

Contents

1 Basic	1	6.9 Factorial without Prime Factor	14
1.1 Windows	1	6.10 Berlekamp Massey . .	14
1.2 Debug Macro	1	6.11 Simplex	15
1.3 Pragma / FastIO . . .	1	6.12 Euclidean	15
1.4 Divide	1	6.13 Linear Programming Construction	15
2 Data Structure	1	6.14 Theorem	15
2.1 Leftist Tree	1	6.15 Estimation	16
2.2 Splay Tree	1	6.16 General Purpose Numbers	16
2.3 Link Cut Tree	2	7 Polynomial	16
2.4 Treap	3	7.1 Number Theoretic Transform	16
2.5 vEB Tree	3	7.2 Fast Fourier Transform	16
3 Flow / Matching	3	7.3 Primes	16
3.1 Dinic	3	7.4 Polynomial Operations	16
3.2 Min Cost Max Flow . .	4	7.5 Fast Linear Recursion	18
3.3 Kuhn Munkres	4	7.6 Fast Walsh Transform	18
3.4 Hopcroft Karp	5	8 Geometry	18
3.5 SW Min Cut	5	8.1 Basic	18
3.6 Gomory Hu Tree	5	8.2 SVG Writer	18
3.7 Blossom	5	8.3 Heart	19
3.8 Min Cost Circulation	6	8.4 External Bisector . .	19
3.9 Weighted Blossom . . .	6	8.5 Intersection of Segments	19
3.10 Flow Model	7	8.6 Intersection of Circle and Line	19
4 Graph	7	8.7 Intersection of Circles	19
4.1 Heavy-Light Decomposition	7	8.8 Intersection of Polygon and Circle	19
4.2 Centroid Decomposition	8	8.9 Tangent Lines of Polygon and Point	19
4.3 Edge BCC	8	8.10 Tangent Lines of Circle and Point	19
4.4 Vertex BCC / Round Square Tree	8	8.11 Tangent Lines of Circles	19
4.5 SCC	8	8.12 Point In Convex . . .	20
4.6 2SAT	9	8.13 Point In Circle . . .	20
4.7 Virtual Tree	9	8.14 Point Segment Distance	20
4.8 Directed MST	9	8.15 Convex Hull	20
4.9 Dominator Tree	9	8.16 Minimum Enclosing Circle	20
4.10 Bipartite Edge Coloring	10	8.17 Union of Circles . .	20
4.11 Edge Coloring	10	8.18 Union of Polygons . .	20
4.12 Maximum Clique	10	8.19 Rotating SweepLine	21
5 String	11	8.20 Half Plane Intersection	21
5.1 Aho-Corasick Automaton	11	8.21 Minkowski Sum	21
5.2 KMP Algorithm	11	8.22 Vector In Polygon . .	21
5.3 Z Algorithm	11	8.23 Delaunay Triangulation	21
5.4 Manacher	11	8.24 Triangulation Voronoi	22
5.5 Suffix Array	11	8.25 3D Point	22
5.6 SAIS	11	8.26 3D Convex Hull	23
5.7 Suffix Automaton	12	9 Else	23
5.8 Minimum Rotation	12	9.1 Pbds	23
5.9 Palindrome Tree	12	9.2 Bit Hack	24
5.10 Lyndon Factorization	12	9.3 Smawk Algorithm	24
5.11 Main Lorentz	13	9.4 Slope Trick	24
6 Math	13	9.5 ALL LCS	24
6.1 Miller Rabin / Pollard Rho	13	9.6 Hilbert Curve	24
6.2 Ext GCD	13	9.7 Line Container	25
6.3 Chinese Remainder Theorem	13	9.8 Min Plus Convolution	25
6.4 PiCount	13	9.9 Matroid Intersection	25
6.5 Linear Function Mod Min	14	9.10 Simulated Annealing .	25
6.6 Floor Sum	14	9.11 Bitset LCS	25
6.7 Quadratic Residue	14	9.12 Binary Search On Fraction	25
6.8 Discrete Log	14	9.13 Cyclic Ternary Search	25
		9.14 Tree Hash	25
		9.15 Python Misc	25

1 Basic

1.1 Windows

```
@echo off
g++ -std=c++17 -DABS -Wall -Wextra -Wshadow -O2 %1.cpp
-o %1.exe && %1.exe
for %i in (A B C ... L) do copy tem.cpp %i.cpp
for /l %i in (1,1,100) do (
python gen.py > in.txt
bad.exe < in.txt > out1.txt
good.exe < in.txt > out2.txt
fc out1.txt out2.txt || goto :end
)
:end
```

1.2 Debug Macro [6636fe]

```
void db() { cerr << endl; }
template <typename T, typename ...U>
void db(T i, U ...j) { cerr << i << ' ', db(j...); }
#ifdef ABS
#define bug(x...) db("[ " + string(#x) + "]", x)
```

```
#define safe cerr << __PRETTY_FUNCTION__ << " Line " <<
__LINE__ << " safe" << endl
#else
#define bug(x...) void(0)
#define safe void(0)
#endif
```

1.3 Pragma / FastIO

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
static char buf[65536], *p = buf, *q = buf;
return p == q && (q = (p = buf) + read(0, buf, 65536)
) == buf ? -1 : *p++;
}
inline int R() {
static char c;
while((c = RC()) < '0'); int a = c ^ '0';
while((c = RC()) >= '0') a *= 10, a += c ^ '0';
return a;
}
inline void W(int n) {
static char buf[12], p;
if (n == 0) OB[OP++] = '0'; p = 0;
while (n) buf[p++] = '0' + (n % 10), n /= 10;
for (--p; p >= 0; --p) OB[OP++] = buf[p];
if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.4 Divide

```
ll floor(ll a, ll b) {return a / b - (a < 0 && a % b);}
ll ceil(ll a, ll b) {return a / b + (a > 0 && a % b);}
a / b < x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x < a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)
```

2 Data Structure

2.1 Leftist Tree [414ab9]

```
struct node {
ll rk, data, sz, sum;
node *l, *r;
node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k)
} {}
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
if (!a || !b) return a ? a : b;
if (a->data < b->data) swap(a, b);
a->r = merge(a->r, b);
if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
a->rk = rk(a->r) + 1;
a->sz = sz(a->l) + sz(a->r) + 1;
a->sum = sum(a->l) + sum(a->r) + a->data;
return a;
}
void pop(node *o) {
node *tmp = o;
o = merge(o->l, o->r);
delete tmp;
}
```

2.2 Splay Tree [21142b]

```
struct Splay {
int pa[N], ch[N][2], sz[N], rt, _id;
ll v[N];
Splay() {}
void init() {
rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
sz[0] = 1, v[0] = inf;
}
```

```

int newnode(int p, int x) {
    int id = _id++;
    v[id] = x, pa[id] = p;
    ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    return id;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int gp = pa[p], c = ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
}
void splay(int i) {
    while (~pa[i]) {
        int p = pa[i];
        if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
            == i ? i : p);
        rotate(i);
    }
    rt = i;
}
int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
        if (v[i] == x) return splay(i), i;
        if (v[i] > x) {
            last = i;
            if (ch[i][0] == -1) break;
            i = ch[i][0];
        }
        else {
            if (ch[i][1] == -1) break;
            i = ch[i][1];
        }
    }
    splay(i);
    return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};

```

2.3 Link Cut Tree [bca367]

```

// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N];
    int sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
    // virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {

```

```

        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0]) {
            sz[i] += sz[ch[i][0]];
            mx[i] = max(mx[i], mx[ch[i][0]]);
        }
        if (ch[i][1]) {
            sz[i] += sz[ch[i][1]];
            mx[i] = max(mx[i], mx[ch[i][1]]);
        }
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
            rev[i] = false;
    }
    void reverse(int i) {
        if (!i) return;
        swap(ch[i][0], ch[i][1]);
        rev[i] ^= true;
    }
    bool isrt(int i) { // rt of splay
        if (!pa[i]) return true;
        return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i;
        int c = ch[i][!x], gp = pa[p];
        if (ch[gp][0] == p) ch[gp][0] = i;
        else if (ch[gp][1] == p) ch[gp][1] = i;
        pa[i] = gp, ch[i][!x] = p, pa[p] = i;
        ch[p][x] = c, pa[c] = p;
        pull(p), pull(i);
    }
    void splay(int i) {
        vector<int> anc;
        anc.push_back(i);
        while (!isrt(anc.back()))
            anc.push_back(pa[anc.back()]);
        while (!anc.empty())
            push(anc.back()), anc.pop_back();
        while (!isrt(i)) {
            int p = pa[i];
            if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
                == p ? i : p);
            rotate(i);
        }
    }
    void access(int i) {
        int last = 0;
        while (i) {
            splay(i);
            if (ch[i][1])
                sz2[i] += sz[ch[i][1]];
            sz2[i] -= sz[last];
            ch[i][1] = last;
            pull(i), last = i, i = pa[i];
        }
    }
    void makert(int i) {
        access(i), splay(i), reverse(i);
    }
    void link(int i, int j) {
        // assert(findrt(i) != findrt(j));
        makert(i);
        makert(j);
        pa[i] = j;
        sz2[j] += sz[i];
        pull(j);
    }
    void cut(int i, int j) {
        makert(i), access(j), splay(i);
        // assert(sz[i] == 2 && ch[i][1] == j);
        ch[i][1] = pa[j] = 0, pull(i);
    }
    int findrt(int i) {
        access(i), splay(i);
        while (ch[i][0]) push(i), i = ch[i][0];
        splay(i);
        return i;
    }
};

```

2.4 Treap [9d5c2a]

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
// delete default code sz
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    o->down(), split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
    o->up();
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c; // [l, r)
    o->down();
    split2(o, a, b, l), split2(b, b, c, r - l);
    // operate
    o = merge(a, merge(b, c)), o->up();
}

```

2.5 vEB Tree [087d11]

```

using u64=uint64_t;
constexpr int lsb(u64 x)
{ return x?__builtin_ctzll(x):1<<30; }
constexpr int msb(u64 x)
{ return x?63-__builtin_clzll(x):-1; }
template<int N, class T=void>
struct veb{
    static const int M=N>>1;
    veb<M> ch[1<<N-M];

```

```

    veb<N-M> aux;
    int mn,mx;
    veb():mn(1<<30),mx(-1){}
    constexpr int mask(int x){return x&((1<<M)-1);}
    bool empty(){return mx==-1;}
    int min(){return mn;}
    int max(){return mx;}
    bool have(int x){
        return x==mn?true:ch[x>>M].have(mask(x));
    }
    void insert_in(int x){
        if(empty()) return mn=mx=x,void();
        if(x<mn) swap(x,mn);
        if(x>mx) mx=x;
        if(ch[x>>M].empty()) aux.insert_in(x>>M);
        ch[x>>M].insert_in(mask(x));
    }
    void erase_in(int x){
        if(mn==mx) return mn=1<<30,mx=-1,void();
        if(x==mn) mn=x=(aux.min()<<M)^ch[aux.min()].min();
        ch[x>>M].erase_in(mask(x));
        if(ch[x>>M].empty()) aux.erase_in(x>>M);
        if(x==mx){
            if(aux.empty()) mx=mn;
            else mx=(aux.max()<<M)^ch[aux.max()].max();
        }
    }
    void insert(int x){
        if(!have(x)) insert_in(x);
    }
    void erase(int x){
        if(have(x)) erase_in(x);
    }
    int next(int x){// >=x
        if(x>mx) return 1<<30;
        if(x<=mn) return mn;
        if(mask(x)<=ch[x>>M].max())
            return ((x>>M)<<M)^ch[x>>M].next(mask(x));
        int y=aux.next((x>>M)+1);
        return (y<<M)^ch[y].min();
    }
    int prev(int x){// <x
        if(x<=mn) return -1;
        if(x>mx) return mx;
        if(x<=(aux.min()<<M)+ch[aux.min()].min())
            return mn;
        if(mask(x)>ch[x>>M].min())
            return ((x>>M)<<M)^ch[x>>M].prev(mask(x));
        int y=aux.prev(x>>M);
        return (y<<M)^ch[y].max();
    }
};
template<int N>
struct veb<N,typename enable_if<N<=6>::type>{
    u64 a;
    veb():a(0){}
    void insert_in(int x){a|=1ull<<x;}
    void insert(int x){a|=1ull<<x;}
    void erase_in(int x){a&=~(1ull<<x);}
    void erase(int x){a&=~(1ull<<x);}
    bool have(int x){return a>>x&1;}
    bool empty(){return a==0;}
    int min(){return lsb(a);}
    int max(){return msb(a);}
    int next(int x){return lsb(a&~((1ull<<x)-1));}
    int prev(int x){return msb(a&((1ull<<x)-1));}
};

```

3 Flow / Matching

3.1 Dinic [8898fb]

```

template <typename T>
struct Dinic { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    struct edge { int to, rev; T cap, flow; };
    int n, s, t;
    vector <vector <edge>> g;
    vector <int> dis, cur;
    T dfs(int u, T cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)g[u].size(); ++i) {

```

```

    edge &e = g[u][i];
    if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
        T df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
            e.flow += df;
            g[e.to][e.rev].flow -= df;
            return df;
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill(all(dis), -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (auto &u : g[v])
            if (!dis[u.to] && u.flow != u.cap) {
                q.push(u.to);
                dis[u.to] = dis[v] + 1;
            }
    }
    return dis[t] != -1;
}
T solve(int _s, int _t) {
    s = _s, t = _t;
    T flow = 0, df;
    while (bfs()) {
        fill(all(cur), 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
void reset() {
    for (int i = 0; i < n; ++i)
        for (auto &j : g[i]) j.flow = 0;
}
void add_edge(int u, int v, T cap) {
    g[u].pb(edge{v, (int)g[v].size(), cap, 0});
    g[v].pb(edge{u, (int)g[u].size() - 1, 0, 0});
}
Dinic (int _n) : n(_n), g(n), dis(n), cur(n) {}
};

```

3.2 Min Cost Max Flow [8083d7]

```

template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
    const T1 INF1 = numeric_limits<T1>::max() / 2;
    const T2 INF2 = numeric_limits<T2>::max() / 2;
    struct edge {
        int v; T1 f; T2 c;
    };
    int n, s, t;
    vector<vector<int>> g;
    vector<edge> e;
    vector<T2> dis, pot;
    vector<int> rt, vis;
    // bool DAG()...
    bool SPFA() {
        fill(all(rt), -1), fill(all(dis), INF2);
        fill(all(vis), false);
        queue<int> q;
        q.push(s), dis[s] = 0, vis[s] = true;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            vis[v] = false;
            for (int id : g[v]) {
                auto [u, f, c] = e[id];
                T2 ndis = dis[v] + c + pot[v] - pot[u];
                if (f > 0 && dis[u] > ndis) {
                    dis[u] = ndis, rt[u] = id;
                    if (!vis[u]) vis[u] = true, q.push(u);
                }
            }
        }
        return dis[t] != INF2;
    } // d9b0f8
    bool dijkstra() {

```

```

        fill(all(rt), -1), fill(all(dis), INF2);
        priority_queue<pair<T2, int>, vector<pair<T2, int>, greater<pair<T2, int>>> pq;
        dis[s] = 0, pq.emplace(dis[s], s);
        while (!pq.empty()) {
            auto [d, v] = pq.top(); pq.pop();
            if (dis[v] < d) continue;
            for (int id : g[v]) {
                auto [u, f, c] = e[id];
                T2 ndis = dis[v] + c + pot[v] - pot[u];
                if (f > 0 && dis[u] > ndis) {
                    dis[u] = ndis, rt[u] = id;
                    pq.emplace(ndis, u);
                }
            }
        }
        return dis[t] != INF2;
    }
}
vector<pair<T1, T2>> solve(int _s, int _t) {
    s = _s, t = _t, fill(all(pot), 0);
    vector<pair<T1, T2>> ans; bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; ++i)
            dis[i] += pot[i] - pot[s];
        T1 add = INF1;
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            add = min(add, e[rt[i]].f);
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            e[rt[i]].f -= add, e[rt[i] ^ 1].f += add;
        ans.emplace_back(add, dis[t]), fr = false;
        for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return ans;
}
void reset() {
    for (int i = 0; i < (int)e.size(); ++i) e[i].f = 0;
}
void add_edge(int u, int v, T1 f, T2 c) {
    g[u].pb((int)e.size(), e.pb({v, f, c}));
    g[v].pb((int)e.size(), e.pb({u, 0, -c}));
}
MCMF (int _n) : n(_n), g(n), e(), dis(n), pot(n),
    rt(n), vis(n) {} // 05becb
};

```

3.3 Kuhn Munkres [b880ad]

```

template <typename T>
struct KM { // 0-based, remember to init
    const T INF = numeric_limits<T>::max() / 2;
    int n; vector<vector<T>> w;
    vector<T> hl, hr, slk;
    vector<int> fl, fr, vl, vr, pre;
    queue<int> q;
    bool check(int x) {
        if (vl[x] = 1, ~fl[x])
            return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill(all(slk), INF), fill(all(vl), 0);
        fill(all(vr), 0);
        while (!q.empty()) q.pop();
        q.push(s), vr[s] = 1;
        while (true) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x) {
                    d = hl[x] + hr[y] - w[x][y];
                    if (!vl[x] && slk[x] >= d) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
                    }
                }
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;

```

```

        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
}
T solve() {
    fill(all(fl), -1), fill(all(fr), -1);
    fill(all(hr), 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(all(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
void add_edge(int a, int b, T wei) { w[a][b] = wei; }
KM (int _n) : n(_n), w(n, vector<T>(n, -INF)), hl(n),
    hr(n), slk(n), fl(n), fr(n), vl(n), vr(n), pre(n){}
};

```

3.4 Hopcroft Karp [33c68d]

```

struct HopcroftKarp { // 0-based
    const int INF = 1 << 30;
    int n, m;
    vector<vector<int>> g;
    vector<int> match, dis, matched, vis;
    bool dfs(int x) {
        vis[x] = true;
        for (int y : g[x])
            if (match[y] == -1 || (dis[match[y]] == dis[x] +
                1 && !vis[match[y]] && dfs(match[y]))) {
                match[y] = x, matched[x] = true;
                return true;
            }
        return false;
    }
    bool bfs() {
        fill(all(dis), -1);
        queue<int> q;
        for (int x = 0; x < n; ++x) if (!matched[x])
            dis[x] = 0, q.push(x);
        int mx = INF;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int y : g[x]) {
                if (match[y] == -1) {
                    mx = dis[x];
                    break;
                } else if (dis[match[y]] == -1)
                    dis[match[y]] = dis[x] + 1, q.push(match[y]);
            }
        }
        return mx < INF;
    }
    int solve() {
        int res = 0;
        fill(all(match), -1);
        fill(all(matched), 0);
        while (bfs()) {
            fill(all(vis), 0);
            for (int x = 0; x < n; ++x) if (!matched[x])
                res += dfs(x);
        }
        return res;
    }
    void add_edge(int x, int y) { g[x].pb(y); }
    HopcroftKarp (int _n, int _m) : n(_n), m(_m), g(n),
        match(m), dis(n), matched(n), vis(n) {}
};

```

3.5 SW Min Cut [b9af94]

```

template<typename T>
struct SW { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    vector<vector<T>> g;
    vector<T> sum;
    vector<bool> vis, dead;
    int n;

```

```

    T solve() {
        T ans = INF;
        for (int r = 0; r + 1 < n; ++r) {
            fill(all(vis), 0), fill(all(sum), 0);
            int num = 0, s = -1, t = -1;
            while (num < n - r) {
                int now = -1;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i] &&
                        (now == -1 || sum[now] > sum[i])) now = i;
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i]) sum[i] += g[now][i];
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i)
                g[i][s] += g[i][t], g[s][i] += g[t][i];
            dead[t] = true;
        }
        return ans;
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w;
    }
    SW (int _n) : n(_n), g(n, vector<T>(n)), vis(n),
        sum(n), dead(n) {}
};

```

3.6 Gomory Hu Tree [90ead2]

```

vector<array<int, 3>> GomoryHu(Dinic<int> flow) {
    // Tree edge min = mcut (0-based)
    int n = flow.n;
    vector<array<int, 3>> ans;
    vector<int> rt(n);
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset();
        ans.pb({i, t, flow.solve(i, t)});
        flow.bfs();
        for (int j = i + 1; j < n; ++j)
            if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
    }
    return ans;
}

```

3.7 Blossom [6092d8]

```

struct Matching { // 0-based
    int n, tk;
    vector<vector<int>> g;
    vector<int> fa, pre, match, s, t;
    queue<int> q;
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (t[x] == tk) return x;
                t[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
    void blossom(int x, int y, int l) {
        while (Find(x) != l) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            if (fa[x] == x) fa[x] = l;
            if (fa[y] == y) fa[y] = l;
            x = pre[y];
        }
    }
    bool bfs(int r) {
        iota(all(fa), 0), fill(all(s), -1);
        while (!q.empty()) q.pop();
        q.push(r);
        s[r] = 0;
        while (!q.empty()) {

```

```

    int x = q.front(); q.pop();
    for (int u : g[x]) {
        if (s[u] == -1) {
            pre[u] = x, s[u] = 1;
            if (match[u] == n) {
                for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                    last = match[b], match[b] = a, match[a] = b;
                return true;
            }
            q.push(match[u]);
            s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
            int l = lca(u, x);
            blossom(x, u, l);
            blossom(u, x, l);
        }
    }
    return false;
}

int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) {
        if (match[x] == n) res += bfs(x);
    }
    return res;
}

void add_edge(int u, int v) {
    g[u].push_back(v), g[v].push_back(u);
}

Matching (int _n) : n(_n), tk(0), g(n), fa(n + 1),
    pre(n + 1, n), match(n + 1, n), s(n + 1, t(n)) {}
};

```

3.8 Min Cost Circulation [bd1e15]

```

struct MinCostCirculation { // 0-base
    struct Edge {
        ll from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    ll dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
        void try_edge(Edge &cur) {
            if (cur.cap > cur.flow) return ++cur.cap, void();
            BellmanFord(cur.to);
            if (dis[cur.from] + cur.cost < 0) {
                ++cur.flow, --G[cur.to][cur.rev].flow;
                for (int i = cur.from; past[i]; i = past[i]->from) {
                    auto &e = *past[i];
                    ++e.flow, --G[e.to][e.rev].flow;
                }
            }
            ++cur.cap;
        }
        void solve(int mxlg) {
            for (int b = mxlg; b >= 0; --b) {
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])
                        e.cap *= 2, e.flow *= 2;
                for (int i = 0; i < n; ++i)
                    for (auto &e : G[i])

```

```

                        if (e.fcap >> b & 1)
                            try_edge(e);
            }
        }
        void init(int _n) { n = _n;
            for (int i = 0; i < n; ++i) G[i].clear();
        }
        void add_edge(ll a, ll b, ll cap, ll cost) {
            G[a].pb(Edge{a, b, 0, cap, 0, cost, sz(G[b]) + (a == b)});
            G[b].pb(Edge{b, a, 0, 0, 0, -cost, sz(G[a]) - 1});
        }
    } mcmf; // O(VE * ELogC)
}

```

3.9 Weighted Blossom [dc42e4]

```

#define pb emplace_back
#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slack, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int _n) : n(_n), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
        match = st = pa = S = vis = slack;
        REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
    }
    int ED(edge e) {
        return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
    }
    void update_slack(int u, int x, int &s) {
        if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
    }
    void set_slack(int x) {
        slack[x] = 0;
        REP(u, 1, n)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x, slack[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q_push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(all(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + all(f), it = f.end() - pr);
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    } // 7bb859
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        REP(i, 0, int(z.size())-1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), all(z));
    }
    void augment(int u, int v) {
        for (;;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(v = xnv, u = st[pa[xnv]]);
        }
    }
    int lca(int u, int v) {
        static int t = 0; ++t;
        for (++t; u || v; swap(u, v)) if (u) {
            if (vis[u] == t) return u;
            vis[u] = t; u = st[match[u]];
            if (u) u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u, int o, int v) {
        int b = int(find(n + 1 + all(st), 0) - begin(st));

```



```

lab[b] = 0, S[b] = 0; match[b] = match[o];
vector<int> f = {o};
for (int x : {u, v}) {
    for (int y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y) = st[match[x]], q_push(y);
    reverse(1 + all(f));
}
flo[b] = f; set_st(b, b);
REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
REP(x, 1, n) flo_from[b][x] = 0;
for (int xs : flo[b]) {
    REP(x, 1, nx)
        if (g[b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]))
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    REP(x, 1, n)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
}
set_slack(b);
}
void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
        if (xs == -1) { xs = x; continue; }
        pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
        slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
    }
    for (int x : flo[b])
        if (x == xr) S[x] = 1, pa[x] = pa[b];
        else S[x] = -1, set_slack(x);
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    }
    else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
} // 82ea63
bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    REP(x, 1, nx) if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            REP(v, 1, n)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (ED(g[u][v]) != 0)
                        update_slack(u, st[v], slack[st[v]]);
                    else if (on_found_edge(g[u][v])) return true;
                }
        }
        int d = inf;
        REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
            d = min(d, lab[b] / 2);
        REP(x, 1, nx)
            if (int s = slack[x]; st[x] == x && s && S[x] <= 0)
                d = min(d, ED(g[s][x]) / (S[x] + 2));
        REP(u, 1, n)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {
                if (lab[u] <= d) return false;
                lab[u] -= d;
            }
        REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
            lab[b] += d * (2 - 4 * S[b]);
        REP(x, 1, nx)
            if (int s = slack[x]; st[x] == x &&
                s && st[s] != x && ED(g[s][x]) == 0)
                if (on_found_edge(g[s][x])) return true;
        REP(b, n + 1, nx)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)

```

```

        expand_blossom(b);
    }
    return false;
}
pair<ll, int> solve() {
    fill(all(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    REP(u, 1, n) lab[u] = w_max;
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void set_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w; } // c78909
};

```

3.10 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - Change the weight of each edge to $\mu(u) + \mu(v) - w(u, v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Let the maximum weight matching of the graph be x , the answer will be $\sum \mu(v) - x$.

4 Graph

4.1 Heavy-Light Decomposition [9ec77f]

```

struct HLD { // 0-based, remember to build
    int n, _id;
    vector<vector<int>> g;
    vector<int> dep, pa, tsz, ch, hd, id;
    void dfs(int v, int p) {
        dep[v] = ~p ? dep[p] + 1 : 0;
        pa[v] = p, tsz[v] = 1, ch[v] = -1;
    }
};

```

```

    for (int u : g[v]) if (u != p) {
        dfs(u, v);
        if (ch[v] == -1 || tsz[ch[v]] < tsz[u])
            ch[v] = u;
        tsz[v] += tsz[u];
    }
}
void hld(int v, int p, int h) {
    hd[v] = h, id[v] = _id++;
    if (~ch[v]) hld(ch[v], v, h);
    for (int u : g[v]) if (u != p && u != ch[v])
        hld(u, v, u);
}
vector<pii> query(int u, int v) {
    vector<pii> ans;
    while (hd[u] != hd[v]) {
        if (dep[hd[u]] > dep[hd[v]]) swap(u, v);
        ans.emplace_back(id[hd[v]], id[v] + 1);
        v = pa[hd[v]];
    }
    if (dep[u] > dep[v]) swap(u, v);
    ans.emplace_back(id[u], id[v] + 1);
    return ans;
}
void build() {
    for (int i = 0; i < n; ++i) if (id[i] == -1)
        dfs(i, -1, hld(i, -1, i));
}
void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u);
}
HLD (int _n) : n(_n), _id(0), g(n), dep(n), pa(n),
    tsz(n), ch(n), hd(n), id(n, -1) {}
};

```

4.2 Centroid Decomposition [28b80a]

```

struct CD { // 0-based, remember to build
    int n, lg; // pa, dep are centroid tree attributes
    vector<vector<int>> g, dis;
    vector<int> pa, tsz, dep, vis;
    void dfs1(int v, int p) {
        tsz[v] = 1;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs1(u, v), tsz[v] += tsz[u];
    }
    int dfs2(int v, int p, int _n) {
        for (int u : g[v])
            if (u != p && !vis[u] && tsz[u] > _n / 2)
                return dfs2(u, v, _n);
        return v;
    }
    void dfs3(int v, int p, int d) {
        dis[v][d] = ~p ? dis[p][d] + 1 : 0;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs3(u, v, d);
    }
    void cd(int v, int p, int d) {
        dfs1(v, -1), v = dfs2(v, -1, tsz[v]);
        vis[v] = true, pa[v] = p, dep[v] = d;
        dfs3(v, -1, d);
        for (int u : g[v]) if (!vis[u])
            cd(u, v, d + 1);
    }
    void build() { cd(0, -1, 0); }
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u);
    }
    CD (int _n) : n(_n), lg(_lg(n) + 1), g(n),
        dis(n, vector<int>(lg)), pa(n), tsz(n),
        dep(n), vis(n) {}
};

```

4.3 Edge BCC [cf5e55]

```

struct EBCC { // 0-based, remember to build
    int n, m, nbcc;
    vector<vector<pii>> g;
    vector<int> pa, low, dep, bcc_id, stk, is_bridge;
    void dfs(int v, int p, int f) {
        low[v] = dep[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (auto [u, e] : g[v]) {
            if (low[u] == -1)

```

```

                dfs(u, v, e), low[v] = min(low[v], low[u]);
            else if (e != f)
                low[v] = min(low[v], dep[u]);
        }
        if (low[v] == dep[v]) {
            if (~f) is_bridge[f] = true;
            int id = nbcc++, x;
            do {
                x = stk.back(), stk.pop_back();
                bcc_id[x] = id;
            } while (x != v);
        }
    }
    void build() {
        is_bridge.assign(m, 0);
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i, -1, -1);
    }
    void add_edge(int u, int v) {
        g[u].emplace_back(v, m), g[v].emplace_back(u, m++);
    }
    EBCC (int _n) : n(_n), m(0), nbcc(0), g(n), pa(n),
        low(n, -1), dep(n), bcc_id(n), stk() {}
};

```

4.4 Vertex BCC / Round Square Tree [3818e9]

```

struct BCC { // 0-based, remember to build
    int n, nbcc; // note for isolated point
    vector<vector<int>> g, _g; // id >= n: bcc
    vector<int> pa, dep, low, stk, pa2, dep2;
    void dfs(int v, int p) {
        dep[v] = low[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (int u : g[v]) if (u != p) {
            if (low[u] == -1) {
                dfs(u, v), low[v] = min(low[v], low[u]);
                if (low[u] >= dep[v]) {
                    int id = nbcc++, x;
                    do {
                        x = stk.back(), stk.pop_back();
                        _g[id + n].pb(x), _g[x].pb(id + n);
                    } while (x != u);
                    _g[id + n].pb(v), _g[v].pb(id + n);
                } else low[v] = min(low[v], dep[u]);
            }
        }
        bool is_cut(int x) { return (_g[x].size() != 1); }
        vector<int> bcc(int id) { return _g[id + n]; }
        int bcc_id(int u, int v) {
            return pa2[dep2[u] < dep2[v] ? v : u] - n;
        }
        void dfs2(int v, int p) {
            dep2[v] = ~p ? dep2[p] + 1 : 0, pa2[v] = p;
            for (int u : _g[v]) if (u != p) dfs2(u, v);
        }
    }
    void build() {
        low.assign(n, -1);
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i, -1, dfs2(i, -1));
    }
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u);
    }
    BCC (int _n) : n(_n), nbcc(0), g(n), _g(2 * n),
        pa(n), dep(n), low(n), stk(), pa2(n * 2),
        dep2(n * 2) {}
};

```

4.5 SCC [9bee8c]

```

struct SCC {
    int n, nscc, _id;
    vector<vector<int>> g;
    vector<int> dep, low, scc_id, stk;
    void dfs(int v) {
        dep[v] = low[v] = _id++, stk.pb(v);
        for (int u : g[v]) if (scc_id[u] == -1) {
            if (low[u] == -1) dfs(u);
            low[v] = min(low[v], low[u]);
        }
        if (low[v] == dep[v]) {
            int id = nscc++, x;

```



```

    do {
        x = stk.back(), stk.pop_back(), scc_id[x] = id;
    } while (x != v);
}
}
void build() {
    for (int i = 0; i < n; ++i) if (low[i] == -1)
        dfs(i);
}
void add_edge(int u, int v) { g[u].pb(v); }
SCC (int _n) : n(_n), nscc(0), _id(0), g(n), dep(n),
    low(n, -1), scc_id(n, -1), stk() {}
};

```

4.6 2SAT [938072]

```

struct SAT { // 0-based, need SCC
    int n; vector<pii> edge; vector<int> is;
    int rev(int x) { return x < n ? x + n : x - n; }
    void add_ifthen(int x, int y) {
        add_clause(rev(x), y); }
    void add_clause(int x, int y) {
        edge.emplace_back(rev(x), y);
        edge.emplace_back(rev(y), x); }
    bool solve() {
        // is[i] = true -> i, is[i] = false -> -i
        SCC scc(2 * n);
        for (auto [u, v] : edge) scc.add_edge(u, v);
        scc.build();
        for (int i = 0; i < n; ++i) {
            if (scc.scc_id[i] == scc.scc_id[i + n])
                return false;
            is[i] = scc.scc_id[i] < scc.scc_id[i + n];
        }
        return true;
    }
    SAT (int _n) : n(_n), edge(), is(n) {}
};

```

4.7 Virtual Tree [9e4a93]

```

// need lca, in, out
vector<pii> virtual_tree(vector<int> &v) {
    auto cmp = [&](int x, int y) { return in[x] < in[y]; };
    sort(all(v), cmp);
    int sz = (int)v.size();
    for (int i = 0; i + 1 < sz; ++i)
        v.pb(lca(v[i], v[i + 1]));
    sort(all(v), cmp);
    v.resize(unique(all(v)) - v.begin());
    vector<int> stk(1, v[0]);
    vector<pii> res;
    for (int i = 1; i < (int)v.size(); ++i) {
        int x = v[i];
        while (out[stk.back()] < out[x]) stk.pop_back();
        res.emplace_back(stk.back(), x), stk.pb(x);
    }
    return res;
}

```

4.8 Directed MST [d6cf86]

```

using D = int;
struct edge { int u, v; D w; };
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
    using T = pair<D, int>;
    using PQ = pair<priority_queue<T, vector<T>,
        greater<T>>, D>;
    auto push = [](PQ &pq, T v) {
        pq.first.emplace(v.first - pq.second, v.second);
    };
    auto top = [](const PQ &pq) -> T {
        auto r = pq.first.top();
        return {r.first + pq.second, r.second};
    };
    auto join = [&push, &top](PQ &a, PQ &b) {
        if (a.first.size() < b.first.size()) swap(a, b);
        while (!b.first.empty())
            push(a, top(b)), b.first.pop();
    };
    vector<PQ> h(n * 2);
    for (int i = 0; i < e.size(); ++i)

```

```

        push(h[e[i].v], {e[i].w, i});
    vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
        n * 2);
    iota(all(a), 0);
    auto o = [&](int x) { int y;
        for (y = x; a[y] != y; y = a[y]);
        for (int ox = x; x != y; ox = x)
            x = a[x], a[ox] = y;
        return y;
    };
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
            p]].u)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].second = -h[q].first.top().first;
                    join(h[pa[q]] = a[q] = p, h[q]);
                } while ((q = o(e[r[q]].u)) != p);
            }
            v[p] = i;
            while (!h[p].first.empty() && o(e[top(h[p]).
                second].u) == p)
                h[p].first.pop();
            r[p] = top(h[p]).second;
        }
    }
    vector<int> ans;
    for (int i = pc - 1; i >= 0; i--)
        if (i != root && v[i] != n) {
            for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
                pa[f]) v[f] = n;
            ans.pb(r[i]);
        }
    return ans;
}

```

4.9 Dominator Tree [49fc16]

```

struct DominatorTree {
    int n, id;
    vector<vector<int>> g, rg, bucket;
    vector<int> sd, dom, vis, rev, pa, rt, mn, res;
    // dom[s] = s, dom[v] = -1 if s -> v not exists
    int query(int v, int x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], 1);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sd[mn[v]] > sd[mn[rt[v]]])
            mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sd[id] = id, id++;
        for (int u : g[v]) {
            if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];
            rg[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : rg[i]) {
                sd[i] = min(sd[i], sd[query(u, 0)]);
            }
            if (i) bucket[sd[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, 0);
                dom[u] = sd[p] == i ? i : p;
            }
            if (i) rt[i] = pa[i];
        }
        fill(all(res), -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sd[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i)
            res[rev[i]] = rev[dom[i]];
    }
};

```

```

    res[s] = s;
    for (int i = 0; i < n; ++i) dom[i] = res[i];
}
void add_edge(int u, int v) { g[u].pb(v); }
Dominator_tree(int _n) : n(_n), id(0), g(n), rg(n),
    bucket(n), sdom(n), dom(n, -1), vis(n, -1),
    rev(n), pa(n), rt(n), mn(n), res(n) {}
};

```

4.10 Bipartite Edge Coloring [a22d96]

```

struct BipartiteEdgeColoring { // 1-based
    // returns edge coloring in adjacent matrix G
    int n, m;
    vector<vector<int>> col, G;
    int find_col(int x) {
        int c = 1;
        while (col[x][c]++) c++;
        return c;
    }
    void dfs(int v, int c1, int c2) {
        if (!col[v][c1]) return col[v][c2] = 0, void(0);
        int u = col[v][c1];
        dfs(u, c2, c1);
        col[v][c1] = 0, col[v][c2] = u, col[u][c2] = v;
    }
    void solve() {
        for (int i = 1; i <= n + m; ++i)
            for (int j = 1; j <= max(n, m); ++j)
                if (col[i][j])
                    G[i][col[i][j]] = G[col[i][j]][i] = j;
    } // u = left index, v = right index
    void add_edge(int u, int v) {
        int c1 = find_col(u), c2 = find_col(v + n);
        dfs(u, c2, c1);
        col[u][c2] = v + n, col[v + n][c2] = u;
    }
    BipartiteEdgeColoring(int _n, int _m) : n(_n),
        m(_m), col(n + m + 1, vector<int>(max(n, m) + 1)),
        G(n + m + 1, vector<int>(n + m + 1)) {}
};

```

4.11 Edge Coloring [60e200]

```

struct Vizing { // 1-based
    // returns edge coloring in adjacent matrix G
    int n;
    vector<vector<int>> C, G;
    vector<int> X, vst;
    vector<pii> E;
    void solve() {
        auto update = [&](int u)
        { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
            return p;
        };
        auto flip = [&](int u, int c1, int c2) {
            int p = C[u][c1];
            swap(C[u][c1], C[u][c2]);
            if (p) G[u][p] = G[p][u] = c2;
            if (!C[u][c1]) X[u] = c1;
            if (!C[u][c2]) X[u] = c2;
            return p;
        };
        fill(1 + all(X), 1);
        for (int t = 0; t < (int)E.size(); ++t) {
            auto [u, v0] = E[t];
            int v = v0, c0 = X[u], c = c0, d;
            vector<pii> L;
            fill(1 + all(vst), 0);
            while (!G[u][v0]) {
                L.emplace_back(v, d = X[v]);
                if (!C[v][c]) {
                    for (int a = sz(L) - 1; a >= 0; --a)
                        c = color(u, L[a].first, c);
                } else if (!C[u][d]) {

```

```

                    for (int a = sz(L) - 1; a >= 0; --a)
                        color(u, L[a].first, L[a].second);
                } else if (vst[d]) break;
                else vst[d] = 1, v = C[u][d];
            }
            if (!G[u][v0]) {
                for (; v; v = flip(v, c, d), swap(c, d));
                if (int a; C[u][c0]) {
                    for (a = sz(L) - 2;
                        a >= 0 && L[a].second != c; --a);
                    for (; a >= 0; --a)
                        color(u, L[a].first, L[a].second);
                } else --t;
            }
        }
        void add_edge(int u, int v) { E.emplace_back(u, v); }
        Vizing(int _n) : n(_n), C(n + 1, vector<int>(n + 1)),
            G(n + 1, vector<int>(n + 1)), X(n + 1), vst(n + 1) {}
};

```

4.12 Maximum Clique [f99a13]

```

struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void add_edge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0;
        int m = r.size();
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++)
            for (int p = cs[k]._Find_first(); p < N;
                p = cs[k]._Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
        bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<N> nmask = mask & a[p];
            for (int i : r)
                if (a[p][i]) nr.push_back(i);
            if (!nr.empty()) {
                if (l < 4) {
                    for (int i : nr)
                        d[i] = (a[i] & nmask).count();
                    sort(nr.begin(), nr.end(),
                        [&](int x, int y) { return d[x] > d[y]; });
                }
                csort(nr, nc), dfs(nr, nc, l + 1, nmask);
            } else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), q--;
        }
    }
    int solve(bitset<N> mask = bitset<N>(),
        string(N, '1')) { // vertex mask
        vector<int> r, c;
        ans = q = 0;
        for (int i = 0; i < n; i++)
            if (mask[i]) r.push_back(i);
        for (int i = 0; i < n; i++)
            d[i] = (a[i] & mask).count();
    }
};

```

```

    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
};

```

5 String

5.1 Aho-Corasick Automaton [d208c9]

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    // vector<int> g[N];
    int cnt[N];
    AC () {sz = 0, extend();}
    void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
        return ch[u][v];
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) now = nxt(now, c - 'a');
        cnt[now]++;
        return now;
    }
    void build_fail() {
        queue<int> q;
        for (int i = 0; i < 26; ++i) if (ch[0][i]) {
            q.push(ch[0][i]);
            // g[0].push_back(ch[0][i]);
            to[0][i] = ch[0][i];
        }
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int j = 0; j < 26; ++j) {
                to[v][j] = ch[v][j] ? ch[v][j] : to[fail[v]][j];
            }
            for (int i = 0; i < 26; ++i) if (ch[v][i]) {
                int u = ch[v][i], k = fail[v];
                while (k && !ch[k][i]) k = fail[k];
                if (ch[k][i]) k = ch[k][i];
                fail[u] = k, cnt[u] += cnt[k];
                // g[k].push_back(u);
                q.push(u);
            }
        }
    }
    // int match(string &s) {
    //     int now = 0, ans = 0;
    //     for (char c : s) {
    //         now = to[now][c - 'a'];
    //         ans += cnt[now];
    //     }
    //     return ans;
    // }
};

```

5.2 KMP Algorithm [f379fc]

```

vector<int> build_fail(string s) {
    vector<int> f(s.size() + 1, 0);
    int k = 0;
    for (int i = 1; i < (int)s.size(); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}
int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < (int)s.size(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == (int)t.size()) ans++, k = f[k];
    }
    return ans;
}

```

5.3 Z Algorithm [7d5c7c]

```

vector<int> buildZ(string s) {
    int n = (int)s.size(), l = 0, r = 0;
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]]) {
            l = i, r = i + Z[i], Z[i]++;
        }
    }
    return Z;
}

```

5.4 Manacher [c18d8b]

```

// return value only consider string tmp, not s
vector<int> manacher(string tmp) {
    string s = "&";
    for (char c : tmp) s.pb(c), s.pb('%');
    int l = 0, r = 0, n = (int)s.size();
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
        while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
        if (Z[i] + i > r) l = i, r = Z[i] + i;
    }
    for (int i = 0; i < n; ++i) {
        Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
    }
    return Z;
}

```

5.5 Suffix Array [ba4998]

```

int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.size();
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]] = s[i]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k)
            y[p++] = sa[i] - k;
        for (int i = n - 1; ~i; --i)
            sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!(x[a] == x[b] && a + k < n && b + k < n && x[a + k] == x[b + k])) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}
void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = query_lcp_min[rk[i] + 1, rk[j] + 1)
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i] == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}

```

5.6 SAIS [fbc167]

```

int sa[N << 1], rk[N], lcp[N];
// string ASCII value need > 0
namespace sfx {
bool _t[N << 1];
int _s[N << 1], _c[N << 1], x[N], _p[N], _q[N << 1];
void pre(int *sa, int *c, int n, int z) {
    fill_n(sa, n, 0), copy_n(c, z, x);
}
void induce(int *sa, int *c, int *s, bool *t, int n,
            int z) {
    copy_n(c, z - 1, x + 1);
    for (int i = 0; i < n; ++i)
        if (sa[i] && !t[sa[i] - 1])
            sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    for (int i = n - 1; i >= 0; --i)
        if (sa[i] && t[sa[i] - 1])
            sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int
          *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        if (s[i] == s[i + 1]) t[i] = t[i + 1];
        else t[i] = s[i] < s[i + 1];
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || !equal(s + sa[i], s + p[q[
                sa[i]] + 1], s + last);
            ns[q[last = sa[i]]] = nmzx += neq;
        }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx +
        1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
void buildSA(string s) {
    int n = s.length();
    for (int i = 0; i < n; ++i) _s[i] = s[i];
    _s[n] = 0;
    sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
    for (int i = 1; i <= n; ++i) sa[i - 1] = sa[i];
} // buildLCP()...
}

```

5.7 Suffix Automaton [7228e9]

```

struct SAM {
    int ch[N][26], len[N], link[N], pos[N], cnt[N], sz;
    // node -> strings with the same endpos set
    // length in range [len(link) + 1, len]
    // node's endpos set -> pos in the subtree of node
    // link -> longest suffix with different endpos set
    // len -> longest suffix
    // pos -> end position
    // cnt -> size of endpos set
    SAM () {len[0] = 0, link[0] = -1, pos[0] = 0, cnt[0]
        = 0, sz = 1;}
    void build(string s) {
        int last = 0;
        for (int i = 0; i < s.length(); ++i) {
            char c = s[i];
            int cur = sz++;
            len[cur] = len[last] + 1, pos[cur] = i + 1;
            int p = last;
            while (~p && !ch[p][c - 'a'])

```

```

                ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) link[cur] = 0;
            else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q];
                    pos[nxt] = 0;
                    for (int j = 0; j < 26; ++j)
                        ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q)
                        ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
            cnt[cur]++;
            last = cur;
        }
        // vector <int> p(sz);
        // iota(all(p), 0);
        // sort(all(p),
        //      [](int i, int j) {return len[i] > len[j];});
        // for (int i = 0; i < sz; ++i)
        //     cnt[link[p[i]]] += cnt[p[i]];
    }
} sam;

```

5.8 Minimum Rotation [aa3a61]

```

string rotate(const string &s) {
    int n = (int)s.size(), i = 0, j = 1;
    string t = s + s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

5.9 Palindrome Tree [0518a5]

```

struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM () {}
    void init(string s) {
        sz = 0, extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst] - 1] != s[i])
                lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
                int now = fail[lst];
                while (s[i - len[now] - 1] != s[i])
                    now = fail[now];
                fail[idx] = ch[now][s[i] - 'a'];
                ch[lst][s[i] - 'a'] = idx;
            }
            lst = ch[lst][s[i] - 'a'], cnt[lst]++;
        }
    }
    void build_count() {
        for (int i = sz - 1; i > 1; --i)
            cnt[fail[i]] += cnt[i];
    }
    int extend() {
        fill(ch[sz], ch[sz] + 26, 0), sz++;
        return sz - 1;
    }
};

```

5.10 Lyndon Factorization [a9eeb0]

```
// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
vector<string> duval(const string &s) {
    vector<string> ans;
    for (int n = (int)s.size(), i = 0, j, k; i < n; ) {
        for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
            k = (s[k] < s[j] ? i : k + 1);
        for (; i <= k; i += j - k)
            ans.pb(s.substr(i, j - k)); // s.substr(L, len)
    }
    return ans;
}
```

5.11 Main Lorentz [f3da14]

```
int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, len.
// substr([l, r], len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l);
    string sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i)
        to_right[i] = Z[r - m + 1 + i - 1];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = 1; i < m; ++i)
        to_left[i] = Z[m - i - 1];
    reverse(all(sl));
    for (int i = 1; i + 1 < m; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = m - i - 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2), tr = min(len - 1, k1);
        if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
    }
    Z = buildZ(sr);
    for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
    reverse(all(sl)), reverse(all(sr));
    Z = buildZ(sl + "#" + sr);
    for (int i = m; i < r; ++i)
        to_left[i] = Z[m - 1 + 1 + r - i - 1];
    reverse(all(sl)), reverse(all(sr));
    for (int i = m; i + 1 < r; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = i - m + 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2, 1), tr = min(len - 1, k1);
        if (tl <= tr)
            rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
    }
    Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i)
        if (Z[r - m + 1 + i - 1] >= m - i)
            rep.pb({i, i, m - i});
}
```

6 Math

6.1 Miller Rabin / Pollard Rho [6c9c33]

```
ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long
    double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022};
ll Pow(ll a, ll b, ll n) {
    ll res = 1;
    for (; b >>= 1, a = mul(a, a, n))
        if (b & 1) res = mul(res, a, n);
    return res;
}
bool check(ll a, ll d, int s, ll n) {
    a = Pow(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
}
```

```
return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(ll n) {
    if (n == 1) return;
    if (IsPrime(n)) return ++res[n], void(0);
    ll d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
}
```

6.2 Ext GCD [a4b22d]

```
//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    if (b == 0) return {1, 0};
    auto [y, x] = extgcd(b, a % b);
    return pair<ll, ll>(x, y - (a / b) * x);
}
```

6.3 Chinese Remainder Theorem [90d2ce]

```
pair<ll, ll> CRT(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return make_pair(-1, -1); // no sol
    m1 /= g, m2 /= g;
    pair<ll, ll> p = extgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return make_pair((res % lcm + lcm) % lcm, lcm);
}
```

6.4 PiCount [1db46f]

```
const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x) {
    for (int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for (int i = 2; i * i < x; ++i) if (isp[i])
        for (int j = i * i; j < x; j += i) isp[j] = false;
    for (int i = 2; i < x; ++i) if (isp[i]) primes.pb(i);
}
void init() {
    sieve(V);
    small_pi[0] = 0;
    for (int i = 1; i < V; ++i)
        small_pi[i] = small_pi[i - 1] + isp[i];
    for (int i = 0; i < M; ++i) dp[0][i] = i;
    for (int i = 1; i < N; ++i) for (int j = 0; j < M; ++j)
```



```

    dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i - 1]];
}
ll phi(ll n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(1ll * primes[a - 1] * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
ll PiCount(ll n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
        small_pi[y];
    ll res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n /
        primes[a]) - PiCount(primes[a]) + 1, 0ll);
    return res;
}

```

6.5 Linear Function Mod Min [5552e3]

```

ll topos(ll x, ll m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    Log m)
ll min_rem(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    for (ll g = __gcd(a, m); g > 1; ) return g * min_rem(
        n / g, m / g, a / g, b / g) + (b % g);
    for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
        = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            ll lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1],
//also return min x to get the value. O(Log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
        % (m / g);
    return {mn, x};
}

```

6.6 Floor Sum [49de67]

```

// sum_{n=1}^n floor((a * i + b) / m) in Log(n + m + a
    + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

6.7 Quadratic Residue [51ec55]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);

```

```

        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1ll * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (p + 1) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
            g1 = (g0 * f1 + g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
        f1 = (2 * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.8 Discrete Log [8f7f93]

```

ll DiscreteLog(ll a, ll b, ll m) { // a^x = b (mod m)
    const int B = 35000;
    ll k = 1 % m, ans = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k) return ans;
        if (b % g) return -1;
        b /= g, m /= g, ans++, k = (k * a / g) % m;
    }
    if (b == k) return ans;
    unordered_map<ll, int> m1;
    ll tot = 1;
    for (int i = 0; i < B; ++i)
        m1[tot * b % m] = i, tot = tot * a % m;
    ll cur = k * tot % m;
    for (int i = 1; i <= B; ++i, cur = cur * tot % m)
        if (m1.count(cur)) return i * B - m1[cur] + ans;
    return -1;
}

```

6.9 Factorial without Prime Factor [c324f3]

```

// O(p^k + Log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
}
// (n! without factor p) % p^k

```

6.10 Berlekamp Massey [f867ec]

```

// need add, sub, mul
vector<int> BerlekampMassey(vector<int> a) {
    // find min |c| such that a_n = sum c_j * a_{n - j -
        1}, 0-based
    // O(N^2), if |c| = k, |a| >= 2k sure correct
    auto f = [&](vector<int> v, ll c) {
        for (int &x : v) x = mul(x, c);
        return v;
    };
    vector<int> c, best;
    int pos = 0, n = (int)a.size();

```

```

for (int i = 0; i < n; ++i) {
    int error = a[i];
    for (int j = 0; j < (int)c.size(); ++j)
        error = sub(error, mul(c[j], a[i - 1 - j]));
    if (error == 0) continue;
    int inv = Pow(error, mod - 2);
    if (c.empty()) {
        c.resize(i + 1, pos = i, best.pb(inv));
    } else {
        vector<int> fix = f(best, error);
        fix.insert(fix.begin(), i - pos - 1, 0);
        if (fix.size() >= c.size()) {
            best = f(c, sub(0, inv));
            best.insert(best.begin(), inv);
            pos = i, c.resize(fix.size());
        }
        for (int j = 0; j < (int)fix.size(); ++j)
            c[j] = add(c[j], fix[j]);
    }
}
return c;
}

```

6.11 Simplex [b68fb9]

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
    bool ls(T a, T b) {return a < b && !eq(a, b);}
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) a[i][j] = 0;
        for (int i = 0; i < m; ++i) b[i] = 0;
        for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
    }
    void pivot(int x, int y) {
        swap(Left[x], Down[y]);
        T k = a[x][y]; a[x][y] = 1;
        vector<int> nz;
        for (int i = 0; i < n; ++i) {
            a[x][i] /= k;
            if (!eq(a[x][i], 0)) nz.push_back(i);
        }
        b[x] /= k;
        for (int i = 0; i < m; ++i) {
            if (i == x || eq(a[i][y], 0)) continue;
            k = a[i][y], a[i][y] = 0;
            b[i] -= k * b[x];
            for (int j : nz) a[i][j] -= k * a[x][j];
        }
        if (eq(c[y], 0)) return;
        k = c[y], c[y] = 0, v += k * b[x];
        for (int i : nz) c[i] -= k * a[x][i];
    }
    // 0: found solution, 1: no feasible solution, 2:
    // unbounded
    int solve() {
        for (int i = 0; i < n; ++i) Down[i] = i;
        for (int i = 0; i < m; ++i) Left[i] = n + i;
        while (true) {
            int x = -1, y = -1;
            for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
            if (x == -1) break;
            for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
            if (y == -1) return 1;
            pivot(x, y);
        }
        while (true) {
            int x = -1, y = -1;
            for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
            if (y == -1) break;

```

```

        for (int i = 0; i < m; ++i)
            if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
        if (x == -1) return 2;
        pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n)
        sol[Left[i]] = b[i];
    return 0;
}
};

```

6.12 Euclidean

$$m = \lfloor \frac{an+b}{c} \rfloor$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

6.13 Linear Programming Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.
 Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.
 \bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.14 Theorem

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Erdős-Gallai Theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

- Burnside's Lemma

Let X be a set and G be a group that acts on X . For $g \in G$, denote by X^g the elements fixed by g :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called bigraphic if there is a labeled simple bipartite graph such that a and b is the degree sequence of this bipartite graph.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called digraphic if there is a labeled simple directed graph such that each vertex v_i has indegree a_i and outdegree b_i .

- Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume = $\pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
- Area = $2\pi r h = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

6.15 Estimation

n	2	3	4	5	6	7	8	9	20	30	40	50	100		
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8		
n	100	1e3	1e6				1e9		1e12		1e15		1e18		
$d(i)$	12	32	240				1344		6720		26880		103680		
arg	60	840	720720	735134400	963761198400	866421317361600	897612484786617600								
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8
n	2	3	4	5	6	7	8	9	10	11	12	13			
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7			

6.16 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

7 Polynomial

7.1 Number Theoretic Transform [536cc5]

```
// mul, add, sub, Pow
struct NTT {
    int w[N];
    NTT() {
        int dw = Pow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i)
            w[i] = mul(w[i - 1], dw);
    }
    void operator()(vector<int>& a, bool inv = false) {
        // 0 <= a[i] < P
        int x = 0, n = a.size();
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (x ^ k) < k; k >>= 1);
            if (j < x) swap(a[x], a[j]);
        }
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    int tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            int invn = Pow(n, mod - 2);
            for (int i = 0; i < n; ++i)
                a[i] = mul(a[i], invn);
        }
    }
} ntt;
```

7.2 Fast Fourier Transform [6f906d]

```
using T = complex<double>;
const double PI = acos(-1);
struct FFT {
    T w[N];
    FFT() {
        T dw = {cos(2 * PI / N), sin(2 * PI / N)};
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
    }
    void operator()(vector<T>& a, bool inv = false) {
        // see NTT, replace ll with T
        if (inv) {
            reverse(a.begin() + 1, a.end());
            T invn = 1.0 / n;
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
        }
    }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	9223372036737335297	3

7.4 Polynomial Operations [9be4e4]

```
typedef vector<int> Poly;
Poly Mul(Poly a, Poly b, int bound = N) { // d02e42
    int m = a.size() + b.size() - 1, n = 1;
    while (n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    Poly out(n);
```

```

    for (int i = 0; i < n; ++i) out[i] = mul(a[i], b[i]);
    ntt(out, true), out.resize(min(m, bound));
    return out;
}
Poly Inverse(Poly a) { // b137d5
    // O(NLogN), a[0] != 0
    int n = a.size();
    Poly res(1, Pow(a[0], mod - 2));
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly v1(a.begin(), a.begin() + m * 2), v2 = res;
        v1.resize(m * 4), v2.resize(m * 4);
        ntt(v1), ntt(v2);
        for (int i = 0; i < m * 4; ++i)
            v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
        ntt(v1, true);
        res.resize(m * 2);
        for (int i = 0; i < m; ++i)
            res[i] = add(res[i], res[i]);
        for (int i = 0; i < m * 2; ++i)
            res[i] = sub(res[i], v1[i]);
    }
    res.resize(n);
    return res;
}
pair <Poly, Poly> Divide(Poly a, Poly b) {
    // a = bQ + R, O(NLogN), b.back() != 0
    int n = a.size(), m = b.size(), k = n - m + 1;
    if (n < m) return {{0}, a};
    Poly ra = a, rb = b;
    reverse(all(ra)), ra.resize(k);
    reverse(all(rb)), rb.resize(k);
    Poly Q = Mul(ra, Inverse(rb), k);
    reverse(all(Q));
    Poly res = Mul(b, Q), R(m - 1);
    for (int i = 0; i < m - 1; ++i)
        R[i] = sub(a[i], res[i]);
    return {Q, R};
}
Poly SqrtImpl(Poly a) { // a642f6
    if (a.empty()) return {0};
    int z = QuadraticResidue(a[0], mod), n = a.size();
    if (z == -1) return {-1};
    Poly q(1, z);
    const int inv2 = (mod + 1) / 2;
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        q.resize(m * 2);
        Poly f2 = Mul(q, q, m * 2);
        for (int i = 0; i < m * 2; ++i)
            f2[i] = sub(f2[i], a[i]);
        f2 = Mul(f2, Inverse(q), m * 2);
        for (int i = 0; i < m * 2; ++i)
            q[i] = sub(q[i], mul(f2[i], inv2));
    }
    q.resize(n);
    return q;
}
Poly Sqrt(Poly a) { // 0dae9c
    // O(NLogN), return {-1} if not exists
    int n = a.size(), m = 0;
    while (m < n && a[m] == 0) m++;
    if (m == n) return Poly(n);
    if (m & 1) return {-1};
    Poly s = SqrtImpl(Poly(a.begin() + m, a.end()));
    if (s[0] == -1) return {-1};
    Poly res(n);
    for (int i = 0; i < s.size(); ++i)
        res[i + m / 2] = s[i];
    return res;
}
Poly Derivative(Poly a) { // 26f29b
    int n = a.size();
    Poly res(n - 1);
    for (int i = 0; i < n - 1; ++i)
        res[i] = mul(a[i + 1], i + 1);
    return res;
}
Poly Integral(Poly a) { // f18ba1
    int n = a.size();
    Poly res(n + 1);
    for (int i = 0; i < n; ++i)
        res[i + 1] = mul(a[i], Pow(i + 1, mod - 2));
    return res;
}
Poly Ln(Poly a) { // 0c1381
    // O(NLogN), a[0] = 1
    int n = a.size();
    if (n == 1) return {0};
    Poly d = Derivative(a);
    a.pop_back();
    return Integral(Mul(d, Inverse(a), n - 1));
}
Poly Exp(Poly a) { // d2b129
    // O(NLogN), a[0] = 0
    int n = a.size();
    Poly q(1, 1);
    a[0] = add(a[0], 1);
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly g(a.begin(), a.begin() + m * 2), h(all(q));
        h.resize(m * 2), h = Ln(h);
        for (int i = 0; i < m * 2; ++i)
            g[i] = sub(g[i], h[i]);
        q = Mul(g, q, m * 2);
    }
    q.resize(n);
    return q;
}
Poly PolyPow(Poly a, ll k) { // d50135
    int n = a.size(), m = 0;
    Poly ans(n, 0);
    while (m < n && a[m] == 0) m++;
    if (k && m && (k >= n || k * m >= n)) return ans;
    if (m == n) return ans[0] = 1, ans;
    int lead = m * k;
    Poly b(a.begin() + m, a.end());
    int base = Pow(b[0], k), inv = Pow(b[0], mod - 2);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], inv);
    b = Ln(b);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], k % mod);
    b = Exp(b);
    for (int i = lead; i < n; ++i)
        ans[i] = mul(b[i - lead], base);
    return ans;
}
vector <int> Evaluate(Poly a, vector <int> x) {
    if (x.empty()) return {}; // e28f67
    int n = x.size();
    vector <Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector <Poly> down(n * 2);
    down[1] = Divide(a, up[1]).second;
    for (int i = 2; i < n * 2; ++i)
        down[i] = Divide(down[i >> 1], up[i]).second;
    Poly y(n);
    for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
    return y;
}
Poly Interpolate(vector <int> x, vector <int> y) {
    int n = x.size(); // 743f56
    vector <Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    Poly a = Evaluate(Derivative(up[1]), x);
    for (int i = 0; i < n; ++i)
        a[i] = mul(y[i], Pow(a[i], mod - 2));
    vector <Poly> down(n * 2);
    for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
    for (int i = n - 1; i > 0; --i) {
        Poly lhs = Mul(down[i * 2], up[i * 2 + 1]);
        Poly rhs = Mul(down[i * 2 + 1], up[i * 2]);
        down[i].resize(lhs.size());
        for (int j = 0; j < lhs.size(); ++j)
            down[i][j] = add(lhs[j], rhs[j]);
    }
    return down[1];
}

```

```

}
Poly TaylorShift(Poly a, int c) { // b59bef
    // return sum a_i(x + c)^i;
    // fac[i] = i!, facp[i] = inv(i!)
    int n = a.size();
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
    reverse(all(a));
    Poly b(n);
    int w = 1;
    for (int i = 0; i < n; ++i)
        b[i] = mul(facp[i], w), w = mul(w, c);
    a = Mul(a, b, n), reverse(all(a));
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    return a;
}
vector<int> SamplingShift(vector<int> a, int c, int m){
    // given f(0), f(1), ..., f(n - 1)
    // return f(c), f(c + 1), ..., f(c + m - 1)
    int n = a.size(); // 4d649d
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    Poly b(n);
    for (int i = 0; i < n; ++i) {
        b[i] = facp[i];
        if (i & 1) b[i] = sub(0, b[i]);
    }
    a = Mul(a, b, n);
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
    reverse(all(a));
    int w = 1;
    for (int i = 0; i < n; ++i)
        b[i] = mul(facp[i], w), w = mul(w, sub(c, i));
    a = Mul(a, b, n);
    reverse(all(a));
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    a.resize(m), b.resize(m);
    for (int i = 0; i < m; ++i) b[i] = facp[i];
    a = Mul(a, b, m);
    for (int i = 0; i < m; ++i) a[i] = mul(a[i], fac[i]);
    return a;
}

```

7.5 Fast Linear Recursion [3f8e4e]

```

int FastLinearRecursion(vector<int> a, vector<int> c,
    ll k) {
    // a_n = sigma c_j * a_{n - j - 1}, 0-based
    // O(NLogNLogK), |a| = |c|
    int n = a.size();
    if (k < n) return a[k];
    vector<int> base(n + 1, 1);
    for (int i = 0; i < n; ++i)
        base[i] = sub(0, c[n - i - 1]);
    vector<int> poly(n);
    (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
    auto calc = [&](vector<int> p1, vector<int> p2) {
        // O(n^2) brute force or O(n log n) NTT
        return Divide(Mul(p1, p2), base).second;
    };
    vector<int> res(n, 0); res[0] = 1;
    for (; k; k >>= 1, poly = calc(poly, poly)) {
        if (k & 1) res = calc(res, poly);
    }
    int ans = 0;
    for (int i = 0; i < n; ++i)
        ans = add(ans, mul(res[i], a[i]));
    return ans;
}

```

7.6 Fast Walsh Transform

```

void fwt(vector<int> &a, bool inv = false) {
    // and : x += y * (1, -1)
    // or  : y += x * (1, -1)
    // xor : x = (x + y) * (1, 1/2)
    //      y = (x - y) * (1, 1/2)
    int n = __lg(a.size());
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
            int x = a[j ^ (1 << i)], y = a[j];
            // do something
        }
    }
}

```

```

}
vector<int> subs_conv(vector<int> a, vector<int> b) {
    // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
    int n = __lg(a.size());
    vector ha(n + 1, vector<int>(1 << n));
    vector hb(n + 1, vector<int>(1 << n));
    vector c(n + 1, vector<int>(1 << n));
    for (int i = 0; i < 1 << n; ++i) {
        ha[__builtin_popcount(i)][i] = a[i];
        hb[__builtin_popcount(i)][i] = b[i];
    }
    for (int i = 0; i <= n; ++i)
        or_fwt(ha[i]), or_fwt(hb[i]);
    for (int i = 0; i <= n; ++i)
        for (int j = 0; i + j <= n; ++j)
            for (int k = 0; k < 1 << n; ++k)
                c[i + j][k] = add(c[i + j][k],
                    mul(ha[i][k], hb[j][k]));
    for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
    vector<int> ans(1 << n);
    for (int i = 0; i < 1 << n; ++i)
        ans[i] = c[__builtin_popcount(i)][i];
    return ans;
}

```

8 Geometry

8.1 Basic

```

template<typename T> struct P {};
using Pt = P<double>;
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 &&
    sign((c - a) * (c - b)) <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
int cmp(Pt a, Pt b)
{ return pos(a) == pos(b) ? sign(a ^ b) > 0 :
    pos(a) < pos(b); }
double area(Pt a, Pt b, Pt c)
{ return fabs((a - b) ^ (a - c)) / 2; }
double angle(Pt a, Pt b)
{ return normalize(atan2(b.y - a.y, b.x - a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) { // CCW
    double c = cos(o), s = sin(o);
    return Pt(c * a.x - s * a.y, s * a.x + c * a.y);
}
Pt perp(Pt a) { return Pt(-a.y, a.x); }
Pt proj_vec(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vec(a, b, c) + a;
}

```

8.2 SVG Writer

```

#ifndef ABS
class SVG { // SVG("test.svg", 0, 0, 10, 10)
    void p(string_view s) { o << s; }
    void p(string_view s, auto v, auto... vs) {
        auto i = s.find('$');
        o << s.substr(0, i) << v, p(s.substr(i + 1), vs...)
            ;
    }
    ofstream o; string c = "red";
public:
    SVG(auto f, auto x1, auto y1, auto x2, auto y2) : o(f) {
        p("<svg xmlns='http://www.w3.org/2000/svg' "
            "viewBox='$ $ $ $'>\n"
            "<style>{*stroke-width:0.5%;}</style>\n",
            x1, -y2, x2 - x1, y2 - y1);
    }
    ~SVG() { p("</svg>\n"); }
    void color(string nc) { c = nc; }
    void line(auto x1, auto y1, auto x2, auto y2) {

```



```

    p("<Line x1='$' y1='$' x2='$' y2='$' stroke='$' />\n",
      x1, -y1, x2, -y2, c); }
void circle(auto x, auto y, auto r) {
  p("<circle cx='$' cy='$' r='$' stroke='$' "
    "fill='none' />\n", x, -y, r, c); }
void text(auto x, auto y, string s, int w = 12) {
  p("<text x='$' y='$' font-size='$px'>${s}</text>\n",
    x, -y, w, s); }
}; // write wrapper for complex if use complex
#else
struct SVG { SVG(auto ...) {} }; // you know how to
#endif

```

8.3 Heart [043c0d]

```

Pt circenter(Pt p0, Pt p1, Pt p2) {
  // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
  double m = 2. * (x1 * y2 - y1 * x2);
  Pt center(0, 0);
  center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
    y1 - y2)) / m;
  center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
    y2 * y2) / m;
  return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) {
  // radius = area / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
    - p2);
  double s = a + b + c;
  return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3)
{ return (p1 + p2 + p3) / 3; }
Pt orthocenter(Pt p1, Pt p2, Pt p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
  p3) * 2; }

```

8.4 External Bisector [caf92]

```

Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
  Pt L1 = p2 - p1, L2 = p3 - p1;
  L2 = L2 * abs(L1) / abs(L2);
  return L1 + L2;
}

```

8.5 Intersection of Segments [e59919]

```

Pt LinesInter(Line a, Line b) {
  double abc = (a.b - a.a) ^ (b.a - a.a);
  double abd = (a.b - a.a) ^ (b.b - a.a);
  if (sign(abc - abd) == 0) return b.b; // no inter
  return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
  if (btw(a.a, a.b, b.a)) return {b.a};
  if (btw(a.a, a.b, b.b)) return {b.b};
  if (btw(b.a, b.b, a.a)) return {a.a};
  if (btw(b.a, b.b, a.b)) return {a.b};
  if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 &&
    ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
    return {LinesInter(a, b)};
  return {};
}

```

8.6 Intersection of Circle and Line [75bb3e]

```

vector<Pt> CircleLineInter(Cir c, Line l) {
  Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
    ) / abs2(l.b - l.a);
  double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
    - s * s / abs2(l.b - l.a);
  if (sign(h2) == -1) return {};
  if (sign(h2) == 0) return {p};
  Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
  return {p - h, p + h};
}

```

8.7 Intersection of Circles [373889]

```

vector<Pt> CirclesInter(Cir c1, Cir c2) {
  double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
  if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
    + c2.r) return {};
  Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
    c2.r - c1.r * c1.r) / (2 * d2));
  double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
    * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
  Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2
    * d2);
  if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
  return {u + v, u - v};
}

```

8.8 Intersection of Polygon and Circle [e005c9]

```

double _area(Pt pa, Pt pb, double r) {
  if (abs(pa) < abs(pb)) swap(pa, pb);
  if (abs(pb) < eps) return 0;
  double S, h, theta;
  double a = abs(pb), b = abs(pa), c = abs(pb - pa);
  double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
  double cosC = (pa * pb) / a / b, C = acos(cosC);
  if (a > r) {
    S = (C / 2) * r * r;
    h = a * b * sin(C) / c;
    if (h < r && B < pi / 2) S -= (acos(h / r) * r * r
      - h * sqrt(r * r - h * h));
  } else if (b > r) {
    theta = pi - B - asin(sin(B) / r * a);
    S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
      * r;
  } else S = 0.5 * sin(C) * a * b;
  return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
  ) {
  double S = 0; int n = poly.size();
  for (int i = 0; i < n; ++i)
    S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
      ori(O, poly[i], poly[(i + 1) % n]);
  return fabs(S);
}

```

8.9 Tangent Lines of Polygon and Point [b569e5]

```

/* The point should be strictly out of hull
  return arbitrary point on the tangent line */
pii get_tangent(vector<Pt> &C, Pt p) {
  auto gao = [&](int s) {
    return cyc_tsearch(C.size(), [&](int x, int y)
      { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

8.10 Tangent Lines of Circle and Point [15bf9b]

```

vector<Line> tangent(Cir c, Pt p) {
  vector<Line> z;
  double d = abs(p - c.o);
  if (sign(d - c.r) == 0) {
    Pt i = rot(p - c.o, pi / 2);
    z.push_back({p, p + i});
  } else if (d > c.r) {
    double o = acos(c.r / d);
    Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
      i, -o) * c.r;
    z.push_back({c.o + j, p});
    z.push_back({c.o + k, p});
  }
  return z;
}

```

8.11 Tangent Lines of Circles [4bf589]

```

vector<Line> tangent(Cir c1, Cir c2, int sign1) {
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.o - c2.o);

```

```

if (sign(d_sq) == 0) return ret;
double d = sqrt(d_sq);
Pt v = (c2.o - c1.o) / d;
double c = (c1.r - sign1 * c2.r) / d;
if (c * c > 1) return ret;
double h = sqrt(max(0.0, 1.0 - c * c));
for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
        sign2 * h * v.x);
    Pt p1 = c1.o + n * c1.r;
    Pt p2 = c2.o + n * (c2.r * sign1);
    if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
        0)
        p2 = p1 + perp(c2.o - c1.o);
    ret.pb({p1, p2});
}
return ret;
}

```

8.12 Point In Convex [771a90]

```

bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
    // only works when no three points are collinear
    int a = 1, b = int(C.size()) - 1, r = !strict;
    if (C.size() == 0) return false;
    if (C.size() < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r) return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}

```

8.13 Point In Circle [960672]

```

// return p4 is strictly in circumcircle of tri(p1,p2,
    p3)
ll sqr(ll x) { return x * x; }
bool in_cc(const Pt &p1, const Pt &p2, const Pt &p3,
    const Pt &p4) {
    ll u11 = p1.x - p4.x; ll u12 = p1.y - p4.y;
    ll u21 = p2.x - p4.x; ll u22 = p2.y - p4.y;
    ll u31 = p3.x - p4.x; ll u32 = p3.y - p4.y;
    ll u13 = sqr(p1.x) - sqr(p4.x) + sqr(p1.y) - sqr(p4.y);
    ll u23 = sqr(p2.x) - sqr(p4.x) + sqr(p2.y) - sqr(p4.y);
    ll u33 = sqr(p3.x) - sqr(p4.x) + sqr(p3.y) - sqr(p4.y);
    __int128 det = (__int128)-u13 * u22 * u31 + (__int128)
        u12 * u23 * u31 + (__int128)u13 * u21 * u32 - (
            __int128)u11 * u23 * u32 - (__int128)u12 * u21 *
            u33 + (__int128)u11 * u22 * u33;
    return det > 0;
}

```

8.14 Point Segment Distance [651335]

```

double PointSegDist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
        * (p - q1)) >= 0)
        return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

8.15 Convex Hull [eae9b2]

```

vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b)
        {return a.x == b.x ? a.y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2],
                ans.back(), pt[i]) <= 0) ans.pop_back();
            ans.pb(pt[i]);
        }
    }
}

```

```

}
reverse(all(pt));
}
if (ans.size() > 1) ans.pop_back();
return ans;
}

```

8.16 Minimum Enclosing Circle [1f5028]

```

Cir min_enclosing(vector<Pt> &p) {
    random_shuffle(all(p));
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < p.size(); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i], r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}

```

8.17 Union of Circles [53b8f9]

```

vector<pair<double, double>> CoverSegment(Cir a, Cir b)
{
    double d = abs(a.o - b.o);
    vector<pair<double, double>> res;
    if (sign(a.r + b.r - d) == 0);
    else if (d <= abs(a.r - b.r) + eps) {
        if (a.r < b.r) res.emplace_back(0, 2 * pi);
    } else if (d < abs(a.r + b.r) - eps) {
        double o = acos((a.r * a.r + d * d - b.r * b.r) /
            (2 * a.r * d));
        double z = norm(atan2((b.o - a.o).y, (b.o - a.o).x)
            );
        double l = norm(z - o), r = norm(z + o);
        if (l > r) res.emplace_back(l, 2 * pi), res.
            emplace_back(0, r);
        else res.emplace_back(l, r);
    }
    return res;
}
double CircleUnionArea(vector<Cir> c) { // circle
    should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; w = 0, i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
            for (auto &e : z) s.push_back(e);
        }
        sort(s.begin(), s.end());
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
        for (auto &e : s) {
            if (e.first > w) a += F(e.first) - F(w);
            w = max(w, e.second);
        }
    }
    return a * 0.5;
}

```

8.18 Union of Polygons [1eca7c]

```

double polyUnion(vector<vector<Pt>> poly) {
    int n = poly.size();
    double ans = 0;
    auto solve = [&](Pt a, Pt b, int cid) {
        vector<pair<Pt, int>> event;
        for (int i = 0; i < n; ++i) {
            int st = 0, sz = poly[i].size();
            while (st < sz && ori(poly[i][st], a, b) != 1)
                st++;
            if (st == sz) continue;
        }
    }
}

```

```

for (int j = 0; j < sz; ++j) {
    Pt c = poly[i][(j + st) % sz];
    Pt d = poly[i][(j + st + 1) % sz];
    if (sign((a - b) ^ (c - d)) != 0) {
        int ok1 = ori(c, a, b) == 1;
        int ok2 = ori(d, a, b) == 1;
        if (ok1 ^ ok2) event.emplace_back(LinesInter
            ({a, b}, {c, d}), ok1 ? 1 : -1);
    } else if (ori(c, a, b) == 0 && sign((a - b) *
        (c - d)) > 0 && i <= cid) {
        event.emplace_back(c, -1);
        event.emplace_back(d, 1);
    }
}
sort(all(event), [&](pair<Pt, int> i, pair<Pt,
    int> j) {
    return ((a - i.first) * (a - b)) < ((a - j.first)
        * (a - b));
});
int now = 0;
Pt lst = a;
for (auto [x, y] : event) {
    if (btw(a, b, lst) && btw(a, b, x) && !now)
        ans += lst ^ x;
    now += y, lst = x;
}
};
for (int i = 0; i < n; ++i) {
    int sz = poly[i].size();
    for (int j = 0; j < sz; ++j)
        solve(poly[i][j], poly[i][(j + 1) % sz], i);
}
return ans / 2;
}

```

8.19 Rotating SweepLine [5e4c3d]

```

struct Event {
    Pt d; int u, v;
    bool operator < (const Event &b) const {
        return sign(d ^ b.d) > 0;
    }
};
Pt ref(Pt o) {return pos(o) == 1 ? Pt(-o.x, -o.y) : o;}
void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> ord(n), pos(n);
    vector<Event> e;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j) if (i ^ j)
            e.pb({ref(pt[i] - pt[j]), i, j});
    sort(all(e));
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) {
        return (sign(pt[i].y - pt[j].y) == 0 ?
            pt[i].x < pt[j].x : pt[i].y < pt[j].y); });
    for (int i = 0; i < n; ++i) pos[ord[i]] = i;
    const auto makeReverse = [](auto &v) {
        sort(all(v)); v.resize(unique(all(v)) - v.begin());
        vector<pii> segs;
        for (int i = 0, j = 0; i < v.size(); i = j) {
            for (; j < v.size() && v[j] - v[i] <= j - i; ++j);
            segs.emplace_back(v[i], v[j - 1] + 1 + 1);
        }
        return segs;
    };
    for (int i = 0, j = 0; i < e.size(); i = j) {
        vector<int> tmp;
        for (; j < e.size() && !(e[i] < e[j]); j++)
            tmp.pb(min(pos[e[j].u], pos[e[j].v]));
        for (auto [l, r] : makeReverse(tmp)) {
            reverse(ord.begin() + l, ord.begin() + r);
            for (int t = l; t < r; ++t) pos[ord[t]] = t;
            // update value here
        }
    }
}

```

8.20 Half Plane Intersection [58ae6c]

```

pair<ll, ll> area_pair(Line a, Line b)
{ return {(a.b - a.a) ^ (b.a - a.a), (a.b - a.a) ^ (b.b
    - a.a)}; }

```

```

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return a02Y * a12X - a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.a --^-- Line.b --^-- */
vector<Line> HalfPlaneInter(vector<Line> arr) {
    sort(all(arr), [&](Line a, Line b) {
        Pt A = a.b - a.a, B = b.b - b.a;
        if (pos(A) != pos(B)) return pos(A) < pos(B);
        if (sign(A ^ B) != 0) return sign(A ^ B) > 0;
        return ori(a.a, a.b, b.b) < 0;
    });
    deque<Line> dq(1, arr[0]);
    auto same = [&](Pt a, Pt b)
    { return sign(a ^ b) == 0 && pos(a) == pos(b); };
    for (auto p : arr) {
        if (same(dq.back().b - dq.back().a, p.b - p.a))
            continue;
        while (sz(dq) >= 2 && !isin(p, dq[sz(dq) - 2], dq.
            back())) dq.pop_back();
        while (sz(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(p);
    }
    while (sz(dq) >= 3 && !isin(dq[0], dq[sz(dq) - 2], dq
        .back())) dq.pop_back();
    while (sz(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(all(dq));
}

```

8.21 Minkowski Sum [6e64eb]

```

void reorder(vector<Pt> &P) {
    rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
        { return make_pair(a.y, a.x) < make_pair(b.y, b.x);
        })), P.end());
}
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    // P, Q: convex polygon, CCW order
    reorder(P), reorder(Q);
    int n = P.size(), m = Q.size();
    P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
    vector<Pt> ans;
    for (int i = 0, j = 0; i < n || j < m; ) {
        ans.pb(P[i] + Q[j]);
        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
        if (val >= 0) i++;
        if (val <= 0) j++;
    }
    return ans;
}

```

8.22 Vector In Polygon [6dac08]

```

// ori(a, b, c) >= 0, valid: "strict" angle from a-b to
// a-c
bool btwangle(Pt a, Pt b, Pt c, Pt p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >=
        strict;
}
// whether vector{cur, p} in counter-clockwise order
// prv, cur, nxt
bool inside(Pt prv, Pt cur, Pt nxt, Pt p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}

```

8.23 Delaunay Triangulation [52180a]

```

const ll inf = MAXC * MAXC * 100; // Lower_bound unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {

```

```

Pt p[3];
Edge edge[3];
Tri* chd[3];
Tri() {}
Tri(const Pt &p0, const Pt &p1, const Pt &p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
}
bool has_chd() const { return chd[0] != 0; }
int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
}
bool contains(const Pt &q) const {
    for (int i = 0; i < 3; ++i)
        if (ori(p[i], p[(i + 1) % 3], q) < 0)
            return 0;
    return 1;
}
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
        new(tris++) Tri(Pt(-inf, -inf), Pt(inf + inf, -
inf), Pt(-inf, inf + inf));
    }
    Tri* find(Pt p) { return find(the_root, p); }
    void add_point(const Pt &p) { add_point(find(the_root
, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const Pt &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, Pt const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root->p[i], root->p[(i +
1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
        for (int i = 0; i < 3; ++i)
            root->chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!lin_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri(tri->p[(pi + 1) % 3],
trj->p[pj], tri->p[pi]);
        Tri* trl = new(tris++) Tri(trj->p[(pj + 1) % 3],
tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri->chd[0] = trk; tri->chd[1] = trl; tri->chd[2] =
0;
        trj->chd[0] = trk; trj->chd[1] = trl; trj->chd[2] =
0;
        flip(trk, 1); flip(trk, 2);
    }
}

```

```

        flip(trl, 1); flip(trl, 2);
    }
}
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now->has_chd())
        return triang.pb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(vector<Pt> &arr) { // build triangulation
    int n = arr.size();
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(arr.begin(), arr.end());
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(arr[i]);
    go(tri.the_root);
}

```

8.24 Triangulation Voronoi [5c6634]

```

vector<Line> ls[N];
Line make_line(Pt p, Line l) {
    Pt d = l.b - l.a; d = perp(d);
    Pt m = (l.a + l.b) / 2; // remember to *2
    l = {m, m + d};
    if (ori(l.a, l.b, p) < 0) swap(l.a, l.b);
    return l;
}
void solve(vector<Pt> &oarr) {
    int n = oarr.size();
    map<pair<ll, ll>, int> mp;
    vector<Pt> arr = oarr;
    for (int i = 0; i < n; ++i)
        mp[{arr[i].x, arr[i].y}] = i;
    build(arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i) {
            pair<ll, ll> tmp = {t->p[i].x, t->p[i].y};
            if (mp.count(tmp)) p.pb(mp[tmp]);
        }
        for (int i = 0; i < sz(p); ++i)
            for (int j = i + 1; j < sz(p); ++j) {
                Line l = {oarr[p[i]], oarr[p[j]]};
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
        for (int i = 0; i < n; ++i)
            ls[i] = HalfPlaneInter(ls[i]);
    }
}

```

8.25 3D Point

```

struct Pt {
    double x, y, z;
    Pt(double _x = 0, double _y = 0, double _z = 0): x(_x
), y(_y), z(_z){}
    Pt operator + (const Pt &o) const
    { return Pt(x + o.x, y + o.y, z + o.z); }
    Pt operator - (const Pt &o) const
    { return Pt(x - o.x, y - o.y, z - o.z); }
    Pt operator * (const double &k) const
    { return Pt(x * k, y * k, z * k); }
    Pt operator / (const double &k) const
    { return Pt(x / k, y / k, z / k); }
    double operator * (const Pt &o) const
    { return x * o.x + y * o.y + z * o.z; }
    Pt operator ^ (const Pt &o) const
    { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x
* o.y - y * o.x)}; }
};
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
Pt cross3(Pt a, Pt b, Pt c)
{ return (b - a) ^ (c - a); }

```

```
double area(Pt a, Pt b, Pt c)
{ return abs(cross3(a, b, c)); }
double volume(Pt a, Pt b, Pt c, Pt d)
{ return cross3(a, b, c) * (d - a); }
bool coplaner(Pt a, Pt b, Pt c, Pt d)
{ return sign(volume(a, b, c, d)) == 0; }
Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
{ Pt n = cross3(a, b, c);
  return o - n * ((o - a) * (n / abs2(n))); }
Pt LinePlaneInter(Pt u, Pt v, Pt a, Pt b, Pt c) {
  // intersection of line uv and plane abc
  Pt n = cross3(a, b, c);
  double s = n * (u - v);
  if (sign(s) == 0) return {-1, -1, -1}; // not found
  return v + (u - v) * ((n * (a - v)) / s);
}
```

8.26 3D Convex Hull [2c9f0d]

```
struct CH3D {
  struct face{int a, b, c; bool ok;} F[8 * N];
  double dblcmp(Pt &p, face &f)
  {return cross3(P[f.a], P[f.b], P[f.c]) * (p - P[f.a])
    ;}
  int g[N][N], num, n;
  Pt P[N];
  void deal(int p, int a, int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
      if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
    } else
      add.a = b, add.b = a, add.c = p, add.ok = 1, g[
        p][b] = g[a][p] = g[b][a] = num, F[num++] =
        add;
  }
  void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
      now].b), deal(p, F[now].a, F[now].c);
  }
  bool same(int s, int t){
    Pt &a = P[F[s].a];
    Pt &b = P[F[s].b];
    Pt &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps &&
      fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
        volume(a, b, c, P[F[t].c])) < eps;
  }
  void init(int _n){n = _n, num = 0;}
  void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&](){
      for (int i = 1; i < n; ++i)
        if (abs(P[0] - P[i]) > eps)
          return swap(P[1], P[i]), 0;
      return 1;
    }() || [&](){
      for (int i = 2; i < n; ++i)
        if (abs(cross3(P[i], P[0], P[1])) > eps)
          return swap(P[2], P[i]), 0;
      return 1;
    }() || [&](){
      for (int i = 3; i < n; ++i)
        if (fabs(((P[0] - P[1]) ^ (P[1] - P[2])) * (P
          [0] - P[i])) > eps)
          return swap(P[3], P[i]), 0;
      return 1;
    }())return;
    for (int i = 0; i < 4; ++i) {
      add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
        (i + 3) % 4, add.ok = true;
      if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
      g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
        a] = num;
      F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
      for (int j = 0; j < num; ++j)
```

```
    if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
      dfs(i, j);
      break;
    }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
      if (F[i].ok) F[num++] = F[i];
  }
  double get_area() {
    double res = 0.0;
    if (n == 3)
      return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
      res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
  }
  double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
      res += volume(Pt(0, 0, 0), P[F[i].a], P[F[i].b],
        P[F[i].c]);
    return fabs(res / 6.0);
  }
  int triangle() {return num;}
  int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag
      , flag = 1)
      for (int j = 0; j < i && flag; ++j)
        flag &= !same(i, j);
    return res;
  }
  Pt getcent(){
    Pt ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
      if (F[i].ok == true) {
        Pt p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].
          c];
        t2 = volume(temp, p1, p2, p3) / 6.0;
        if (t2 > 0)
          ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
            ans.y += (p1.y + p2.y + p3.y + temp.y) *
              t2, ans.z += (p1.z + p2.z + p3.z + temp.z
                ) * t2, v += t2;
      }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
      );
    return ans;
  }
  double pointmindis(Pt p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
      if(F[i].ok == true) {
        Pt p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].
          c];
        double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
          z - p1.z) * (p3.y - p1.y);
        double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
          x - p1.x) * (p3.z - p1.z);
        double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
          y - p1.y) * (p3.x - p1.x);
        double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
          ;
        double temp = fabs(a * p.x + b * p.y + c * p.z
          + d) / sqrt(a * a + b * b + c * c);
        rt = min(rt, temp);
      }
    return rt;
  }
};
```

9 Else

9.1 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue<int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
```



```
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1); // 4 0
bitset<100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope<int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```
ll next_perm(ll v) { ll t = v | (v - 1);
    return (t + 1) |
        (((~t & ~t) - 1) >> (__builtin_ctz(v) + 1)); }
```

9.3 Smawk Algorithm [5a33b4]

```
ll f(int l, int r) { }
bool select(int r, int u, int v) {
    // if f(r, v) is better than f(r, u), return true
    return f(r, u) < f(r, v);
}
// For all 2x2 submatrix: (x < y => y is better than x)
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
vector<int> solve(vector<int> &r, vector<int> &c) {
    const int n = r.size();
    if (n == 0) return {};
    vector<int> c2;
    for (const int &i : c) {
        while (!c2.empty() && select(r[c2.size() - 1], c2.back(), i)) c2.pop_back();
        if (c2.size() < n) c2.pb(i);
    }
    vector<int> r2;
    for (int i = 1; i < n; i += 2) r2.pb(r[i]);
    const auto a2 = solve(r2, c2);
    vector<int> ans(n);
    for (int i = 0; i < a2.size(); i++)
        ans[i * 2 + 1] = a2[i];
    int j = 0;
    for (int i = 0; i < n; i += 2) {
        ans[i] = c2[j];
        const int end = i + 1 == n ? c2.back() : ans[i + 1];
        while (c2[j] != end) {
            j++;
            if (select(r[i], ans[i], c2[j])) ans[i] = c2[j];
        }
    }
    return ans;
}
vector<int> smawk(int n, int m) {
    vector<int> row(n), col(m);
    iota(all(row), 0), iota(all(col), 0);
    return solve(row, col);
}
```

9.4 Slope Trick [d51078]

```
template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
    slope_trick_convex() {left.push(numeric_limits<T>::min() / 2), right.push(numeric_limits<T>::max() / 2);}
    void push_left(T x) {left.push(x - ground_l);}
    void push_right(T x) {right.push(x - ground_r);}
    //add a Line with slope 1 to the right starting from x
    void add_right(T x) {
        T l = left.top() + ground_l;
```

```
        if (l <= x) push_right(x);
        else push_left(x), push_right(l), left.pop(), minn += 1 - x;
    }
    //add a Line with slope -1 to the left starting from x
    void add_left(T x) {
        T r = right.top() + ground_r;
        if (r >= x) push_left(x);
        else push_right(x), push_left(r), right.pop(), minn += x - r;
    }
    //val[i]=min(val[j]) for all i-l<=j<=i+r
    void expand(T l, T r) {ground_l -= 1, ground_r += r;}
    void shift_up(T x) {minn += x;}
    T get_val(T x) {
        T l = left.top() + ground_l, r = right.top() + ground_r;
        if (x >= l && x <= r) return minn;
        if (x < l) {
            vector<T> trash;
            T cur_val = minn, slope = 1, res;
            while (1) {
                trash.push_back(left.top());
                left.pop();
                if (left.top() + ground_l <= x) {
                    res = cur_val + slope * (1 - x);
                    break;
                }
                cur_val += slope * (1 - (left.top() + ground_l));
                l = left.top() + ground_l;
                slope += 1;
            }
            for (auto i : trash) left.push(i);
            return res;
        }
        if (x > r) {
            vector<T> trash;
            T cur_val = minn, slope = 1, res;
            while (1) {
                trash.push_back(right.top());
                right.pop();
                if (right.top() + ground_r >= x) {
                    res = cur_val + slope * (x - r);
                    break;
                }
                cur_val += slope * ((right.top() + ground_r) - r);
                r = right.top() + ground_r;
                slope += 1;
            }
            for (auto i : trash) right.push(i);
            return res;
        }
        assert(0);
    }
};
```

9.5 ALL LCS [5ff948]

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(t.size());
    iota(all(h), 0);
    for (int a = 0; a < s.size(); ++a) {
        int v = -1;
        for (int c = 0; c < t.size(); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

9.6 Hilbert Curve [1274a3]

```
ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
```

```

    res += s * 111 * s * ((3 * rx) ^ ry);
    if (ry == 0) {
        if (rx == 1) x = s - 1 - x, y = s - 1 - y;
        swap(x, y);
    }
    return res;
} // n = 2^k

```

9.7 Line Container [673ffd]

```

// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a
        % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) { // ax + b
        auto z = insert({a, b, 0}); y = z++; x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

9.8 Min Plus Convolution [b34de3]

```

// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector
    <int> &b) {
    int n = a.size(), m = b.size();
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from);
        Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}

```

9.9 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.10 Simulated Annealing

```

double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 100000; ++it) {
    // ans: answer, nw: current value, rnd(): mt19937 rnd
    ()
    if (exp(-(nw - ans) / factor) >= (double)rnd() %
        base) / base)
        ans = nw;
    factor *= 0.99995;
}

```

9.11 Bitset LCS

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

9.12 Binary Search On Fraction [765c5a]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```

9.13 Cyclic Ternary Search [9017cc]

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(1, r % n) ? l : r % n;
}

```

9.14 Tree Hash [34aae5]

```

ull seed;
ull shift(ull x) { x ^= x << 13; x ^= x >> 7;
    x ^= x << 17; return x; }
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u]) if (i != f)
        sum += shift(dfs(i, u));
    return sum;
}

```

9.15 Python Misc

[illegible]