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1 Basic

1.1 Shell Script

```
#!/usr/bin/env bash
g++ -std=c++17 -DABS -O2 -Wall -Wextra -Wshadow $1.cpp
-o $1 && ./$1
for i in {A..J}; do cp tem.cpp $i.cpp; done;
cpp hash.cpp -dD -P -fpreprocessed | tr -d "[:space:]"
| md5sum | cut -c -6
```

1.2 Debug Macro [d41d8c]

```
void db() { cout << endl; }
template <typename T, typename ...U>
void db(T i, U ...j) { cout << i << ' ', db(j...); }
#ifdef ABS
#define bug(x...) db("[ " + string(#x) + "]", x)
#else
#define bug(x...) void(0)
#endif
```

1.3 Pragma / FastIO

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)
        ) == buf ? -1 : *p++;
}
inline int R() {
    static char c;
    while((c = RC()) < '0'); int a = c ^ '0';
    while((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.4 Divide

```
ll floor(ll a, ll b) {return a / b - (a < 0 && a % b);}
ll ceil(ll a, ll b) {return a / b + (a > 0 && a % b);}
a / b < x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x < a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)
```

2 Data Structure

2.1 Leftist Tree [d41d8c]

```
struct node {
    ll rk, data, sz, sum;
    node *l, *r;
    node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1;
    a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}
```

2.2 Splay Tree [d41d8c]

```
struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    ll v[N];
    Splay() {}
    void init() {
        rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
        sz[0] = 1, v[0] = inf;
    }
    int newnode(int p, int x) {
        int id = _id++;
        v[id] = x, pa[id] = p;
        ch[id][0] = ch[id][1] = -1, sz[id] = 1;
        return id;
    }
    void rotate(int i) {
```

```

int p = pa[i], x = ch[p][1] == i;
int gp = pa[p], c = ch[i][!x];
sz[p] -= sz[i], sz[i] += sz[p];
if (~c) sz[p] += sz[c], pa[c] = p;
ch[p][x] = c, pa[p] = i;
pa[i] = gp, ch[i][!x] = p;
if (~gp) ch[gp][ch[gp][1] == p] = i;
}
void splay(int i) {
    while (~pa[i]) {
        int p = pa[i];
        if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
            == i ? i : p);
        rotate(i);
    }
    rt = i;
}
int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
        if (v[i] == x) return splay(i), i;
        if (v[i] > x) {
            last = i;
            if (ch[i][0] == -1) break;
            i = ch[i][0];
        }
        else {
            if (ch[i][1] == -1) break;
            i = ch[i][1];
        }
    }
    splay(i);
    return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};

```

2.3 Link Cut Tree [d41d8c]

```

// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N];
    int sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
    // virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0]) {
            sz[i] += sz[ch[i][0]];
            mx[i] = max(mx[i], mx[ch[i][0]]);
        }
        if (ch[i][1]) {

```

```

            sz[i] += sz[ch[i][1]];
            mx[i] = max(mx[i], mx[ch[i][1]]);
        }
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
            rev[i] = false;
    }
    void reverse(int i) {
        if (!i) return;
        swap(ch[i][0], ch[i][1]);
        rev[i] ^= true;
    }
    bool isrt(int i) { // rt of splay
        if (!pa[i]) return true;
        return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i;
        int c = ch[i][!x], gp = pa[p];
        if (ch[gp][0] == p) ch[gp][0] = i;
        else if (ch[gp][1] == p) ch[gp][1] = i;
        pa[i] = gp, ch[i][!x] = p, pa[p] = i;
        ch[p][x] = c, pa[c] = p;
        pull(p), pull(i);
    }
    void splay(int i) {
        vector<int> anc;
        anc.push_back(i);
        while (!isrt(anc.back()))
            anc.push_back(pa[anc.back()]);
        while (!anc.empty())
            push(anc.back()), anc.pop_back();
        while (!isrt(i)) {
            int p = pa[i];
            if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
                == p ? i : p);
            rotate(i);
        }
    }
    void access(int i) {
        int last = 0;
        while (i) {
            splay(i);
            if (ch[i][1])
                sz2[i] += sz[ch[i][1]];
            sz2[i] -= sz[last];
            ch[i][1] = last;
            pull(i), last = i, i = pa[i];
        }
    }
    void makert(int i) {
        access(i), splay(i), reverse(i);
    }
    void link(int i, int j) {
        // assert(findrt(i) != findrt(j));
        makert(i);
        makert(j);
        pa[i] = j;
        sz2[j] += sz[i];
        pull(j);
    }
    void cut(int i, int j) {
        makert(i), access(j), splay(i);
        // assert(sz[i] == 2 && ch[i][1] == j);
        ch[i][1] = pa[j] = 0, pull(i);
    }
    int findrt(int i) {
        access(i), splay(i);
        while (ch[i][0]) push(i), i = ch[i][0];
        splay(i);
        return i;
    }
};

```

2.4 Treap [d41d8c]

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {

```

```

    sz = 1;
    if (l) sz += l->sz;
    if (r) sz += r->sz;
}
void down() {}
};
// delete default code sz
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    o->down(), split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
    o->up();
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c; // [l, r)
    o->down();
    split2(o, a, b, l), split2(b, b, c, r - l);
    // operate
    o = merge(a, merge(b, c)), o->up();
}

```

2.5 2D Segment Tree [d41d8c]

```

// 2D range add, range sum in Log^2
struct seg {
    int l, r;
    ll sum, lz;
    seg *ch[2]{};
    seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
    void push() {
        if (lz) ch[0]->add(l, r, lz), ch[1]->add(l, r, lz),
            lz = 0;
    }
    void pull() { sum = ch[0]->sum + ch[1]->sum; }
    void add(int _l, int _r, ll d) {
        if (_l <= l && r <= _r) {
            sum += d * (r - l), lz += d;
            return;
        }
    }

```

```

    }
    if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
        new seg(l + r >> 1, r);
    push();
    if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
    pull();
}
ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(l, _l));
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
}
};
struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2]{};
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N) {
        if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch[1] =
            new seg2(l + r >> 1, r);
    }
    void add(int _l, int _r, int _l2, int _r2, ll d) {
        v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
        if (_l <= l && r <= _r)
            return lz.add(_l2, _r2, d), void(0);
        if (_l < l + r >> 1)
            ch[0]->add(_l, _r, _l2, _r2, d);
        if (l + r >> 1 < _r)
            ch[1]->add(_l, _r, _l2, _r2, d);
    }
    ll qsum(int _l, int _r, int _l2, int _r2) {
        if (_l <= l && r <= _r) return v.qsum(_l2, _r2);
        ll d = min(r, _r) - max(l, _l);
        ll res = lz.qsum(_l2, _r2) * d;
        if (_l < l + r >> 1)
            res += ch[0]->qsum(_l, _r, _l2, _r2);
        if (l + r >> 1 < _r)
            res += ch[1]->qsum(_l, _r, _l2, _r2);
        return res;
    }
};

```

2.6 vEB Tree [d41d8c]

```

using u64=uint64_t;
constexpr int lsb(u64 x)
{ return x?__builtin_ctzll(x):1<<30; }
constexpr int msb(u64 x)
{ return x?63-__builtin_clzll(x):-1; }
template<int N, class T=void>
struct veb{
    static const int M=N>>1;
    veb<M> ch[1<<N-M];
    veb<N-M> aux;
    int mn,mx;
    veb():mn(1<<30),mx(-1){}
    constexpr int mask(int x){return x&((1<<M)-1);}
    bool empty(){return mx==-1;}
    int min(){return mn;}
    int max(){return mx;}
    bool have(int x){
        return x==mn?true:ch[x>>M].have(mask(x));
    }
    void insert_in(int x){
        if(empty()) return mn=mx=x,void();
        if(x<mn) swap(x,mn);
        if(x>mx) mx=x;
        if(ch[x>>M].empty()) aux.insert_in(x>>M);
        ch[x>>M].insert_in(mask(x));
    }
    void erase_in(int x){
        if(mn==mx) return mn=1<<30,mx=-1,void();
        if(x==mn) mn=x=(aux.min()<<M)^ch[aux.min()].min();
        ch[x>>M].erase_in(mask(x));
        if(ch[x>>M].empty()) aux.erase_in(x>>M);
        if(x==mx){

```

```

    if(aux.empty()) mx=mn;
    else mx=(aux.max()<<M)^ch[aux.max()].max();
}
}
void insert(int x){
    if(!have(x)) insert_in(x);
}
void erase(int x){
    if(have(x)) erase_in(x);
}
int next(int x){// >=x
    if(x>mx) return 1<<30;
    if(x<=mn) return mn;
    if(mask(x)<=ch[x>>M].max())
        return ((x>>M)<<M)^ch[x>>M].next(mask(x));
    int y=aux.next((x>>M)+1);
    return (y<<M)^ch[y].min();
}
int prev(int x){// <x
    if(x<=mn) return -1;
    if(x>mx) return mx;
    if(x<=(aux.min()<<M)+ch[aux.min()].min())
        return mn;
    if(mask(x)>ch[x>>M].min())
        return ((x>>M)<<M)^ch[x>>M].prev(mask(x));
    int y=aux.prev(x>>M);
    return (y<<M)^ch[y].max();
}
}
};
template<int N>
struct veb<N,typename enable_if<N<=6>::type>{
    u64 a;
    veb():a(0){}
    void insert_in(int x){a|=1ull<<x;}
    void insert(int x){a|=1ull<<x;}
    void erase_in(int x){a&=~(1ull<<x);}
    void erase(int x){a&=~(1ull<<x);}
    bool have(int x){return a>>x&1;}
    bool empty(){return a==0;}
    int min(){return lsb(a);}
    int max(){return msb(a);}
    int next(int x){return lsb(a&~((1ull<<x)-1));}
    int prev(int x){return msb(a&((1ull<<x)-1));}
};

```

2.7 Range Set [d41d8c]

```

struct RangeSet { // [L, r)
    set<pii> S;
    void cut(int x) {
        auto it = S.lower_bound({x + 1, -1});
        if (it == S.begin()) return;
        auto [l, r] = *prev(it);
        if (l >= x || x >= r) return;
        S.erase(prev(it));
        S.insert({l, x});
        S.insert({x, r});
    }
    vector<pii> split(int l, int r) {
        // remove and return ranges in [L, r)
        cut(l), cut(r);
        vector<pii> res;
        while (true) {
            auto it = S.lower_bound({l, -1});
            if (it == S.end() || r <= it->first) break;
            res.pb(*it), S.erase(it);
        }
        return res;
    }
    void insert(int l, int r) {
        // add a range [L, r), [L, r) not in S
        auto it = S.lower_bound({l, r});
        if (it != S.begin() && prev(it)->second == l)
            l = prev(it)->first, S.erase(prev(it));
        if (it != S.end() && r == it->first)
            r = it->second, S.erase(it);
        S.insert({l, r});
    }
    bool count(int x) {
        auto it = S.lower_bound({x + 1, -1});
        return it != S.begin() && prev(it)->first <= x
            && x < prev(it)->second;
    }
};

```

```

}
};

```

3 Flow / Matching

3.1 Dinic [d41d8c]

```

template <typename T>
struct Dinic { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    struct edge { int to, rev; T cap, flow; };
    int n, s, t;
    vector<vector<edge>> g;
    vector<int> dis, cur;
    T dfs(int u, T cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)g[u].size(); ++i) {
            edge &e = g[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                T df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    g[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill(all(dis), -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (auto &u : g[v])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[v] + 1;
                }
        }
        return dis[t] != -1;
    }
    T solve(int _s, int _t) {
        s = _s, t = _t;
        T flow = 0, df;
        while (bfs()) {
            fill(all(cur), 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : g[i]) j.flow = 0;
    }
    void add_edge(int u, int v, T cap) {
        g[u].pb(edge{v, (int)g[v].size(), cap, 0});
        g[v].pb(edge{u, (int)g[u].size() - 1, 0, 0});
    }
    Dinic(int _n) : n(_n), g(n), dis(n), cur(n) {}
};

```

3.2 Min Cost Max Flow [d41d8c]

```

template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
    const T1 INF1 = numeric_limits<T1>::max() / 2;
    const T2 INF2 = numeric_limits<T2>::max() / 2;
    struct edge {
        int v; T1 f; T2 c;
    };
    int n, s, t;
    vector<vector<int>> g;
    vector<edge> e;
    vector<T2> dis, pot;
    vector<int> rt, vis;
    // bool DAG()...
    bool SPFA() {
        fill(all(rt), -1), fill(all(dis), INF2);
        fill(all(vis), false);
    }
};

```

```

queue <int> q;
q.push(s), dis[s] = 0, vis[s] = true;
while (!q.empty()) {
    int v = q.front(); q.pop();
    vis[v] = false;
    for (int id : g[v]) {
        auto [u, f, c] = e[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
            dis[u] = ndis, rt[u] = id;
            if (!vis[u]) vis[u] = true, q.push(u);
        }
    }
}
return dis[t] != INF2;
}
bool dijkstra() {
    fill(all(rt), -1), fill(all(dis), INF2);
    priority_queue <pair <T2, int>, vector <pair <T2,
        int>>, greater <pair <T2, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
        auto [d, v] = pq.top(); pq.pop();
        if (dis[v] < d) continue;
        for (int id : g[v]) {
            auto [u, f, c] = e[id];
            T2 ndis = dis[v] + c + pot[v] - pot[u];
            if (f > 0 && dis[u] > ndis) {
                dis[u] = ndis, rt[u] = id;
                pq.emplace(ndis, u);
            }
        }
    }
    return dis[t] != INF2;
}
vector <pair <T1, T2>> solve(int _s, int _t) {
    s = _s, t = _t, fill(all(pot), 0);
    vector <pair <T1, T2>> ans; bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; ++i)
            dis[i] += pot[i] - pot[s];
        T1 add = INF1;
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            add = min(add, e[rt[i]].f);
        for (int i = t; i != s; i = e[rt[i] ^ 1].v)
            e[rt[i]].f -= add, e[rt[i] ^ 1].f += add;
        ans.emplace_back(add, dis[t]), fr = false;
        for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return ans;
}
void reset() {
    for (int i = 0; i < (int)e.size(); ++i) e[i].f = 0;
}
void add_edge(int u, int v, T1 f, T2 c) {
    g[u].pb((int)e.size(), e.pb({v, f, c}));
    g[v].pb((int)e.size(), e.pb({u, 0, -c}));
}
MCMF (int _n) : n(_n), g(n), e(), dis(n), pot(n),
    rt(n), vis(n) {}
};

```

3.3 Kuhn Munkres [d41d8c]

```

template <typename T>
struct KM { // 0-based, remember to init
    const T INF = numeric_limits<T>::max() / 2;
    int n; vector <vector <T>> w;
    vector <T> hl, hr, slk;
    vector <int> fl, fr, vl, vr, pre;
    queue <int> q;
    bool check(int x) {
        if (vl[x] = 1, ~fl[x])
            return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill(all(slk), INF), fill(all(vl), 0);
        fill(all(vr), 0);
        while (!q.empty()) q.pop();
        q.push(s), vr[s] = 1;
    }
};

```

```

while (true) {
    T d;
    while (!q.empty()) {
        int y = q.front(); q.pop();
        for (int x = 0; x < n; ++x) {
            d = hl[x] + hr[y] - w[x][y];
            if (!vl[x] && slk[x] >= d) {
                if (pre[x] = y, d) slk[x] = d;
                else if (!check(x)) return;
            }
        }
    }
    d = INF;
    for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
    for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
}
T solve() {
    fill(all(fl), -1), fill(all(fr), -1);
    fill(all(hr), 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(all(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
void add_edge(int a, int b, T wei) { w[a][b] = wei; }
KM (int _n) : n(_n), w(n, vector<T>(n, -INF)), hl(n),
    hr(n), slk(n), fl(n), fr(n), vl(n), vr(n), pre(n) {}
};

```

3.4 Hopcroft Karp [d41d8c]

```

struct HopcroftKarp { // 0-based
    const int INF = 1 << 30;
    int n, m;
    vector <vector <int>> g;
    vector <int> match, dis, matched, vis;
    bool dfs(int x) {
        vis[x] = true;
        for (int y : g[x])
            if (match[y] == -1 || (dis[match[y]] == dis[x] +
                1 && !vis[match[y]] && dfs(match[y]))) {
                match[y] = x, matched[x] = true;
                return true;
            }
        return false;
    }
    bool bfs() {
        fill(all(dis), -1);
        queue <int> q;
        for (int x = 0; x < n; ++x) if (!matched[x])
            dis[x] = 0, q.push(x);
        int mx = INF;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int y : g[x]) {
                if (match[y] == -1) {
                    mx = dis[x];
                    break;
                } else if (dis[match[y]] == -1)
                    dis[match[y]] = dis[x] + 1, q.push(match[y]);
            }
        }
        return mx < INF;
    }
    int solve() {
        int res = 0;
        fill(all(match), -1);
        fill(all(matched), 0);
        while (bfs()) {
            fill(all(vis), 0);
            for (int x = 0; x < n; ++x) if (!matched[x])
                res += dfs(x);
        }
    }
};

```

```

    }
    return res;
}
void add_edge(int x, int y) { g[x].pb(y); }
HopcroftKarp (int _n, int _m) : n(_n), m(_m), g(n),
    match(m), dis(n), matched(n), vis(n) {}
};

```

3.5 SW Min Cut [d41d8c]

```

template <typename T>
struct SW { // 0-based
    const T INF = numeric_limits<T>::max() / 2;
    vector<vector<T>> g;
    vector<T> sum;
    vector<bool> vis, dead;
    int n;
    T solve() {
        T ans = INF;
        for (int r = 0; r + 1 < n; ++r) {
            fill(all(vis), 0), fill(all(sum), 0);
            int num = 0, s = -1, t = -1;
            while (num < n - r) {
                int now = -1;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i] &&
                        (now == -1 || sum[now] > sum[i])) now = i;
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i]) sum[i] += g[now][i];
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i)
                g[i][s] += g[i][t], g[s][i] += g[t][i];
            dead[t] = true;
        }
        return ans;
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w; }
    SW (int _n) : n(_n), g(n, vector<T>(n)), vis(n),
        sum(n), dead(n) {}
};

```

3.6 Gomory Hu Tree [d41d8c]

```

vector<array<int, 3>> GomoryHu(Dinic<int> flow) {
    // Tree edge min = mincut (0-based)
    int n = flow.n;
    vector<array<int, 3>> ans;
    vector<int> rt(n);
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset();
        ans.pb({i, t, flow.solve(i, t)});
        flow.bfs();
        for (int j = i + 1; j < n; ++j)
            if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
    }
    return ans;
}

```

3.7 Blossom [d41d8c]

```

struct Matching { // 0-based
    int n, tk;
    vector<vector<int>> g;
    vector<int> fa, pre, match, s, t;
    queue<int> q;
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (t[x] == tk) return x;
                t[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
};

```

```

}
void blossom(int x, int y, int l) {
    while (Find(x) != l) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.push(y), s[y] = 0;
        if (fa[x] == x) fa[x] = 1;
        if (fa[y] == y) fa[y] = 1;
        x = pre[y];
    }
}
bool bfs(int r) {
    iota(all(fa), 0), fill(all(s), -1);
    while (!q.empty()) q.pop();
    q.push(r);
    s[r] = 0;
    while (!q.empty()) {
        int x = q.front(); q.pop();
        for (int u : g[x]) {
            if (s[u] == -1) {
                pre[u] = x, s[u] = 1;
                if (match[u] == n) {
                    for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                        last = match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]);
                s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = lca(u, x);
                blossom(x, u, l);
                blossom(u, x, l);
            }
        }
    }
    return false;
}
int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) {
        if (match[x] == n) res += bfs(x);
    }
    return res;
}
void add_edge(int u, int v) {
    g[u].push_back(v), g[v].push_back(u);
}
Matching (int _n) : n(_n), tk(0), g(n), fa(n + 1),
    pre(n + 1, n), match(n + 1, n), s(n + 1, t(n) {}
};

```

3.8 Min Cost Circulation [d41d8c]

```

struct MinCostCirculation { // 0-base
    struct Edge {
        ll from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    ll dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(Edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {

```



```

++cur.flow, --G[cur.to][cur.rev].flow;
for (int i = cur.from; past[i]; i = past[i]->from) {
    auto &e = *past[i];
    ++e.flow, --G[e.to][e.rev].flow;
}
}
++cur.cap;
}
void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                e.cap *= 2, e.flow *= 2;
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                if (e.fcap >> b & 1)
                    try_edge(e);
    }
}
void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
}
void add_edge(int a, int b, int cap, int cost) {
    G[a].pb(Edge{a, b, 0, cap, 0, cost, sz(G[b]) + (a == b)});
    G[b].pb(Edge{b, a, 0, 0, 0, -cost, sz(G[a]) - 1});
}
} mcmf; // O(VE * ElogC)

```

3.9 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 - Construct super source S and sink T
 - For each edge (x, y, c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 - For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 - For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 - For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 - Flow from S to T , the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T
 - Construct a max flow model, let K be the sum of all weights
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K
 - For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 - For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
 - T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 - Change the weight of each edge to $\mu(u) + \mu(v) - w(u, v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 - Let the maximum weight matching of the graph be x , the answer will be $\sum \mu(v) - x$.
- Project selection problem
 - If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 - Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .

- The mincut is equivalent to the maximum profit of a subset of projects.

- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
- Create edge (x, y) with capacity c_{xy} .
- Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4 Graph

4.1 Heavy-Light Decomposition [d41d8c]

```

struct HLD { // 0-based, remember to build
    int n, _id;
    vector<vector<int>> g;
    vector<int> dep, pa, tsz, ch, hd, id;
    void dfs(int v, int p) {
        dep[v] = ~p ? dep[p] + 1 : 0;
        pa[v] = p, tsz[v] = 1, ch[v] = -1;
        for (int u : g[v]) if (u != p) {
            dfs(u, v);
            if (ch[v] == -1 || tsz[ch[v]] < tsz[u])
                ch[v] = u;
            tsz[v] += tsz[u];
        }
    }
    void hld(int v, int p, int h) {
        hd[v] = h, id[v] = _id++;
        if (~ch[v]) hld(ch[v], v, h);
        for (int u : g[v]) if (u != p && u != ch[v])
            hld(u, v, u);
    }
    vector<pii> query(int u, int v) {
        vector<pii> ans;
        while (hd[u] != hd[v]) {
            if (dep[hd[u]] > dep[hd[v]]) swap(u, v);
            ans.emplace_back(id[hd[v]], id[v] + 1);
            v = pa[hd[v]];
        }
        if (dep[u] > dep[v]) swap(u, v);
        ans.emplace_back(id[u], id[v] + 1);
        return ans;
    }
    void build() {
        for (int i = 0; i < n; ++i) if (id[i] == -1)
            dfs(i, -1), hld(i, -1, i);
    }
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u);
    }
    HLD(int _n) : n(_n), _id(0), g(n), dep(n), pa(n),
        tsz(n), ch(n), hd(n), id(n, -1) {}
};

```

4.2 Centroid Decomposition [d41d8c]

```

struct CD { // 0-based, remember to build
    int n, lg; // pa, dep are centroid tree attributes
    vector<vector<int>> g, dis;
    vector<int> pa, tsz, dep, vis;
    void dfs1(int v, int p) {
        tsz[v] = 1;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs1(u, v), tsz[v] += tsz[u];
    }
    int dfs2(int v, int p, int _n) {
        for (int u : g[v])
            if (u != p && !vis[u] && tsz[u] > _n / 2)
                return dfs2(u, v, _n);
        return v;
    }
    void dfs3(int v, int p, int d) {
        dis[v][d] = ~p ? dis[p][d] + 1 : 0;
        for (int u : g[v]) if (u != p && !vis[u])
            dfs3(u, v, d);
    }
    void cd(int v, int p, int d) {
        dfs1(v, -1), v = dfs2(v, -1, tsz[v]);
        vis[v] = true, pa[v] = p, dep[v] = d;
    }
};

```

```

    dfs3(v, -1, d);
    for (int u : g[v]) if (!vis[u])
        cd(u, v, d + 1);
}
void build() { cd(0, -1, 0); }
void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
CD (int _n) : n(_n), lg(_lg(n) + 1), g(n),
    dis(n, vector<int>(lg)), pa(n), tsz(n),
    dep(n), vis(n) {}
};

```

4.3 Edge BCC [d41d8c]

```

struct EBCC { // 0-based, remember to build
    int n, m, nbcc;
    vector<vector<pii>> g;
    vector<int> pa, low, dep, bcc_id, stk, is_bridge;
    void dfs(int v, int p, int f) {
        low[v] = dep[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (auto [u, e] : g[v]) {
            if (low[u] == -1)
                dfs(u, v, e), low[v] = min(low[v], low[u]);
            else if (e != f)
                low[v] = min(low[v], dep[u]);
        }
        if (low[v] == dep[v]) {
            if (~f) is_bridge[f] = true;
            int id = nbcc++, x;
            do {
                x = stk.back(), stk.pop_back();
                bcc_id[x] = id;
            } while (x != v);
        }
    }
    void build() {
        is_bridge.assign(m, 0);
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i, -1, -1);
    }
    void add_edge(int u, int v) {
        g[u].emplace_back(v, m), g[v].emplace_back(u, m++);
    }
    EBCC (int _n) : n(_n), m(0), nbcc(0), g(n), pa(n),
        low(n, -1), dep(n), bcc_id(n), stk() {}
};

```

4.4 Vertex BCC / Round Square Tree [d41d8c]

```

struct BCC { // 0-based, remember to build
    int n, nbcc; // note for isolated point
    vector<vector<int>> g, _g; // id >= n: bcc
    vector<int> pa, dep, low, stk, pa2, dep2;
    void dfs(int v, int p) {
        dep[v] = low[v] = ~p ? dep[p] + 1 : 0;
        stk.pb(v), pa[v] = p;
        for (int u : g[v]) if (u != p) {
            if (low[u] == -1) {
                dfs(u, v), low[v] = min(low[v], low[u]);
                if (low[u] >= dep[v]) {
                    int id = nbcc++, x;
                    do {
                        x = stk.back(), stk.pop_back();
                        _g[id + n].pb(x), _g[x].pb(id + n);
                    } while (x != u);
                    _g[id + n].pb(v), _g[v].pb(id + n);
                }
            } else low[v] = min(low[v], dep[u]);
        }
    }
    bool is_cut(int x) { return (int)_g[x].size() != 1; }
    vector<int> bcc(int id) { return _g[id + n]; }
    int bcc_id(int u, int v) {
        return pa2[dep2[u] < dep2[v] ? v : u] - n; }
    void dfs2(int v, int p) {
        dep2[v] = ~p ? dep2[p] + 1 : 0, pa2[v] = p;
        for (int u : _g[v]) if (u != p) dfs2(u, v);
    }
    void build() {
        low.assign(n, -1);
        for (int i = 0; i < n; ++i) if (low[i] == -1)

```

```

        dfs(i, -1), dfs2(i, -1);
    }
    void add_edge(int u, int v) {
        g[u].pb(v), g[v].pb(u); }
    BCC (int _n) : n(_n), nbcc(0), g(n), _g(2 * n),
        pa(n), dep(n), low(n), stk(), pa2(n * 2),
        dep2(n * 2) {}
};

```

4.5 SCC [d41d8c]

```

struct SCC {
    int n, nscc, _id;
    vector<vector<int>> g;
    vector<int> dep, low, scc_id, stk;
    void dfs(int v) {
        dep[v] = low[v] = _id++, stk.pb(v);
        for (int u : g[v]) if (scc_id[u] == -1) {
            if (low[u] == -1) dfs(u);
            low[v] = min(low[v], low[u]);
        }
        if (low[v] == dep[v]) {
            int id = nscc++, x;
            do {
                x = stk.back(), stk.pop_back(), scc_id[x] = id;
            } while (x != v);
        }
    }
    void build() {
        for (int i = 0; i < n; ++i) if (low[i] == -1)
            dfs(i);
    }
    void add_edge(int u, int v) { g[u].pb(v); }
    SCC (int _n) : n(_n), nscc(0), _id(0), g(n), dep(n),
        low(n, -1), scc_id(n, -1), stk() {}
};

```

4.6 2SAT [d41d8c]

```

struct SAT { // 0-based, need SCC
    int n; vector<pii> edge; vector<int> is;
    int rev(int x) { return x < n ? x + n : x - n; }
    void add_ifthen(int x, int y) {
        add_clause(rev(x), y); }
    void add_clause(int x, int y) {
        edge.emplace_back(rev(x), y);
        edge.emplace_back(rev(y), x); }
    bool solve() {
        // is[i] = true -> i, is[i] = false -> -i
        SCC scc(2 * n);
        for (auto [u, v] : edge) scc.add_edge(u, v);
        scc.build();
        for (int i = 0; i < n; ++i) {
            if (scc.scc_id[i] == scc.scc_id[i + n])
                return false;
            is[i] = scc.scc_id[i] < scc.scc_id[i + n];
        }
        return true;
    }
    SAT (int _n) : n(_n), edge(), is(n) {}
};

```

4.7 Virtual Tree [d41d8c]

```

// need lca, in, out
vector<pii> virtual_tree(vector<int> &v) {
    auto cmp = [&](int x, int y) { return in[x] < in[y]; };
    sort(all(v), cmp);
    int sz = (int)v.size();
    for (int i = 0; i + 1 < sz; ++i)
        v.pb(lca(v[i], v[i + 1]));
    sort(all(v), cmp);
    v.resize(unique(all(v)) - v.begin());
    vector<int> stk(1, v[0]);
    vector<pii> res;
    for (int i = 1; i < (int)v.size(); ++i) {
        int x = v[i];
        while (out[stk.back()] < out[x]) stk.pop_back();
        res.emplace_back(stk.back(), x), stk.pb(x);
    }
    return res;
}

```


4.8 Directed MST [d41d8c]

```
using D = int;
struct edge { int u, v; D w; };
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
    using T = pair<D, int>;
    using PQ = pair<priority_queue<T, vector<T>,
        greater<T>>, D>;
    auto push = [](PQ &pq, T v) {
        pq.first.emplace(v.first - pq.second, v.second);
    };
    auto top = [](const PQ &pq) -> T {
        auto r = pq.first.top();
        return {r.first + pq.second, r.second};
    };
    auto join = [&push, &top](PQ &a, PQ &b) {
        if (a.first.size() < b.first.size()) swap(a, b);
        while (!b.first.empty())
            push(a, top(b)), b.first.pop();
    };
    vector<PQ> h(n * 2);
    for (int i = 0; i < e.size(); ++i)
        push(h[e[i].v], {e[i].w, i});
    vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
        n * 2);
    iota(all(a), 0);
    auto o = [&](int x) { int y;
        for (y = x; a[y] != y; y = a[y]);
        for (int ox = x; x != y; ox = x)
            x = a[x], a[ox] = y;
        return y;
    };
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
            p]].u)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].second = -h[q].first.top().first;
                    join(h[pa[q] = a[q] = p], h[q]);
                } while ((q = o(e[r[q]].u)) != p);
            }
            v[p] = i;
            while (!h[p].first.empty() && o(e[top(h[p]).
                second].u) == p)
                h[p].first.pop();
            r[p] = top(h[p]).second;
        }
    }
    vector<int> ans;
    for (int i = pc - 1; i >= 0; i--)
        if (i != root && v[i] != n) {
            for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
                pa[f]) v[f] = n;
            ans.pb(r[i]);
        }
    return ans;
}
```

4.9 Dominator Tree [d41d8c]

```
struct DominatorTree {
    int n, id;
    vector<vector<int>> g, rg, bucket;
    vector<int> sdom, dom, vis, rev, pa, rt, mn, res;
    // dom[s] = s, dom[v] = -1 if s -> v not exists
    int query(int v, int x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], 1);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]])
            mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : g[v]) {
            if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];

```

```
            rg[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : rg[i]) {
                sdom[i] = min(sdom[i], sdom[query(u, 0)]);
            }
            if (i) bucket[sdom[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, 0);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = pa[i];
        }
        fill(all(res), -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i)
            res[rev[i]] = rev[dom[i]];
        res[s] = s;
        for (int i = 0; i < n; ++i) dom[i] = res[i];
    }
    void add_edge(int u, int v) { g[u].pb(v); }
    Dominator_tree (int _n) : n(_n), id(0), g(n), rg(n),
        bucket(n), sdom(n), dom(n, -1), vis(n, -1),
        rev(n), pa(n), rt(n), mn(n), res(n) {}
};
```

4.10 Bipartite Edge Coloring [d41d8c]

```
struct BipartiteEdgeColoring { // 1-based
    // returns edge coloring in adjacent matrix G
    int n, m;
    vector<vector<int>> col, G;
    int find_col(int x) {
        int c = 1;
        while (col[x][c] c++;
        return c;
    }
    void dfs(int v, int c1, int c2) {
        if (!col[v][c1]) return col[v][c2] = 0, void(0);
        int u = col[v][c1];
        dfs(u, c2, c1);
        col[v][c1] = 0, col[v][c2] = u, col[u][c2] = v;
    }
    void solve() {
        for (int i = 1; i <= n + m; ++i)
            for (int j = 1; j <= max(n, m); ++j)
                if (col[i][j])
                    G[i][col[i][j]] = G[col[i][j]][i] = j;
    } // u = left index, v = right index
    void add_edge(int u, int v) {
        int c1 = find_col(u), c2 = find_col(v + n);
        dfs(u, c2, c1);
        col[u][c2] = v + n, col[v + n][c2] = u;
    }
    BipartiteEdgeColoring (int _n, int _m) : n(_n),
        m(_m), col(n + m + 1, vector<int>(max(n, m) + 1)),
        G(n + m + 1, vector<int>(n + m + 1)) {}
};
```

4.11 Edge Coloring [d41d8c]

```
struct Vizing { // 1-based
    // returns edge coloring in adjacent matrix G
    int n;
    vector<vector<int>> C, G;
    vector<int> X, vst;
    vector<pii> E;
    void solve() {
        auto update = [&](int u)
            { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
        auto color = [&](int u, int v, int c) {
            int p = G[u][v];
            G[u][v] = G[v][u] = c;
            C[u][c] = v, C[v][c] = u;
            C[u][p] = C[v][p] = 0;
            if (p) X[u] = X[v] = p;
            else update(u), update(v);
        };
    }
};
```

```

    return p;
};
auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
};
fill(1 + all(X), 1);
for (int t = 0; t < (int)E.size(); ++t) {
    auto [u, v0] = E[t];
    int v = v0, c0 = X[u], c = c0, d;
    vector<pii> L;
    fill(1 + all(vst), 0);
    while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) {
            for (int a = sz(L) - 1; a >= 0; --a)
                c = color(u, L[a].first, c);
        } else if (!C[u][d]) {
            for (int a = sz(L) - 1; a >= 0; --a)
                color(u, L[a].first, L[a].second);
        } else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (int a; C[u][c0]) {
            for (a = sz(L) - 2;
                 a >= 0 && L[a].second != c; --a);
            for (; a >= 0; --a)
                color(u, L[a].first, L[a].second);
        }
        else --t;
    }
}
}
void add_edge(int u, int v) { E.emplace_back(u, v); }
Vizing(int _n) : n(_n), C(n + 1, vector<int>(n + 1)),
G(n + 1, vector<int>(n + 1)), X(n + 1), vst(n + 1) {}
};

```

4.12 Maximum Clique [d41d8c]

```

struct MaxClique { // Maximum Clique
    bitset<N> a[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; i++) a[i].reset();
    }
    void add_edge(int u, int v) { a[u][v] = a[v][u] = 1; }
    void csort(vector<int> &r, vector<int> &c) {
        int mx = 1, km = max(ans - q + 1, 1), t = 0;
        int m = r.size();
        cs[1].reset(), cs[2].reset();
        for (int i = 0; i < m; i++) {
            int p = r[i], k = 1;
            while ((cs[k] & a[p]).count()) k++;
            if (k > mx) mx++, cs[mx + 1].reset();
            cs[k][p] = 1;
            if (k < km) r[t++] = p;
        }
        c.resize(m);
        if (t) c[t - 1] = 0;
        for (int k = km; k <= mx; k++)
            for (int p = cs[k]._Find_first(); p < N;
                 p = cs[k]._Find_next(p))
                r[t] = p, c[t] = k, t++;
    }
    void dfs(vector<int> &r, vector<int> &c, int l,
            bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr, nc;
            bitset<N> nmask = mask & a[p];

```

```

        for (int i : r)
            if (a[p][i]) nr.push_back(i);
        if (!nr.empty()) {
            if (1 < 4) {
                for (int i : nr)
                    d[i] = (a[i] & nmask).count();
                sort(nr.begin(), nr.end(),
                    [&](int x, int y) { return d[x] > d[y]; });
            }
            csort(nr, nc), dfs(nr, nc, l + 1, nmask);
        } else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), q--;
    }
}
int solve(bitset<N> mask = bitset<N>(
    string(N, '1')) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; i++)
        if (mask[i]) r.push_back(i);
    for (int i = 0; i < n; i++)
        d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
        [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
}
};

```

5 String

5.1 Aho-Corasick Automaton [d41d8c]

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    vector<int> g[N];
    int cnt[N];
    AC () { sz = 0, extend(); }
    void extend() { fill(ch[sz], ch[sz] + 26, 0), sz++; }
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
        return ch[u][v];
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) now = nxt(now, c - 'a');
        cnt[now]++;
        return now;
    }
    void build_fail() {
        queue<int> q;
        for (int i = 0; i < 26; ++i) if (ch[0][i]) {
            q.push(ch[0][i]);
            g[0].push_back(ch[0][i]);
            to[0][i] = ch[0][i];
        }
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int j = 0; j < 26; ++j) {
                to[v][j] = ch[v][j] ? ch[v][j] : to[fail[v]][j];
            }
            for (int i = 0; i < 26; ++i) if (ch[v][i]) {
                int u = ch[v][i], k = fail[v];
                while (k && !ch[k][i]) k = fail[k];
                if (ch[k][i]) k = ch[k][i];
                fail[u] = k;
                cnt[u] += cnt[k], g[k].push_back(u);
                q.push(u);
            }
        }
    }
    int match(string &s) {
        int now = 0, ans = 0;
        for (char c : s) {
            now = to[now][c - 'a'];
            ans += cnt[now];
        }
        return ans;
    }
};

```

5.2 KMP Algorithm [d41d8c]

```
vector<int> build_fail(string s) {
    vector<int> f(s.size() + 1, 0);
    int k = 0;
    for (int i = 1; i < (int)s.size(); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}

int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < (int)s.size(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == (int)t.size()) ans++, k = f[k];
    }
    return ans;
}
```

5.3 Z Algorithm [d41d8c]

```
vector<int> buildZ(string s) {
    int n = (int)s.size(), l = 0, r = 0;
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]]) {
            l = i, r = i + Z[i], Z[i]++;
        }
    }
    return Z;
}
```

5.4 Manacher [d41d8c]

```
// return value only consider string tmp, not s
vector<int> manacher(string tmp) {
    string s = "&";
    for (char c : tmp) s.pb(c), s.pb('%');
    int l = 0, r = 0, n = (int)s.size();
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
        while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
        if (Z[i] + i > r) l = i, r = Z[i] + i;
    }
    for (int i = 0; i < n; ++i) {
        Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
    }
    return Z;
}
```

5.5 Suffix Array [d41d8c]

```
int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.size();
    for (int i = 0; i < n; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]] = s[i]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k)
            y[p++] = sa[i] - k;
        for (int i = n - 1; ~i; --i)
            sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!(x[a] == x[b] && a + k < n && b + k < n && x[a + k] == x[b + k])) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
    }
```

```
swap(x, y), m = p + 1;
}

void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = query_lcp_min[rk[i] + 1, rk[j] + 1)
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i] == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}
```

5.6 SAIS [d41d8c]

```
int sa[N << 1], rk[N], lcp[N];
// string ASCII value need > 0
namespace sfx {
    bool _t[N << 1];
    int _s[N << 1], _c[N << 1], x[N], _p[N], _q[N << 1];
    void pre(int *sa, int *c, int n, int z) {
        fill_n(sa, n, 0), copy_n(c, z, x);
    }
    void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; ++i)
            if (sa[i] && !t[sa[i] - 1])
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; --i)
            if (sa[i] && t[sa[i] - 1])
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
        fill_n(c, z, 0);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            if (s[i] == s[i + 1]) t[i] = t[i + 1];
            else t[i] = s[i] < s[i + 1];
        pre(sa, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(sa, c, s, t, n, z);
        for (int i = 0; i < n; ++i)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                bool neq = last < 0 || !equal(s + sa[i], s + p[q[sa[i]] + 1], s + last);
                ns[q[last = sa[i]]] = nmzx += neq;
            }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
        pre(sa, c, n, z);
        for (int i = nn - 1; i >= 0; --i)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        induce(sa, c, s, t, n, z);
    }
    void buildSA(string s) {
        int n = s.length();
        for (int i = 0; i < n; ++i) _s[i] = s[i];
        _s[n] = 0;
        sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
        for (int i = 1; i <= n; ++i) sa[i - 1] = sa[i];
    }
    // buildLCP()...
```

5.7 Suffix Automaton [d41d8c]

```
struct SAM {
    int ch[N][26], len[N], link[N], pos[N], cnt[N], sz;
    // node -> strings with the same endpos set
    // length in range [len(link) + 1, len]
    // node's endpos set -> pos in the subtree of node
    // link -> longest suffix with different endpos set
    // len -> longest suffix
    // pos -> end position
    // cnt -> size of endpos set
    SAM () {len[0] = 0, link[0] = -1, pos[0] = 0, cnt[0]
        = 0, sz = 1;}
    void build(string s) {
        int last = 0;
        for (int i = 0; i < s.length(); ++i) {
            char c = s[i];
            int cur = sz++;
            len[cur] = len[last] + 1, pos[cur] = i + 1;
            int p = last;
            while (~p && !ch[p][c - 'a'])
                ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) link[cur] = 0;
            else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q];
                    pos[nxt] = 0;
                    for (int j = 0; j < 26; ++j)
                        ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q)
                        ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
            cnt[cur]++;
            last = cur;
        }
        vector<int> p(sz);
        iota(all(p), 0);
        sort(all(p),
            [&](int i, int j) {return len[i] > len[j];});
        for (int i = 0; i < sz; ++i)
            cnt[link[p[i]]] += cnt[p[i]];
    }
} sam;
```

5.8 Minimum Rotation [d41d8c]

```
string rotate(const string &s) {
    int n = (int)s.size(), i = 0, j = 1;
    string t = s + s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}
```

5.9 Palindrome Tree [d41d8c]

```
struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM () {}
    void init(string s) {
        sz = 0, extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst]] != s[i])
                lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
            }
        }
    }
}
```

```
int now = fail[lst];
while (s[i - len[now]] != s[i])
    now = fail[now];
fail[idx] = ch[now][s[i] - 'a'];
ch[lst][s[i] - 'a'] = idx;
}
lst = ch[lst][s[i] - 'a'], cnt[lst]++;
}
}
void build_count() {
    for (int i = sz - 1; i > 1; --i)
        cnt[fail[i]] += cnt[i];
}
int extend() {
    fill(ch[sz], ch[sz] + 26, 0), sz++;
    return sz - 1;
}
};
```

5.10 Lyndon Factorization [d41d8c]

```
// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
vector<string> duval(const string &s) {
    vector<string> ans;
    for (int n = (int)s.size(), i = 0, j, k; i < n; ) {
        for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
            k = (s[k] < s[j] ? i : k + 1);
        for (; i <= k; i += j - k)
            ans.pb(s.substr(i, j - k)); // s.substr(L, len)
        return ans;
    }
}
```

5.11 Main Lorentz [d41d8c]

```
int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, len.
// substr( [L, r], len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l);
    string sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = l; i < m; ++i)
        to_right[i] = Z[r - m + 1 + i - l];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = l; i < m; ++i)
        to_left[i] = Z[m - i - l];
    reverse(all(sl));
    for (int i = l; i + 1 < m; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = m - i - 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2), tr = min(len - 1, k1);
        if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
    }
    Z = buildZ(sr);
    for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
    reverse(all(sl)), reverse(all(sr));
    Z = buildZ(sl + "#" + sr);
    for (int i = m; i < r; ++i)
        to_left[i] = Z[m - 1 + 1 + r - i - 1];
    reverse(all(sl)), reverse(all(sr));
    for (int i = m; i + 1 < r; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = i - m + 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(len - k2, 1), tr = min(len - 1, k1);
        if (tl <= tr)
            rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
    }
    Z = buildZ(sr + "#" + sl);
    for (int i = l; i < m; ++i)
        if (Z[r - m + 1 + i - l] >= m - i)
            rep.pb({i, i, m - i});
}
```

6 Math

6.1 Miller Rabin / Pollard Rho [d41d8c]

```

11 mul(11 x, 11 y, 11 p) {return (x * y - (11)((long
    double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022};
11 Pow(11 a, 11 b, 11 n) {
    11 res = 1;
    for (; b >>= 1, a = mul(a, a, n))
        if (b & 1) res = mul(res, a, n);
    return res;
}
bool check(11 a, 11 d, int s, 11 n) {
    a = Pow(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool IsPrime(11 n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    11 d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (11 i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
11 FindFactor(11 n) {
    if (IsPrime(n)) return 1;
    for (11 p : small) if (n % p == 0) return p;
    11 x, y = 2, d, t = 1;
    auto f = [&](11 a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            11 g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(11 n) {
    if (n == 1) return;
    if (IsPrime(n)) return ++res[n], void(0);
    11 d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
}

```

6.2 Ext GCD [d41d8c]

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(11 a, 11 b) {
    if (b == 0) return {1, 0};
    auto [y, x] = extgcd(b, a % b);
    return pair<ll, ll>(x, y - (a / b) * x);
}

```

6.3 Chinese Remainder Theorem [d41d8c]

```

pair<ll, ll> CRT(11 x1, 11 m1, 11 x2, 11 m2) {
    11 g = gcd(m1, m2);
    if ((x2 - x1) % g) return make_pair(-1, -1); // no sol
    m1 /= g, m2 /= g;
    pair<ll, ll> p = extgcd(m1, m2);
    11 lcm = m1 * m2 * g;
    11 res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return make_pair((res % lcm + lcm) % lcm, lcm);
}

```

6.4 PiCount [d41d8c]

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i])
        for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.pb(i);
}
void init(){
    sieve(V);
    small_pi[0] = 0;
    for(int i = 1; i < V; ++i)
        small_pi[i] = small_pi[i - 1] + isp[i];
    for(int i = 0; i < M; ++i) dp[0][i] = i;
    for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
        dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i - 1]];
}
11 phi(11 n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(11l * primes[a - 1] * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
11 PiCount(11 n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
        small_pi[y];
    11 res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n /
        primes[a]) - PiCount(primes[a]) + 1, 011);
    return res;
}

```

6.5 Linear Function Mod Min [d41d8c]

```

11 topos(11 x, 11 m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    Log m)
11 min_rem(11 n, 11 m, 11 a, 11 b) {
    a = topos(a, m), b = topos(b, m);
    for (11 g = __gcd(a, m); g > 1; ) return g * min_rem(n
        , m / g, a / g, b / g) + (b % g);
    for (11 nn, nm, na, nb; a; n = nn, m = nm, a = na, b
        = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            11 lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(Log m)
//{value, x}
pair<ll, ll> min_rem_pos(11 n, 11 m, 11 a, 11 b) {
    a = topos(a, m), b = topos(b, m);
    11 mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    11 x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
        % (m / g);
    return {mn, x};
}

```

6.6 Determinant [d41d8c]

```

int Det(vector<vector<int>> a) {

```



```

int n = (int)a.size(), det = 1;
for (int i = 0; i < n; ++i) {
    if (!a[i][i]) {
        det = sub(0, det);
        for (int j = i + 1; j < n; ++j) if (a[j][i]) {
            swap(a[j], a[i]);
            break;
        }
        if (!a[i][i]) return 0;
    }
    det = mul(det, a[i][i]);
    int tmp = Pow(a[i][i], mod - 2);
    for (int j = 0; j < n; ++j)
        a[i][j] = mul(a[i][j], tmp);
    for (int j = 0; j < n; ++j) if (i ^ j) {
        tmp = a[j][i];
        for (int k = 0; k < n; ++k) {
            a[j][k] = sub(a[j][k], mul(a[i][k], tmp));
        }
    }
}
return det;
}

```

6.7 Floor Sum [d41d8c]

```

// sum^{n-1}_0 floor((a * i + b) / m) in Log(n + m + a + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

6.8 Quadratic Residue [d41d8c]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (111 * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (p + 1) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
            g1 = (g0 * f1 + g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
        f1 = (2 * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.9 Discrete Log [d41d8c]

```

ll DiscreteLog(ll a, ll b, ll m) { // a^x = b (mod m)
    const int B = 35000;

```

```

    ll k = 1 % m, ans = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k) return ans;
        if (b % g) return -1;
        b /= g, m /= g, ans++, k = (k * a / g) % m;
    }
    if (b == k) return ans;
    unordered_map<ll, int> m1;
    ll tot = 1;
    for (int i = 0; i < B; ++i)
        m1[tot * b % m] = i, tot = tot * a % m;
    ll cur = k * tot % m;
    for (int i = 1; i <= B; ++i, cur = cur * tot % m)
        if (m1.count(cur)) return i * B - m1[cur] + ans;
    return -1;
}

```

6.10 Factorial without Prime Factor

[d41d8c]

```

// O(p^k + Log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k

```

6.11 Berlekamp Massey [d41d8c]

```

// need add, sub, mul
vector<int> BerlekampMassey(vector<int> a) {
    // find min |c| such that a_n = sum c_j * a_{n-j-1}, 0-based
    // O(N^2), if |c| = k, |a| >= 2k sure correct
    auto f = [&](vector<int> v, ll c) {
        for (int &x : v) x = mul(x, c);
        return v;
    };
    vector<int> c, best;
    int pos = 0, n = (int)a.size();
    for (int i = 0; i < n; ++i) {
        int error = a[i];
        for (int j = 0; j < (int)c.size(); ++j)
            error = sub(error, mul(c[j], a[i - 1 - j]));
        if (error == 0) continue;
        int inv = Pow(error, mod - 2);
        if (c.empty()) {
            c.resize(i + 1), pos = i, best.pb(inv);
        } else {
            vector<int> fix = f(best, error);
            fix.insert(fix.begin(), i - pos - 1, 0);
            if (fix.size() >= c.size()) {
                best = f(c, sub(0, inv));
                best.insert(best.begin(), inv);
                pos = i, c.resize(fix.size());
            }
            for (int j = 0; j < (int)fix.size(); ++j)
                c[j] = add(c[j], fix[j]);
        }
    }
    return c;
}

```

6.12 Simplex [d41d8c]

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
};

```

```

bool ls(T a, T b) {return a < b && !eq(a, b);}
void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
    for (int i = 0; i < m; ++i)
        for (int j = 0; j < n; ++j) a[i][j] = 0;
    for (int i = 0; i < m; ++i) b[i] = 0;
    for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
}
void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector<int> nz;
    for (int i = 0; i < n; ++i) {
        a[x][i] /= k;
        if (!eq(a[x][i], 0)) nz.push_back(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
        if (i == x || eq(a[i][y], 0)) continue;
        k = a[i][y], a[i][y] = 0;
        b[i] -= k * b[x];
        for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
}
// 0: found solution, 1: no feasible solution, 2:
// unbounded
int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (true) {
        int x = -1, y = -1;
        for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
        if (x == -1) break;
        for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
        if (y == -1) return 1;
        pivot(x, y);
    }
    while (true) {
        int x = -1, y = -1;
        for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
        if (y == -1) break;
        for (int i = 0; i < m; ++i)
            if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
        if (x == -1) return 2;
        pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n)
        sol[Left[i]] = b[i];
    return 0;
}
};

```

6.13 Linear Programming Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.

Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.

\bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$

- $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
- $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$

4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.14 Euclidean

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

6.15 Theorem

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there are

$$\frac{(n-2)!}{(d_1-1)!(d_2-1)! \cdots (d_n-1)!}$$

spanning trees.

- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős-Gallai Theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

- Burnside's Lemma

Let X be a set and G be a group that acts on X . For $g \in G$, denote by X^g the elements fixed by g :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called bigraphic if there is a labeled simple bipartite graph such that a and b is the degree sequence of this bipartite graph.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$. Sequences a and b called digraphic if there is a labeled simple directed graph such that each vertex v_i has indegree a_i and outdegree b_i .

- Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

- Möbius inversion formula

$$\begin{aligned} - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos \theta)(1 - \cos \theta)^2/3$.
- Area $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos \theta)$.

6.16 Estimation

- The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200000 for $n < 1e19$.
- The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for $n = 0 \sim 9$, 627 for $n = 20$, $\sim 2e5$ for $n = 50$, $\sim 2e8$ for $n = 100$.
- Total number of partitions of n distinct elements: $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \dots$

6.17 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

7 Polynomial

7.1 Number Theoretic Transform [d41d8c]

```
// mul, add, sub, Pow
struct NTT {
    int w[N];
    NTT() {
        int dw = Pow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i)
            w[i] = mul(w[i - 1], dw);
    }
    void operator()(vector<int>& a, bool inv = false) {
        // 0 <= a[i] < P
        int x = 0, n = a.size();
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (x ^= k) < k; k >>= 1);
            if (j < x) swap(a[x], a[j]);
        }
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    int tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            int invn = Pow(n, mod - 2);
            for (int i = 0; i < n; ++i)
                a[i] = mul(a[i], invn);
        }
    }
} ntt;
```

7.2 Fast Fourier Transform [d41d8c]

```
using T = complex<double>;
const double PI = acos(-1);
struct NTT {
    T w[N];
    FFT() {
        T dw = {cos(2 * PI / N), sin(2 * PI / N)};
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
    }
    void operator()(vector<T>& a, bool inv = false) {
        // see NTT, replace ll with T
        if (inv) {
            reverse(a.begin() + 1, a.end());
            T invn = 1.0 / n;
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
        }
    }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	194555039024054273	5
2748779069441	3	9223372036737335297	3

7.4 Polynomial Operations [d41d8c]

```
typedef vector<int> Poly;
Poly Mul(Poly a, Poly b, int bound = N) {
    int m = a.size() + b.size() - 1, n = 1;
    while (n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    Poly out(n);
```

```

    for (int i = 0; i < n; ++i) out[i] = mul(a[i], b[i]);
    ntt(out, true), out.resize(min(m, bound));
    return out;
}

Poly Inverse(Poly a) {
    // O(NLogN), a[0] != 0
    int n = a.size();
    Poly res(1, Pow(a[0], mod - 2));
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly v1(a.begin(), a.begin() + m * 2), v2 = res;
        v1.resize(m * 4), v2.resize(m * 4);
        ntt(v1), ntt(v2);
        for (int i = 0; i < m * 4; ++i)
            v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
        ntt(v1, true);
        res.resize(m * 2);
        for (int i = 0; i < m; ++i)
            res[i] = add(res[i], res[i]);
        for (int i = 0; i < m * 2; ++i)
            res[i] = sub(res[i], v1[i]);
    }
    res.resize(n);
    return res;
}

pair <Poly, Poly> Divide(Poly a, Poly b) {
    // a = bQ + R, O(NLogN), b.back() != 0
    int n = a.size(), m = b.size(), k = n - m + 1;
    if (n < m) return {{0}, a};
    Poly ra = a, rb = b;
    reverse(all(ra)), ra.resize(k);
    reverse(all(rb)), rb.resize(k);
    Poly Q = Mul(ra, Inverse(rb), k);
    reverse(all(Q));
    Poly res = Mul(b, Q), R(m - 1);
    for (int i = 0; i < m - 1; ++i)
        R[i] = sub(a[i], res[i]);
    return {Q, R};
}

Poly SqrtImpl(Poly a) {
    if (a.empty()) return {0};
    int z = QuadraticResidue(a[0], mod), n = a.size();
    if (z == -1) return {-1};
    Poly q(1, z);
    const int inv2 = (mod + 1) / 2;
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        q.resize(m * 2);
        Poly f2 = Mul(q, q, m * 2);
        for (int i = 0; i < m * 2; ++i)
            f2[i] = sub(f2[i], a[i]);
        f2 = Mul(f2, Inverse(q), m * 2);
        for (int i = 0; i < m * 2; ++i)
            q[i] = sub(q[i], mul(f2[i], inv2));
    }
    q.resize(n);
    return q;
}

Poly Sqrt(Poly a) {
    // O(NLogN), return {-1} if not exists
    int n = a.size(), m = 0;
    while (m < n && a[m] == 0) m++;
    if (m == n) return Poly(n);
    if (m & 1) return {-1};
    Poly s = SqrtImpl(Poly(a.begin() + m, a.end()));
    if (s[0] == -1) return {-1};
    Poly res(n);
    for (int i = 0; i < s.size(); ++i)
        res[i + m / 2] = s[i];
    return res;
}

Poly Derivative(Poly a) {
    int n = a.size();
    Poly res(n - 1);
    for (int i = 0; i < n - 1; ++i)
        res[i] = mul(a[i + 1], i + 1);
    return res;
}

Poly Integral(Poly a) {
    int n = a.size();
    Poly res(n + 1);
    for (int i = 0; i < n; ++i)
        res[i + 1] = mul(a[i], Pow(i + 1, mod - 2));
    return res;
}

Poly Ln(Poly a) {
    // O(NLogN), a[0] = 1
    int n = a.size();
    if (n == 1) return {0};
    Poly d = Derivative(a);
    a.pop_back();
    return Integral(Mul(d, Inverse(a), n - 1));
}

Poly Exp(Poly a) {
    // O(NLogN), a[0] = 0
    int n = a.size();
    Poly q(1, 1);
    a[0] = add(a[0], 1);
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        Poly g(a.begin(), a.begin() + m * 2), h(all(q));
        h.resize(m * 2), h = Ln(h);
        for (int i = 0; i < m * 2; ++i)
            g[i] = sub(g[i], h[i]);
        q = Mul(g, q, m * 2);
    }
    q.resize(n);
    return q;
}

Poly PolyPow(Poly a, ll k) {
    int n = a.size(), m = 0;
    Poly ans(n, 0);
    while (m < n && a[m] == 0) m++;
    if (k && m && (k >= n || k * m >= n)) return ans;
    if (m == n) return ans[0] = 1, ans;
    int lead = m * k;
    Poly b(a.begin() + m, a.end());
    int base = Pow(b[0], k), inv = Pow(b[0], mod - 2);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], inv);
    b = Ln(b);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], k % mod);
    b = Exp(b);
    for (int i = lead; i < n; ++i)
        ans[i] = mul(b[i - lead], base);
    return ans;
}

vector <int> Evaluate(Poly a, vector <int> x) {
    if (x.empty()) return {};
    int n = x.size();
    vector <Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector <Poly> down(n * 2);
    down[1] = Divide(a, up[1]).second;
    for (int i = 2; i < n * 2; ++i)
        down[i] = Divide(down[i >> 1], up[i]).second;
    Poly y(n);
    for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
    return y;
}

Poly Interpolate(vector <int> x, vector <int> y) {
    int n = x.size();
    vector <Poly> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    Poly a = Evaluate(Derivative(up[1]), x);
    for (int i = 0; i < n; ++i)
        a[i] = mul(y[i], Pow(a[i], mod - 2));
    vector <Poly> down(n * 2);
    for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
    for (int i = n - 1; i > 0; --i) {
        Poly lhs = Mul(down[i * 2], up[i * 2 + 1]);
        Poly rhs = Mul(down[i * 2 + 1], up[i * 2]);
        down[i].resize(lhs.size());
        for (int j = 0; j < lhs.size(); ++j)
            down[i][j] = add(lhs[j], rhs[j]);
    }
    return down[1];
}

```

```

}
Poly TaylorShift(Poly a, int c) {
    // return sum a_i(x + c)^i;
    // fac[i] = i!, facp[i] = inv(i!)
    int n = a.size();
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
    reverse(all(a));
    Poly b(n);
    int w = 1;
    for (int i = 0; i < n; ++i)
        b[i] = mul(facp[i], w), w = mul(w, c);
    a = Mul(a, b, n), reverse(all(a));
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    return a;
}
vector<int> SamplingShift(vector<int> a, int c, int m){
    // given f(0), f(1), ..., f(n - 1)
    // return f(c), f(c + 1), ..., f(c + m - 1)
    int n = a.size();
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    Poly b(n);
    for (int i = 0; i < n; ++i) {
        b[i] = facp[i];
        if (i & 1) b[i] = sub(0, b[i]);
    }
    a = Mul(a, b, n);
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
    reverse(all(a));
    int w = 1;
    for (int i = 0; i < n; ++i)
        b[i] = mul(facp[i], w), w = mul(w, sub(c, i));
    a = Mul(a, b, n);
    reverse(all(a));
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], facp[i]);
    a.resize(m), b.resize(m);
    for (int i = 0; i < m; ++i) b[i] = facp[i];
    a = Mul(a, b, m);
    for (int i = 0; i < m; ++i) a[i] = mul(a[i], fac[i]);
    return a;
}

```

7.5 Fast Linear Recursion [d41d8c]

```

int FastLinearRecursion(vector<int> a, vector<int> c,
    ll k) {
    // a_n = sigma c_j * a_{n - j - 1}, 0-based
    // O(NLogNLogK), |a| = |c|
    int n = a.size();
    if (k < n) return a[k];
    vector<int> base(n + 1, 1);
    for (int i = 0; i < n; ++i)
        base[i] = sub(0, c[n - i - 1]);
    vector<int> poly(n);
    (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
    auto calc = [&](vector<int> p1, vector<int> p2) {
        // O(n^2) brute force or O(n log n) NTT
        return Divide(Mul(p1, p2), base).second;
    };
    vector<int> res(n, 0); res[0] = 1;
    for (; k; k >>= 1, poly = calc(poly, poly)) {
        if (k & 1) res = calc(res, poly);
    }
    int ans = 0;
    for (int i = 0; i < n; ++i)
        ans = add(ans, mul(res[i], a[i]));
    return ans;
}

```

7.6 Fast Walsh Transform

```

void fwt(vector<int> &a, bool inv = false) {
    // and : x += y * (1, -1)
    // or  : y += x * (1, -1)
    // xor : x = (x + y) * (1, 1/2)
    //      y = (x - y) * (1, 1/2)
    int n = __lg(a.size());
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
            int x = a[j ^ (1 << i)], y = a[j];
            // do something
        }
    }
}

```

```

}
vector<int> subs_conv(vector<int> a, vector<int> b) {
    // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
    int n = __lg(a.size());
    vector ha(n + 1, vector<int>(1 << n));
    vector hb(n + 1, vector<int>(1 << n));
    vector c(n + 1, vector<int>(1 << n));
    for (int i = 0; i < 1 << n; ++i) {
        ha[__builtin_popcount(i)][i] = a[i];
        hb[__builtin_popcount(i)][i] = b[i];
    }
    for (int i = 0; i <= n; ++i)
        or_fwt(ha[i]), or_fwt(hb[i]);
    for (int i = 0; i <= n; ++i)
        for (int j = 0; i + j <= n; ++j)
            for (int k = 0; k < 1 << n; ++k)
                c[i + j][k] = add(c[i + j][k],
                    mul(ha[i][k], hb[j][k]));
    for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
    vector<int> ans(1 << n);
    for (int i = 0; i < 1 << n; ++i)
        ans[i] = c[__builtin_popcount(i)][i];
    return ans;
}

```

8 Geometry

8.1 Basic

```

const double eps = 1e-8, PI = acos(-1);
int sign(double x)
{ return fabs(x) <= eps ? 0 : (x > 0 ? 1 : -1); }
double normalize(double x) {
    while (x < -eps) x += PI * 2;
    while (x > PI * 2 + eps) x -= PI * 2;
    return x;
}
template<typename T>
struct P {
    T x, y;
    P<T>(T _x, T _y) : x(_x), y(_y) {}
    P<T> operator + (P<T> o) {
        return P<T>(x + o.x, y + o.y);
    }
    P<T> operator - (P<T> o) {
        return P<T>(x - o.x, y - o.y);
    }
    P<T> operator * (T k) { return P<T>(x * k, y * k); }
    P<T> operator / (T k) { return P<T>(x / k, y / k); }
    T operator * (P<T> o) { return x * o.x + y * o.y; }
    T operator ^ (P<T> o) { return x * o.y - y * o.x; }
};
using Pt = P<double>;
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 &&
    sign((c - a) * (c - b)) <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
int cmp(Pt a, Pt b)
{ return pos(a) == pos(b) ? sign(a ^ b) > 0 :
    pos(a) < pos(b); }
double area(Pt a, Pt b, Pt c)
{ return fabs((a - b) ^ (a - c)) / 2; }
double angle(Pt a, Pt b)
{ return normalize(atan2(b.y - a.y, b.x - a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) { // CCW
    double c = cos(o), s = sin(o);
    return Pt(c * a.x - s * a.y, s * a.x + c * a.y);
}
Pt perp(Pt a) { return Pt(-a.y, a.x); }
Pt proj_vec(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vec(a, b, c) + a;
}

```


8.2 Heart [d41d8c]

```
Pt circenter(Pt p0, Pt p1, Pt p2) {
    // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    double m = 2. * (x1 * y2 - y1 * x2);
    Pt center(0, 0);
    center.x = (x1 * x1 * x2 - x2 * x2 * y1 + y1 * y2 * (
        y1 - y2)) / m;
    center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
        y2 * y2) / m;
    return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) {
    // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
        - p2);
    double s = a + b + c;
    return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3)
{ return (p1 + p2 + p3) / 3; }
Pt orthocenter(Pt p1, Pt p2, Pt p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
    p3) * 2; }
```

8.3 External Bisector [d41d8c]

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
    Pt L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}
```

8.4 Intersection of Segments [d41d8c]

```
Pt LinesInter(Line a, Line b) {
    double abc = (a.b - a.a) ^ (b.a - a.a);
    double abd = (a.b - a.a) ^ (b.b - a.a);
    if (sign(abc - abd) == 0) return b.b; // no inter
    return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
    if (btw(a.a, a.b, b.a)) return {b.a};
    if (btw(a.a, a.b, b.b)) return {b.b};
    if (btw(b.a, b.b, a.a)) return {a.a};
    if (btw(b.a, b.b, a.b)) return {a.b};
    if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 &&
        ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
        return {LinesInter(a, b)};
    return {};
}
```

8.5 Intersection of Circle and Line [d41d8c]

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
        ) / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
        - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
}
```

8.6 Intersection of Circles [d41d8c]

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
        + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
        c2.r - c1.r * c1.r) / (2 * d2));
    double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
        * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
    Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2
        * d2);
    if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
    return {u + v, u - v};
}
```

8.7 Intersection of Polygon and Circle [d41d8c]

```
double _area(Pt pa, Pt pb, double r){
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
    double cosC = (pa * pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < pi / 2) S -= (acos(h / r) * r * r
            - h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = pi - B - asin(sin(B) / r * a);
        S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
            * r;
    } else S = 0.5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
    double S = 0; int n = poly.size();
    for (int i = 0; i < n; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
            ori(O, poly[i], poly[(i + 1) % n]);
    return fabs(S);
}
```

8.8 Tangent Lines of Circle and Point [d41d8c]

```
vector<Line> tangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sign(d - c.r) == 0) {
        Pt i = rot(p - c.o, pi / 2);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
            i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}
```

8.9 Tangent Lines of Circles [d41d8c]

```
vector<Line> tangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if (sign(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
            sign2 * h * v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
            0)
            p2 = p1 + perp(c2.o - c1.o);
        ret.pb({p1, p2});
    }
    return ret;
}
```

8.10 Point In Convex [d41d8c]

```
bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
    // only works when no three points are collinear
    int a = 1, b = int(C.size()) - 1, r = !strict;
    if (C.size() == 0) return false;
    if (C.size() < 3) return r && btw(C[0], C.back(), p);
```

```

if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
    -r) return false;
while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
}
return ori(C[a], C[b], p) < r;
}

```

8.11 Point In Circle [d41d8c]

```

// return p4 is strictly in circumcircle of tri(p1,p2,
// p3)
ll sqr(ll x) { return x * x; }
bool in_cc(const Pt &p1, const Pt &p2, const Pt &p3,
    const Pt &p4) {
    ll u11 = p1.x - p4.x; ll u12 = p1.y - p4.y;
    ll u21 = p2.x - p4.x; ll u22 = p2.y - p4.y;
    ll u31 = p3.x - p4.x; ll u32 = p3.y - p4.y;
    ll u13 = sqr(p1.x) - sqr(p4.x) + sqr(p1.y) - sqr(p4.y);
    ll u23 = sqr(p2.x) - sqr(p4.x) + sqr(p2.y) - sqr(p4.y);
    ll u33 = sqr(p3.x) - sqr(p4.x) + sqr(p3.y) - sqr(p4.y);
    __int128 det = (__int128)u13 * u22 * u31 + (__int128)
        u12 * u23 * u31 + (__int128)u13 * u21 * u32 - (
        __int128)u11 * u23 * u32 - (__int128)u12 * u21 *
        u33 + (__int128)u11 * u22 * u33;
    return det > 0;
}

```

8.12 Point Segment Distance [d41d8c]

```

double PointSegDist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
        * (p - q1)) >= 0))
        return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

8.13 Convex Hull [d41d8c]

```

vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b)
        {return a.x == b.x ? a.y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2],
                ans.back(), pt[i]) <= 0) ans.pop_back();
            ans.pb(pt[i]);
        }
        reverse(all(pt));
    }
    if (ans.size() > 1) ans.pop_back();
    return ans;
}

```

8.14 Minimum Enclosing Circle [d41d8c]

```

Cir min_enclosing(vector<Pt> &p) {
    random_shuffle(all(p));
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < p.size(); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i], r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}

```

8.15 Union of Circles [d41d8c]

```

vector<pair<double, double>> CoverSegment(Cir a, Cir b)
{
    double d = abs(a.o - b.o);
    vector<pair<double, double>> res;
    if (sign(a.r + b.r - d) == 0);
    else if (d <= abs(a.r - b.r) + eps) {
        if (a.r < b.r) res.emplace_back(0, 2 * pi);
    } else if (d < abs(a.r + b.r) - eps) {
        double o = acos((a.r * a.r + d * d - b.r * b.r) /
            (2 * a.r * d));
        double z = norm(atan2((b.o - a.o).y, (b.o - a.o).x)
            );
        double l = norm(z - o), r = norm(z + o);
        if (l > r) res.emplace_back(l, 2 * pi), res.
            emplace_back(0, r);
        else res.emplace_back(l, r);
    }
    return res;
}

double CircleUnionArea(vector<Cir> c) { // circle
    // should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
            for (auto &e : z) s.push_back(e);
        }
        sort(s.begin(), s.end());
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
        for (auto &e : s) {
            if (e.first > w) a += F(e.first) - F(w);
            w = max(w, e.second);
        }
    }
    return a * 0.5;
}

```

8.16 Union of Polygons [d41d8c]

```

double polyUnion(vector<vector<Pt>> poly) {
    int n = poly.size();
    double ans = 0;
    auto solve = [&](Pt a, Pt b, int cid) {
        vector<pair<Pt, int>> event;
        for (int i = 0; i < n; ++i) {
            int st = 0, sz = poly[i].size();
            while (st < sz && ori(poly[i][st], a, b) != 1)
                st++;
            if (st == sz) continue;
            for (int j = 0; j < sz; ++j) {
                Pt c = poly[i][(j + st) % sz];
                Pt d = poly[i][(j + st + 1) % sz];
                if (sign((a - b) ^ (c - d)) != 0) {
                    int ok1 = ori(c, a, b) == 1;
                    int ok2 = ori(d, a, b) == 1;
                    if (ok1 ^ ok2) event.emplace_back(LinesInter
                        ({a, b}, {c, d}), ok1 ? 1 : -1);
                } else if (ori(c, a, b) == 0 && sign((a - b) *
                    (c - d)) > 0 && i <= cid) {
                    event.emplace_back(c, -1);
                    event.emplace_back(d, 1);
                }
            }
        }
    };
    sort(all(event), [&](pair<Pt, int> i, pair<Pt,
        int> j) {
        return ((a - i.first) * (a - b)) < ((a - j.first)
            * (a - b));
    });
    int now = 0;
    Pt lst = a;
    for (auto [x, y] : event) {
        if (btw(a, b, lst) && btw(a, b, x) && !now)
            ans += lst ^ x;
        now += y, lst = x;
    }
    for (int i = 0; i < n; ++i) {

```

```

    int sz = poly[i].size();
    for (int j = 0; j < sz; ++j)
        solve(poly[i][j], poly[i][(j + 1) % sz], i);
}
return ans / 2;
}

```

8.17 Rotating SweepLine [d41d8c]

```

struct Event {
    Pt d; int u, v;
    bool operator < (const Event &b) const {
        return sign(d ^ b.d) > 0;
    };
};
Pt ref(Pt o) {return pos(o) == 1 ? Pt(-o.x, -o.y) : o;}
void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> ord(n), pos(n);
    vector<Event> e;
    for (int i = 0; i < n; ++i)
        for (int j = i + 1; j < n; ++j) if (i ^ j)
            e.pb({ref(pt[i] - pt[j]), i, j});
    sort(all(e));
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) {
        return (sign(pt[i].y - pt[j].y) == 0 ?
            pt[i].x < pt[j].x : pt[i].y < pt[j].y); });
    for (int i = 0; i < n; ++i) pos[ord[i]] = i;
    const auto makeReverse = [&](auto &v) {
        sort(all(v)); v.resize(unique(all(v)) - v.begin());
        vector<pii> segs;
        for (int i = 0, j = 0; i < v.size(); i = j) {
            for (; j < v.size() && v[j] - v[i] <= j - i; ++j)
                ;
            segs.emplace_back(v[i], v[j - 1] + 1 + 1);
        }
        return segs;
    };
    for (int i = 0, j = 0; i < e.size(); i = j) {
        vector<int> tmp;
        for (; j < e.size() && !(e[i] < e[j]); j++)
            tmp.pb(min(pos[e[j].u], pos[e[j].v]));
        for (auto [l, r] : makeReverse(tmp)) {
            reverse(ord.begin() + l, ord.begin() + r);
            for (int t = l; t < r; ++t) pos[ord[t]] = t;
            // update value here
        }
    }
}

```

8.18 Half Plane Intersection [d41d8c]

```

pair<ll, ll> area_pair(Line a, Line b)
{ return {(a.b - a.a) ^ (b.a - a.a), (a.b - a.a) ^ (b.b - a.a)}; }
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return a02Y * a12X - a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* ---^--- Line.a ---^--- Line.b ---^--- */
vector<Line> HalfPlaneInter(vector<Line> arr) {
    sort(all(arr), [&](Line a, Line b) {
        Pt A = a.b - a.a, B = b.b - b.a;
        if (pos(A) != pos(B)) return pos(A) < pos(B);
        if (sign(A ^ B) != 0) return sign(A ^ B) > 0;
        return ori(a.a, a.b, b.b) < 0;
    });
    deque<Line> dq(1, arr[0]);
    auto same = [&](Pt a, Pt b)
    { return sign(a ^ b) == 0 && pos(a) == pos(b); };
    for (auto p : arr) {
        if (same(dq.back().b - dq.back().a, p.b - p.a))
            continue;
        while (sz(dq) >= 2 && !isin(p, dq[sz(dq) - 2], dq.back())) dq.pop_back();
        while (sz(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(p);
    }
}

```

```

}
while (sz(dq) >= 3 && !isin(dq[0], dq[sz(dq) - 2], dq.back())) dq.pop_back();
while (sz(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
return vector<Line>(all(dq));
}

```

8.19 Minkowski Sum [d41d8c]

```

void reorder(vector<Pt> &P) {
    rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
        { return make_pair(a.y, a.x) < make_pair(b.y, b.x);
        })), P.end());
}
vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    // P, Q: convex polygon, CCW order
    reorder(P), reorder(Q);
    int n = P.size(), m = Q.size();
    P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
    vector<Pt> ans;
    for (int i = 0, j = 0; i < n || j < m; ) {
        ans.pb(P[i] + Q[j]);
        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
        if (val >= 0) i++;
        if (val <= 0) j++;
    }
    return ans;
}

```

8.20 Vector In Polygon [d41d8c]

```

// ori(a, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(Pt a, Pt b, Pt c, Pt p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector{cur, p} in counter-clockwise order
// prv, cur, nxt
bool inside(Pt prv, Pt cur, Pt nxt, Pt p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}

```

8.21 Delaunay Triangulation [d41d8c]

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
    Pt p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const Pt &p0, const Pt &p1, const Pt &p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(const Pt &q) const {
        for (int i = 0; i < 3; ++i)

```

```

    if (ori(p[i], p[(i + 1) % 3], q) < 0)
        return 0;
    return 1;
}
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
        new(tris++) Tri(Pt(-inf, -inf), Pt(inf + inf, -
inf), Pt(-inf, inf + inf));
    }
    Tri* find(Pt p) { return find(the_root, p); }
    void add_point(const Pt &p) { add_point(find(the_root
, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const Pt &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, Pt const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root->p[i], root->p[(i +
1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
        for (int i = 0; i < 3; ++i)
            root->chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri(tri->p[(pi + 1) % 3],
trj->p[pj], tri->p[pi]);
        Tri* trl = new(tris++) Tri(trj->p[(pj + 1) % 3],
tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri->chd[0] = trk; tri->chd[1] = trl; tri->chd[2] =
0;
        trj->chd[0] = trk; trj->chd[1] = trl; trj->chd[2] =
0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now->has_chd())
        return triang.pb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}

```

```

void build(vector<Pt> &arr) { // build triangulation
    int n = arr.size();
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(all(arr));
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(arr[i]);
    go(tri.the_root);
}

```

8.22 Triangulation Voronoi [d41d8c]

```

vector<Line> ls[N];
Line make_line(Pt p, Line l) {
    Pt d = l.b - l.a; d = perp(d);
    Pt m = (l.a + l.b) / 2; // remember to *2
    l = {m, m + d};
    if (ori(l.a, l.b, p) < 0) swap(l.a, l.b);
    return l;
}
void solve(vector<Pt> &oarr) {
    int n = oarr.size();
    map<pair<ll, ll>, int> mp;
    vector<Pt> arr = oarr;
    for (int i = 0; i < n; ++i)
        mp[{arr[i].x, arr[i].y}] = i;
    build(arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i) {
            pair<ll, ll> tmp = {t->p[i].x, t->p[i].y};
            if (mp.count(tmp)) p.pb(mp[tmp]);
        }
        for (int i = 0; i < sz(p); ++i)
            for (int j = i + 1; j < sz(p); ++j) {
                Line l = {oarr[p[i]], oarr[p[j]]};
                ls[p[i]].pb(make_line(oarr[p[i]], l));
                ls[p[j]].pb(make_line(oarr[p[j]], l));
            }
        for (int i = 0; i < n; ++i)
            ls[i] = HalfPlaneInter(ls[i]);
    }
}

```

8.23 3D Point

```

struct Pt {
    double x, y, z;
    Pt(double _x = 0, double _y = 0, double _z = 0): x(_x
), y(_y), z(_z){}
    Pt operator + (const Pt &o) const
    { return Pt(x + o.x, y + o.y, z + o.z); }
    Pt operator - (const Pt &o) const
    { return Pt(x - o.x, y - o.y, z - o.z); }
    Pt operator * (const double &k) const
    { return Pt(x * k, y * k, z * k); }
    Pt operator / (const double &k) const
    { return Pt(x / k, y / k, z / k); }
    double operator * (const Pt &o) const
    { return x * o.x + y * o.y + z * o.z; }
    Pt operator ^ (const Pt &o) const
    { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x
* o.y - y * o.x)}; }
};
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
Pt cross3(Pt a, Pt b, Pt c)
{ return (b - a) ^ (c - a); }
double area(Pt a, Pt b, Pt c)
{ return abs(cross3(a, b, c)); }
double volume(Pt a, Pt b, Pt c, Pt d)
{ return cross3(a, b, c) * (d - a); }
bool coplaner(Pt a, Pt b, Pt c, Pt d)
{ return sign(volume(a, b, c, d)) == 0; }
Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
{ Pt n = cross3(a, b, c);
  return o - n * ((o - a) * (n / abs2(n))); }
Pt LinePlaneInter(Pt u, Pt v, Pt a, Pt b, Pt c) {
    // intersection of line uv and plane abc
    Pt n = cross3(a, b, c);
    double s = n * (u - v);
    if (sign(s) == 0) return {-1, -1, -1}; // not found
}

```

```

    return v + (u - v) * ((n * (a - v)) / s);
}

```

8.24 3D Convex Hull [d41d8c]

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];
    double dblcmp(Pt &p, face &f)
    {return cross3(P[f.a], P[f.b], P[f.c]) * (p - P[f.a])
    ;}
    int g[N][N], num, n;
    Pt P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        } else
            add.a = b, add.b = a, add.c = p, add.ok = 1, g[
                p][b] = g[a][p] = g[b][a] = num, F[num++] =
                add;
    }
    void dfs(int p, int now) {
        F[now].ok = 0;
        deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
            now].b), deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t){
        Pt &a = P[F[s].a];
        Pt &b = P[F[s].b];
        Pt &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
                volume(a, b, c, P[F[t].c])) < eps;
    }
    void init(int _n){n = _n, num = 0;}
    void solve() {
        face add;
        num = 0;
        if(n < 4) return;
        if([&](){
            for (int i = 1; i < n; ++i)
                if (abs(P[0] - P[i]) > eps)
                    return swap(P[1], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 2; i < n; ++i)
                if (abs(cross3(P[i], P[0], P[1])) > eps)
                    return swap(P[2], P[i]), 0;
            return 1;
        }() || [&](){
            for (int i = 3; i < n; ++i)
                if (fabs(((P[0] - P[1]) ^ (P[1] - P[2])) * (P
                    [0] - P[i])) > eps)
                    return swap(P[3], P[i]), 0;
            return 1;
        }())return;
        for (int i = 0; i < 4; ++i) {
            add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
                (i + 3) % 4, add.ok = true;
            if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
            g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
                a] = num;
            F[num++] = add;
        }
        for (int i = 4; i < n; ++i)
            for (int j = 0; j < num; ++j)
                if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                    dfs(i, j);
                    break;
                }
        for (int tmp = num, i = (num = 0); i < tmp; ++i)
            if (F[i].ok) F[num++] = F[i];
    }
    double get_area() {
        double res = 0.0;
        if (n == 3)
            return abs(cross3(P[0], P[1], P[2])) / 2.0;
        for (int i = 0; i < num; ++i)
            res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
        return res / 2.0;
    }
}

```

```

}
double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
        res += volume(Pt(0, 0, 0), P[F[i].a], P[F[i].b],
            P[F[i].c]);
    return fabs(res / 6.0);
}
int triangle() {return num;}
int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag
        , flag = 1)
        for (int j = 0; j < i && flag; ++j)
            flag &= !same(i, j);
    return res;
}
Pt getcent(){
    Pt ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Pt p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].
                c];
            t2 = volume(temp, p1, p2, p3) / 6.0;
            if (t2 > 0)
                ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                ans.y += (p1.y + p2.y + p3.y + temp.y) *
                    t2, ans.z += (p1.z + p2.z + p3.z + temp.z
                        ) * t2, v += t2;
        }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
        );
    return ans;
}
double pointmindis(Pt p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
        if(F[i].ok == true) {
            Pt p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].
                c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
                z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
                x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
                y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
                ;
            double temp = fabs(a * p.x + b * p.y + c * p.z
                + d) / sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
}

```

9 Else

9.1 Pbds

```

#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue <int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1); // 4 0
bitset <100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope <int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);

```



```
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```
ll next_perm(ll v) { ll t = v | (v - 1);
  return (t + 1) |
    (((~t & ~t) - 1) >> (__builtin_ctz(v) + 1)); }
```

9.3 Dynamic Programming Condition

9.3.1 Totally Monotone (Concave/Convex)

$$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$$

$$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$$

9.3.2 Monge Condition (Concave/Convex)

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$$

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$$

9.3.3 Optimal Split Point

If $B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$

then $H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$

9.4 Smawk Algorithm [d41d8c]

```
ll f(int l, int r) { }
bool select(int r, int u, int v) {
  // if f(r, v) is better than f(r, u), return true
  return f(r, u) < f(r, v);
}
// For all 2x2 submatrix:
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
vector<int> solve(vector<int> &r, vector<int> &c) {
  const int n = r.size();
  if (n == 0) return {};
  vector<int> c2;
  for (const int &i : c) {
    while (!c2.empty() && select(r[c2.size() - 1], c2.back(), i)) c2.pop_back();
    if (c2.size() < n) c2.pb(i);
  }
  vector<int> r2;
  for (int i = 1; i < n; i += 2) r2.pb(r[i]);
  const auto a2 = solve(r2, c2);
  vector<int> ans(n);
  for (int i = 0; i < a2.size(); i++)
    ans[i * 2 + 1] = a2[i];
  int j = 0;
  for (int i = 0; i < n; i += 2) {
    ans[i] = c2[j];
    const int end = i + 1 == n ? c2.back() : ans[i + 1];
    while (c2[j] != end) {
      j++;
      if (select(r[i], ans[i], c2[j])) ans[i] = c2[j];
    }
  }
  return ans;
}
vector<int> smawk(int n, int m) {
  vector<int> row(n), col(m);
  iota(all(row), 0), iota(all(col), 0);
  return solve(row, col);
}
```

9.5 Slope Trick [d41d8c]

```
template<typename T>
struct slope_trick_convex {
  T minn = 0, ground_l = 0, ground_r = 0;
  priority_queue<T, vector<T>, less<T>> left;
  priority_queue<T, vector<T>, greater<T>> right;
  slope_trick_convex() {left.push(numeric_limits<T>::min() / 2), right.push(numeric_limits<T>::max() / 2);}
  void push_left(T x) {left.push(x - ground_l);}
  void push_right(T x) {right.push(x - ground_r);}
}
```

```
//add a line with slope 1 to the right starting from x
void add_right(T x) {
  T l = left.top() + ground_l;
  if (l <= x) push_right(x);
  else push_left(x), push_right(l), left.pop(), minn += 1 - x;
}
//add a line with slope -1 to the left starting from x
void add_left(T x) {
  T r = right.top() + ground_r;
  if (r >= x) push_left(x);
  else push_right(x), push_left(r), right.pop(), minn += x - r;
}
//val[i]=min(val[j]) for all i-l<=j<=i+r
void expand(T l, T r) {ground_l -= l, ground_r += r;}
void shift_up(T x) {minn += x;}
T get_val(T x) {
  T l = left.top() + ground_l, r = right.top() + ground_r;
  if (x >= l && x <= r) return minn;
  if (x < l) {
    vector<T> trash;
    T cur_val = minn, slope = 1, res;
    while (1) {
      trash.push_back(left.top());
      left.pop();
      if (left.top() + ground_l <= x) {
        res = cur_val + slope * (1 - x);
        break;
      }
      cur_val += slope * (1 - (left.top() + ground_l));
      l = left.top() + ground_l;
      slope += 1;
    }
    for (auto i : trash) left.push(i);
    return res;
  }
  if (x > r) {
    vector<T> trash;
    T cur_val = minn, slope = 1, res;
    while (1) {
      trash.push_back(right.top());
      right.pop();
      if (right.top() + ground_r >= x) {
        res = cur_val + slope * (x - r);
        break;
      }
      cur_val += slope * ((right.top() + ground_r) - r);
      r = right.top() + ground_r;
      slope += 1;
    }
    for (auto i : trash) right.push(i);
    return res;
  }
  assert(0);
}
};
```

9.6 ALL LCS [d41d8c]

```
void all_lcs(string s, string t) { // 0-base
  vector<int> h(t.size());
  iota(all(h), 0);
  for (int a = 0; a < s.size(); ++a) {
    int v = -1;
    for (int c = 0; c < t.size(); ++c)
      if (s[a] == t[c] || h[c] < v)
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

9.7 Hilbert Curve [d41d8c]

```
ll hilbert(int n, int x, int y) {
```

```

11 res = 0;
for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 111 * s * ((3 * rx) ^ ry);
    if (ry == 0) {
        if (rx == 1) x = s - 1 - x, y = s - 1 - y;
        swap(x, y);
    }
}
return res;
} // n = 2^k

```

9.8 Random

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t a) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(i + FIXED_RANDOM);
    }
};
unordered_map<int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());

```

9.9 Line Container [d41d8c]

```

// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator<(const Line &rhs) const { return a <
        rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a
        % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) { // ax + b
        auto z = insert({a, b, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

9.10 Min Plus Convolution [d41d8c]

```

// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector
    <int> &b) {
    int n = a.size(), m = b.size();
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from);
        Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}

```

9.11 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.12 Simulated Annealing

```

double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 100000; ++it) {
    // ans: answer, nw: current value, rnd(): mt19937 rnd
    ()
    if (exp(-(nw - ans) / factor) >= (double)(rnd() %
        base) / base)
        ans = nw;
    factor *= 0.99995;
}

```

9.13 Bitset LCS

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; ++i) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

9.14 Binary Search On Fraction

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p, q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}

```

9.15 Python Misc

```

from [decimal, fractions, math, random] import *
arr = list(map(int, input().split())) # input
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
    ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
S = set(), S.add((a, b)), S.remove((a, b)) # set
if not (a, b) in S:
    D = dict(), D[(a, b)] = 1, del D[(a, b)] # dict
for (a, b) in D.items():
    arr = [randint(1, C) for i in range(N)]
    choice([8, 6, 4, 1]) # random pick one
    shuffle(arr)
    print(*arr, sep=' ')

```