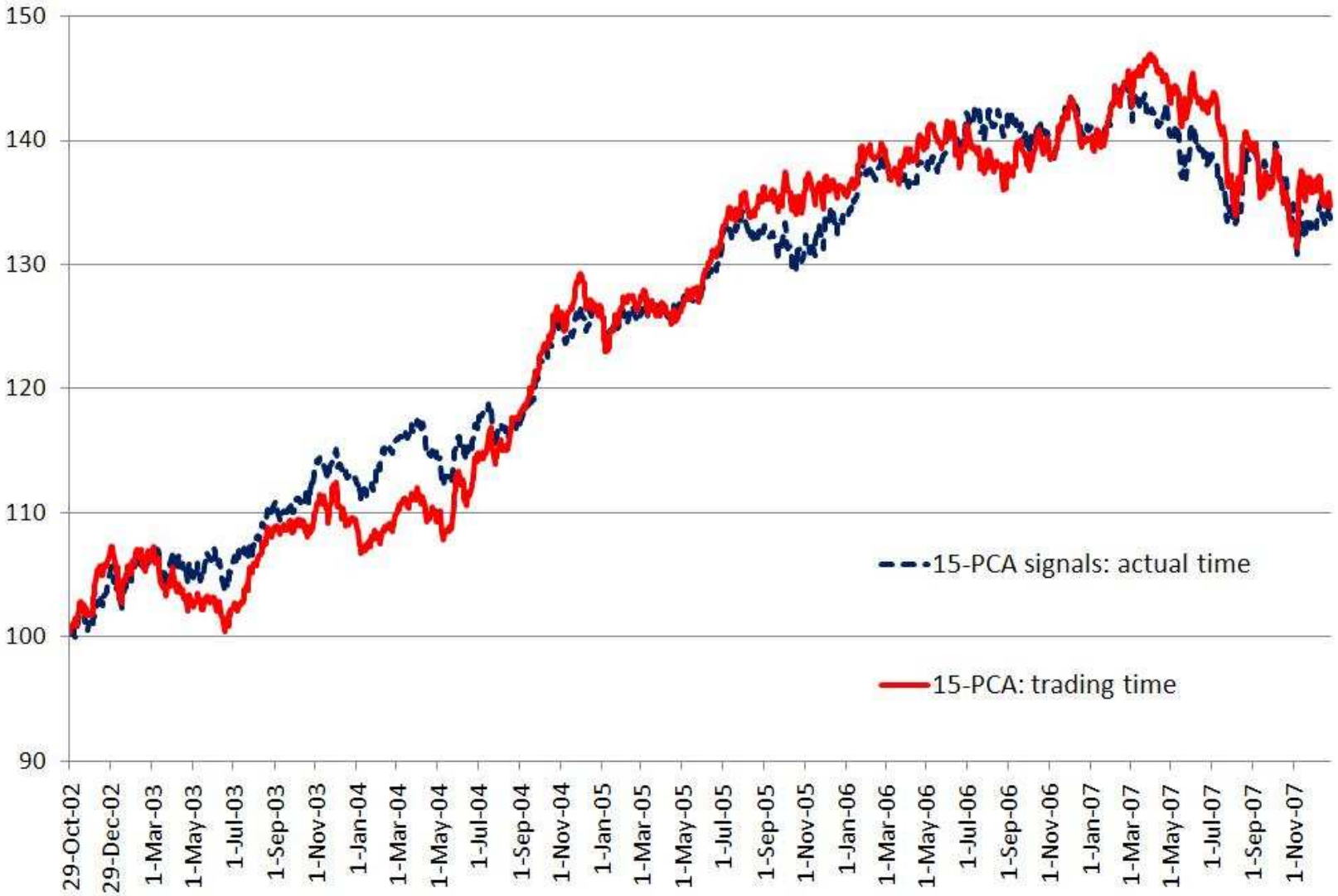
# ETF signals: trading time vs. actual time



# ETF signals in trading time: Sharpe Ratios



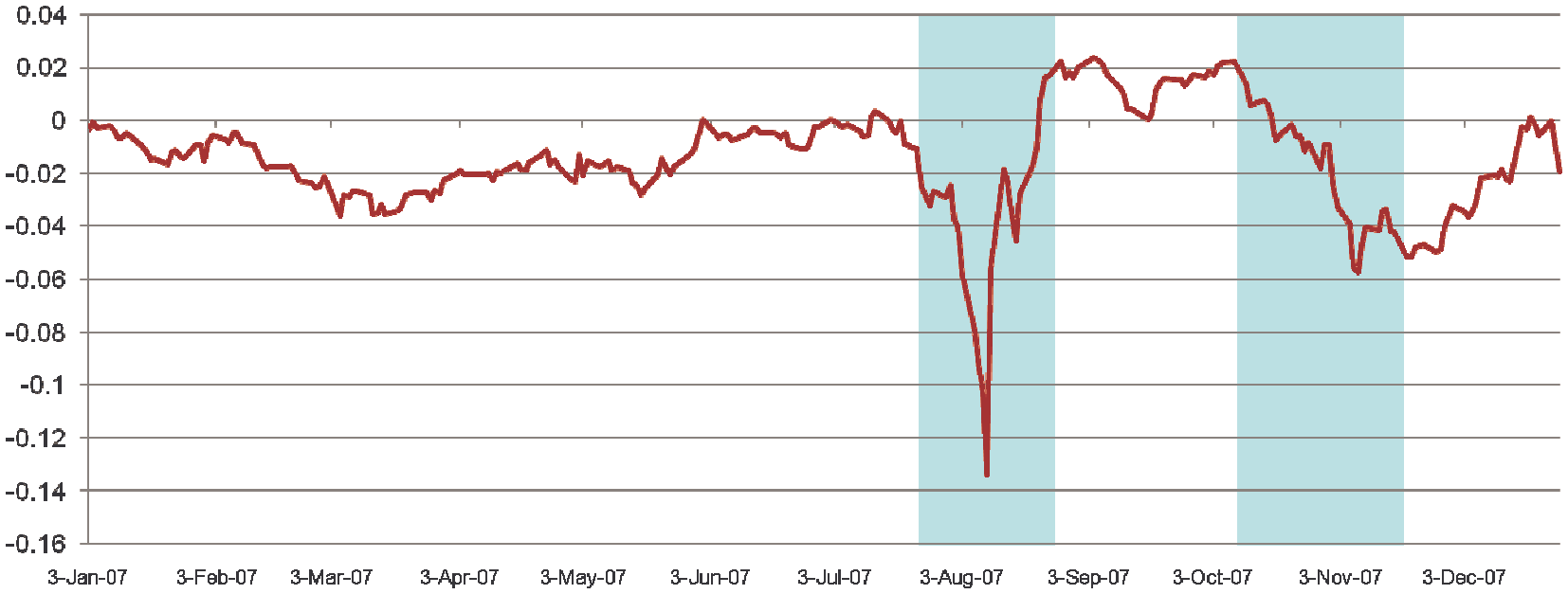
# PCA: trading time vs. actual time



# 15 PCA signals in trading time: Sharpe Ratios

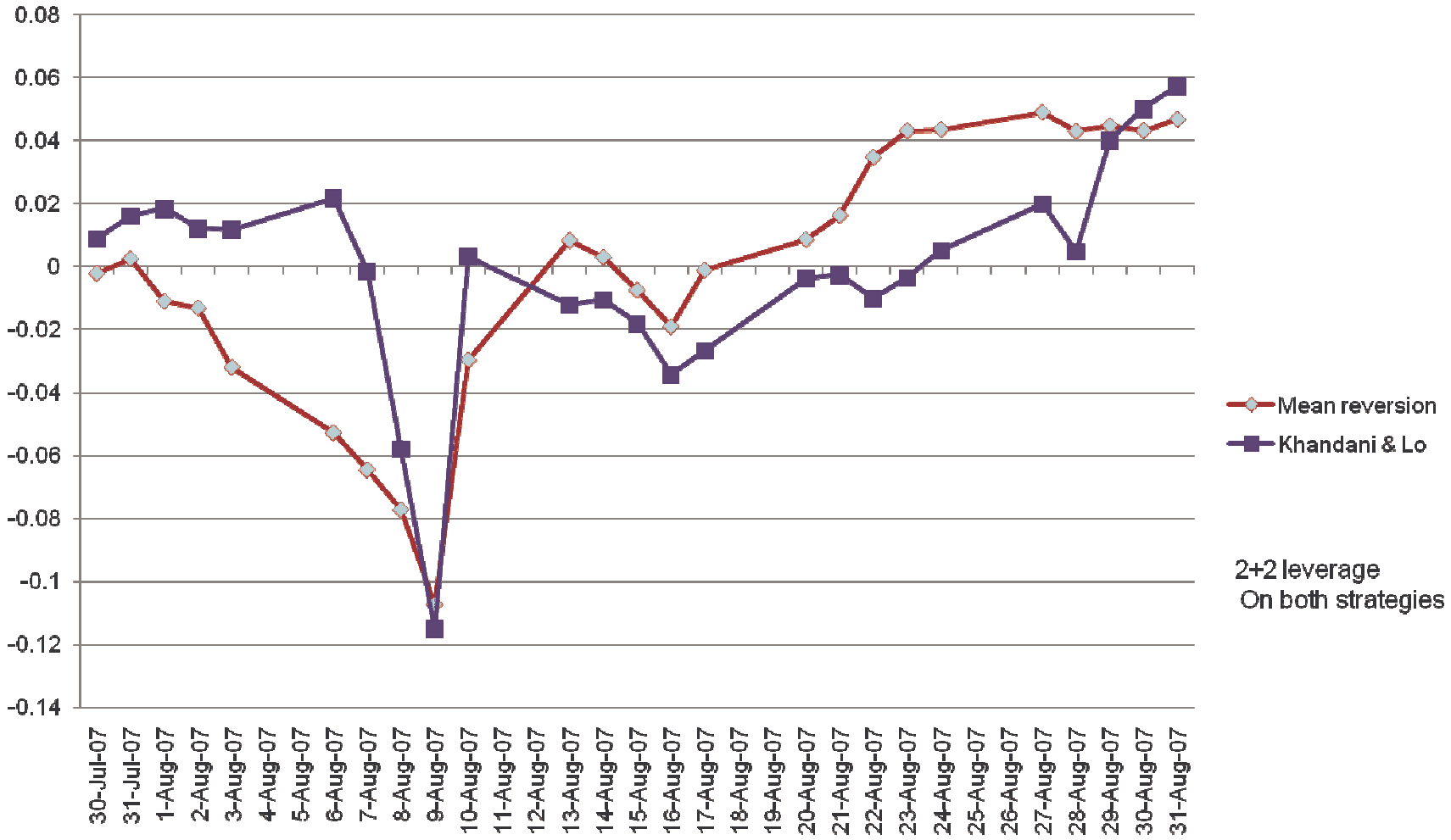
| | HHH | IYR | IYT | OIH | RKH | RTH | SMH | UTH | XLE | XLF | XLI | XLK | XLP | XLV | XLY | Porfolio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2003** | 0.9 | (0.1) | (0.0) | 0.6 | 0.4 | 0.6 | (1.7) | (1.3) | (0.0) | 0.9 | 0.3 | 1.7 | (0.5) | (0.4) | 1.5 | 0.2 |
| **2004** | 1.5 | 1.8 | 0.6 | 0.9 | 0.9 | 0.8 | (0.5) | 0.2 | 1.7 | 1.9 | 1.1 | 1.4 | 0.7 | 0.1 | 1.4 | 2.4 |
| **2005** | (1.1) | (0.8) | 0.8 | 1.0 | 0.1 | 1.9 | 0.4 | 0.6 | 1.4 | 0.8 | 1.4 | 1.5 | (1.7) | 1.7 | (0.6) | 1.2 |
| **2006** | 0.3 | 1.2 | (0.8) | 1.1 | 0.9 | (0.9) | (0.1) | 0.8 | (0.5) | 0.2 | 0.4 | 0.0 | (0.1) | 0.3 | 0.3 | 0.6 |
| **2007** | (0.2) | (0.7) | (0.6) | 1.0 | (1.1) | (1.9) | 0.2 | 0.8 | 0.9 | (1.7) | 2.6 | 0.9 | 0.4 | 0.9 | (1.4) | (0.5) |
| | 0.29 | 0.27 | (0.00) | 0.93 | 0.22 | 0.13 | (0.35) | 0.21 | 0.69 | 0.42 | 1.16 | 1.10 | (0.23) | 0.50 | 0.24 | 0.80 |

# Focus on the crash of 2007

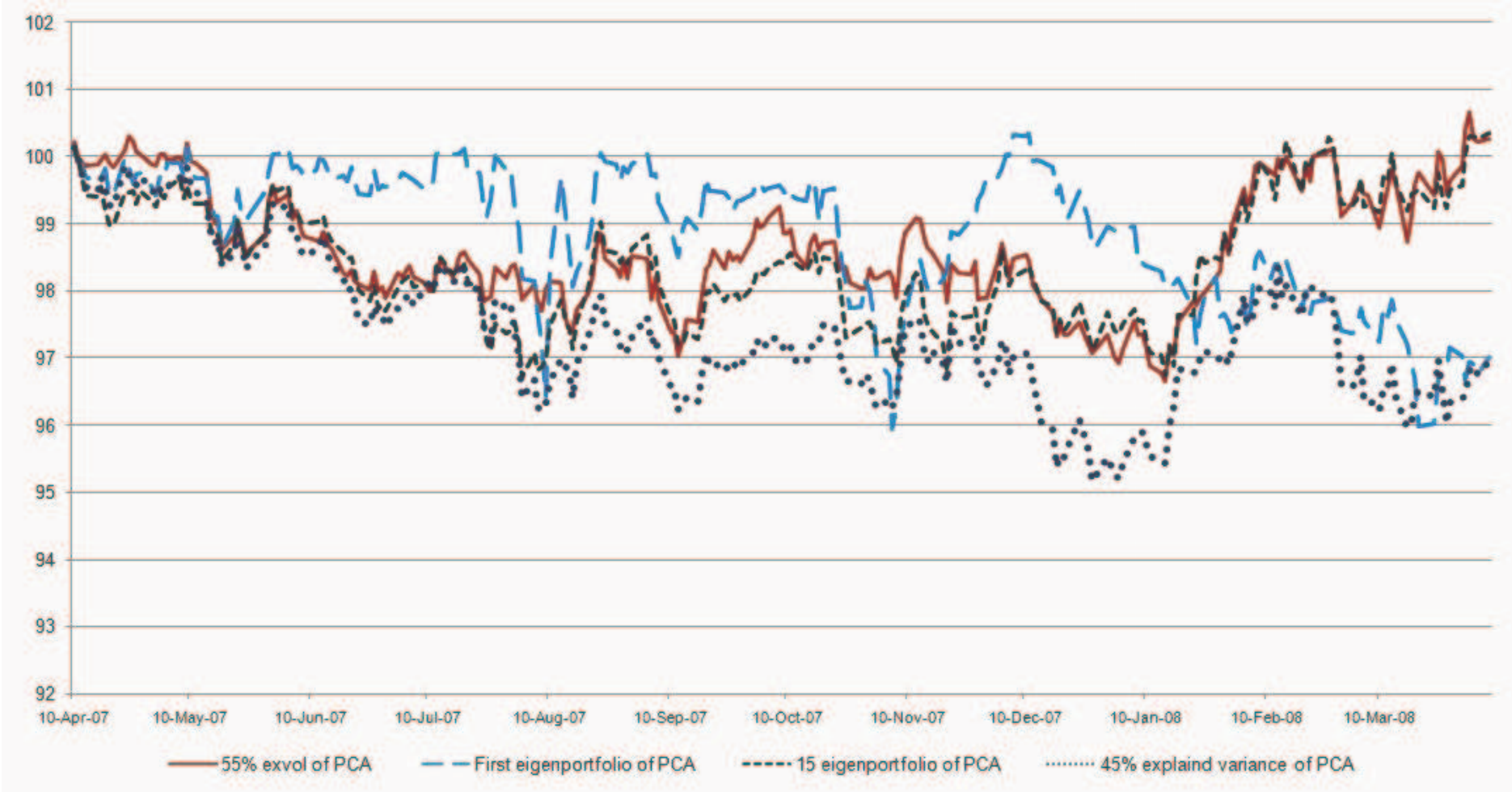



2007: Flat/small losses year with ``cataclysmic'' dips in August and November. Leverage=2+2

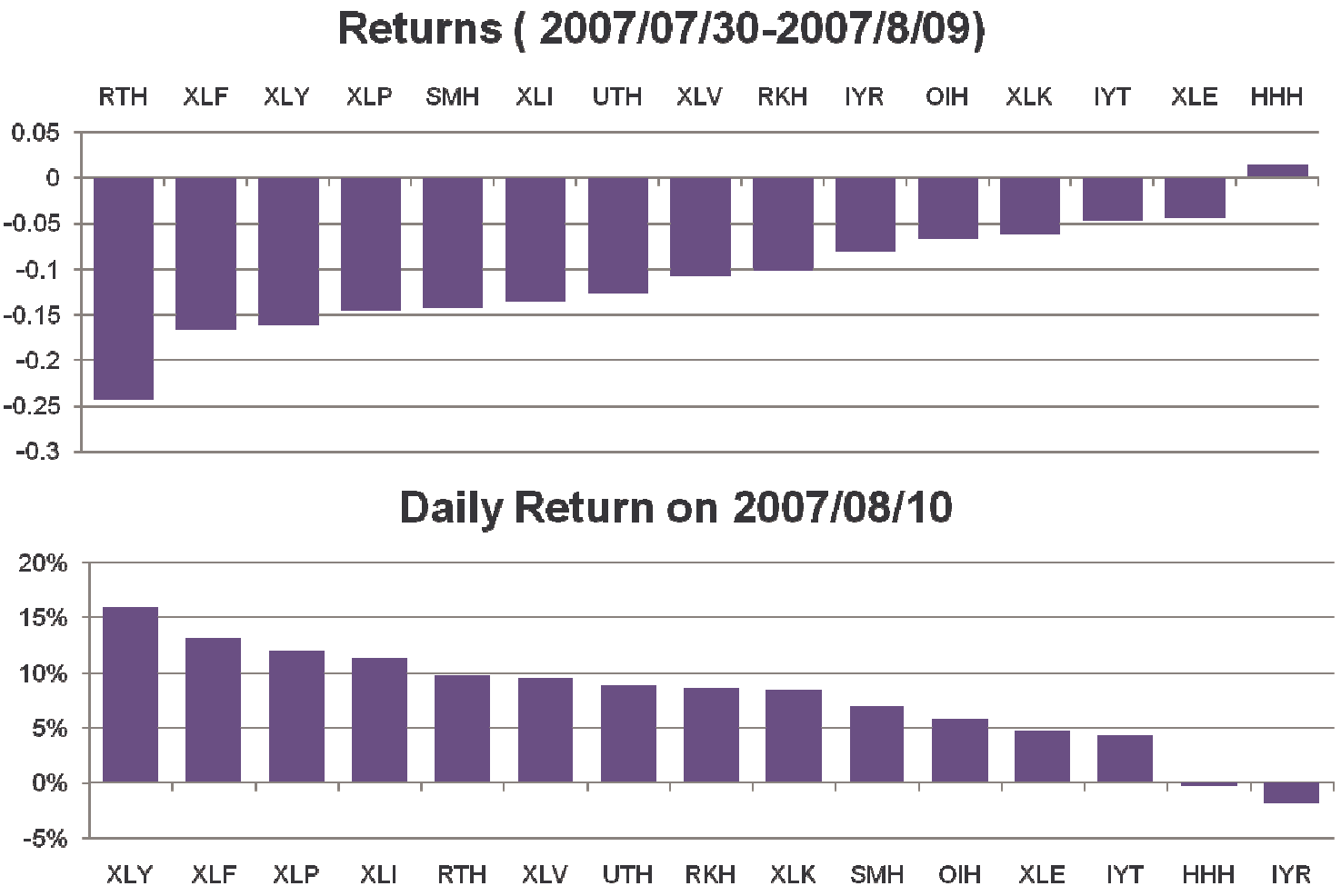
# August 2007: comparison with Khandani & Lo



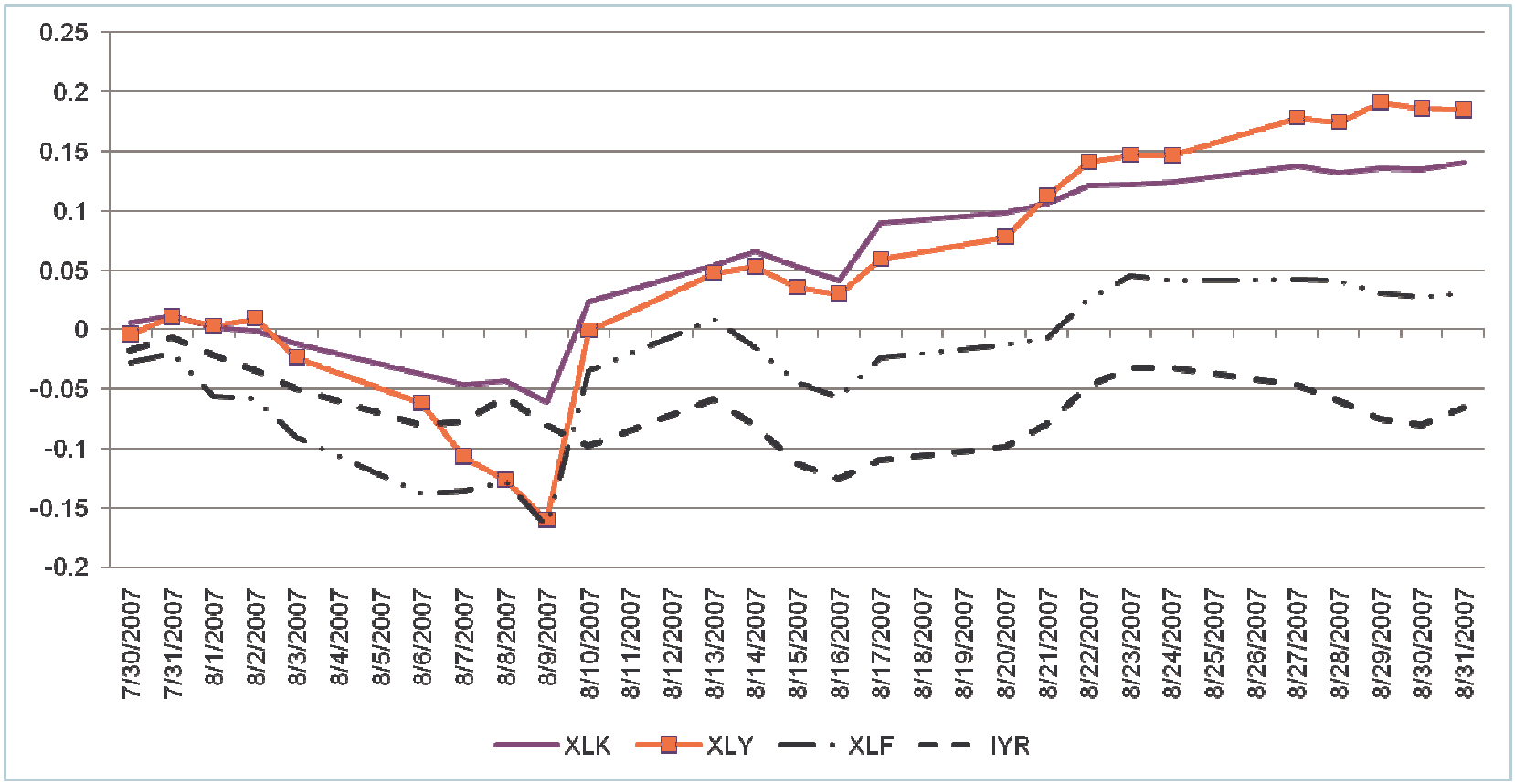
# Comparing performance of MR strategies Jan 2007-March 2008



# Sector view in Aug 2007

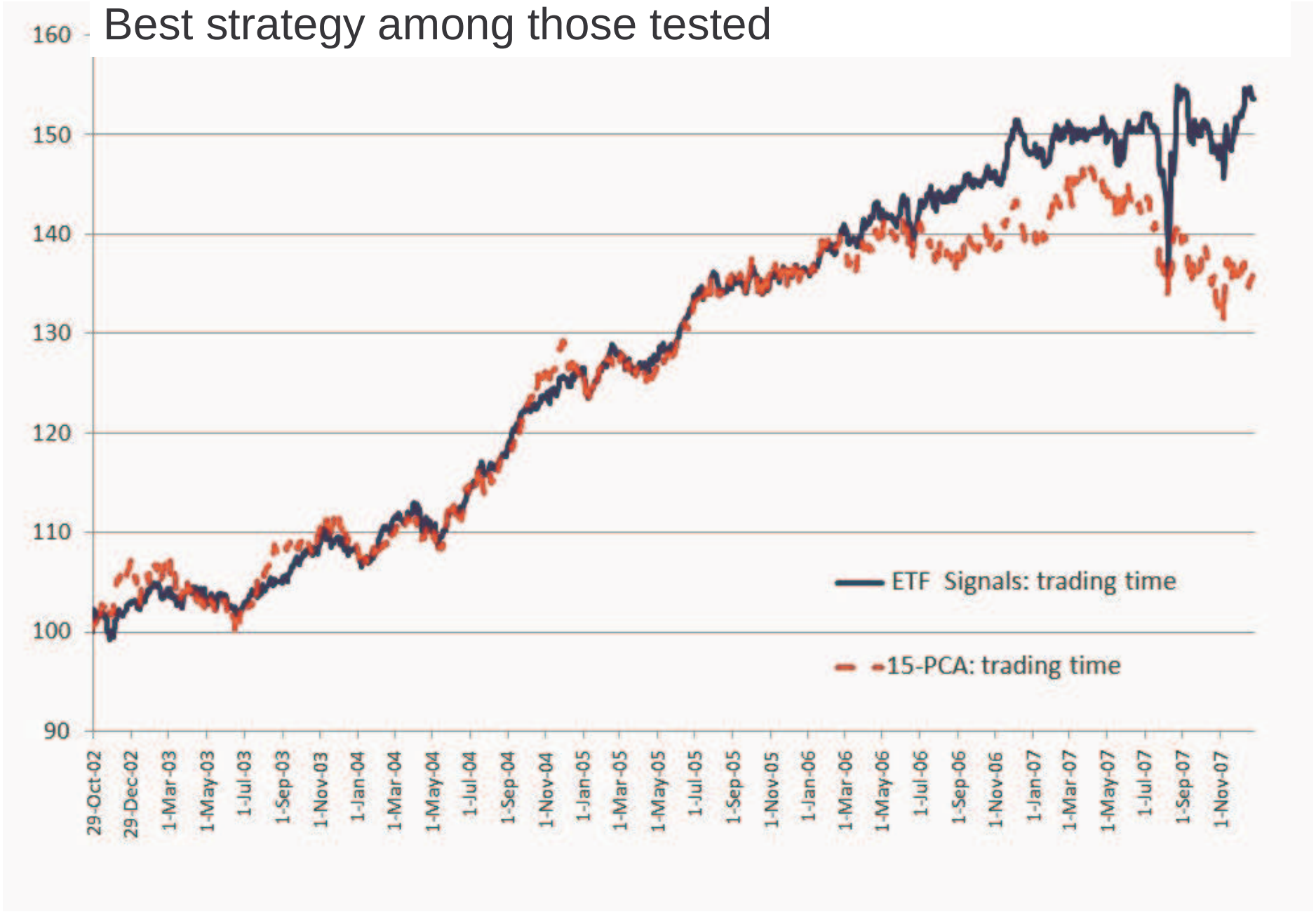


# Tech & Consumer vs. Financials & Real Estate (Aug 2007)




Consumer discretionary L/S more volatile than Financials

ETFs in trading time outperformed PCAs after 2005.
Best strategy among those tested



# Comparing performance of MR strategies Jan 2007-March 2008