Institute of Electrical Control Engineering
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內插之物理意義
以時域高階微分特徵擷取與DAC為例

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前言: 動機

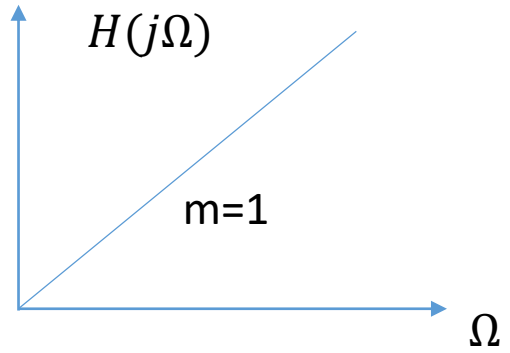
- 當需要時域高階微分做特徵時，高或低fps實驗結果差異不少。
- 差異來源為何?
- 有人提可以做內插? 真的有用嗎?
- 內插不是說沒有增加資訊量? 還可能失真?

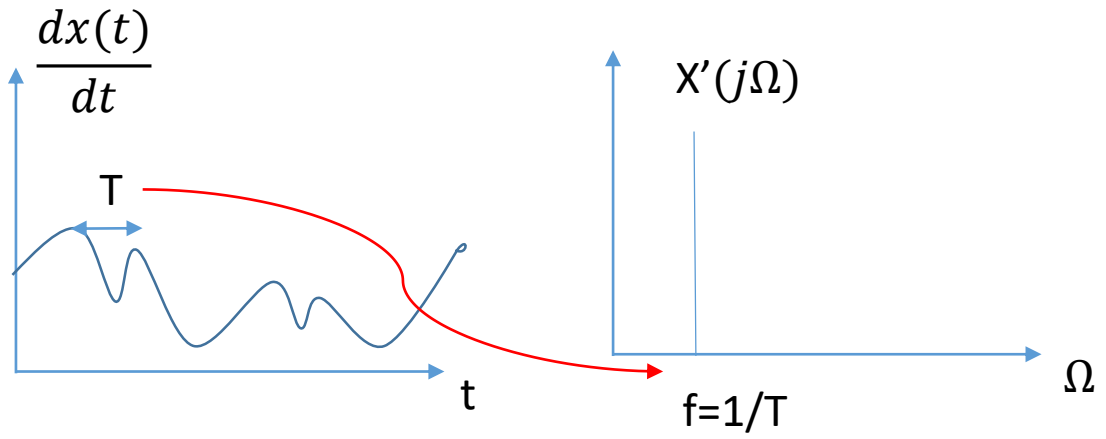
Outline

- 時域高階微分特徵擷取, ADC/DAC
  - 例子: 訊號從哪裡消失/失真? 時域特徵 →倍數降低FPS需求
  - Quantization Noise.

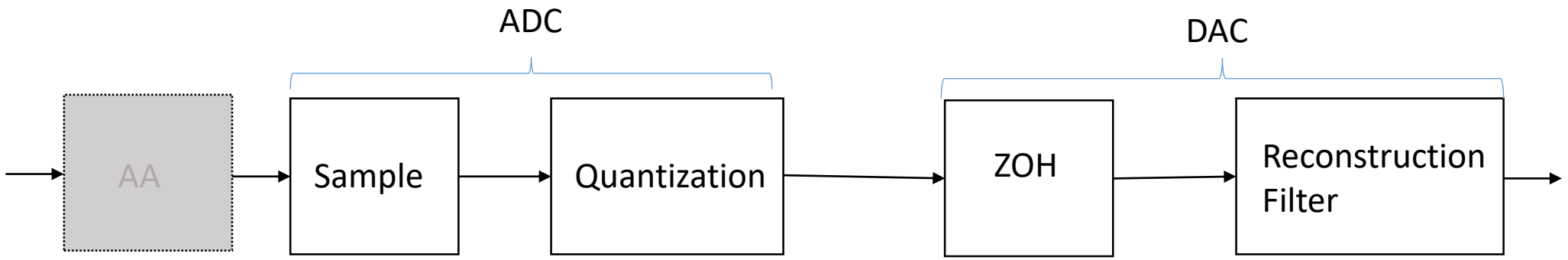
連續特徵

- 微分之頻域響應(analog):
  - $sin'(\Omega t) = \Omega \cos(\Omega t)$
- 時頻對應:
  - 時域特徵間隔T↔存在頻率為1/T之訊號
- 結論:
  - $\frac{d^n x}{dt^n}$ 存在間隔T特徵 ↔ x(t)存在1/T之訊號
  - Nots: T約等於0.1sec

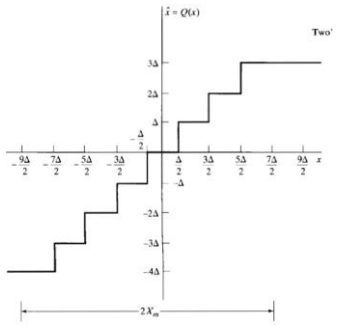


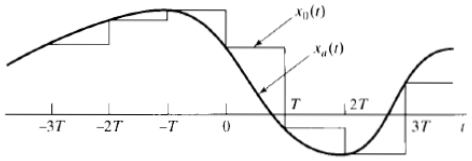


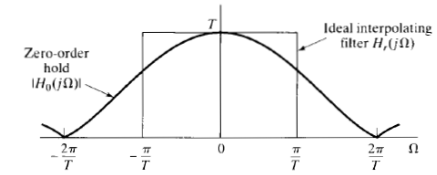
ADC/DAC

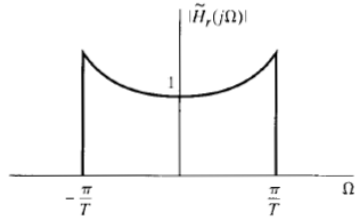


- Anti-Aliasing
- Nyquist Rate
- White Noise







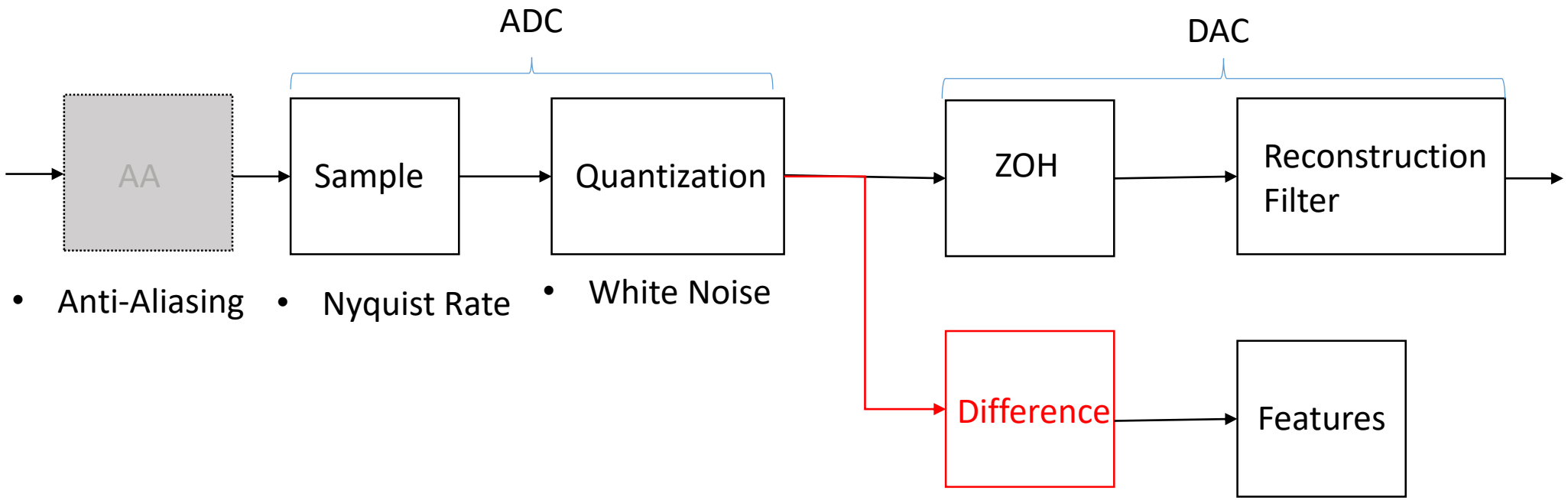


Nyquist Theorem: Fs只要>2*Fmax, 即可完整reconstruct

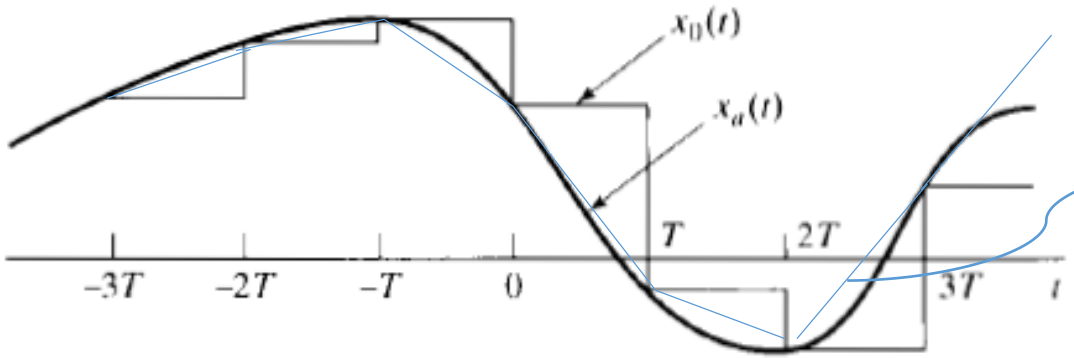
→儀器不精確會造成雜訊, 但不會使訊號消失

→為何PPG 1500Hz 效果比rPPG 30Hz好這麼多?

ADC/DAC: 目前特徵取法:



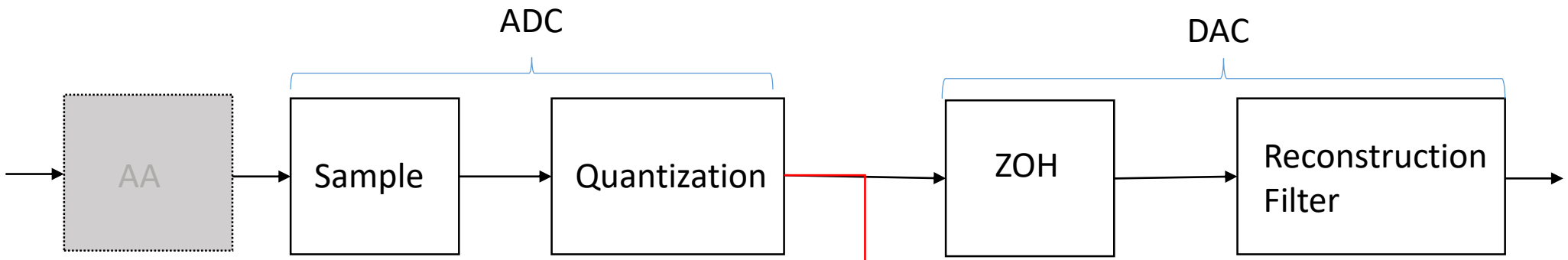
- Anti-Aliasing
- Nyquist Rate
- White Noise



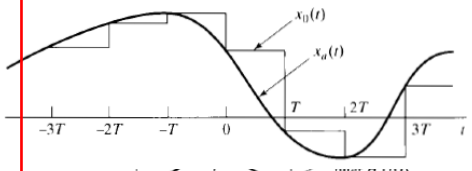
e'(n)=$\frac{dx}{dt}$ -[$x(n) - x(n-1)$]

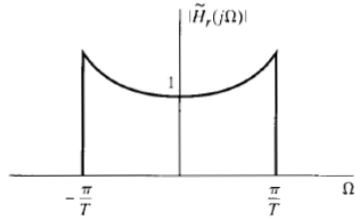
→Error propagation when higher order.

Simulated DAC (Ideal DAC)



- Anti-Aliasing
- Nyquist Rate
- White Noise

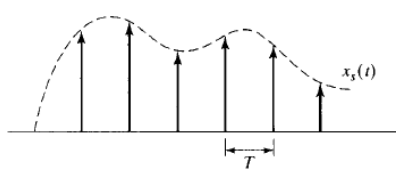


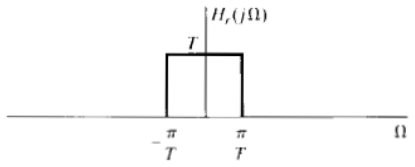


Simulated DAC

Nyquist Theorem:

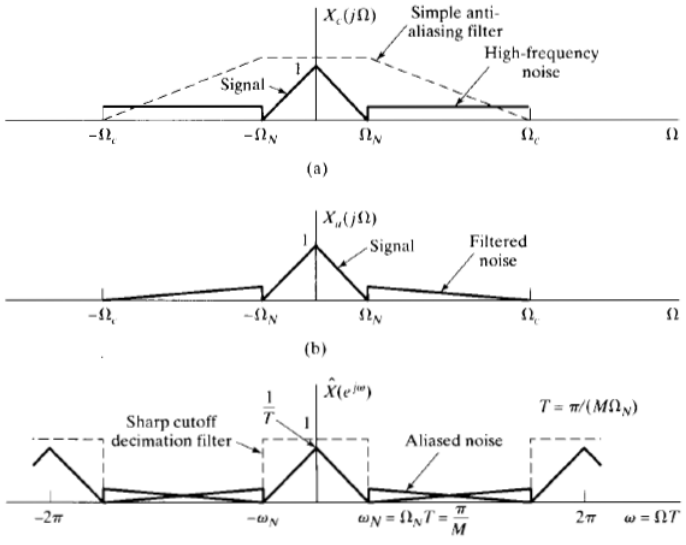
Fs只要>2*Fmax, 即可完整reconstruct





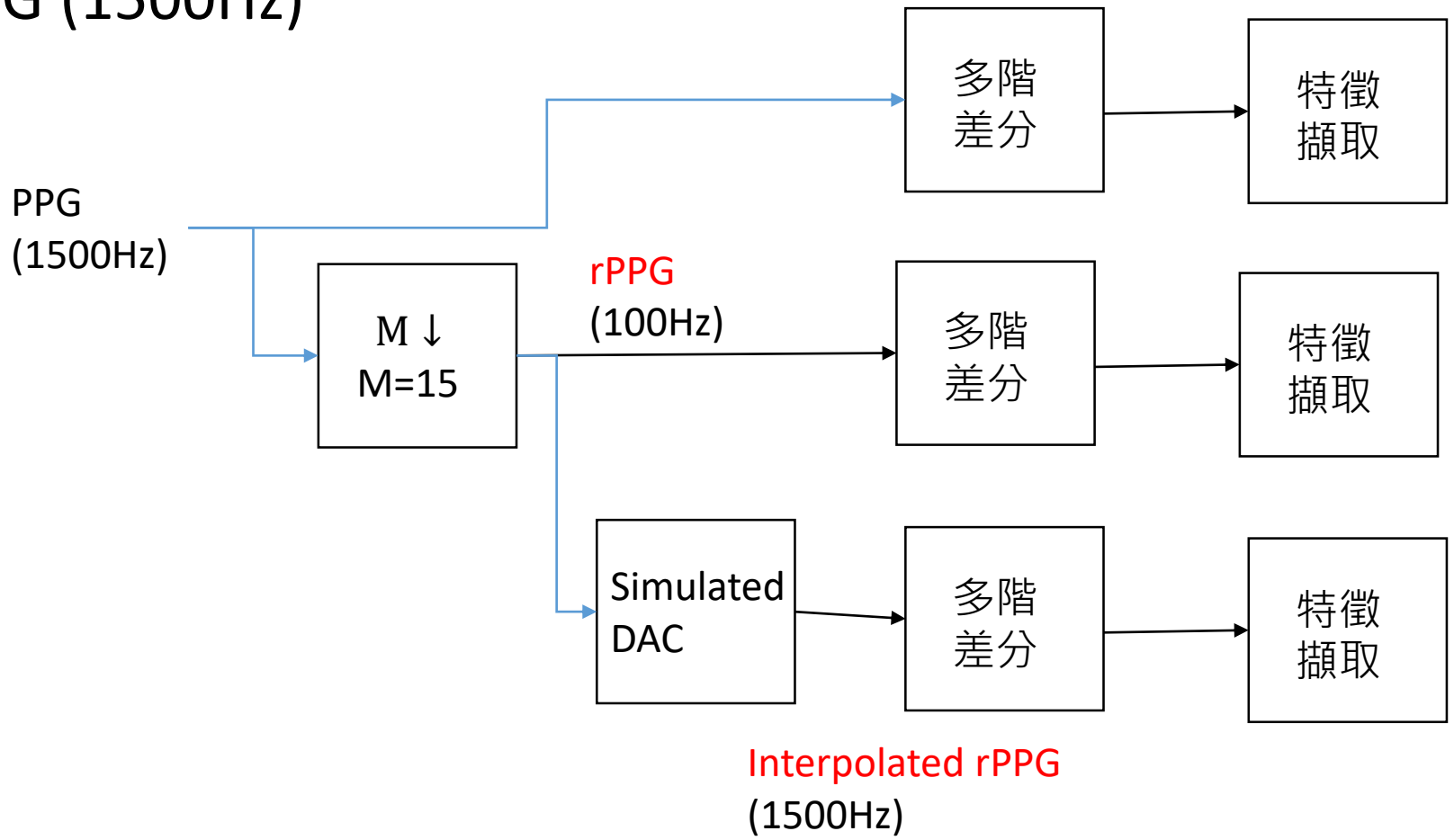
那為何要Oversample?

- 若**Nyquist Theorem實務上為真,**語音44kHz足以,為何要196kHz?
  - 更簡單的AA
  - 更簡單的Reconstruction Filters
  - 較高SNR
- 但高低與否不影響訊號存在與否

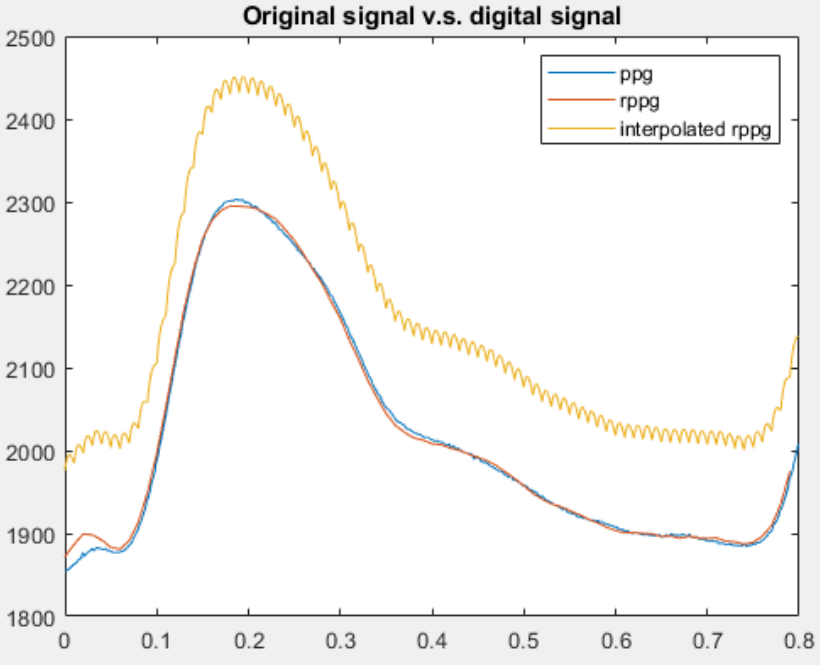


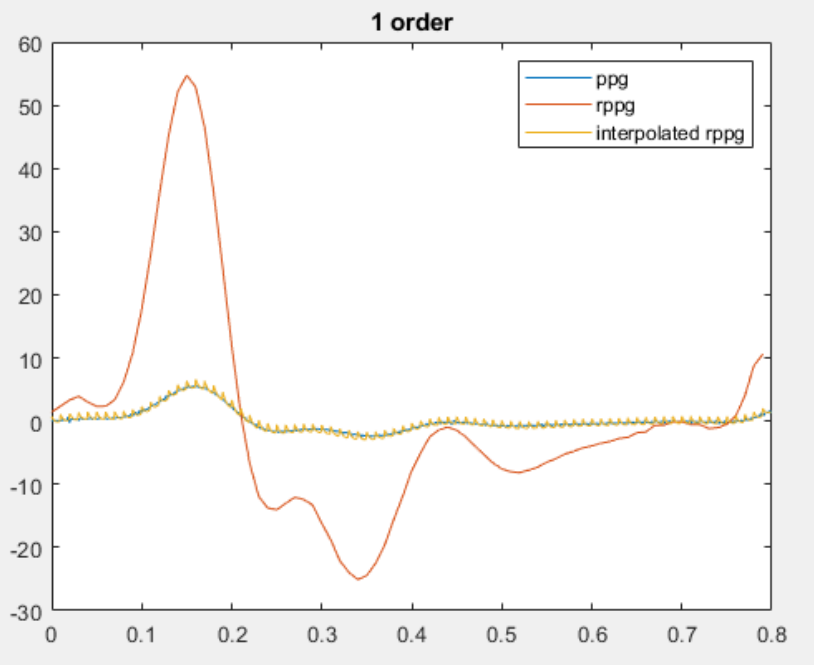
實驗設計:

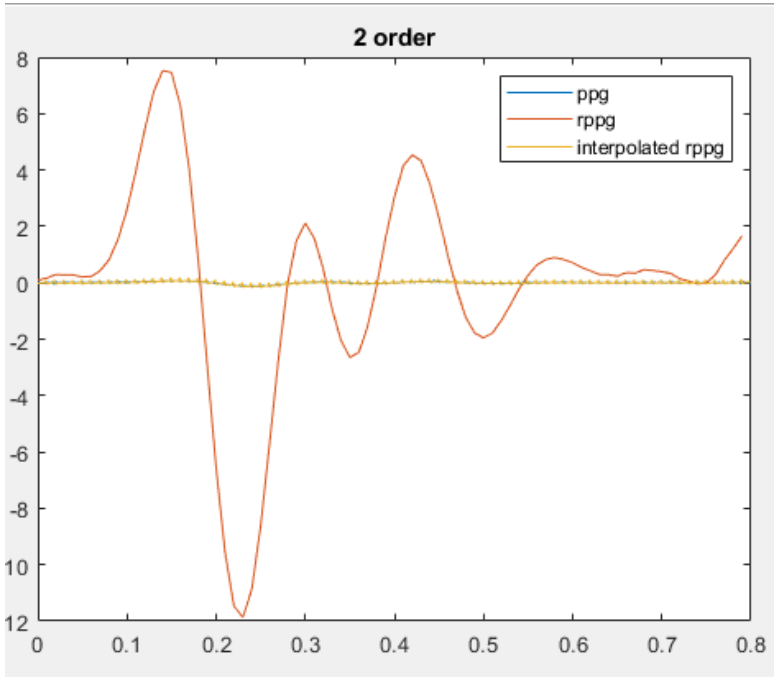
- PPG (1500Hz)



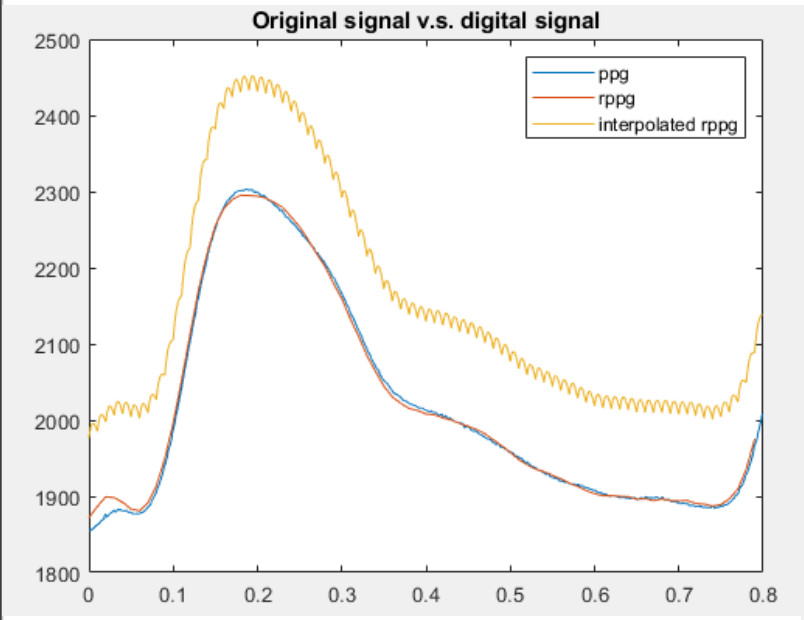
結果

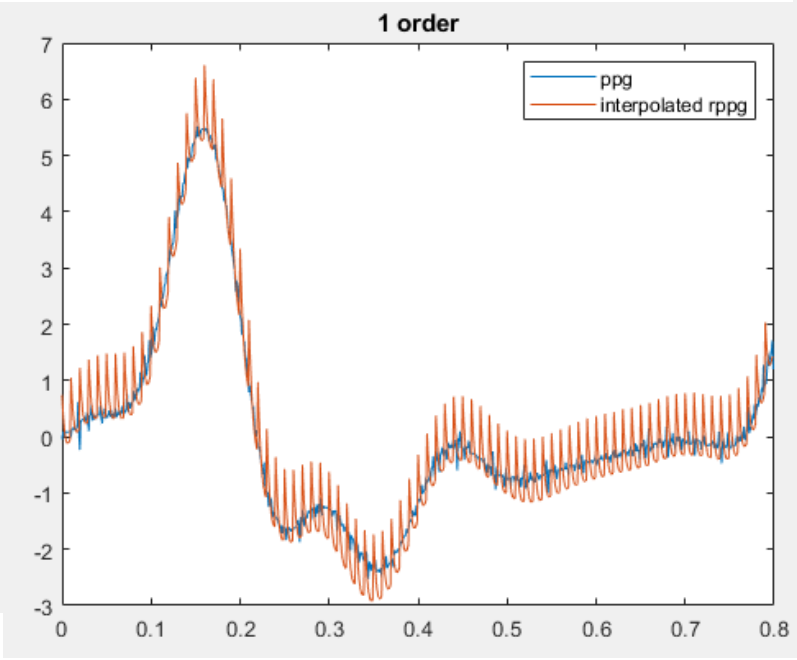

Original signal v.s. digital signal
ppg
rppg
interpolated rppg
2500
2400
2300
2200
2100
2000
1900
1800
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

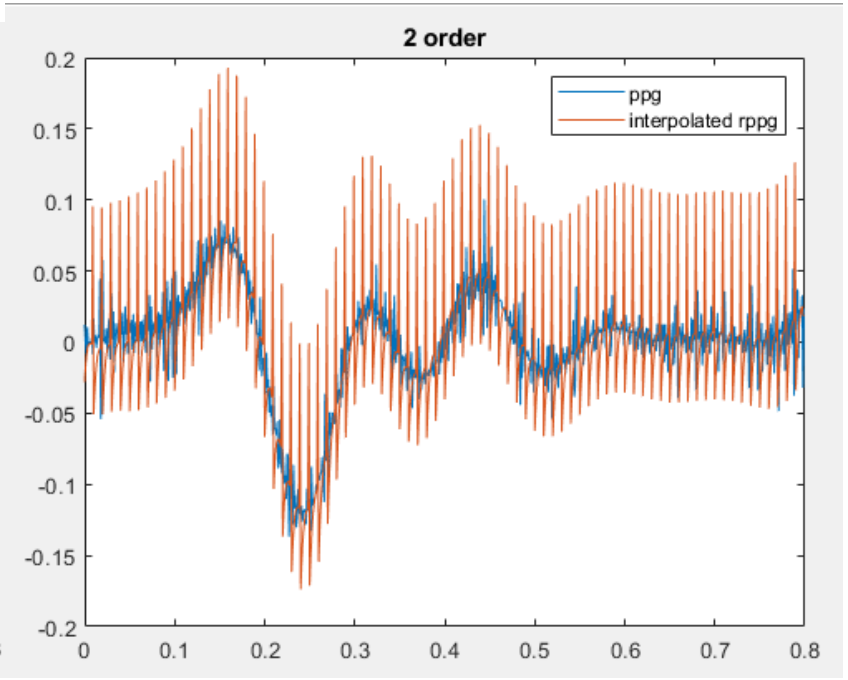

1 order
ppg
rppg
interpolated rppg
60
50
40
30
20
10
0
-10
-20
-30
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


2 order
ppg
rppg
interpolated rppg
8
6
4
2
0
-2
-4
-6
-8
10
12
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

結果(續)

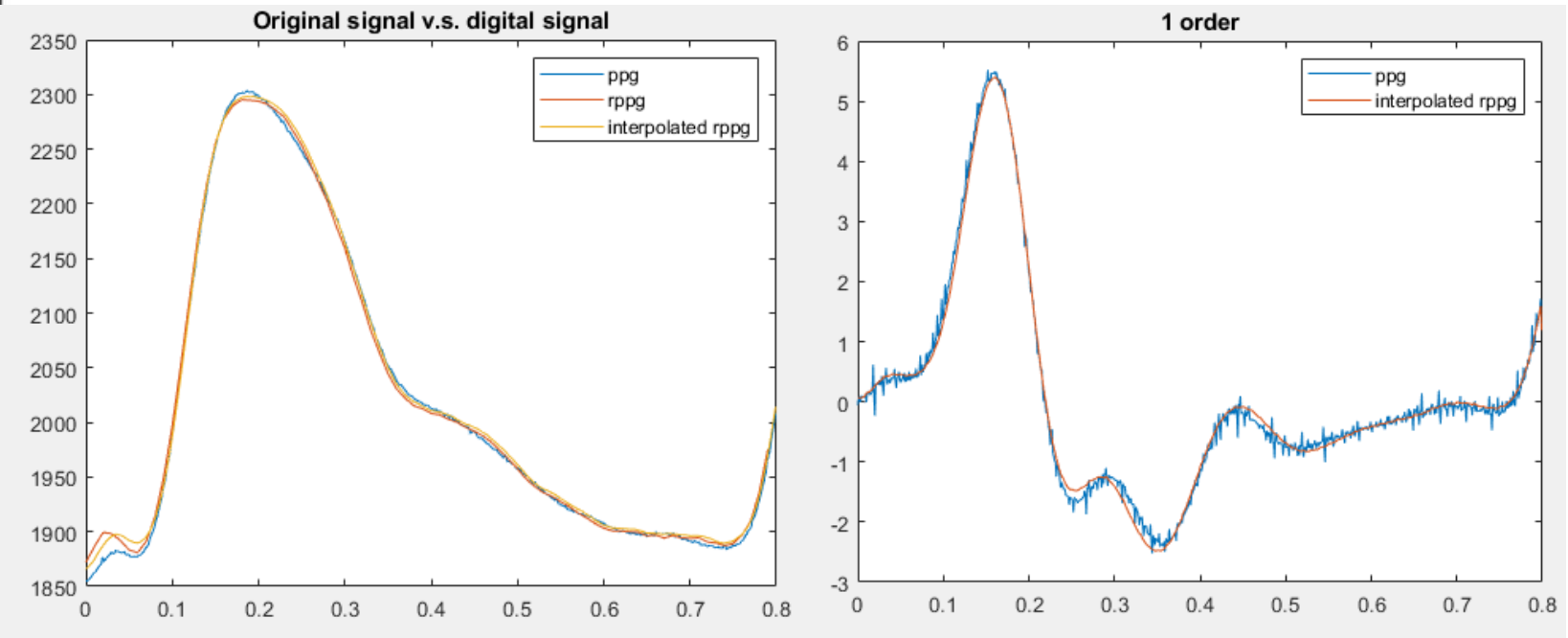

Original signal v.s. digital signal
ppg
rppg
interpolated rppg
2500
2400
2300
2200
2100
2000
1900
1800
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


1 order
ppg
interpolated rppg
7
6
5
4
3
2
1
0
-1
-2
-3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


2 order
ppg
interpolated rppg
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

利用Oversample, 濾掉高頻雜訊, 提高SNR

- LPF: BW改成 -pi/4 ←→pi/4
- 只考慮取樣頻率: 100Hz與1500Hz時域特徵可做到雷同



DAC小總結:

- 特性: LTI (Linear Phase)
- 可以做到
  - 增加時域特徵
- 不可以做到
  - 降噪
  - 改變頻域響應

運算量考量

- DAC後運算量激增?
  - 轉換恆等式
- 直觀想法: upsample一堆0, 有辦法避免
- 解法: Polyphase Decomposition
  - 原本: NL乘法 & NL-1加法
  - 後來: N乘法 & L(N/L-1)加法

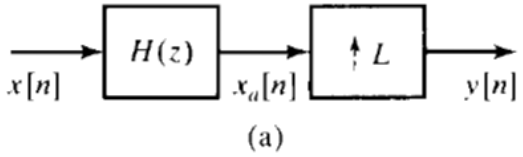
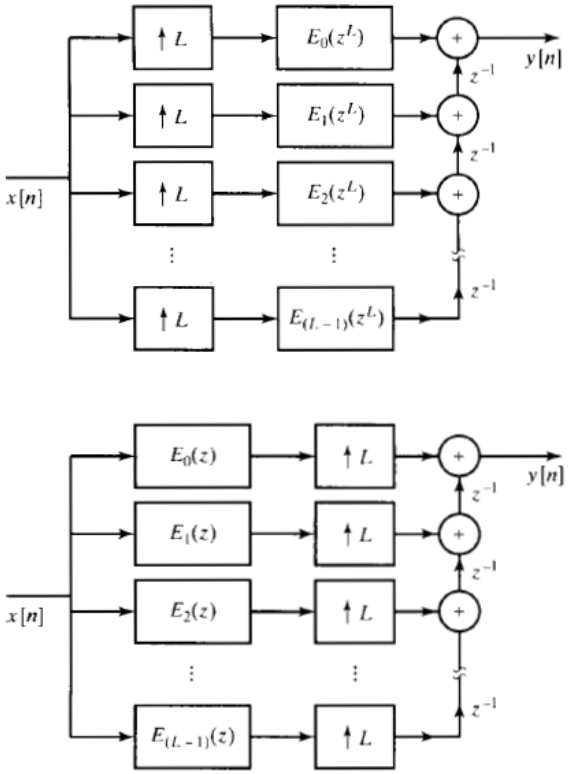
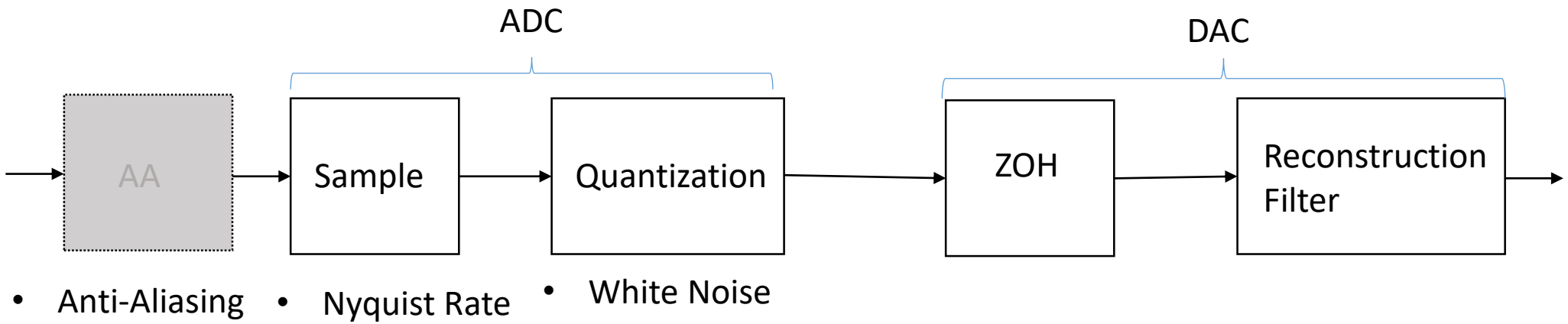


Figure 4.31 Two equivalent systems based on upsampling identities.



AD/DA的雜訊來源:



- Quantization
- Filter誤差
  - AA的誤差
  - Reconstruction Filter的誤差
- **仍需考慮: 攝影機與ADC之差異**

Quantization Noise:

- 1bit = M*4 = X_m/4 = 6dB
- M: oversample ratio
- X_m→如右圖
- If sigma delta ADC is allowed, we can save more.

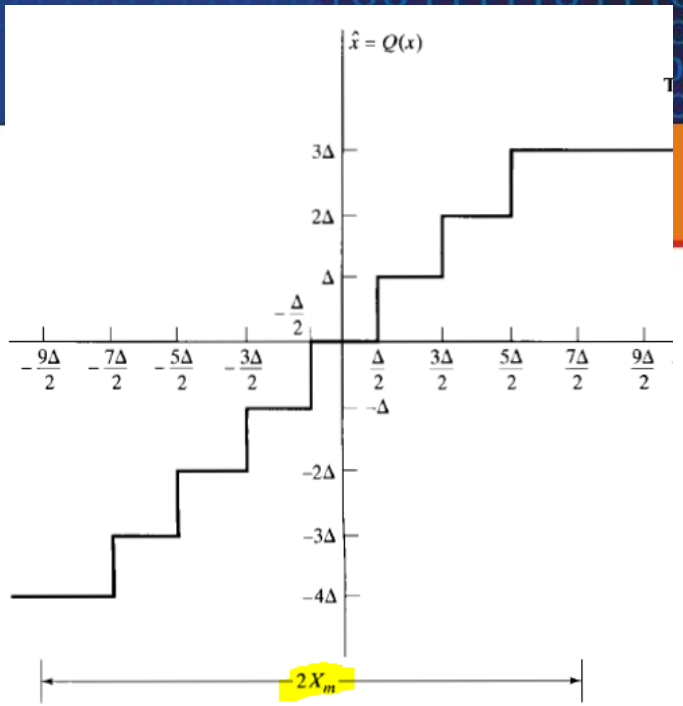


Figure 4.48 Typical quantizer for A/D conversion.