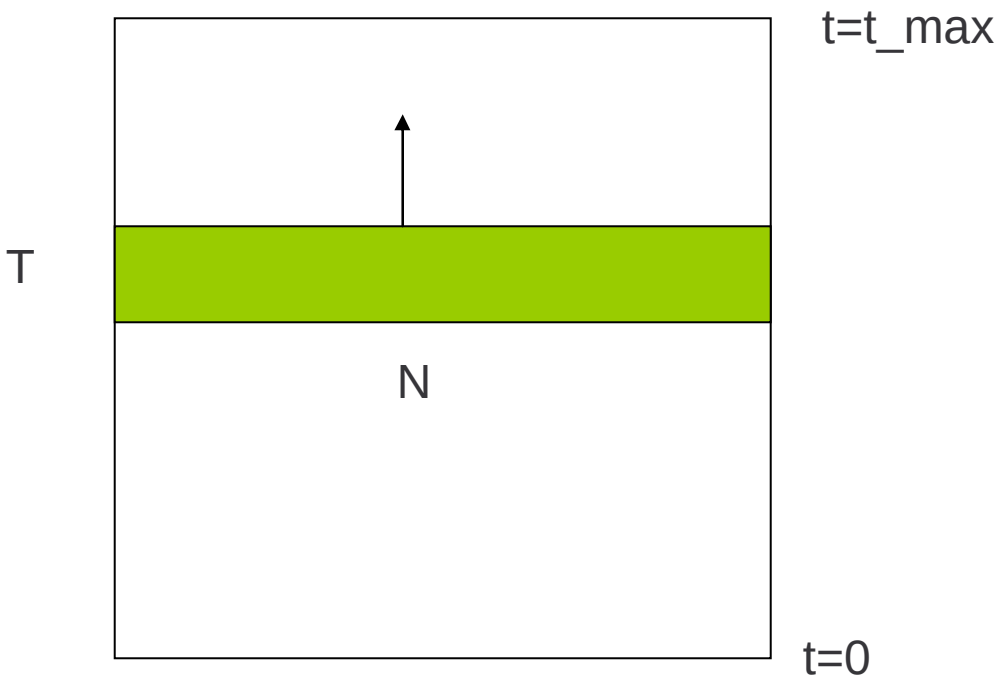
Lecture 3:
Dynamic PCA & More Examples

Marco Avellaneda

G63.2936.001

Spring Semester 2009

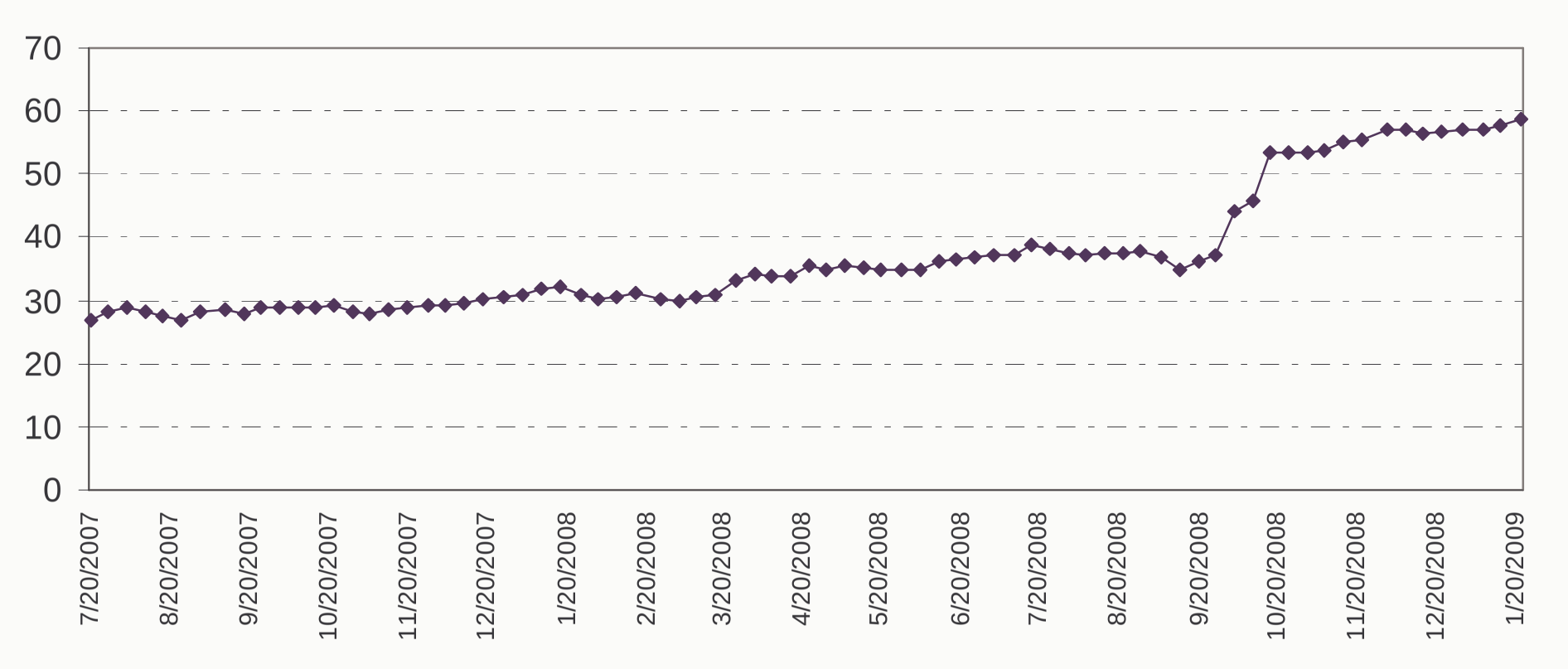
Dynamic PCA Analysis



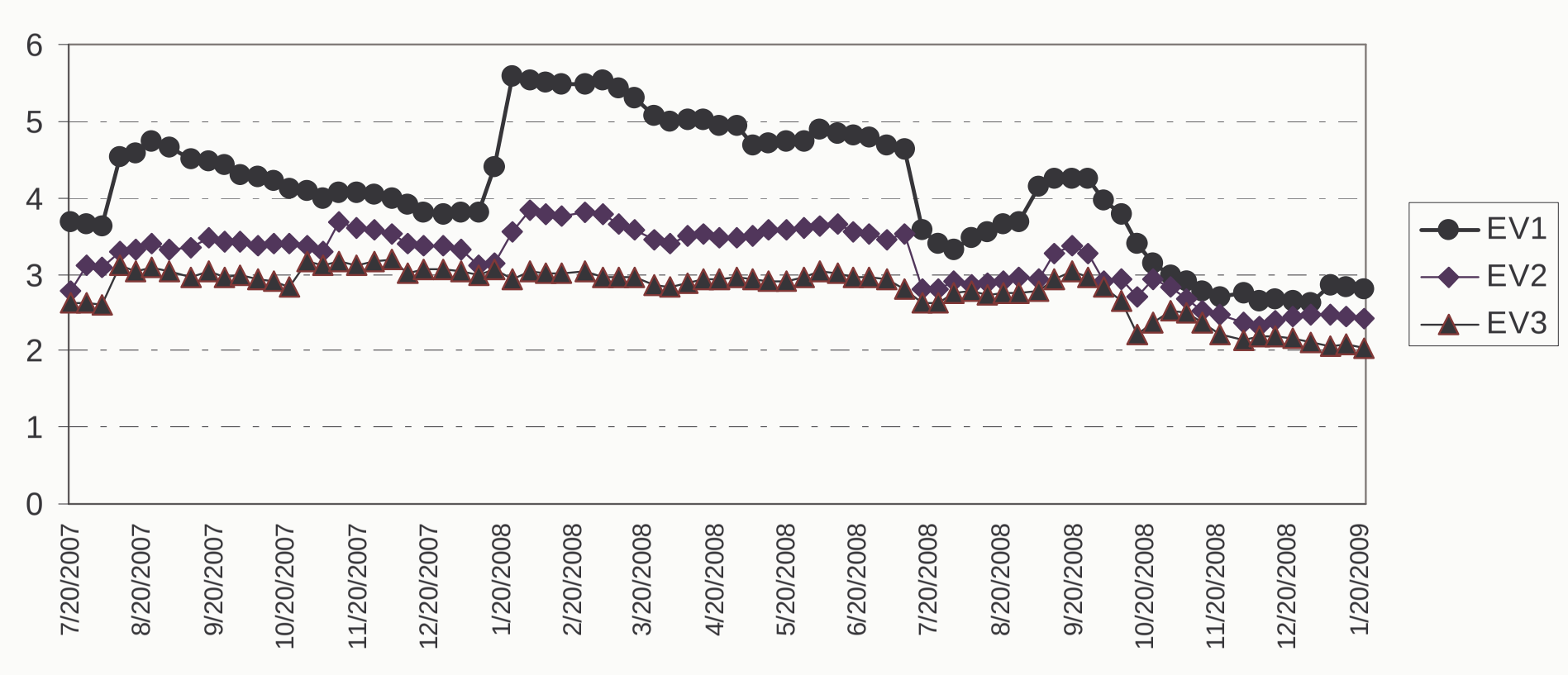
Rolling window of T days
Exponential moving avg.

$$C_{ij}(t) = \frac{1}{T}\sum_{s=1}^{T}\left(R_{i,t-s} - \left\langle R_{i,t-s+1}\right\rangle_1^T\right)\left(R_{j,t-s} - \left\langle R_{j,t-s+1}\right\rangle_1^T\right)$$

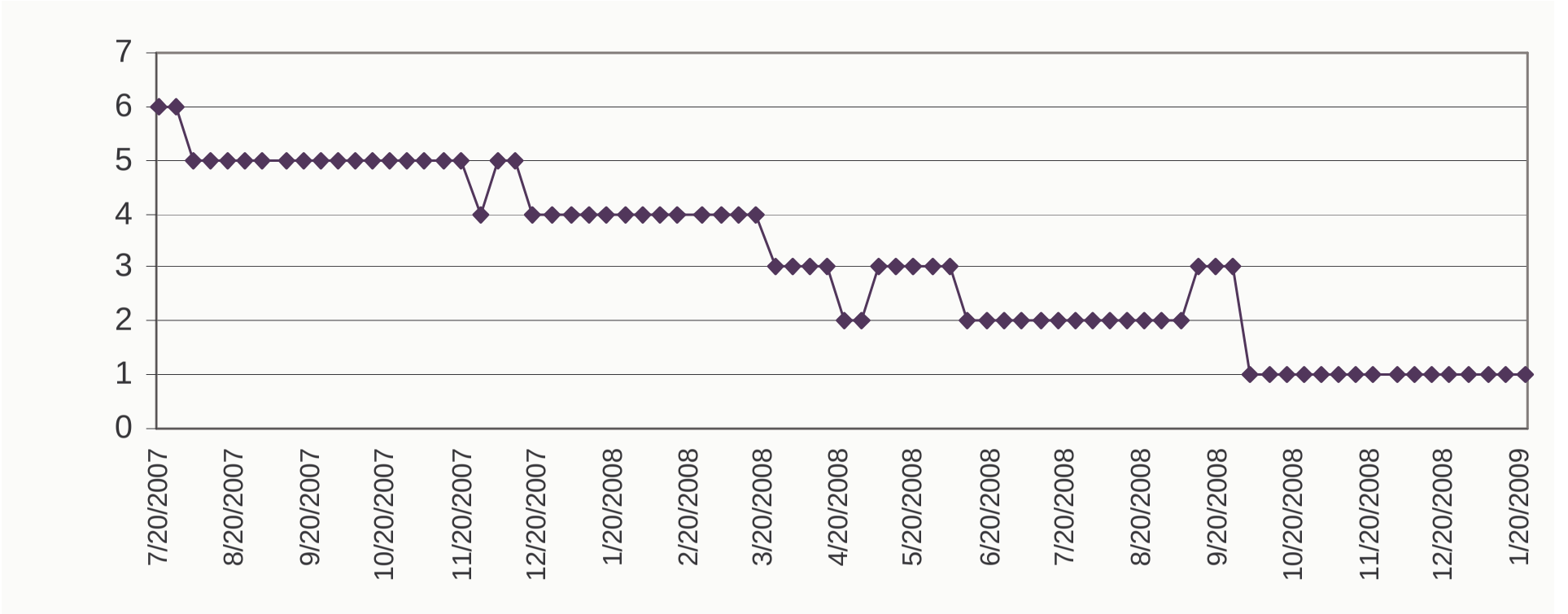
First Eigenvalue of NDX correlation matrix (7/20/07 to 1/20/09) 6-month trailing window



EV2, EV3, EV4 for the same period



Number of eigenvalues needed to explain 40% of the variance



Risk-factors seen dynamically

$$F_t^{(k)} = \sum_{j=1}^{N} V_j^{(k)}(t) \frac{R_{jt}}{\sqrt{C_{jj}(t)}}$$

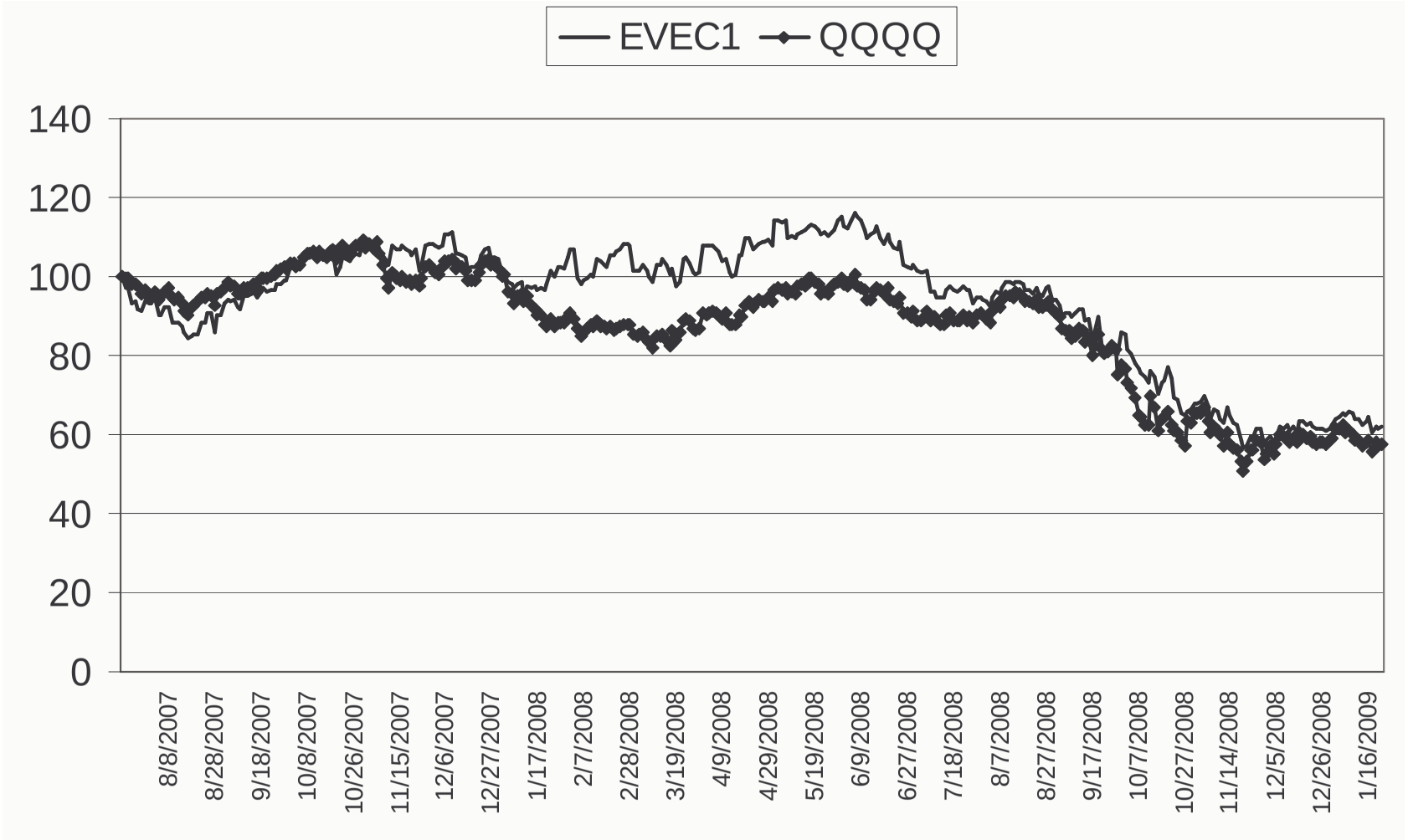
Backward estimation of correlation; use last returns as ``shocks''

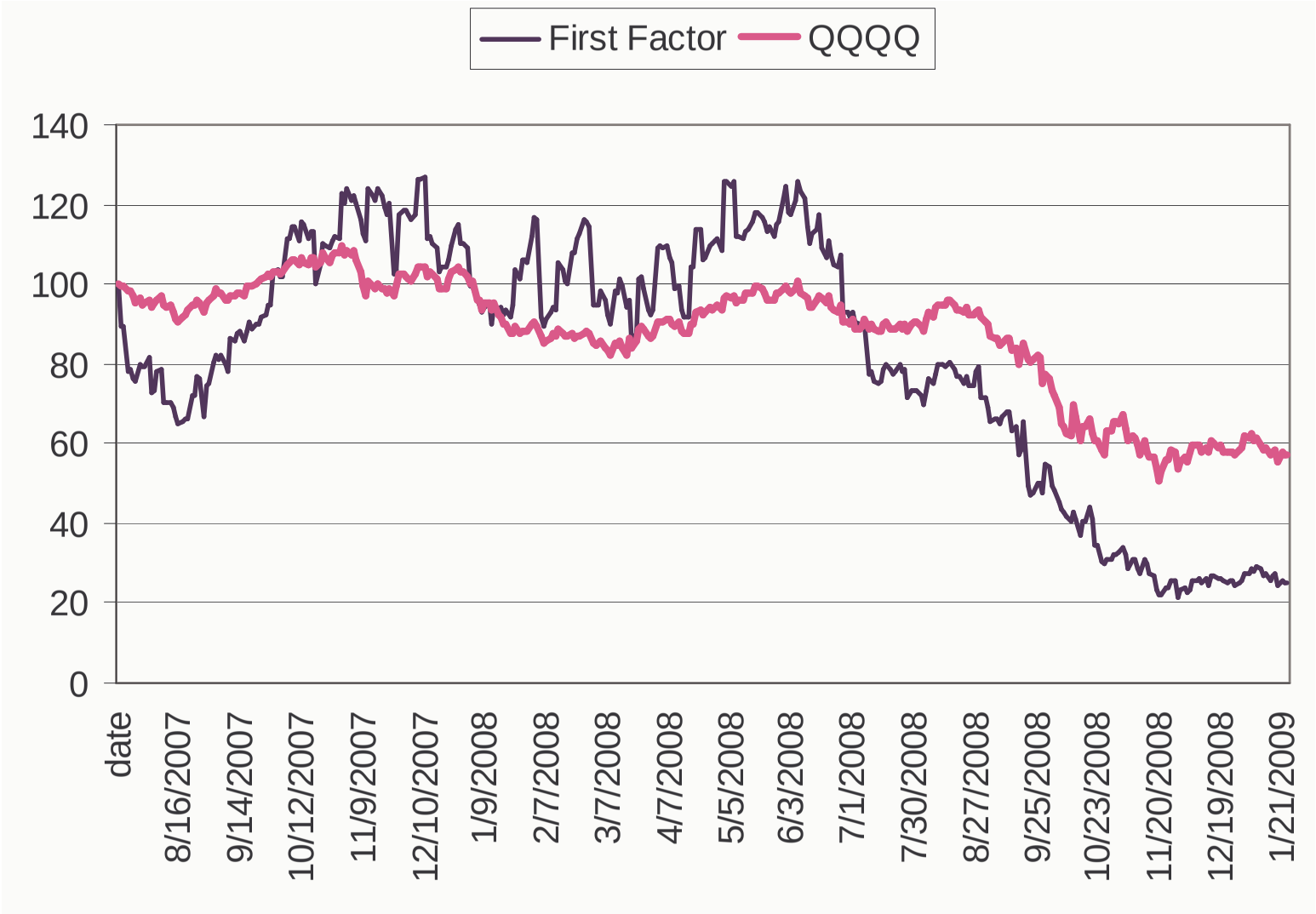
$$Var\left(F_t^{(k)}\right) \sim \lambda_k$$

$$G_t^{(k)} = 100 \cdot \prod_{s=1}^{t} \left(1 + \alpha \cdot F_s^{(k)}\right),$$

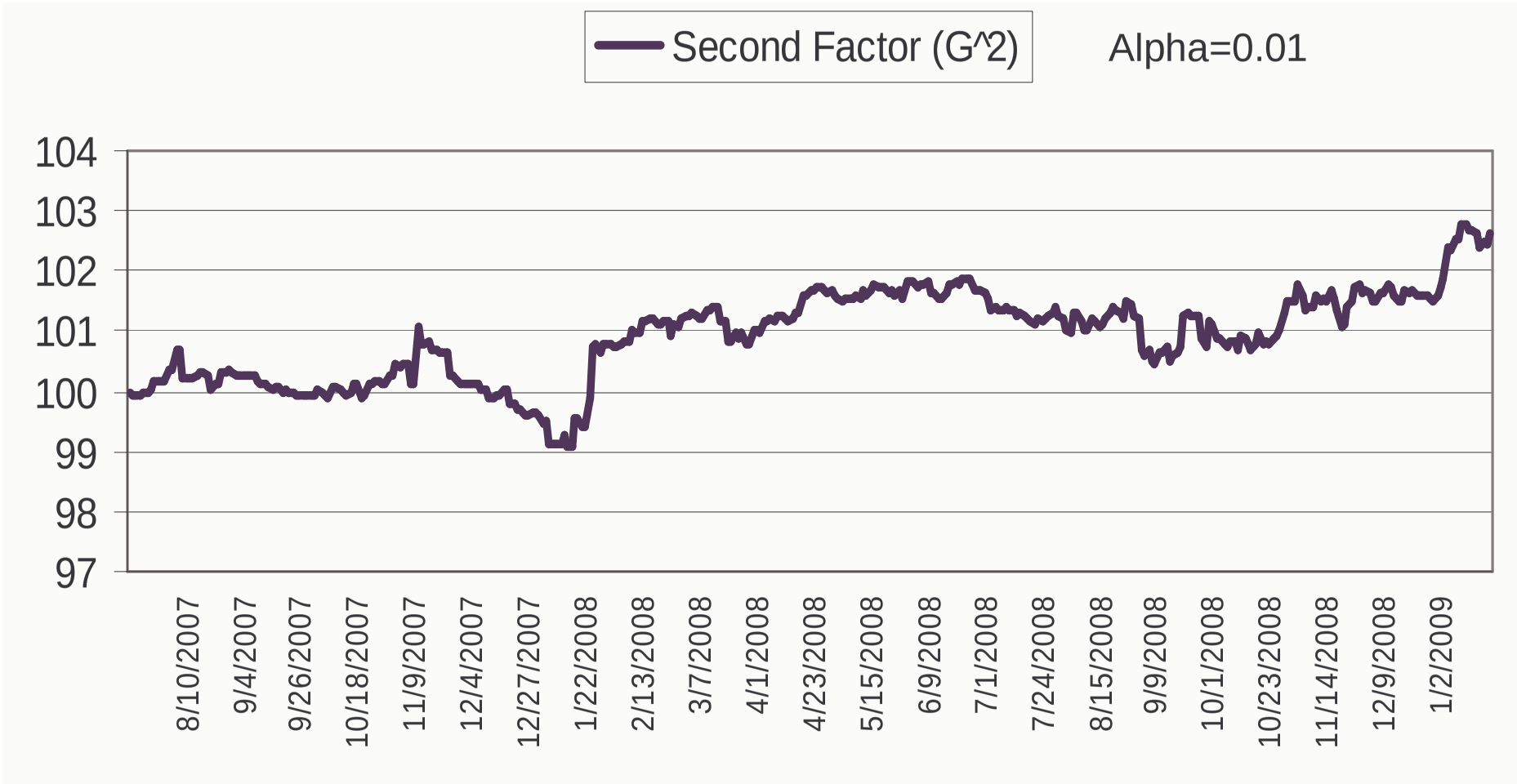
Accumulated returns from investing in the RF

First Factor vs. QQQQ assuming equal volatility

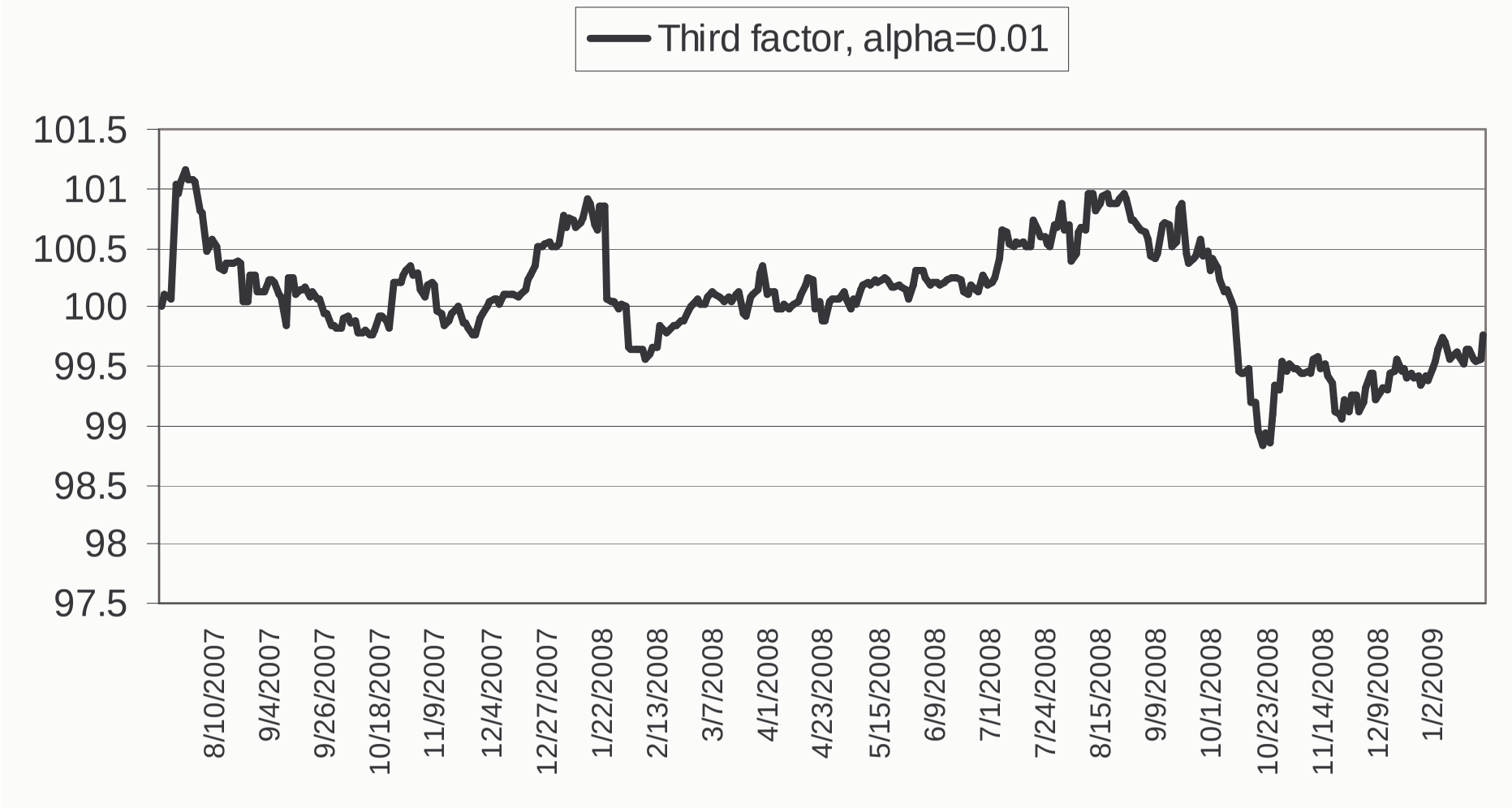



First Factor
QQQQ
140
120
100
80
60
40
20
0
date
8/16/2007
9/14/2007
10/12/2007
11/9/2007
12/10/2007
1/9/2008
2/7/2008
3/7/2008
4/7/2008
5/5/2008
6/3/2008
7/1/2008
7/30/2008
8/27/2008
9/25/2008
10/23/2008
11/20/2008
12/19/2008
1/21/2009

Second Factor



Third factor



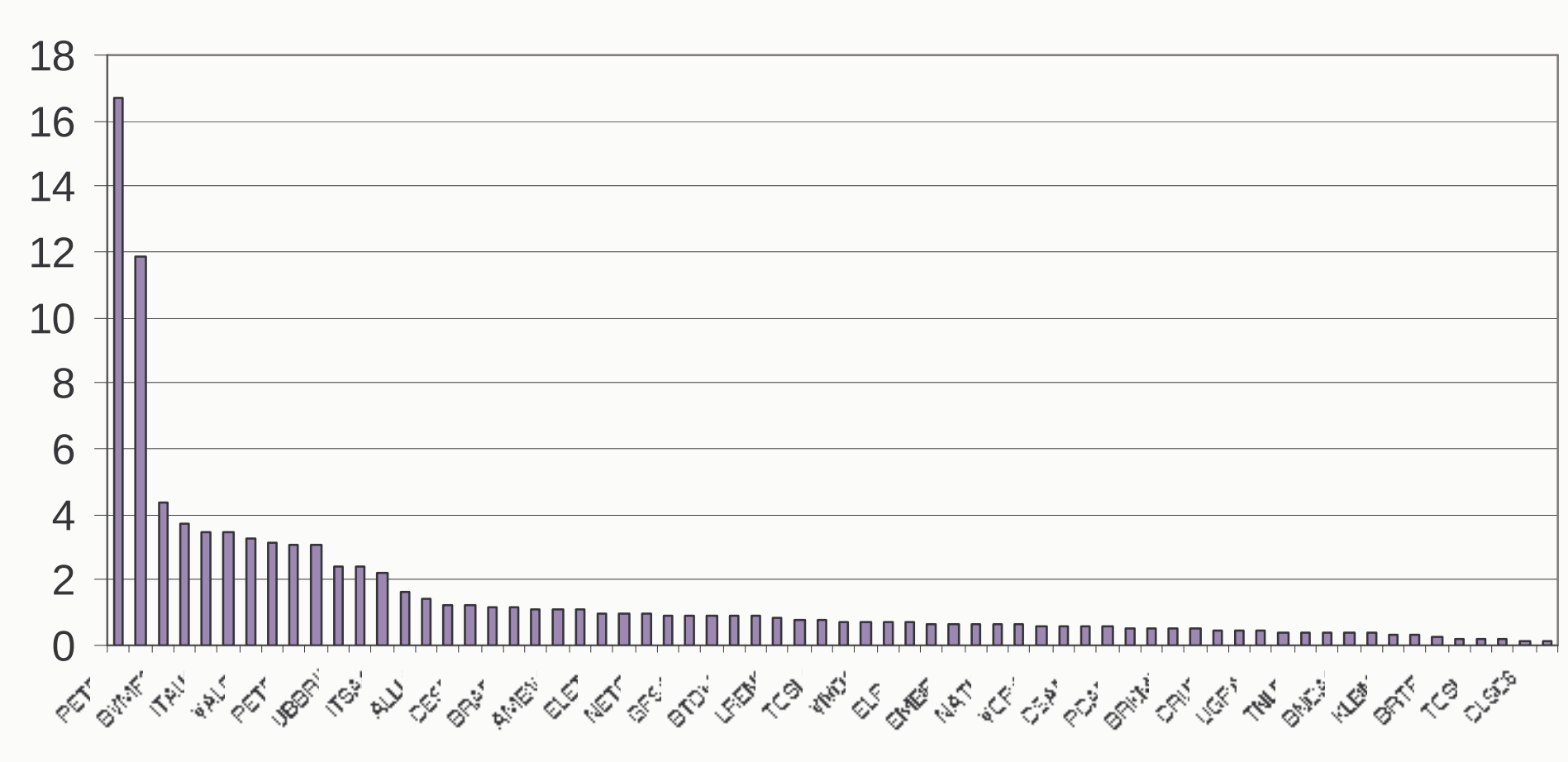
Top 20 Brazilian listed companies (**Bovespa**)

Stock	Sector
Petrobras - Petroleo Bras-pr	Energy
Petrobras - Petroleo Bras	Energy
CIA Vale DO RIO Doce-pref A	Materials
CIA Vale DO RIO Doce-adr	Materials
Banco Bradesco -pref	Financials
Banco Itau Holding Fin-pref	Financials
Unibanco-units	Financials
CIA DE Bebidas DAS Ame-pref	Consumer Staples
CIA Siderurgica Nacional SA	Materials
Gerdau Sa-pref	Materials
CIA Energetica Minas Ger-prf	Utilities
Itausa-investimentos Itau-pr	Financials
Usinas Sider Minas Ger-pf A	Materials
Bm&f Bovespa SA	Financials
Tele Norte Leste Part-pref	Telecommunication Services
Banco DO Brasil SA	Financials
Gerdau Metalurgica Sa-pref	Materials
Redecard SA	Information Technology
Centrais Eletricas Brasilier	Utilities
Cpfl Energia SA	Utilities
Empresa Bras DE Aeronautica	Industrials
Bradespar SA -pref	Financials
Centrais Eletricas Bras-pr B	Utilities
Souza Cruz SA	Consumer Staples
ALL America Latina Logistica	Industrials
Perdigao SA	Consumer Staples

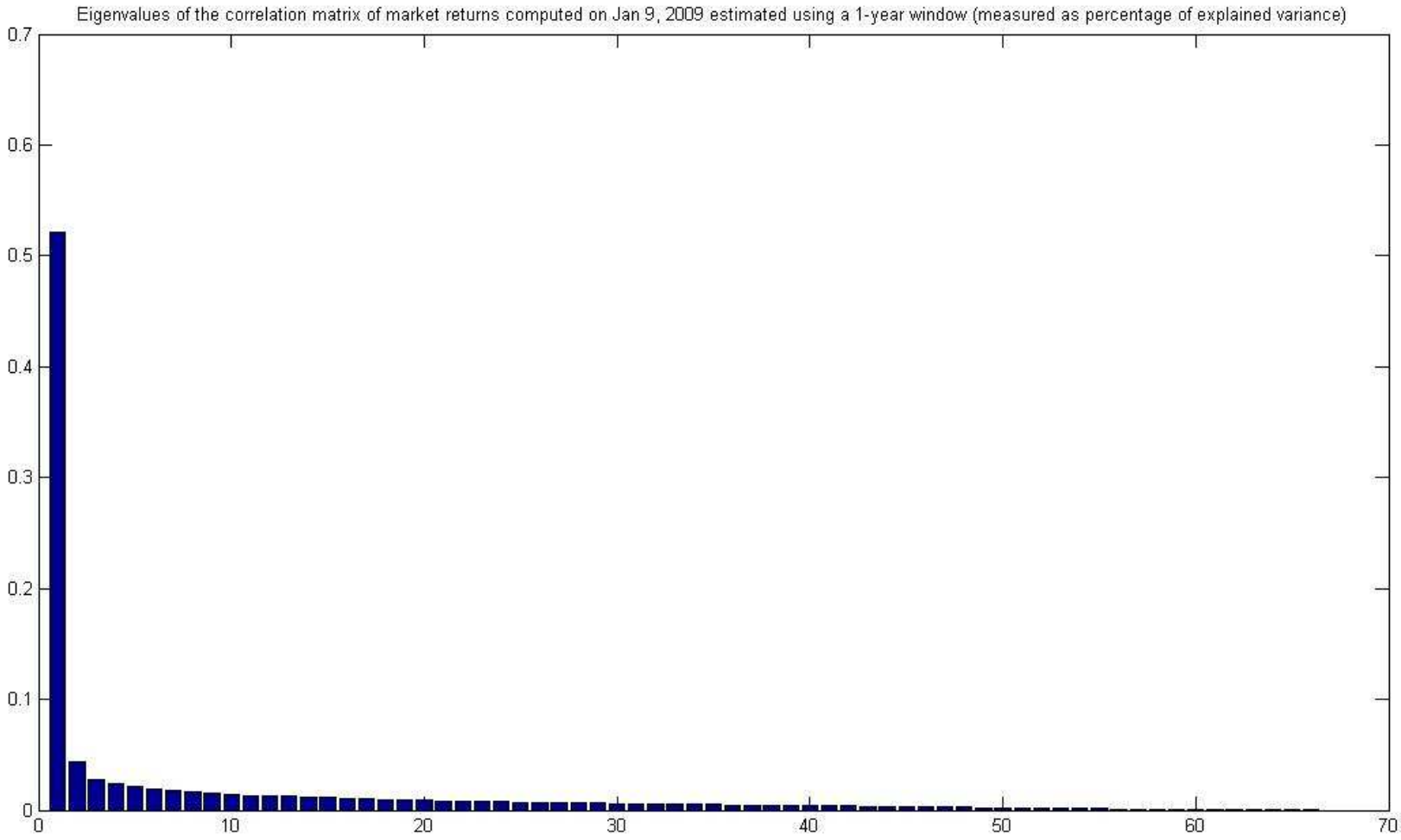
TICKER	COMPANY	CLASS	WEIGHT
PETR4	PETROBRAS	PN EJ	16.715
VALE5	VALE R DOCE	PNA N1	11.884
BVMF3	BMF BOVESPA	ON EJ NM	4.360
BBDC4	BRADESCO	PN N1	3.700
ITAU4	ITAUBANCO	PN EDJ N1	3.471
CSNA3	SID NACIONAL	ON	3.454
VALE3	VALE R DOCE	ON N1	3.27
GGBR4	GERDAU	PN N1	3.1
PETR3	PETROBRAS	ON EJ	3.061
USIM5	USIMINAS	PNA EJ N1	3.047
UBBR11	UNIBANCO	UNT EDJ N1	2.417
BBAS3	BRASIL	ON EJ NM	2.404
ITSA4	ITAUSA	PN EJ N1	2.238
CMIG4	CEMIG	PN N1	1.642
ALLL11	ALL AMER LAT	UNT N2	1.415
CYRE3	CYRELA REALT	ON NM	1.257
CESP6	CESP	PNB N1	1.215
LAME4	LOJAS AMERIC	PN INT	1.2
BRAP4	BRADESPAR	PN N1	1.199
SDIA4	SADIA S/A	PN N1	1.139
AMBV4	AMBEV	PN	1.122
TNLP4	TELEMAR	PN	1.093
ELET3	ELETROBRAS	ON N1	0.952
ELET6	ELETROBRAS	PNB N1	0.951
NETC4	NET	PN N2	0.946
RDCD3	REDECARD	ON EJ NM	0.933
GFSA3	GAFISA	ON NM	0.93
GOAU4	GERDAU MET	PN N1	0.916
BTOW3	B2W VAREJO	ON NM	0.904
PRGA3	PERDIGAO S/A	ON NM	0.887
LREN3	LOJAS RENNER	ON NM	0.858
ARCZ6	ARACRUZ	PNB N1	0.777
TCSL4	TIM PART S/A	PN	0.758

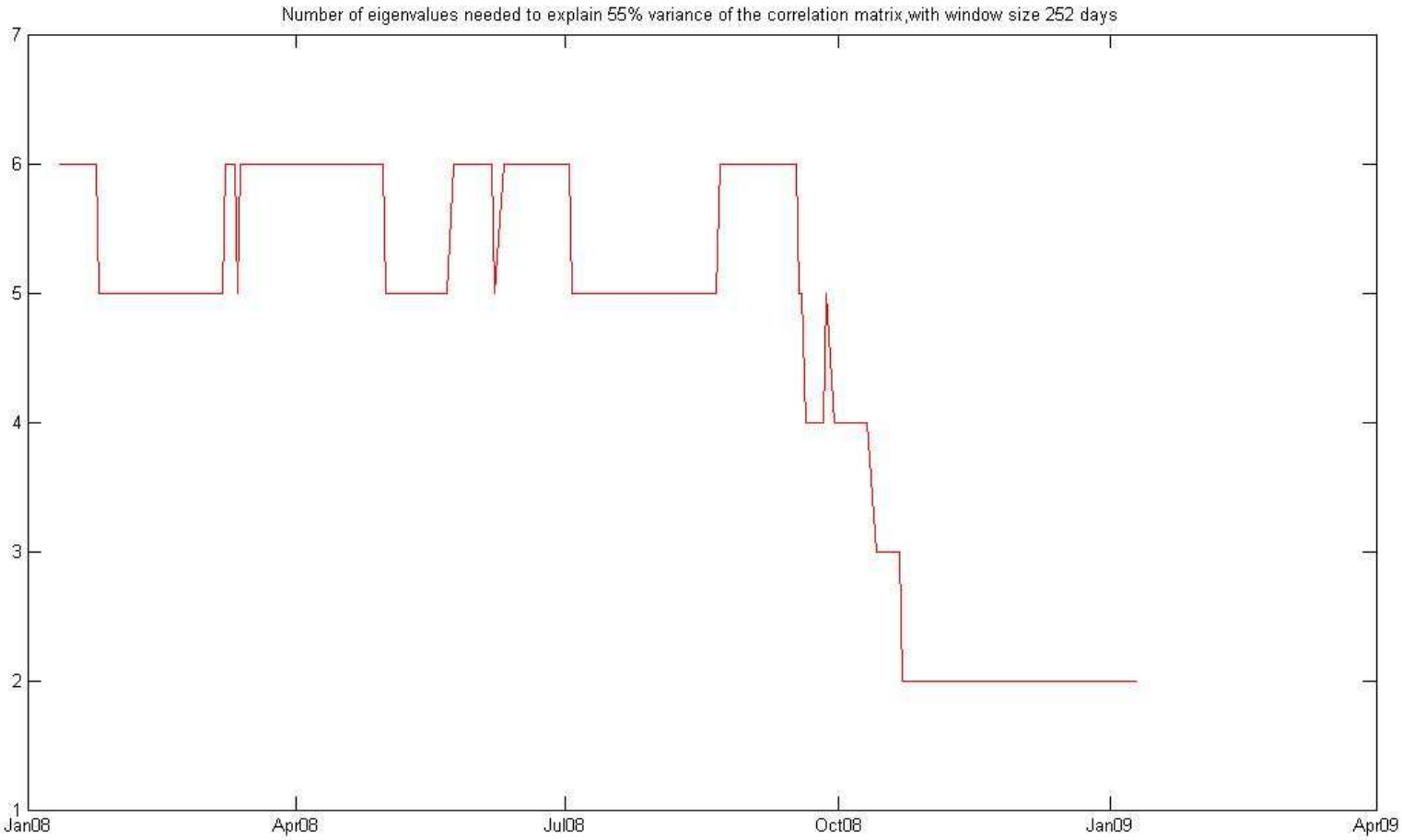
TICKER	COMPANY	TYPE	WEIGHT
GOLL4	GOL	PN N2	0.737
VIVO4	VIVO	PN EJ	0.722
TAMM4	TAM S/A	PN N2	0.711
ELPL6	ELETROPAULO	PNB EJ N2	0.691
JBSS3	JBS	ON NM	0.658
EMBR3	EMBRAER	ON NM	0.656
CPLE6	COPEL	PNB N1	0.651
NATU3	NATURA	ON NM	0.638
USIM3	USIMINAS	ON EJ N1	0.62
VCPA4	V C P	PN N1	0.618
CCRO3	CCR RODOVIAS	ON NM	0.611
CSAN3	COSAN	ON NM	0.603
RSID3	ROSSI RESID	ON NM	0.569
PCAR4	P.ACUCAR-CBD	PN N1	0.553
DURA4	DURATEX	PN N1	0.551
BRKM5	BRASKEM	PNA N1	0.545
CPFE3	CPFL ENERGIA	ON NM	0.522
CRUZ3	SOUZA CRUZ	ON EJ	0.487
BRTP4	BRASIL T PAR	PN EJ N1	0.437
UGPA4	ULTRAPAR	PN N1	0.437
BRTO4	BRASIL TELEC	PN EJ N1	0.42
TNLP3	TELEMAR	ON	0.418
SBSP3	SABESP	ON NM	0.403
BNCA3	NOSSA CAIXA	ON EJ NM	0.397
TRPL4	TRAN PAULIST	PN N1	0.371
KLBN4	KLABIN S/A	PN N1	0.355
TMAR5	TELEMAR N L	PNA EJ	0.296
BRTP3	BRASIL T PAR	ON EJ N1	0.284
LIGT3	LIGHT S/A	ON NM	0.203
TCSL3	TIM PART S/A	ON	0.197
TLPP4	TELESP	PN EJ	0.191
CLSC6	CELESC	PNB N2	0.121
CGAS5	COMGAS	PNA	0.114

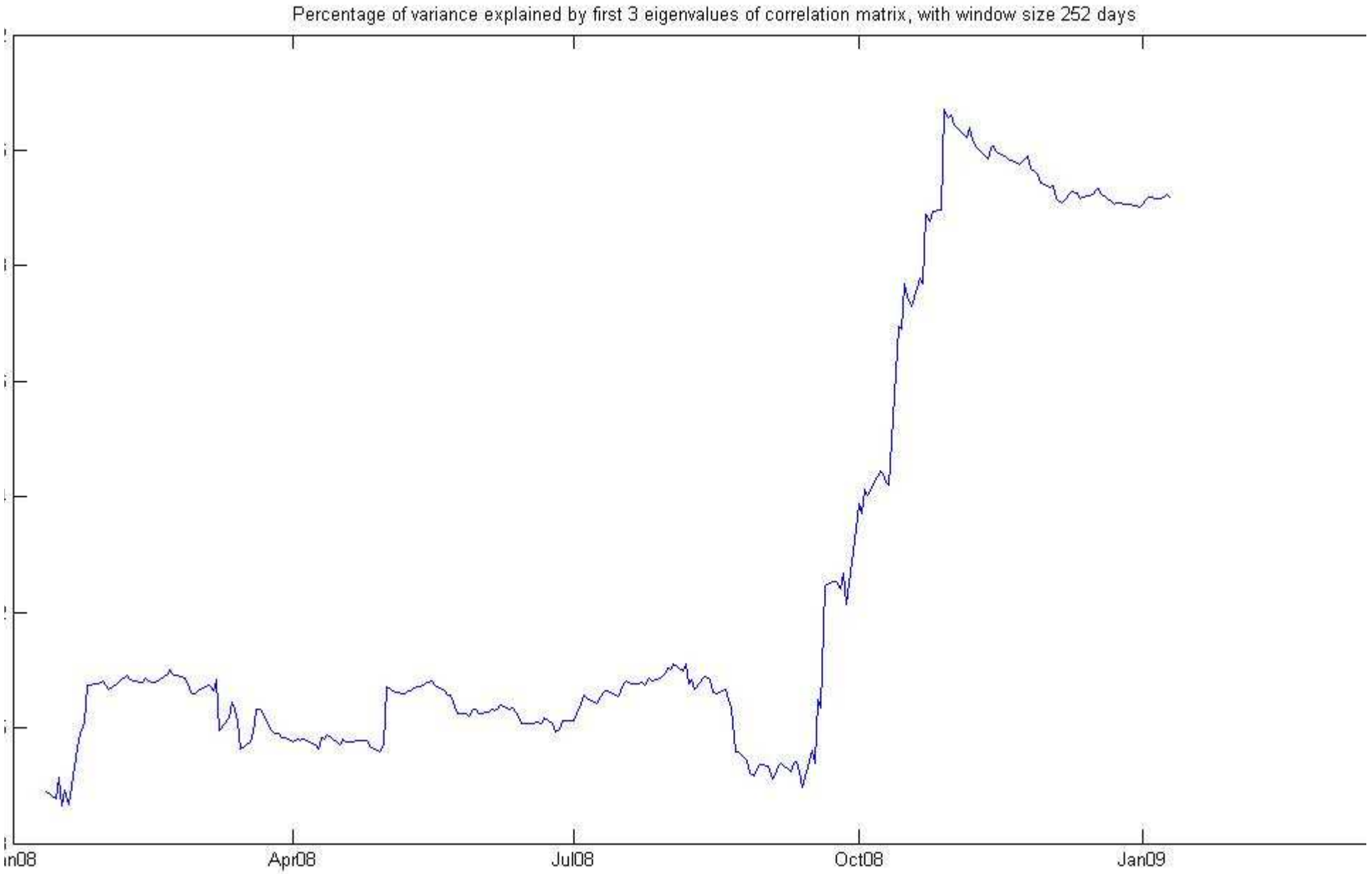
Capitalization of IBOVESPA components



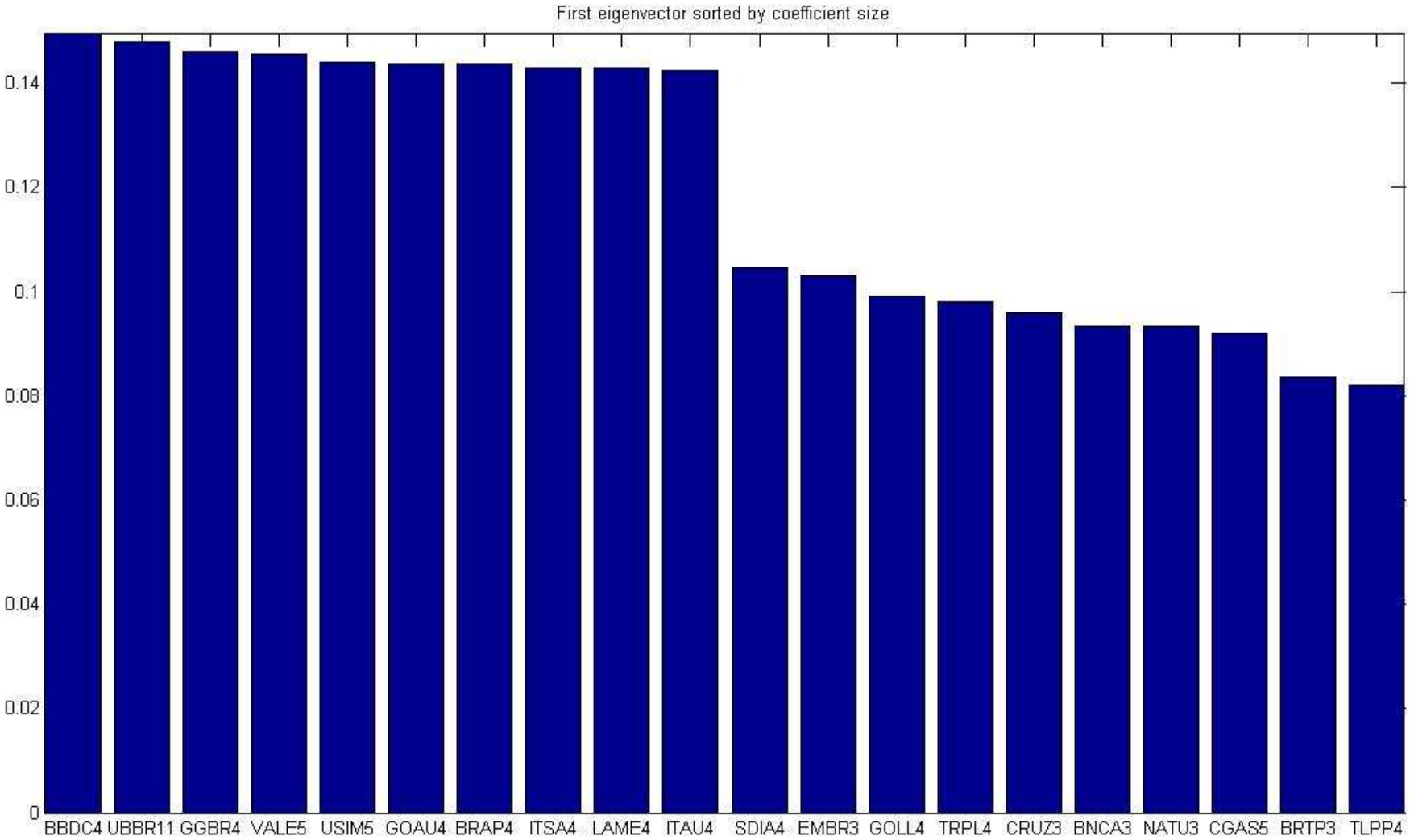
IBOVESPA Correlation Spectrum



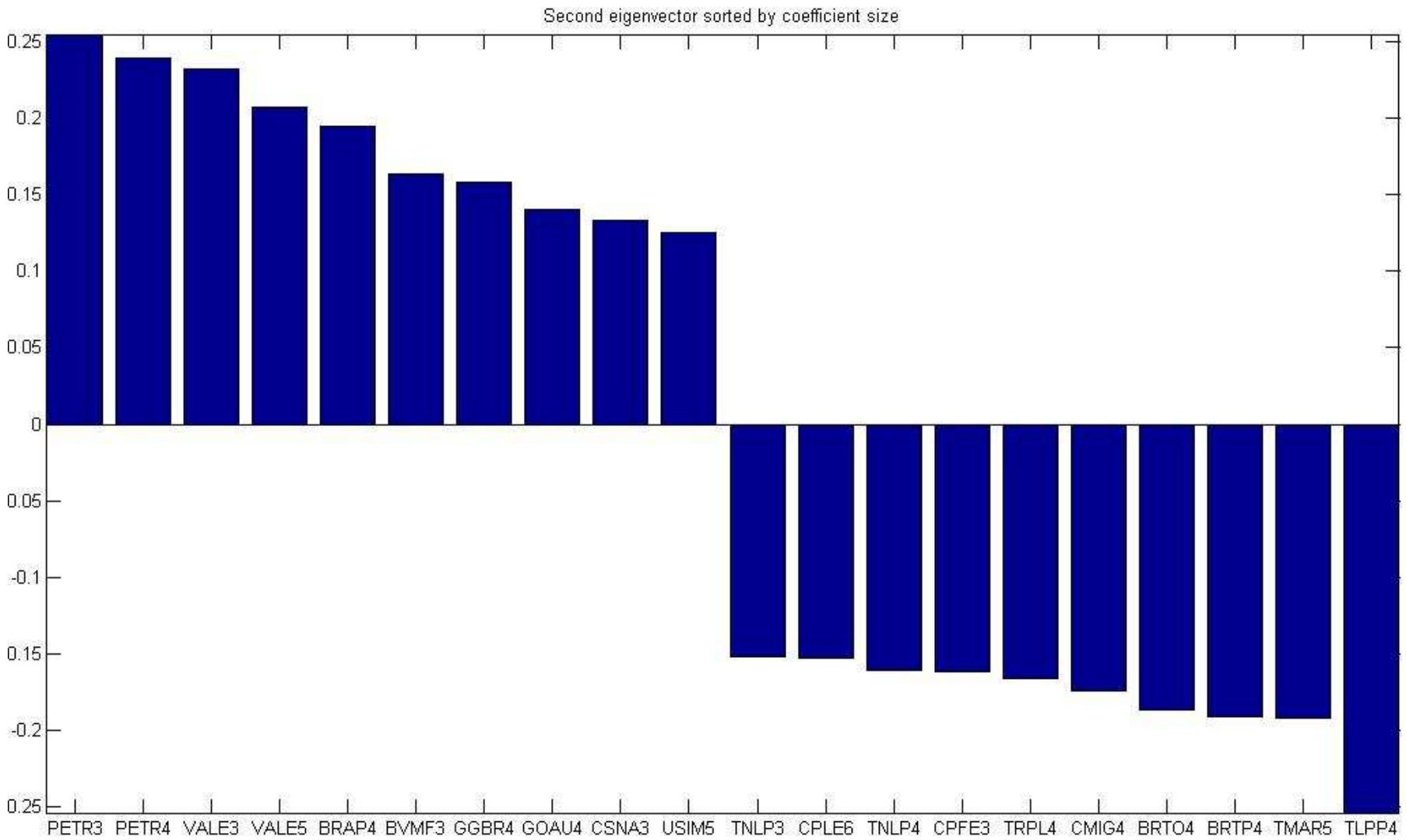

Number of eigenvalues needed to explain 55% variance of the correlation matrix,with window size 252 days
7
6
5
4
3
2
1
Jan08
Apr08
Jul08
Oct08
Jan09
Apr09


Percentage of variance explained by first 3 eigenvalues of correlation matrix, with window size 252 days
Apr08
Jul08
Oct08
Jan09

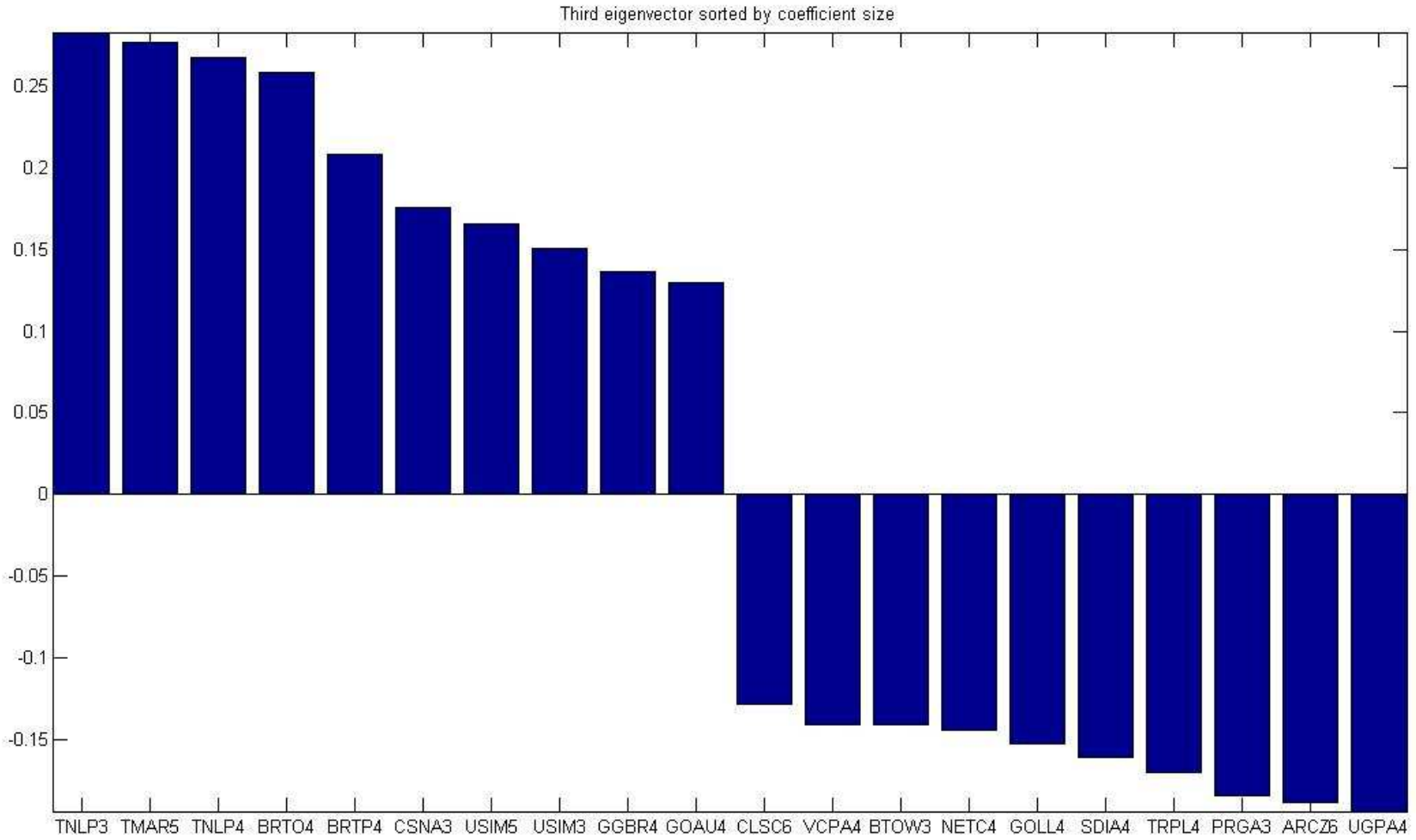
First Eigenvector



Second eigenvector



Third eigenvector

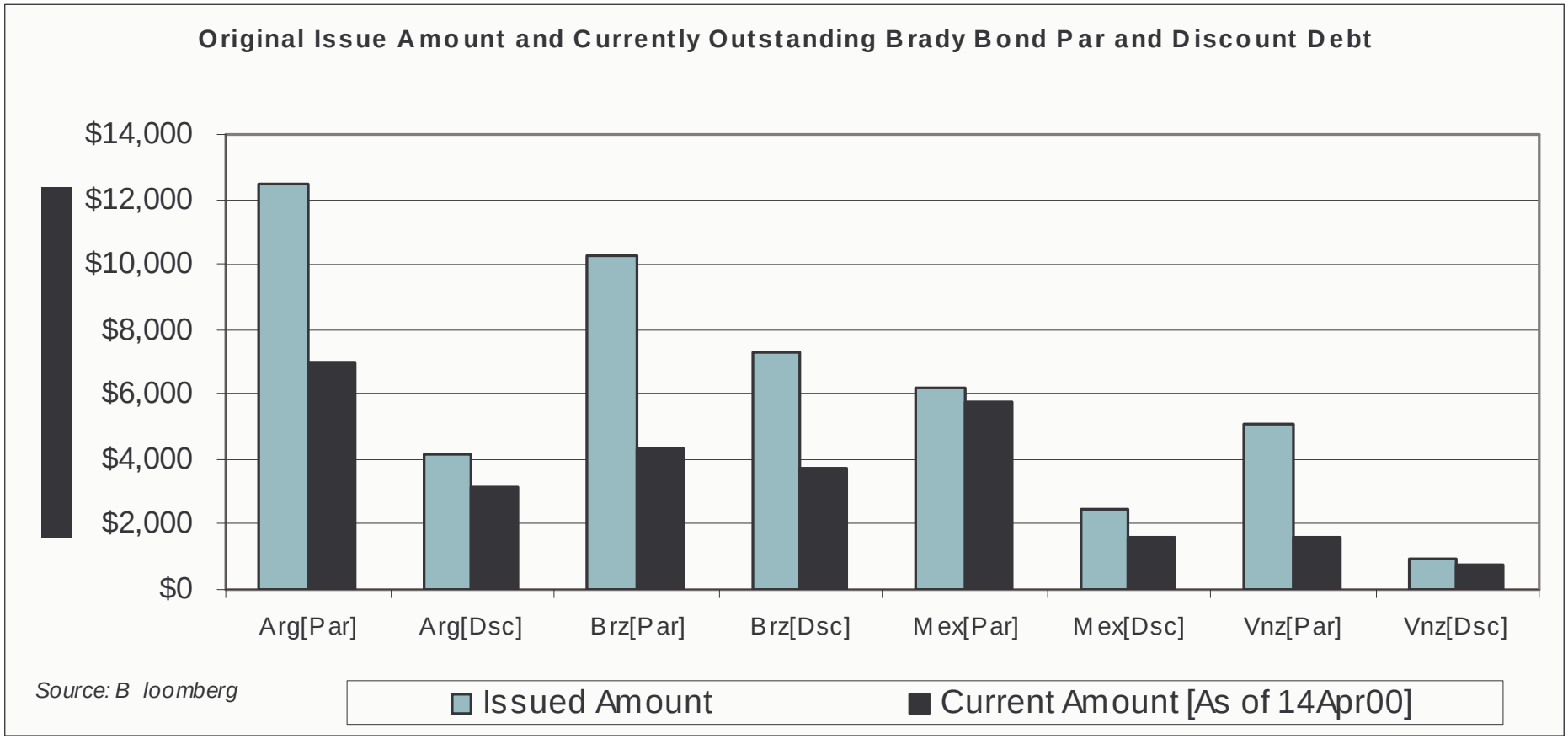


Sovereign Bond Spreads
Brady Bonds from the 1990s

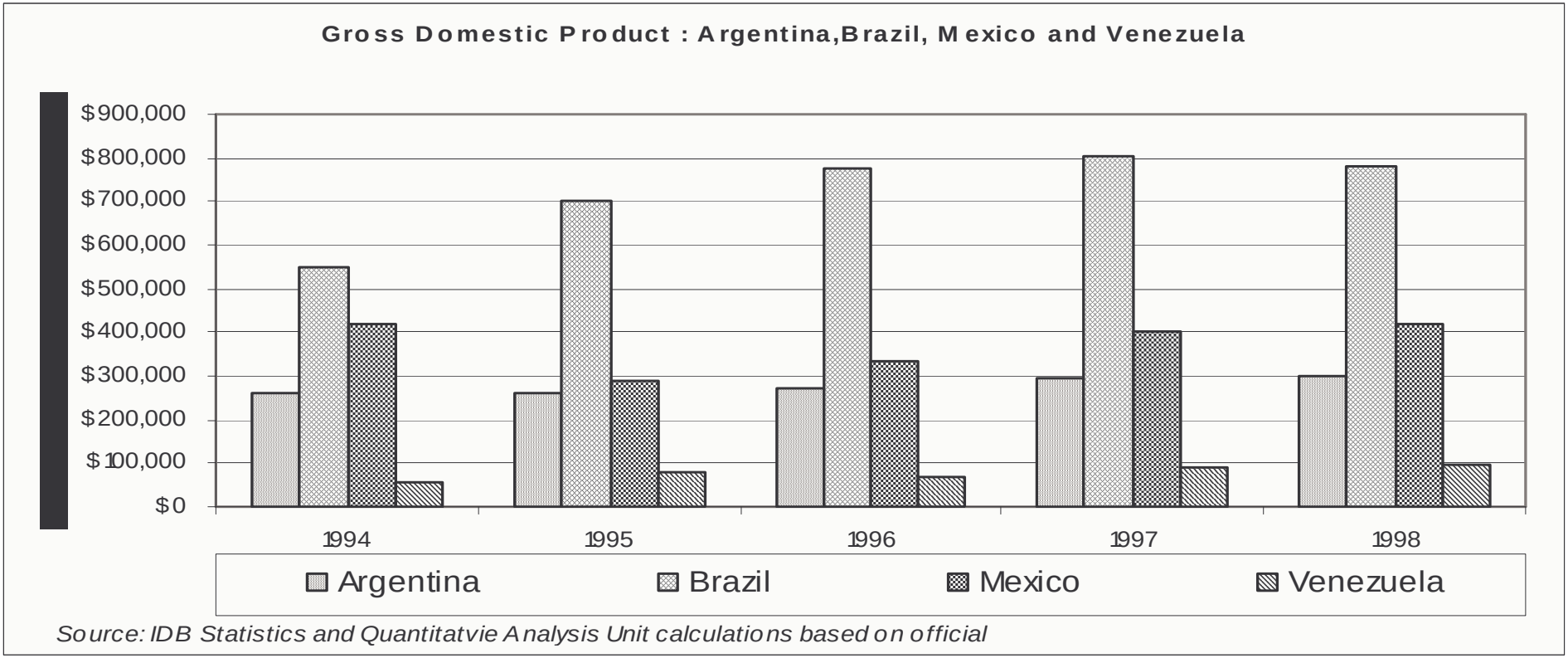
Major Brady Bond Issuers

Mexico
Venezuela
Brazil
Argentina

Brady Bond Issuance by Country



GDP of Issuers



Source: IDB Statistics and Quantitatvie Analysis Unit calculations based on official

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CGAS5	COMGAS	PNA	0.114

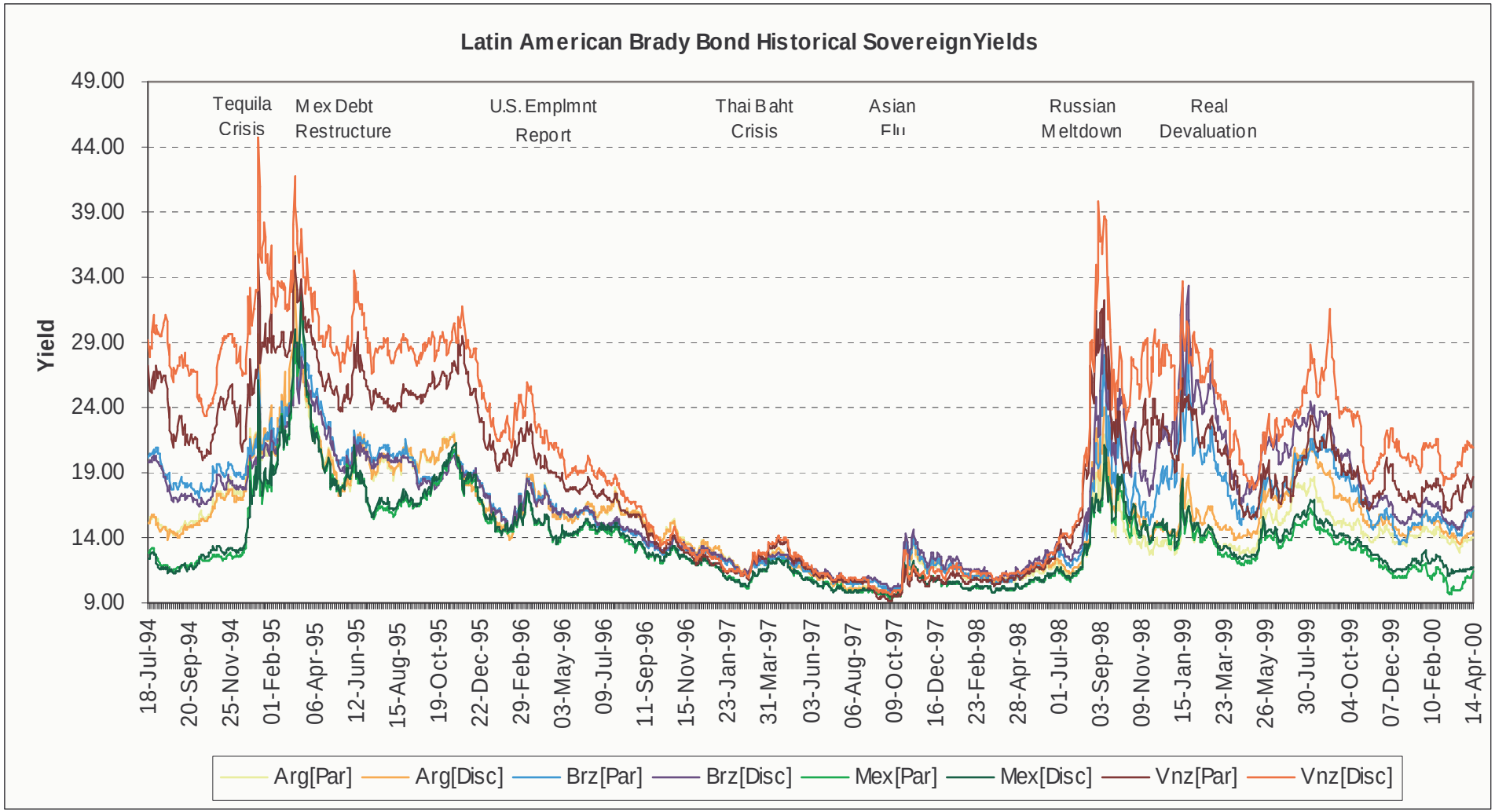
Characteristics of Brady Bonds

Rolling Guarantee: Principal and next two/ three coupons are collateralized with US Treasury securities

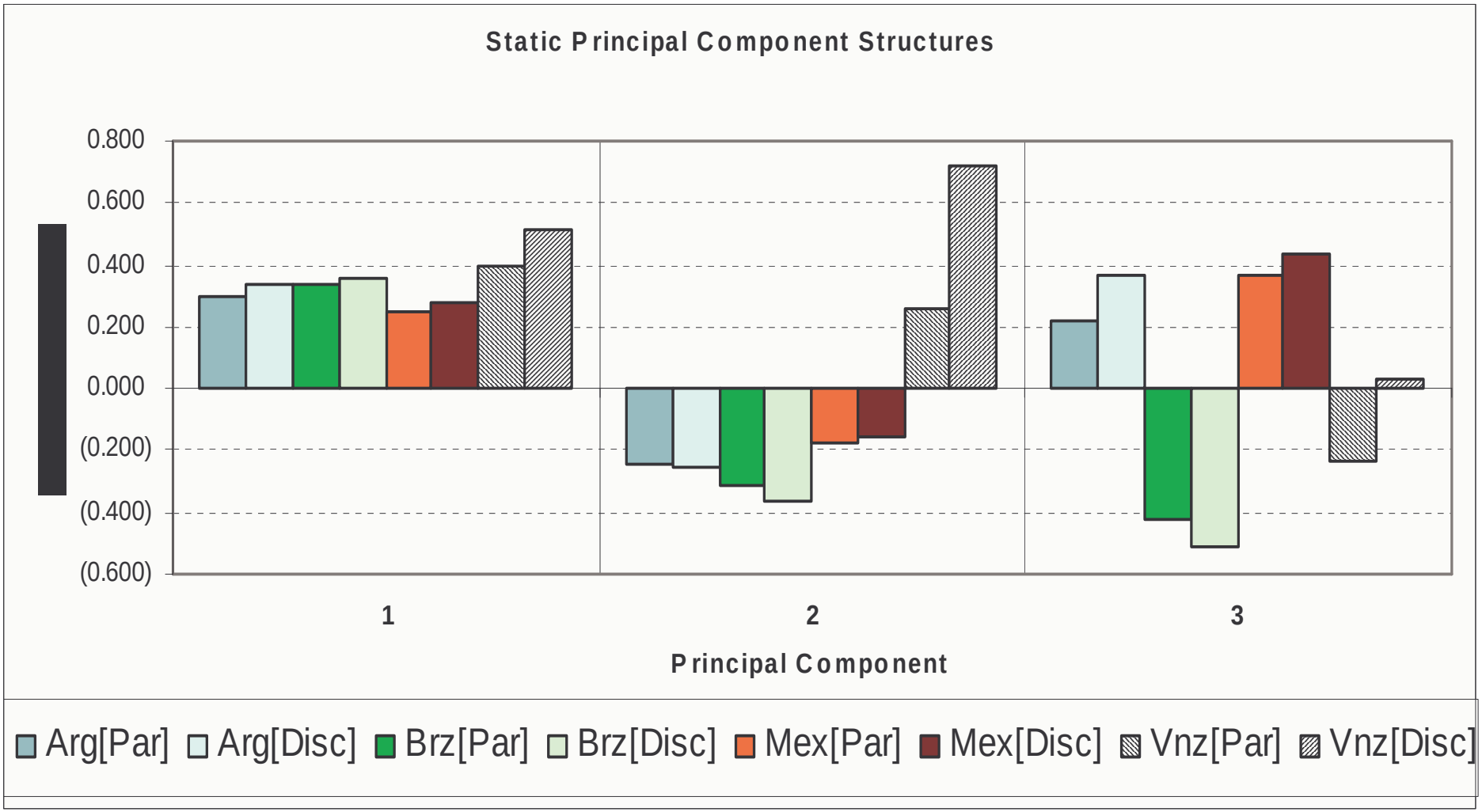
Par Bonds: Fixed rate, step-up coupon

Discount Bonds: Floating rate, amortizing schedule for principal

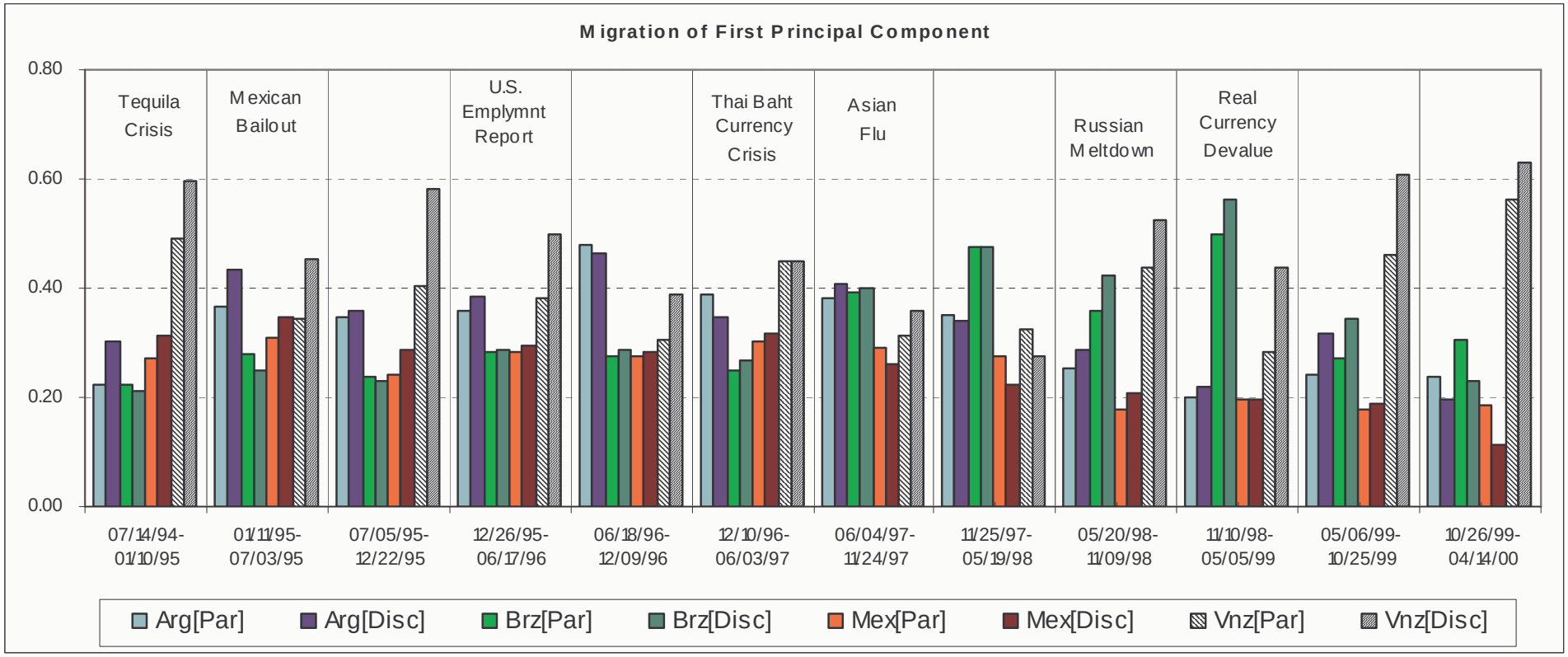
Evolution of Bond Yields



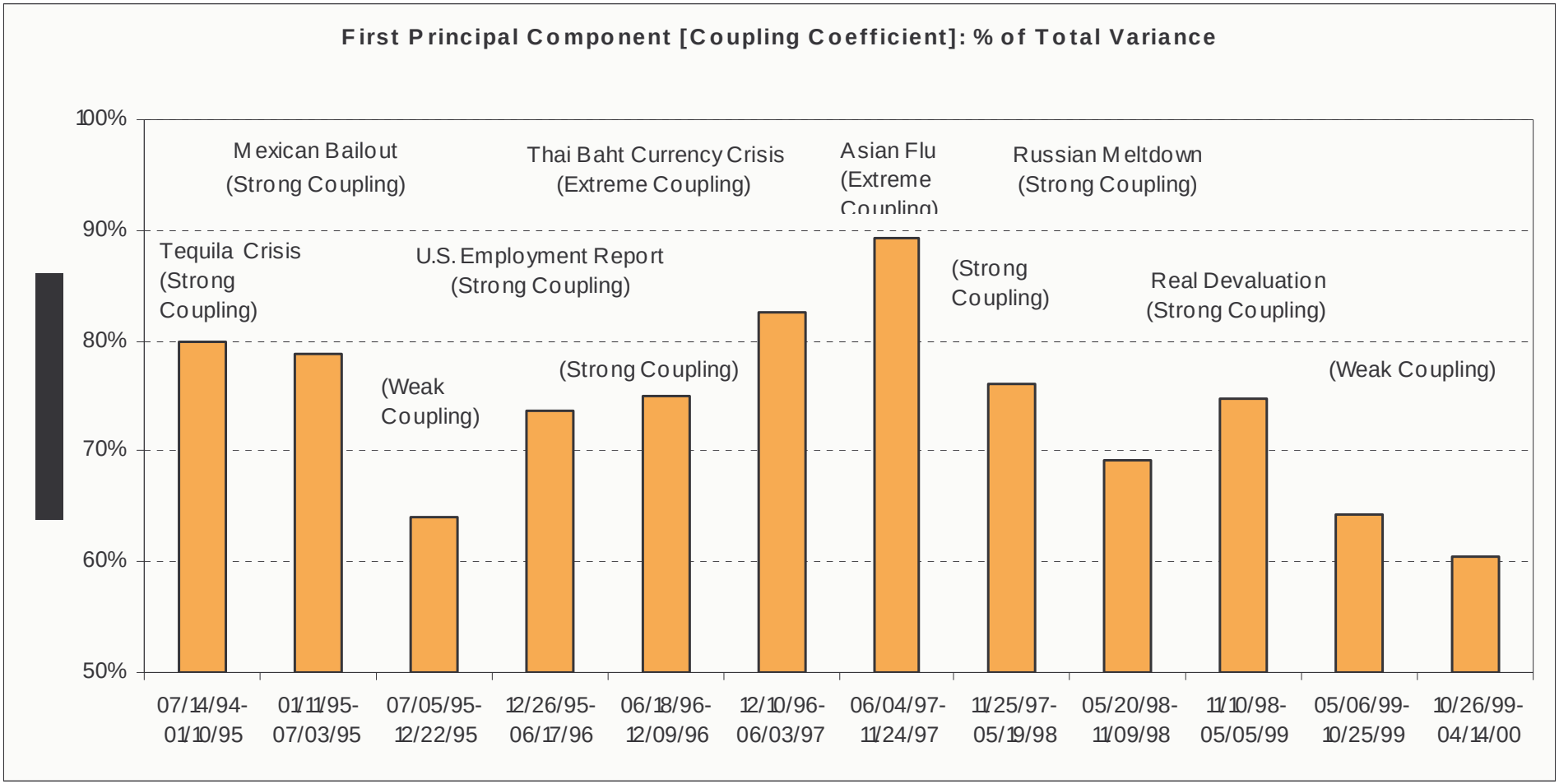
Static PCA: first three factors



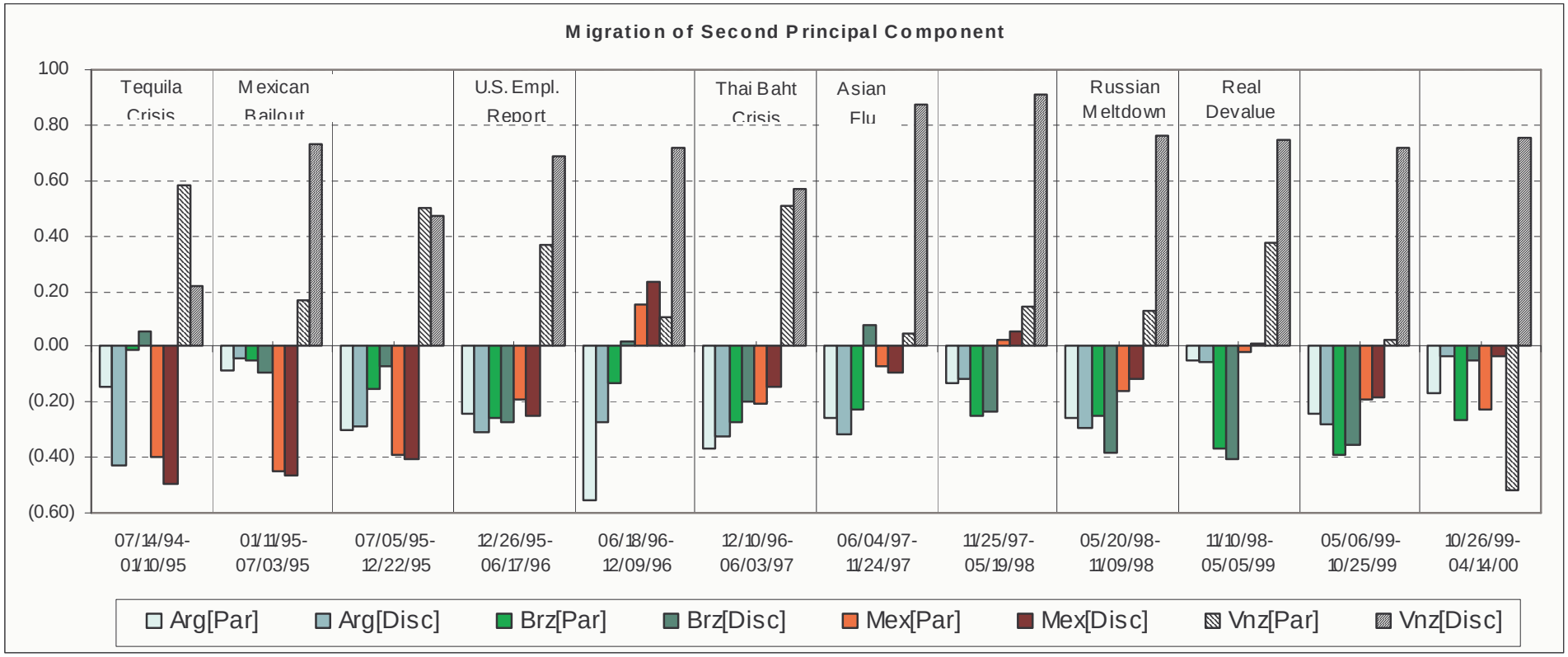
Dynamic PCA: Evolution of the first Eigenvector



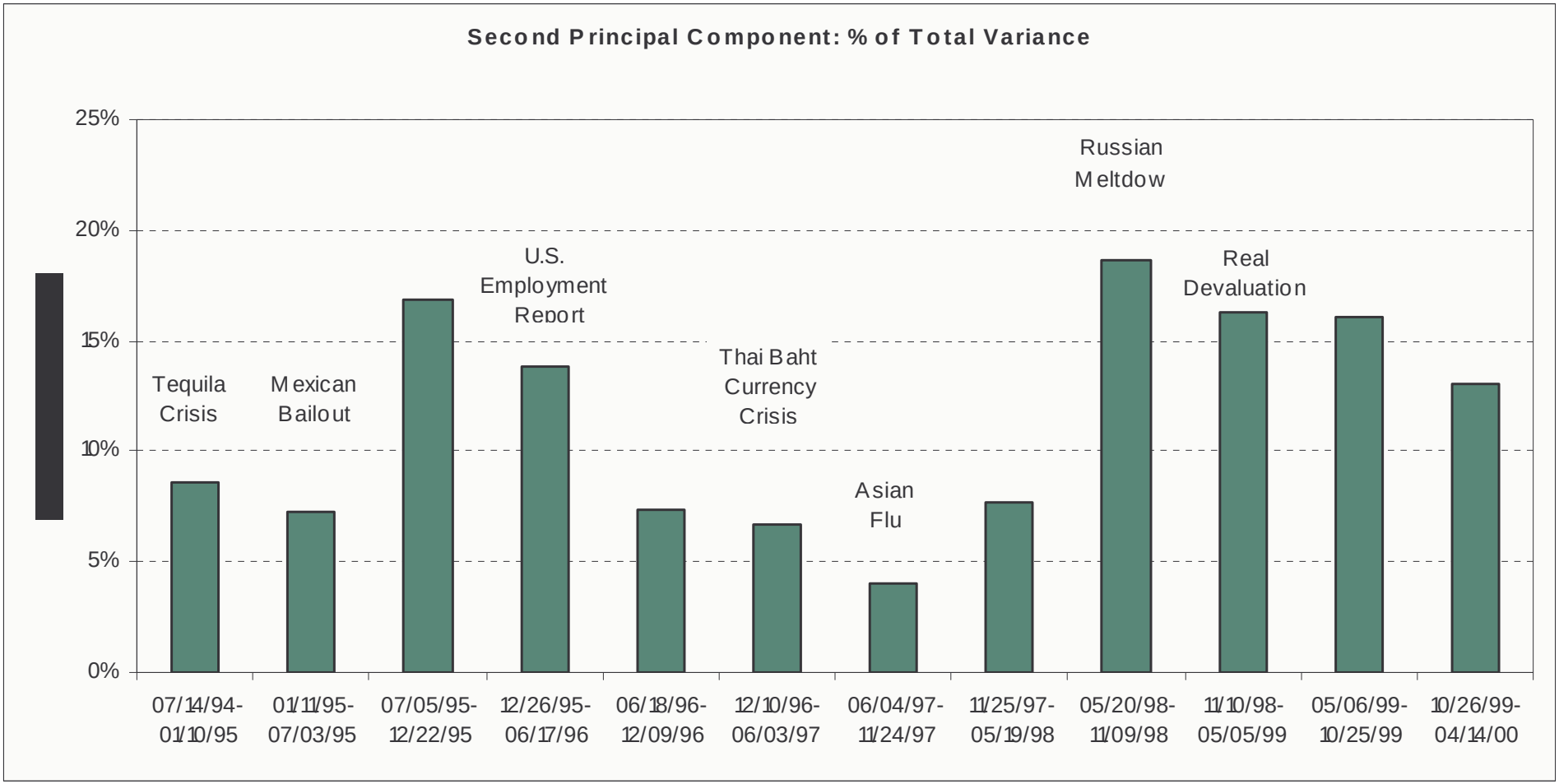
Evolution of the First Eigenvalue



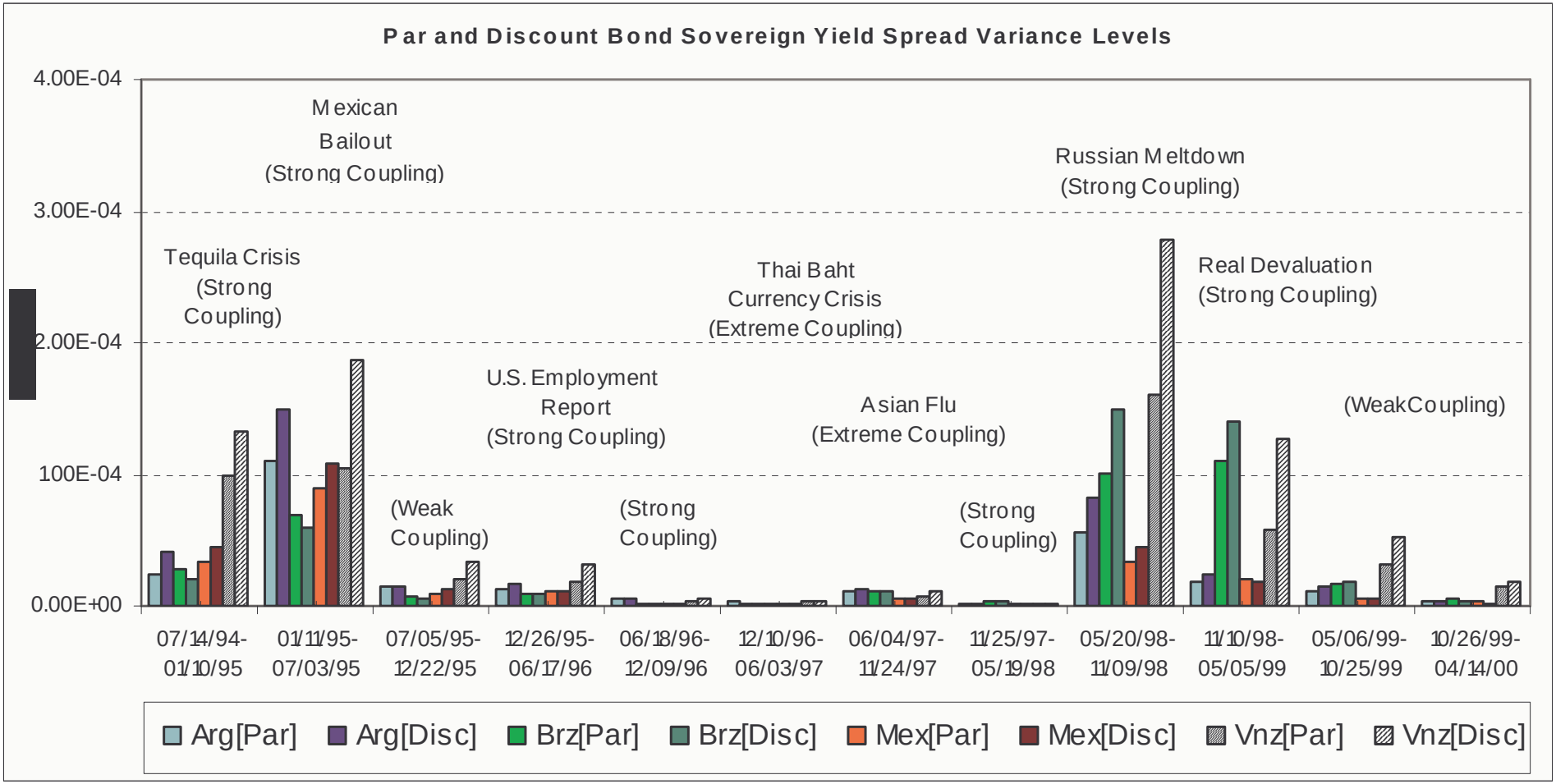
Second Eigenvalue



Second Eigenvalue



Yield Variance



Second Eigenvalue

Second PCA Structure	Number of Observations
Venezuela vs. Argentina/Mexico/Brazil	5
Venezuela vs. Argentina/Brazil	2
Venezuela vs. Argentina/Mexico	1
Venezuela vs. Brazil or Mexico	1 each

Correlation Matrices and Risk Management

Portfolio weights $Q_1, Q_2, ..., Q_N$

$$R(Q_1, Q_2, ..., Q_N) = \sum_{k=1}^{N_{factors}} \lambda_k \left(\sum_{i=1}^{N} V_i^{(k)} \sigma_i Q_i \right)^2 + \sum_{i=1}^{N} \nu_i \sigma_i^2 Q_i^2$$

$$\nu_i = 1 - \frac{\sum_{k=1}^{N_{factors}} \lambda_k}{N} \sum_{k=1}^{N_{factors}} \left(V_i^{(k)} \right)^2$$