Contents

1 Basic

1.1 Shell Script

```
#!/usr/bin/env bash
g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
    $1 && ./$1
for i in {A..J}; do cp tem.cpp $i.cpp; done;
```

1.2 Default Code

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define pb push_back
#define pii pair<int, int>
#define all(a) a.begin(), a.end()
#define sz(a) ((int)a.size())
```

1.3 Increase Stack Size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bk = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bk));
```

1.4 Debug Macro

```
void db() { cout << endl; }
template <typename T, typename ...U>
void db(T i, U ...j) { cout << i << ' ', db(j...); }
#define test(x...) db("[" + string(x) + "]", x)
```

1.5 Pragma / FastIO

```
#pragma GCC optimize("Ofast,inline,unroll-loops")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
  static char buf[65536], *p = buf, *q = buf;
  return p == q && (q = (p = buf) + read(0, buf, 65536)
      ) == buf ? -1 : *p++;
}
inline int R() {
  static char c;
  while((c = RC()) < '0'); int a = c ^ '0';
  while((c = RC()) >= '0') a *= 10, a += c ^ '0';
  return a;
}
inline void W(int n) {
  static char buf[12], p;
  if (n == 0) OB[OP++]='0'; p = 0;
  while (n) buf[p++] = '0' + (n % 10), n /= 10;
  for (--p; p >= 0; --p) OB[OP++] = buf[p];
  if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.6 Divide*

```
ll floor(ll a, ll b) {
  return a / b - (a < 0 && a % b);
}
ll ceil(ll a, ll b) {
  return a / b + (a > 0 && a % b);
}
a / b <  x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x <  a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)
```

2 Data Structure

2.1 Leftist Tree

```
struct node {
  ll rk, data, sz, sum;
  node *l, *r;
  node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k
      ) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (a->data < b->data) swap(a, b);
  a->r = merge(a->r, b);
  if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
  a->rk = rk(a->r) + 1;
  a->sz = sz(a->l) + sz(a->r) + 1;
  a->sum = sum(a->l) + sum(a->r) + a->data;
  return a;
}
void pop(node *&o) {
  node *tmp = o;
  o = merge(o->l, o->r);
  delete tmp;
}
```

2.2 Splay Tree

```
struct Splay {
  int pa[N], ch[N][2], sz[N], rt, _id;
  ll v[N];
  Splay() {}
  void init() {
    rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
    sz[0] = 1, v[0] = inf;
  }
  int newnode(int p, int x) {
    int id = _id++;
    v[id] = x, pa[id] = p;
    ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    return id;
  }
  void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int gp = pa[p], c = ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
  }
  void splay(int i) {
    while (~pa[i]) {
      int p = pa[i];
      if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
          == i ? i : p);
      rotate(i);
    }
    rt = i;
  }
  int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
      if (v[i] == x) return splay(i), i;
      if (v[i] > x) {
        last = i;
        if (ch[i][0] == -1) break;
        i = ch[i][0];
      }
      else {
        if (ch[i][1] == -1) break;
        i = ch[i][1];
      }
    }
    splay(i);
    return last; // -1 if not found
  }
  void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
      // assert(ch[rt][1] == -1);
      int id = newnode(rt, x);
      ch[rt][1] = id, ++sz[rt];
      splay(id);
    }
    else if (v[i] != x) {
      splay(i);
      int id = newnode(rt, x), c = ch[rt][0];
      ch[rt][0] = id;
      ch[id][0] = c;
      if (~c) pa[c] = id, sz[id] += sz[c];
      ++sz[rt];
      splay(id);
    }
  }
};
```

2.3 Link Cut Tree

```
// weighted subtree size, weighted path max
struct LCT {
  int ch[N][2], pa[N], v[N], sz[N];
  int sz2[N], w[N], mx[N], _id;
  // sz := sum of v in splay, sz2 := sum of v in
      virtual subtree
  // mx := max w in splay
  bool rev[N];
  LCT() : _id(1) {}
  int newnode(int _v, int _w) {
    int x = _id++;
    ch[x][0] = ch[x][1] = pa[x] = 0;
    v[x] = sz[x] = _v;
    sz2[x] = 0;
    w[x] = mx[x] = _w;
    rev[x] = false;
    return x;
  }
  void pull(int i) {
    sz[i] = v[i] + sz2[i];
    mx[i] = w[i];
    if (ch[i][0]) {
      sz[i] += sz[ch[i][0]];
      mx[i] = max(mx[i], mx[ch[i][0]]);
    }
    if (ch[i][1]) {
      sz[i] += sz[ch[i][1]];
      mx[i] = max(mx[i], mx[ch[i][1]]);
    }
  }
  void push(int i) {
    if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
        rev[i] = false;
  }
  void reverse(int i) {
    if (!i) return;
    swap(ch[i][0], ch[i][1]);
    rev[i] ^= true;
  }
  bool isrt(int i) {// rt of splay
    if (!pa[i]) return true;
    return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
  }
  void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int c = ch[i][!x], gp = pa[p];
    if (ch[gp][0] == p) ch[gp][0] = i;
    else if (ch[gp][1] == p) ch[gp][1] = i;
```

```
    pa[i] = gp, ch[i][!x] = p, pa[p] = i;
    ch[p][x] = c, pa[c] = p;
    pull(p), pull(i);
  }
  void splay(int i) {
    vector<int> anc;
    anc.push_back(i);
    while (!isrt(anc.back()))
      anc.push_back(pa[anc.back()]);
    while (!anc.empty())
      push(anc.back()), anc.pop_back();
    while (!isrt(i)) {
      int p = pa[i];
      if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
           == p ? i : p);
      rotate(i);
    }
  }
  void access(int i) {
    int last = 0;
    while (i) {
      splay(i);
      if (ch[i][1])
        sz2[i] += sz[ch[i][1]];
      sz2[i] -= sz[last];
      ch[i][1] = last;
      pull(i), last = i, i = pa[i];
    }
  }
  void makert(int i) {
    access(i), splay(i), reverse(i);
  }
  void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
  }
  void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
  }
  int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
  }
};
```

2.4 Treap

```
struct node {
  int data, sz;
  node *l, *r;
  node(int k) : data(k), sz(1), l(0), r(0) {}
  void up() {
    sz = 1;
    if (l) sz += l->sz;
    if (r) sz += r->sz;
  }
  void down() {}
};
// delete default code sz
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (rand() % (sz(a) + sz(b)) < sz(a))
    return a->down(), a->r = merge(a->r, b), a->up(),a;
  return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
  if (!o) return a = b = 0, void();
  o->down();
  if (o->data <= k)
    a = o, split(o->r, a->r, b, k), a->up();
  else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
  if (sz(o) <= k) return a = o, b = 0, void();
  o->down();
  if (sz(o->l) + 1 <= k)
    a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
  else b = o, split2(o->l, a, b->l, k);
  o->up();
}
node *kth(node *o, int k) {
  if (k <= sz(o->l)) return kth(o->l, k);
  if (k == sz(o->l) + 1) return o;
  return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
  if (!o) return 0;
  if (o->data < key)
    return sz(o->l) + 1 + Rank(o->r, key);
  else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
  if (!o) return 0;
  if (o->data == k) {
    node *t = o;
    o->down(), o = merge(o->l, o->r);
    delete t;
    return 1;
  }
  node *&t = k < o->data ? o->l : o->r;
  return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
  node *a, *b;
  o->down(), split(o, a, b, k),
  o = merge(a, merge(new node(k), b));
  o->up();
}
void interval(node *&o, int l, int r) {
  node *a, *b, *c; // [l, r)
  o->down();
  split2(o, a, b, l), split2(b, b, c, r - l);
  // operate
  o = merge(a, merge(b, c)), o->up();
}
```

2.5 2D Segment Tree*

```
// 2D range add, range sum in log^2
struct seg {
  int l, r;
  ll sum, lz;
  seg *ch[2]{};
  seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
  void push() {
    if (lz) ch[0]->add(l, r, lz), ch[1]->add(l, r, lz),
         lz = 0;
  }
  void pull() { sum = ch[0]->sum + ch[1]->sum; }
  void add(int _l, int _r, ll d) {
    if (_l <= l && r <= _r) {
      sum += d * (r - l), lz += d;
      return;
    }
    if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
         new seg(l + r >> 1, r);
    push();
    if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
    pull();
  }
  ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(l, _l));
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
  }
};
struct seg2 {
  int l, r;
  seg v, lz;
  seg2 *ch[2]{};
```

```
  seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N
      ) {
    if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch
        [1] = new seg2(l + r >> 1, r);
  }
  void add(int _l, int _r, int _l2, int _r2, ll d) {
    v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
    if (_l <= l && r <= _r)
      return lz.add(_l2, _r2, d), void(0);
    if (_l < l + r >> 1)
        ch[0]->add(_l, _r, _l2, _r2, d);
    if (l + r >> 1 < _r)
        ch[1]->add(_l, _r, _l2, _r2, d);
  }
  ll qsum(int _l, int _r, int _l2, int _r2) {
    if (_l <= l && r <= _r) return v.qsum(_l2, _r2);
    ll d = min(r, _r) - max(l, _l);
    ll res = lz.qsum(_l2, _r2) * d;
    if (_l < l + r >> 1)
        res += ch[0]->qsum(_l, _r, _l2, _r2);
    if (l + r >> 1 < _r)
        res += ch[1]->qsum(_l, _r, _l2, _r2);
    return res;
  }
};
```

2.6 Range Set*

```
struct RangeSet { // [l, r)
  set <pii> S;
  void cut(int x) {
    auto it = S.lower_bound({x + 1, -1});
    if (it == S.begin()) return;
    auto [l, r] = *prev(it);
    if (l >= x || x >= r) return;
    S.erase(prev(it));
    S.insert({l, x});
    S.insert({x, r});
  }
  vector <pii> split(int l, int r) {
    // remove and return ranges in [l, r)
    cut(l), cut(r);
    vector <pii> res;
    while (true) {
      auto it = S.lower_bound({l, -1});
      if (it == S.end() || r <= it->first) break;
      res.pb(*it), S.erase(it);
    }
    return res;
  }
  void insert(int l, int r) {
    // add a range [l, r), [l, r) not in S
    auto it = S.lower_bound({l, r});
    if (it != S.begin() && prev(it)->second == l)
      l = prev(it)->first, S.erase(prev(it));
    if (it != S.end() && r == it->first)
      r = it->second, S.erase(it);
    S.insert({l, r});
  }
  bool count(int x) {
    auto it = S.lower_bound({x + 1, -1});
    return it != S.begin() && prev(it)->first <= x
            && x < prev(it)->second;
  }
};
```

2.7 vEB Tree*

```
using u64=uint64_t;
constexpr int lsb(u64 x)
{ return x?__builtin_ctzll(x):1<<30; }
constexpr int msb(u64 x)
{ return x?63-__builtin_clzll(x):-1; }
template<int N, class T=void>
struct veb{
  static const int M=N>>1;
  veb<M> ch[1<<N-M];
  veb<N-M> aux;
  int mn,mx;
  veb():mn(1<<30),mx(-1){}
  constexpr int mask(int x){return x&((1<<M)-1);}
  bool empty(){return mx==-1;}
  int min(){return mn;}
  int max(){return mx;}
  bool have(int x){
    return x==mn?true:ch[x>>M].have(mask(x));
  }
  void insert_in(int x){
    if(empty()) return mn=mx=x,void();
    if(x<mn) swap(x,mn);
    if(x>mx) mx=x;
    if(ch[x>>M].empty()) aux.insert_in(x>>M);
    ch[x>>M].insert_in(mask(x));
  }
  void erase_in(int x){
    if(mn==mx) return mn=1<<30,mx=-1,void();
    if(x==mn) mn=x=(aux.min()<<M)^ch[aux.min()].min();
    ch[x>>M].erase_in(mask(x));
    if(ch[x>>M].empty()) aux.erase_in(x>>M);
    if(x==mx){
      if(aux.empty()) mx=mn;
      else mx=(aux.max()<<M)^ch[aux.max()].max();
    }
  }
  void insert(int x){
    if(!have(x)) insert_in(x);
  }
  void erase(int x){
    if(have(x)) erase_in(x);
  }
  int next(int x){// >=x
    if(x>mx) return 1<<30;
    if(x<=mn) return mn;
    if(mask(x)<=ch[x>>M].max())
      return ((x>>M)<<M)^ch[x>>M].next(mask(x));
    int y=aux.next((x>>M)+1);
    return (y<<M)^ch[y].min();
  }
  int prev(int x){// <x
    if(x<=mn) return -1;
    if(x>mx) return mx;
    if(x<=(aux.min()<<M)+ch[aux.min()].min())
      return mn;
    if(mask(x)>ch[x>>M].min())
      return ((x>>M)<<M)^ch[x>>M].prev(mask(x));
    int y=aux.prev(x>>M);
    return (y<<M)^ch[y].max();
  }
};
template<int N>
struct veb<N,typename enable_if<N<=6>::type>{
  u64 a;
  veb():a(0){}
  void insert_in(int x){a|=1ull<<x;}
  void insert(int x){a|=1ull<<x;}
  void erase_in(int x){a&=~(1ull<<x);}
  void erase(int x){a&=~(1ull<<x);}
  bool have(int x){return a>>x&1;}
  bool empty(){return a==0;}
  int min(){return lsb(a);}
  int max(){return msb(a);}
  int next(int x){return lsb(a&~((1ull<<x)-1));}
  int prev(int x){return msb(a&((1ull<<x)-1));}
};
```

3 Flow / Matching

3.1 Dinic

```
template <typename T>
struct Dinic { // 0-base
  const T INF = 1 << 30;
  struct edge {
    int to, rev;
    T cap, flow;
  };
  vector<edge> adj[N];
  int s, t, dis[N], cur[N], n;
  T dfs(int u, T cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < adj[u].size(); ++i) {
      edge &e = adj[u][i];
      if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
```

```
        T df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df;
          adj[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int tmp = q.front();
      q.pop();
      for (auto &u : adj[tmp])
        if (!~dis[u.to] && u.flow != u.cap) {
          q.push(u.to);
          dis[u.to] = dis[tmp] + 1;
        }
    }
    return dis[t] != -1;
  }
  T solve(int _s, int _t) {
    s = _s, t = _t;
    T flow = 0, df;
    while (bfs()) {
      fill_n(cur, n, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].clear();
  }
  void reset() {
    for (int i = 0; i < n; ++i)
      for (auto &j : adj[i]) j.flow = 0;
  }
  void add_edge(int u, int v, T cap) {
    adj[u].pb(edge{v, (int)adj[v].size(), cap, 0});
    adj[v].pb(edge{u, (int)adj[u].size() - 1, 0, 0});
  }
};
```

3.2 Min Cost Max Flow

```
template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
  const T1 INF1 = 1 << 30;
  const T2 INF2 = 1 << 30;
  struct edge {
    int v; T1 f; T2 c;
  } E[M << 1];
  vector <int> adj[N];
  T2 dis[N], pot[N];
  int rt[N], vis[N], n, m, s, t;
  // bool DAG()...
  bool SPFA() {
    fill_n(rt, n, -1), fill_n(dis, n, INF2);
    fill_n(vis, n, false);
    queue <int> q;
    q.push(s), dis[s] = 0, vis[s] = true;
    while (!q.empty()) {
      int v = q.front(); q.pop();
      vis[v] = false;
      for (int id : adj[v]) {
        auto [u, f, c] = E[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
          dis[u] = ndis, rt[u] = id;
          if (!vis[u]) vis[u] = true, q.push(u);
        }
      }
    }
    return dis[t] != INF2;
  }
  bool dijkstra() {
    fill_n(rt, n, -1), fill_n(dis, n, INF2);
    priority_queue <pair <T2, int>, vector <pair <T2,
      int>>, greater <pair <T2, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
      auto [d, v] = pq.top(); pq.pop();
      if (dis[v] < d) continue;
      for (int id : adj[v]) {
        auto [u, f, c] = E[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
          dis[u] = ndis, rt[u] = id;
          pq.emplace(ndis, u);
        }
      }
    }
    return dis[t] != INF2;
  }
  pair <T1, T2> solve(int _s, int _t) {
    s = _s, t = _t, fill_n(pot, n, 0);
    T1 flow = 0; T2 cost = 0; bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
      for (int i = 0; i < n; i++)
        dis[i] += pot[i] - pot[s];
      T1 add = INF1;
      for (int i = t; i != s; i = E[rt[i] ^ 1].v)
        add = min(add, E[rt[i]].f);
      for (int i = t; i != s; i = E[rt[i] ^ 1].v)
        E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
      flow += add, cost += add * dis[t], fr = false;
      for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return make_pair(flow, cost);
  }
  void init(int _n) {
    n = _n, m = 0;
    for (int i = 0; i < n; ++i) adj[i].clear();
  }
  void reset() {
    for (int i = 0; i < m; ++i) E[i].f = 0;
  }
  void add_edge(int u, int v, T1 f, T2 c) {
    adj[u].pb(m), E[m++] = {v, f, c};
    adj[v].pb(m), E[m++] = {u, 0, -c};
  }
};
```

3.3 Kuhn Munkres

```
template <typename T>
struct KM { // 0-based
  const T INF = 1 << 30;
  T w[N][N], hl[N], hr[N], slk[N];
  int fl[N], fr[N], pre[N], n;
  bool vl[N], vr[N];
  queue <int> q;
  KM () {}
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j) w[i][j] = -INF;
  }
  void add_edge(int a, int b, T wei) { w[a][b] = wei; }
  bool check(int x) {
    if (vl[x] = 1, ~fl[x])
      return q.push(fl[x]), vr[fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    fill(slk, slk + n, INF), fill(vl, vl + n, 0);
    fill(vr, vr + n, 0);
    while (!q.empty()) q.pop();
    q.push(s), vr[s] = 1;
    while (true) {
      T d;
      while (!q.empty()) {
        int y = q.front(); q.pop();
        for (int x = 0; x < n; ++x)
          if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] -
              w[x][y]))
            if (pre[x] = y, d) slk[x] = d;
```

```
          else if (!check(x)) return;
      }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
    }
  }
  T solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1);
    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
};
```

3.4 SW Min Cut

```
template <typename T>
struct SW { // 0-based
  const T INF = 1 << 30;
  T g[N][N], sum[N]; int n;
  bool vis[N], dead[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) fill_n(g[i], n, 0);
    fill(dead, dead + n, false);
  }
  void add_edge(int u, int v, T w) {
    g[u][v] += w, g[v][u] += w;
  }
  T solve() {
    T ans = INF;
    for (int round = 0; round + 1 < n; ++round) {
      fill(vis, vis + n, false), fill(sum, sum + n, 0);
      int num = 0, s = -1, t = -1;
      while (num < n - round) {
        int now = -1;
        for (int i = 0; i < n; ++i)
          if (!vis[i] && !dead[i] &&
            (now == -1 || sum[now] > sum[i])) now = i;
        s = t, t = now;
        vis[now] = true, num++;
        for (int i = 0; i < n; ++i)
          if (!vis[i] && !dead[i]) sum[i] += g[now][i];
      }
      ans = min(ans, sum[t]);
      for (int i = 0; i < n; ++i)
        g[i][s] += g[i][t], g[s][i] += g[t][i];
      dead[t] = true;
    }
    return ans;
  }
};
```

3.5 Gomory Hu Tree

```
vector <array <int, 3>> GomoryHu(Dinic <int> flow) {
  // Tree edge min = mincut (0-based)
  int n = flow.n;
  vector <array <int, 3>> ans;
  vector <int> rt(n);
  for (int i = 1; i < n; ++i) {
    int t = rt[i];
    flow.reset();
    ans.pb({i, t, flow.solve(i, t)});
    flow.bfs();
    for (int j = i + 1; j < n; ++j)
      if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
  }
  return ans;
}
```

3.6 Blossom

```
struct Matching { // 0-based
  int fa[N], pre[N], match[N], s[N], v[N], n, tk;
  vector <int> g[N];
  queue <int> q;
  Matching (int _n) : n(_n), tk(0) {
    for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
    for (int i = 0; i < n; ++i) g[i].clear();
  }
  void add_edge(int u, int v) {
    g[u].push_back(v), g[v].push_back(u);
  }
  int Find(int u) {
    return u == fa[u] ? u : fa[u] = Find(fa[u]);
  }
  int lca(int x, int y) {
    tk++;
    x = Find(x), y = Find(y);
    for (; ; swap(x, y)) {
      if (x != n) {
        if (v[x] == tk) return x;
        v[x] = tk;
        x = Find(pre[match[x]]);
      }
    }
  }
  void blossom(int x, int y, int l) {
    while (Find(x) != l) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      if (fa[x] == x) fa[x] = l;
      if (fa[y] == y) fa[y] = l;
      x = pre[y];
    }
  }
  bool bfs(int r) {
    for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
    while (!q.empty()) q.pop();
    q.push(r);
    s[r] = 0;
    while (!q.empty()) {
      int x = q.front(); q.pop();
      for (int u : g[x]) {
        if (s[u] == -1) {
          pre[u] = x, s[u] = 1;
          if (match[u] == n) {
            for (int a = u, b = x, last; b != n; a =
                last, b = pre[a])
              last = match[b], match[b] = a, match[a] =
                  b;
            return true;
          }
          q.push(match[u]);
          s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = lca(u, x);
          blossom(x, u, l);
          blossom(u, x, l);
        }
      }
    }
    return false;
  }
  int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) {
      if (match[x] == n) res += bfs(x);
    }
    return res;
  }
};
```

3.7 Weighted Blossom

```
struct WeightGraph { // 1-based
  static const int inf = INT_MAX;
  static const int maxn = 514;
  struct edge {
    int u, v, w;
    edge(){}
    edge(int u, int v, int w): u(u), v(v), w(w) {}
```

```
};
int n, n_x;
edge g[maxn * 2][maxn * 2];
int lab[maxn * 2];
int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
    pa[maxn * 2];
int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
vector<int> flo[maxn * 2];
queue<int> q;
int e_delta(const edge &e) { return lab[e.u] + lab[e.
    v] - g[e.u][e.v].w * 2; }
void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
    [x] = u; }
void set_slack(int x) {
  slack[x] = 0;
  for (int u = 1; u <= n; ++u)
    if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
      update_slack(u, x);
}
void q_push(int x) {
  if (x <= n) q.push(x);
  else for (size_t i = 0; i < flo[x].size(); i++)
      q_push(flo[x][i]);
}
void set_st(int x, int b) {
  st[x] = b;
  if (x > n) for (size_t i = 0; i < flo[x].size(); ++
      i) set_st(flo[x][i], b);
}
int get_pr(int b, int xr) {
  int pr = find(flo[b].begin(), flo[b].end(), xr) -
      flo[b].begin();
  if (pr % 2 == 1) {
    reverse(flo[b].begin() + 1, flo[b].end());
    return (int)flo[b].size() - pr;
  }
  return pr;
}
void set_match(int u, int v) {
  match[u] = g[u][v].v;
  if (u <= n) return;
  edge e = g[u][v];
  int xr = flo_from[u][e.u], pr = get_pr(u, xr);
  for (int i = 0; i < pr; ++i) set_match(flo[u][i],
      flo[u][i ^ 1]);
  set_match(xr, v);
  rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
      end());
}
void augment(int u, int v) {
  for (; ; ) {
    int xnv = st[match[u]];
    set_match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    u = st[pa[xnv]], v = xnv;
  }
}
int get_lca(int u, int v) {
  static int t = 0;
  for (++t; u || v; swap(u, v)) {
    if (u == 0) continue;
    if (vis[u] == t) return u;
    vis[u] = t;
    u = st[match[u]];
    if (u) u = st[pa[u]];
  }
  return 0;
}
void add_blossom(int u, int lca, int v) {
  int b = n + 1;
  while (b <= n_x && st[b]) ++b;
  if (b > n_x) ++n_x;
  lab[b] = 0, S[b] = 0;
  match[b] = match[lca];
  flo[b].clear();
  flo[b].push_back(lca);
  for (int x = u, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[
        match[x]]), q_push(y);
  reverse(flo[b].begin() + 1, flo[b].end());
  for (int x = v, y; x != lca; x = st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y = st[
        match[x]]), q_push(y);
  set_st(b, b);
  for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
      w = 0;
  for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
  for (size_t i = 0; i < flo[b].size(); ++i) {
    int xs = flo[b][i];
    for (int x = 1; x <= n_x; ++x)
      if (g[b][x].w == 0 || e_delta(g[xs][x]) <
          e_delta(g[b][x]))
        g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    for (int x = 1; x <= n; ++x)
      if (flo_from[xs][x]) flo_from[b][x] = xs;
  }
  set_slack(b);
}
void expand_blossom(int b) {
  for (size_t i = 0; i < flo[b].size(); ++i)
    set_st(flo[b][i], flo[b][i]);
  int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
      xr);
  for (int i = 0; i < pr; i += 2) {
    int xs = flo[b][i], xns = flo[b][i + 1];
    pa[xs] = g[xns][xs].u;
    S[xs] = 1, S[xns] = 0;
    slack[xs] = 0, set_slack(xns);
    q_push(xns);
  }
  S[xr] = 1, pa[xr] = pa[b];
  for (size_t i = pr + 1; i < flo[b].size(); ++i) {
    int xs = flo[b][i];
    S[xs] = -1, set_slack(xs);
  }
  st[b] = 0;
}
bool on_found_edge(const edge &e) {
  int u = st[e.u], v = st[e.v];
  if (S[v] == -1) {
    pa[v] = e.u, S[v] = 1;
    int nu = st[match[v]];
    slack[v] = slack[nu] = 0;
    S[nu] = 0, q_push(nu);
  } else if (S[v] == 0) {
    int lca = get_lca(u, v);
    if (!lca) return augment(u,v), augment(v,u), true
        ;
    else add_blossom(u, lca, v);
  }
  return false;
}
bool matching() {
  memset(S + 1, -1, sizeof(int) * n_x);
  memset(slack + 1, 0, sizeof(int) * n_x);
  q = queue<int>();
  for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
        q_push(x);
  if (q.empty()) return false;
  for (; ; ) {
    while (q.size()) {
      int u = q.front(); q.pop();
      if (S[st[u]] == 1) continue;
      for (int v = 1; v <= n; ++v)
        if (g[u][v].w > 0 && st[u] != st[v]) {
          if (e_delta(g[u][v]) == 0) {
            if (on_found_edge(g[u][v])) return true;
          } else update_slack(u, st[v]);
        }
    }
    int d = inf;
    for (int b = n + 1; b <= n_x; ++b)
      if (st[b] == b && S[b] == 1) d = min(d, lab[b]
          / 2);
    for (int x = 1; x <= n_x; ++x)
      if (st[x] == x && slack[x]) {
        if (S[x] == -1) d = min(d, e_delta(g[slack[x
            ]][x]));
        else if (S[x] == 0) d = min(d, e_delta(g[
            slack[x]][x]) / 2);
```

```
        }
      for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
          if (lab[u] <= d) return 0;
          lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
      }
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b) {
          if (S[st[b]] == 0) lab[b] += d * 2;
          else if (S[st[b]] == 1) lab[b] -= d * 2;
        }
      q = queue<int>();
      for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && slack[x] && st[slack[x]] != x
             && e_delta(g[slack[x]][x]) == 0)
          if (on_found_edge(g[slack[x]][x])) return
             true;
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
  }
  pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
      }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
      if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
  }
  void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
       g[vi][ui].w = wi; }
  void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v)
        g[u][v] = edge(u, v, 0);
  }
};
```

3.8 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x\bar{y} + x'\bar{y'})$$

  can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4 Graph

4.1 Binary Lifting

```
int dep[N], pa[N], to[N]; // pa[rt] = rt, to[rt] = rt
int lift(int x, int k) {
  k = dep[x] - k;
  while (dep[x] > k)
    x = dep[to[x]] < k ? pa[x] : to[x];
  return x;
}
void add(int p, int v) {
  dep[v] = dep[p] + 1, par[v] = p;
  to[v] = dep[p] - dep[to[p]] == dep[to[p]] - dep[to[to
       [p]]] ? to[to[p]] : p;
}
```

4.2 Heavy-Light Decomposition

```
vector <int> g[N];
int dep[N], pa[N], sz[N], ch[N], hd[N], id[N], _id;
void dfs(int i, int p) {
  dep[i] = ~p ? dep[p] + 1 : 0;
  pa[i] = p, sz[i] = 1, ch[i] = -1;
  for (int j : g[i]) if (j != p) {
    dfs(j, i);
    if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
    sz[i] += sz[j];
  }
}
void hld(int i, int p, int h) {
  hd[i] = h;
  id[i] = _id++;
  if (~ch[i]) hld(ch[i], i, h);
  for (int j : g[i]) if (j != p && j != ch[i])
    hld(j, i, j);
}
void query(int i, int j) {
  // query2 -> [l, r)
  while (hd[i] != hd[j]) {
    if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
    query2(id[hd[i]], id[i] + 1), i = pa[hd[i]];
  }
  if (dep[i] < dep[j]) swap(i, j);
  query2(id[j], id[i] + 1);
}
```

4.3 Centroid Decomposition

```
vector <int> g[N];
int dis[N][logN], pa[N], sz[N], dep[N];
bool vis[N];
void dfs_sz(int i, int p) {
  sz[i] = 1;
  for (int j : g[i]) if (j != p && !vis[j])
    dfs_sz(j, i), sz[i] += sz[j];
}
int cen(int i, int p, int _n) {
  for (int j : g[i])
    if (j != p && !vis[j] && sz[j] > _n / 2)
      return cen(j, i, _n);
  return i;
}
void dfs_dis(int i, int p, int d) {
  // from i to ancestor with depth d
  dis[i][d] = ~p ? dis[p][d] + 1 : 0;
  for (int j : g[i]) if (j != p && !vis[j])
    dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
  dfs_sz(i, -1), i = cen(i, -1, sz[i]);
  vis[i] = true, pa[i] = p, dep[i] = d;
  dfs_dis(i, -1, d);
  for (int j : g[i]) if (!vis[j])
    cd(j, i, d + 1);
}
```

4.4 Edge BCC

```
vector <int> g[N], _g[N];
// Notice Multiple Edges
int pa[N], low[N], dep[N], bcc_id[N], _id;
vector <int> stk, bcc[N];
bool vis[N], is_bridge[N];
void dfs(int i, int p = -1) {
  low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
  stk.pb(i), pa[i] = p, vis[i] = true;
  for (int j : g[i]) if (j != p) {
    if (!vis[j])
      dfs(j, i), low[i] = min(low[i], low[j]);
    else low[i] = min(low[i], dep[j]);
  }
  if (low[i] == dep[i]) {
    if (~p) is_bridge[i] = true; // (i, pa[i])
    int id = _id++, x;
    do {
      x = stk.back(), stk.pop_back();
      bcc_id[x] = id, bcc[id].pb(x);
    } while (x != i);
  }
}
void build(int n) {
  for (int i = 0; i < n; ++i) if (!vis[i])
    dfs(i);
  for (int i = 0; i < n; ++i) if (is_bridge[i]) {
    int u = bcc_id[i], v = bcc_id[pa[i]];
    _g[u].pb(v), _g[v].pb(u);
  }
}
```

4.5 Vertex BCC / Round Square Tree

```
vector <int> g[N], _g[N << 1];
// _g: index >= N: bcc, index < N: original vertex
int pa[N], dep[N], low[N], _id;
bool vis[N];
vector <int> stk;
void dfs(int i, int p = -1) {
  dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
  stk.pb(i), pa[i] = p, vis[i] = true;
  for (int j : g[i]) if (j != p) {
    if (!vis[j]) {
      dfs(j, i), low[i] = min(low[i], low[j]);
      if (low[j] >= dep[i]) {
        int id = _id++, x;
        do {
          x = stk.back(), stk.pop_back();
          _g[id + N].pb(x), _g[x].pb(id + N);
        } while (x != j);
        _g[id + N].pb(i), _g[i].pb(id + N);
      }
    } else low[i] = min(low[i], dep[j]);
  }
}
bool is_cut(int x) {return _g[x].size() != 1;}
vector <int> bcc(int x) {return _g[x + N];}
int pa2[N << 1], dep2[N << 1];
void dfs2(int i, int p = -1) {
  dep2[i] = ~p ? dep2[p] + 1 : 0, pa2[i] = p;
  for (int j : _g[i]) if (j != p) {
    dfs2(j, i);
  }
}
int bcc_id(int u, int v) {
  if (dep2[u] < dep2[v]) swap(u, v);
  return pa2[u] - N;
}
void build(int n) {
  for (int i = 0; i < n; ++i) if (!vis[i])
    dfs(i), dfs2(i);
}
```

4.6 SCC / 2SAT

```
struct SAT {
  vector <int> g[N << 1], stk;
  int dep[N << 1], low[N << 1], scc_id[N << 1];
  int n, _id, _t;
  bool is[N];
  SAT() {}
  void init(int _n) {
    n = _n, _id = _t = 0;
    for (int i = 0; i < 2 * n; ++i)
      g[i].clear(), dep[i] = scc_id[i] = -1;
    stk.clear();
  }
  void add_edge(int x, int y) { g[x].push_back(y); }
  int rev(int i) { return i < n ? i + n : i - n; }
  void add_ifthen(int x, int y)
  { add_clause(rev(x), y); }
  void add_clause(int x, int y)
  { add_edge(rev(x), y), add_edge(rev(y), x); }
  void dfs(int i) {
    dep[i] = low[i] = _t++, stk.pb(i);
    for (int j : g[i]) if (scc_id[j] == -1) {
      if (dep[j] == -1) dfs(j);
      low[i] = min(low[i], low[j]);
    }
    if (low[i] == dep[i]) {
      int id = _id++, x;
      do {
        x = stk.back(), stk.pop_back(), scc_id[x] = id;
      } while (x != i);
    }
  }
  bool solve() {
    // is[i] = true -> i, is[i] = false -> -i
    for (int i = 0; i < 2 * n; ++i) if (dep[i] == -1)
      dfs(i);
    for (int i = 0; i < n; ++i) {
      if (scc_id[i] == scc_id[i + n]) return false;
      if (scc_id[i] < scc_id[i + n]) is[i] = true;
      else is[i] = false;
    }
    return true;
  }
};
```

4.7 Virtual Tree

```
// need lca
vector <int> _g[N], stk;
int st[N], ed[N];
void solve(vector<int> v) {
  auto cmp = [&](int x, int y) {return st[x] < st[y];};
  sort(all(v), cmp);
  int sz = v.size();
  for (int i = 0; i < sz - 1; ++i)
    v.pb(lca(v[i], v[i + 1]));
  sort(all(v), cmp);
  v.resize(unique(all(v)) - v.begin());
  stk.clear(), stk.pb(v[0]);
  for (int i = 1; i < v.size(); ++i) {
```

```
    int x = v[i];
    while (ed[stk.back()] < ed[x]) stk.pop_back();
    _g[stk.back()].pb(x), stk.pb(x);
  }
  // do something
  for (int i : v) _g[i].clear();
}
```

4.8 Directed MST

```
using D = int;
struct edge {
  int u, v; D w;
};
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
  using T = pair <D, int>;
  using PQ = pair <priority_queue <T, vector <T>,
      greater <T>>, D>;
  auto push = [](PQ &pq, T v) {
    pq.first.emplace(v.first - pq.second, v.second);
  };
  auto top = [](const PQ &pq) -> T {
    auto r = pq.first.top();
    return {r.first + pq.second, r.second};
  };
  auto join = [&push, &top](PQ &a, PQ &b) {
    if (a.first.size() < b.first.size()) swap(a, b);
    while (!b.first.empty())
      push(a, top(b)), b.first.pop();
  };
  vector<PQ> h(n * 2);
  for (int i = 0; i < e.size(); ++i)
    push(h[e[i].v], {e[i].w, i});
  vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
      n * 2);
  iota(all(a), 0);
  auto o = [&](int x) { int y;
    for (y = x; a[y] != y; y = a[y]);
    for (int ox = x; x != y; ox = x)
      x = a[x], a[ox] = y;
    return y;
  };
  v[root] = n + 1;
  int pc = n;
  for (int i = 0; i < n; ++i) if (v[i] == -1) {
    for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
        p]].u)) {
      if (v[p] == i) {
        int q = p; p = pc++;
        do {
          h[q].second = -h[q].first.top().first;
          join(h[pa[q] = a[q] = p], h[q]);
        } while ((q = o(e[r[q]].u)) != p);
      }
      v[p] = i;
      while (!h[p].first.empty() && o(e[top(h[p]).
          second].u) == p)
        h[p].first.pop();
      r[p] = top(h[p]).second;
    }
  }
  vector<int> ans;
  for (int i = pc - 1; i >= 0; i--)
    if (i != root && v[i] != n) {
      for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
          pa[f]) v[f] = n;
      ans.pb(r[i]);
    }
  return ans;
}
```

4.9 Dominator Tree

```
struct Dominator_tree {
  int n, id, sdom[N], dom[N];
  vector <int> adj[N], radj[N], bucket[N];
  int vis[N], rev[N], pa[N], rt[N], mn[N], res[N];
  // dom[s] = s, dom[v] = -1 if s -> v not exists
  Dominator_tree () {}
  void init(int _n) {
    n = _n, id = 0;
    for (int i = 0; i < n; ++i)
      adj[i].clear(), radj[i].clear(), bucket[i].clear
          ();
    fill_n(dom, n, -1), fill_n(vis, n, -1);
  }
  void add_edge(int u, int v) {adj[u].pb(v);}
  int query(int v, int x) {
    if (rt[v] == v) return x ? -1 : v;
    int p = query(rt[v], 1);
    if (p == -1) return x ? rt[v] : mn[v];
    if (sdom[mn[v]] > sdom[mn[rt[v]]])
      mn[v] = mn[rt[v]];
    rt[v] = p;
    return x ? p : mn[v];
  }
  void dfs(int v) {
    vis[v] = id, rev[id] = v;
    rt[id] = mn[id] = sdom[id] = id, id++;
    for (int u : adj[v]) {
      if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];
      radj[vis[u]].pb(vis[v]);
    }
  }
  void build(int s) {
    dfs(s);
    for (int i = id - 1; ~i; --i) {
      for (int u : radj[i]) {
        sdom[i] = min(sdom[i], sdom[query(u, 0)]);
      }
      if (i) bucket[sdom[i]].pb(i);
      for (int u : bucket[i]) {
        int p = query(u, 0);
        dom[u] = sdom[p] == i ? i : p;
      }
      if (i) rt[i] = pa[i];
    }
    fill_n(res, n, -1);
    for (int i = 1; i < id; ++i) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    }
    for (int i = 1; i < id; ++i)
        res[rev[i]] = rev[dom[i]];
    res[s] = s;
    for (int i = 0; i < n; ++i) dom[i] = res[i];
  }
};
```

4.10 Vizing

```
struct Vizing { // 1-based
  // returns edge coloring in adjacent matrix G
  int C[N][N], G[N][N], X[N], vst[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j)
        C[i][j] = G[i][j] = 0;
  }
  void solve(vector<pii> &E) {
    auto update = [&](int u)
    { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
    auto color = [&](int u, int v, int c) {
      int p = G[u][v];
      G[u][v] = G[v][u] = c;
      C[u][c] = v, C[v][c] = u;
      C[u][p] = C[v][p] = 0;
      if (p) X[u] = X[v] = p;
      else update(u), update(v);
      return p;
    };
    auto flip = [&](int u, int c1, int c2) {
      int p = C[u][c1];
      swap(C[u][c1], C[u][c2]);
      if (p) G[u][p] = G[p][u] = c2;
      if (!C[u][c1]) X[u] = c1;
      if (!C[u][c2]) X[u] = c2;
      return p;
    };
    fill_n(X + 1, n, 1);
    for (int t = 0; t < E.size(); ++t) {
      auto [u, v0] = E[t];
      int v = v0, c0 = X[u], c = c0, d;
```

```
      vector<pii> L;
      fill_n(vst + 1, n, 0);
      while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) {
          for (int a = sz(L) - 1; a >= 0; --a)
            c = color(u, L[a].first, c);
        } else if (!C[u][d]) {
          for (int a = sz(L) - 1; a >= 0; --a)
            color(u, L[a].first, L[a].second);
        } else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
      }
      if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (int a; C[u][c0]) {
          for (a = sz(L) - 2; a >= 0 && L[a].second !=
              c; --a);
          for (; a >= 0; --a) color(u, L[a].first, L[a
              ].second);
        }
        else --t;
      }
    }
  }
};
```

5 String

5.1 Aho-Corasick Automaton

```
struct AC {
  int ch[N][26], to[N][26], fail[N], sz;
  vector <int> g[N];
  int cnt[N];
  AC () {sz = 0, extend();}
  void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
  int nxt(int u, int v) {
    if (!ch[u][v]) ch[u][v] = sz, extend();
    return ch[u][v];
  }
  int insert(string s) {
    int now = 0;
    for (char c : s) now = nxt(now, c - 'a');
    cnt[now]++;
    return now;
  }
  void build_fail() {
    queue <int> q;
    for (int i = 0; i < 26; ++i) if (ch[0][i]) {
      q.push(ch[0][i]);
      g[0].push_back(ch[0][i]);
    }
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (int j = 0; j < 26; ++j) {
        to[v][j] = ch[v][j] ? v : to[fail[v]][j];
      }
      for (int i = 0; i < 26; ++i) if (ch[v][i]) {
        int u = ch[v][i], k = fail[v];
        while (k && !ch[k][i]) k = fail[k];
        if (ch[k][i]) k = ch[k][i];
        fail[u] = k;
        cnt[u] += cnt[k], g[k].push_back(u);
        q.push(u);
      }
    }
  }
  int match(string &s) {
    int now = 0, ans = 0;
    for (char c : s) {
      now = to[now][c - 'a'];
      if (ch[now][c - 'a']) now = ch[now][c - 'a'];
      ans += cnt[now];
    }
    return ans;
  }
};
```

5.2 KMP Algorithm

```
vector <int> build_fail(string s) {
  vector <int> f(s.length() + 1, 0);
  int k = 0;
  for (int i = 1; i < s.length(); ++i) {
    while (k && s[k] != s[i]) k = f[k];
    if (s[k] == s[i]) k++;
    f[i + 1] = k;
  }
  return f;
}
int match(string s, string t) {
  vector <int> f = build_fail(t);
  int k = 0, ans = 0;
  for (int i = 0; i < s.length(); ++i) {
    while (k && s[i] != t[k]) k = f[k];
    if (s[i] == t[k]) k++;
    if (k == t.length()) ans++, k = f[k];
  }
  return ans;
}
```

5.3 Z Algorithm

```
vector <int> buildZ(string s) {
  int n = s.length();
  vector <int> Z(n);
  int l = 0, r = 0;
  for (int i = 0; i < n; ++i) {
    Z[i] = max(min(Z[i - l], r - i), 0);
    while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]]) {
      l = i, r = i + Z[i], Z[i]++;
    }
  }
  return Z;
}
```

5.4 Manacher

```
// return value only consider string tmp, not s
vector <int> manacher(string tmp) {
  string s = "&";
  for (char c : tmp) s.pb(c), s.pb('%');
  int l = 0, r = 0, n = s.size();
  vector <int> Z(n);
  for (int i = 0; i < n; ++i) {
    Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
    while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
    if (Z[i] + i > r) l = i, r = Z[i] + i;
  }
  for (int i = 0; i < n; ++i) {
    Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
  }
  return Z;
}
```

5.5 Suffix Array

```
int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
  int *x = tmp[0], *y = tmp[1], m = 256, n = s.size();
  for (int i = 0; i < m; ++i) c[i] = 0;
  for (int i = 0; i < n; ++i) c[x[i] = s[i]]++;
  for (int i = 1; i < m; ++i) c[i] += c[i - 1];
  for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
  for (int k = 1; k < n; k <<= 1) {
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    int p = 0;
    for (int i = n - k; i < n; ++i) y[p++] = i;
    for (int i = 0; i < n; ++i) if (sa[i] >= k)
      y[p++] = sa[i] - k;
    for (int i = n - 1; ~i; --i)
      sa[--c[x[y[i]]]] = y[i];
    y[sa[0]] = p = 0;
    for (int i = 1; i < n; ++i) {
      int a = sa[i], b = sa[i - 1];
      if (!(x[a] == x[b] && a + k < n && b + k < n && x
          [a + k] == x[b + k])) p++;
      y[sa[i]] = p;
    }
    if (n == p + 1) break;
```

```cpp
    swap(x, y), m = p + 1;
  }
}
void buildLCP(string s) {
  // lcp[i] = LCP(sa[i - 1], sa[i])
  // lcp(i, j) = query_lcp_min [rk[i] + 1, rk[j] + 1)
  int n = s.length(), val = 0;
  for (int i = 0; i < n; ++i) rk[sa[i]] = i;
  for (int i = 0; i < n; ++i) {
    if (!rk[i]) lcp[rk[i]] = 0;
    else {
      if (val) val--;
      int p = sa[rk[i] - 1];
      while (val + i < n && val + p < n && s[val + i]
          == s[val + p]) val++;
      lcp[rk[i]] = val;
    }
  }
}
```

5.6 SAIS

```cpp
int sa[N << 1], rk[N], lcp[N];
// string ASCII value need > 0
namespace sfx {
bool _t[N << 1];
int _s[N << 1], _c[N << 1], x[N], _p[N], _q[N << 1];
void pre(int *sa, int *c, int n, int z) {
  fill_n(sa, n, 0), copy_n(c, z, x);
}
void induce(int *sa, int *c, int *s, bool *t, int n,
    int z) {
  copy_n(c, z - 1, x + 1);
  for (int i = 0; i < n; ++i)
    if (sa[i] && !t[sa[i] - 1])
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
  copy_n(c, z, x);
  for (int i = n - 1; i >= 0; --i)
    if (sa[i] && t[sa[i] - 1])
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int
     *c, int n, int z) {
  bool uniq = t[n - 1] = true;
  int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
      last = -1;
  fill_n(c, z, 0);
  for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
  partial_sum(c, c + z, c);
  if (uniq) {
    for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
    return;
  }
  for (int i = n - 2; i >= 0; --i)
    if (s[i] == s[i + 1]) t[i] = t[i + 1];
    else t[i] = s[i] < s[i + 1];
  pre(sa, c, n, z);
  for (int i = 1; i <= n - 1; ++i)
    if (t[i] && !t[i - 1])
      sa[--x[s[i]]] = p[q[i] = nn++] = i;
  induce(sa, c, s, t, n, z);
  for (int i = 0; i < n; ++i)
    if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
      bool neq = last < 0 || !equal(s + sa[i], s + p[q[
          sa[i]] + 1], s + last);
      ns[q[last = sa[i]]] = nmxz += neq;
    }
  sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz +
       1);
  pre(sa, c, n, z);
  for (int i = nn - 1; i >= 0; --i)
    sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
  induce(sa, c, s, t, n, z);
}
void buildSA(string s) {
  int n = s.length();
  for (int i = 0; i < n; ++i) _s[i] = s[i];
  _s[n] = 0;
  sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
  for (int i = 1; i <= n; ++i) sa[i - 1] = sa[i];
} // buildLCP()...
}
```

5.7 Suffix Automaton

```cpp
struct SAM {
  int ch[N][26], len[N], link[N], pos[N], cnt[N], sz;
  // node -> strings with the same endpos set
  // length in range [len(link) + 1, len]
  // node's endpos set -> pos in the subtree of node
  // link -> longest suffix with different endpos set
  // len  -> longest suffix
  // pos  -> end position
  // cnt  -> size of endpos set
  SAM () {len[0] = 0, link[0] = -1, pos[0] = 0, cnt[0]
      = 0, sz = 1;}
  void build(string s) {
    int last = 0;
    for (int i = 0; i < s.length(); ++i) {
      char c = s[i];
      int cur = sz++;
      len[cur] = len[last] + 1, pos[cur] = i + 1;
      int p = last;
      while (~p && !ch[p][c - 'a'])
        ch[p][c - 'a'] = cur, p = link[p];
      if (p == -1) link[cur] = 0;
      else {
        int q = ch[p][c - 'a'];
        if (len[p] + 1 == len[q]) {
          link[cur] = q;
        } else {
          int nxt = sz++;
          len[nxt] = len[p] + 1, link[nxt] = link[q];
          pos[nxt] = 0;
          for (int j = 0; j < 26; ++j)
            ch[nxt][j] = ch[q][j];
          while (~p && ch[p][c - 'a'] == q)
            ch[p][c - 'a'] = nxt, p = link[p];
          link[q] = link[cur] = nxt;
        }
      }
      cnt[cur]++;
      last = cur;
    }
    vector <int> p(sz);
    iota(all(p), 0);
    sort(all(p),
      [&](int i, int j) {return len[i] > len[j];});
    for (int i = 0; i < sz; ++i)
      cnt[link[p[i]]] += cnt[p[i]];
  }
} sam;
```

5.8 Minimum Rotation

```cpp
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && t[i + k] == t[j + k]) ++k;
    if (t[i + k] <= t[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return t.substr(pos, n);
}
```

5.9 Palindrome Tree

```cpp
struct PAM {
  int ch[N][26], cnt[N], fail[N], len[N], sz;
  string s;
  // 0 -> even root, 1 -> odd root
  PAM () {}
  void init(string s) {
    sz = 0, extend(), extend();
    len[0] = 0, fail[0] = 1, len[1] = -1;
    int lst = 1;
    for (int i = 0; i < s.length(); ++i) {
      while (s[i - len[lst] - 1] != s[i])
        lst = fail[lst];
      if (!ch[lst][s[i] - 'a'])  {
```

```
        int idx = extend();
        len[idx] = len[lst] + 2;
        int now = fail[lst];
        while (s[i - len[now] - 1] != s[i])
          now = fail[now];
        fail[idx] = ch[now][s[i] - 'a'];
        ch[lst][s[i] - 'a'] = idx;
      }
      lst = ch[lst][s[i] - 'a'], cnt[lst]++;
    }
  }
  void build_count() {
    for (int i = sz - 1; i > 1; --i)
      cnt[fail[i]] += cnt[i];
  }
  int extend() {
    fill(ch[sz], ch[sz] + 26, 0), sz++;
    return sz - 1;
  }
};
```

5.10 Main Lorentz

```
int to_left[N], to_right[N];
vector <array <int, 3>> rep; // l, r, len.
// substr( [l, r], len * 2) are tandem
void findRep(string &s, int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  findRep(s, l, m), findRep(s, m, r);
  string sl = s.substr(l, m - l);
  string sr = s.substr(m, r - m);
  vector <int> Z = buildZ(sr + "#" + sl);
  for (int i = l; i < m; ++i)
    to_right[i] = Z[r - m + 1 + i - l];
  reverse(all(sl));
  Z = buildZ(sl);
  for (int i = l; i < m; ++i)
    to_left[i] = Z[m - i - 1];
  reverse(all(sl));
  for (int i = l; i + 1 < m; ++i) {
    int k1 = to_left[i], k2 = to_right[i + 1];
    int len = m - i - 1;
    if (k1 < 1 || k2 < 1 || len < 2) continue;
    int tl = max(1, len - k2), tr = min(len - 1, k1);
    if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl,len});
  }
  Z = buildZ(sr);
  for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
  reverse(all(sl)), reverse(all(sr));
  Z = buildZ(sl + "#" + sr);
  for (int i = m; i < r; ++i)
    to_left[i] = Z[m - l + 1 + r - i - 1];
  reverse(all(sl)), reverse(all(sr));
  for (int i = m; i + 1 < r; ++i) {
    int k1 = to_left[i], k2 = to_right[i + 1];
    int len = i - m + 1;
    if (k1 < 1 || k2 < 1 || len < 2) continue;
    int tl = max(len - k2, 1), tr = min(len - 1, k1);
    if (tl <= tr)
      rep.pb({i + 1 - len - tr, i + 1 - len - tl,len});
  }
  Z = buildZ(sr + "#" + sl);
  for (int i = l; i < m; ++i)
    if (Z[r - m + 1 + i - l] >= m - i)
      rep.pb({i, i, m - i});
}
```

6 Math

6.1 Miller Rabin / Pollard Rho

```
ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long
    double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
     1795265022};
ll Pow(ll a, ll b, ll n) {
  ll res = 1;
  for (; b; b >>= 1, a = mul(a, a, n))
    if (b & 1) res = mul(res, a, n);
  return res;
}
bool check(ll a, ll d, int s, ll n) {
  a = Pow(a, d, n);
  if (a <= 1) return 1;
  for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
    if (a == 1) return 0;
    if (a == n - 1) return 1;
  }
  return 0;
}
bool IsPrime(ll n) {
  if (n < 2) return 0;
  if (n % 2 == 0) return n == 2;
  ll d = n - 1, s = 0;
  while (d % 2 == 0) d >>= 1, ++s;
  for (ll i : chk) if (!check(i, d, s, n)) return 0;
  return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
  if (IsPrime(n)) return 1;
  for (ll p : small) if (n % p == 0) return p;
  ll x, y = 2, d, t = 1;
  auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
  for (int l = 2; ; l <<= 1) {
    x = y;
    int m = min(l, 32);
    for (int i = 0; i < l; i += m) {
      d = 1;
      for (int j = 0; j < m; ++j) {
        y = f(y), d = mul(d, abs(x - y), n);
      }
      ll g = __gcd(d, n);
      if (g == n) {
        l = 1, y = 2, ++t;
        break;
      }
      if (g != 1) return g;
    }
  }
}
map <ll, int> res;
void PollardRho(ll n) {
  if (n == 1) return;
  if (IsPrime(n)) return ++res[n], void(0);
  ll d = FindFactor(n);
  PollardRho(n / d), PollardRho(d);
}
```

6.2 Ext GCD

```
//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
  pair<ll, ll> res, tmp;
  ll f = 1, g = 1;
  if (a < 0) a *= -1, f *= -1;
  if (b < 0) b *= -1, g *= -1;
  if (b == 0) return {f, 0};
  tmp = extgcd(b, a % b);
  res.first = tmp.second * f;
  res.second = (tmp.first - tmp.second * (a / b)) * g;
  return res;
}
```

6.3 Chinese Remainder Theorem

```
ll CRT(ll x1, ll m1, ll x2, ll m2) {
  ll g = gcd(m1, m2);
  if ((x2 - x1) % g) return -1; // no sol
  m1 /= g, m2 /= g;
  pair <ll, ll> p = extgcd(m1, m2);
  ll lcm = m1 * m2 * g;
  ll res = p.first * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return (res % lcm + lcm) % lcm;
}
```

6.4 PiCount

```
const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
```

```cpp
int small_pi[V], dp[N][M];
void sieve(int x){
  for(int i = 2; i < x; ++i) isp[i] = true;
  isp[0] = isp[1] = false;
  for(int i = 2; i * i < x; ++i) if(isp[i])
    for(int j = i * i; j < x; j += i) isp[j] = false;
  for(int i = 2; i < x; ++i) if(isp[i]) primes.pb(i);
}
void init(){
  sieve(V);
  small_pi[0] = 0;
  for(int i = 1; i < V; ++i)
    small_pi[i] = small_pi[i - 1] + isp[i];
  for(int i = 0; i < M; ++i) dp[0][i] = i;
  for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
    dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i -
        1]];
}
ll phi(ll n, int a){
  if(!a) return n;
  if(n < M && a < N) return dp[a][n];
  if(primes[a - 1] > n) return 1;
  if(1ll * primes[a - 1] * primes[a - 1] >= n && n < V)
    return small_pi[n] - a + 1;
  return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
ll PiCount(ll n){
  if(n < V) return small_pi[n];
  int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
      small_pi[y];
  ll res = phi(n, a) + a - 1;
  for(; primes[a] <= s; ++a) res -= max(PiCount(n /
      primes[a]) - PiCount(primes[a]) + 1, 0ll);
  return res;
}
```

6.5 Linear Function Mod Min

```cpp
ll topos(ll x, ll m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    log m)
ll min_rem(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  for (ll g = __gcd(a, m); g > 1;) return g * min_rem(n
      , m / g, a / g, b / g) + (b % g);
  for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
      = nb) {
    if (a <= m - a) {
      nn = (a * (n - 1) + b) / m;
      if (!nn) break;
      nn += (b < a);
      nm = a, na = topos(-m, a);
      nb = b < a ? b : topos(b - m, a);
    } else {
      ll lst = b - (n - 1) * (m - a);
      if (lst >= 0) {b = lst; break;}
      nn = -(lst / m) + (lst % m < -a) + 1;
      nm = m - a, na = m % (m - a), nb = b % (m - a);
    }
  }
  return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
  //ax = (mn - b) (mod m)
  ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
      % (m / g);
  return {mn, x};
}
```

6.6 Determinant*

```cpp
ll Det(vector <vector <ll>> a) {
  int n = a.size();
  ll det = 1;
  for (int i = 0; i < n; ++i) {
    if (!a[i][i]) {
      det = -det;
      if (det < 0) det += mod;
      for (int j = i + 1; j < n; ++j) if (a[j][i]) {
        swap(a[j], a[i]);
        break;
      }
      if (!a[i][i]) return 0;
    }
    det = det * a[i][i] % mod;
    ll mul = mpow(a[i][i], mod - 2);
    for (int j = 0; j < n; ++j)
      a[i][j] = a[i][j] * mul % mod;
    for (int j = 0; j < n; ++j) if (i ^ j) {
      ll mul = a[j][i];
      for (int k = 0; k < n; ++k) {
        a[j][k] -= a[i][k] * mul % mod;
        if (a[j][k] < 0) a[j][k] += mod;
      }
    }
  }
  return det;
}
```

6.7 Floor Sum

```cpp
// sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a
    + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
  ll ans = 0;
  if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
  if (b >= m) ans += n * (b / m), b %= m;
  ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
  if (y_max == 0) return ans;
  ans += (n - (x_max + a - 1) / a) * y_max;
  ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
  return ans;
}
```

6.8 Quadratic Residue

```cpp
int Jacobi(int a, int m) {
  int s = 1;
  for (; m > 1; ) {
    a %= m;
    if (a == 0) return 0;
    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
  }
  return s;
}
int QuadraticResidue(int a, int p) {
  if (p == 2) return a & 1;
  const int jc = Jacobi(a, p);
  if (jc == 0) return 0;
  if (jc == -1) return -1;
  int b, d;
  for (; ; ) {
    b = rand() % p;
    d = (1ll * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
  }
  ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
  for (int e = (p + 1) >> 1; e; e >>= 1) {
    if (e & 1) {
      tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
      g1 = (g0 * f1 + g1 * f0) % p;
      g0 = tmp;
    }
    tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
    f1 = (2 * f0 * f1) % p;
    f0 = tmp;
  }
  return g0;
}
```

6.9 Simplex

```cpp
struct Simplex { // 0-based
  using T = long double;
```

```
  static const int N = 410, M = 30010;
  const T eps = 1e-7;
  int n, m;
  int Left[M], Down[N];
  // Ax <= b, max c^T x
  // result : v, xi = sol[i]
  T a[M][N], b[M], c[N], v, sol[N];
  bool eq(T a, T b) {return fabs(a - b) < eps;}
  bool ls(T a, T b) {return a < b && !eq(a, b);}
  void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
    for (int i = 0; i < m; ++i)
      for (int j = 0; j < n; ++j) a[i][j] = 0;
    for (int i = 0; i < m; ++i) b[i] = 0;
    for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
  }
  void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector <int> nz;
    for (int i = 0; i < n; ++i) {
      a[x][i] /= k;
      if (!eq(a[x][i], 0)) nz.push_back(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
      if (i == x || eq(a[i][y], 0)) continue;
      k = a[i][y], a[i][y] = 0;
      b[i] -= k * b[x];
      for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
  }
  // 0: found solution, 1: no feasible solution, 2:
      unbounded
  int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (true) {
      int x = -1, y = -1;
      for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x
          == -1 || b[i] < b[x])) x = i;
      if (x == -1) break;
      for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) &&
           (y == -1 || a[x][i] < a[x][y])) y = i;
      if (y == -1) return 1;
      pivot(x, y);
    }
    while (true) {
      int x = -1, y = -1;
      for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y
          == -1 || c[i] > c[y])) y = i;
      if (y == -1) break;
      for (int i = 0; i < m; ++i)
        if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y
            ] < b[x] / a[x][y])) x = i;
      if (x == -1) return 2;
      pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n)
      sol[Left[i]] = b[i];
    return 0;
  }
};
```

6.10 Berlekamp Massey

```
// need add, sub, mul
vector <ll> BerlekampMassey(vector <ll> a) {
  // find min |c| such that a_n = sum c_j * a_{n - j -
      1}, 0-based
  // O(N^2), if |c| = k, |a| >= 2k sure correct
  auto f = [&](vector<ll> v, ll c) {
    for (ll &x : v) x = mul(x, c);
    return v;
  };
  vector <ll> c, best;
  int pos = 0, n = a.size();
  for (int i = 0; i < n; ++i) {
    ll error = a[i];
    for (int j = 0; j < c.size(); ++j)
      error = sub(error, mul(c[j], a[i - 1 - j]));
    if (error == 0) continue;
    ll inv = mpow(error, mod - 2);
    if (c.empty()) {
      c.resize(i + 1), pos = i, best.pb(inv);
    } else {
      vector <ll> fix = f(best, error);
      fix.insert(fix.begin(), i - pos - 1, 0);
      if (fix.size() >= c.size()) {
        best = f(c, sub(0, inv));
        best.insert(best.begin(), inv);
        pos = i, c.resize(fix.size());
      }
      for (int j = 0; j < fix.size(); ++j)
        c[j] = add(c[j], fix[j]);
    }
  }
  return c;
}
```

6.11 Linear Programming Construction

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y} \ge \mathbf{c}$ and $\mathbf{y} \ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i \in [1,n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^{m} A_{ji}\bar{y}_j = c_i$ holds and for all $i \in [1,m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^{n} A_{ij}\bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \le i \le n} A_{ji}x_i \ge b_j \to \sum_{1 \le i \le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1 \le i \le n} A_{ji}x_i = b_j$
   - $\sum_{1 \le i \le n} A_{ji}x_i \le b_j$
   - $\sum_{1 \le i \le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.12 Euclidean

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor = \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) & \\ +f(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a,b,c,n) = \sum_{i=0}^{n} i\lfloor \frac{ai+b}{c} \rfloor = \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) & \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2 = \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) & \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) & \\ -2f(c, c-b-1, a, m-1) - f(a,b,c,n), & \text{otherwise} \end{cases}$$

6.13 Theorem

- Kirchhoff's Theorem

  Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i,j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

  Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i,j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Cayley's Formula
  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each *labeled* vertices, there are
  $$\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$$
  spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős–Gallai Theorem

  A sequence of non-negative integers $d_1 \ge d_2 \ge \ldots \ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and
  $$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$
  holds for all $1 \le k \le n$.

- Burnside's Lemma

  Let X be a set and G be a group that acts on X. For $g \in G$, denote by X^g the elements fixed by g:
  $$X^g = \{x \in X \mid gx \in X\}$$
  Then
  $$|X/G| = \frac{1}{|G|}\sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

  A pair of sequences of nonnegative integers $a_1 \ge \cdots \ge a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$.

- Fulkerson-Chen-Anstee theorem

  A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \ge \cdots \ge a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$.

- Möbius inversion formula
  - $f(n) = \sum_{d \mid n} g(d) \Leftrightarrow g(n) = \sum_{d \mid n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n \mid d} g(d) \Leftrightarrow g(n) = \sum_{n \mid d} \mu(\frac{d}{n}) f(d)$

- Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

6.14 Estimation

- The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200000 for $n < 1e19$.
- The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. $1, 1, 2, 3, 5, 7, 11, 15, 22, 30$ for $n = 0 \sim 9$, 627 for $n = 20$, $\sim 2e5$ for $n = 50$, $\sim 2e8$ for $n = 100$.
- Total number of partitions of n distinct elements: $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \ldots$.

6.15 General Purpose Numbers

- Bernoulli numbers

  $B_0 - 1, B_1^{\pm} = \pm\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$

  $\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$, EGF is $B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$.

  $S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k^{+} n^{m+1-k}$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

  $S(n,k) = S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1$

  $S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i} i^n$

  $x^n = \sum_{i=0}^{n} S(n,i)(x)_i$

- Pentagonal number theorem
  $$\prod_{n=1}^{\infty}(1-x^n) = 1 + \sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2} + x^{k(3k-1)/2}\right)$$

- Catalan numbers

  $C_n^{(k)} = \frac{1}{(k-1)n+1}\binom{kn}{n}$

  $C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$

- Eulerian numbers

  Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

  $E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$

  $E(n,0) = E(n,n-1) = 1$

  $E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

6.16 Tips for Generating Funtion

- Ordinary Generating Function $A(x) = \sum_{i \ge 0} a_i x^i$
  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x)' \Rightarrow na_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^{n} a_i$

- Exponential Generating Function $A(x) = \sum_{i \ge 0} \frac{a_i}{i!} x_i$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n}\binom{n}{i} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1, i_2, \ldots, i_k} a_{i_1} a_{i_2} \ldots a_{i_k}$
  - $xA(x) \Rightarrow na_n$

- Special Generating Function
  - $(1+x)^n = \sum_{i \ge 0}\binom{n}{i} x^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \ge 0}\binom{i}{n-1} x^i$

7 Polynomial

7.1 Number Theoretic Transform

```
// mul, add, sub, mpow
// ll -> int if too slow
struct NTT {
  ll w[N];
  NTT() {
    ll dw = mpow(G, (mod - 1) / N);
    w[0] = 1;
    for (int i = 1; i < N; ++i)
      w[i] = mul(w[i - 1], dw);
  }
  void operator()(vector<ll>& a, bool inv = false) { //
      0 <= a[i] < P
    int x = 0, n = a.size();
    for (int j = 1; j < n - 1; ++j) {
      for (int k = n >> 1; (x ^= k) < k; k >>= 1);
      if (j < x) swap(a[x], a[j]);
    }
    for (int L = 2; L <= n; L <<= 1) {
      int dx = N / L, dl = L >> 1;
      for (int i = 0; i < n; i += L) {
        for (int j = i, x = 0; j < i + dl; ++j, x += dx
            ) {
          ll tmp = mul(a[j + dl], w[x]);
          a[j + dl] = sub(a[j], tmp);
          a[j] = add(a[j], tmp);
        }
      }
    }
    if (inv) {
      reverse(a.begin() + 1, a.end());
```

```
      ll invn = mpow(n, mod - 2);
      for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], invn);
    }
  }
} ntt;
```

7.2 Fast Fourier Transform

```
using T = complex <double>;
const double PI = acos(-1);
struct NTT {
  T w[N];
  FFT() {
    T dw = {cos(2 * PI / N), sin(2 * PI / N)};
    w[0] = 1;
    for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
  }
  void operator()(vector<T>& a, bool inv = false) {
    // see NTT, replace ll with T
    if (inv) {
      reverse(a.begin() + 1, a.end());
      T invn = 1.0 / n;
      for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
    }
  }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	9223372036737335297	3

7.4 Polynomial Operations

```
vector <ll> Mul(vector <ll> a, vector <ll> b, int bound
     = N) {
  int m = a.size() + b.size() - 1, n = 1;
  while (n < m) n <<= 1;
  a.resize(n), b.resize(n);
  ntt(a), ntt(b);
  vector <ll> out(n);
  for (int i = 0; i < n; ++i) out[i] = mul(a[i], b[i]);
  ntt(out, true), out.resize(min(m, bound));
  return out;
}
vector <ll> Inverse(vector <ll> a) {
  // O(NLogN), a[0] != 0
  int n = a.size();
  vector <ll> res(1, mpow(a[0], mod - 2));
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    vector <ll> v1(a.begin(), a.begin() + m * 2), v2 =
        res;
    v1.resize(m * 4), v2.resize(m * 4);
    ntt(v1), ntt(v2);
    for (int i = 0; i < m * 4; ++i)
      v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
    ntt(v1, true);
    res.resize(m * 2);
    for (int i = 0; i < m; ++i)
      res[i] = add(res[i], res[i]);
    for (int i = 0; i < m * 2; ++i)
      res[i] = sub(res[i], v1[i]);
  }
  res.resize(n);
  return res;
}
pair <vector <ll>, vector <ll>> Divide(vector <ll> a,
    vector <ll> b) {
  // a = bQ + R, O(NLogN), b.back() != 0
  int n = a.size(), m = b.size(), k = n - m + 1;
  if (n < m) return {{0}, a};
  vector <ll> ra = a, rb = b;
  reverse(all(ra)), ra.resize(k);
  reverse(all(rb)), rb.resize(k);
  vector <ll> Q = Mul(ra, Inverse(rb), k);
  reverse(all(Q));
  vector <ll> res = Mul(b, Q), R(m - 1);
  for (int i = 0; i < m - 1; ++i)
    R[i] = sub(a[i], res[i]);
  return {Q, R};
}
vector <ll> SqrtImpl(vector <ll> a) {
  if (a.empty()) return {0};
  int z = QuadraticResidue(a[0], mod), n = a.size();
  if (z == -1) return {-1};
  vector <ll> q(1, z);
  const int inv2 = (mod + 1) / 2;
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    q.resize(m * 2);
    vector <ll> f2 = Mul(q, q, m * 2);
    for (int i = 0; i < m * 2; ++i)
      f2[i] = sub(f2[i], a[i]);
    f2 = Mul(f2, Inverse(q), m * 2);
    for (int i = 0; i < m * 2; ++i)
      q[i] = sub(q[i], mul(f2[i], inv2));
  }
  q.resize(n);
  return q;
}
vector <ll> Sqrt(vector <ll> a) {
  // O(NLogN), return {-1} if not exists
  int n = a.size(), m = 0;
  while (m < n && a[m] == 0) m++;
  if (m == n) return vector <ll>(n);
  if (m & 1) return {-1};
  vector <ll> s = SqrtImpl(vector <ll>(a.begin() + m, a
      .end()));
  if (s[0] == -1) return {-1};
  vector <ll> res(n);
  for (int i = 0; i < s.size(); ++i)
    res[i + m / 2] = s[i];
  return res;
}
vector <ll> Derivative(vector <ll> a) {
  int n = a.size();
  vector <ll> res(n - 1);
  for (int i = 0; i < n - 1; ++i)
    res[i] = mul(a[i + 1], i + 1);
  return res;
}
vector <ll> Integral(vector <ll> a) {
  int n = a.size();
  vector <ll> res(n + 1);
  for (int i = 0; i < n; ++i)
    res[i + 1] = mul(a[i], mpow(i + 1, mod - 2));
  return res;
}
vector <ll> Ln(vector <ll> a) {
  // O(NLogN), a[0] = 1
  int n = a.size();
  if (n == 1) return {0};
  vector <ll> d = Derivative(a);
  a.pop_back();
  return Integral(Mul(d, Inverse(a), n - 1));
}
vector <ll> Exp(vector <ll> a) {
  // O(NLogN), a[0] = 0
  int n = a.size();
  vector <ll> q(1, 1);
  a[0] = add(a[0], 1);
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    vector <ll> g(a.begin(), a.begin() + m * 2), h(all(
        q));
    h.resize(m * 2), h = Ln(h);
    for (int i = 0; i < m * 2; ++i)
      g[i] = sub(g[i], h[i]);
    q = Mul(g, q, m * 2);
  }
  q.resize(n);
  return q;
}
vector <ll> Pow(vector <ll> a, ll k) {
```

```
  int n = a.size(), m = 0;
  vector <ll> ans(n, 0);
  while (m < n && a[m] == 0) m++;
  if (k && m && (k >= n || k * m >= n)) return ans;
  if (m == n) return ans[0] = 1, ans;
  ll lead = m * k;
  vector <ll> b(a.begin() + m, a.end());
  ll base = mpow(b[0], k), inv = mpow(b[0], mod - 2);
  for (int i = 0; i < n - m; ++i)
    b[i] = mul(b[i], inv);
  b = Ln(b);
  for (int i = 0; i < n - m; ++i)
    b[i] = mul(b[i], k % mod);
  b = Exp(b);
  for (int i = lead; i < n; ++i)
    ans[i] = mul(b[i - lead], base);
  return ans;
}
vector <ll> Evaluate(vector <ll> a, vector <ll> x) {
  if (x.empty()) return {};
  int n = x.size();
  vector <vector <ll>> up(n * 2);
  for (int i = 0; i < n; ++i)
    up[i + n] = {sub(0, x[i]), 1};
  for (int i = n - 1; i > 0; --i)
    up[i] = Mul(up[i * 2], up[i * 2 + 1]);
  vector <vector <ll>> down(n * 2);
  down[1] = Divide(a, up[1]).second;
  for (int i = 2; i < n * 2; ++i)
    down[i] = Divide(down[i >> 1], up[i]).second;
  vector <ll> y(n);
  for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
  return y;
}
vector <ll> Interpolate(vector <ll> x, vector <ll> y) {
  int n = x.size();
  vector <vector <ll>> up(n * 2);
  for (int i = 0; i < n; ++i)
    up[i + n] = {sub(0, x[i]), 1};
  for (int i = n - 1; i > 0; --i)
    up[i] = Mul(up[i * 2], up[i * 2 + 1]);
  vector <ll> a = Evaluate(Derivative(up[1]), x);
  for (int i = 0; i < n; ++i)
    a[i] = mul(y[i], mpow(a[i], mod - 2));
  vector <vector <ll>> down(n * 2);
  for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
  for (int i = n - 1; i > 0; --i) {
    vector <ll> lhs = Mul(down[i * 2], up[i * 2 + 1]);
    vector <ll> rhs = Mul(down[i * 2 + 1], up[i * 2]);
    down[i].resize(lhs.size());
    for (int j = 0; j < lhs.size(); ++j)
      down[i][j] = add(lhs[j], rhs[j]);
  }
  return down[1];
}
```

7.5 Fast Linear Recursion

```
ll FastLinearRecursion(vector <ll> a, vector <ll> c, ll
     k) {
  // a_n = sigma c_j * a_{n - j - 1}, 0-based
  // O(NLogNLogK), |a| = |c|
  int n = a.size();
  if (k < n) return a[k];
  vector <ll> base(n + 1, 1);
  for (int i = 0; i < n; ++i)
    base[i] = sub(0, c[n - i - 1]);
  vector <ll> poly(n);
  (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
  auto calc = [&](vector <ll> p1, vector <ll> p2) {
    // O(n^2) bruteforce or O(nlogn) NTT
    return Divide(Mul(p1, p2), base).second;
  };
  vector <ll> res(n, 0); res[0] = 1;
  for (; k; k >>= 1, poly = calc(poly, poly)) {
    if (k & 1) res = calc(res, poly);
  }
  ll ans = 0;
  for (int i = 0; i < n; ++i)
    (ans += res[i] * a[i]) %= mod;
  return ans;
}
```

7.6 Fast Walsh Transform

```
void fwt(vector <int> &a) {
  // and : x += y * (1, -1)
  // or  : y += x * (1, -1)
  // xor : x = (x + y) * (1, 1/2)
  //       y = (x - y) * (1, 1/2)
  int n = __lg(a.size());
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
      int x = a[j ^ (1 << i)], y = a[j];
      // do something
    }
  }
}
vector<int> subs_conv(vector<int> a, vector<int> b) {
  // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
  int n = __lg(a.size());
  vector<vector<int>> ha(n + 1, vector<int>(1 << n));
  vector<vector<int>> hb(n + 1, vector<int>(1 << n));
  vector<vector<int>> c(n + 1, vector<int>(1 << n));
  for (int i = 0; i < 1 << n; ++i) {
    ha[__builtin_popcount(i)][i] = a[i];
    hb[__builtin_popcount(i)][i] = b[i];
  }
  for (int i = 0; i <= n; ++i)
    or_fwt(ha[i]), or_fwt(hb[i]);
  for (int i = 0; i <= n; ++i)
    for (int j = 0; i + j <= n; ++j)
      for (int k = 0; k < 1 << n; ++k)
        // mind overflow
        c[i + j][k] += ha[i][k] * hb[j][k];
  for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
  vector <int> ans(1 << n);
  for (int i = 0; i < 1 << n; ++i)
    ans[i] = c[__builtin_popcount(i)][i];
  return ans;
}
```

8 Geometry

8.1 Basic

```
const double eps = 1e-8, PI = acos(-1);
int sign(double x)
{ return fabs(x) <= eps ? 0 : (x > 0 ? 1 : -1); }
double norm(double x) {
  while (x < -eps) x += PI * 2;
  while (x > PI * 2 + eps) x -= PI * 2;
  return x;
}
struct Pt {
  double x, y;
  Pt (double _x, double _y) : x(_x), y(_y) {}
  Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
  Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
  Pt operator * (double k) {return Pt(x * k, y * k);}
  Pt operator / (double k) {return Pt (x / k, y / k);}
  double operator * (Pt o) {return x * o.x + y * o.y;}
  double operator ^ (Pt o) {return x * o.y - y * o.x;}
};
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 && sign((c - a) * (c - b))
     <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
double area(Pt a, Pt b, Pt c)
{ return fabs((a - b) ^ (a - c)) / 2; }
double angle(Pt a, Pt b)
{ return norm(atan2(b.y - a.y, b.x - a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) { // CCW
  double c = cos(o), s = sin(o);
  return Pt(c * a.x - s * a.y, s * a.x + c * a.y);
}
```

```
Pt perp(Pt a) {return Pt(-a.y, a.x);}
Pt proj_vec(Pt a, Pt b, Pt c) { // vector ac proj to ab
  return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
  return proj_vec(a, b, c) + a;
}
```

8.2 Heart

```
Pt circenter(Pt p0, Pt p1, Pt p2) {
  // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
  double m = 2. * (x1 * y2 - y1 * x2);
  Pt center(0, 0);
  center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
      y1 - y2)) / m;
  center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
       y2 * y2) / m;
  return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) {
  // radius = area / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
       - p2);
  double s = a + b + c;
  return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3)
{ return (p1 + p2 + p3) / 3; }
Pt orthocenter(Pt p1, Pt p2, Pt p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
     p3) * 2; }
```

8.3 External Bisector

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
  Pt L1 = p2 - p1, L2 = p3 - p1;
  L2 = L2 * abs(L1) / abs(L2);
  return L1 + L2;
}
```

8.4 Intersection of Segments

```
Pt LinesInter(Line a, Line b) {
  double abc = (a.b - a.a) ^ (b.a - a.a);
  double abd = (a.b - a.a) ^ (b.b - a.a);
  if (sign(abc - abd) == 0) return b.b;// no inter
  return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
  if (btw(a.a, a.b, b.a)) return {b.a};
  if (btw(a.a, a.b, b.b)) return {b.b};
  if (btw(b.a, b.b, a.a)) return {a.a};
  if (btw(b.a, b.b, a.b)) return {a.b};
  if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 &&
      ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
    return {LinesInter(a, b)};
  return {};
}
```

8.5 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
  Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
      ) / abs2(l.b - l.a);
  double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
      - s * s / abs2(l.b - l.a);
  if (sign(h2) == -1) return {};
  if (sign(h2) == 0) return {p};
  Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
  return {p - h, p + h};
}
```

8.6 Intersection of Circles

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
  double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
  if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
       + c2.r) return {};
  Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
      c2.r - c1.r * c1.r) / (2 * d2));
  double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
       * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
  Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2
       * d2);
  if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
  return {u + v, u - v};
}
```

8.7 Intersection of Polygon and Circle

```
double _area(Pt pa, Pt pb, double r){
  if (abs(pa) < abs(pb)) swap(pa, pb);
  if (abs(pb) < eps) return 0;
  double S, h, theta;
  double a = abs(pb), b = abs(pa), c = abs(pb - pa);
  double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
  double cosC = (pa * pb) / a / b, C = acos(cosC);
  if (a > r) {
    S = (C / 2) * r * r;
    h = a * b * sin(C) / c;
    if (h < r && B < pi / 2) S -= (acos(h / r) * r * r
        - h * sqrt(r * r - h * h));
  } else if (b > r) {
    theta = pi - B - asin(sin(B) / r * a);
    S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
        * r;
  } else S = 0.5 * sin(C) * a * b;
  return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
  double S = 0; int n = poly.size();
  for (int i = 0; i < n; ++i)
    S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
         ori(O, poly[i], poly[(i + 1) % n]);
  return fabs(S);
}
```

8.8 Tangent Lines of Circle and Point

```
vector<Line> tangent(Cir c, Pt p) {
  vector<Line> z;
  double d = abs(p - c.o);
  if (sign(d - c.r) == 0) {
    Pt i = rot(p - c.o, pi / 2);
    z.push_back({p, p + i});
  } else if (d > c.r) {
    double o = acos(c.r / d);
    Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
        i, -o) * c.r;
    z.push_back({c.o + j, p});
    z.push_back({c.o + k, p});
  }
  return z;
}
```

8.9 Tangent Lines of Circles

```
vector <Line> tangent(Cir c1, Cir c2, int sign1) {
  // sign1 = 1 for outer tang, -1 for inter tang
  vector <Line> ret;
  double d_sq = abs2(c1.o - c2.o);
  if (sign(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  Pt v = (c2.o - c1.o) / d;
  double c = (c1.r - sign1 * c2.r) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
        sign2 * h * v.x);
    Pt p1 = c1.o + n * c1.r;
    Pt p2 = c2.o + n * (c2.r * sign1);
    if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
        0)
      p2 = p1 + perp(c2.o - c1.o);
    ret.pb({p1, p2});
  }
  return ret;
}
```

8.10 Point In Convex

```
bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
  int a = 1, b = int(C.size()) - 1, r = !strict;
  if (C.size() == 0) return false;
  if (C.size() < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
      -r) return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}
```

8.11 Point Segment Distance

```
double PointSegDist(Pt q0, Pt q1, Pt p) {
  if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
  if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
       * (p - q1)) >= 0)
    return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

8.12 Convex Hull

```
vector <Pt> ConvexHull(vector <Pt> pt) {
  int n = pt.size();
  sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.
      y < b.y : a.x < b.x;});
  vector <Pt> ans = {pt[0]};
  for (int t : {0, 1}) {
    int m = ans.size();
    for (int i = 1; i < n; ++i) {
      while (ans.size() > m && ori(ans[ans.size() - 2],
           ans.back(), pt[i]) <= 0)
        ans.pop_back();
      ans.pb(pt[i]);
    }
    reverse(all(pt));
  }
  ans.pop_back();
  return ans;
}
```

8.13 Convex Hull Distance

```
double ConvexHullDist(vector<Pt> A, vector<Pt> B) {
  Pt O(0, 0);
  for (auto &p : B) p = O - p;
  auto C = Minkowski(A, B); // assert SZ(C) > 0
  if (PointInConvex(C, O)) return 0;
  double ans = PointSegDist(C.back(), C[0], O);
  for (int i = 0; i + 1 < C.size(); ++i)
    ans = min(ans, PointSegDist(C[i], C[i + 1], O));
  return ans;
}
```

8.14 Minimum Enclosing Circle

```
Cir min_enclosing(vector<Pt> &p) {
  random_shuffle(all(p));
  double r = 0.0;
  Pt cent = p[0];
  for (int i = 1; i < p.size(); ++i) {
    if (abs2(cent - p[i]) <= r) continue;
    cent = p[i], r = 0.0;
    for (int j = 0; j < i; ++j) {
      if (abs2(cent - p[j]) <= r) continue;
      cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
      for (int k = 0; k < j; ++k) {
        if (abs2(cent - p[k]) <= r) continue;
        cent = circenter(p[i], p[j], p[k]);
        r = abs2(p[k] - cent);
      }
    }
  }
  return {cent, sqrt(r)};
}
```

8.15 Union of Circles

```
vector<pair<double, double>> CoverSegment(Cir a, Cir b)
    {
  double d = abs(a.o - b.o);
  vector<pair<double, double>> res;
  if (sign(a.r + b.r - d) == 0);
  else if (d <= abs(a.r - b.r) + eps) {
    if (a.r < b.r) res.emplace_back(0, 2 * pi);
  } else if (d < abs(a.r + b.r) - eps) {
    double o = acos((a.r * a.r + d * d - b.r * b.r) /
        (2 * a.r * d));
    double z = norm(atan2((b.o - a.o).y, (b.o - a.o).x)
        );
    double l = norm(z - o), r = norm(z + o);
    if (l > r) res.emplace_back(l, 2 * pi), res.
        emplace_back(0, r);
    else res.emplace_back(l, r);
  }
  return res;
}
double CircleUnionArea(vector<Cir> c) { // circle
    should be identical
  int n = c.size();
  double a = 0, w;
  for (int i = 0; w = 0, i < n; ++i) {
    vector<pair<double, double>> s = {{2 * pi, 9}}, z;
    for (int j = 0; j < n; ++j) if (i != j) {
      z = CoverSegment(c[i], c[j]);
      for (auto &e : z) s.push_back(e);
    }
    sort(s.begin(), s.end());
    auto F = [&] (double t) { return c[i].r * (c[i].r *
         t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
    for (auto &e : s) {
      if (e.first > w) a += F(e.first) - F(w);
      w = max(w, e.second);
    }
  }
  return a * 0.5;
}
```

8.16 Union of Polygons

```
double polyUnion(vector <vector <Pt>> poly) {
  int n = poly.size();
  double ans = 0;
  auto solve = [&](Pt a, Pt b, int cid) {
    vector <pair <Pt, int>> event;
    for (int i = 0; i < n; ++i) {
      int st = 0, sz = poly[i].size();
      while (st < sz && ori(poly[i][st], a, b) != 1)
        st++;
      if (st == sz) continue;
      for (int j = 0; j < sz; ++j) {
        Pt c = poly[i][(j + st) % sz];
        Pt d = poly[i][(j + st + 1) % sz];
        if (sign((a - b) ^ (c - d)) != 0) {
          int ok1 = ori(c, a, b) == 1;
          int ok2 = ori(d, a, b) == 1;
          if (ok1 ^ ok2) event.emplace_back(LinesInter
              ({a, b}, {c, d}), ok1 ? 1 : -1);
        } else if (ori(c, a, b) == 0 && sign((a - b) *
            (c - d)) > 0 && i <= cid) {
          event.emplace_back(c, -1);
          event.emplace_back(d, 1);
        }
      }
    }
    sort(all(event), [&](pair <Pt, int> i, pair <Pt,
        int> j) {
      return ((a - i.first) * (a - b)) < ((a - j.first)
           * (a - b));
    });
    int now = 0;
    Pt lst = a;
    for (auto [x, y] : event) {
      if (btw(a, b, lst) && btw(a, b, x) && !now)
        ans += lst ^ x;
      now += y, lst = x;
    }
  };
  for (int i = 0; i < n; ++i) {
    int sz = poly[i].size();
```

```
    for (int j = 0; j < sz; ++j)
      solve(poly[i][j], poly[i][(j + 1) % sz], i);
  }
  return ans / 2;
}
```

8.17 Rotating SweepLine

```
void RotatingSweepLine(vector <Pt> &pt) {
  int n = pt.size();
  vector <int> ord(n), cur(n);
  vector <pii> line;
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j) if (i ^ j)
      line.emplace_back(i, j);
  sort(all(line), [&](pii i, pii j) {
    Pt a = pt[i.second] - pt[i.first];
    Pt b = pt[j.second] - pt[j.first];
    if (pos(a) == pos(b)) return sign(a ^ b) > 0;
    return pos(a) < pos(b);
  });
  iota(all(ord), 0);
  sort(all(ord), [&](int i, int j) {
    return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt
        [j].x : pt[i].y < pt[j].y);
  });
  for (int i = 0; i < n; ++i) cur[ord[i]] = i;
  for (auto [i, j] : line) {
    // point sort by the distance to line(i, j)
    tie(cur[i], cur[j], ord[cur[i]], ord[cur[j]]) =
        make_tuple(cur[j], cur[i], j, i);
  }
}
```

8.18 Half Plane Intersection

```
vector<Pt> HalfPlaneInter(vector<Line> vec) {
  //         x
  // line.a -------> line.b
  int n = vec.size();
  sort(all(vec), [&](Line a, Line b) {
    Pt A = a.b - a.a, B = b.b - b.a;
    if (pos(A) == pos(B)) {
      if (sign(A ^ B) == 0)
        return sign((b.a - a.a) ^ (b.b - a.a)) > 0;
      return sign(A ^ B) > 0;
    }
    return pos(A) < pos(B);
  });
  auto same = [&](Line a, Line b) {
    return sign((a.b - a.a) ^ (b.b - b.a)) == 0 &&
           sign((a.b - a.a) * (b.b - b.a)) > 0;
  };
  deque <Pt> inter;
  deque <Line> seg;
  for (int i = 0; i < n; ++i) if (!i || !same(vec[i -
      1], vec[i])) {
    while (seg.size() >= 2 && sign((vec[i].b - inter.
        back()) ^ (vec[i].a - inter.back())) == 1)
      seg.pop_back(), inter.pop_back();
    while (seg.size() >= 2 && sign((vec[i].b - inter.
        front()) ^ (vec[i].a - inter.front())) == 1)
      seg.pop_front(), inter.pop_front();
    seg.pb(vec[i]);
    if (seg.size() >= 2) inter.pb(LinesInter(seg[seg.
        size() - 2], seg.back()));
  }
  while (seg.size() >= 2 && sign((seg.front().b - inter
      .back()) ^ (seg.front().a - inter.back())) == 1)
    seg.pop_back(), inter.pop_back();
  inter.pb(LinesInter(seg.front(), seg.back()));
  return vector<Pt>(all(inter));
}
```

8.19 Minkowski Sum

```
void reorder(vector <Pt> &P) {
  rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
       { return make_pair(a.y, a.x) < make_pair(b.y, b.
      x); }), P.end());
}
vector <Pt> Minkowski(vector <Pt> P, vector <Pt> Q) {
  // P, Q: convex polygon, CCW order
  reorder(P), reorder(Q);
  int n = P.size(), m = Q.size();
  P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
  vector <Pt> ans;
  for (int i = 0, j = 0; i < n || j < m; ) {
    ans.pb(P[i] + Q[j]);
    auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
    if (val >= 0) i++;
    if (val <= 0) j++;
  }
  return ans;
}
```

8.20 Delaunay Triangulation

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // lower_bound
    unknown
struct Tri;
struct Edge {
  Tri* tri; int side;
  Edge(): tri(0), side(0){}
  Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
  pll p[3];
  Edge edge[3];
  Tri* chd[3];
  Tri() {}
  Tri(const pll& p0, const pll& p1, const pll& p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
  }
  bool has_chd() const { return chd[0] != 0; }
  int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
  }
  bool contains(pll const& q) const {
    for (int i = 0; i < 3; ++i)
      if (ori(p[i], p[(i + 1) % 3], q) < 0)
        return 0;
    return 1;
  }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
  if(a.tri) a.tri->edge[a.side] = b;
  if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
  Trig() {
    the_root = // Tri should at least contain all
        points
      new(tris++) Tri(pll(-inf, -inf), pll(inf + inf, -
          inf), pll(-inf, inf + inf));
  }
  Tri* find(pll p) { return find(the_root, p); }
  void add_point(const pll &p) { add_point(find(
      the_root, p), p); }
  Tri* the_root;
  static Tri* find(Tri* root, const pll &p) {
    while (1) {
      if (!root->has_chd())
        return root;
      for (int i = 0; i < 3 && root->chd[i]; ++i)
        if (root->chd[i]->contains(p)) {
          root = root->chd[i];
          break;
        }
    }
```

```
    assert(0); // "point not found"
  }
  void add_point(Tri* root, pll const& p) {
    Tri* t[3];
    /* split it into three triangles */
    for (int i = 0; i < 3; ++i)
      t[i] = new(tris++) Tri(root->p[i], root->p[(i +
          1) % 3], p);
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
    for (int i = 0; i < 3; ++i)
      root->chd[i] = t[i];
    for (int i = 0; i < 3; ++i)
      flip(t[i], 2);
  }
  void flip(Tri* tri, int pi) {
    Tri* trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
        pj])) return;
    /* flip edge between tri,trj */
    Tri* trk = new(tris++) Tri(tri->p[(pi + 1) % 3],
        trj->p[pj], tri->p[pi]);
    Tri* trl = new(tris++) Tri(trj->p[(pj + 1) % 3],
        tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->chd[0] = trk; tri->chd[1] = trl; tri->chd[2] =
         0;
    trj->chd[0] = trk; trj->chd[1] = trl; trj->chd[2] =
         0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
  }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
  if (vst.find(now) != vst.end())
    return;
  vst.insert(now);
  if (!now->has_chd())
    return triang.pb(now);
  for (int i = 0; i < now->num_chd(); ++i)
    go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
  tris = pool; triang.clear(); vst.clear();
  random_shuffle(ps, ps + n);
  Trig tri; // the triangulation structure
  for (int i = 0; i < n; ++i)
    tri.add_point(ps[i]);
  go(tri.the_root);
}
```

8.21 Triangulation Vonoroi

```
vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
  pdd d = l.Y - l.X; d = perp(d);
  pdd m = (l.X + l.Y) / 2;
  l = Line(m, m + d);
  if (ori(l.X, l.Y, p) < 0)
    l = Line(m + d, m);
  return l;
}
double calc_area(int id) {
  // use to calculate the area of point "strictly in
      the convex hull"
  vector<Line> hpi = halfPlaneInter(ls[id]);
  vector<pdd> ps;
  for (int i = 0; i < SZ(hpi); ++i)
    ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1) %
        SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
  double rt = 0;
  for (int i = 0; i < SZ(ps); ++i)
    rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
  return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
  map<pll, int> mp;
  for (int i = 0; i < n; ++i)
    arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
  build(n, arr); // Triangulation
  for (auto *t : triang) {
    vector<int> p;
    for (int i = 0; i < 3; ++i)
      if (mp.find(t->p[i]) != mp.end())
        p.pb(mp[t->p[i]]);
    for (int i = 0; i < SZ(p); ++i)
      for (int j = i + 1; j < SZ(p); ++j) {
        Line l(oarr[p[i]], oarr[p[j]]);
        ls[p[i]].pb(make_line(oarr[p[i]], l));
        ls[p[j]].pb(make_line(oarr[p[j]], l));
      }
  }
}
```

8.22 3D Point

```
struct Point {
  double x, y, z;
  Point(double _x = 0, double _y = 0, double _z = 0): x
      (_x), y(_y), z(_z){}
  Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);
    }
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
      p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(const Point &a)
{ return sqrt(dot(a, a)); }
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
pdd proj(Point a, Point b, Point c, Point u) {
// proj. u to the plane of a, b, and c
  Point e1 = b - a;
  Point e2 = c - a;
  e1 = e1 / abs(e1);
  e2 = e2 - e1 * dot(e2, e1);
  e2 = e2 / abs(e2);
  Point p = u - a;
  return pdd(dot(p, e1), dot(p, e2));
}
```

8.23 3D Convex Hull

```
struct CH3D {
  struct face{int a, b, c; bool ok;} F[8 * N];
  double dblcmp(Point &p,face &f)
  {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a
      ]);}
  int g[N][N], num, n;
  Point P[N];
  void deal(int p,int a,int b) {
    int f = g[a][b];
    face add;
    if (F[f].ok) {
      if (dblcmp(P[p],F[f]) > eps) dfs(p,f);
      else
        add.a = b, add.b = a, add.c = p, add.ok = 1, g[
            p][b] = g[a][p] = g[b][a] = num, F[num++]=
            add;
    }
  }
  void dfs(int p, int now) {
    F[now].ok = 0;
    deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
        now].b), deal(p, F[now].a, F[now].c);
```

```cpp
  }
  bool same(int s,int t){
    Point &a = P[F[s].a];
    Point &b = P[F[s].b];
    Point &c = P[F[s].c];
    return fabs(volume(a, b, c, P[F[t].a])) < eps &&
        fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
        volume(a, b, c, P[F[t].c])) < eps;
  }
  void init(int _n){n = _n, num = 0;}
  void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&](){
        for (int i = 1; i < n; ++i)
        if (abs(P[0] - P[i]) > eps)
        return swap(P[1], P[i]), 0;
        return 1;
        }() || [&](){
        for (int i = 2; i < n; ++i)
        if (abs(cross3(P[i], P[0], P[1])) > eps)
        return swap(P[2], P[i]), 0;
        return 1;
        }() || [&](){
        for (int i = 3; i < n; ++i)
        if (fabs(dot(cross(P[0] - P[1], P[1] - P[2]), P
            [0] - P[i])) > eps)
        return swap(P[3], P[i]), 0;
        return 1;
        }())return;
    for (int i = 0; i < 4; ++i) {
      add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
          (i + 3) % 4, add.ok = true;
      if (dblcmp(P[i],add) > 0) swap(add.b, add.c);
      g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
          a] = num;
      F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
      for (int j = 0; j < num; ++j)
        if (F[j].ok && dblcmp(P[i],F[j]) > eps) {
          dfs(i, j);
          break;
        }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
      if (F[i].ok) F[num++] = F[i];
  }
  double get_area() {
    double res = 0.0;
    if (n == 3)
      return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
      res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
  }
  double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
      res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b
          ], P[F[i].c]);
    return fabs(res / 6.0);
  }
  int triangle() {return num;}
  int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag
        , flag = 1)
      for (int j = 0; j < i && flag; ++j)
        flag &= !same(i,j);
    return res;
  }
  Point getcent(){
    Point ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
      if (F[i].ok == true) {
        Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
            i].c];
        t2 = volume(temp, p1, p2, p3) / 6.0;
        if (t2>0)
          ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
              ans.y += (p1.y + p2.y + p3.y + temp.y) *
              t2, ans.z += (p1.z + p2.z + p3.z + temp.z
              ) * t2, v += t2;
      }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
        );
    return ans;
  }
  double pointmindis(Point p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
      if(F[i].ok == true) {
        Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
            i].c];
        double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.
            z - p1.z) * (p3.y - p1.y);
        double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.
            x - p1.x) * (p3.z - p1.z);
        double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.
            y - p1.y) * (p3.x - p1.x);
        double d = 0 - (a * p1.x + b * p1.y + c * p1.z)
            ;
        double temp = fabs(a * p.x + b * p.y + c * p.z
            + d) / sqrt(a * a + b * b + c * c);
        rt = min(rt, temp);
      }
    return rt;
  }
};
```

9 Else

9.1 Pbds

```cpp
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue <int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1);// 4 0
bitset <100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope <int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```cpp
long long next_perm(long long v) {
  long long t = v | (v - 1);
  return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(
      v) + 1));
}
void subset(long long s) {
  long long sub = s;
  while (sub) sub = (sub - 1) & s;
}
```

9.3 Dynamic Programming Condition

9.3.1 Totally Monotone (Concave/Convex)

$$\forall i < i', j < j',\ B[i][j] \le B[i'][j] \implies B[i][j'] \le B[i'][j']$$
$$\forall i < i', j < j',\ B[i][j] \ge B[i'][j] \implies B[i][j'] \ge B[i'][j']$$

9.3.2 Monge Condition (Concave/Convex)

$$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \ge B[i][j'] + B[i'][j]$$
$$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \le B[i][j'] + B[i'][j]$$

9.3.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$$

9.4 Smawk Algorithm

```cpp
ll query(int l, int r) {
  // ...
}
struct SMAWK {
  // Condition:
  // If M[1][0] < M[1][1] then M[0][0] < M[0][1]
  // If M[1][0] == M[1][1] then M[0][0] <= M[0][1]
  // For all i, find r_i s.t. M[i][r_i] is maximum ||
      minimum.
  int ans[N], tmp[N];
  void interpolate(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    vector <int> nl;
    for (int i = 1; i < n; i += 2) {
      nl.push_back(l[i]);
    }
    run(nl, r);
    for (int i = 1, j = 0; i < n; i += 2) {
      while (j < m && r[j] < ans[l[i]])
        j++;
      assert(j < m && ans[l[i]] == r[j]);
      tmp[l[i]] = j;
    }
    for (int i = 0; i < n; i += 2) {
      int curl = 0, curr = m - 1;
      if (i)
        curl = tmp[l[i - 1]];
      if (i + 1 < n)
        curr = tmp[l[i + 1]];
      ll res = query(l[i], r[curl]);
      ans[l[i]] = r[curl];
      for (int j = curl + 1; j <= curr; ++j) {
        ll nxt = query(l[i], r[j]);
        if (res < nxt)
          res = nxt, ans[l[i]] = r[j];
      }
    }
  }
  void reduce(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    vector <int> nr;
    for (int j : r) {
      while (!nr.empty()) {
        int i = nr.size() - 1;
        if (query(l[i], nr.back()) <= query(l[i], j))
          nr.pop_back();
        else
          break;
      }
      if (nr.size() < n)
        nr.push_back(j);
    }
    run(l, nr);
  }
  void run(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    if (max(n, m) <= 2) {
      for (int i : l) {
        ans[i] = r[0];
        if (m > 1) {
          if (query(i, r[0]) < query(i, r[1]))
            ans[i] = r[1];
        }
      }
    } else if (n >= m) {
      interpolate(l, r);
    } else {
      reduce(l, r);
    }
  }
};
```

9.5 Slope Trick

```cpp
template<typename T>
struct slope_trick_convex {
  T minn = 0, ground_l = 0, ground_r = 0;
  priority_queue<T, vector<T>, less<T>> left;
  priority_queue<T, vector<T>, greater<T>> right;
  slope_trick_convex() {left.push(numeric_limits<T>::
      min() / 2), right.push(numeric_limits<T>::max() /
       2);}
  void push_left(T x) {left.push(x - ground_l);}
  void push_right(T x) {right.push(x - ground_r);}
  //add a line with slope 1 to the right starting from
      x
  void add_right(T x) {
    T l = left.top() + ground_l;
    if (l <= x) push_right(x);
    else push_left(x), push_right(l), left.pop(), minn
        += l - x;
  }
  //add a line with slope -1 to the left starting from
      x
  void add_left(T x) {
    T r = right.top() + ground_r;
    if (r >= x) push_left(x);
    else push_right(x), push_left(r), right.pop(), minn
         += x - r;
  }
  //val[i]=min(val[j]) for all i-l<=j<=i+r
  void expand(T l, T r) {ground_l -= l, ground_r += r;}
  void shift_up(T x) {minn += x;}
  T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() +
        ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
      vector<T> trash;
      T cur_val = minn, slope = 1, res;
      while (1) {
        trash.push_back(left.top());
        left.pop();
        if (left.top() + ground_l <= x) {
          res = cur_val + slope * (l - x);
          break;
        }
        cur_val += slope * (l - (left.top() + ground_l)
            );
        l = left.top() + ground_l;
        slope += 1;
      }
      for (auto i : trash) left.push(i);
      return res;
    }
    if (x > r) {
      vector<T> trash;
      T cur_val = minn, slope = 1, res;
      while (1) {
        trash.push_back(right.top());
        right.pop();
        if (right.top() + ground_r >= x) {
          res = cur_val + slope * (x - r);
          break;
        }
        cur_val += slope * ((right.top() + ground_r) -
            r);
        r = right.top() + ground_r;
        slope += 1;
      }
      for (auto i : trash) right.push(i);
      return res;
    }
    assert(0);
  }
};
```

9.6 ALL LCS

```cpp
void all_lcs(string s, string t) { // 0-base
  vector<int> h(t.size());
  iota(all(h), 0);
  for (int a = 0; a < s.size(); ++a) {
    int v = -1;
    for (int c = 0; c < t.size(); ++c)
      if (s[a] == t[c] || h[c] < v)
```

```
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

9.7 Hilbert Curve

```
ll hilbert(int n, int x, int y) {
  ll res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
} // n = 2^k
```

9.8 Random

```
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t a) const {
    static const uint64_t FIXED_RANDOM = chrono::
        steady_clock::now().time_since_epoch().count();
    return splitmix64(i + FIXED_RANDOM);
  }
};
unordered_map <int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());
```

9.9 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S. Otherwise for each $x \in S, y \notin S$, create edges

- $x \to y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \to x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2, and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.10 Python Misc

```
from [decimal, fractions, math, random] import *
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
     ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
print(*[randint(1, C) for i in range(0, N)], sep=' ')
```