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## 1 Basic

### 1.1 Compiler Shell

```

1 if [ $# -ne 2 ] ; then
1     g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
1         $1
1 else
1     g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
1         $1 -fsanitize=address
2 fi
2 ./ $1
2 chmod +x ./run.sh
2 ./run.sh main [1]
3

```

### 1.2 Default Code

```

4 #include <bits/stdc++.h>
4 using namespace std;
5 #define ll long long
5 #define pb push_back
5 #define all(x) x.begin(), x.end()
5

```

### 1.3 Testing Todo List

0. choose editor
1. shell script
2. \_\_int128, \_\_lg, \_\_builtin\_popcount
3. judge speed v.s. local speed
4. CE penalty?

### 1.4 Debug Macro

```

10 void db() {cout << endl;}
10 template <typename T, typename ...U> void db(T i, U ...
10     j) {
11     cout << i << ' ', db(j...);
11 }
11 #define test(x...) db("[ " + string(x) + " ]", x)
11

```

### 1.5 Stress Test Shell

```

12 g++ $1.cpp -o $1
12 g++ $2.cpp -o $2
12 g++ $3.cpp -o $3
13 for i in {1..100} ; do
13     ./ $3 > input.txt
13     # st=$(date +%s%N)
13     ./ $1 < input.txt > output1.txt
13     # echo "$((($(date +%s%N) - $st)/1000000))ms"
14     ./ $2 < input.txt > output2.txt
14     if cmp --silent -- "output1.txt" "output2.txt" ; then
14         continue
14     fi
14     echo Input:
14     cat input.txt
14     echo Your Output:
14     cat output1.txt
14     echo Correct Output:
14     cat output2.txt
14     break
15 done
15 echo OK!
15 ./stress.sh main good gen

```

### 1.6 Pragma

```

17 #pragma GCC optimize("Ofast,inline,unroll-loops")
17 #pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")

```

### 1.7 Fast IO

```

18 #include <unistd.h>
18 char OB[65536]; int OP;
18 inline char RC() {
18     static char buf[65536], *p = buf, *q = buf;
18     return p == q && (q = (p = buf) + read(0, buf, 65536)
18         ) == buf ? -1 : *p++;
18 }
19 inline int R() {
19     static char c;
20

```

```

while((c = RC()) < '0'); int a = c ^ '0';
while((c = RC()) >= '0') a *= 10, a += c ^ '0';
return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}

```

## 2 Data Structure

### 2.1 Leftist Tree

```

struct node {
    ll rk, data, sz, sum;
    node *l, *r;
    node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

### 2.2 Splay Tree

```

struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    ll v[N];
    Splay() {}
    void init() {
        rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
        sz[0] = 1, v[0] = inf;
    }
    int newNode(int p, int x) {
        int id = _id++;
        v[id] = x, pa[id] = p;
        ch[id][0] = ch[id][1] = -1, sz[id] = 1;
        return id;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i, gp = pa[p], c = ch[i][!x];
        sz[p] -= sz[i], sz[i] += sz[p];
        if (~c) sz[p] += sz[c], pa[c] = p;
        ch[p][x] = c, pa[p] = i;
        pa[i] = gp, ch[i][!x] = p;
        if (~gp) ch[gp][ch[gp][1] == p] = i;
    }
    void splay(int i) {
        while (~pa[i]) {
            int p = pa[i];
            if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1] == i ? i : p);
            rotate(i);
        }
        rt = i;
    }
    int lower_bound(int x) {
        int i = rt, last = -1;
        while (true) {
            if (v[i] == x) return splay(i), i;
            if (v[i] > x) {
                last = i;
            }
        }
    }
}

```

```

        if (ch[i][0] == -1) break;
        i = ch[i][0];
    }
    else {
        if (ch[i][1] == -1) break;
        i = ch[i][1];
    }
}
splay(i);
return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newNode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newNode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};

```

### 2.3 Link Cut Tree

```

// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N], sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newNode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0])
            sz[i] += sz[ch[i][0]], mx[i] = max(mx[i], mx[ch[i][0]]);
        if (ch[i][1])
            sz[i] += sz[ch[i][1]], mx[i] = max(mx[i], mx[ch[i][1]]);
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
            rev[i] = false;
    }
    void reverse(int i) {
        if (!i) return;
        swap(ch[i][0], ch[i][1]);
        rev[i] ^= true;
    }
    bool isrt(int i) { // rt of splay
        if (!pa[i]) return true;
        return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i, c = ch[i][!x], gp = pa[p];
        if (ch[gp][0] == p) ch[gp][0] = i;
        else if (ch[gp][1] == p) ch[gp][1] = i;
        pa[i] = gp, ch[i][!x] = p, pa[p] = i;
        ch[p][x] = c, pa[c] = p;
        pull(p), pull(i);
    }
}

```

```

}
void splay(int i) {
    vector<int> anc;
    anc.push_back(i);
    while (!isrt(anc.back())) anc.push_back(pa[anc.back()]);
    while (!anc.empty()) push(anc.back(), anc.pop_back());
    while (!isrt(i)) {
        int p = pa[i];
        if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1] == p ? i : p);
        rotate(i);
    }
}
void access(int i) {
    int last = 0;
    while (i) {
        splay(i);
        if (ch[i][1])
            sz2[i] += sz[ch[i][1]];
        sz2[i] -= sz[last];
        ch[i][1] = last;
        pull(i), last = i, i = pa[i];
    }
}
void makert(int i) {
    access(i), splay(i), reverse(i);
}
void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
}
void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
}
int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
}
};

```

## 2.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)

```

```

        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}

```

## 2.5 Persistent Segment Tree

```

struct Seg {
    // Persistent Segment Tree, single point modify,
    // range query sum
    // 0-indexed, [l, r)
    static Seg mem[M], *pt;
    int l, r, m, val;
    Seg* ch[2];
    Seg() = default;
    Seg(int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
        val(0) {
        if (r - l > 1) {
            ch[0] = new (pt++) Seg(l, m);
            ch[1] = new (pt++) Seg(m, r);
        }
    }
    void pull() { val = ch[0]->val + ch[1]->val; }
    Seg* modify(int p, int v) {
        Seg *now = new (pt++) Seg(*this);
        if (r - l == 1) {
            now->val = v;
        } else {
            now->ch[p >= m] = ch[p >= m]->modify(p, v);
            now->pull();
        }
        return now;
    }
    int query(int a, int b) {
        if (a <= l && r <= b) return val;
        int ans = 0;
        if (a < m) ans += ch[0]->query(a, b);
        if (m < b) ans += ch[1]->query(a, b);
        return ans;
    }
} Seg::mem[M], *Seg::pt = mem;
// Init Tree
Seg *root = new (Seg::pt++) Seg(0, n);

```

## 2.6 2D Segment Tree

```

// 2D range add, range sum in Log^2
struct seg {
    int l, r;

```

```

ll sum, lz;
seg *ch[2]{};
seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
void push() {
    if (lz) ch[0]->add(l, r, lz), ch[1]->modify(l, r,
        lz), lz = 0;
}
void pull() {sum = ch[0]->sum + ch[1]->sum;}
void add(int _l, int _r, ll d) {
    if (_l <= l && r <= _r) {
        sum += d * (r - l + 1);
        lz += d;
        return;
    }
    if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
        new seg(l + r >> 1, r);
    push();
    if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
    pull();
}
ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(l, _l) + 1);
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
}
};

struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2]{};
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N) {}
    if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch[1] =
        new seg2(l + r >> 1, r);
}
void add(int _l, int _r, int _l2, int _r2, ll d) {
    v.add(_l2, _r2, d * (min(r, _r) - max(l, _l) + 1));
    if (_l <= l && r <= _r) {
        lz.add(_l2, _r2, d);
        return;
    }
    if (_l < l + r >> 1) ch[0]->add(_l, _r, _l2, _r2, d);
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, _l2, _r2, d);
}
ll qsum(int _l, int _r, int _l2, int _r2) {
    ll res = v.qsum(_l2, _r2);
    if (_l <= l && r <= _r) return res;
    res += lz.qsum(_l2, _r2) * (min(r, _r) - max(l, _l) + 1);
    if (_l < l + r >> 1) res += ch[0]->query(_l, _r, _l2, _r2);
    if (l + r >> 1 < _r) res += ch[1]->query(_l, _r, _l2, _r2);
    return res;
}
};

```

## 2.7 Zkw

```

ll mx[N << 1], sum[N << 1], lz[N << 1];
void add(int l, int r, ll d) { // [l, r), 0-based
    int len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <<= 1) {
        sum[l] += cntl * d, sum[r] += cntr * d;
        if (len > 1) {
            mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
            mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
        }
        if (~l & 1)
            sum[l ^ 1] += d * len, mx[l ^ 1] += d, lz[l ^ 1]
                += d, cntl += len;
        if (r & 1)
            sum[r ^ 1] += d * len, mx[r ^ 1] += d, lz[r ^ 1]
                += d, cntr += len;
    }
}

```

```

}
sum[l] += cntl * d, sum[r] += cntr * d;
if (len > 1) {
    mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
    mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
}
cntl += cntr;
for (l >>= 1; l; l >>= 1) {
    sum[l] += cntl * d;
    mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
}
}
ll qsum(int l, int r) {
    ll res = 0, len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <<= 1) {
        res += cntl * lz[l] + cntr * lz[r];
        if (~l & 1) res += sum[l ^ 1], cntl += len;
        if (r & 1) res += sum[r ^ 1], cntr += len;
    }
    res += cntl * lz[l] + cntr * lz[r];
    cntl += cntr;
    for (l >>= 1; l; l >>= 1) res += cntl * lz[l];
    return res;
}
ll qmax(int l, int r) {
    ll maxl = -INF, maxr = -INF;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1) {
        maxl += lz[l], maxr += lz[r];
        if (~l & 1) maxl = max(maxl, mx[l ^ 1]);
        if (r & 1) maxr = max(maxr, mx[r ^ 1]);
    }
    maxl = max(maxl + lz[l], maxr + lz[r]);
    for (l >>= 1; l; l >>= 1) maxl += lz[l];
    return maxl;
}
}

```

## 2.8 Chtholly Tree

```

struct ChthollyTree {
    struct interval {
        int l, r;
        ll v;
        interval(int _l, int _r, ll _v) : l(_l), r(_r), v(_v) {}
    };
    struct cmp {
        bool operator () (const interval &a, const interval
            &b) const {
            return a.l < b.l;
        }
    };
    set<interval, cmp> s;
    vector<interval> split(int l, int r) {
        // split into [0, l), [l, r), [r, n) and return [l,
            r)
        vector<interval> del, ans, re;
        auto it = s.lower_bound(interval(l, -1, 0));
        if (it != s.begin() && (it == s.end() || l < it->l)) {
            --it;
            del.pb(*it);
            if (r < it->r) {
                re.pb(interval(it->l, l, it->v));
                ans.pb(interval(l, r, it->v));
                re.pb(interval(r, it->r, it->v));
            } else {
                re.pb(interval(it->l, l, it->v));
                ans.pb(interval(l, it->r, it->v));
            }
            ++it;
        }
        for (; it != s.end() && it->r <= r; ++it) {
            ans.pb(*it);
            del.pb(*it);
        }
        if (it != s.end() && it->l < r) {
            del.pb(*it);
            ans.pb(interval(it->l, r, it->v));
            re.pb(interval(r, it->r, it->v));
        }
    }
}

```

```

    for (interval &i : del)
        s.erase(i);
    for (interval &i : re)
        s.insert(i);
    return ans;
}
void merge(vector <interval> a) {
    for (interval &i : a)
        s.insert(i);
}
};

```

## 2.9 Incremental Min Sum

```

struct IncrementalMinSum {
    multiset <int, greater <int>> in;
    multiset <int> out;
    ll sum; int cap;
    DS () : sum(0), cap(0) {}
    void enlarge() {
        if (!out.empty()) {
            int mx = *out.begin();
            sum += mx, in.insert(mx), out.erase(out.begin());
        }
        cap++;
    }
    void insert(int x) {
        if (!cap) {
            out.insert(x);
            return;
        }
        if (in.size() < cap) {
            in.insert(x), sum += x;
            return;
        }
        int mx = *in.begin();
        if (x < mx) {
            sum -= mx, out.insert(mx), in.erase(in.begin());
            sum += x, in.insert(x);
        } else {
            out.insert(x);
        }
    }
    void erase(int x) {
        if (out.find(x) != out.end()) {
            out.erase(out.lower_bound(x));
        } else {
            in.erase(in.lower_bound(x)), sum -= x;
            if (!out.empty()) {
                int mx = *out.begin();
                sum += mx, out.erase(out.begin()), in.insert(mx);
            }
        }
    }
};

```

## 3 Flow / Matching

### 3.1 Dinic

```

struct Dinic { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> adj[N];
    int s, t, dis[N], cur[N], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)adj[u].size(); ++i) {
            edge &e = adj[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    adj[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
    }
};

```

```

    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int tmp = q.front();
        q.pop();
        for (auto &u : adj[tmp])
            if (!dis[u.to] && u.flow != u.cap) {
                q.push(u.to);
                dis[u.to] = dis[tmp] + 1;
            }
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].clear();
}
void reset() {
    for (int i = 0; i < n; ++i)
        for (auto &j : adj[i]) j.flow = 0;
}
void add_edge(int u, int v, int cap) {
    adj[u].pb(edge{v, cap, 0, (int)adj[v].size()});
    adj[v].pb(edge{u, 0, 0, (int)adj[u].size() - 1});
}
};

```

### 3.2 Min Cost Max Flow

```

template <typename T>
struct MCMF {
    const T INF = 1ll << 60;
    struct edge {
        int v;
        T f, c;
        edge (int _v, T _f, T _c) : v(_v), f(_f), c(_c) {}
    };
    vector <edge> E;
    vector <vector <int>> adj;
    vector <T> dis, pot;
    vector <int> rt;
    int n, s, t;
    MCMF (int _n, int _s, int _t) : n(_n), s(_s), t(_t) {
        adj.resize(n);
    }
    void add_edge(int u, int v, T f, T c) {
        adj[u].pb(E.size()), E.pb(edge(v, f, c));
        adj[v].pb(E.size()), E.pb(edge(u, 0, -c));
    }
    bool SPFA() {
        rt.assign(n, -1), dis.assign(n, INF);
        vector <bool> vis(n, false);
        queue <int> q;
        q.push(s), dis[s] = 0, vis[s] = true;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            vis[v] = false;
            for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
                dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v];
                rt[E[id].v] = id;
                if (!vis[E[id].v]) vis[E[id].v] = true, q.push(E[id].v);
            }
        }
        return dis[t] != INF;
    }
};

```

```

bool dijkstra() {
    rt.assign(n, -1), dis.assign(n, INF);
    priority_queue <pair <T, int>, vector <pair <T, int>
        >>, greater <pair <T, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
        int d, v; tie(d, v) = pq.top(); pq.pop();
        if (dis[v] < d) continue;
        for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
            dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v], rt[E[id].v] = id;
            pq.emplace(dis[E[id].v], E[id].v);
        }
    }
    return dis[t] != INF;
}

pair <T, T> solve() {
    pot.assign(n, 0);
    T cost = 0, flow = 0;
    bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; i++) {
            dis[i] += pot[i] - pot[s];
        }
        T add = INF;
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            add = min(add, E[rt[i]].f);
        }
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
        }
        flow += add, cost += add * dis[t];
        fr = false;
        swap(dis, pot);
    }
    return make_pair(flow, cost);
}
};

```

### 3.3 Kuhn Munkres

```

template <typename T>
struct KM { // 0-based
    T w[N][N], hl[N], hr[N], slk[N];
    T fl[N], fr[N], pre[N]; int n;
    bool vl[N], vr[N];
    const T INF = 1e9;
    queue <int> q;
    KM(int _n) : n(_n) {
        for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)
            w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x]) return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill(slk, slk + n, INF), fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        q.push(s), vr[s] = 1;
        while (1) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {

```

```

                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x) if (!vl[x] && !slk[x] && !check(x)) return;
        }
    }
    T solve() {
        fill(fl, fl + n, -1), fill(fr, fr + n, -1), fill(hr, hr + n, 0);
        for (int i = 0; i < n; ++i) hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) bfs(i);
        T res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

### 3.4 SW Min Cut

```

template <typename T>
struct SW { // 0-based
    T g[N][N], sum[N]; int n;
    bool vis[N], dead[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) fill(g[i], g[i] + n, 0);
        fill(dead, dead + n, false);
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w;
    }
    T solve() {
        T ans = 1 << 30;
        for (int round = 0; round + 1 < n; ++round) {
            fill(vis, vis + n, false), fill(sum, sum + n, 0);
            int num = 0, s = -1, t = -1;
            while (num < n - round) {
                int now = -1;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    if (now == -1 || sum[now] < sum[i]) now = i;
                }
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    sum[i] += g[now][i];
                }
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i) {
                g[i][s] += g[i][t];
                g[s][i] += g[t][i];
            }
            dead[t] = true;
        }
        return ans;
    }
};

```

### 3.5 Gomory Hu Tree

```

vector <array <int, 3>> GomoryHu(vector <vector <pii>>
    adj, int n) {
    // Tree edge min -> mincut (0-based)
    Dinic flow(n);
    for (int i = 0; i < n; ++i) for (auto [j, w] : adj[i])
        flow.add_edge(i, j, w);
    flow.record();
    vector <array <int, 3>> ans;
    vector <int> rt(n);
    for (int i = 0; i < n; ++i) rt[i] = 0;
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset(); // clear flows on all edge
        ans.push_back({i, t, flow.solve(i, t)});
    }
}

```

```

    flow.runbfs(i);
    for (int j = i + 1; j < n; ++j) if (rt[j] == t &&
        flow.vis[j]) {
        rt[j] = i;
    }
}
return ans;
}

```

### 3.6 Blossom

```

struct Matching { // 0-based
    int fa[N], pre[N], match[N], s[N], v[N], n, tk;
    vector<int> g[N];
    queue<int> q;
    Matching(int _n) : n(_n), tk(0) {
        for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
        for (int i = 0; i < n; ++i) g[i].clear();
    }
    void add_edge(int u, int v) {
        g[u].push_back(v), g[v].push_back(u);
    }
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (v[x] == tk) return x;
                v[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
    void blossom(int x, int y, int l) {
        while (Find(x) != l) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            if (fa[x] == x) fa[x] = 1;
            if (fa[y] == y) fa[y] = 1;
            x = pre[y];
        }
    }
    bool bfs(int r) {
        for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
        while (!q.empty()) q.pop();
        q.push(r);
        s[r] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int u : g[x]) {
                if (s[u] == -1) {
                    pre[u] = x, s[u] = 1;
                    if (match[u] == n) {
                        for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;
                        return true;
                    }
                    q.push(match[u]);
                    s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = lca(u, x);
                    blossom(x, u, l);
                    blossom(u, x, l);
                }
            }
        }
        return false;
    }
    int solve() {
        int res = 0;
        for (int x = 0; x < n; ++x) {
            if (match[x] == n) res += bfs(x);
        }
        return res;
    }
};

```

### 3.7 Weighted Blossom

```

struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w) : u(u), v(v), w(w) {}
    };
    int n, n_x;
    edge g[maxn * 2][maxn * 2];
    int lab[maxn * 2];
    int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
        pa[maxn * 2];
    int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
        maxn * 2];
    vector<int> flo[maxn * 2];
    queue<int> q;
    int e_delta(const edge &e) { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slack(int u, int x) { if (!slack[x] ||
        e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
        [x] = u; }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (size_t i = 0; i < flo[x].size(); i++)
            q_push(flo[x][i]);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (size_t i = 0; i < flo[x].size(); ++
            i) set_st(flo[x][i], b);
    }
    int get_pr(int b, int xr) {
        int pr = find(flo[b].begin(), flo[b].end(), xr) -
            flo[b].begin();
        if (pr % 2 == 1) {
            reverse(flo[b].begin() + 1, flo[b].end());
            return (int)flo[b].size() - pr;
        }
        return pr;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        edge e = g[u][v];
        int xr = flo_from[u][e.u], pr = get_pr(u, xr);
        for (int i = 0; i < pr; ++i) set_match(flo[u][i],
            flo[u][i ^ 1]);
        set_match(xr, v);
        rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
            end());
    }
    void augment(int u, int v) {
        for (; ; ) {
            int xnv = st[match[u]];
            set_match(u, v);
            if (!xnv) return;
            set_match(xnv, st[pa[xnv]]);
            u = st[pa[xnv]], v = xnv;
        }
    }
    int get_lca(int u, int v) {
        static int t = 0;
        for (++t; u || v; swap(u, v)) {
            if (u == 0) continue;
            if (vis[u] == t) return u;
            vis[u] = t;
            u = st[match[u]];
            if (u) u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u, int lca, int v) {
        int b = n + 1;
    }
};

```



```

while (b <= n_x && st[b]) ++b;
if (b > n_x) ++n_x;
lab[b] = 0, S[b] = 0;
match[b] = match[lca];
flo[b].clear();
flo[b].push_back(lca);
for (int x = u, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[
        match[x]]), q_push(y);
reverse(flo[b].begin() + 1, flo[b].end());
for (int x = v, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[
        match[x]]), q_push(y);
set_st(b, b);
for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
    w = 0;
for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
for (size_t i = 0; i < flo[b].size(); ++i) {
    int xs = flo[b][i];
    for (int x = 1; x <= n_x; ++x)
        if (g[b][x].w == 0 || e_delta(g[xs][x]) <
            e_delta(g[b][x]))
            g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
}
set_slack(b);
}
void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
}
bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
            q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
    }
    int d = inf;

```

```

for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1) d = min(d, lab[b]
        / 2);
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x]) {
        if (S[x] == -1) d = min(d, e_delta(g[slack[x]]
            [x]));
        else if (S[x] == 0) d = min(d, e_delta(g[
            slack[x]][x]) / 2);
    }
for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
        if (lab[u] <= d) return 0;
        lab[u] -= d;
    } else if (S[st[u]] == 1) lab[u] += d;
}
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b) {
        if (S[st[b]] == 0) lab[b] += d * 2;
        else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
q = queue<int>();
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x
        && e_delta(g[slack[x]][x]) == 0)
        if (on_found_edge(g[slack[x]][x])) return
            true;
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand_blossom(b);
}
return false;
}
pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
    g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
}
};

```

### 3.8 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source  $S$  and sink  $T$ .
  2. For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  3. For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  5. The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.



- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  1. Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  2. DFS from unmatched vertices in  $X$ .
  3.  $x \in X$  is chosen iff  $x$  is unvisited.
  4.  $y \in Y$  is chosen iff  $y$  is visited.
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer  $T$
  2. Construct a max flow model, let  $K$  be the sum of all weights
  3. Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$
  4. For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  5. For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6.  $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  1. For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  2. Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  3. Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  1. If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  2. Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  3. The mincut is equivalent to the maximum profit of a subset of projects.

- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

1. Create edge  $(x, t)$  with capacity  $c_x$  and create edge  $(s, y)$  with capacity  $c_y$ .
2. Create edge  $(x, y)$  with capacity  $c_{xy}$ .
3. Create edge  $(x, y)$  and edge  $(x', y')$  with capacity  $c_{xyx'y'}$ .

## 4 Graph

### 4.1 Heavy-Light Decomposition

```
vector<int> dep, pa, sz, ch, hd, id;
int _id;
void dfs(int i, int p) {
    dep[i] = ~p ? dep[p] + 1 : 0;
    pa[i] = p, sz[i] = 1, ch[i] = -1;
    for (int j : g[i])
        if (j != p) {
            dfs(j, i);
            if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
            sz[i] += sz[j];
        }
}
void hld(int i, int p, int h) {
    hd[i] = h;
    id[i] = _id++;
    if (~ch[i]) hld(ch[i], i, h);
    for (int j : g[i]) if (j != p && j != ch[i])
        hld(j, i, j);
}
void query(int i, int j) {
    while (hd[i] != hd[j]) {
        if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
        query2(id[hd[i]], id[i] + 1, i = pa[hd[i]]);
    }
    if (dep[i] < dep[j]) swap(i, j);
    query2(id[j], id[i] + 1);
}
```

### 4.2 Centroid Decomposition

```
vector<vector<int>> dis;
vector<int> pa, sz;
vector<bool> vis;
void dfs_sz(int i, int p) {
    sz[i] = 1;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_sz(j, i), sz[i] += sz[j];
}
void cen(int i, int p, int _n) {
```

```
    for (int j : g[i]) if (j != p && !vis[j] && sz[j] >
        _n / 2)
        return cen(j, i, _n);
    return i;
}
void dfs_dis(int i, int p, int d) { // from i to
    ancestor with depth d
    dis[i][d] = ~p ? dis[p][d] + 1 : 0;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
    dfs_sz(i), i = cen(i, -1, sz[i]);
    vis[i] = true, pa[i] = p;
    dfs_dis(i, -1, d);
    for (int j : g[i]) if (!vis[j])
        cd(j, i, d + 1);
}
```

### 4.3 Edge BCC

```
vector<int> low, dep, bcc_id, stk;
vector<bool> vis;
int _id;
void dfs(int i, int p) {
    low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    vis[i] = true;
    for (int j : g[i])
        if (j != p) {
            if (!vis[j])
                dfs(j, i), low[i] = min(low[i], low[j]);
            else
                low[i] = min(low[i], dep[j]);
        }
    if (low[i] == dep[i]) {
        int id = _id++;
        while (stk.back() != i) {
            int x = stk.back();
            stk.pop_back();
            bcc_id[x] = id;
        }
        stk.pop_back();
        bcc_id[i] = id;
    }
}
```

### 4.4 Block Cut Tree

```
vector<vector<int>> g, _g;
vector<int> dep, low, stk;
void dfs(int i, int p) {
    dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    for (int j : g[i]) if (j != p) {
        if (dep[j] == -1) {
            dfs(j, i), low[i] = min(low[i], low[j]);
            if (low[j] >= dep[i]) {
                int id = _g.size();
                _g.emplace_back();
                while (stk.back() != j) {
                    int x = stk.back();
                    stk.pop_back();
                    _g[x].push_back(id), _g[id].push_back(x);
                }
                stk.pop_back();
                _g[j].push_back(id), _g[id].push_back(j);
                _g[i].push_back(id), _g[id].push_back(i);
            }
        } else low[i] = min(low[i], dep[j]);
    }
}
```

### 4.5 SCC / 2SAT

```
struct SAT {
    vector<vector<int>> g;
    vector<int> dep, low, scc_id;
    vector<bool> is;
    vector<int> stk;
    int n, _id, _t;
    SAT() {}
```

```

void init(int _n) {
    n = _n, _id = _t = 0;
    g.assign(2 * n, vector<int>());
    dep.assign(2 * n, -1), low.assign(2 * n, -1);
    scc_id.assign(2 * n, -1), is.assign(2 * n, false);
    stk.clear();
}
void add_edge(int x, int y) {g[x].push_back(y);}
int rev(int i) {return i < n ? i + n : i - n;}
void add_ifthen(int x, int y) {add_clause(rev(x), y);}
void add_clause(int x, int y) {
    add_edge(rev(x), y);
    add_edge(rev(y), x);
}
void dfs(int i) {
    dep[i] = low[i] = _t++;
    stk.push_back(i);
    for (int j : g[i])
        if (scc_id[j] == -1) {
            if (dep[j] == -1)
                dfs(j);
            low[i] = min(low[i], low[j]);
        }
    if (low[i] == dep[i]) {
        int id = _id++;
        while (stk.back() != i) {
            int x = stk.back();
            stk.pop_back();
            scc_id[x] = id;
        }
        stk.pop_back();
        scc_id[i] = id;
    }
}
bool solve() {
    for (int i = 0; i < 2 * n; ++i)
        if (dep[i] == -1)
            dfs(i);
    for (int i = 0; i < n; ++i) {
        if (scc_id[i] == scc_id[i + n]) return false;
        if (scc_id[i] < scc_id[i + n])
            is[i] = true;
        else
            is[i + n] = true;
    }
    return true;
}
};

```

## 4.6 Negative Cycle

```

vector<pair<int, long long>> adj[N];
template<typename T>
struct NegativeCycle {
    vector<T> dis;
    vector<int> rt;
    int n; T INF;
    vector<int> cycle;
    NegativeCycle() = default;
    NegativeCycle(int _n) : n(_n), INF(numeric_limits<T>::max()) {
        dis.assign(n, 0), rt.assign(n, -1);
        int relax = -1;
        for (int t = 0; t < n; ++t) {
            relax = -1;
            for (int i = 0; i < n; ++i) {
                for (auto [j, w] : adj[i]) if (dis[j] > dis[i] + w) {
                    dis[j] = dis[i] + w, rt[j] = i;
                    relax = j;
                }
            }
        }
        if (relax != -1) {
            int s = relax;
            for (int i = 0; i < n; ++i) s = rt[s];
            vector<bool> vis(n, false);
            while (!vis[s]) {
                cycle.push_back(s), vis[s] = true;
                s = rt[s];
            }
        }
    }
};

```

```

        reverse(cycle.begin(), cycle.end());
    }
};

```

## 4.7 Virtual Tree

```

vector<vector<int>> _g;
vector<int> st, ed, stk;
void solve(vector<int> v) {
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    int sz = v.size();
    for (int i = 0; i < sz - 1; ++i)
        v.push_back(lca(v[i], v[i + 1]));
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    v.resize(unique(all(v)) - v.begin());
    stk.clear(); stk.push_back(v[0]);
    for (int i = 1; i < v.size(); ++i) {
        int x = v[i];
        while (ed[stk.back()] < ed[x]) stk.pop_back();
        _g[stk.back()].push_back(x), stk.push_back(x);
    }
    // do something
    for (int i : v) _g[i].clear();
}

```

## 4.8 Directed MST

```

template<typename T> struct DMST { // 1-based
    T g[maxn][maxn], fw[maxn];
    int n, fr[maxn];
    bool vis[maxn], inc[maxn];
    void clear() {
        for (int i = 0; i < maxn; ++i) {
            for (int j = 0; j < maxn; ++j) g[i][j] = inf;
            vis[i] = inc[i] = false;
        }
    }
    void addedge(int u, int v, T w) {
        g[u][v] = min(g[u][v], w);
    }
    T query(int root, int _n) {
        n = _n;
        if (dfs(root) != n) return -1;
        T ans = 0;
        while (true) {
            for (int i = 1; i <= n; ++i) fw[i] = inf, fr[i] = i;
            for (int i = 1; i <= n; ++i) if (!inc[i]) {
                for (int j = 1; j <= n; ++j) {
                    if (!inc[j] && i != j && g[j][i] < fw[i]) {
                        fw[i] = g[j][i];
                        fr[i] = j;
                    }
                }
            }
            int x = -1;
            for (int i = 1; i <= n; ++i) if (i != root && !inc[i]) {
                int j = i, c = 0;
                while (j != root && fr[j] != i && c <= n) ++c, j = fr[j];
                if (j == root || c > n) continue;
                else { x = i; break; }
            }
            if (!~x) {
                for (int i = 1; i <= n; ++i) if (i != root && !inc[i]) ans += fw[i];
                return ans;
            }
            int y = x;
            for (int i = 1; i <= n; ++i) vis[i] = false;
            do { ans += fw[y]; y = fr[y]; vis[y] = inc[y] = true; } while (y != x);
            inc[x] = false;
            for (int k = 1; k <= n; ++k) if (vis[k]) {
                for (int j = 1; j <= n; ++j) if (!vis[j]) {
                    if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
                    if (g[j][k] < inf && g[j][k] - fw[k] < g[j][x]) g[j][x] = g[j][k] - fw[k];
                }
            }
        }
    }
};

```

```

    }
    }
    return ans;
}
int dfs(int now) {
    int r = 1;
    vis[now] = true;
    for (int i = 1; i <= n; ++i) if (g[now][i] < inf && !vis[i]) r += dfs(i);
    return r;
}
};

```

## 4.9 Dominator Tree

```

struct Dominator_tree {
    int n, id;
    vector<vector<int>> adj, radj, bucket;
    vector<int> sdom, dom, vis, rev, par, rt, mn;
    Dominator_tree(int _n) : n(_n), id(0) {
        adj.resize(n), radj.resize(n), bucket.resize(n);
        sdom.resize(n), dom.resize(n, -1), vis.resize(n, -1);
        rev.resize(n), rt.resize(n), mn.resize(n), par.resize(n);
    }
    void add_edge(int u, int v) {adj[u].pb(v);}
    int query(int v, bool x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], true);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]]) mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : adj[v]) {
            if (vis[u] == -1) dfs(u), par[vis[u]] = vis[v];
            radj[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : radj[i]) {
                sdom[i] = min(sdom[i], sdom[query(u, false)]);
            }
            if (i) bucket[sdom[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, false);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = par[i];
        }
        vector<int> res(n, -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i) res[rev[i]] = rev[dom[i]];
        res[s] = s;
        dom = res;
    }
};

```

## 5 String

### 5.1 Aho-Corasick Automaton

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    vector<int> g[N];
    int cnt[N];
    AC() {sz = 0, extend();}
    void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
    }
};

```

```

return ch[u][v];
}
int insert(string s) {
    int now = 0;
    for (char c : s) now = nxt(now, c - 'a');
    cnt[now]++;
    return now;
}
void build_fail() {
    queue<int> q;
    for (int i = 0; i < 26; ++i) if (ch[0][i]) {
        to[0][i] = ch[0][i];
        q.push(ch[0][i]);
        g[0].push_back(ch[0][i]);
    }
    while (!q.empty()) {
        int v = q.front(); q.pop();
        for (int j = 0; j < 26; ++j) {
            to[v][j] = ch[v][j] ? ch[v][j] : to[fail[v]][j];
        }
        for (int i = 0; i < 26; ++i) if (ch[v][i]) {
            int u = ch[v][i], k = fail[v];
            while (k && !ch[k][i]) k = fail[k];
            if (ch[k][i]) k = ch[k][i];
            fail[u] = k;
            cnt[u] += cnt[k], g[k].push_back(u);
            q.push(u);
        }
    }
}
int match(string &s) {
    int now = 0, ans = 0;
    for (char c : s) {
        now = to[now][c - 'a'];
        if (ch[now][c - 'a']) now = ch[now][c - 'a'];
        ans += cnt[now];
    }
    return ans;
}
};

```

### 5.2 KMP Algorithm

```

vector<int> build_fail(string s) {
    vector<int> f(s.length() + 1, 0);
    int k = 0;
    for (int i = 1; i < s.length(); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}
int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < s.length(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == t.length()) ans++, k = f[k];
    }
    return ans;
}

```

### 5.3 Z Algorithm

```

vector<int> build(string s) {
    int n = s.length();
    vector<int> Z(n);
    int l = 0, r = 0;
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < s.size() && s[Z[i]] == s[i + Z[i]]) {
            l = i, r = i + Z[i], Z[i]++;
        }
    }
    return Z;
}

```

### 5.4 Manacher

```
vector<int> manacher(string &s) {
    string t = "^#";
    for (char c : s) t += c, t += '#';
    t += '&';
    int n = t.length();
    vector<int> r(n, 0);
    int C = 0, R = 0;
    for (int i = 1; i < n - 1; ++i) {
        int mirror = 2 * C - i;
        r[i] = (i < R ? min(r[mirror], R - i) : 0);
        while (t[i - 1 - r[i]] == t[i + 1 + r[i]]) r[i]++;
        if (i + r[i] > R) R = i + r[i], C = i;
    }
    return r;
}
```

## 5.5 Suffix Array

```
int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.length();
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++] = sa[i] - k;
        for (int i = n - 1; ~i; --i) sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!((x[a] == x[b] && a + k < n && b + k < n && x[a + k] == x[b + k]))) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}
void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = min(lcp[rk[i] + 1], lcp[rk[i] + 2], ..., lcp[rk[j]])
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i] == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}
```

## 5.6 SAIS

```
namespace sfx {
    bool _t[N * 2];
    int SA[N * 2], H[N], RA[N];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
    void pre(int *sa, int *c, int n, int z) {
        fill_n(sa, n, 0), copy_n(c, z, x);
    }
    void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; ++i) if (sa[i] && !t[sa[i] - 1]) sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; --i) if (sa[i] && t[sa[i] - 1]) sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
    }
}
```

```
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || !equal(s + sa[i], s + p[q[sa[i]] + 1], s + last);
            ns[q[last = sa[i]]] = nmzx += neq;
        }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
vector<int> build(int *s, int n) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    for (int i = 0; i < n; ++i)
        sa[i] = SA[i + 1];
    return sa;
}
```

## 5.7 Suffix Automaton

```
struct SAM {
    int ch[N][26], len[N], link[N], cnt[N], sz;
    // link -> suffix endpos
    SAM() {len[0] = 0, link[0] = -1, sz = 1;}
    void build(string s) {
        int last = 0;
        for (char c : s) {
            int cur = sz++;
            len[cur] = len[last] + 1;
            int p = last;
            while (~p && !ch[p][c - 'a']) ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) {
                link[cur] = 0;
            } else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q];
                    for (int j = 0; j < 26; ++j) ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q) ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
            cnt[cur]++;
            last = cur;
        }
        vector<int> p(sz);
        iota(all(p), 0);
        sort(all(p), [&](int i, int j) {return len[i] > len[j]});
    }
}
```

```

    for (int i = 0; i < sz; ++i) cnt[link[p[i]]] += cnt[p[i]];
}
};

```

## 5.8 Minimum Rotation

```

string rotate(const string &s) {
    int n = s.length();
    string t = s + s;
    int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

## 5.9 Palindrome Tree

```

struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM(string _s) : s(_s) {
        sz = 0;
        extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst] - 1] != s[i]) lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
                int now = fail[lst];
                while (s[i - len[now] - 1] != s[i]) now = fail[now];
                fail[idx] = ch[now][s[i] - 'a'];
                ch[lst][s[i] - 'a'] = idx;
            }
            lst = ch[lst][s[i] - 'a'], cnt[lst]++;
        }
        void build_count() {
            for (int i = sz - 1; i > 1; --i)
                cnt[fail[i]] += cnt[i];
        }
        int extend() {
            fill(ch[sz], ch[sz] + 26, 0), sz++;
            return sz - 1;
        }
    }
};

```

## 5.10 Main Lorentz

```

int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, Len.
// substr(L ~ r, Len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l), sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i) to_right[i] = Z[r - m + 1 + i - 1];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = 1; i < m; ++i) to_left[i] = Z[m - i - 1];
    reverse(all(sl));
    for (int i = 1; i + 1 < m; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1], len = m - i - 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2), tr = min(len - 1, k1);

```

```

        if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
    }
    Z = buildZ(sr);
    for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
    reverse(all(sl)), reverse(all(sr));
    Z = buildZ(sl + "#" + sr);
    for (int i = m; i < r; ++i) to_left[i] = Z[m - 1 + 1 + r - i - 1];
    reverse(all(sl)), reverse(all(sr));
    for (int i = m; i + 1 < r; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1], len = i - m + 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(len - k2, 1), tr = min(len - 1, k1);
        if (tl <= tr) rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
    }
    Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i) {
        if (Z[r - m + 1 + i - 1] >= m - i) {
            rep.pb({i, i, m - i});
        }
    }
}

```

## 6 Math

### 6.1 Fraction

```

struct fraction {
    ll n, d;
    fraction(const ll _n=0, const ll _d=1): n(_n), d(_d) {
        {
            ll t = gcd(n, d);
            n /= t, d /= t;
            if (d < 0) n = -n, d = -d;
        }
    }
    fraction operator-(const fraction &b) const {
        return fraction(-n, d);
    }
    fraction operator+(const fraction &b) const {
        return fraction(n * b.d + b.n * d, d * b.d);
    }
    fraction operator-(const fraction &b) const {
        return fraction(n * b.d - b.n * d, d * b.d);
    }
    fraction operator*(const fraction &b) const {
        return fraction(n * b.n, d * b.d);
    }
    fraction operator/(const fraction &b) const {
        return fraction(n * b.d, d * b.n);
    }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};

```

### 6.2 Miller Rabin / Pollard Rho

```

ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long double)x / p * y) * p + p) % p;}
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
ll Pow(ll a, ll b, ll n) {ll res = 1; for (; b; b >>= 1, a = mul(a, a, n)) if (b & 1) res = mul(res, a, n); return res;}
bool check(ll a, ll d, int s, ll n) {
    a = Pow(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};

```

```

11 FindFactor(11 n) {
    if (IsPrime(n)) return 1;
    for (11 p : small) if (n % p == 0) return p;
    11 x, y = 2, d, t = 1;
    auto f = [&](11 a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            11 g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<11, int> PollardRho(11 n) {
    map<11, int> res;
    if (n == 1) return res;
    if (IsPrime(n)) return ++res[n], res;
    11 d = FindFactor(n);
    res = PollardRho(n / d);
    auto res2 = PollardRho(d);
    for (auto [x, y] : res2) res[x] += y;
    return res;
}

```

### 6.3 Ext GCD

```

//a * p.first + b * p.second = gcd(a, b)
pair<11, 11> extgcd(11 a, 11 b) {
    pair<11, 11> res;
    if (a < 0) {
        res = extgcd(-a, b);
        res.first *= -1;
        return res;
    }
    if (b < 0) {
        res = extgcd(a, -b);
        res.second *= -1;
        return res;
    }
    if (b == 0) return {1, 0};
    res = extgcd(b, a % b);
    return {res.second, res.first - res.second * (a / b)};
}

```

### 6.4 PiCount

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i]) for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.push_back(i);
}
void init(){
    sieve(V);
    small_pi[0] = 0;
    for(int i = 1; i < V; ++i) small_pi[i] = small_pi[i - 1] + isp[i];
    for(int i = 0; i < M; ++i) dp[0][i] = i;
    for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
        dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i - 1]];
}
11 phi(11 n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;

```

```

    if(((11)primes[a - 1]) * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    11 de = phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
    return de;
}
11 PiCount(11 n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a = small_pi[y];
    11 res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n / primes[a]) - PiCount(primes[a]) + 1, 0);
    return res;
}

```

### 6.5 Linear Function Mod Min

```

11 topos(11 x, 11 m) {x %= m; if (x < 0) x += m; return x;}
//min value of ax + b (mod m) for x \in [0, n - 1]. O(log m)
11 min_rem(11 n, 11 m, 11 a, 11 b) {
    a = topos(a, m), b = topos(b, m);
    for (11 g = __gcd(a, m); g > 1;) return g * min_rem(n / g, a / g, b / g) + (b % g);
    for (11 nn, nm, na, nb; a; n = nn, m = nm, a = na, b = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            11 lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1], also return min x to get the value. O(log m)
//{value, x}
pair<11, 11> min_rem_pos(11 n, 11 m, 11 a, 11 b) {
    a = topos(a, m), b = topos(b, m);
    11 mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    11 x = (extgcd(a, m).first + m) * ((mn - b + m) / g) % (m / g);
    return {mn, x};
}

```

### 6.6 Floor Sum

```

// sum^{n-1}_{0} floor((a * i + b) / m) in log(n + m + a + b)
11 floor_sum(11 n, 11 m, 11 a, 11 b) {
    11 ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    11 y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

### 6.7 Simplex

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]. 1 based
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
}

```



```

bool ls(T a, T b) {return a < b && !eq(a, b);}
void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
    for (int i = 0; i < m; ++i) for (int j = 0; j < n; ++j) a[i][j] = 0;
    for (int i = 0; i < m; ++i) b[i] = 0;
    for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
}
void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector<int> nz;
    for (int i = 0; i < n; ++i) {
        a[x][i] /= k;
        if (!eq(a[x][i], 0)) nz.push_back(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
        if (i == x || eq(a[i][y], 0)) continue;
        k = a[i][y], a[i][y] = 0;
        b[i] -= k * b[x];
        for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
}
// 0: found solution, 1: no feasible solution, 2:
// unbounded
int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (1) {
        int x = -1, y = -1;
        for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
        if (x == -1) break;
        for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
        if (y == -1) return 1;
        pivot(x, y);
    }
    while (1) {
        int x = -1, y = -1;
        for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
        if (y == -1) break;
        for (int i = 0; i < m; ++i) if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
        if (x == -1) return 2;
        pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n) sol[Left[i]] = b[i];
    return 0;
}
};

```

## 6.8 Linear Programming Construction

Standard form: maximize  $\mathbf{c}^T \mathbf{x}$  subject to  $A\mathbf{x} \leq \mathbf{b}$  and  $\mathbf{x} \geq 0$ .

Dual LP: minimize  $\mathbf{b}^T \mathbf{y}$  subject to  $A^T \mathbf{y} \geq \mathbf{c}$  and  $\mathbf{y} \geq 0$ .

$\mathbf{x}$  and  $\mathbf{y}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.9 Theorem

### • Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .

- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

### • Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

### • Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are

$$\frac{(n-2)!}{(d_1-1)!(d_2-1)! \cdots (d_n-1)!}$$

spanning trees.

- Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

### • Erdős-Gallai Theorem

A sequence of non-negative integers  $d_1 \geq d_2 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if  $d_1 + d_2 + \dots + d_n$  is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all  $1 \leq k \leq n$ .

### • Burnside's Lemma

Let  $X$  be a set and  $G$  be a group that acts on  $X$ . For  $g \in G$ , denote by  $X^g$  the elements fixed by  $g$ :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

### • Gale-Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k b_i$  holds for every  $1 \leq k \leq n$ .

### • Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

### • Möbius inversion formula

- $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
- $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

### • Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r-h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos \theta)(1 - \cos \theta)^2/3$ .
- Area  $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos \theta)$ .

### • Chinese Remainder Theorem

- $x \equiv a_i \pmod{m_i}$
- $M = \prod m_i, M_i = M/m_i$
- $t_i M_i \equiv 1 \pmod{m_i}$
- $x = \sum a_i t_i M_i \pmod{M}$

## 7 Geometry

### 7.1 Basic

```
int sign(double x) {return abs(x) <= eps ? 0 : (x > 0 ?
    1 : -1);}
struct Pt {
    double x, y;
    Pt (double _x, double _y) : x(_x), y(_y) {}
    Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
    Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
    Pt operator * (double k) {return Pt(x * k, y * k);}
    Pt operator / (double k) {return Pt(x / k, y / k);}
    double operator * (Pt o) {return x * o.x + y * o.y;}
    double operator ^ (Pt o) {return x * o.y - y * o.x;}
};
double abs2(Pt o) {return o.x * o.x + o.y * o.y;}
double abs(Pt o) {return sqrt(abs2(o));}
int ori(Pt o, Pt a, Pt b) {return sign((o - a) ^ (o - b));}
bool btw(Pt a, Pt b, Pt c) { // c on segment ab?
    return ori(a, b, c) == 0 && sign((c - a) * (c - b))
        <= 0;
}
double area(Pt a, Pt b, Pt c) {return abs((a - b) ^ (a - c)) / 2;}
Pt proj_vector(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / ((b - a) * (b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vector(a, b, c) + a;
}
struct Line {
    Pt a, b;
};
struct Cir {
    Pt o; double r;
};
```

### 7.2 Intersection of Segments

```
Pt LinesInter(Line a, Line b) {
    double abc = (a.b - a.a) ^ (b.a - a.a);
    double abd = (a.b - a.a) ^ (b.b - a.a);
    if (sign(abc - abd) == 0) return b.b; // no inter
    return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
    if (btw(a.a, a.b, b.a)) return {b.a};
    if (btw(a.a, a.b, b.b)) return {b.b};
    if (btw(b.a, b.b, a.a)) return {a.a};
    if (btw(b.a, b.b, a.b)) return {a.b};
    if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 &&
        ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
        return {LinesInter(a, b)};
    return {};
}
```

### 7.3 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a))
        / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
}
```

### 7.4 Intersection of Circles

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r * c2.r - c1.r * c1.r) / (2 * d2));
```

```
double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
    * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2 * d2);
if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
return {u + v, u - v};
}
```

### 7.5 Convex Hull

```
vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2], ans.back(), pt[i]) <= 0)
                ans.pop_back();
            ans.push_back(pt[i]);
        }
        reverse(all(pt));
    }
    ans.pop_back();
    return ans;
}
```

### 7.6 PolarAngle Sort

```
void PolarAngleSort(vector<Pt> &pts) {
    auto pos = [&](Pt a) {return sign(a.y) == 0 ? sign(a.x) < 0 : sign(a.y) > 0;};
    sort(all(pts), [&](Pt a, Pt b) {return pos(a) == pos(b) ? sign(a ^ b) > 0 : pos(a) < pos(b);});
}
```

### 7.7 Rotating Caliper

```
void RotatingCaliper(vector<Pt> &pts) {
    int n = pts.size();
    for (int i = 0, j = 2; i < n; ++i) {
        int ni = (i + 1) % n;
        while (true) {
            int nj = (j + 1) % n;
            if (area(pts[j], pts[i], pts[ni]) < area(pts[nj], pts[i], pts[ni])) {
                j = nj;
            } else {
                break;
            }
        }
        // do something
    }
}
```

### 7.8 Rotating SweepLine

```
void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> id(n), pos(n);
    vector<pair<int, int>> line;
    for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j) if (i ^ j) line.emplace_back(i, j);
    sort(line.begin(), line.end(), [&](pair<int, int> i, pair<int, int> j) {
        Pt a = pt[i.second] - pt[i.first], b = pt[j.second] - pt[j.first];
        return (a.pos() == b.pos() ? sign(a ^ b) > 0 : a.pos() < b.pos());
    });
    iota(id.begin(), id.end(), 0);
    sort(id.begin(), id.end(), [&](int i, int j) {
        return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt[j].x : pt[i].y < pt[j].y);
    });
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (auto [i, j] : line) {
        // point sort by the distance to line(i, j)
        // do something.
    }
}
```

```

    tie(pos[i], pos[j], id[pos[i]], id[pos[j]]) =
        make_tuple(pos[j], pos[i], j, i);
}
}

```

## 7.9 Half Plane Intersection

```

vector<Pt> HalfPlaneInter(vector<pair<Pt, Pt>> vec)
{
    //          x
    // first -----> second
    auto pos = [&](Pt a) {return sign(a.y) == 0 ? sign(a
        .x) < 0 : sign(a.y) > 0;};
    sort(all(vec), [&](pair<Pt, Pt> a, pair<Pt, Pt> b)
        {
            Pt A = a.second - a.first, B = b.second - b.first;
            if (pos(A) == pos(B)) {
                if (sign(A ^ B) == 0) return sign((b.first - a
                    first) * (b.second - a.first)) > 0;
                return sign(A ^ B) > 0;
            }
            return pos(A) < pos(B);
        });
    deque<Pt> inter;
    deque<pair<Pt, Pt>> seg;
    int n = vec.size();
    auto get = [&](pair<Pt, Pt> a, pair<Pt, Pt> b) {
        return intersect(a.first, a.second, b.first, b
            second);};
    for (int i = 0; i < n; ++i) if (!i || vec[i] != vec[i
        - 1]) {
        while (seg.size() >= 2 && sign((vec[i].second -
            inter.back()) ^ (vec[i].first - inter.back()))
            == 1) seg.pop_back(), inter.pop_back();
        while (seg.size() >= 2 && sign((vec[i].second -
            inter.front()) ^ (vec[i].first - inter.front()))
            == 1) seg.pop_front(), inter.pop_front();
        seg.push_back(vec[i]);
        if (seg.size() >= 2) inter.pb(get(seg[seg.size() -
            2], seg.back()));
    }
    while (seg.size() >= 2 && sign((seg.front().second -
        inter.back()) ^ (seg.front().first - inter.back())
        ) == 1) seg.pop_back(), inter.pop_back();
    inter.push_back(get(seg.front(), seg.back()));
    return vector<Pt>(all(inter));
}

```

## 7.10 Minkowski Sum

```

vector<Pt> Minkowski(vector<Pt> a, vector<Pt> b) {
    a = ConvexHull(a), b = ConvexHull(b);
    int n = a.size(), m = b.size();
    vector<Pt> c = {a[0] + b[0]}, s1, s2;
    for (int i = 0; i < n; ++i)
        s1.pb(a[(i + 1) % n] - a[i]);
    for (int i = 0; i < m; i++)
        s2.pb(b[(i + 1) % m] - b[i]);
    for (int p1 = 0, p2 = 0; p1 < n || p2 < m;)
        if (p2 == m || (p1 < n && sign(s1[p1] ^ s2[p2]) >=
            0))
            c.pb(c.back() + s1[p1++]);
        else
            c.pb(c.back() + s2[p2++]);
    return ConvexHull(c);
}

```

## 8 Polynomial

### 8.1 Number Theoretic Transform

```

const int N = 1 << 18, mod = 998244353, G = 3;
struct NTT {
    ll w[N];
    NTT() {
        ll dw = mpow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw %
            mod;
    }
    void bitrev(vector<ll>& a, int n) {

```

```

        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(vector<ll>& a, int n, bool inv =
        false) { // 0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx
                    ) {
                    ll tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            ll invn = mpow(n, mod - 2);
            for (int i = 0; i < n; ++i) a[i] = mul(a[i], invn
                );
        }
    }
};
vector<ll> mul(vector<ll> a, vector<ll> b, int M = N /
    2) {
    int m = a.size() + b.size() - 1, n = 1;
    while (n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a, n), ntt(b, n);
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], b[i]);
    ntt(a, n, 1);
    a.resize(min(m, M));
    return a;
}

```

### 8.2 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

### 8.3 Inverse of Polynomial

```

vector<ll> inv(vector<ll> a) {
    int m = a.size();
    vector<ll> res(1, modpow(a[0], mod - 2));
    for (int n = 2; n / 2 < m; n <= 1) {
        if (a.size() < n) a.resize(n);
        vector<ll> v1(a.begin(), a.begin() + n), v2 = res;
        v1.resize(n * 2), v2.resize(n * 2);
        ntt(v1, n * 2), ntt(v2, n * 2);
        for (int i = 0; i < n * 2; ++i) v1[i] = mul(mul(v1[i]
            ], v2[i]), v2[i]);
        ntt(v1, n * 2, 1);
        vector<ll> nres(n);
        for (int i = 0; i < n / 2; ++i) nres[i] = add(res[i]
            ], res[i]);
        for (int i = 0; i < n; ++i) nres[i] = sub(nres[i],
            v1[i]);
        res = nres;
    }
    res.resize(m);
    return res;
}

```

### 8.4 Fast Walsh Transform

```

void fwt(vector<int> &a) {
    // and : a[j] += x;
    //      : a[j] -= x;
    // or : a[j] ^ (1 << i) += y;
    //      : a[j] ^ (1 << i) -= y;

```

```
// xor : a[j] = x - y, a[j ^ (1 << i)] = x + y;
//      : a[j] = (x - y) / 2, a[j ^ (1 << i)] = (x + y) / 2;
int n = __lg(a.size());
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
        int x = a[j ^ (1 << i)], y = a[j];
        // do something
    }
}
```

## 9 Else

### 9.1 Bit Hack

```
long long next_perm(long long v) {
    long long t = v | (v - 1);
    return (t + 1) | (((~t & ~t) - 1) >> (__builtin_ctz(v) + 1));
}
void subset(long long s) {
    long long sub = s;
    while (sub) sub = (sub - 1) & s;
}
```

### 9.2 Slope Trick

```
template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
    slope_trick_convex() {left.push(numeric_limits<T>::min() / 2), right.push(numeric_limits<T>::max() / 2);}
    void push_left(T x) {left.push(x - ground_l);}
    void push_right(T x) {right.push(x - ground_r);}
    //add a line with slope 1 to the right starting from x
    void add_right(T x) {
        T l = left.top() + ground_l;
        if (l <= x) push_right(x);
        else push_left(x), push_right(l), left.pop(), minn += 1 - x;
    }
    //add a line with slope -1 to the left starting from x
    void add_left(T x) {
        T r = right.top() + ground_r;
        if (r >= x) push_left(x);
        else push_right(x), push_left(r), right.pop(), minn += x - r;
    }
    //val[i]=min(val[j]) for all i-l<=j<=i+r
    void expand(T l, T r) {ground_l -= l, ground_r += r;}
    void shift_up(T x) {minn += x;}
    T get_val(T x) {
        T l = left.top() + ground_l, r = right.top() + ground_r;
        if (x >= l && x <= r) return minn;
        if (x < l) {
            vector<T> trash;
            T cur_val = minn, slope = 1, res;
            while (1) {
                trash.push_back(left.top());
                left.pop();
                if (left.top() + ground_l <= x) {
                    res = cur_val + slope * (l - x);
                    break;
                }
            }
            cur_val += slope * (l - (left.top() + ground_l));
            l = left.top() + ground_l;
            slope += 1;
        }
        for (auto i : trash) left.push(i);
        return res;
    }
    if (x > r) {
        vector<T> trash;
```

```
T cur_val = minn, slope = 1, res;
while (1) {
    trash.push_back(right.top());
    right.pop();
    if (right.top() + ground_r >= x) {
        res = cur_val + slope * (x - r);
        break;
    }
    cur_val += slope * ((right.top() + ground_r) - r);
    r = right.top() + ground_r;
    slope += 1;
}
for (auto i : trash) right.push(i);
return res;
}
assert(0);
};
```

### 9.3 Hilbert Curve

```
long long hilbertOrder(int x, int y, int pow, int rotate) {
    rotate {
        if (pow == 0) return 0;
        int hpow = 1 << (pow-1);
        int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow) ? 1 : 2);
        seg = (seg + rotate) & 3;
        const int rotateDelta[4] = {3, 0, 0, 1};
        int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
        int nrot = (rotate + rotateDelta[seg]) & 3;
        long long subSquareSize = 1ll << (pow * 2 - 2);
        long long ans = seg * subSquareSize;
        long long add = hilbertOrder(nx, ny, pow - 1, nrot);
        ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
        return ans;
    }
}
```

### 9.4 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
int main () {
    __gnu_pbds::priority_queue <int> pq1, pq2;
    pq1.join(pq2); // pq1 += pq2, pq2 = {}
    cc_hash_table<int, int> m1;
    tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> oset;
    oset.insert(2), oset.insert(4);
    cout << *oset.find_by_order(1) << ' ' << oset.order_of_key(1) << '\n'; // 4 0
    bitset <100> BS;
    BS.flip(3), BS.flip(5);
    cout << BS._Find_first() << ' ' << BS._Find_next(3) << '\n'; // 3 5
    rope <int> rp1, rp2;
    rp1.push_back(1), rp1.push_back(3);
    rp1.insert(0, 2); // pos, num
    rp1.erase(0, 2); // pos, len
    rp1.substr(0, 2); // pos, len
    rp2.push_back(4);
    rp1 += rp2, rp2 = rp1;
    cout << rp2[0] << ' ' << rp2[1] << '\n'; // 3 4
}
```

### 9.5 Random

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t a) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().time_since_epoch().count();
```

```

    return splitmix64(i + FIXED_RANDOM);
}
};
unordered_map<int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());

```

## 9.6 Mo's Algorithm

```

struct MoSolver {
    struct query {
        int l, r, id;
        bool operator < (const query &o) {
            if (l / C == o.l / C) return (l / C) & 1 ? r > o.r : r < o.r;
            return l / C < o.l / C;
        }
    };
    int cur_ans;
    vector<int> ans;
    void add(int x) {
        // do something
    }
    void sub(int x) {
        // do something
    }
    vector<query> Q;
    void add_query(int l, int r, int id) {
        // [l, r)
        Q.push_back({l, r, id});
        ans.push_back(0);
    }
    void run() {
        sort(Q.begin(), Q.end());
        int pl = 0, pr = 0;
        cur_ans = 0;
        for (query &i : Q) {
            while (pl > i.l)
                add(a[--pl]);
            while (pr < i.r)
                add(a[pr++]);
            while (pl < i.l)
                sub(a[pl++]);
            while (pr > i.r)
                sub(a[--pr]);
            ans[i.id] = cur;
        }
    }
};

```

## 9.7 Smawk Algorithm

```

ll query(int l, int r) {
    // ...
}
struct SMAWK {
    // Condition:
    // If  $M[1][0] < M[1][1]$  then  $M[0][0] < M[0][1]$ 
    // If  $M[1][0] == M[1][1]$  then  $M[0][0] <= M[0][1]$ 
    // For all  $i$ , find  $r_i$  s.t.  $M[i][r_i]$  is maximum //
    // minimum.
    int ans[N], tmp[N];
    void interpolate(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nl;
        for (int i = 1; i < n; i += 2) {
            nl.push_back(l[i]);
        }
        run(nl, r);
        for (int i = 1, j = 0; i < n; i += 2) {
            while (j < m && r[j] < ans[l[i]])
                j++;
            assert(j < m && ans[l[i]] == r[j]);
            tmp[l[i]] = j;
        }
        for (int i = 0; i < n; i += 2) {
            int curl = 0, curr = m - 1;
            if (i)
                curl = tmp[l[i - 1]];
            if (i + 1 < n)
                curr = tmp[l[i + 1]];
            ll res = query(l[i], r[curl]);
            ans[l[i]] = r[curl];
        }
    }
};

```

```

    for (int j = curl + 1; j <= curr; ++j) {
        ll nxt = query(l[i], r[j]);
        if (res < nxt)
            res = nxt, ans[l[i]] = r[j];
    }
}
}
void reduce(vector<int> l, vector<int> r) {
    int n = l.size(), m = r.size();
    vector<int> nr;
    for (int j : r) {
        while (!nr.empty()) {
            int i = nr.size() - 1;
            if (query(l[i], nr.back()) <= query(l[i], j))
                nr.pop_back();
            else
                break;
        }
        if (nr.size() < n)
            nr.push_back(j);
    }
    run(l, nr);
}
void run(vector<int> l, vector<int> r) {
    int n = l.size(), m = r.size();
    if (max(n, m) <= 2) {
        for (int i : l) {
            ans[i] = r[0];
            if (m > 1) {
                if (query(i, r[0]) < query(i, r[1]))
                    ans[i] = r[1];
            }
        }
    } else if (n >= m) {
        interpolate(l, r);
    } else {
        reduce(l, r);
    }
}
};

```

## 9.8 Two Dimension Add Sum

```

struct TwoDimensionAddAndSum {
    // 0-index, [l, r)
    struct Seg {
        int l, r, m;
        ll vala, valb, lza, lzb;
        Seg* ch[2];
        Seg(int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
            vala(0), valb(0), lza(0), lzb(0) {
            if (r - l > 1) {
                ch[0] = new Seg(l, m);
                ch[1] = new Seg(m, r);
            }
        }
        void pull() { vala = ch[0]->vala + ch[1]->vala, valb =
            ch[0]->valb + ch[1]->valb; }
        void give(ll a, ll b) {
            lza += a, lzb += b;
            vala += a * (r - l), valb += b * (r - l);
        }
        void push() {
            ch[0]->give(lza, lzb), ch[1]->give(lza, lzb), lza =
                lzb = 0;
        }
        void add(int a, int b, ll va, ll vb) {
            if (a <= l && r <= b)
                give(va, vb);
            else {
                push();
                if (a < m) ch[0]->add(a, b, va, vb);
                if (m < b) ch[1]->add(a, b, va, vb);
                pull();
            }
        }
        long long query(int a, int b, int v) {
            if (a <= l && r <= b) return vala * v + valb;
            push();
            long long ans = 0;
            if (a < m) ans += ch[0]->query(a, b, v);
            if (m < b) ans += ch[1]->query(a, b, v);
        }
    };
};

```

```

        return ans;
    }
};
// note integer overflow.
vector <array <int, 4>> E[N];
vector <array <int, 4>> Q[N];
vector <ll> ans;
void add_event(int x1, int y1, int x2, int y2, ll v)
{
    E[x1].pb({y1, y2, v, -v * x1});
    E[x2].pb({y1, y2, -v, v * x2});
}
void add_query(int x1, int y1, int x2, int y2, int id
) {
    Q[x1].pb({y1, y2, -1, id});
    Q[x2].pb({y1, y2, 1, id});
    ans.pb(0);
}
void solve(int n) {
    Seg root(0, n);
    for (int i = 0; i <= n; ++i) {
        for (auto j : E[i]) root.add(j[0], j[1], j[2], j
[3]);
        for (auto j : Q[i]) ans[j[3]] += j[2] * root.
query(j[0], j[1], i);
    }
}
};

```

## 9.9 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.