

# Contents

<b>1 Basic</b>	<b>1</b>		
1.1 Shell Script	1		
1.2 Default Code	1		
1.3 Increase Stack Size	1		
1.4 Debug Macro	1		
1.5 Stress Test Shell*	1		
1.6 Pragma / FastIO	2		
1.7 Divide*	2		
<b>2 Data Structure</b>	<b>2</b>		
2.1 Leftist Tree	2		
2.2 Splay Tree	2		
2.3 Link Cut Tree	2		
2.4 Treap	3		
2.5 2D Segment Tree*	3		
2.6 Zkw*	4		
2.7 Chtholly Tree*	4		
2.8 vEB Tree*	5		
<b>3 Flow / Matching</b>	<b>5</b>		
3.1 Dinic	5		
3.2 Min Cost Max Flow	5		
3.3 Kuhn Munkres	6		
3.4 SW Min Cut	6		
3.5 Gomory Hu Tree	7		
3.6 Blossom	7		
3.7 Weighted Blossom	7		
3.8 Flow Model	9		
<b>4 Graph</b>	<b>9</b>		
4.1 Heavy-Light Decomposition	9		
4.2 Centroid Decomposition	9		
4.3 Edge BCC	9		
4.4 Block Cut Tree	9		
4.5 SCC / 2SAT	10		
4.6 Virtual Tree	10		
4.7 Directed MST	10		
4.8 Dominator Tree	11		
<b>5 String</b>	<b>11</b>		
5.1 Aho-Corasick Automaton	11		
5.2 KMP Algorithm	11		
5.3 Z Algorithm	11		
5.4 Manacher	11		
5.5 Suffix Array	12		
5.6 SAIS	12		
5.7 Suffix Automaton	12		
5.8 Minimum Rotation	13		
5.9 Palindrome Tree	13		
5.10 Main Lorentz	13		
<b>6 Math</b>	<b>13</b>		
6.1 Miller Rabin / Pollard Rho	13		
6.2 Ext GCD	14		
6.3 PiCount	14		
6.4 Linear Function Mod Min	14		
6.5 Determinant	14		
6.6 Floor Sum	14		
6.7 Quadratic Residue	14		
6.8 Simplex	15		
6.9 Berlekamp Massey	15		
6.10 Linear Programming Construction	15		
6.11 Euclidean	15		
6.12 Theorem	16		
6.13 Estimation	16		
6.14 General Purpose Numbers	16		
6.15 Tips for Generating Function	16		
<b>7 Polynomial</b>	<b>17</b>		
7.1 Number Theoretic Transform	17		
7.2 Fast Fourier Transform	17		
7.3 Primes	17		
7.4 Polynomial Operations	17		
7.5 Fast Linear Recursion	18		
7.6 Fast Walsh Transform	18		
<b>8 Geometry</b>	<b>19</b>		
8.1 Basic	19		
8.2 Heart	19		
8.3 External Bisector	19		
8.4 Intersection of Segments	19		
8.5 Intersection of Circle and Line	19		
8.6 Intersection of Circles	19		
8.7 Intersection of Polygon and Circle	19		
8.8 Tangent Lines of Circle and Point	20		
8.9 Tangent Lines of Circles	20		
8.10 Point In Convex	20		
8.11 Point Segment Distance	20		
8.12 Convex Hull	20		
8.13 Convex Hull Distance	20		
8.14 Minimum Enclosing Circle	20		
8.15 Union of Circles	20		
8.16 Union of Polygons	21		
8.17 Polar Angle Sort	21		
8.18 Rotating Caliper	21		
8.19 Rotating Sweepline	21		
8.20 Half Plane Intersection	21		
8.21 Minkowski Sum	21		
8.22 Delaunay Triangulation	22		
8.23 Triangulation Voronoi	22		
<b>9 Else</b>	<b>23</b>		
9.1 Bit Hack	23		
9.2 Dynamic Programming Condition	23		
9.2.1 Totally Monotone (Concave/Convex)	23		
9.2.2 Monge Condition (Concave/Convex)	23		
9.2.3 Optimal Split Point	23		
9.3 Slope Trick	23		
9.4 Manhattan MST	23		
9.5 Dynamic MST	24		
9.6 ALL LCS	24		
9.7 Hilbert Curve	24		
9.8 Pbds	24		
9.9 Random	24		
9.10 Smawk Algorithm	25		
9.11 Matroid Intersection	25		
9.12 Python Misc	25		

## 1 Basic

### 1.1 Shell Script

```
g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
    $1 && ./ $1
for i in {A..J}; do cp tem.cpp $i.cpp; done;
```

### 1.2 Default Code

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define pb push_back
#define pii pair<int, int>
#define all(a) a.begin(), a.end()
#define sz(a) ((int)a.size())
```

### 1.3 Increase Stack Size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bak = (char*)rsp
;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

### 1.4 Debug Macro

```
void db() {cout << endl;}
template <typename T, typename ...U> void db(T i, U ...
    j) {
    cout << i << ' ', db(j...);
}
#define test(x...) db("[ " + string(x) + " ]", x)
```

### 1.5 Stress Test Shell\*

```
#!/usr/bin/env bash
g++ $1.cpp -o $1
g++ $2.cpp -o $2
g++ $3.cpp -o $3
for i in {1..100}; do
    ./ $3 > input.txt
    # st=$(date +%s%N)
    ./ $1 < input.txt > output1.txt
    # echo "$(((date +%s%N) - $st)/1000000))ms"
    ./ $2 < input.txt > output2.txt
    if cmp --silent -- "output1.txt" "output2.txt" ; then
        continue
    fi
    echo Input:
    cat input.txt
    echo Your Output:
    cat output1.txt
    echo Correct Output:
    cat output2.txt
    exit 1
done
echo OK!
./stress.sh main good gen
```

## 1.6 Pragma / FastIO

```
#pragma GCC optimize("Ofast,inline,unroll-loops")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)
        ) == buf ? -1 : *p++;
}
inline int R() {
    static char c;
    while((c = RC()) < '0'); int a = c ^ '0';
    while((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

## 1.7 Divide\*

```
ll divdown(ll a, ll b) {
    return a / b - (a < 0 && a % b);
}
ll divup(ll a, ll b) {
    return a / b + (a > 0 && a % b);
}
a / b < x -> divdown(a, b) + 1 <= x
a / b <= x -> divup(a, b) <= x
x < a / b -> x <= divup(a, b) - 1
x <= a / b -> x <= divdown(a, b)
```

# 2 Data Structure

## 2.1 Leftist Tree

```
struct node {
    ll rk, data, sz, sum;
    node *l, *r;
    node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}
```

## 2.2 Splay Tree

```
struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    ll v[N];
    Splay() {}
    void init() {
        rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
        sz[0] = 1, v[0] = inf;
    }
    int newnode(int p, int x) {
        int id = _id++;
        v[id] = x, pa[id] = p;
        ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    }
```

```
    return id;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i, gp = pa[p], c =
        ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
}
void splay(int i) {
    while (~pa[i]) {
        int p = pa[i];
        if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
            == i ? i : p);
        rotate(i);
    }
    rt = i;
}
int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
        if (v[i] == x) return splay(i), i;
        if (v[i] > x) {
            last = i;
            if (ch[i][0] == -1) break;
            i = ch[i][0];
        }
        else {
            if (ch[i][1] == -1) break;
            i = ch[i][1];
        }
    }
    splay(i);
    return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};
```

## 2.3 Link Cut Tree

```
// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N], sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
    // virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0])
```

```

    sz[i] += sz[ch[i][0]], mx[i] = max(mx[i], mx[ch[i][0]]);
    if (ch[i][1])
        sz[i] += sz[ch[i][1]], mx[i] = max(mx[i], mx[ch[i][1]]);
}
void push(int i) {
    if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
        rev[i] = false;
}
void reverse(int i) {
    if (!i) return;
    swap(ch[i][0], ch[i][1]);
    rev[i] ^= true;
}
bool isrt(int i) { // rt of splay
    if (!pa[i]) return true;
    return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i, c = ch[i][!x], gp = pa[p];
    if (ch[gp][0] == p) ch[gp][0] = i;
    else if (ch[gp][1] == p) ch[gp][1] = i;
    pa[i] = gp, ch[i][!x] = p, pa[p] = i;
    ch[p][x] = c, pa[c] = p;
    pull(p), pull(i);
}
void splay(int i) {
    vector<int> anc;
    anc.push_back(i);
    while (!isrt(anc.back())) anc.push_back(pa[anc.back()]);
    while (!anc.empty()) push(anc.back()), anc.pop_back();
    while (!isrt(i)) {
        int p = pa[i];
        if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1] == p ? i : p);
        rotate(i);
    }
}
void access(int i) {
    int last = 0;
    while (i) {
        splay(i);
        if (ch[i][1])
            sz2[i] += sz[ch[i][1]];
        sz2[i] -= sz[last];
        ch[i][1] = last;
        pull(i), last = i, i = pa[i];
    }
}
void makert(int i) {
    access(i), splay(i), reverse(i);
}
void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
}
void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
}
int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
}
};

```

## 2.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;

```

```

    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    o->down(), split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
    o->up();
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c; // [l, r)
    o->down();
    split2(o, a, b, l), split2(b, b, c, r - l);
    // operate
    o = merge(a, merge(b, c)), o->up();
}

```

## 2.5 2D Segment Tree\*

```

// 2D range add, range sum in Log^2
struct seg {
    int l, r;
    ll sum, lz;
    seg *ch[2]{};
    seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
    void push() {
        if (lz) ch[0]->add(l, r, lz), ch[1]->modify(l, r,
            lz), lz = 0;
    }
    void pull() {sum = ch[0]->sum + ch[1]->sum;}
    void add(int _l, int _r, ll d) {
        if (_l <= l && r <= _r) {

```

```

    sum += d * (r - 1);
    lz += d;
    return;
}
if (!ch[0]) ch[0] = new seg(1, 1 + r >> 1), ch[1] =
    new seg(1 + r >> 1, r);
push();
if (_l < 1 + r >> 1) ch[0]->add(_l, _r, d);
if (1 + r >> 1 < _r) ch[1]->add(_l, _r, d);
pull();
}
ll qsum(int _l, int _r) {
    if (_l <= 1 && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(1, _l));
    push();
    ll res = 0;
    if (_l < 1 + r >> 1) res += ch[0]->qsum(_l, _r);
    if (1 + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
}
};
struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2];
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N) {
        if (1 < r - 1) ch[0] = new seg2(1, 1 + r >> 1), ch[1] =
            new seg2(1 + r >> 1, r);
    }
    void add(int _l, int _r, int _l2, int _r2, ll d) {
        v.add(_l2, _r2, d * (min(r, _r) - max(1, _l)));
        if (_l <= 1 && r <= _r) {
            lz.add(_l2, _r2, d);
            return;
        }
        if (_l < 1 + r >> 1) ch[0]->add(_l, _r, _l2, _r2, d);
        if (1 + r >> 1 < _r) ch[1]->add(_l, _r, _l2, _r2, d);
    }
    ll qsum(int _l, int _r, int _l2, int _r2) {
        ll res = v.qsum(_l2, _r2);
        if (_l <= 1 && r <= _r) return res;
        res += lz.qsum(_l2, _r2) * (min(r, _r) - max(1, _l));
        if (_l < 1 + r >> 1) res += ch[0]->query(_l, _r, _l2, _r2);
        if (1 + r >> 1 < _r) res += ch[1]->query(_l, _r, _l2, _r2);
        return res;
    }
};

```

## 2.6 Zkw\*

```

ll mx[N << 1], sum[N << 1], lz[N << 1];
void add(int l, int r, ll d) { // [L, r), 0-based
    int len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <= 1) {
        sum[l] += cntl * d, sum[r] += cntr * d;
        if (len > 1) {
            mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
            mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
        }
        if (~l & 1)
            sum[l ^ 1] += d * len, mx[l ^ 1] += d, lz[l ^ 1]
                += d, cntl += len;
        if (r & 1)
            sum[r ^ 1] += d * len, mx[r ^ 1] += d, lz[r ^ 1]
                += d, cntr += len;
    }
    sum[l] += cntl * d, sum[r] += cntr * d;
    if (len > 1) {
        mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
        mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
    }
    cntl += cntr;
    for (l >>= 1; l; l >>= 1) {
        sum[l] += cntl * d;
        mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
    }
}

```

```

}
}
ll qsum(int l, int r) {
    ll res = 0, len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <= 1) {
        res += cntl * lz[l] + cntr * lz[r];
        if (~l & 1) res += sum[l ^ 1], cntl += len;
        if (r & 1) res += sum[r ^ 1], cntr += len;
    }
    res += cntl * lz[l] + cntr * lz[r];
    cntl += cntr;
    for (l >>= 1; l; l >>= 1) res += cntl * lz[l];
    return res;
}
ll qmax(int l, int r) {
    ll maxl = -INF, maxr = -INF;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1)
        {
            maxl += lz[l], max[r] += lz[r];
            if (~l & 1) maxl = max(maxl, mx[l ^ 1]);
            if (r & 1) maxr = max(maxr, mx[r ^ 1]);
        }
    maxl = max(maxl + lz[l], maxr + lz[r]);
    for (l >>= 1; l; l >>= 1) maxl += lz[l];
    return maxl;
}

```

## 2.7 Chtholly Tree\*

```

struct ChthollyTree {
    struct interval {
        int l, r;
        ll v;
        interval(int _l, int _r, ll _v) : l(_l), r(_r), v(_v) {}
    };
    struct cmp {
        bool operator () (const interval &a, const interval
            & b) const {
            return a.l < b.l;
        }
    };
    set <interval, cmp> s;
    vector <interval> split(int l, int r) {
        // split into [0, l), [l, r), [r, n) and return [L,
        // r)
        vector <interval> del, ans, re;
        auto it = s.lower_bound(interval(l, -1, 0));
        if (it != s.begin() && (it == s.end() || l < it->l))
            {
                --it;
                del.pb(*it);
                if (r < it->r) {
                    re.pb(interval(it->l, l, it->v));
                    ans.pb(interval(l, r, it->v));
                    re.pb(interval(r, it->r, it->v));
                } else {
                    re.pb(interval(it->l, l, it->v));
                    ans.pb(interval(l, it->r, it->v));
                }
                ++it;
            }
        for (; it != s.end() && it->r <= r; ++it) {
            ans.pb(*it);
            del.pb(*it);
        }
        if (it != s.end() && it->l < r) {
            del.pb(*it);
            ans.pb(interval(it->l, r, it->v));
            re.pb(interval(r, it->r, it->v));
        }
        for (interval &i : del)
            s.erase(i);
        for (interval &i : re)
            s.insert(i);
        return ans;
    }
    void merge(vector <interval> a) {
        for (interval &i : a)
            s.insert(i);
    }
}

```

};

## 2.8 vEB Tree\*

```

using u64=uint64_t;
constexpr int lsb(u64 x){return x?__builtin_ctzll(x)
:1<<30;}
constexpr int msb(u64 x){return x?63-__builtin_clzll(x)
:-1;}
template<int N, class T=void>
struct veb{
    static const int M=N>>1;
    veb<M> ch[1<<N-M];
    veb<N-M> aux;
    int mn,mx;
    veb():mn(1<<30),mx(-1){}
    constexpr int mask(int x){return x&((1<<M)-1);}
    bool empty(){return mx==-1;}
    int min(){return mn;}
    int max(){return mx;}
    bool have(int x){
        if(x==mn) return true;
        return ch[x>>M].have(mask(x));
    }
    void insert_in(int x){
        if(empty()) return mn=mx=x,void();
        if(x<mn) swap(x,mn);
        if(x>mx) mx=x;
        if(ch[x>>M].empty()) aux.insert_in(x>>M);
        ch[x>>M].insert_in(mask(x));
    }
    void erase_in(int x){
        if(mn==mx) return mn=1<<30,mx=-1,void();
        if(x==mn) mn=x=(aux.min()<<M)^ch[aux.min()].min();
        ch[x>>M].erase_in(mask(x));
        if(ch[x>>M].empty()) aux.erase_in(x>>M);
        if(x==mx){
            if(aux.empty()) mx=mn;
            else mx=(aux.max()<<M)^ch[aux.max()].max();
        }
    }
    void insert(int x){
        if(have(x)) return;
        insert_in(x);
    }
    void erase(int x){
        if(!have(x)) return;
        erase_in(x);
    }
    int next(int x){// >=x
        if(x>mx) return 1<<30;
        if(x<=mn) return mn;
        if(mask(x)<=ch[x>>M].max()) return ((x>>M)<<M)^ch[x>>M].next(mask(x));
        int y=aux.next((x>>M)+1);
        return (y<<M)^ch[y].min();
    }
    int prev(int x){// <x
        if(x<=mn) return -1;
        if(x>mx) return mx;
        if(x<=(aux.min()<<M)+ch[aux.min()].min()) return mn;
        if(mask(x)>ch[x>>M].min()) return ((x>>M)<<M)^ch[x>>M].prev(mask(x));
        int y=aux.prev(x>>M);
        return (y<<M)^ch[y].max();
    }
};
template<int N>
struct veb<N,typename enable_if<N<=6>::type>{
    u64 a;
    veb():a(0){}
    void insert_in(int x){a|=1ull<<x;}
    void insert(int x){a|=1ull<<x;}
    void erase_in(int x){a&=~(1ull<<x);}
    void erase(int x){a&=~(1ull<<x);}
    bool have(int x){return a>>x&1;}
    bool empty(){return a==0;}
    int min(){return lsb(a);}
    int max(){return msb(a);}
    int next(int x){return lsb(a&~((1ull<<x)-1));}
    int prev(int x){return msb(a&((1ull<<x)-1));}
};

```

};

## 3 Flow / Matching

### 3.1 Dinic

```

const int INF = 1 << 30;
struct Dinic { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> adj[N];
    int s, t, dis[N], cur[N], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)adj[u].size(); ++i)
        {
            edge &e = adj[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    adj[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : adj[tmp])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) adj[i].clear();
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : adj[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        adj[u].pb(edge{v, cap, 0, (int)adj[v].size()});
        adj[v].pb(edge{u, 0, 0, (int)adj[u].size() - 1});
    }
};

```

### 3.2 Min Cost Max Flow

```

template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
    const T1 INF1 = 1 << 30;
    const T2 INF2 = 1 << 30;
    struct edge {
        int v; T1 f; T2 c;
    };
    E[M << 1];
    vector <int> adj[N];
    T2 dis[N], pot[N];
    int rt[N], vis[N], n, m, s, t;
    bool SPFA() {

```

```

fill_n(rt, n, -1), fill_n(dis, n, INF2);
fill_n(vis, n, false);
queue<int> q;
q.push(s), dis[s] = 0, vis[s] = true;
while (!q.empty()) {
    int v = q.front(); q.pop();
    vis[v] = false;
    for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
        dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v], rt[E[id].v] = id;
        if (!vis[E[id].v]) vis[E[id].v] = true, q.push(E[id].v);
    }
}
return dis[t] != INF2;
}
bool dijkstra() {
    fill_n(rt, n, -1), fill_n(dis, n, INF2);
    priority_queue<pair<T2, int>, vector<pair<T2, int>>, greater<pair<T2, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
        auto [d, v] = pq.top(); pq.pop();
        if (dis[v] < d) continue;
        for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
            dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v], rt[E[id].v] = id;
            pq.emplace(dis[E[id].v], E[id].v);
        }
    }
    return dis[t] != INF2;
}
pair<T1, T2> solve(int _s, int _t) {
    s = _s, t = _t, fill_n(pot, n, 0);
    T1 flow = 0; T2 cost = 0;
    bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; ++i) {
            dis[i] += pot[i] - pot[s];
        }
        T1 add = INF1;
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            add = min(add, E[rt[i]].f);
        }
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
        }
        flow += add, cost += add * dis[t];
        fr = false;
        for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return make_pair(flow, cost);
}
void init(int _n) {
    n = _n, m = 0;
    for (int i = 0; i < n; ++i) adj[i].clear();
}
void reset() {
    for (int i = 0; i < m; ++i) E[i].f = 0;
}
void add_edge(int u, int v, T1 f, T2 c) {
    adj[u].pb(m), E[m++] = {v, f, c};
    adj[v].pb(m), E[m++] = {u, 0, -c};
}
}
};

```

### 3.3 Kuhn Munkres

```

template<typename T>
struct KM { // 0-based
    T w[N][N], hl[N], hr[N], slk[N];
    T fl[N], fr[N], pre[N]; int n;
    bool vl[N], vr[N];
    const T INF = 1e9;
    queue<int> q;
    KM(int _n) : n(_n) {
        for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)

```

```

        w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x]) return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill(slk, slk + n, INF), fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        q.push(s), vr[s] = 1;
        while (1) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x) if (!vl[x] && !slk[x] && !check(x)) return;
        }
    }
    T solve() {
        fill(fl, fl + n, -1), fill(fr, fr + n, -1), fill(hr, hr + n, 0);
        for (int i = 0; i < n; ++i) hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) bfs(i);
        T res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

### 3.4 SW Min Cut

```

template<typename T>
struct SW { // 0-based
    T g[N][N], sum[N]; int n;
    bool vis[N], dead[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) fill(g[i], g[i] + n, 0);
        fill(dead, dead + n, false);
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w;
    }
    T solve() {
        T ans = 1 << 30;
        for (int round = 0; round + 1 < n; ++round) {
            fill(vis, vis + n, false), fill(sum, sum + n, 0);
            int num = 0, s = -1, t = -1;
            while (num < n - round) {
                int now = -1;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    if (now == -1 || sum[now] < sum[i]) now = i;
                }
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    sum[i] += g[now][i];
                }
            }

```



```

    }
    ans = min(ans, sum[t]);
    for (int i = 0; i < n; ++i) {
        g[i][s] += g[i][t];
        g[s][i] += g[t][i];
    }
    dead[t] = true;
}
return ans;
}
};

```

### 3.5 Gomory Hu Tree

```

vector<array<int, 3>> GomoryHu(vector<vector<pii>>
    adj, int n) {
    // Tree edge min -> mincut (0-based)
    Dinic flow(n);
    for (int i = 0; i < n; ++i) for (auto [j, w] : adj[i])
        flow.add_edge(i, j, w);
    flow.record();
    vector<array<int, 3>> ans;
    vector<int> rt(n);
    for (int i = 0; i < n; ++i) rt[i] = 0;
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset(); // clear flows on all edge
        ans.push_back({i, t, flow.solve(i, t)});
        flow.runbfs(i);
        for (int j = i + 1; j < n; ++j) if (rt[j] == t &&
            flow.vis[j]) {
            rt[j] = i;
        }
    }
    return ans;
}

```

### 3.6 Blossom

```

struct Matching { // 0-based
    int fa[N], pre[N], match[N], s[N], v[N], n, tk;
    vector<int> g[N];
    queue<int> q;
    Matching(int _n) : n(_n), tk(0) {
        for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
        for (int i = 0; i < n; ++i) g[i].clear();
    }
    void add_edge(int u, int v) {
        g[u].push_back(v), g[v].push_back(u);
    }
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (v[x] == tk) return x;
                v[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
    void blossom(int x, int y, int l) {
        while (Find(x) != l) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            if (fa[x] == x) fa[x] = l;
            if (fa[y] == y) fa[y] = l;
            x = pre[y];
        }
    }
    bool bfs(int r) {
        for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
        while (!q.empty()) q.pop();
        q.push(r);
        s[r] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int u : g[x]) {

```

```

                if (s[u] == -1) {
                    pre[u] = x, s[u] = 1;
                    if (match[u] == n) {
                        for (int a = u, b = x, last; b != n; a =
                            last, b = pre[a])
                            last = match[b], match[b] = a, match[a] =
                                b;
                        return true;
                    }
                    q.push(match[u]);
                    s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = lca(u, x);
                    blossom(x, u, l);
                    blossom(u, x, l);
                }
            }
        }
        return false;
    }
    int solve() {
        int res = 0;
        for (int x = 0; x < n; ++x) {
            if (match[x] == n) res += bfs(x);
        }
        return res;
    }
};

```

### 3.7 Weighted Blossom

```

struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w) : u(u), v(v), w(w) {}
    };
    int n, n_x;
    edge g[maxn * 2][maxn * 2];
    int lab[maxn * 2];
    int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
        pa[maxn * 2];
    int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
        maxn * 2];
    vector<int> flo[maxn * 2];
    queue<int> q;
    int e_delta(const edge &e) { return lab[e.u] + lab[e.
        v] - g[e.u][e.v].w * 2; }
    void update_slack(int u, int x) { if (!slack[x] ||
        e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
        [x] = u; }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (size_t i = 0; i < flo[x].size(); i++)
            q_push(flo[x][i]);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (size_t i = 0; i < flo[x].size(); ++
            i) set_st(flo[x][i], b);
    }
    int get_pr(int b, int xr) {
        int pr = find(flo[b].begin(), flo[b].end(), xr) -
            flo[b].begin();
        if (pr % 2 == 1) {
            reverse(flo[b].begin() + 1, flo[b].end());
            return (int)flo[b].size() - pr;
        }
        return pr;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        edge e = g[u][v];

```

```

    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i],
        flo[u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
}
void augment(int u, int v) {
    for (; ; ) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
        w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        for (int x = 1; x <= n_x; ++x)
            if (g[b][x].w == 0 || e_delta(g[xs][x]) <
                e_delta(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;

```

```

        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
            q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b]
                / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1) d = min(d, e_delta(g[slack[x]
                    ][x]));
                else if (S[x] == 0) d = min(d, e_delta(g[
                    slack[x]][x]) / 2);
            }
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d) return 0;
                lab[u] -= d;
            } else if (S[st[u]] == 1) lab[u] += d;
        }
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b) {
                if (S[st[b]] == 0) lab[b] += d * 2;
                else if (S[st[b]] == 1) lab[b] -= d * 2;
            }
        q = queue<int>();
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x] && st[slack[x]] != x
                && e_delta(g[slack[x]][x]) == 0)
                if (on_found_edge(g[slack[x]][x])) return
                    true;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)
                expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}

```



```

void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
    g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
};

```

### 3.8 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$ .
  - Construct a max flow model, let  $K$  be the sum of all weights.
  - Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$ .
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$ .
  - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$ .
  - $T$  is a valid answer if the maximum flow  $f < K|V|$ .
- Minimum weight edge cover
  - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  - Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  - The mincut is equivalent to the maximum profit of a subset of projects.

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge  $(x, t)$  with capacity  $c_x$  and create edge  $(s, y)$  with capacity  $c_y$ .
- Create edge  $(x, y)$  with capacity  $c_{xy}$ .
- Create edge  $(x, y)$  and edge  $(x', y')$  with capacity  $c_{xyx'y'}$ .

## 4 Graph

### 4.1 Heavy-Light Decomposition

```

vector<int> dep, pa, sz, ch, hd, id;
int _id;
void dfs(int i, int p) {
    dep[i] = ~p ? dep[p] + 1 : 0;
    pa[i] = p, sz[i] = 1, ch[i] = -1;
    for (int j : g[i])
        if (j != p) {
            dfs(j, i);
            if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
            sz[i] += sz[j];
        }
}

```

```

}
void hld(int i, int p, int h) {
    hd[i] = h;
    id[i] = _id++;
    if (~ch[i]) hld(ch[i], i, h);
    for (int j : g[i]) if (j != p && j != ch[i])
        hld(j, i, j);
}
void query(int i, int j) {
    while (hd[i] != hd[j]) {
        if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
        query2(id[hd[i]], id[i] + 1, i = pa[hd[i]]);
    }
    if (dep[i] < dep[j]) swap(i, j);
    query2(id[j], id[i] + 1);
}

```

### 4.2 Centroid Decomposition

```

vector<vector<int>> dis; // dis[n][Logn]
vector<int> pa, sz, dep;
vector<bool> vis;
void dfs_sz(int i, int p) {
    sz[i] = 1;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_sz(j, i), sz[i] += sz[j];
}
int cen(int i, int p, int _n) {
    for (int j : g[i]) if (j != p && !vis[j] && sz[j] >
        _n / 2)
        return cen(j, i, _n);
    return i;
}
void dfs_dis(int i, int p, int d) { // from i to
    ancestor with depth d
    dis[i][d] = ~p ? dis[p][d] + 1 : 0;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
    dfs_sz(i, -1), i = cen(i, -1, sz[i]);
    vis[i] = true, pa[i] = p, dep[i] = d;
    dfs_dis(i, -1, d);
    for (int j : g[i]) if (!vis[j])
        cd(j, i, d + 1);
}

```

### 4.3 Edge BCC

```

vector<int> low, dep, bcc_id, stk;
vector<bool> vis;
int _id;
void dfs(int i, int p) {
    low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    vis[i] = true;
    for (int j : g[i])
        if (j != p) {
            if (!vis[j])
                dfs(j, i), low[i] = min(low[i], low[j]);
            else
                low[i] = min(low[i], dep[j]);
        }
    if (low[i] == dep[i]) {
        int id = _id++;
        while (stk.back() != i) {
            int x = stk.back();
            stk.pop_back();
            bcc_id[x] = id;
        }
        stk.pop_back();
        bcc_id[i] = id;
    }
}

```

### 4.4 Block Cut Tree

```

vector<vector<int>> g, _g;
vector<int> dep, low, stk;
void dfs(int i, int p) {
    dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
}

```

```

for (int j : g[i]) if (j != p) {
    if (dep[j] == -1) {
        dfs(j, i), low[i] = min(low[i], low[j]);
        if (low[j] >= dep[i]) {
            int id = _g.size();
            _g.emplace_back();
            while (stk.back() != j) {
                int x = stk.back();
                stk.pop_back();
                _g[x].push_back(id), _g[id].push_back(x);
            }
            stk.pop_back();
            _g[j].push_back(id), _g[id].push_back(j);
            _g[i].push_back(id), _g[id].push_back(i);
        }
    } else low[i] = min(low[i], dep[j]);
}
}

```

## 4.5 SCC / 2SAT

```

struct SAT {
    vector<vector<int>> g;
    vector<int> dep, low, scc_id;
    vector<bool> is;
    vector<int> stk;
    int n, _id, _t;
    SAT() {}
    void init(int _n) {
        n = _n, _id = _t = 0;
        g.assign(2 * n, vector<int>());
        dep.assign(2 * n, -1), low.assign(2 * n, -1);
        scc_id.assign(2 * n, -1), is.assign(2 * n, false);
        stk.clear();
    }
    void add_edge(int x, int y) {g[x].push_back(y);}
    int rev(int i) {return i < n ? i + n : i - n;}
    void add_ifthen(int x, int y) {add_clause(rev(x), y);}
    void add_clause(int x, int y) {
        add_edge(rev(x), y);
        add_edge(rev(y), x);
    }
    void dfs(int i) {
        dep[i] = low[i] = _t++;
        stk.push_back(i);
        for (int j : g[i])
            if (scc_id[j] == -1) {
                if (dep[j] == -1)
                    dfs(j);
                low[i] = min(low[i], low[j]);
            }
        if (low[i] == dep[i]) {
            int id = _id++;
            while (stk.back() != i) {
                int x = stk.back();
                stk.pop_back();
                scc_id[x] = id;
            }
            stk.pop_back();
            scc_id[i] = id;
        }
    }
    bool solve() {
        for (int i = 0; i < 2 * n; ++i)
            if (dep[i] == -1)
                dfs(i);
        for (int i = 0; i < n; ++i) {
            if (scc_id[i] == scc_id[i + n]) return false;
            if (scc_id[i] < scc_id[i + n])
                is[i] = true;
            else
                is[i + n] = true;
        }
        return true;
    }
};

```

## 4.6 Virtual Tree

```

vector<vector<int>> _g;
vector<int> st, ed, stk;

```

```

void solve(vector<int> v) {
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    int sz = v.size();
    for (int i = 0; i < sz - 1; ++i)
        v.push_back(lca(v[i], v[i + 1]));
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    v.resize(unique(all(v)) - v.begin());
    stk.clear(); stk.push_back(v[0]);
    for (int i = 1; i < v.size(); ++i) {
        int x = v[i];
        while (ed[stk.back()] < ed[x]) stk.pop_back();
        _g[stk.back()].push_back(x), stk.push_back(x);
    }
    // do something
    for (int i : v) _g[i].clear();
}

```

## 4.7 Directed MST

```

template <typename T> struct DMST { // 1-based
    T g[maxn][maxn], fw[maxn];
    int n, fr[maxn];
    bool vis[maxn], inc[maxn];
    void clear() {
        for (int i = 0; i < maxn; ++i) {
            for (int j = 0; j < maxn; ++j) g[i][j] = inf;
            vis[i] = inc[i] = false;
        }
    }
    void addedge(int u, int v, T w) {
        g[u][v] = min(g[u][v], w);
    }
    T query(int root, int _n) {
        n = _n;
        if (dfs(root) != n) return -1;
        T ans = 0;
        while (true) {
            for (int i = 1; i <= n; ++i) fw[i] = inf, fr[i] = i;
            for (int i = 1; i <= n; ++i) if (!inc[i]) {
                for (int j = 1; j <= n; ++j) {
                    if (!inc[j] && i != j && g[j][i] < fw[i]) {
                        fw[i] = g[j][i];
                        fr[i] = j;
                    }
                }
            }
            int x = -1;
            for (int i = 1; i <= n; ++i) if (i != root && !inc[i]) {
                int j = i, c = 0;
                while (j != root && fr[j] != i && c <= n) ++c, j = fr[j];
                if (j == root || c > n) continue;
                else { x = i; break; }
            }
            if (!~x) {
                for (int i = 1; i <= n; ++i) if (i != root && !inc[i]) ans += fw[i];
                return ans;
            }
            int y = x;
            for (int i = 1; i <= n; ++i) vis[i] = false;
            do { ans += fw[y]; y = fr[y]; vis[y] = inc[y] = true; } while (y != x);
            inc[x] = false;
            for (int k = 1; k <= n; ++k) if (vis[k]) {
                for (int j = 1; j <= n; ++j) if (!vis[j]) {
                    if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
                    if (g[j][k] < inf && g[j][k] - fw[k] < g[j][x]) g[j][x] = g[j][k] - fw[k];
                }
            }
        }
        return ans;
    }
    int dfs(int now) {
        int r = 1;
        vis[now] = true;
    }
};

```

```

    for (int i = 1; i <= n; ++i) if (g[now][i] < inf &&
        !vis[i]) r += dfs(i);
    return r;
}
};

```

## 4.8 Dominator Tree

```

struct Dominator_tree {
    int n, id;
    vector<vector<int>> adj, radj, bucket;
    vector<int> sdom, dom, vis, rev, par, rt, mn;
    Dominator_tree (int _n) : n(_n), id(0) {
        adj.resize(n), radj.resize(n), bucket.resize(n);
        sdom.resize(n), dom.resize(n, -1), vis.resize(n,
            -1);
        rev.resize(n), rt.resize(n), mn.resize(n), par.
            resize(n);
    }
    void add_edge(int u, int v) {adj[u].pb(v);}
    int query(int v, bool x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], true);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]]) mn[v] = mn[rt[v]
            ];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : adj[v]) {
            if (vis[u] == -1) dfs(u), par[vis[u]] = vis[v];
            radj[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : radj[i]) {
                sdom[i] = min(sdom[i], sdom[query(u, false)]);
            }
            if (i) bucket[sdom[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, false);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = par[i];
        }
        vector<int> res(n, -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i) res[rev[i]] = rev[dom[
            i]];
        res[s] = s;
        dom = res;
    }
};

```

## 5 String

### 5.1 Aho-Corasick Automaton

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    vector<int> g[N];
    int cnt[N];
    AC () {sz = 0, extend();}
    void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
        return ch[u][v];
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) now = nxt(now, c - 'a');
        cnt[now]++;
        return now;
    }
};

```

```

void build_fail() {
    queue<int> q;
    for (int i = 0; i < 26; ++i) if (ch[0][i]) {
        q.push(ch[0][i]);
        g[0].push_back(ch[0][i]);
    }
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (int j = 0; j < 26; ++j) {
            to[v][j] = ch[v][j] ? v : to[fail[v]][j];
        }
        for (int i = 0; i < 26; ++i) if (ch[v][i]) {
            int u = ch[v][i], k = fail[v];
            while (k && !ch[k][i]) k = fail[k];
            if (ch[k][i]) k = ch[k][i];
            fail[u] = k;
            cnt[u] += cnt[k], g[k].push_back(u);
            q.push(u);
        }
    }
}

int match(string &s) {
    int now = 0, ans = 0;
    for (char c : s) {
        now = to[now][c - 'a'];
        if (ch[now][c - 'a']) now = ch[now][c - 'a'];
        ans += cnt[now];
    }
    return ans;
};

```

### 5.2 KMP Algorithm

```

vector<int> build_fail(string s) {
    vector<int> f(s.length() + 1, 0);
    int k = 0;
    for (int i = 1; i < s.length(); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}

int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < s.length(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == t.length()) ans++, k = f[k];
    }
    return ans;
}

```

### 5.3 Z Algorithm

```

vector<int> build(string s) {
    int n = s.length();
    vector<int> Z(n);
    int l = 0, r = 0;
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < s.size() && s[Z[i]] == s[i + Z[i]
            ]) {
            l = i, r = i + Z[i], Z[i]++;
        }
    }
    return Z;
}

```

### 5.4 Manacher

```

vector<int> manacher(string &s) {
    string t = "^#";
    for (char c : s) t += c, t += '#';
    t += '&';
    int n = t.length();
    vector<int> r(n, 0);
    int C = 0, R = 0;
    for (int i = 1; i < n - 1; ++i) {
        int mirror = 2 * C - i;

```

```

    r[i] = (i < R ? min(r[mirror], R - i) : 0);
    while (t[i - 1 - r[i]] == t[i + 1 + r[i]]) r[i]++;
    if (i + r[i] > R) R = i + r[i], C = i;
}
return r;
}

```

## 5.5 Suffix Array

```

int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.length();
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]] = s[i]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++] = sa[i] - k;
        for (int i = n - 1; ~i; --i) sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!(x[a] == x[b] && a + k < n && b + k < n && x[a + k] == x[b + k])) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}
void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = min(lcp[rk[i] + 1], lcp[rk[i] + 2], ..., lcp[rk[j]])
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i] == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}

```

## 5.6 SAIS

```

namespace sfx {
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
void pre(int *sa, int *c, int n, int z) {
    fill_n(sa, n, 0), copy_n(c, z, x);
}
void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
    copy_n(c, z - 1, x + 1);
    for (int i = 0; i < n; ++i) if (sa[i] && !t[sa[i] - 1]) sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    for (int i = n - 1; i >= 0; --i) if (sa[i] && t[sa[i] - 1]) sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
}

```

```

if (uniq) {
    for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
    return;
}
for (int i = n - 2; i >= 0; --i)
    t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
pre(sa, c, n, z);
for (int i = 1; i <= n - 1; ++i)
    if (t[i] && !t[i - 1])
        sa[--x[s[i]]] = p[q[i] = nn++] = i;
induce(sa, c, s, t, n, z);
for (int i = 0; i < n; ++i)
    if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
        bool neq = last < 0 || !equal(s + sa[i], s + p[q[sa[i]] + 1], s + last);
        ns[q[last = sa[i]]] = nmzx += neq;
    }
sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
pre(sa, c, n, z);
for (int i = nn - 1; i >= 0; --i)
    sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
induce(sa, c, s, t, n, z);
}
vector<int> build(int *s, int n) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    for (int i = 0; i < n; ++i)
        sa[i] = SA[i + 1];
    return sa;
}
}

```

## 5.7 Suffix Automaton

```

struct SAM {
    int ch[N][26], len[N], link[N], pos[N], cnt[N], sz;
    // node -> strings with the same endpos set
    // length in range [len(link) + 1, len]
    // node's endpos set -> pos in the subtree of node
    // link -> longest suffix with different endpos set
    // len -> longest suffix
    // pos -> end position
    // cnt -> size of endpos set
    SAM() {len[0] = 0, link[0] = -1, pos[0] = 0, cnt[0] = 0, sz = 1;}
    void build(string s) {
        int last = 0;
        for (int i = 0; i < s.length(); ++i) {
            char c = s[i];
            int cur = sz++;
            len[cur] = len[last] + 1, pos[cur] = i + 1;
            int p = last;
            while (~p && !ch[p][c - 'a']) ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) {
                link[cur] = 0;
            } else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q], pos[nxt] = 0;
                    for (int j = 0; j < 26; ++j) ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q) ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
            cnt[cur]++;
            last = cur;
        }
        vector<int> p(sz);
        iota(all(p), 0);
        sort(all(p), [&](int i, int j) {return len[i] > len[j]});
    }
}

```

```

    for (int i = 0; i < sz; ++i) cnt[link[p[i]]] += cnt[p[i]];
}
} sam;

```

## 5.8 Minimum Rotation

```

string rotate(const string &s) {
    int n = s.length();
    string t = s + s;
    int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

## 5.9 Palindrome Tree

```

struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM(string _s) : s(_s) {
        sz = 0;
        extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst] - 1] != s[i]) lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
                int now = fail[lst];
                while (s[i - len[now] - 1] != s[i]) now = fail[now];
                fail[idx] = ch[now][s[i] - 'a'];
                ch[lst][s[i] - 'a'] = idx;
            }
            lst = ch[lst][s[i] - 'a'], cnt[lst]++;
        }
        void build_count() {
            for (int i = sz - 1; i > 1; --i)
                cnt[fail[i]] += cnt[i];
        }
        int extend() {
            fill(ch[sz], ch[sz] + 26, 0), sz++;
            return sz - 1;
        }
    };
}

```

## 5.10 Main Lorentz

```

int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, Len.
// substr(L ~ r, Len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l), sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i) to_right[i] = Z[r - m + 1 + i - 1];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = 1; i < m; ++i) to_left[i] = Z[m - i - 1];
    reverse(all(sl));
    for (int i = 1; i + 1 < m; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1], len = m - i - 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2), tr = min(len - 1, k1);

```

```

        if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
    }
    Z = buildZ(sr);
    for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
    reverse(all(sl)), reverse(all(sr));
    Z = buildZ(sl + "#" + sr);
    for (int i = m; i < r; ++i) to_left[i] = Z[m - 1 + 1 + r - i - 1];
    reverse(all(sl)), reverse(all(sr));
    for (int i = m; i + 1 < r; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1], len = i - m + 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(len - k2, 1), tr = min(len - 1, k1);
        if (tl <= tr) rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
    }
    Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i) {
        if (Z[r - m + 1 + i - 1] >= m - i) {
            rep.pb({i, i, m - i});
        }
    }
}

```

## 6 Math

### 6.1 Miller Rabin / Pollard Rho

```

ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long double)x / p * y) * p + p) % p;}
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
ll Pow(ll a, ll b, ll n) {ll res = 1; for (; b; b >>= 1, a = mul(a, a, n)) if (b & 1) res = mul(res, a, n); return res;}
bool check(ll a, ll d, int s, ll n) {
    a = Pow(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; l <= 1) {
        x = y;
        int m = min(1, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(ll n) {
    if (n == 1) return;
    if (IsPrime(n)) return ++res[n], void(0);
}

```

```

    ll d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
}

```

## 6.2 Ext GCD

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    pair<ll, ll> res;
    if (a < 0) {
        res = extgcd(-a, b);
        res.first *= -1;
        return res;
    }
    if (b < 0) {
        res = extgcd(a, -b);
        res.second *= -1;
        return res;
    }
    if (b == 0) return {1, 0};
    res = extgcd(b, a % b);
    return {res.second, res.first - res.second * (a / b)};
}

```

## 6.3 PiCount

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i]) for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.push_back(i);
}
void init(){
    sieve(V);
    small_pi[0] = 0;
    for(int i = 1; i < V; ++i) small_pi[i] = small_pi[i - 1] + isp[i];
    for(int i = 0; i < M; ++i) dp[0][i] = i;
    for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j) dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i] - 1];
}
ll phi(ll n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(((ll)primes[a - 1] * primes[a - 1] >= n && n < V)) return small_pi[n] - a + 1;
    ll de = phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
    return de;
}
ll PiCount(ll n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a = small_pi[y];
    ll res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n / primes[a]) - PiCount(primes[a]) + 1, 0ll);
    return res;
}

```

## 6.4 Linear Function Mod Min

```

ll topos(ll x, ll m) {x %= m; if (x < 0) x += m; return x;}
//min value of ax + b (mod m) for x \in [0, n - 1]. O(Log m)
ll min_rem(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    for (ll g = __gcd(a, m); g > 1; ) return g * min_rem(n / g, m / g, a / g, b / g) + (b % g);
    for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b = nb) {
        if (a <= m - a) {

```

```

            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            ll lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1], also return min x to get the value. O(Log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g) % (m / g);
    return {mn, x};
}

```

## 6.5 Determinant

```

ll Det(vector<vector<ll>> a) {
    int n = a.size();
    ll det = 1;
    for (int i = 0; i < n; ++i) {
        if (!a[i][i]) {
            det = -det;
            if (det < 0) det += mod;
            for (int j = i + 1; j < n; ++j) if (a[j][i]) {
                swap(a[j], a[i]);
                break;
            }
            if (!a[i][i]) return 0;
        }
        det = det * a[i][i] % mod;
        ll mul = mpow(a[i][i], mod - 2);
        for (int j = 0; j < n; ++j) a[i][j] = a[i][j] * mul % mod;
        for (int j = 0; j < n; ++j) if (i ^ j) {
            ll mul = a[j][i];
            for (int k = 0; k < n; ++k) {
                a[j][k] -= a[i][k] * mul % mod;
                if (a[j][k] < 0) a[j][k] += mod;
            }
        }
    }
    return det;
}

```

## 6.6 Floor Sum

```

// sum^{n-1}_0 floor((a * i + b) / m) in Log(n + m + a + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

## 6.7 Quadratic Residue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;

```



```

    swap(a, m);
}
return s;
}
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 6.8 Simplex

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
    bool ls(T a, T b) {return a < b && !eq(a, b);}
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;
        for (int i = 0; i < m; ++i) for (int j = 0; j < n; ++j) a[i][j] = 0;
        for (int i = 0; i < m; ++i) b[i] = 0;
        for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
    }
    void pivot(int x, int y) {
        swap(Left[x], Down[y]);
        T k = a[x][y]; a[x][y] = 1;
        vector<int> nz;
        for (int i = 0; i < n; ++i) {
            a[x][i] /= k;
            if (!eq(a[x][i], 0)) nz.push_back(i);
        }
        b[x] /= k;
        for (int i = 0; i < m; ++i) {
            if (i == x || eq(a[i][y], 0)) continue;
            k = a[i][y], a[i][y] = 0;
            b[i] -= k * b[x];
            for (int j : nz) a[i][j] -= k * a[x][j];
        }
        if (eq(c[y], 0)) return;
        k = c[y], c[y] = 0, v += k * b[x];
        for (int i : nz) c[i] -= k * a[x][i];
    }
    // 0: found solution, 1: no feasible solution, 2:
    // unbounded
    int solve() {
        for (int i = 0; i < n; ++i) Down[i] = i;
        for (int i = 0; i < m; ++i) Left[i] = n + i;
        while (1) {
            int x = -1, y = -1;
            for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
            if (x == -1) break;
            for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;

```

```

            if (y == -1) return 1;
            pivot(x, y);
        }
        while (1) {
            int x = -1, y = -1;
            for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
            if (y == -1) break;
            for (int i = 0; i < m; ++i) if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
            if (x == -1) return 2;
            pivot(x, y);
        }
        for (int i = 0; i < m; ++i) if (Left[i] < n) sol[Left[i]] = b[i];
        return 0;
    }
};

```

## 6.9 Berlekamp Massey

```

vector<ll> BerlekampMassey(vector<ll> a) {
    // find min |c| such that a_n = sum c_j * a_{n-j-1}, 0-based
    // O(N^2), if |c| = k, |a| >= 2k sure correct
    auto f = [&](vector<ll> v, ll c) {
        for (ll &x : v) x = mul(x, c);
        return v;
    };
    vector<ll> c, best;
    int pos = 0, n = a.size();
    for (int i = 0; i < n; ++i) {
        ll error = a[i];
        for (int j = 0; j < c.size(); ++j) error = sub(error, mul(c[j], a[i - 1 - j]));
        if (error == 0) continue;
        ll inv = mpow(error, mod - 2);
        if (c.empty()) {
            c.resize(i + 1);
            pos = i;
            best.pb(inv);
        } else {
            vector<ll> fix = f(best, error);
            fix.insert(fix.begin(), i - pos - 1, 0);
            if (fix.size() >= c.size()) {
                best = f(c, sub(0, inv));
                best.insert(best.begin(), inv);
                pos = i;
                c.resize(fix.size());
            }
            for (int j = 0; j < fix.size(); ++j) c[j] = add(c[j], fix[j]);
        }
    }
    return c;
}

```

## 6.10 Linear Programming Construction

Standard form: maximize  $c^T x$  subject to  $Ax \leq b$  and  $x \geq 0$ .  
 Dual LP: minimize  $b^T y$  subject to  $A^T y \geq c$  and  $y \geq 0$ .  
 $x$  and  $y$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.11 Euclidean

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + \left\lfloor \frac{b}{c} \right\rfloor \cdot (n+1) & a \geq c \vee b \geq c \\ +f(a \bmod c, b \bmod c, c, n), & a < c \vee a = 0 \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) \\ + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.12 Theorem

### • Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### • Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### • Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each *Labeled* vertices, there are

$$\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$$

spanning trees.

- Let  $T_{n,k}$  be the number of *Labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

### • Erdős-Gallai Theorem

A sequence of non-negative integers  $d_1 \geq d_2 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + d_2 + \dots + d_n$  is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all  $1 \leq k \leq n$ .

### • Burnside's Lemma

Let  $X$  be a set and  $G$  be a group that acts on  $X$ . For  $g \in G$ , denote by  $X^g$  the elements fixed by  $g$ :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

### • Gale-Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^n a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### • Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### • Möbius inversion formula

$$\begin{aligned} - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

### • Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$ .
- Area  $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$ .

### • Chinese Remainder Theorem

- $x \equiv a_i \pmod{m_i}$
- $M = \prod m_i, M_i = M/m_i$
- $t_i M_i \equiv 1 \pmod{m_i}$
- $x = \sum a_i t_i M_i \pmod{M}$

## 6.13 Estimation

- The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200000 for  $n < 1e19$ .
- The number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for  $n = 0 \sim 9$ , 627 for  $n = 20$ ,  $\sim 2e5$  for  $n = 50$ ,  $\sim 2e8$  for  $n = 100$ .
- Total number of partitions of  $n$  distinct elements:  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \dots$

## 6.14 General Purpose Numbers

### • Bernoulli numbers

$$B_0 = 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

### • Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

### • Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

### • Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 6.15 Tips for Generating Function

### • Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x)' \Rightarrow n a_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

### • Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k} \binom{n}{k} a_i b_{n-i}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x) \Rightarrow n a_n$

### • Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{n-1}{i} x^i$

## 7 Polynomial

### 7.1 Number Theoretic Transform

```
// mul, add, sub, mpow
// LL -> int if too slow
struct NTT {
    ll w[N];
    NTT() {
        ll dw = mpow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = mul(w[i - 1], dw);
    }
    void operator()(vector<ll>& a, bool inv = false) { //
        0 <= a[i] < P
        int x = 0, n = a.size();
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (x ^= k) < k; k >>= 1);
            if (j < x) swap(a[x], a[j]);
        }
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            ll invn = mpow(n, mod - 2);
            for (int i = 0; i < n; ++i) a[i] = mul(a[i], invn);
        }
    }
} ntt;
```

### 7.2 Fast Fourier Transform

```
using T = complex<double>;
const double PI = acos(-1);
struct NTT {
    T w[N];
    FFT() {
        T dw = {cos(2 * PI / N), sin(2 * PI / N)};
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
    }
    void operator()(vector<T>& a, bool inv = false) {
        // see NTT, replace LL with T
        if (inv) {
            reverse(a.begin() + 1, a.end());
            T invn = 1.0 / n;
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
        }
    }
} ntt;
```

### 7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	9223372036737335297	3

### 7.4 Polynomial Operations

```
vector<ll> Mul(vector<ll> a, vector<ll> b, int bound
    = N) {
    int m = a.size() + b.size() - 1, n = 1;
    while (n < m) n <= 1;
```

```
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    vector<ll> out(n);
    for (int i = 0; i < n; ++i) out[i] = mul(a[i], b[i]);
    ntt(out, true), out.resize(min(m, bound));
    return out;
}

vector<ll> Inverse(vector<ll> a) {
    // O(NlogN), a[0] != 0
    int n = a.size();
    vector<ll> res(1, mpow(a[0], mod - 2));
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        vector<ll> v1(a.begin(), a.begin() + m * 2), v2 = res;
        v1.resize(m * 4), v2.resize(m * 4);
        ntt(v1), ntt(v2);
        for (int i = 0; i < m * 4; ++i) v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
        ntt(v1, true);
        res.resize(m * 2);
        for (int i = 0; i < m; ++i) res[i] = add(res[i], res[i]);
        for (int i = 0; i < m * 2; ++i) res[i] = sub(res[i], v1[i]);
    }
    res.resize(n);
    return res;
}

pair<vector<ll>, vector<ll>> Divide(vector<ll> a,
    vector<ll> b) {
    // a = bQ + R, O(NlogN), b.back() != 0
    int n = a.size(), m = b.size(), k = n - m + 1;
    if (n < m) return {{0}, a};
    vector<ll> ra = a, rb = b;
    reverse(all(ra)), ra.resize(k);
    reverse(all(rb)), rb.resize(k);
    vector<ll> Q = Mul(ra, Inverse(rb), k);
    reverse(all(Q));
    vector<ll> res = Mul(b, Q), R(m - 1);
    for (int i = 0; i < m - 1; ++i) R[i] = sub(a[i], res[i]);
    return {Q, R};
}

vector<ll> SqrtImpl(vector<ll> a) {
    if (a.empty()) return {0};
    int z = QuadraticResidue(a[0], mod), n = a.size();
    if (z == -1) return {-1};
    vector<ll> q(1, z);
    const int inv2 = (mod + 1) / 2;
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        q.resize(m * 2);
        vector<ll> f2 = Mul(q, q, m * 2);
        for (int i = 0; i < m * 2; ++i) f2[i] = sub(f2[i], a[i]);
        f2 = Mul(f2, Inverse(q), m * 2);
        for (int i = 0; i < m * 2; ++i) q[i] = sub(q[i], mul(f2[i], inv2));
    }
    q.resize(n);
    return q;
}

vector<ll> Sqrt(vector<ll> a) {
    // O(NlogN), return {-1} if not exists
    int n = a.size(), m = 0;
    while (m < n && a[m] == 0) m++;
    if (m == n) return vector<ll>(n);
    if (m & 1) return {-1};
    vector<ll> s = SqrtImpl(vector<ll>(a.begin() + m, a.end()));
    if (s[0] == -1) return {-1};
    vector<ll> res(n);
    for (int i = 0; i < s.size(); ++i) res[i + m / 2] = s[i];
    return res;
}

vector<ll> Derivative(vector<ll> a) {
    int n = a.size();
    vector<ll> res(n - 1);
    for (int i = 0; i < n - 1; ++i) res[i] = mul(a[i + 1], i + 1);
```

```

    return res;
}
vector<ll> Integral(vector<ll> a) {
    int n = a.size();
    vector<ll> res(n + 1);
    for (int i = 0; i < n; ++i) {
        res[i + 1] = mul(a[i], mpow(i + 1, mod - 2));
    }
    return res;
}
vector<ll> Ln(vector<ll> a) {
    // O(NlogN), a[0] = 1
    int n = a.size();
    if (n == 1) return {0};
    vector<ll> d = Derivative(a);
    a.pop_back();
    return Integral(Mul(d, Inverse(a), n - 1));
}
vector<ll> Exp(vector<ll> a) {
    // O(NlogN), a[0] = 0
    int n = a.size();
    vector<ll> q(1, 1);
    a[0] = add(a[0], 1);
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        vector<ll> g(a.begin(), a.begin() + m * 2), h(all(
            q));
        h.resize(m * 2), h = Ln(h);
        for (int i = 0; i < m * 2; ++i) {
            g[i] = sub(g[i], h[i]);
        }
        q = Mul(g, q, m * 2);
    }
    q.resize(n);
    return q;
}
vector<ll> Pow(vector<ll> a, ll k) {
    int n = a.size(), m = 0;
    vector<ll> ans(n, 0);
    while (m < n && a[m] == 0) m++;
    if (k && m && (k >= n || k * m >= n)) return ans;
    if (m == n) return ans[0] = 1, ans;
    ll lead = m * k;
    vector<ll> b(a.begin() + m, a.end());
    ll base = mpow(b[0], k), inv = mpow(b[0], mod - 2);
    for (int i = 0; i < n - m; ++i) b[i] = mul(b[i], inv);
    b = Ln(b);
    for (int i = 0; i < n - m; ++i) b[i] = mul(b[i], k % mod);
    b = Exp(b);
    for (int i = lead; i < n; ++i) ans[i] = mul(b[i - lead], base);
    return ans;
}
vector<ll> Evaluate(vector<ll> a, vector<ll> x) {
    if (x.empty()) return {};
    int n = x.size();
    vector<vector<ll>> up(n * 2);
    for (int i = 0; i < n; ++i) up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i) up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector<vector<ll>> down(n * 2);
    down[1] = Divide(a, up[1]).second;
    for (int i = 2; i < n * 2; ++i) down[i] = Divide(down[i >> 1], up[i]).second;
    vector<ll> y(n);
    for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
    return y;
}
vector<ll> Interpolate(vector<ll> x, vector<ll> y) {
    int n = x.size();
    vector<vector<ll>> up(n * 2);
    for (int i = 0; i < n; ++i) up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i) up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector<ll> a = Evaluate(Derivative(up[1]), x);
    for (int i = 0; i < n; ++i) {
        a[i] = mul(y[i], mpow(a[i], mod - 2));
    }
}

```

```

vector<vector<ll>> down(n * 2);
for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
for (int i = n - 1; i > 0; --i) {
    vector<ll> lhs = Mul(down[i * 2], up[i * 2 + 1]);
    vector<ll> rhs = Mul(down[i * 2 + 1], up[i * 2]);
    down[i].resize(lhs.size());
    for (int j = 0; j < lhs.size(); ++j) {
        down[i][j] = add(lhs[j], rhs[j]);
    }
}
return down[1];
}

```

## 7.5 Fast Linear Recursion

```

ll FastLinearRecursion(vector<ll> a, vector<ll> c, ll k) {
    // a_n = sigma c_j * a_{n-j-1}, 0-based
    // O(NlogNlogK), |a| = |c|
    int n = a.size();
    if (k < n) return a[k];
    vector<ll> base(n + 1, 1);
    for (int i = 0; i < n; ++i) base[i] = sub(0, c[n - i - 1]);
    vector<ll> poly(n);
    (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
    auto calc = [&](vector<ll> p1, vector<ll> p2) {
        // O(n^2) brute force or O(nlogn) NTT
        return Divide(Mul(p1, p2), base).second;
    };
    vector<ll> res(n, 0); res[0] = 1;
    for (; k >= 1, poly = calc(poly, poly)) {
        if (k & 1) res = calc(res, poly);
    }
    ll ans = 0;
    for (int i = 0; i < n; ++i) {
        (ans += res[i] * a[i]) %= mod;
    }
    return ans;
}

```

## 7.6 Fast Walsh Transform

```

void fwt(vector<int> &a) {
    // and : x += y * (1, -1)
    // or  : y += x * (1, -1)
    // xor : x = (x + y) * (1, 1/2)
    //      y = (x - y) * (1, 1/2)
    int n = __lg(a.size());
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
            int x = a[j ^ (1 << i)], y = a[j];
            // do something
        }
    }
}
vector<int> subs_conv(vector<int> a, vector<int> b) {
    // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
    int n = __lg(a.size());
    vector<vector<int>> ha(n + 1, vector<int>(1 << n));
    vector<vector<int>> hb(n + 1, vector<int>(1 << n));
    vector<vector<int>> c(n + 1, vector<int>(1 << n));
    for (int i = 0; i < 1 << n; ++i) {
        ha[__builtin_popcount(i)][i] = a[i];
        hb[__builtin_popcount(i)][i] = b[i];
    }
    for (int i = 0; i <= n; ++i) or_fwt(ha[i]), or_fwt(hb[i]);
    for (int i = 0; i <= n; ++i)
        for (int j = 0; i + j <= n; ++j)
            for (int k = 0; k < 1 << n; ++k)
                // mind overflow
                c[i + j][k] += ha[i][k] * hb[j][k];
    for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
    vector<int> ans(1 << n);
    for (int i = 0; i < 1 << n; ++i)
        ans[i] = c[__builtin_popcount(i)][i];
    return ans;
}

```

## 8 Geometry

### 8.1 Basic

```
const double eps = 1e-8, PI = acos(-1);
int sign(double x) {return abs(x) <= eps ? 0 : (x > 0 ? 1 : -1);}
double norm(double x) {
    while (x < -eps) x += PI * 2;
    while (x > PI * 2 + eps) x -= PI * 2;
    return x;
}
struct Pt {
    double x, y;
    Pt(double _x, double _y) : x(_x), y(_y) {}
    Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
    Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
    Pt operator * (double k) {return Pt(x * k, y * k);}
    Pt operator / (double k) {return Pt(x / k, y / k);}
    double operator * (Pt o) {return x * o.x + y * o.y;}
    double operator ^ (Pt o) {return x * o.y - y * o.x;}
};
struct Line {
    Pt a, b;
};
struct Cir {
    Pt o; double r;
};
double abs2(Pt o) {return o.x * o.x + o.y * o.y;}
double abs(Pt o) {return sqrt(abs2(o));}
int ori(Pt o, Pt a, Pt b) {return sign((o - a) ^ (o - b));}
bool btw(Pt a, Pt b, Pt c) { // c on segment ab?
    return ori(a, b, c) == 0 && sign((c - a) * (c - b)) <= 0;
}
double area(Pt a, Pt b, Pt c) {return abs((a - b) ^ (a - c)) / 2;}
double angle(Pt a, Pt b) {return norm(atan2(b.y - a.y, b.x - a.x));}
Pt unit(Pt o) {return o / abs(o);}
Pt rot(Pt a, double o) { // CCW
    double c = cos(o), s = sin(o);
    return Pt(c * a.x - s * a.y, s * a.x + c * a.y);
}
Pt perp(Pt a) {return Pt(-a.y, a.x);}
Pt proj_vector(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vector(a, b, c) + a;
}
```

### 8.2 Heart

```
Pt circenter(Pt p0, Pt p1, Pt p2) { // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    double m = 2. * (x1 * y2 - y1 * x2);
    Pt center(0, 0);
    center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) { // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3) {
    return (p1 + p2 + p3) / 3;
}
Pt orthocenter(Pt p1, Pt p2, Pt p3) {
    return masscenter(p1, p2, p3) * 3 - circenter(p1, p2, p3);
}
```

### 8.3 External Bisector

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
    Pt L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}
```

### 8.4 Intersection of Segments

```
Pt LinesInter(Line a, Line b) {
    double abc = (a.b - a.a) ^ (b.a - a.a);
    double abd = (a.b - a.a) ^ (b.b - a.a);
    if (sign(abc - abd) == 0) return b.b; // no inter
    return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
    if (btw(a.a, a.b, b.a)) return {b.a};
    if (btw(a.a, a.b, b.b)) return {b.b};
    if (btw(b.a, b.b, a.a)) return {a.a};
    if (btw(b.a, b.b, a.b)) return {a.b};
    if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 && ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
        return {LinesInter(a, b)};
    return {};
}
```

### 8.5 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)) / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
}
```

### 8.6 Intersection of Circles

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r * c2.r - c1.r * c1.r) / (2 * d2));
    double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d) * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
    Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2 * d2);
    if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
    return {u + v, u - v};
}
```

### 8.7 Intersection of Polygon and Circle

```
double _area(Pt pa, Pt pb, double r) {
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
    double cosC = (pa * pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < pi / 2) S -= (acos(h / r) * r * r - h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = pi - B - asin(sin(B) / r * a);
        S = .5 * a * r * sin(theta) + (C - theta) / 2 * r * r;
    } else S = .5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r) {
    double S = 0; int n = poly.size();
    for (int i = 0; i < n; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) * ori(O, poly[i], poly[(i + 1) % n]);
}
```

```
    return fabs(S);
}
```

## 8.8 Tangent Lines of Circle and Point

```
vector<Line> tangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sign(d - c.r) == 0) {
        Pt i = rot(p - c.o, pi / 2);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
            i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}
```

## 8.9 Tangent Lines of Circles

```
vector<Line> tangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if (sign(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
            sign2 * h * v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
            0)
            p2 = p1 + perp(c2.o - c1.o);
        ret.pb({p1, p2});
    }
    return ret;
}
```

## 8.10 Point In Convex

```
bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
    int a = 1, b = int(C.size()) - 1, r = !strict;
    if (C.size() == 0) return false;
    if (C.size() < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}
```

## 8.11 Point Segment Distance

```
double PointSegDist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
        * (p - q1)) >= 0)
        return fabs((q1 - q0) ^ (p - q0)) / abs(q0 - q1);
    return min(abs(p - q0), abs(p - q1));
}
```

## 8.12 Convex Hull

```
vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.
        y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
```

```
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2],
                ans.back(), pt[i]) <= 0)
                ans.pop_back();
            ans.push_back(pt[i]);
        }
        reverse(all(pt));
    }
    ans.pop_back();
    return ans;
}
```

## 8.13 Convex Hull Distance

```
double ConvexHullDist(vector<Pt> A, vector<Pt> B) {
    for (auto &p : B) p = Pt(0, 0) - p;
    auto C = Minkowski(A, B); // assert SZ(C) > 0
    if (PointInConvex(C, Pt(0, 0))) return 0;
    double ans = PointSegDist(C.back(), C[0], Pt(0, 0));
    for (int i = 0; i + 1 < C.size(); ++i) {
        ans = min(ans, PointSegDist(C[i], C[i + 1], Pt(0,
            0)));
    }
    return ans;
}
```

## 8.14 Minimum Enclosing Circle

```
Cir min_enclosing(vector<Pt> &p) {
    random_shuffle(p.begin(), p.end());
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < p.size(); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i];
        r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2;
            r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}
```

## 8.15 Union of Circles

```
vector<pair<double, double>> CoverSegment(Cir a, Cir b)
{
    double d = abs(a.o - b.o);
    vector<pair<double, double>> res;
    if (sign(a.r + b.r - d) == 0);
    else if (d <= abs(a.r - b.r) + eps) {
        if (a.r < b.r) res.emplace_back(0, 2 * pi);
    } else if (d < abs(a.r + b.r) - eps) {
        double o = acos((a.r * a.r + d * d - b.r * b.r) /
            (2 * a.r * d)), z = atan2((b.o - a.o).y, (b.o -
            a.o).x);
        if (z < 0) z += 2 * pi;
        double l = z - o, r = z + o;
        if (l < 0) l += 2 * pi;
        if (r > 2 * pi) r -= 2 * pi;
        if (l > r) res.emplace_back(l, 2 * pi), res.
            emplace_back(0, r);
        else res.emplace_back(l, r);
    }
    return res;
}

double CircleUnionArea(vector<Cir> c) { // circle
    should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; w = 0, i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
```



```

    for (auto &e : z) s.push_back(e);
}
sort(s.begin(), s.end());
auto F = [&](double t) { return c[i].r * (c[i].r *
    t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
for (auto &e : s) {
    if (e.first > w) a += F(e.first) - F(w);
    w = max(w, e.second);
}
}
return a * 0.5;
}

```

## 8.16 Union of Polygons

```

double polyUnion(vector<vector<Pt>> poly) {
    int n = poly.size();
    double ans = 0;
    auto solve = [&](Pt a, Pt b, int cid) {
        vector<pair<Pt, int>> event;
        for (int i = 0; i < n; ++i) {
            int st = 0, sz = poly[i].size();
            while (st < sz && ori(poly[i][st], a, b) != 1) st
                ++;
            if (st == sz) continue;
            for (int j = 0; j < sz; ++j) {
                Pt c = poly[i][(j + st) % sz], d = poly[i][(j +
                    st + 1) % sz];
                if (sign((a - b) ^ (c - d)) != 0) {
                    int ok1 = ori(c, a, b) == 1, ok2 = ori(d, a,
                        b) == 1;
                    if (ok1 ^ ok2) event.emplace_back(LinesInter
                        ({a, b}, {c, d}), ok1 ? 1 : -1);
                } else if (ori(c, a, b) == 0 && sign((a - b) *
                    (c - d)) > 0 && i <= cid) {
                    event.emplace_back(c, -1);
                    event.emplace_back(d, 1);
                }
            }
        }
        sort(all(event), [&](pair<Pt, int> i, pair<Pt,
            int> j) {
            return ((a - i.first) * (a - b)) < ((a - j.first)
                * (a - b));
        });
        int now = 0;
        Pt lst = a;
        for (auto [x, y] : event) {
            if (btw(a, b, lst) && btw(a, b, x) && !now) ans
                += lst ^ x;
            now += y, lst = x;
        }
    };
    for (int i = 0; i < n; ++i) for (int j = 0; j < poly[
        i].size(); ++j) {
        solve(poly[i][j], poly[i][(j + 1) % poly[i].
            size()], i);
    }
    return ans / 2;
}

```

## 8.17 Polar Angle Sort

```

void PolarAngleSort(vector<Pt> &pts) {
    auto pos = [&](Pt a) { return sign(a.y) == 0 ? sign(a.
        x) < 0 : sign(a.y) > 0; };
    sort(all(pts), [&](Pt a, Pt b) { return pos(a) == pos(
        b) ? sign(a ^ b) > 0 : pos(a) < pos(b); });
}

```

## 8.18 Rotating Caliper

```

void RotatingCaliper(vector<Pt> &pts) {
    int n = pts.size();
    for (int i = 0, j = 2; i < n; ++i) {
        int ni = (i + 1) % n;
        while (true) {
            int nj = (j + 1) % n;
            if (area(pts[j], pts[i], pts[ni]) < area(pts[nj],
                pts[i], pts[ni])) {
                j = nj;
            } else {

```

```

                break;
            }
        }
        // do something
    }
}

```

## 8.19 Rotating SweepLine

```

void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> ord(n), cur(n);
    vector<pii> line;
    for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++
        j) if (i ^ j)
        line.emplace_back(i, j);
    auto pos = [&](Pt a) { return sign(a.y) == 0 ? sign(a.
        x) < 0 : sign(a.y) > 0; };
    sort(all(line), [&](pii i, pii j) {
        Pt a = pt[i.second] - pt[i.first], b = pt[j.second]
            - pt[j.first];
        return (pos(a) == pos(b) ? sign(a ^ b) > 0 : pos(a)
            < pos(b));
    });
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) {
        return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt
            [j].x : pt[i].y < pt[j].y);
    });
    for (int i = 0; i < n; ++i) cur[ord[i]] = i;
    for (auto [i, j] : line) {
        // point sort by the distance to Line(i, j)
        tie(cur[i], cur[j], ord[cur[i]], ord[cur[j]]) =
            make_tuple(cur[j], cur[i], j, i);
    }
}

```

## 8.20 Half Plane Intersection

```

vector<Pt> HalfPlaneInter(vector<Line> vec) {
    //      x
    // Line.a -----> Line.b
    int n = vec.size();
    auto pos = [&](Pt a) { return sign(a.y) == 0 ? sign(a.
        x) < 0 : sign(a.y) > 0; };
    sort(all(vec), [&](Line a, Line b) {
        Pt A = a.b - a.a, B = b.b - b.a;
        if (pos(A) == pos(B)) {
            if (sign(A ^ B) == 0) return sign((b.a - a.a) * (
                b.b - a.a)) > 0;
            return sign(A ^ B) > 0;
        }
        return pos(A) < pos(B);
    });
    deque<Pt> inter;
    deque<Line> seg;
    for (int i = 0; i < n; ++i) if (!i || !(vec[i] == vec
        [i - 1])) {
        while (seg.size() >= 2 && sign((vec[i].b - inter.
            back()) ^ (vec[i].a - inter.back())) == 1) seg.
            pop_back(), inter.pop_back();
        while (seg.size() >= 2 && sign((vec[i].b - inter.
            front()) ^ (vec[i].a - inter.front())) == 1)
            seg.pop_front(), inter.pop_front();
        seg.pb(vec[i]);
        if (seg.size() >= 2) inter.pb(LinesInter(seg[seg.
            size() - 2], seg.back()));
    }
    while (seg.size() >= 2 && sign((seg.front().b - inter.
        back()) ^ (seg.front().a - inter.back())) == 1)
        seg.pop_back(), inter.pop_back();
    inter.pb(LinesInter(seg.front(), seg.back()));
    return vector<Pt>(all(inter));
}

```

## 8.21 Minkowski Sum

```

void reorder(vector<Pt> &P) {
    rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
        { return make_pair(a.y, a.x) < make_pair(b.y, b.
            x); }), P.end());
}

```

```
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    // P, Q: convex polygon
    reorder(P), reorder(Q);
    int n = P.size(), m = Q.size();
    P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
    vector<Pt> ans;
    for (int i = 0, j = 0; i < n || j < m; ) {
        ans.pb(P[i] + Q[j]);
        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
        if (val >= 0) i++;
        if (val <= 0) j++;
    }
    return ans;
}
```

## 8.22 Delaunay Triangulation

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
        new(tris++) Tri(pll(-inf, -inf), pll(inf + inf, -
            inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
    }
}
```

```

    }
    assert(0); // "point not found"
}
void add_point(Tri* root, pll const& p) {
    Tri* t[3];
    /* split it into three triangles */
    for (int i = 0; i < 3; ++i)
        t[i] = new(tris++) Tri(root->p[i], root->p[(i +
            1) % 3], p);
    for (int i = 0; i < 3; ++i)
        edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
    for (int i = 0; i < 3; ++i)
        edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
    for (int i = 0; i < 3; ++i)
        root->chd[i] = t[i];
    for (int i = 0; i < 3; ++i)
        flip(t[i], 2);
}
void flip(Tri* tri, int pi) {
    Tri* trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
        pj])) return;
    /* flip edge between tri, trj */
    Tri* trk = new(tris++) Tri(tri->p[(pi + 1) % 3],
        trj->p[pj], tri->p[pi]);
    Tri* trl = new(tris++) Tri(trj->p[(pj + 1) % 3],
        tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->chd[0] = trk; tri->chd[1] = trl; tri->chd[2] =
        0;
    trj->chd[0] = trk; trj->chd[1] = trl; trj->chd[2] =
        0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
}
}
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now->has_chd())
        return triang.pb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}
```

## 8.23 Triangulation Voronoi

```
vector<Line> ls[N];
pll arr[N];
Line make_line(pll p, Line l) {
    pll d = l.Y - l.X; d = perp(d);
    pll m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pll> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1) %
            SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
}
```

```

double rt = 0;
for (int i = 0; i < SZ(ps); ++i)
    rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t->p[i]) != mp.end())
                p.pb(mp[t->p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
    }
}

```

## 9 Else

### 9.1 Bit Hack

```

long long next_perm(long long v) {
    long long t = v | (v - 1);
    return (t + 1) | (((~t & ~t) - 1) >> (__builtin_ctz(
        v) + 1));
}
void subset(long long s) {
    long long sub = s;
    while (sub) sub = (sub - 1) & s;
}

```

### 9.2 Dynamic Programming Condition

#### 9.2.1 Totally Monotone (Concave/Convex)

$$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$$

$$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$$

#### 9.2.2 Monge Condition (Concave/Convex)

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$$

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$$

#### 9.2.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$$

### 9.3 Slope Trick

```

template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
    slope_trick_convex() {left.push(numeric_limits<T>::
        min() / 2), right.push(numeric_limits<T>::max() /
        2);}
    void push_left(T x) {left.push(x - ground_l);}
    void push_right(T x) {right.push(x - ground_r);}
    //add a line with slope 1 to the right starting from
    x
    void add_right(T x) {
        T l = left.top() + ground_l;
        if (l <= x) push_right(x);
        else push_left(x), push_right(l), left.pop(), minn
            += 1 - x;
    }
    //add a line with slope -1 to the left starting from
    x
    void add_left(T x) {
        T r = right.top() + ground_r;
        if (r >= x) push_left(x);
    }
}

```

```

else push_right(x), push_left(r), right.pop(), minn
    += x - r;
}
//val[i]=min(val[j]) for all i-l<=j<=i+r
void expand(T l, T r) {ground_l -= l, ground_r += r;}
void shift_up(T x) {minn += x;}
T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() +
        ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(left.top());
            left.pop();
            if (left.top() + ground_l <= x) {
                res = cur_val + slope * (1 - x);
                break;
            }
            cur_val += slope * (1 - (left.top() + ground_l)
                );
            l = left.top() + ground_l;
            slope += 1;
        }
        for (auto i : trash) left.push(i);
        return res;
    }
    if (x > r) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(right.top());
            right.pop();
            if (right.top() + ground_r >= x) {
                res = cur_val + slope * (x - r);
                break;
            }
            cur_val += slope * ((right.top() + ground_r) -
                r);
            r = right.top() + ground_r;
            slope += 1;
        }
        for (auto i : trash) right.push(i);
        return res;
    }
    assert(0);
}
};

```

### 9.4 Manhattan MST

```

void solve(int n) {
    init();
    vector<int> v(n), ds;
    for (int i = 0; i < n; ++i) {
        v[i] = i;
        ds.push_back(x[i] - y[i]);
    }
    sort(ds.begin(), ds.end());
    ds.resize(unique(ds.begin(), ds.end()) - ds.begin());
    sort(v.begin(), v.end(), [&](int i, int j) { return x
        [i] == x[j] ? y[i] > y[j] : x[i] > x[j]; });
    int j = 0;
    for (int i = 0; i < n; ++i) {
        int p = lower_bound(ds.begin(), ds.end(), x[v[i]] -
            y[v[i]]) - ds.begin() + 1;
        pair<int, int> q = query(p);
        // query return prefix minimum
        if (~q.second) add_edge(v[i], q.second);
        add(p, make_pair(x[v[i]] + y[v[i]], v[i]));
    }
}
void make_graph() {
    solve(n);
    for (int i = 0; i < n; ++i) swap(x[i], y[i]);
    solve(n);
    for (int i = 0; i < n; ++i) x[i] = -x[i];
    solve(n);
    for (int i = 0; i < n; ++i) swap(x[i], y[i]);
    solve(n);
}

```

## 9.5 Dynamic MST

```

int cnt[maxn], cost[maxn], st[maxn], ed[maxn];
pair<int, int> qr[maxn];
// qr[i].first = id of edge to be changed, qr[i].second
// = weight after operation
// cnt[i] = number of operation on edge i
// call solve(0, q - 1, v, 0), where v contains edges i
// such that cnt[i] == 0
void contract(int l, int r, vector<int> v, vector<int>
    &x, vector<int> &y) {
    sort(v.begin(), v.end(), [&](int i, int j) {
        if (cost[i] == cost[j]) return i < j;
        return cost[i] < cost[j];
    });
    djs.save();
    for (int i = 1; i <= r; ++i) djs.merge(st[qr[i].first],
        ed[qr[i].first]);
    for (int i = 0; i < (int)v.size(); ++i) {
        if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
            x.push_back(v[i]);
            djs.merge(st[v[i]], ed[v[i]]);
        }
    }
    djs.undo();
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) djs.merge(st[
        x[i]], ed[x[i]]);
    for (int i = 0; i < (int)v.size(); ++i) {
        if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
            y.push_back(v[i]);
            djs.merge(st[v[i]], ed[v[i]]);
        }
    }
    djs.undo();
}
void solve(int l, int r, vector<int> v, long long c) {
    if (l == r) {
        cost[qr[l].first] = qr[l].second;
        if (st[qr[l].first] == ed[qr[l].first]) {
            printf("%lld\n", c);
            return;
        }
        int minv = qr[l].second;
        for (int i = 0; i < (int)v.size(); ++i) minv = min(
            minv, cost[v[i]]);
        printf("%lld\n", c + minv);
        return;
    }
    int m = (l + r) >> 1;
    vector<int> lv = v, rv = v;
    vector<int> x, y;
    for (int i = m + 1; i <= r; ++i) {
        cnt[qr[i].first]--;
        if (cnt[qr[i].first] == 0) lv.push_back(qr[i].first);
    }
    contract(l, m, lv, x, y);
    long long lc = c, rc = c;
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        lc += cost[x[i]];
        djs.merge(st[x[i]], ed[x[i]]);
    }
    solve(l, m, y, lc);
    djs.undo();
    x.clear(), y.clear();
    for (int i = m + 1; i <= r; ++i) cnt[qr[i].first]++;
    for (int i = 1; i <= m; ++i) {
        cnt[qr[i].first]--;
        if (cnt[qr[i].first] == 0) rv.push_back(qr[i].first);
    }
    contract(m + 1, r, rv, x, y);
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        rc += cost[x[i]];
        djs.merge(st[x[i]], ed[x[i]]);
    }
    solve(m + 1, r, y, rc);
    djs.undo();
    for (int i = 1; i <= m; ++i) cnt[qr[i].first]++;
}

```

```

}

```

## 9.6 ALL LCS

```

void all_lcs(string s, string t) { // 0-base
    vector<int> h(t.size());
    iota(all(h), 0);
    for (int a = 0; a < s.size(); ++a) {
        int v = -1;
        for (int c = 0; c < t.size(); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS[s[0, a], t[b, c]] =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}

```

## 9.7 Hilbert Curve

```

long long hilbertOrder(int x, int y, int pow, int
    rotate) {
    if (pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y <
        hpow) ? 1 : 2);
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    long long subSquareSize = 1ll << (pow * 2 - 2);
    long long ans = seg * subSquareSize;
    long long add = hilbertOrder(nx, ny, pow - 1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}

```

## 9.8 Pbds

```

#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue<int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1); // 4 0
bitset<100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope<int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4

```

## 9.9 Random

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t a) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(i + FIXED_RANDOM);
    }
};
unordered_map<int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());

```

## 9.10 Smawk Algorithm

```

11 query(int l, int r) {
    // ...
}
struct SMAWK {
    // Condition:
    // If  $M[l][0] < M[l][1]$  then  $M[0][0] < M[0][1]$ 
    // If  $M[l][0] == M[l][1]$  then  $M[0][0] \leq M[0][1]$ 
    // For all  $i$ , find  $r_i$  s.t.  $M[i][r_i]$  is maximum //
    // minimum.
    int ans[N], tmp[N];
    void interpolate(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nl;
        for (int i = 1; i < n; i += 2) {
            nl.push_back(l[i]);
        }
        run(nl, r);
        for (int i = 1, j = 0; i < n; i += 2) {
            while (j < m && r[j] < ans[l[i]])
                j++;
            assert(j < m && ans[l[i]] == r[j]);
            tmp[l[i]] = j;
        }
        for (int i = 0; i < n; i += 2) {
            int curl = 0, curr = m - 1;
            if (i)
                curl = tmp[l[i - 1]];
            if (i + 1 < n)
                curr = tmp[l[i + 1]];
            ll res = query(l[i], r[curl]);
            ans[l[i]] = r[curl];
            for (int j = curl + 1; j <= curr; ++j) {
                ll nxt = query(l[i], r[j]);
                if (res < nxt)
                    res = nxt, ans[l[i]] = r[j];
            }
        }
    }
    void reduce(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nr;
        for (int j : r) {
            while (!nr.empty()) {
                int i = nr.size() - 1;
                if (query(l[i], nr.back()) <= query(l[i], j))
                    nr.pop_back();
                else
                    break;
            }
            if (nr.size() < n)
                nr.push_back(j);
        }
        run(l, nr);
    }
    void run(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        if (max(n, m) <= 2) {
            for (int i : l) {
                ans[i] = r[0];
                if (m > 1) {
                    if (query(i, r[0]) < query(i, r[1]))
                        ans[i] = r[1];
                }
            }
        } else if (n >= m) {
            interpolate(l, r);
        } else {
            reduce(l, r);
        }
    }
};

```

- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.12 Python Misc

```

from [decimal, fractions, math, random] import *
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
    ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
print(*[randint(1, C) for i in range(0, N)], sep=' ')

```

## 9.11 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .