

Impedance Matching Using Smith Charts

Guide

This document is for references only. Please, email zafar@bu.edu for comments, errors, typos, etc.

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Most of the material provided in this guide is part of EC782 course at Boston University (special thanks to Prof. Ronald Knepper).

Introduction

Impedance matching is the practice of designing the input (output) impedance of an electrical load (source) to maximize the power transfer and minimize reflections. Impedance matching is not only used in electronics. It also could be used in photonics (where impedance is equivalent to refractive indices), acoustics (sound energy is reflected when passing from one material to another), as well as in mechanics (mass becomes somewhat equivalent to impedance due to energy-mass relationship).

In this reference we will show a more intuitive approach to electrical impedance matching using Smith charts (figure 1). Usually, as a standard, electrical engineers try to match the source/loads to either 50 or 75 Ohms. However, you can set the impedance to anything you want, as long as the source and load at any given node are equal (figure 2).

There are two Smith charts – Y and Z (admittance and resistance). We will be using the Z-Smith chart only, however we will use the Y-circles as well while matching (you will have to imagine them). The Smith chart is composed of impedance (Z) or admittance (Y) circles of constant resistance, and curved lines, also called constant reactance segments (Z- and Y-). The numbers on the chart are normalized to the goal impedance (i.e. 50 Ohm)

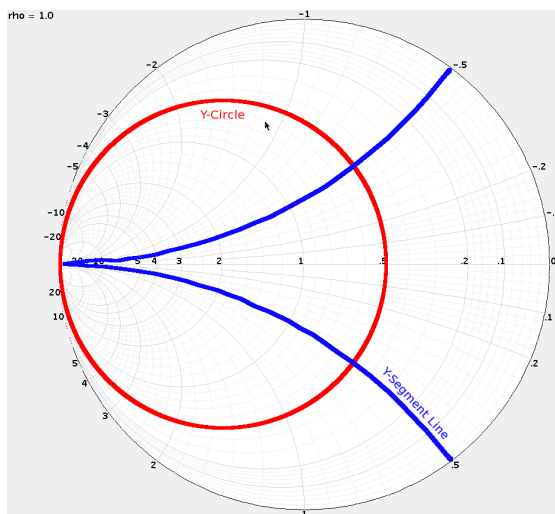


Figure 1a. Admittance Chart (Y-Smith)

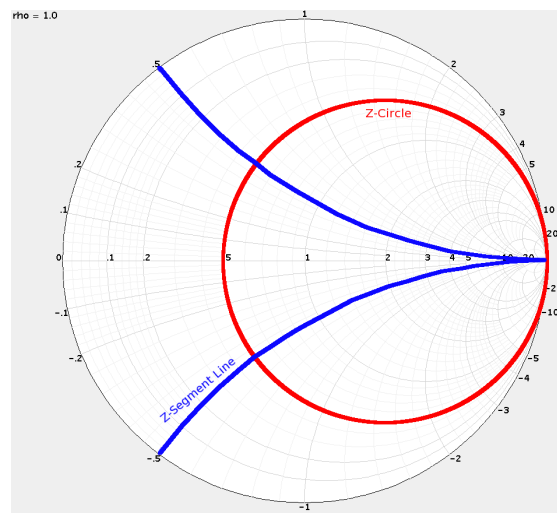


Figure 1b. Resistance Chart (Z-Smith)

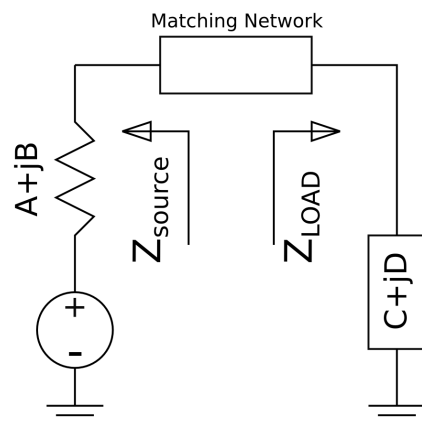


Figure 2. Basic diagram of impedance matching.

General Approach

The general approach is fairly simple, and can be done by taking the following steps:

1. Find real and imaginary parts of the source/load impedance
 - We will use Spectre simulation tool in this reference to find the impedance values.
2. Find the point of the unmatched network on the Z-Smith chart
 - In this reference we will use the Cadence tools to plot the values
3. Decide if you need any DC blocks and/or chokes
 - Usually it is a bad idea to connect the output of some designs directly to the load, as there might be a direct path to ground. Thus a DC blocking capacitor should be used. The optimum value for the DC blocking capacitor could be calculated, but as a rule of thumb – the bigger the better (check below to see why)
4. Choose your matching network. This is the main part, and exactly the place where the magic begins.
 - You can choose from three different passive components, each could be set in series or in parallel (shunt) with the matching network, so six different setups (see table in the appendix):
 1. **Series resistor**
Bigger value moves the current point **to the right**, along the **Z-segments**
 2. **Shunt resistor**
Bigger value moves the current point **to the left**, along the **Y-segments**
 3. **Series capacitor**
Smaller value moves the current point **counter-clockwise**, along the **Z-circle**
 4. **Shunt capacitor**
Bigger value moves the current point **clockwise**, along the **Y-circle**
 5. **Series inductor**
Bigger value moves the current point **clockwise**, along the **Z-circle**
 6. **Shunt inductor**
Smaller value moves the current point **counter-clockwise**, along the **Y-circle**
 - We said that the DC blocking capacitor should be big. This is because the bigger the series capacitor, the less impact it has on the S-parameters (see “*Series Capacitor*” in the previous bullet). Also, if you decide to use a choke, the bigger inductor value in parallel would have less impact on the S-parameters and matching (see “*Shunt Inductor*” in the previous bullet).
 - Resistors are frequency independent, while capacitors and inductors are frequency dependent
 - You can also use matched transmission lines (not given in the current reference). This is an advanced technique; you can take one of the advanced analog courses (i.e. EC782 at Boston University with Prof. Knepper) if you want to learn it.
 - See examples below...

Examples

1. Simple amplifier¹

Let's assume you have a design that looks like that:

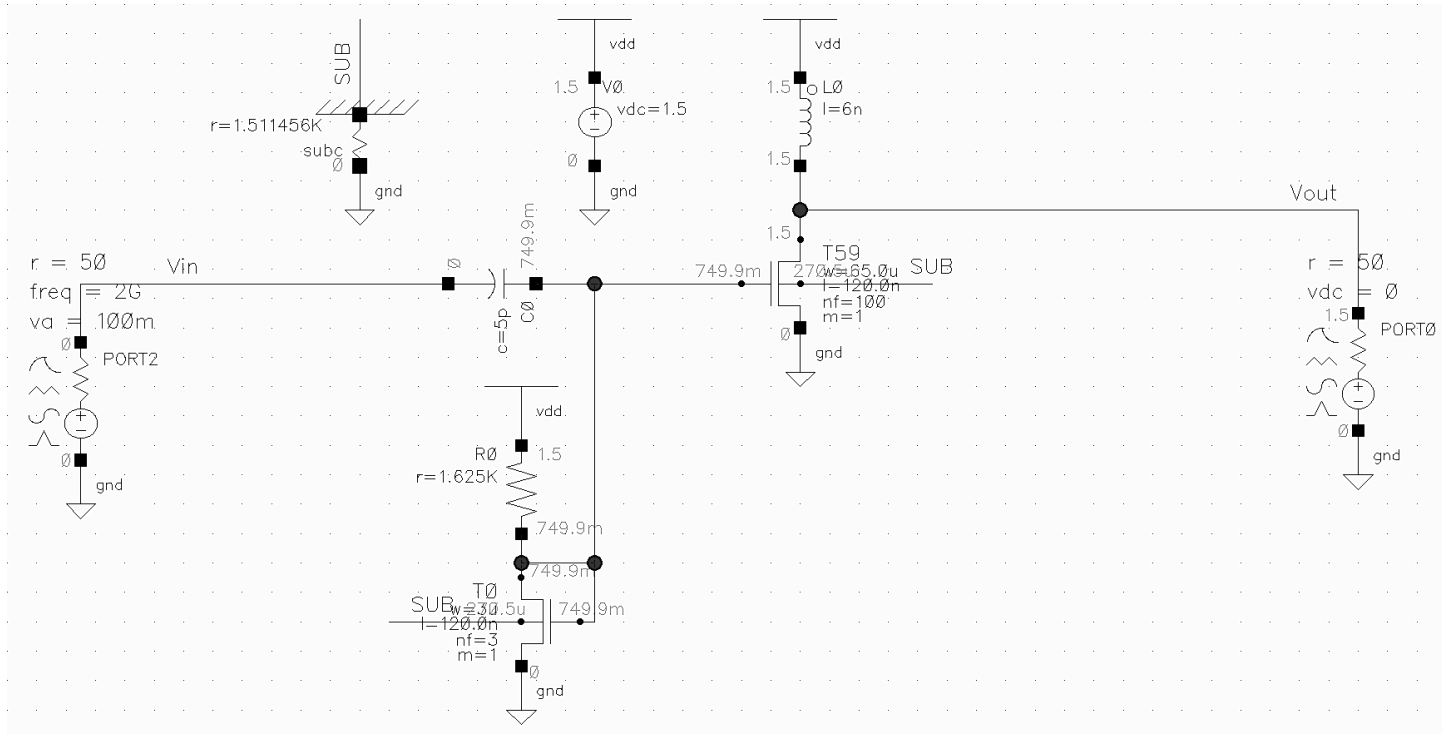


Figure 1.1. Amplifier + Biasing

On figure 1.1 above we have an amplifier + biasing network. I already put a 5pF DC-blocking capacitor at the input, in order for the biasing to work properly. Note that I didn't put a DC-block in output, as I want to show that we can use matching network to be DC-blocking as well. I also put the sizing I used (130nm technology), so you would be able to reproduce the experiment.

The amplifier in the given example is not that great (it distorts the output signal) and is given only as an example. If you need a real amplifier, you will have to design it yourself.

The first and the second steps are to find the real and imaginary components of the impedances – both input and output and find their position on the Smith chart.

As we see on the figure 1.2a, the impedances are (multiply by 50, because we are matching to 50 Ohm):

$$Z_{11} = 50 \cdot (6.83 - j3.31) = 341.5 - j165.5$$

$$Z_{22} = 50 \cdot (.5596 + j1.5345) = 27.98 + j76.725$$

¹ Note: matching a simple amplifier is much more difficult than a complex design, because input and output are not really isolated. That causes any matching that we do on one side of the design affect the matching on the other side

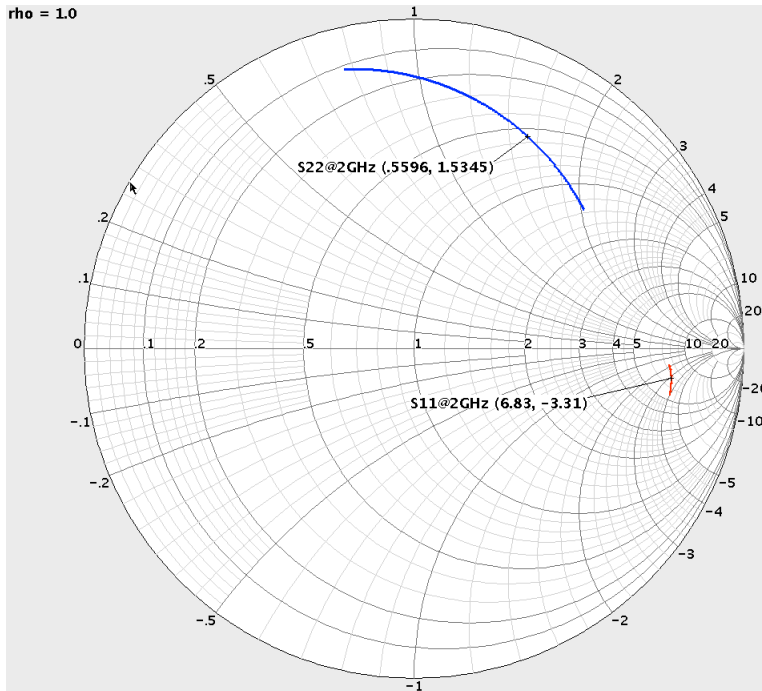


Figure 1.2a. Z-Smith Chart with input and output impedances (real, imaginary)

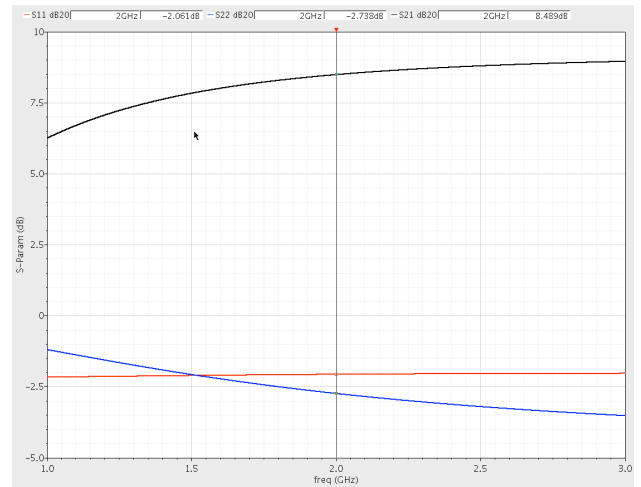


Figure 1.2b. S-parameters before matching (S11=-2.06dB, S22=-2.74dB, S12=8.49dB)

Let's start with the S11 matching. From figure 1.2a we can see that if we move the point clockwise along the Y-circle (not shown, imagine Y-Smith chart), we will eventually end up at the unity circle of the Z-Smith chart (getting on the unity circle as the first step is always a good idea). To move clockwise on Y-circles we need to use a shunt capacitor. The value of the capacitor could be either calculated, or found using a "parametric analysis" tool in Cadence (or you can use binary search technique to manually find it).

I suggest calculating the raw number, and then using a "parametric analysis" tool to fine-tune it. As an example I will show how to calculate it. For the given example we decided to use a shunt capacitor first, so the current point will be in parallel with a capacitor. To find the value of the capacitor we equate the real part to 50 (as the unity circle represents constant resistance of 50 Ohm):

$$\begin{aligned}
 \text{Re}(Z'_{11}) &= \text{Re}\left(Z_{11} \parallel \frac{1}{j\omega C_{\text{shunt}}}\right) = 50 \\
 \Rightarrow \text{Re}\left(\frac{Z_{11} \cdot \frac{1}{j\omega C_{\text{shunt}}}}{Z_{11} + \frac{1}{j\omega C_{\text{shunt}}}}\right) &= 50 \\
 \Rightarrow \text{Re}\left(\frac{\frac{341.5 - j165.5}{j2\pi \cdot 2e9 \cdot C_{\text{shunt}}}}{341.5 - j165.5 + \frac{1}{j2\pi \cdot 2e9 \cdot C_{\text{shunt}}}}\right) &= 50 \\
 \Rightarrow C_{\text{shunt}} &\cong 423.1e-15
 \end{aligned}$$

After adding a shunt capacitor with a value of 423.1fF, we find the point of interest exactly at the unity circle (figure 1.3a show $Z_{11}=50-j136.35$). The next step is to move the point clockwise along the Z-circle (series inductor). To calculate the raw value, we need to put inductor in series with the new impedance value and equate it to 50 (not the real part, but the whole thing, as our goal is exactly 50 Ohm):

$$\begin{aligned} Z''_{11} &= Z'_{11} + j\omega L = 50 \\ &\Rightarrow 50 - j136.35 + j2\pi \cdot 2e9 L = 50 \\ &\Rightarrow L \cong 10.82e-9 \end{aligned}$$

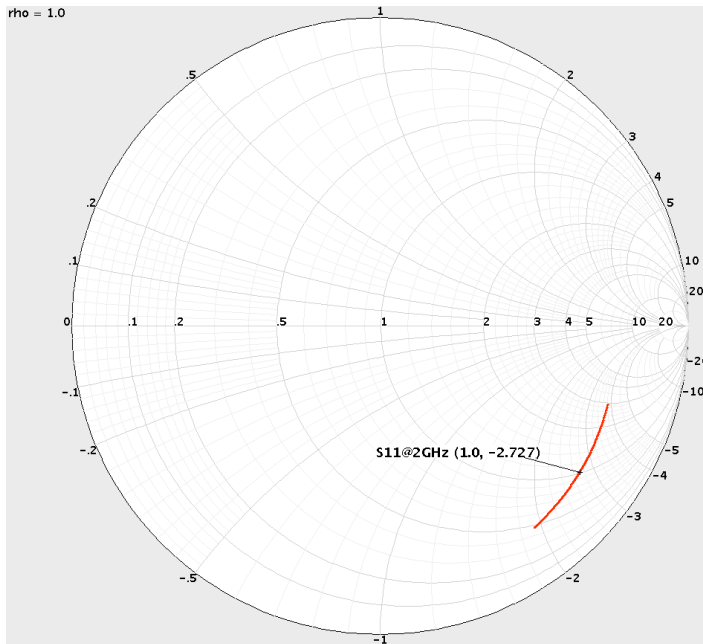


Figure 1.3a. Z-Smith chart after adding a shunt capacitor

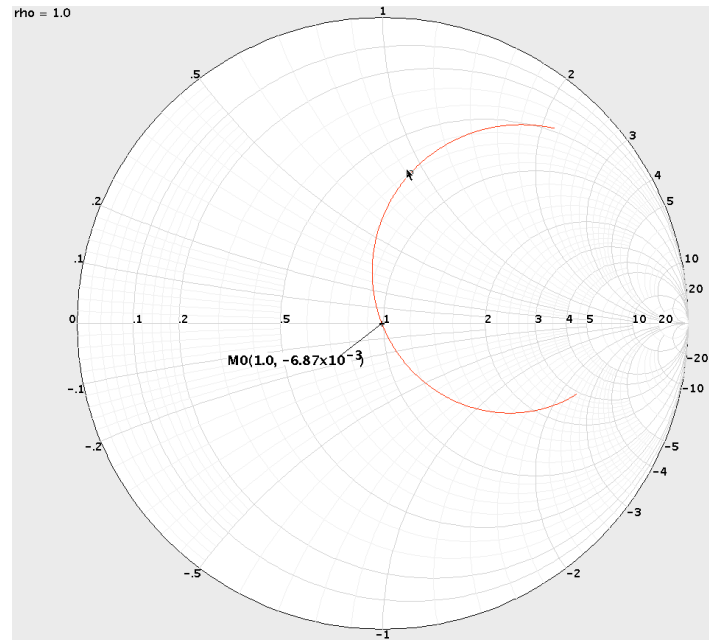


Figure 1.3b. Z-Smith chart after completing the input matching

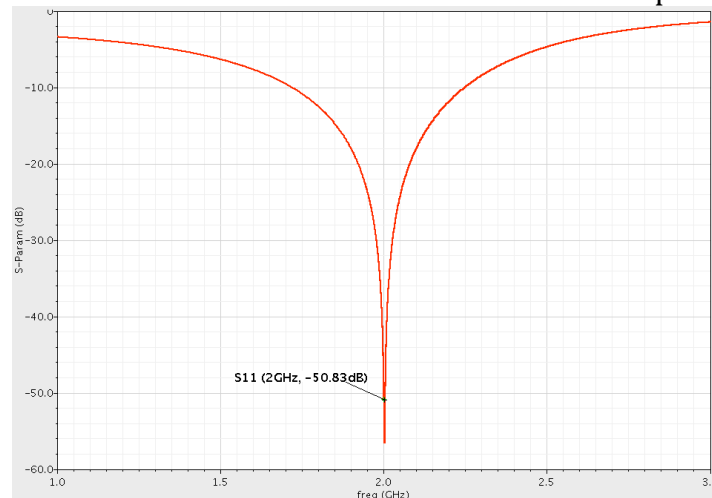


Figure 1.3c. S11 after impedance matching (-50.83dB).

The minimum of the graph is not exactly at 2GHz due to approximations

After matching the input we can start matching the output. However, after the output is matched as well, we will need to repeat the input matching process because of the poor isolation of single stage amplifiers.

To match the output we re-simulate the s-parameters, as the output impedance probably changed (because of the input matching). The new S22 results are shown on figure 1.4.

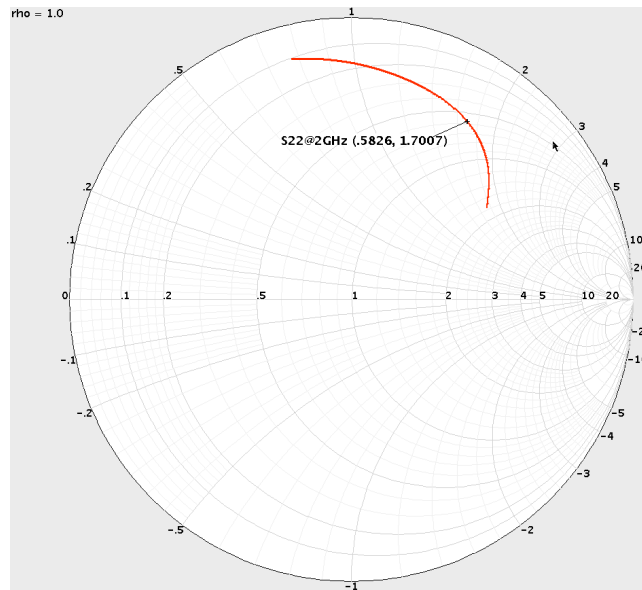


Figure 1.4. Output impedance after matching the input

As suggested before, first try moving the given impedance point to the unity circle. Moving the point clockwise along Y-circle could do this (use shunt capacitor). After that the point could be moved counter-clockwise along the Z-circle (capacitor in series – also used as a DC-block). Figures 1.5a and 1.5b show the Smith charts after placing the shunt only and the shunt + series capacitors respectively ($C_{\text{shunt}} \sim 225.72\text{fF}$; $C_{\text{series}} \sim 746.51\text{fF}$; calculations not shown).



Figure 1.5a. Output impedance after placing a shunt capacitor

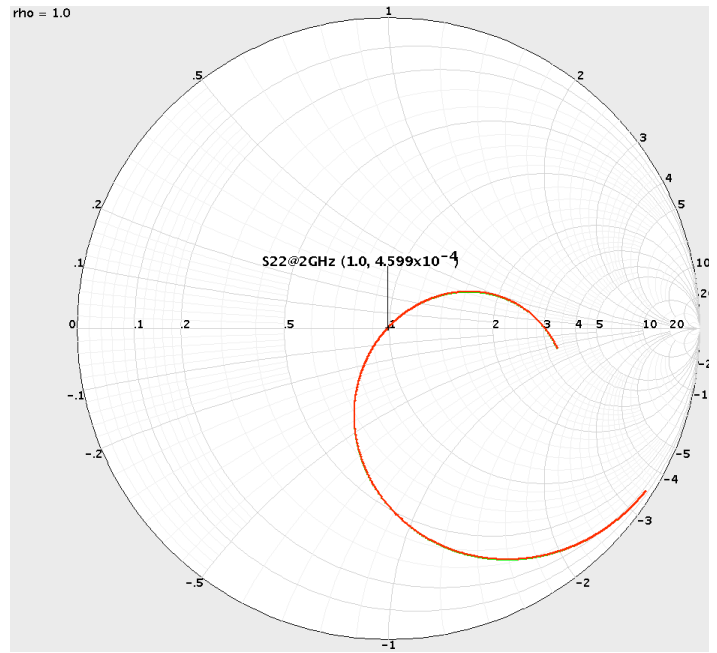


Figure 1.5b. Output impedance after completing the matching

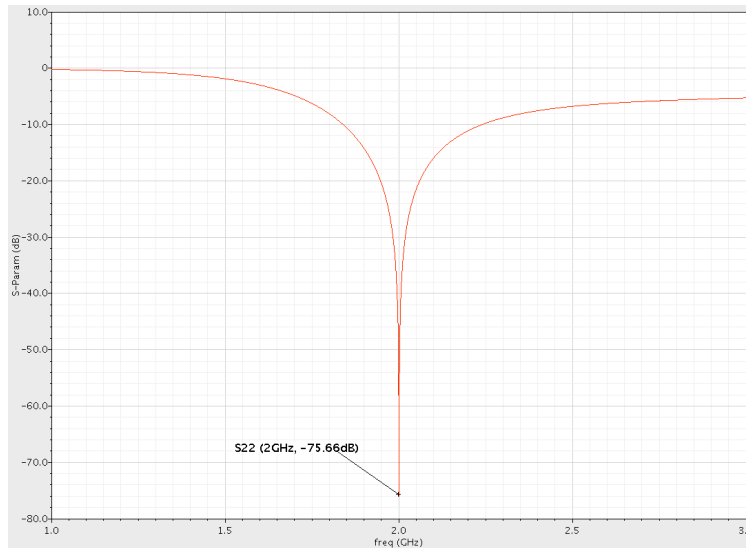


Figure 1.5c. S22 after impedance matching (-75.66dB).

Now, if you look at the S11, you will notice that the impedance shifted, we can fix it by lowering the value of the shunt capacitor at the input. The process will not be shown in the current guide, and is left as reader's exercise.

After the matching is complete, you can jump back and forth between input and output impedances to match both to their minimum values (fine-tuning). You can use a parametric analysis tool to achieve the best s-parameters at this point.

Figure 1.6 shows the final design, and figure 1.7 shows the S-parameters of the matched amplifier. Note that the overall gain of the amplifier is improved.

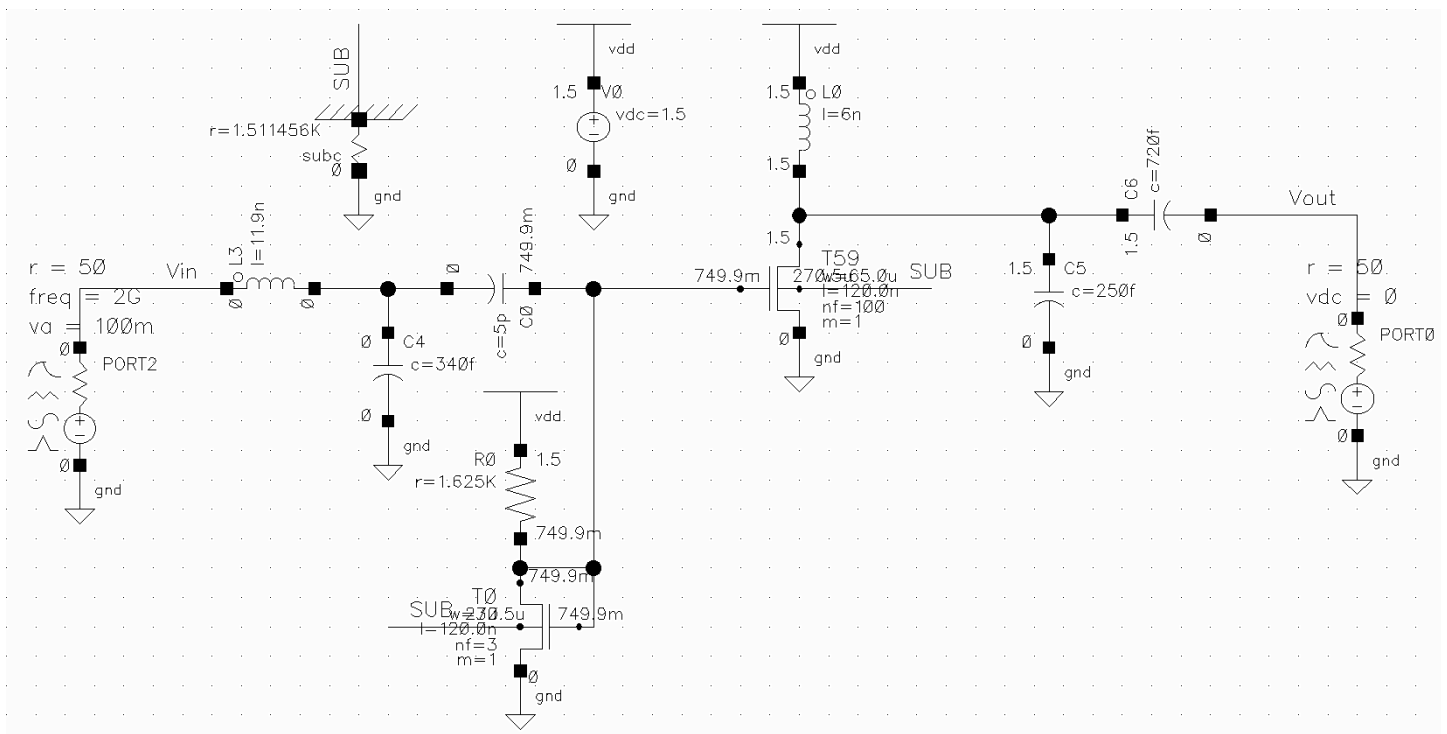


Figure 1.6. Matched amplifier (50 Ohm @ 2GHz)

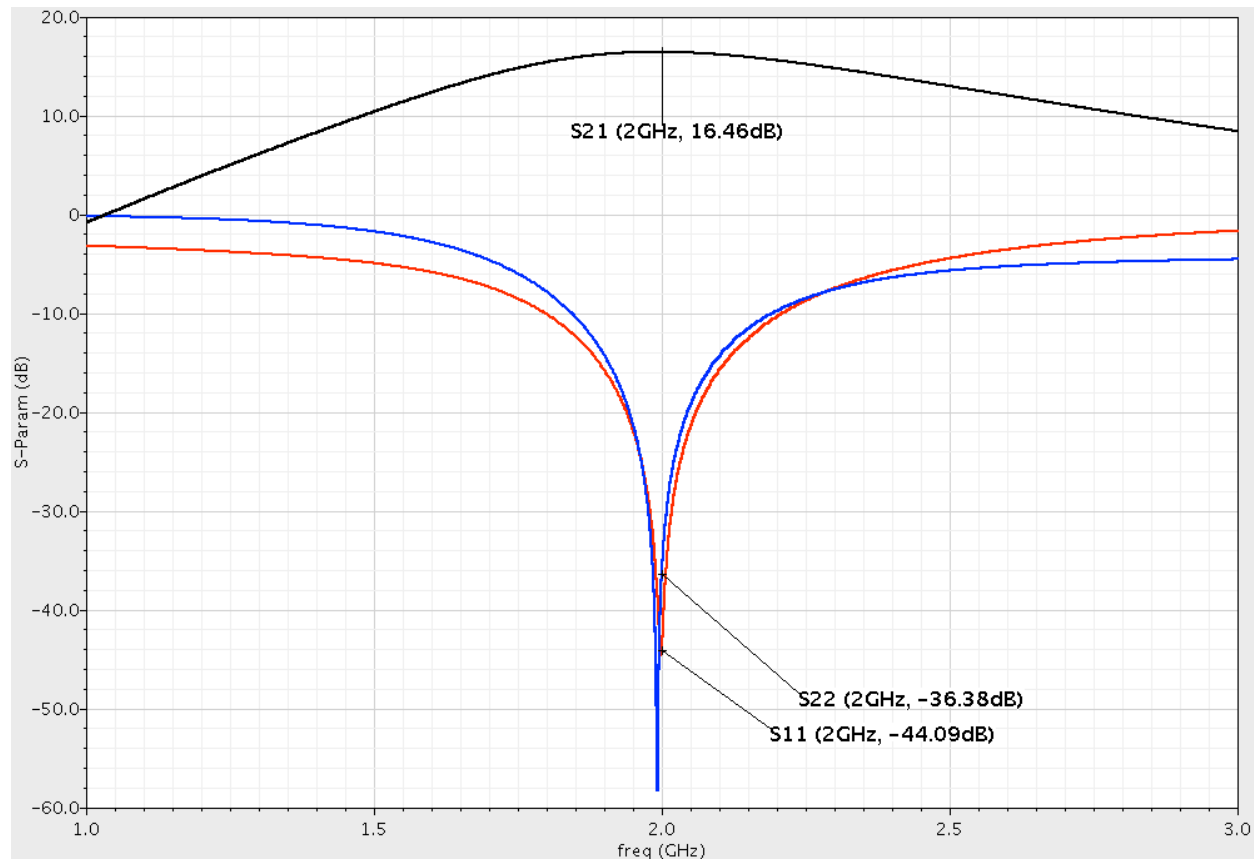


Figure 1.7. Final results for S-parameters

2. Amplifier (resistive matching)

Consider the previous example (shown on figure 2.1). If looking at its input impedance, one might notice that the unmatched input impedance point (figure 2.2) lies on one of the Y-segment lines (line of constant reactance), which passes very close to the unity point (50 Ohm point). If area is of a big concern, and fair matching is enough, we could use a resistor in parallel to get a decent input s-matching.

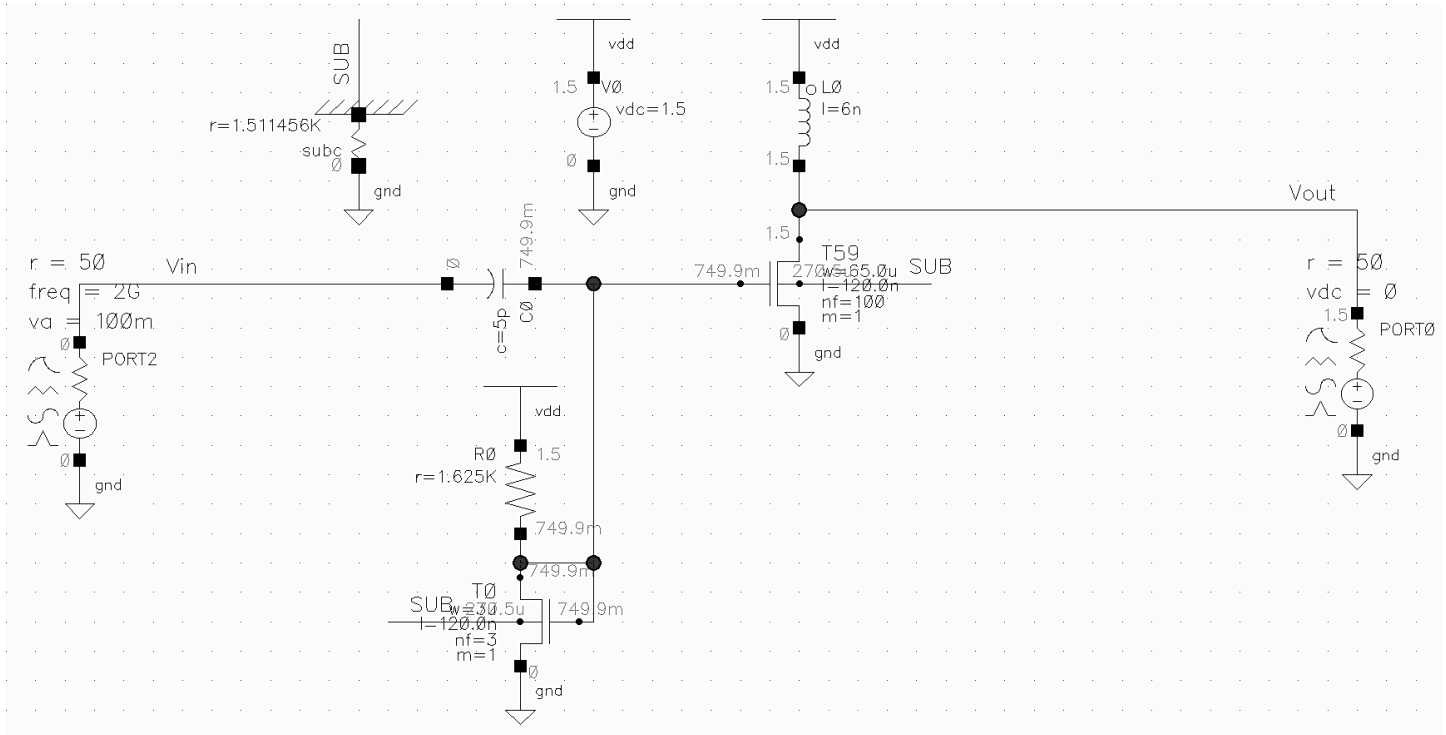


Figure 2.1. Amplifier + Biasing

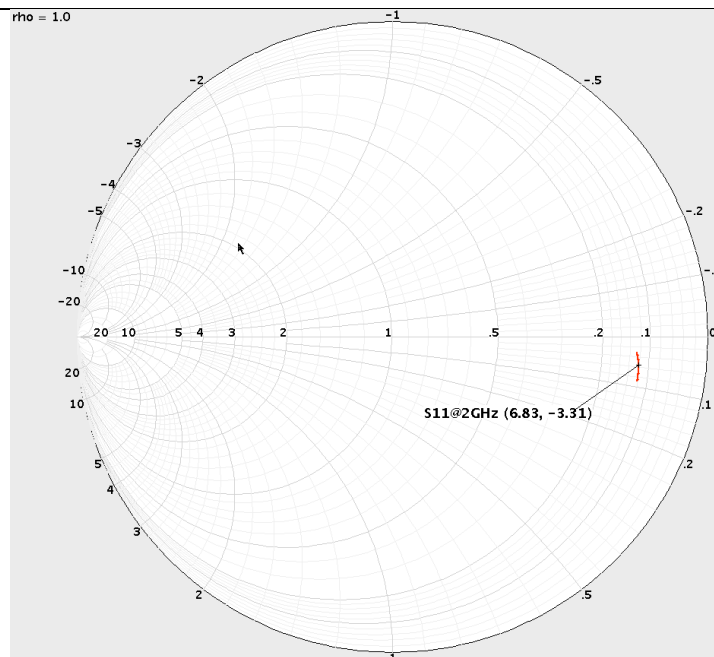


Figure 2.2. Y-Smith chart with input impedance shown
(Note that the values represent impedance, not admittance!)

The calculation is simple; we just use the given impedance in parallel with an unknown resistor in parallel, and equate it to 50 Ohm (don't forget to use only the real part):

$$\begin{aligned}\operatorname{Re}(Z_{11} \parallel R_{shunt}) &= \operatorname{Re}\left(\frac{Z_{11}R}{Z_{11} + R}\right) = 50 \\ \Rightarrow \operatorname{Re}\left(\frac{(341.5 - j165.5)R}{341.5 - j165.5 + R}\right) &= 50 \\ \Rightarrow R &\cong 56.94\end{aligned}$$

Be advised, that resistors suffer from thermal noise, and their use is discouraged in the low-noise amplifiers. Also the output matching network will be slightly different (left as an exercise for the reader).

The results of the input matching are shown on figures 2.3a and 2.3b

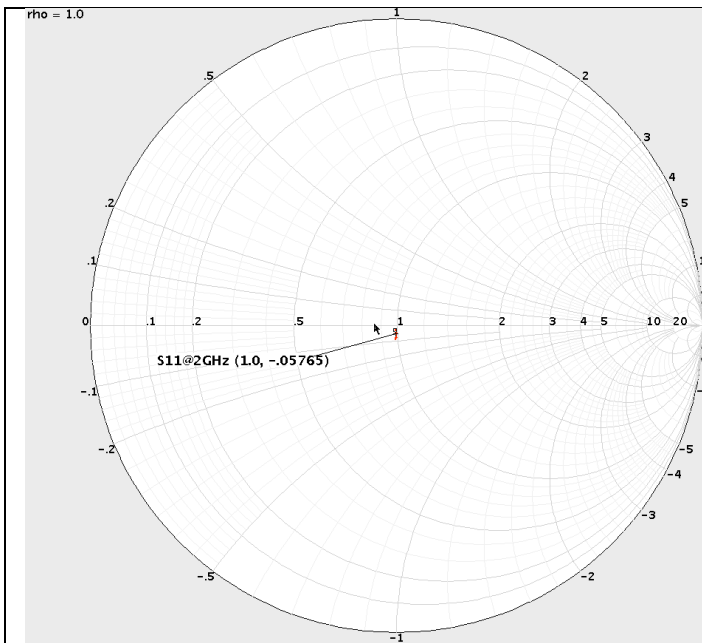


Figure 2.3a. Z-Smith showing the achieved impedance (Real: 1; Imag: -0.05765)

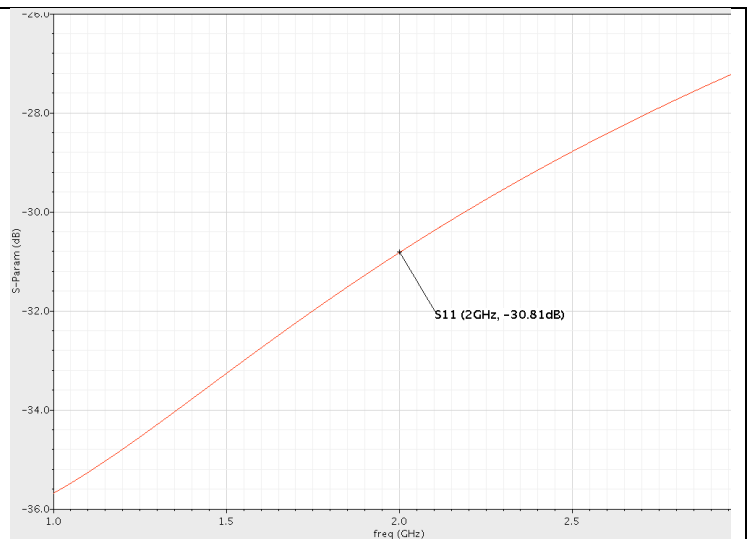


Figure 2.3b. S11 after matching (parallel resistor)

Appendix

| | Series | Shunt |
|------------------|---|---|
| Resistor | → ∞ Right along Z-Segments | → ∞ Left along Y-Segments |
| Capacitor | → 0 Counter-clockwise along Z-circle | → ∞ Clockwise along Y-circle |
| Inductor | → ∞ Clockwise along Z-circle | → 0 Counter-clockwise along Y-circle |

→ means “approaches”