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Assignment Title:	Workshop 3
Subject Number:	ELEN 90062
Subject Name:	High Speed Electronics
Student Name:	Xuetong Wu , Zhaoxuan Huang, Shiyu Wang
Lecturer/Tutor:	Workshop Time: Tuesday 11:00 - 1:00
Due Date:	14/09/2018

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University of Melbourne

ELEN90062 High Speed Electronics

WORKSHOP

Workshop 3&4 Report
Calibration and Measurement

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September 9, 2018

Contents

1 Calibration

Executive Summary

The objective of this workshop is to calibrate the vector network analyser (VNA) and use the calibrated VNA to measure the unknown shunt for its S-parameters. The calibration is based on Short, Open, Load and Through calibration circuit board. Finally, the unknown shunt can be found by calculating its S-parameters for its impedances.

1.1 Task 1

Now we have a two port network as follows:

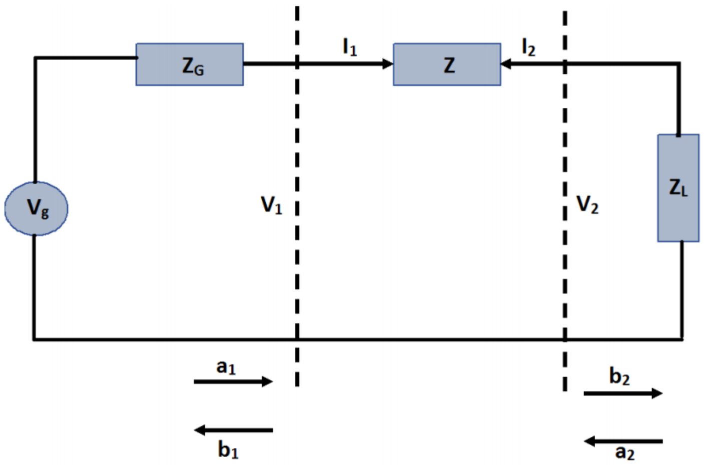


Figure 1: Two Port Network

Where a_1 and a_2 are input waves while b_1 and b_2 are defined as reflected waves, we have the equations:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

And the S-parameters are defined as:

$$S_{11} = \frac{b_1}{a_1}|_{a_2=0} \quad S_{12} = \frac{b_1}{a_2}|_{a_1=0}$$
$$S_{21} = \frac{b_2}{a_1}|_{a_2=0} \quad S_{22} = \frac{b_2}{a_2}|_{a_1=0}$$

Where S_{11} is the input reflection coefficient, S_{12} is the forward transmission gain, S_{21} is the reverse transmission gain and S_{22} is the output reflection coefficient. And for a_i and

b_i, we can show that:

$$a_i = \frac{V_i + I_i Z_0}{2\sqrt{Z_0}}$$
$$b_i = \frac{V_i - I_i Z_0}{2\sqrt{Z_0}}$$

In task 1, we consider the two-port network shown in the above, here, $Z_G = Z_0$, $Z = 0$, we can calculate the S_{11}:

$$S_{11} = \frac{b_1}{a_1}|_{a_2=0}$$

Where:

$$b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}, a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

Hence:

$$S_{11} = \frac{b_1}{a_1}|_{a_2=0} = \frac{V_1 - I_1 Z_0}{V_1 + I_1 Z_0} = \frac{\frac{V_1}{I_1} - Z_0}{\frac{V_1}{I_1} + Z_0}$$

As $Z = 0$:

$$\frac{V_1}{I_1} = Z_L + Z = Z_L$$

Finally, we reach:

$$S_{11} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

1.2 Task 2

Similar to **Task 1**, S_{21} is defined as:

$$S_{21} = \frac{b_2}{a_1}|_{a_2=0}$$

Where:

$$b_1 = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}, a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

In this case, $Z_G = Z_0$, $Z \neq 0$, then:

$$S_{21} = \frac{b_2}{a_1}|_{a_2=0} = \frac{V_2 - I_2 Z_0}{V_1 + I_1 Z_0}$$

From the voltage-current relationship, we have:

$$V_2 = -I_2 Z_L$$
$$V_1 = I_1(Z_L + Z)$$

By substituting V_1 and V_2, we reach:

$$S_{21} = \frac{b_2}{a_1}|_{a_2=0} = \frac{V_2 - I_2 Z_0}{V_1 + I_1 Z_0} = \frac{-I_2(Z_L + Z_0)}{I_1(Z_0 + Z + Z_L)}$$

As $I_1 = -I_2$, and if $Z_L = Z_0$, the result will be:

$$S_{21} = \frac{b_2}{a_1}|_{a_2=0} = \frac{V_2 - I_2 Z_0}{V_1 + I_1 Z_0} = \frac{-I_2(Z_L + Z_0)}{I_1(Z_0 + Z + Z_L)} = \frac{Z_L + Z_0}{Z_0 + Z + Z_L} = \frac{2Z_0}{Z + 2Z_0}$$

1.3 Task 3

Due to the storage error, we did not save the measurement result to the correct usb disk, hence we do not have the measurement results, to explain the calibration clearer, we use matlab to draw Smith Chart to simulate the measurement results.

For the short circuit, when calibrated, the source and load impedance should be both 0, by this the S_{11} and S_{22} can be calculated as:

$$S_{11} = \frac{Z_S - Z_0}{Z_S + Z0} = -1$$

$$S_{22} = \frac{Z_L - Z_0}{Z_L + Z0} = -1$$

With full reflection and infinite through load, the transmission coefficients S_{12} and S_{21} should be 0, thus they are in the center point in the Smith Chart. The measurement result is as follows:

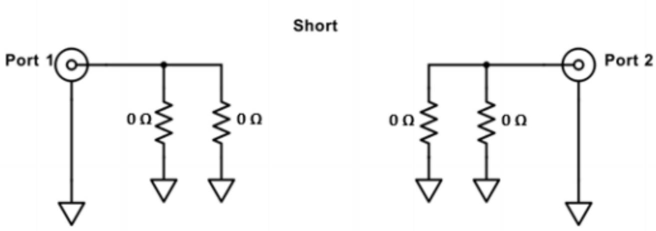


Figure 2: Short Circuit

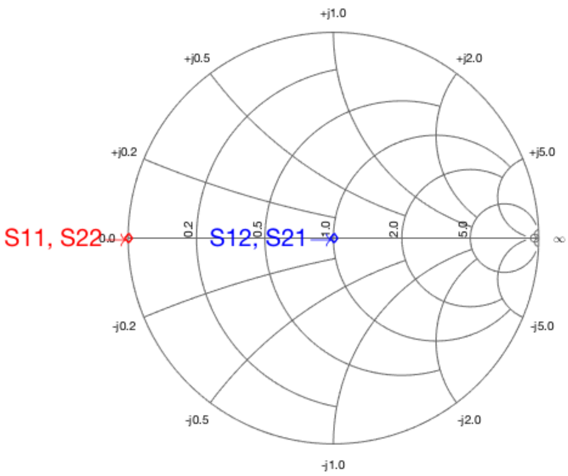


Figure 3: Short Circuit Calibration Result

Thus in the Smith Chart, S_{11} and S_{22} should be approximately at $(-1, 0)$ while S_{11} and

S_{22} locate at $(0, 0)$.
Also for the open circuit, when calibrated, the source and load impedance should be both infinite, by this the S_{11} and S_{22} can be calculated as:

$$S_{11} = \frac{Z_S - Z_0}{Z_S + Z0} = 1$$

$$S_{22} = \frac{Z_L - Z_0}{Z_L + Z0} = 1$$

With full reflection and infinite through load, the transmission coefficients S_{12} and S_{21} should be 0, thus they are in the center point in the Smith Chart.

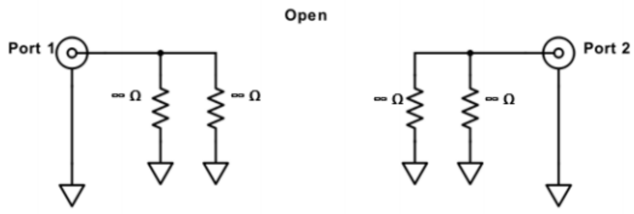


Figure 4: Open Circuit

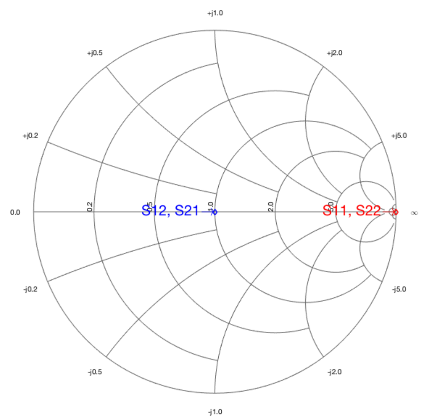


Figure 5: Open Circuit Calibration Result

Thus in the Smith Chart, S_{11} and S_{22} should be approximately at $(1, 0)$ while S_{11} and S_{22} locate at $(0, 0)$.
For load circuit, as shown as follows:

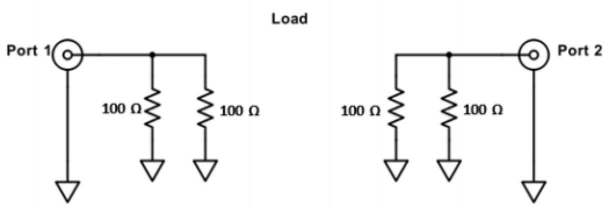


Figure 6: Load Circuit

The equivalent source impedance and load impedance are:

$$Z_S = Z_L = (100//100)\Omega = 50\Omega, Z = \infty$$

Hence:

$$S_{11} = \frac{Z_S - Z_0}{Z_S + Z0} = 0$$

$$S_{22} = \frac{Z_L - Z_0}{Z_L + Z0} = 0$$

$$S_{21} = S_{12} = \frac{2Z_0}{Z + 2Z_0} = 0$$

In this case, all calibration S-parameters are in the center of the Smith Chart.

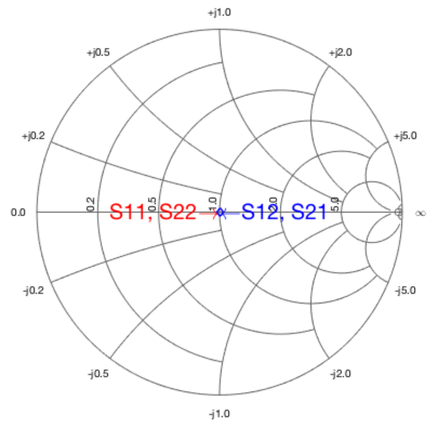


Figure 7: Load Circuit Calibration Result

Finally, we take a look at through circuits:

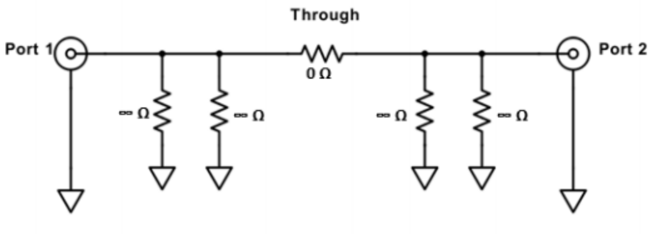


Figure 8: Through Circuit

The equivalent source impedance and load impedance are:

$$Z_S = Z_L = \infty\Omega, Z = 0$$

Hence:

$$S_{11} = \frac{Z_S - Z_0}{Z_S + Z0} = 1$$

$$S_{22} = \frac{Z_L - Z_0}{Z_L + Z0} = 1$$

$$S_{21} = S_{12} = \frac{2Z_0}{Z + 2Z_0} = 1$$

In this case, all calibration S-parameters are located at $(1, 0)$ of the Smith Chart.

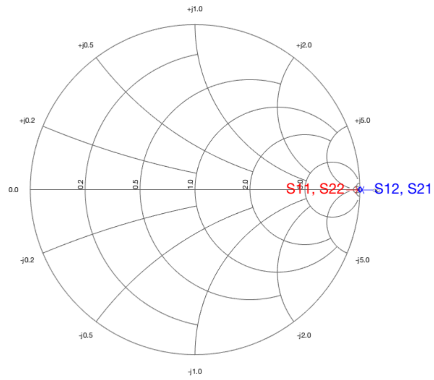


Figure 9: Through Circuit Calibration Result

1.4 Task 4

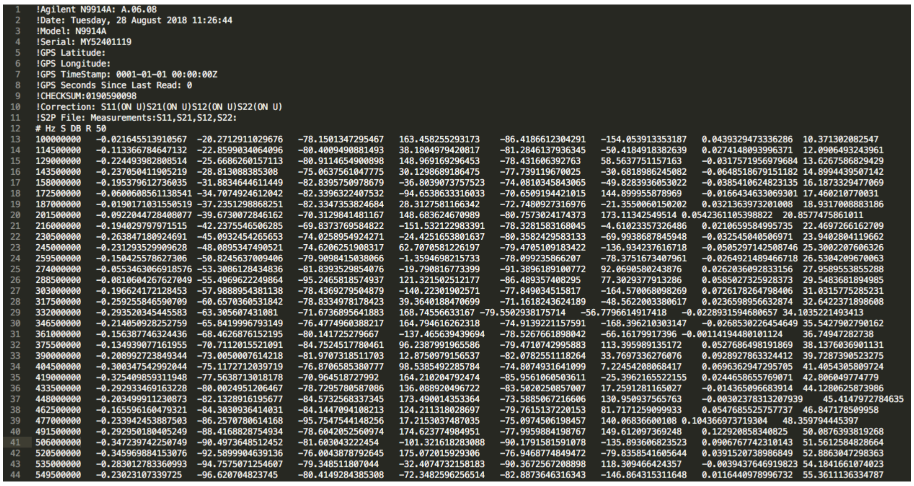
We have two unknown shunt circuits, as it is single port, we only care about the **reflection coefficient** S_{11} as it is:

$$S_{11} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At 500MHz, by reading the $s2p$ data, here the format for .s2p files are: in $*.s2p$ files each record contains 1 frequency value and 4 S-parameters (total of 9 values), just before the data section of an $s2p$ file, there should be a line that looks like this:

$$Hz\ S\ DB\ R\ 50$$

Here, Hz means the operating frequency, S means S-Parameter, DB stands for the unit of the value(decibel/angle), and R 50 indicates that the load impedance is 50Ω. And for the 9 values, they represent:
Frequency, Magnitude(S11), Phase(S11), Magnitude(S21), Phase(S21), Magnitude(S12), Phase(S12), Magnitude(S22), Phase(S22)
The table looks like below:



```
!Agilent N9914A: A.06.08
!Date: Tuesday, 28 August 2018 11:26:44
!Model: N9914A
!Serial: MY52401119
!GPS Latitude:
!GPS Longitude:
!GPS TimeStamp: 0001-01-01 00:00:00Z
!GPS Seconds Since Last Read: 0
!CHECKSUM:0190590098
!Correction: S11(ON U)S21(ON U)S12(ON U)S22(ON U)
!S2P File: Measurements:S11,S21,S12,S22:
# Hz S DB R 50
100000000   -0.021645513910567  -20.2712911029676   -78.1501347295467   163.458255293173    -86.4186612304291   -154.053913353187   0.0439329473336286  10.371302082547
114500000   -0.113366784647132  -22.8599034064096   -80.4009490881493   38.1804979420817    -81.2846137936345   -50.4184918382639   0.0274148093996371  12.0906493243961
129000000   -0.224493982808514  -25.6686260157113   -80.9114654900898   148.969169296453    -78.431606392763    58.5637751157163    -0.0317571956979684 13.6267586829429
143500000   -0.237050411905219  -28.813088385308    -75.0637561047775   30.1298689186475    -77.739119670025    -30.6818986245082   -0.0648518679151182 14.8994439507142
158000000   -0.195379612736035  -31.8834644611449   -82.8395750978679   -36.8039073757523   -74.0810345843065   -49.8283936053022   -0.0385410624823135 16.1873329477069
172500000   -0.0600608561138541 -34.7074924612042   -82.3396322407532   -94.6538633316033   -70.6509194421015   144.899955878969    -0.0166434633069301 17.460210770031
187000000   -0.0190171031550519 -37.2351298868251   -82.3347353824684   28.3127581166342    -72.7480927316976   -21.3550060150202   0.0321363973201008  18.9317008883186
201500000   -0.0922044728408077 -39.6730072846162   -70.3129841481167   148.683624670989    -80.7573024174373   173.1134254951 4 0.0542361105398822  20.8577475861011
216000000   -0.194029797971515  -42.2375546506285   -69.8373769584822   -151.532122983391   -78.3281583168045   -4.61023357326486   0.0210659584995735  22.4697266162709
230500000   -0.263847180924691  -45.0932454265653   -74.0258954924271   -24.4251653801637   -80.3582429583133   -69.9938687845948   -0.032545040506971  23.9402804119662
245000000   -0.231293529909628  -48.0895347490521   -74.6206251908317   62.7070581226197    -79.4705109183422   -136.934237616718   -0.0505297142508746 25.3002207606326
259500000   -0.150425578627306  -50.8245637009406   -79.9098415038066   -1.3594698215733    -78.099235866207    -78.3751673407961   -0.0264921489466718 26.5304209670063
274000000   -0.0553463066918576 -53.3086128434836   -81.8393529854076   -19.790816773399    -91.3896189100772   92.0690580243876    0.0262036092833156  27.9589553855288
288500000   -0.0810604267627049 -55.4969622249864   -95.2465818574937   121.321502512177    -86.489357408295    77.3029377913286    0.0585027325928373  29.5483681894985
303000000   -0.196624172128453  -57.9888954381138   -78.4369279504879   -140.22301902571    -77.849034515817    -164.570068098269   0.0726178264798406  31.0315775285231
317500000   -0.259255846590709  -60.6570360531842   -78.8334978178423   39.3640188470699    -71.1618243624189   -48.5622003380617   0.0236598956632874  32.6422371898608
332000000   -0.293520345445583  -63.305607431081    -71.6736895641883   168.74556633167 -79.5502938175714   -56.7796614917418   -0.022893159468065 7 34.1035221493413
346500000   -0.214050928252759  -65.8419996793149   -76.4774960388217   164.794616262318    -74.9139221157591   -168.396210303147   -0.0268530226454649 35.5427902790162
361000000   -0.156387746324436  -68.4626876152195   -80.141725279667    -137.465639439694   -78.5267661898042   -66.16179917396 -0.00114194480101124    36.74947282738
375500000   -0.134939077161955  -70.7112015521091   -84.7524517780461   96.2387991965586    -79.4710742995883   113.395989135172    0.0527686498191869  38.1376036901131
390000000   -0.208992723849344  -73.0050007614218   -81.9707318511703   12.8750979156537    -82.0782551118264   33.7697336276076    0.0928927863324412  39.7287390523275
404500000   -0.300347542992044  -75.1172712039719   -76.8706585380777   98.5385492285784    -74.8074931641099   7.22454208068417    0.0696362947295705  41.4054305809724
419000000   -0.325409859311948  -77.5638713018178   -70.964518727992    164.210204792474    -85.9561060503611   -25.3962165522155   0.0244658655769071  42.806049774779
433500000   -0.292933469163228  -80.0024951206467   -78.7295780587086   136.088920496722    -83.5020250857007   17.2591281165027    -0.01436509668391 4 44.1280625873986
448000000   -0.203499911230873  -82.1328916195677   -84.5732568337345   173.490014353364    -73.5885067216606   130.950937565763    -0.00302378313207939    45.4147972784635
462500000   -0.165596160479321  -84.3030936414031   -84.1447094108213   124.211318028697    -79.7615137220153   81.7171259099933    0.0547685525757737  46.847178509958
477000000   -0.23394245388750 3 -86.2570780614168  -95.7547544148256   17.2153037487035    -75.0974506198457   140.06836600108 0.104366973719304   48.359794445397
491500000   -0.292950180405249  -88.4168828754934   -78.6042052560974   174.623774984951    -77.9959884198767   149.612097369248    0.12292085834082 5  50.0876393819268
506000000   -0.347239742250749  -90.4973648512452   -81.603043222454    -101.321618283088   -90.1791581591078   -135.893606823523   0.0906767742310143  51.5612584828664
520500000   -0.345969884153076  -92.5899904639136   -76.0043878792645   175.072015929306    -76.9468774849472   -79.8358541605644   0.0391520738986849  52.8863047298363
535000000   -0.283012783360993  -94.7575071254607   -79.348511807044    -32.4074732158183   -90.3672567208898   118.309466424357    -0.0039437646919823 54.1841661074023
549500000   -0.23023107339725   -96.620704823745    -80.4149284385308   -72.3482596256514   -82.8873646316343   -146.864315311648   0.0116440978996732  55.3611136334787
```

Figure 10: *sp*2 format for unknown circuit 1

we can find the according frequency response at a certain frequency. For the unknown shunt circuit 1, the nearest value to the target frequency is 506MHz, and for S_{11}:

$$S_{11,dB} = -0.3472dB \Rightarrow 20\log(|S_{11}|) = -0.3472dB \Rightarrow |S_{11}| = 0.9608$$

$$S_{11,angle} = -90.49^{\circ}$$

Thus $S_{11} = 0.9608\angle-90.49^{\circ}$, which is equivalent to:

$$S_{11,Re} = -0.0082$$

$$S_{11,Im} = -0.9607j$$

The characteristic impedance is 50Ω, therefore, we can compute the load impedance:

$$Z_L = \frac{1+\Gamma}{1-\Gamma}Z_0 = \frac{1+S_{11}}{1-S_{11}}Z_0 = (1.9848 - 49.5356j)\Omega$$

The trace of S-parameter in dB is as follows:

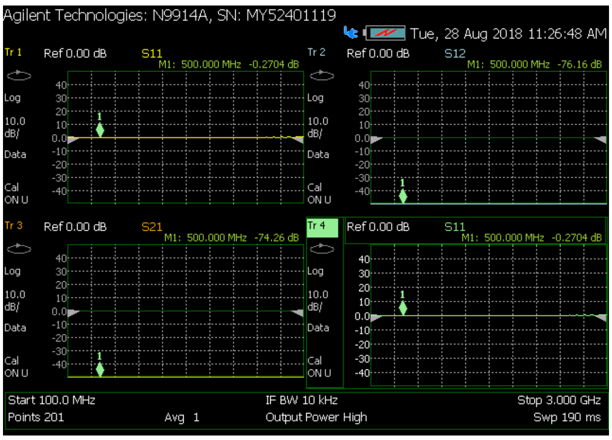


Figure 11: S-parameter display for unknown circuit 1

For unknown shunt 2, we have:

```
!GPS Latitude:
!GPS Longitude:
!GPS TimeStamp: 0001-01-01 00:00:00Z
!GPS Seconds Since Last Read: 0
!CHECKSUM:1109555246
!Correction: S11(ON U)S21(ON U)S12(ON U)S22(ON U)
!S2P File: Measurements:S11,S21,S12,S22:
# Hz S DB R 50
100000000   -9.38111382689657   2.64628259270894   -76.9495903390521   -148.625237680477   -80.1266256628798   -113.459512688857   0.0453852422982092  10.3722522300059
114500000   -9.44567167788452   3.87747921673288   -69.5022217197591   22.7239286476214   -77.8119412072587   -156.559147468157   0.0278034384484848  12.0802432369824
129000000   -9.62959530202886   4.25992951619723   -98.2703850282963   -154.702833611962   -74.4213504420007   42.3054266000498   -0.042183712437913  13.6032206790988
143500000   -9.69693326662984   3.88462517332746   -83.7909276492555   -116.593994522223   -81.3706877303322   139.688288344804   -0.0756208330475731 14.8431021649124
158000000   -9.63624143647742   3.47815465889898   -87.8484250729404   -17.6085576019087   -73.1320260463927   171.655117705038   -0.0483000776921065 16.095753640812
172500000   -9.46196364411892   3.63196112277004   -77.2010131190055   -42.0043433460946   -77.2068332909118   116.450822849716   -0.0202870791512594 17.3383263175544
187000000   -9.34325663274793   4.63428890208254   -86.1207036406172   76.4277707902511   -90.5111336830254   -145.391313228278   0.037041362832146   18.8546765000985
201500000   -9.36151289754627   6.07836041403322   -85.2430362186218   -65.3001487436261   -79.9399149418874   66.7321669703324   0.0566337401873404  20.8332841968859
216000000   -9.50917400534318   6.95911260824169   -79.1054748529456   -117.095746230089   -83.1986197555783   144.476230272648   0.0225241319677772  22.4900554279007
230500000   -9.68518954376473   6.91841450004868   -76.1418160110041   -169.78903949449   -80.603618794376   112.862575187096   -0.0395505935586332 24.0200148515264
245000000   -9.71753730200758   6.47052870816301   -83.5321957337493   -7.43734727954518   -77.9321717861737   178.331390327275   -0.0608977093574109 25.3670573741669
259500000   -9.56896316134164   6.08586317422309   -78.0645646990359   -72.8807847225337   -97.5137845708388   -173.221651071031   -0.0318915015834918 26.5714864089448
274000000   -9.3946834063247   6.82359667927815   -78.399144413609   -43.4475813511323   -82.3801734320484   -10.8585337403369   0.0263604070185647  27.9976714320877
288500000   -9.31641749920157   8.26987213323143   -85.6768949670505   -59.0032267585278   -82.8897313929919   74.2532783682488   0.0667732660791594  29.6352438730554
303000000   -9.44628688724589   9.67014295289016   -76.7629195384031   -90.9723227077362   -73.8852388122424   149.204844771422   0.0783206473552193  31.1297011028963
317500000   -9.6043903090604   9.89046517241798   -82.6950299331577   47.5308627448084   -73.7909761670431   -94.2290002084037   0.02447405760224   32.8186620766306
332000000   -9.75285428313731   9.54903478173947   -71.8235562837712   -68.9122733929529   -75.3064978335781   -60.9809490876462   -0.0291046821443439 34.30337921959
346500000   -9.66305716782688   8.85219296453771   -79.6562282246661   -104.727157769362   -75.0791166326422   111.915826185155   -0.0451219091997855 35.7498257576798
361000000   -9.49760887898806   9.03023082880227   -84.3021801221441   -55.7282512157098   -80.5227819597997   120.773206496869   -0.0134194980714743 36.9053887708099
375500000   -9.33026050688525   10.2257105988597   -77.3200985038631   -97.0737162387121   -84.2670456040275   146.314152441095   0.0525989974367816  38.2468522412609
390000000   -9.3362858777335   11.6320623095373   -75.2422051829552   73.5542332390235   -86.7407719119426   -177.868279816145   0.097853256621974   39.7706714907293
404500000   -9.54097489211165   12.9859009501203   -75.4257028323542   92.6621393547249   -80.5452325579471   -146.248120120829   0.0678070186156558  41.4687328013613
419000000   -9.7280305743891   12.9975681315274   -80.5397743691318   -110.128113722443   -81.0355823145993   34.446601230278 0.0196225070104028  42.8667118241846
433500000   -9.79824146971529   12.2076915133006   -81.1211608241376   -172.330564731269   -80.5590923256551   79.0125700485026   -0.0269295014711464 44.1861421624308
448000000   -9.63499677202383   11.5957342020812   -79.2608120656515   -122.61012493649   -70.4798452646884   -118.256735616847   -0.0159697840131567 45.4805889220209
462500000   -9.43826871298363   12.180408075568 -88.0424265783195   52.9316857041613   -80.1225143822201   -174.867947569143   0.0481013696213539  46.8930313103795
477000000   -9.32227270942757   13.7761216441657   -75.1583003049744   -90.4146316979289   -76.9226541549138   -158.622539978887   0.101765989578732   48.3622100123439
491500000   -9.3870404396477   14.9181444711048   -96.8458724611079   83.0091328369181   -77.7512122640768   114.737488765917   0.124038386913127   50.0657449433098
506000000   -9.61520338234193   15.8987500007969   -75.409070247238   7.24958930658829   -76.2954205415953   125.3208214841  0.0955167950607093  51.4975003074696
520500000   -9.76762026795857   15.7121241915971   -77.0606755516667   147.757287392881   -73.7676507125422   80.3774517014829   0.0337109878649002  52.8170294089252
535000000   -9.77796957996055   14.7884591228728   -82.9576953724341   -16.671468728016   -81.0693907191371   62.8920440724353   -0.0166671979499229 54.1170786265935
549500000   -9.57157939631873   14.530597282877 -73.4833944494222   103.950824508366   -78.0230702793506   78.1788698048671   0.00257555289682011 55.3382174913842
564000000   -9.39715667494453   15.3013783106247   -85.6316207617862   -46.1317148936408   -74.6078891942361   160.345323738199   0.0762678302679277  56.896379894193
578500000   -9.34064346109469   16.8164945504836   -84.3798148749252   -23.6153966065491   -84.3418025636824   -106.068668351289   0.140474868737101   58.6103853994843
593000000   -9.49157537395777   18.3862538304396   -77.5335812848967   -25.8660295001026   -88.1615724276433   97.1449977007644   0.153353031589426   60.1600646056601
607500000   -9.66683094340796   18.8326676401724   -81.0817394672304   -164.412797259849   -78.0053995139121   20.6849924883632   0.107625453972071   61.722594480297
```

Figure 12: *sp*2 format for unknown circuit 2

From the table, yielding:

$$S_{11,dB} = -9.615dB \Rightarrow 20\log(|S_{11}|) = -9.615dB \Rightarrow |S_{11}| = 0.33$$

$$S_{11,angle} = 15.9°$$

Thus $S_{11} = 0.33\angle 15.9°$, which is equivalent to:

$$S_{11,Re} = 0.3174$$

$$S_{11,Im} = 0.09j$$

The characteristic impedance is 50Ω, therefore, we can compute the load impedance:

$$Z_L = \frac{1+\Gamma}{1-\Gamma}Z_0 = \frac{1+S_{11}}{1-S_{11}}Z_0 = (93.97 + 19.07j)\Omega$$

The trace of S-parameter in dB is as follows:

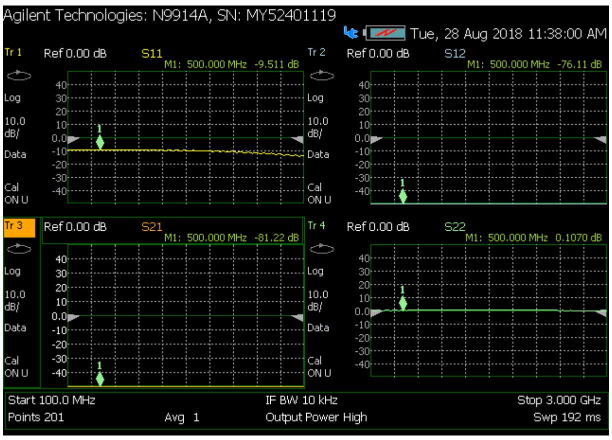


Figure 13: S-parameter display for unknown circuit 2

1.5 Task 5

Based on the *s2p* file, the record values are shown in the figure below:

```
!Agilent N9914A: A.06.08
!Date: Tuesday, 28 August 2018 11:29:47
!Model: N9914A
!Serial: MY52401119
!GPS Latitude:
!GPS Longitude:
!GPS TimeStamp: 0001-01-01 00:00:00Z
!GPS Seconds Since Last Read: 0
!CHECKSUM:0636978779
!Correction: S11(ON U)S21(ON U)S12(ON U)S22(ON U)
!S2P File: Measurements:S11,S21,S12,S22:
# Hz S DB R 50
100000000   -26.2541699619754   78.4395595270556    -0.0697796046567847 -1.5085265560184    -0.0975119791975242 -1.62399428768586   -25.8146988558496   71.7147680942003
114500000   -24.4188790941698   79.9731408873763    -0.0732309366581931 -1.83658949469111   -0.107462965377391  -1.90563101576733   -23.7470179385539   79.9303426595208
129000000   -23.2487382073849   84.700552137281 -0.0795856788558849 -2.24726013072725   -0.108922582542349  -2.19549117328968   -22.7157414704271   90.7978090038081
143500000   -22.316483082941    86.8123898656977    -0.0918608128023368 -2.62714494512114   -0.111736079092987  -2.6306403824314    -22.1191019941447   94.8479837544148
158000000   -21.5808792789483   87.6682252644119    -0.0882357092802899 -2.98740064316748   -0.114549091735701  -3.0097573548937    -22.095991630164    92.6742627820996
172500000   -20.8030518454888   85.6521431762121    -0.058786435205679  -3.1702517256838    -0.0829651616719342 -3.17470518000703   -21.3896611342314   85.7307652271934
187000000   -19.8996492477217   84.2245604597221    -0.065414159634805  -3.34409893776828   -0.0875403110250417 -3.42981431974968   -20.3650138621127   80.2949328757148
201500000   -18.910269426671    83.4814624589985    -0.0935531381761931 -3.57757310355837   -0.121496367477945  -3.71468429183952   -19.0274776995164   80.9069694593562
216000000   -18.2271685317651   84.4457642480696    -0.115264006186044  -3.94858370545877   -0.145610154134844  -3.84683615377245   -18.2690574902902   85.1836463393653
230500000   -17.7717669327722   85.3228317864213    -0.142437422441532  -4.32389487443725   -0.17076551666731   -4.41169146439293   -17.8836001739993   88.863633230134
245000000   -17.3818437608195   85.646256850703 -0.149118029764704  -4.68926771050803   -0.172076980230447  -4.51009720498498   -17.7749755986115   88.7514093554347
259500000   -17.0020522298508   84.3636836199246    -0.135818472152631  -4.92711097983618   -0.160027548628502  -4.968618817712 -17.5452710335997   85.9529521391691
274000000   -16.4401789780911   82.7775637814641    -0.124941105335287  -5.04524548906528   -0.152179183010779  -5.15116380294596   -16.8931128091832   81.5027857359872
288500000   -15.8638811024476   82.0450032174961    -0.145815853261753  -5.21510114173212   -0.176255015270333  -5.05001428013691   -16.0822836759088   80.2920296025026
303000000   -15.3424582976647   82.4838676480533    -0.172851247082939  -5.49281118092859   -0.199467407980957  -5.51841438705095   -15.5470888765782   81.5600860709161
317500000   -14.9536476280811   83.413470163118 -0.203943427355164  -5.85591012768486   -0.230157531538507  -5.94482129812982   -15.124940146217    84.8350146378497
332000000   -14.7351164292257   83.8920750021108    -0.235237844755596  -6.25425686078397   -0.257077526894856  -6.3043425377086    -14.9149661036184   86.9214595428735
346500000   -14.4831395696229   83.0915774926415    -0.22893601092289   -6.49253939157759   -0.252241384990803  -6.48501452020903   -14.8821866692517   85.3967759542531
361000000   -14.1055813307531   81.7975741284967    -0.21357604476939   -6.70414994513193   -0.239542917561185  -6.70911080950685   -14.6641675795161   82.6788588855219
375500000   -13.6869313489963   80.8404866787358    -0.218703150379374  -6.78523272659627   -0.237522431862514  -6.80507182882845   -14.1646420032371   79.4205476093297
390000000   -13.2752327650348   80.6068942710571    -0.235618584545877  -6.98428465530531   -0.258978313832112  -6.99514519464587   -13.6585596268365   79.5095136881479
404500000   -12.9696402323843   81.8469359857411    -0.284646194377014  -7.25798699638738   -0.309630742197102  -7.2801542138956    -13.1424725044779   82.1832915908307
419000000   -12.8130110853355   82.2247931802235    -0.311140686346172  -7.7510509361054    -0.330811991887013  -7.75422888990538   -12.9917279682379   83.9838631274599
433500000   -12.6693367281181   82.1901567396018    -0.327529634378954  -8.09670987865445   -0.345254797695717  -8.1315698899954    -12.9722425267222   84.1209088305606
448000000   -12.425863658339    81.0119497975381    -0.317390391530249  -8.1530807441078    -0.336093526682407  -8.15135839130149   -12.8397686367391   81.7554450539048
462500000   -12.0947036705957   79.7712972365278    -0.311767056458759  -8.20310681550282   -0.333917521243722  -8.2154090372514    -12.5123128482402   78.9647545480512
477000000   -11.7220068253507   79.3585084456322    -0.329028462206005  -8.3713385668643    -0.349106947108048  -8.42504092506048   -12.0774286548952   78.210617811642
491500000   -11.4099143908003   79.0825331036804    -0.363615576438015  -8.73365237234014   -0.385834235727589  -8.81412624599387   -11.6062233265299   79.3605748901836
506000000   -11.2028431041614   79.9319251455592    -0.405570655482585  -9.06612700097362   -0.428872886550292  -9.05967251850841   -11.3359829509396   81.5749556926049
520500000   -11.0809067277188   80.2351785920228    -0.430675403923504  -9.36581493304827   -0.453729396103342  -9.46525809106774   -11.2612524726571   83.107868735363
535000000   -10.9409896684728   79.957080446907 -0.435953975677893  -9.77413206361755   -0.455576071756911  -9.8107630008169    -11.3368140456008   82.1015646499098
549500000   -10.7080763037489   78.7012999017505    -0.432455381033465  -9.72323675094136   -0.45054083286886   -9.80363412011919   -11.1939607552114   79.6277193177363
```

Figure 14: *sp*2 format for unknown through circuit

The S-parameters of the two-port network are calculated as:

$$S_{11} = 0.2753\angle 79.93^\circ \qquad S_{12} = 0.9543\angle -9.07^\circ$$
$$S_{21} = 0.9518\angle -9.06^\circ \qquad S_{22} = 0.2713\angle 81.57^\circ$$

From $S_{21} = \frac{2Z_0}{Z+2Z_0}$, we can calculate:

$$S_{21} = \frac{100}{Z+100} = 0.9518\angle -9.06^\circ$$

$$\Rightarrow Z = \frac{2Z_0}{S_{21}} - 2Z_0 = (3.7534 + 16.5438j)\Omega$$

As a result, the through load is $(3.7534 + 16.5438j)\Omega$.

2 Matching

Executive Summary

The objective of this lab is to use two-port characteristics to design a microwave amplifier to find the maximum gain. A single stub is used to find conjugate matching to maximum the gain. In this lab, two approaches are applied, hand calculation and CST simulation, and the results of both are compared for verification.

2.1 Task 1

Looking from input port, we write:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 tan(\beta l)}{Z_0 + jZ_L tan(\beta l)} = R_{in} + X_{in}$$

Then we reach:

$$P_L = \frac{V_g^2 \times R_{in} \times |1 - |\Gamma_L^2|| \times |1 - |\Gamma_S^2||}{2(Z_g + Z_{in})^2} = \frac{V_g^2 \times R_{in} \times |1 - |\Gamma_L^2|| \times |1 - |\Gamma_S^2||}{2((R_g + R_{in})^2 + (X_g + X_{in})^2)}$$

$$\Rightarrow P_L = \frac{V_g^2 \times R_{in} \times |1 - |\frac{(Z_L - Z_0)^2}{(Z_L + Z_0)^2}|| \times |1 - |\frac{(Z_g - Z_0)^2}{(Z_g + Z_0)^2}||}{2((R_g + R_{in})^2 + (X_g + X_{in})^2)}$$

By this we try to minimise the denominator of P_L:

$$min(2((R_g + R_{in})^2 + (X_g + X_{in})^2))$$

Hence, let $X_g = -X_{in}$ to reduce the effect of reactance, the load power becomes:

$$P_L = \frac{V_g^2 \times R_{in} \times |1 - |\frac{(Z_L - Z_0)^2}{(Z_L + Z_0)^2}|| \times |1 - |\frac{(Z_g - Z_0)^2}{(Z_g + Z_0)^2}||}{2((R_g + R_{in})^2} = \frac{V_g^2 \times |1 - |\frac{(Z_L - Z_0)^2}{(Z_L + Z_0)^2}|| \times |1 - |\frac{(Z_g - Z_0)^2}{(Z_g + Z_0)^2}||}{2(R_{in} + \frac{R_g^2}{R_{in}} + 2R_g)}$$

P_L reaches max when $R_{in} = R_g$, as a result, when:

$$R_{in} = R_g, X_g = -X_{in} \Rightarrow Z_{in} = Z_g^*$$

P_L is maximized.

2.2 Task 2

From the conjugating conditions, we can explicitly derive solutions for Γ_S and Γ_L:

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4C_1^2}}{2C_1}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4C_2^2}}{2C_2}$$

Where:

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$
$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$
$$C_1 = S_{11} - \Delta S_{22}^*$$
$$C_2 = S_{22} - \Delta S_{11}^*$$
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

In this question, the S parameters for a BJT at **2.4GHz** is:

$$S_{11} = 0.912\angle 127.1^\circ$$
$$S_{11} = 0.041\angle 26.0^\circ$$
$$S_{11} = 1.721\angle 35.6^\circ$$
$$S_{11} = 0.75\angle 143.9^\circ$$

With the help of matlab, the calculation code is as followed:

```
%Here is the calculation of S-Parameter
S11 = 0.912*cos(127.1/180*pi) + 0.912*sin(127.1/180*pi)*j;
S12 = 0.041*cos(26/180*pi) + 0.041*sin(26/180*pi)*j;
S21 = 1.721*cos(35.6/180*pi) + 1.721*sin(35.6/180*pi)*j;
S22 = 0.75*cos(143.9/180*pi) + 0.75*sin(143.9/180*pi)*j;

%Calculation for some parameters
DELTA = S11*S22 - S12*S21;
B1 = 1 + abs(S11)*abs(S11) - abs(S22)*abs(S22) - abs(DELTA)*abs(DELTA);
B2 = 1 + abs(S22)*abs(S22) - abs(S11)*abs(S11) - abs(DELTA)*abs(DELTA);
C1 = S11 - DELTA*conj(S22);
C2 = S22 - DELTA*conj(S11);

%Calculation for Gamma
GAMMAS1 = (B1 + sqrt(B1*B1 - 4*abs(C1)*abs(C1)))/(2*C1);
GAMMAS2 = (B1 - sqrt(B1*B1 - 4*abs(C1)*abs(C1)))/(2*C1);
GAMMAL1 = (B2 + sqrt(B2*B2 - 4*abs(C2)*abs(C2)))/(2*C2);
GAMMAL2 = (B2 - sqrt(B2*B2 - 4*abs(C2)*abs(C2)))/(2*C2);

%Calculation for gain
GAMMAIN = S11 + (S12*S21*GAMMAL2)/(1-S22*GAMMAL2);
GT = abs(S21)*abs(S21);
GMAX = abs(S21)*abs(S21)*(1-abs(GAMMAL2)*abs(GAMMAL2))/((1-abs(GAMMAS2)*
    abs(GAMMAS2))*abs(1-S22*GAMMAL2)*S22*GAMMAL2));
Ratio = GMAX/GT;
```

Yielding the result:

$$\Gamma_L = -0.5909 - 0.1240j = 0.6038\angle - 168.14°$$

$$Y_L = 0.6038\angle 11.85°$$

$$\Gamma_S = -0.5889 - 0.6701j = 0.8921\angle - 131.31°$$

$$Y_S = 0.8921\angle 48.69°$$

The original transducer gain G_T is:

$$G_T = |S_{21}|^2 = 2.9618 = 4.716dB$$

Under the maximum gain with conjugate matching:

$$G_{max} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_S|^2)(|1-\Gamma_L S_{22}|^2)} = 24.3144 = 13.859dB$$

Which improves approximately 9.143dB.
The stub-matching for the generator can be implemented using the Smith Chart shown below:

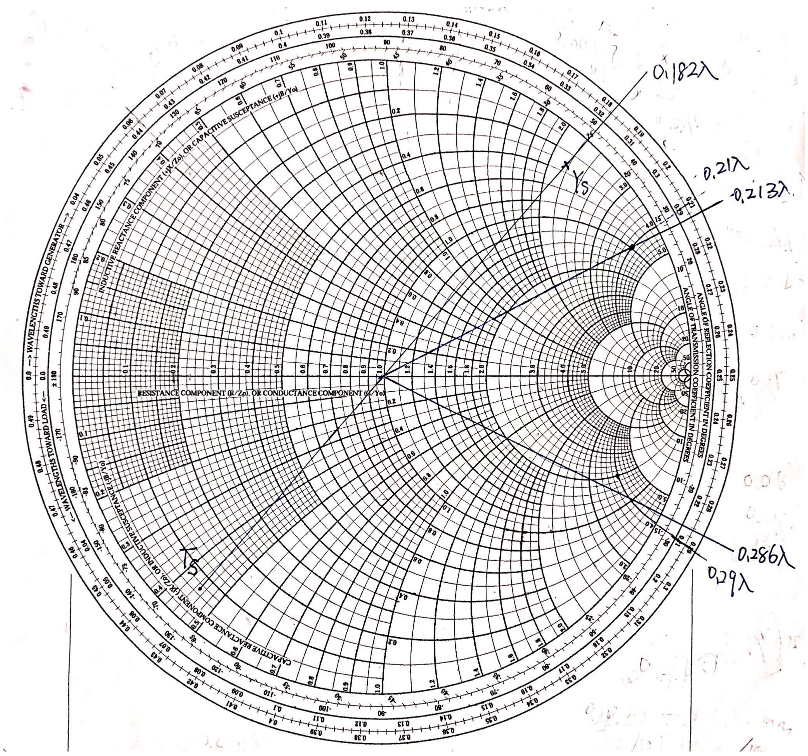


Figure 15: Stub Matching Smith Chart for generator(Γ_S)

Here, from the figure we have two solutions:

$$l_{stub,in1} = 0.21\lambda, l_{in,1} = 0.469\lambda$$

$$l_{stub,in2} = 0.29\lambda, l_{in,2} = 0.396\lambda$$

Similarly, for the load impedance matching:

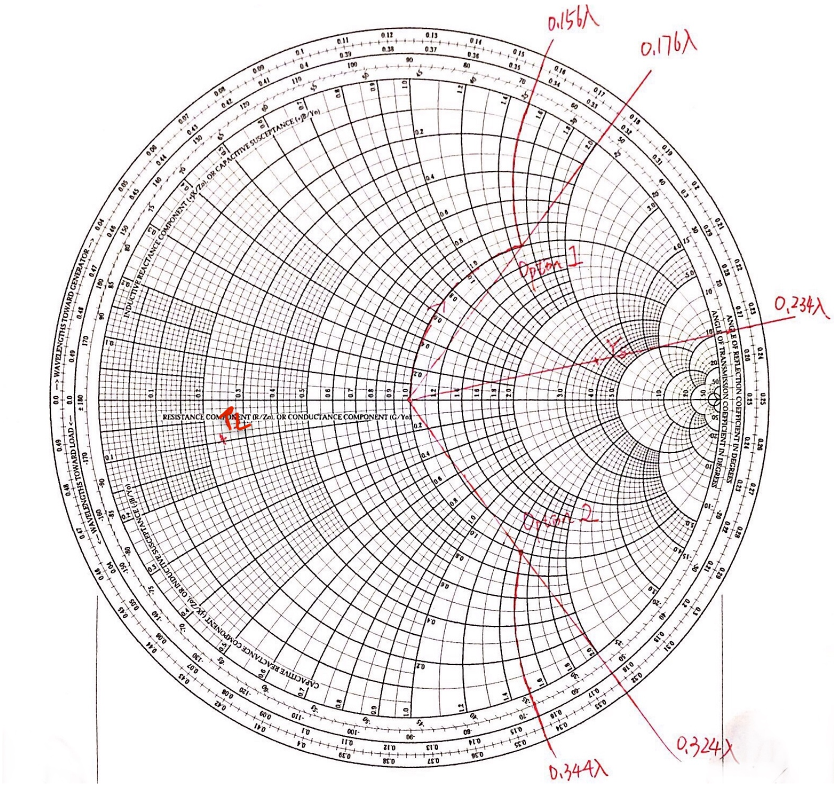


Figure 16: Stub Matching Smith Chart for load(Γ_L)

Here, from the figure we have two solutions:

$$l_{stub,out1} = 0.156\lambda, l_{out,1} = 0.058\lambda$$

$$l_{stub,out2} = 0.344\lambda, l_{out,2} = 0.41\lambda$$

The wavelength of on the strip can be calculated as:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}} = 3.2623$$

$$\lambda = \frac{c}{\sqrt{\varepsilon_{eff}}f} = \frac{3 \times 10^8}{\sqrt{3.2623} \times 2.4 \times 10^9} = 69.2mm$$

Hence:

$$l_{stub,in1} = 0.21\lambda = 14.532mm, l_{in,1} = 0.469\lambda = 32.45mm$$

$$l_{stub,in2} = 0.29\lambda = 20.068mm, l_{in,2} = 0.396\lambda = 27.4mm$$

$$l_{stub,out1} = 0.156\lambda = 10.79mm, l_{out,1} = 0.058\lambda = 4mm$$

$$l_{stub,out2} = 0.344\lambda = 23.8mm, l_{out,2} = 0.41\lambda = 28.372mm$$

2.3 Task 3

2.3.1 Question I

By the simulation, the optimized micro strip width w is 2.31mm shown in the figure below:

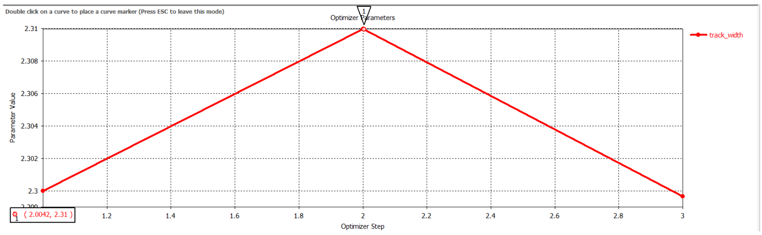


Figure 17: Optimized Microstrip Width w

And the corresponding characteristic impedance response is:

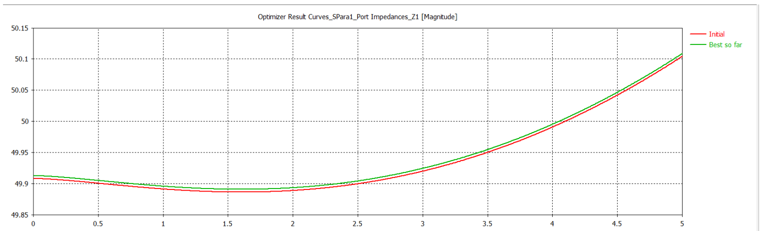


Figure 18: Optimized Microstrip Impedance Response

2.3.2 Question II

The impedance Z_0 follows the following relationship:

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}} ln \frac{5.98h}{0.8w + T} = 50\Omega$$

Where $\varepsilon_r = 4.3, h = 1.2mm, T = 35\mu m$, yielding:

$$w = 2.228mm$$

2.3.3 Question III

The calculated value $w = 2.228mm$ is a bit less than the theoretical value $w = 2.31mm$, but they are much close that we could consider almost the same, the differences are mainly caused by the simulation environment and the calculation approximation, actually the impedance of the microstrip is:

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{2}\sqrt{E_r + 1}} ln(1 + 4(X_1 + X_2)(\frac{H}{W_{eff}}))$$

Where:

$$W_{eff} = W + (\frac{t}{\pi}) ln(\frac{4e}{\sqrt{(\frac{T}{H})^2 + (\frac{T}{W\pi + 1.1T\pi})^2}}) \frac{E_r + 1}{2E_r}$$

$$X_1 = 4(\frac{14E_r + 8}{11E_r})(\frac{H}{W_{eff}})$$

$$X_2 = \sqrt{16(\frac{H}{W_{eff}})^2(\frac{14E_r + 8}{11E_r})^2 + (\frac{E_r + 1}{2E_r})\pi^2}$$

The equation given in **Question II** is also an approximation of the characteristic impedance, hence we may put $2.3mm$ as our optimized width value.

2.4 Task 4

2.4.1 Question I

The S_{11} looking from input side(left side of the left micro strip) is:

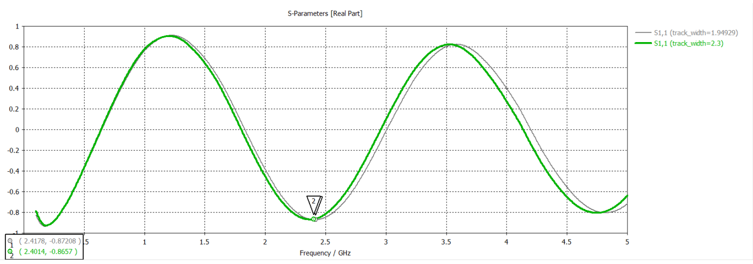


Figure 19: Real Part of S_{11}

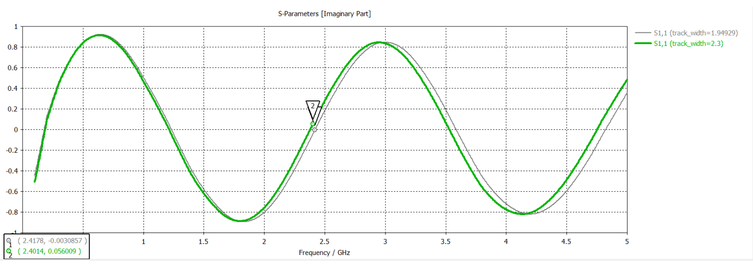


Figure 20: Imaginary Part of S_{11}

Hence:

$$S_{11} = -0.8657 + 0.056j = 0.8675\angle 176.2988°$$

The S_{22} looking from right side(right side of the right micro strip) is:

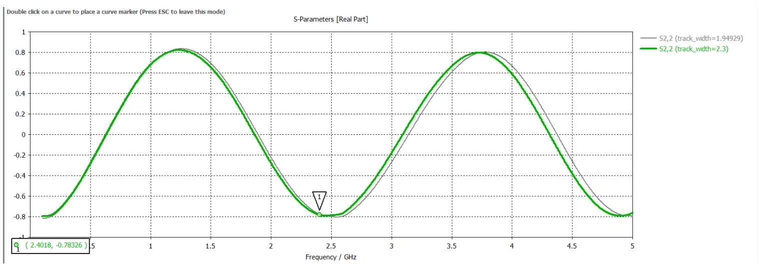


Figure 21: Real Part of S_{22}

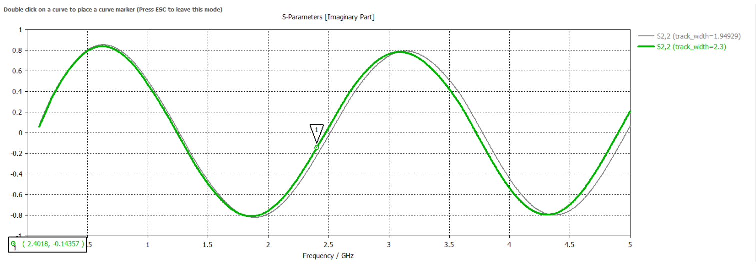


Figure 22: Imaginary Part of S_{22}

Hence:

$$S_{22} = -0.7833 - 0.14357j = 0.7963\angle - 169.6°$$

2.4.2 Question II

As the impedance of the source and load is matched, the gain of the amplifier is:

$$G_T = |S_{21}|^2 = 1.3842^2 = 1.9161$$

Where the S_{21} is:

$$S_{21} = 0.0255 - 1.384j = 1.3842\angle - 88.9°$$

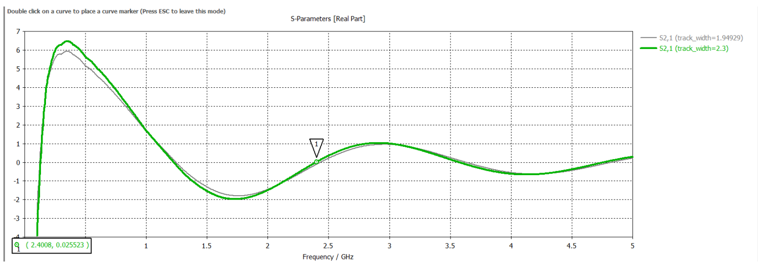


Figure 23: Real Part of S_{21}

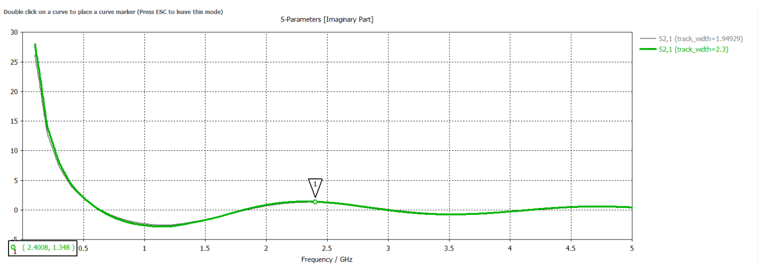


Figure 24: Imaginary Part of S_{21}

2.4.3 Question III

The S-parameters are measured at the connector, as shown in the figure below:

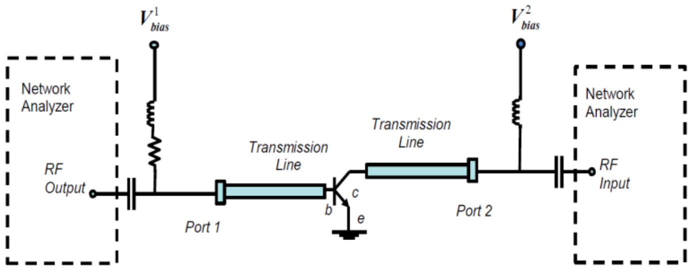


Figure 25: Biasing and decoupling circuit for BJT

the measurement should include the transmission line for simulations, and S-parameter would change due to the transmission line, e.g. Γ_S or Γ_L would rotate in the same VSWR circle, we should take this factor into consideration, as a result, S-parameter should be measured at the connector.

2.5 Task 5

2.5.1 Question I

The parameters are listed below:

$$
\begin{aligned}
l_{stub,in} &= 21mm \\
l_{in} &= 25mm \\
l_{stub,out} &= 7.5mm \\
l_{out} &= 1mm
\end{aligned}
$$

The results are quite different from the calculated values in **Task 2**, then we check the S-parameters in the $s2p$ file and at 2.4GHz:

$$S_{11} = 0.929\angle 129.2°$$

$$S_{21} = 1.442\angle 41.7°$$

$$S_{12} = 0.042\angle 29°$$

$$S_{22} = 0.853\angle 143.3°$$

The S-parameters are different from the given data in task 2, causing different stub and line length, based on this, we can calculate new Γ_S and Γ_L:

$$\Gamma_L = -0.6720 - 0.2284j = 0.71\angle -161.23°$$

$$Y_L = 0.71\angle 18.77°$$

$$\Gamma_S = -0.6253 - 0.6353j = 0.8913\angle -134.54°$$

$$Y_S = 0.8921\angle 45.46°$$

The stub-matching for the generator can be implemented using the Smith Chart shown below:

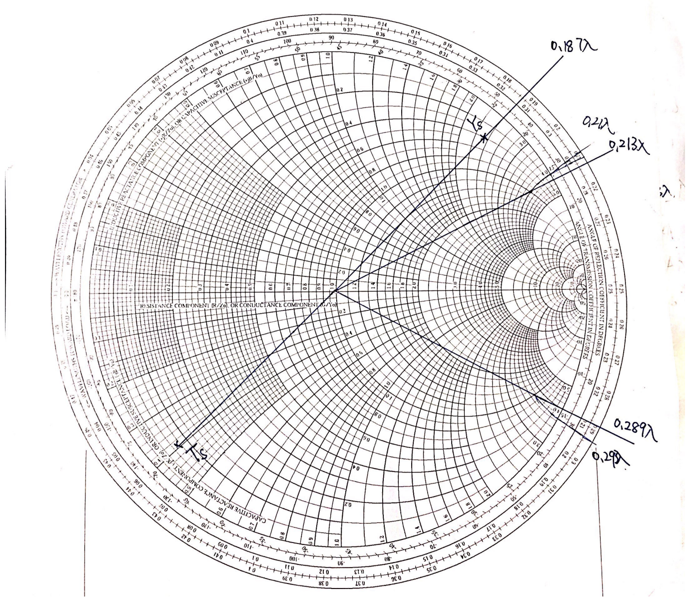


Figure 26: Stub Matching Smith Chart for generator(Γ_S)

Here, from the figure we have two solutions:

$$l_{stub,in1} = 0.21\lambda, l_{in,1} = 0.474\lambda$$

$$l_{stub,in2} = 0.293\lambda, l_{in,2} = 0.398\lambda$$

The stub-matching for the load can be implemented using the Smith Chart shown below:

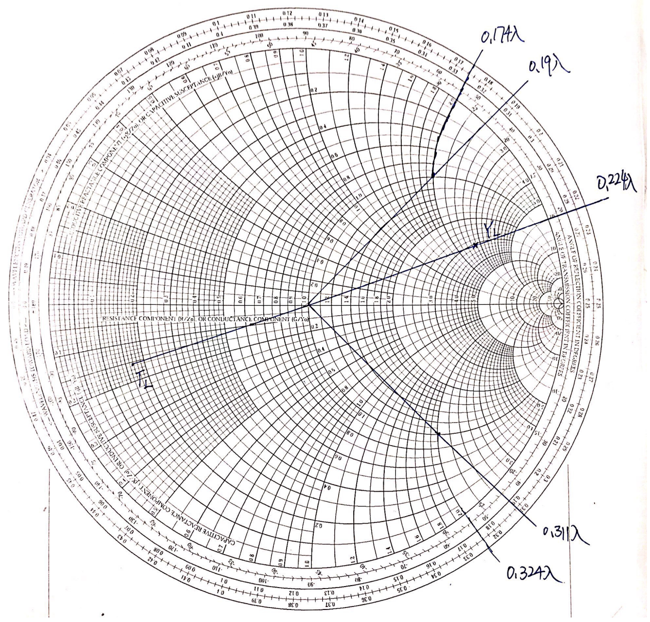


Figure 27: Stub Matching Smith Chart for load(Γ_L)

Also we have two solutions:

$$l_{stub,out1} = 0.174\lambda, l_{out,1} = 0.034\lambda$$

$$l_{stub,out2} = 0.324\lambda, l_{out,2} = 0.413\lambda$$

To conclude:

$$l_{stub,in1} = 0.21\lambda = 14.532mm, l_{in,1} = 0.474\lambda = 32.8mm$$

$$l_{stub,in2} = 0.293\lambda = 20.3mm, l_{in,2} = 0.398\lambda = 27.5416mm$$

$$l_{stub,out1} = 0.174\lambda = 12.04mm, l_{out,1} = 0.034\lambda = 2.353mm$$

$$l_{stub,out2} = 0.324\lambda = 22.42mm, l_{out,2} = 0.413\lambda = 28.58mm$$

The original power gain is about:

$$G_T = 1.9161 = 2.824dB$$

The improved gain should be approximately:

$$G_{T,new} = |S_{21,new}|^2 = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |\Gamma_S|^2)(|1 - \Gamma_L S_{22}|^2)} = 23.42 = 13.7dB$$

2.5.2 Question II

Based on the stub matching, new S_{11} and S_{22} are:

$$S_{11} = -0.0134 + 0.0735j = 0.0747\angle 100.3°$$

$$S_{22} = -0.3541 + 0.2998j = 0.464\angle 139.74°$$

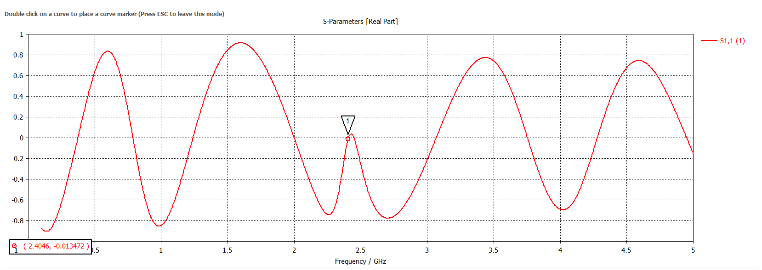


Figure 28: Real Part of S_{11}

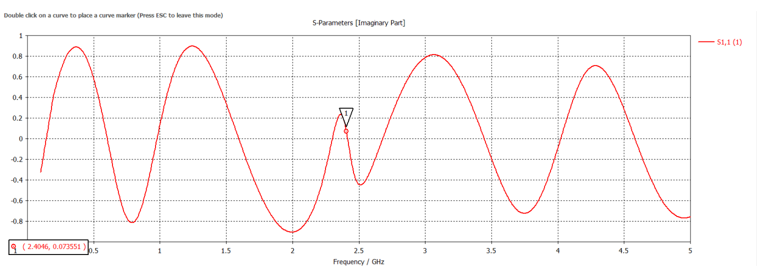


Figure 29: Imaginary Part of S_{11}

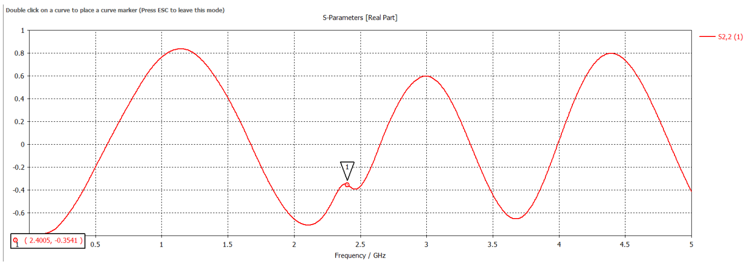


Figure 30: Real Part of S_{22}

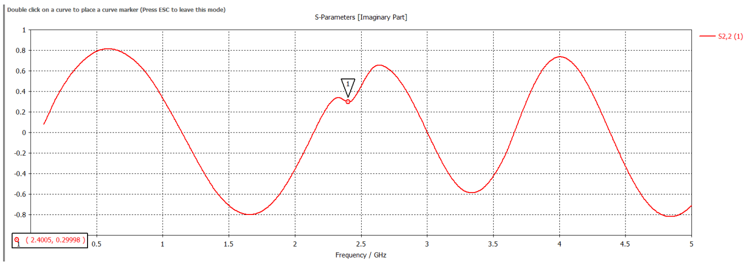


Figure 31: Imaginary Part of S_{22}

2.5.3 Question IIII

The amplifier gain S_{21}^2 is:

$$S_{21} = 2.651 - 0.8j = 2.7691\angle - 16.8^{\circ}$$

$$G_{max} = |S_{21}|^2 = 7.6679 = 8.9dB$$

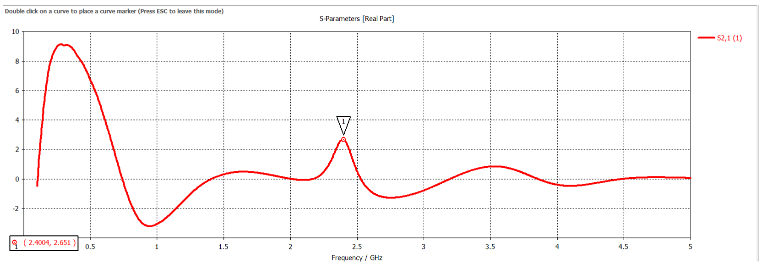


Figure 32: Real Part of S_{21}

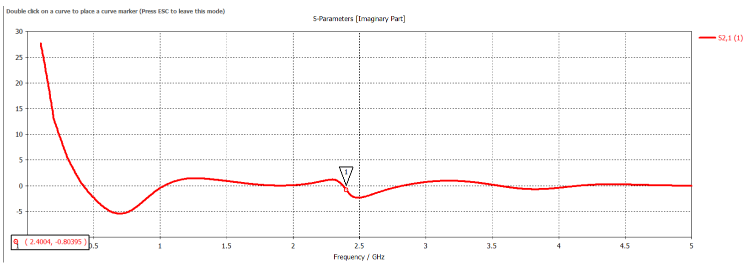


Figure 33: Imaginary Part of S_{21}

The gain is improved but it is not maximised as the input length is not exactly what we calculated, by changing the values to:

$$l_{stub,in} = 21mm, l_{in} = 26mm$$

$$l_{stub,out} = 12.04mm, l_{out} = 2.353mm$$

We can reach:

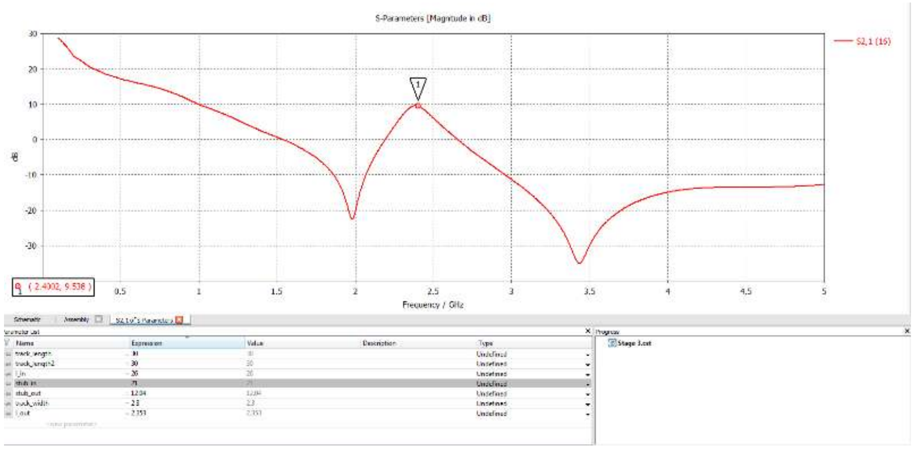


Figure 34: Improved Power Gain S_{21}

S_{21} will improve a little bit to 9.5dB, but still much less than theoretical gain 13.7dB, this is due to the real loss in the transmission line and unmatched impedance as $Z_0 = 49.9\Omega$.Also, the manually measurement of the Smith chart can lead to some error on the input and output length, i.e. $l_{stub,in}$ and $l_{stub,out}$ are not matching the maximum gain. The gain can still grow with more precisely values of length. And the approximation of calculation may contribute to the error as well.