Contents

1 Basic

1.1 Compiler Shell

```
if [ $# -ne 2 ] ; then
  g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
       $1
else
  g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
       $1 -fsanitize=address
fi
./$1
chmod +x ./run.sh
./run.sh main [1]
```

1.2 Default Code

```
#include <bits/stdc++.h>
using namespace std;
#define ll long long
#define pb push_back
#define all(x) x.begin(), x.end()
```

1.3 Testing Todo List

```
0. choose editor
1. shell script
2. __int128, __lg, __builtin_popcount
3. judge speed v.s.local speed
4. CE penalty?
```

1.4 Debug Macro

```
void db() {cout << endl;}
template <typename T, typename ...U> void db(T i, U ...
    j) {
  cout << i << ' ', db(j...);
}
#define test(x...) db("[" + string(x) + "]", x)
```

1.5 Stress Test Shell

```
g++ $1.cpp -o $1
g++ $2.cpp -o $2
g++ $3.cpp -o $3
for i in {1..100} ; do
  ./$3 > input.txt
  # st=$(date +%s%N)
  ./$1 < input.txt > output1.txt
  # echo "$((($(date +%s%N) - $st)/1000000))ms"
  ./$2 < input.txt > output2.txt
  if cmp --silent -- "output1.txt" "output2.txt" ; then
    continue
  fi
  echo Input:
  cat input.txt
  echo Your Output:
  cat output1.txt
  echo Correct Output:
  cat output2.txt
  break
done
echo OK!
./stress.sh main good gen
```

1.6 Pragma

```
#pragma GCC optimize("Ofast,inline,unroll-loops")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")
```

1.7 Fast IO

```
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
  static char buf[65536], *p = buf, *q = buf;
  return p == q && (q = (p = buf) + read(0, buf, 65536)
      ) == buf ? -1 : *p++;
}
inline int R() {
  static char c;
  while((c = RC()) < '0'); int a = c ^ '0';
  while((c = RC()) >= '0') a *= 10, a += c ^ '0';
  return a;
}
inline void W(int n) {
  static char buf[12], p;
  if (n == 0) OB[OP++]='0'; p = 0;
  while (n) buf[p++] = '0' + (n % 10), n /= 10;
  for (--p; p >= 0; --p) OB[OP++] = buf[p];
  if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

2 Data Structure

2.1 Leftist Tree

```
struct node {
  ll rk, data, sz, sum;
  node *l, *r;
  node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k
      ) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (a->data < b->data) swap(a, b);
  a->r = merge(a->r, b);
  if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
  a->rk = rk(a->r) + 1, a->sz = sz(a->l) + sz(a->r) +
      1;
  a->sum = sum(a->l) + sum(a->r) + a->data;
  return a;
}
void pop(node *&o) {
  node *tmp = o;
  o = merge(o->l, o->r);
  delete tmp;
}
```

2.2 Splay Tree

```
struct Splay {
  int pa[N], ch[N][2], sz[N], rt, _id;
  ll v[N];
  Splay() {}
  void init() {
    rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
    sz[0] = 1, v[0] = inf;
  }
  int newnode(int p, int x) {
    int id = _id++;
    v[id] = x, pa[id] = p;
    ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    return id;
  }
  void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i, gp = pa[p], c =
        ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
  }
  void splay(int i) {
    while (~pa[i]) {
      int p = pa[i];
      if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
          == i ? i : p);
      rotate(i);
    }
    rt = i;
  }
  int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
      if (v[i] == x) return splay(i), i;
      if (v[i] > x) {
        last = i;
        if (ch[i][0] == -1) break;
        i = ch[i][0];
      }
      else {
        if (ch[i][1] == -1) break;
        i = ch[i][1];
      }
    }
    splay(i);
    return last; // -1 if not found
  }
  void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
      // assert(ch[rt][1] == -1);
      int id = newnode(rt, x);
      ch[rt][1] = id, ++sz[rt];
      splay(id);
    }
    else if (v[i] != x) {
      splay(i);
      int id = newnode(rt, x), c = ch[rt][0];
      ch[rt][0] = id;
      ch[id][0] = c;
      if (~c) pa[c] = id, sz[id] += sz[c];
      ++sz[rt];
      splay(id);
    }
  }
};
```

2.3 Link Cut Tree

```
// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N], sz2[N], w[N], mx[
        N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
        virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0])
            sz[i] += sz[ch[i][0]], mx[i] = max(mx[i],
                mx[ch[i][0]]);
        if (ch[i][1])
            sz[i] += sz[ch[i][1]], mx[i] = max(mx[i],
                mx[ch[i][1]]);
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i
            ][1]), rev[i] = false;
    }
    void reverse(int i) {
        if (!i) return;
        swap(ch[i][0], ch[i][1]);
        rev[i] ^= true;
    }
    bool isrt(int i) {// rt of splay
        if (!pa[i]) return true;
        return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i, c = ch[i][!x
            ], gp = pa[p];
        if (ch[gp][0] == p) ch[gp][0] = i;
        else if (ch[gp][1] == p) ch[gp][1] = i;
        pa[i] = gp, ch[i][!x] = p, pa[p] = i;
        ch[p][x] = c, pa[c] = p;
        pull(p), pull(i);
    }
    void splay(int i) {
        vector<int> anc;
        anc.push_back(i);
        while (!isrt(anc.back())) anc.push_back(pa[anc.
            back()]);
        while (!anc.empty()) push(anc.back()), anc.
            pop_back();
        while (!isrt(i)) {
            int p = pa[i];
            if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[
                p]][1] == p ? i : p);
            rotate(i);
        }
    }
    void access(int i) {
        int last = 0;
        while (i) {
            splay(i);
            if (ch[i][1])
                sz2[i] += sz[ch[i][1]];
            sz2[i] -= sz[last];
            ch[i][1] = last;
            pull(i), last = i, i = pa[i];
        }
    }
    void makert(int i) {
        access(i), splay(i), reverse(i);
    }
    void link(int i, int j) {
        // assert(findrt(i) != findrt(j));
        makert(i);
        makert(j);
        pa[i] = j;
        sz2[j] += sz[i];
        pull(j);
    }
    void cut(int i, int j) {
        makert(i), access(j), splay(i);
        // assert(sz[i] == 2 && ch[i][1] == j);
        ch[i][1] = pa[j] = 0, pull(i);
    }
    int findrt(int i) {
        access(i), splay(i);
        while (ch[i][0]) push(i), i = ch[i][0];
        splay(i);
        return i;
    }
};
```

2.4 Treap

```
struct node {
  int data, sz;
  node *l, *r;
  node(int k) : data(k), sz(1), l(0), r(0) {}
  void up() {
    sz = 1;
    if (l) sz += l->sz;
    if (r) sz += r->sz;
  }
  void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (rand() % (sz(a) + sz(b)) < sz(a))
    return a->down(), a->r = merge(a->r, b), a->up(), a
      ;
  return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
  if (!o) return a = b = 0, void();
  o->down();
  if (o->data <= k)
    a = o, split(o->r, a->r, b, k), a->up();
  else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
  if (sz(o) <= k) return a = o, b = 0, void();
  o->down();
  if (sz(o->l) + 1 <= k)
    a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
  else b = o, split2(o->l, a, b->l, k);
  o->up();
}
node *kth(node *o, int k) {
  if (k <= sz(o->l)) return kth(o->l, k);
  if (k == sz(o->l) + 1) return o;
  return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
  if (!o) return 0;
  if (o->data < key)
    return sz(o->l) + 1 + Rank(o->r, key);
  else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
  if (!o) return 0;
  if (o->data == k) {
    node *t = o;
    o->down(), o = merge(o->l, o->r);
    delete t;
    return 1;
  }
  node *&t = k < o->data ? o->l : o->r;
  return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
  node *a, *b;
  split(o, a, b, k),
  o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
  node *a, *b, *c;
  split2(o, a, b, l - 1), split2(b, b, c, r);
  // operate
  o = merge(a, merge(b, c));
}
```

2.5 Persistent Segment Tree

```
struct Seg {
  // Persistent Segment Tree, single point modify,
      range query sum
  // 0-indexed, [l, r)
  static Seg mem[M], *pt;
  int l, r, m, val;
  Seg* ch[2];
  Seg () = default;
  Seg (int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
      val(0) {
    if (r - l > 1) {
      ch[0] = new (pt++) Seg(l, m);
      ch[1] = new (pt++) Seg(m, r);
    }
  }
  void pull() {val = ch[0]->val + ch[1]->val;}
  Seg* modify(int p, int v) {
    Seg *now = new (pt++) Seg(*this);
    if (r - l == 1) {
      now->val = v;
    } else {
      now->ch[p >= m] = ch[p >= m]->modify(p, v);
      now->pull();
    }
    return now;
  }
  int query(int a, int b) {
    if (a <= l && r <= b) return val;
    int ans = 0;
    if (a < m) ans += ch[0]->query(a, b);
    if (m < b) ans += ch[1]->query(a, b);
    return ans;
  }
} Seg::mem[M], *Seg::pt = mem;
// Init Tree
Seg *root = new (Seg::pt++) Seg(0, n);
```

2.6 2D Segment Tree

```
// 2D range add, range sum in log^2
struct seg {
  int l, r;
  ll sum, lz;
  seg *ch[2]{};
  seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
  void push() {
    if (lz) ch[0]->add(l, r, lz), ch[1]->modify(l, r,
        lz), lz = 0;
  }
  void pull() {sum = ch[0]->sum + ch[1]->sum;}
  void add(int _l, int _r, ll d) {
    if (_l <= l && r <= _r) {
      sum += d * (r - l);
      lz += d;
      return;
    }
    if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
        new seg(l + r >> 1, r);
    push();
    if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
    pull();
  }
  ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(l, _l));
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
  }
};
struct seg2 {
  int l, r;
  seg v, lz;
  seg2 *ch[2]{};
  seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N
      ) {
    if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch
        [1] = new seg2(l + r >> 1, r);
  }
  void add(int _l, int _r, int _l2, int _r2, ll d) {
    v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
    if (_l <= l && r <= _r) {
      lz.add(_l2, _r2, d);
      return;
    }
    if (_l < l + r >> 1) ch[0]->add(_l, _r, _l2, _r2, d
        );
    if (l + r >> 1 < _r) ch[1]->add(_l, _r, _l2, _r2, d
        );
  }
  ll qsum(int _l, int _r, int _l2, int _r2) {
    ll res = v.qsum(_l2, _r2);
    if (_l <= l && r <= _r) return res;
    res += lz.qsum(_l2, _r2) * (min(r, _r) - max(l, _l)
        );
    if (_l < l + r >> 1) res += ch[0]->query(_l, _r,
        _l2, _r2);
    if (l + r >> 1 < _r) res += ch[1]->query(_l, _r,
        _l2, _r2);
    return res;
  }
};
```

2.7 Zkw

```
ll mx[N << 1], sum[N << 1], lz[N << 1];
void add(int l, int r, ll d) { // [L, r), 0-based
  int len = 1, cntl = 0, cntr = 0;
  for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
      len <<= 1) {
    sum[l] += cntl * d, sum[r] += cnt[r] * d;
    if (len > 1) {
      mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
      mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
    }
    if (~l & 1)
      sum[l ^ 1] += d * len, mx[l ^ 1] += d, lz[l ^ 1]
          += d, cntl += len;
    if (r & 1)
      sum[r ^ 1] += d * len, mx[r ^ 1] += d, lz[r ^ 1]
          += d, cntr += len;
  }
  sum[l] += cntl * d, sum[r] += cntr * d;
  if (len > 1) {
    mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
    mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
  }
  cntl += cntr;
  for (l >>= 1; l; l >>= 1) {
    sum[l] += cntl * d;
    mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
  }
}
ll qsum(int l, int r) {
  ll res = 0, len = 1, cntl = 0, cntr = 0;
  for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
      len <<= 1) {
    res += cntl * lz[l] + cntr * lz[r];
    if (~l & 1) res += sum[l ^ 1], cntl += len;
    if (r & 1) res += sum[r ^ 1], cntr += len;
  }
  res += cntl * lz[l] + cntr * lz[r];
  cntl += cntr;
  for (l >>= 1; l; l >>= 1) res += cntl * lz[l];
  return res;
}
ll qmax(int l, int r) {
  ll maxl = -INF, maxr = -INF;
  for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1)
      {
    maxl += lz[l], max[r] += lz[r];
    if (~l & 1) maxl = max(maxl, mx[l ^ 1]);
    if (r & 1) maxr = max(maxr, mx[r ^ 1]);
  }
  maxl = max(maxl + lz[l], maxr + lz[r]);
  for (l >>= 1; l; l >>= 1) maxl += lz[l];
  return maxl;
}
```

2.8 Chtholly Tree

```
struct ChthollyTree {
  struct interval {
    int l, r;
    ll v;
    interval (int _l, int _r, ll _v) : l(_l), r(_r), v(
        _v) {}
  };
  struct cmp {
    bool operator () (const interval &a, const interval
        & b) const {
      return a.l < b.l;
    }
  };
  set <interval, cmp> s;
  vector <interval> split(int l, int r) {
    // split into [0, l), [l, r), [r, n) and return [l,
        r)
    vector <interval> del, ans, re;
    auto it = s.lower_bound(interval(l, -1, 0));
    if (it != s.begin() && (it == s.end() || l < it->l)
        ) {
      --it;
      del.pb(*it);
      if (r < it->r) {
        re.pb(interval(it->l, l, it->v));
        ans.pb(interval(l, r, it->v));
        re.pb(interval(r, it->r, it->v));
      } else {
        re.pb(interval(it->l, l, it->v));
        ans.pb(interval(l, it->r, it->v));
      }
      ++it;
    }
    for (; it != s.end() && it->r <= r; ++it) {
      ans.pb(*it);
      del.pb(*it);
    }
    if (it != s.end() && it->l < r) {
      del.pb(*it);
      ans.pb(interval(it->l, r, it->v));
      re.pb(interval(r, it->r, it->v));
    }
    for (interval &i : del)
      s.erase(i);
    for (interval &i : re)
      s.insert(i);
    return ans;
  }
  void merge(vector <interval> a) {
    for (interval &i : a)
      s.insert(i);
  }
};
```

2.9 Incremental Min Sum

```
struct IncrementalMinSum {
  multiset <int, greater <int>> in;
  multiset <int> out;
  ll sum; int cap;
  DS () : sum(0), cap(0) {}
  void enlarge() {
    if (!out.empty()) {
      int mx = *out.begin();
      sum += mx, in.insert(mx), out.erase(out.begin());
    }
    cap++;
  }
  void insert(int x) {
    if (!cap) {
      out.insert(x);
      return;
    }
    if (in.size() < cap) {
      in.insert(x), sum += x;
      return;
    }
    int mx = *in.begin();
    if (x < mx) {
      sum -= mx, out.insert(mx), in.erase(in.begin());
      sum += x, in.insert(x);
    } else {
      out.insert(x);
    }
  }
  void erase(int x) {
    if (out.find(x) != out.end()) {
      out.erase(out.lower_bound(x));
    } else {
      in.erase(in.lower_bound(x)), sum -= x;
      if (!out.empty()) {
        int mx = *out.begin();
        sum += mx, out.erase(out.begin()), in.insert(mx
            );
      }
    }
  }
};
```

3 Flow / Matching

3.1 Dinic

```
struct Dinic {
  const int INF = 1 << 30;
  struct edge {
    int v, f;
    edge (int _v, int _f) : v(_v), f(_f) {}
  };
  vector <vector <int>> adj;
  vector <edge> E;
  vector <int> level;
  int n, s, t;
  Dinic (int _n, int _s, int _t) : n(_n), s(_s), t(_t)
      {adj.resize(n);}
  void add_edge(int u, int v, int f) {
    adj[u].pb(E.size()), E.pb(edge(v, f));
    adj[v].pb(E.size()), E.pb(edge(u, 0));
  }
  bool bfs() {
    level.assign(n, -1);
    queue <int> q;
    level[s] = 0, q.push(s);
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (int id : adj[v]) if (E[id].f > 0 && level[E[
          id].v] == -1) {
          level[E[id].v] = level[v] + 1;
          q.push(E[id].v);
        }
    }
    return level[t] != -1;
  }
  int dfs(int v, int minf) {
    if (v == t) return minf;
    int ans = 0;
    for (int id : adj[v]) if (E[id].f > 0 && level[E[id
        ].v] == level[v] + 1) {
        int nxtf = dfs(E[id].v, min(minf, E[id].f));
        minf -= nxtf, E[id].f -= nxtf;
        ans += nxtf, E[id ^ 1].f += nxtf;
        if (!minf) return ans;
      }
    if (!ans) level[v] = -1;
    return ans;
  }
  int solve() {
    int ans = 0;
    while (bfs()) ans += dfs(s, INF);
    return ans;
  }
};
```

3.2 Min Cost Max Flow

```
template <typename T>
struct MCMF {
  const T INF = 1ll << 60;
  struct edge {
    int v;
    T f, c;
    edge (int _v, T _f, T _c) : v(_v), f(_f), c(_c) {}
  };
  vector <edge> E;
  vector <vector <int>> adja;
  vector <T> dis, pot;
  vector <int> rt;
  int n, s, t;
  MCMF (int _n, int _s, int _t) : n(_n), s(_s), t(_t) {
    adj.resize(n);
  }
  void add_edge(int u, int v, T f, T c) {
    adj[u].pb(E.size()), E.pb(edge(v, f, c));
    adj[v].pb(E.size()), E.pb(edge(u, 0, -c));
  }
  bool SPFA() {
    rt.assign(n, -1), dis.assign(n, INF);
    vector <bool> vis(n, false);
    queue <int> q;
    q.push(s), dis[s] = 0, vis[s] = true;
    while (!q.empty()) {
      int v = q.front(); q.pop();
      vis[v] = false;
      for (int id : adj[v]) if (E[id].f > 0 && dis[E[id
          ].v] > dis[v] + E[id].c + pot[v] - pot[E[id].
          v]) {
          dis[E[id].v] = dis[v] + E[id].c + pot[v] -
              pot[E[id].v], rt[E[id].v] = id;
          if (!vis[E[id].v]) vis[E[id].v] = true, q.
              push(E[id].v);
        }
    }
    return dis[t] != INF;
  }
  bool dijkstra() {
    rt.assign(n, -1), dis.assign(n, INF);
    priority_queue <pair <T, int>, vector <pair <T, int
        >>, greater <pair <T, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
      int d, v; tie(d, v) = pq.top(); pq.pop();
      if (dis[v] < d) continue;
      for (int id : adj[v]) if (E[id].f > 0 && dis[E[id
          ].v] > dis[v] + E[id].c + pot[v] - pot[E[id].
          v]) {
```

```
          dis[E[id].v] = dis[v] + E[id].c + pot[v] -
              pot[E[id].v], rt[E[id].v] = id;
          pq.emplace(dis[E[id].v], E[id].v);
        }
    }
    return dis[t] != INF;
  }
  pair <T, T> solve() {
    pot.assign(n, 0);
    T cost = 0, flow = 0;
    bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
      for (int i = 0; i < n; i++) {
        dis[i] += pot[i] - pot[s];
      }
      T add = INF;
      for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
        add = min(add, E[rt[i]].f);
      }
      for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
        E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
      }
      flow += add, cost += add * dis[t];
      fr = false;
      swap(dis, pot);
    }
    return make_pair(flow, cost);
  }
};
```

3.3 Kuhn Munkres

```
template <typename T>
struct KM { // 0-based
  T w[N][N], hl[N], hr[N], slk[N];
  T fl[N], fr[N], pre[N]; int n;
  bool vl[N], vr[N];
  const T INF = 1e9;
  queue <int> q;
  KM (int _n) : n(_n) {
    for (int i = 0; i < n; ++i) for (int j = 0; j < n;
        ++j)
        w[i][j] = -INF;
  }
  void add_edge(int a, int b, int wei) {
    w[a][b] = wei;
  }
  bool check(int x) {
    if (vl[x] = 1, ~fl[x]) return q.push(fl[x]), vr[fl[
        x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    fill(slk, slk + n, INF), fill(vl, vl + n, 0), fill(
        vr, vr + n, 0);
    q.push(s), vr[s] = 1;
    while (1) {
      T d;
      while (!q.empty()) {
        int y = q.front(); q.pop();
        for (int x = 0; x < n; ++x)
          if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] -
              w[x][y]))
            if (pre[x] = y, d) slk[x] = d;
            else if (!check(x)) return;
      }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x) if (!vl[x] && !slk[x]
           && !check(x)) return;
    }
  }
  T solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1), fill(hr
        , hr + n, 0);
    for (int i = 0; i < n; ++i) hl[i] = *max_element(w[
        i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
};
```

3.4 SW Min Cut

```
template <typename T>
struct SW { // 0-based
  T g[N][N], sum[N]; int n;
  bool vis[N], dead[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) fill(g[i], g[i] + n, 0)
        ;
    fill(dead, dead + n, false);
  }
  void add_edge(int u, int v, T w) {
    g[u][v] += w, g[v][u] += w;
  }
  T solve() {
    T ans = 1 << 30;
    for (int round = 0; round + 1 < n; ++round) {
      fill(vis, vis + n, false), fill(sum, sum + n, 0);
      int num = 0, s = -1, t = -1;
      while (num < n - round) {
        int now = -1;
        for (int i = 0; i < n; ++i) if (!vis[i] && !
            dead[i]) {
            if (now == -1 || sum[now] < sum[i]) now = i
                ;
        }
        s = t, t = now;
        vis[now] = true, num++;
        for (int i = 0; i < n; ++i) if (!vis[i] && !
            dead[i]) {
            sum[i] += g[now][i];
        }
      }
      ans = min(ans, sum[t]);
      for (int i = 0; i < n; ++i) {
        g[i][s] += g[i][t];
        g[s][i] += g[t][i];
      }
      dead[t] = true;
    }
    return ans;
  }
};
```

3.5 Gomory Hu Tree

```
vector <array <int, 3>> GomoryHu(vector <vector <pii>>
    adj, int n) {
// Tree edge min -> mincut (0-based)
  Dinic flow(n);
  for (int i = 0; i < n; ++i) for (auto [j, w] : adj[i
      ])
      flow.add_edge(i, j, w);
  flow.record();
  vector <array <int, 3>> ans;
  vector <int> rt(n);
  for (int i = 0; i < n; ++i) rt[i] = 0;
  for (int i = 1; i < n; ++i) {
    int t = rt[i];
    flow.reset(); // clear flows on all edge
    ans.push_back({i, t, flow.solve(i, t)});
    flow.runbfs(i);
    for (int j = i + 1; j < n; ++j) if (rt[j] == t &&
        flow.vis[j]) {
        rt[j] = i;
      }
  }
  return ans;
}
```

3.6 Blossom

```
struct Matching { // 0-based
  int fa[N], pre[N], match[N], s[N], v[N], n, tk;
  vector <int> g[N];
  queue <int> q;
  Matching (int _n) : n(_n), tk(0) {
    for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
    for (int i = 0; i < n; ++i) g[i].clear();
  }
  void add_edge(int u, int v) {
    g[u].push_back(v), g[v].push_back(u);
  }
  int Find(int u) {
    return u == fa[u] ? u : fa[u] = Find(fa[u]);
  }
  int lca(int x, int y) {
    tk++;
    x = Find(x), y = Find(y);
    for (; ; swap(x, y)) {
      if (x != n) {
        if (v[x] == tk) return x;
        v[x] = tk;
        x = Find(pre[match[x]]);
      }
    }
  }
  void blossom(int x, int y, int l) {
    while (Find(x) != l) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      if (fa[x] == x) fa[x] = l;
      if (fa[y] == y) fa[y] = l;
      x = pre[y];
    }
  }
  bool bfs(int r) {
    for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
    while (!q.empty()) q.pop();
    q.push(r);
    s[r] = 0;
    while (!q.empty()) {
      int x = q.front(); q.pop();
      for (int u : g[x]) {
        if (s[u] == -1) {
          pre[u] = x, s[u] = 1;
          if (match[u] == n) {
            for (int a = u, b = x, last; b != n; a =
                last, b = pre[a])
              last = match[b], match[b] = a, match[a] =
                   b;
            return true;
          }
          q.push(match[u]);
          s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = lca(u, x);
          blossom(x, u, l);
          blossom(u, x, l);
        }
      }
    }
    return false;
  }
  int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) {
      if (match[x] == n) res += bfs(x);
    }
    return res;
  }
};
```

3.7 Weighted Blossom

```
struct WeightGraph { // 1-based
  static const int inf = INT_MAX;
  static const int maxn = 514;
  struct edge {
    int u, v, w;
    edge(){}
    edge(int u, int v, int w): u(u), v(v), w(w) {}
  };
  int n, n_x;
  edge g[maxn * 2][maxn * 2];
  int lab[maxn * 2];
  int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
      pa[maxn * 2];
  int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
      maxn * 2];
  vector<int> flo[maxn * 2];
  queue<int> q;
  int e_delta(const edge &e) { return lab[e.u] + lab[e.
      v] - g[e.u][e.v].w * 2; }
  void update_slack(int u, int x) { if (!slack[x] ||
      e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
      [x] = u; }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else for (size_t i = 0; i < flo[x].size(); i++)
      q_push(flo[x][i]);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flo[x].size(); ++
        i) set_st(flo[x][i], b);
  }
  int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) -
        flo[b].begin();
    if (pr % 2 == 1) {
      reverse(flo[b].begin() + 1, flo[b].end());
      return (int)flo[b].size() - pr;
    }
    return pr;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i],
        flo[u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
  }
  void augment(int u, int v) {
    for (; ; ) {
      int xnv = st[match[u]];
      set_match(u, v);
      if (!xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
      if (u == 0) continue;
      if (vis[u] == t) return u;
      vis[u] = t;
      u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
      flo[b].push_back(x), flo[b].push_back(y = st[
          match[x]]), q_push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
```

```
      flo[b].push_back(x), flo[b].push_back(y = st[
          match[x]]), q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
        w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
      int xs = flo[b][i];
      for (int x = 1; x <= n_x; ++x)
        if (g[b][x].w == 0 || e_delta(g[xs][x]) <
            e_delta(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
      set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
         xr);
    for (int i = 0; i < pr; i += 2) {
      int xs = flo[b][i], xns = flo[b][i + 1];
      pa[xs] = g[xns][xs].u;
      S[xs] = 1, S[xns] = 0;
      slack[xs] = 0, set_slack(xns);
      q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
      int xs = flo[b][i];
      S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
      pa[v] = e.u, S[v] = 1;
      int nu = st[match[v]];
      slack[v] = slack[nu] = 0;
      S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
      int lca = get_lca(u, v);
      if (!lca) return augment(u,v), augment(v,u), true
          ;
      else add_blossom(u, lca, v);
    }
    return false;
  }
  bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
      if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
           q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
      while (q.size()) {
        int u = q.front(); q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 && st[u] != st[v]) {
            if (e_delta(g[u][v]) == 0) {
              if (on_found_edge(g[u][v])) return true;
            } else update_slack(u, st[v]);
          }
      }
      int d = inf;
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1) d = min(d, lab[b]
            / 2);
      for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && slack[x]) {
          if (S[x] == -1) d = min(d, e_delta(g[slack[x
              ]][x]));
          else if (S[x] == 0) d = min(d, e_delta(g[
              slack[x]][x]) / 2);
        }
      for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
          if (lab[u] <= d) return 0;
          lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
      }
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b) {
          if (S[st[b]] == 0) lab[b] += d * 2;
          else if (S[st[b]] == 1) lab[b] -= d * 2;
        }
      q = queue<int>();
      for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && slack[x] && st[slack[x]] != x
             && e_delta(g[slack[x]][x]) == 0)
          if (on_found_edge(g[slack[x]][x])) return
              true;
      for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
  }
  pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
      }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
      if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
  }
  void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
       g[vi][ui].w = wi; }
  void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
      for (int v = 1; v <= n; ++v)
        g[u][v] = edge(u, v, 0);
  }
};
```

3.8 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.

- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.

- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w

5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
6. T is a valid answer if the maximum flow $f < K|V|$

- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u, v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s, v) with capacity p_v; otherwise, create edge (v, t) with capacity $-p_v$.
  2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x\bar{y} + \sum_{xyx'y'} c_{xyx'y'}(x\bar{y} + x'\bar{y'})$$

  can be minimized by the mincut of the following graph:

  1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y.
  2. Create edge (x, y) with capacity c_{xy}.
  3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4 Graph

4.1 Heavy-Light Decomposition

```
vector<int> dep, pa, sz, ch, hd, id;
int _id;
void dfs(int i, int p) {
  dep[i] = ~p ? dep[p] + 1 : 0;
  pa[i] = p, sz[i] = 1, ch[i] = -1;
  for (int j : g[i])
    if (j != p) {
      dfs(j, i);
      if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
      sz[i] += sz[j];
    }
}
void hld(int i, int p, int h) {
  hd[i] = h;
  id[i] = _id++;
  if (~ch[i]) hld(ch[i], i, h);
  for (int j : g[i]) if (j != p && j != ch[i])
    hld(j, i, j);
}
void query(int i, int j) {
  while (hd[i] != hd[j]) {
    if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
    query2(id[hd[i]], id[i] + 1), i = pa[hd[i]];
  }
  if (dep[i] < dep[j]) swap(i, j);
  query2(id[j], id[i] + 1);
}
```

4.2 Centroid Decomposition

```
vector<vector<int>> dis;
vector<int> pa, sz;
vector<bool> vis;
void dfs_sz(int i, int p) {
  sz[i] = 1;
  for (int j : g[i]) if (j != p && !vis[j])
    dfs_sz(j, i), sz[i] += sz[j];
}
void cen(int i, int p, int _n) {
  for (int j : g[i]) if (j != p && !vis[j] && sz[j] >
      _n / 2)
    return cen(j, i, _n);
  return i;
}
void dfs_dis(int i, int p, int d) { // from i to
    ancestor with depth d
  dis[i][d] = ~p ? dis[p][d] + 1 : 0;
  for (int j : g[i]) if (j != p && !vis[j])
    dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
  dfs_sz(i), i = cen(i);
  vis[i] = true, pa[i] = p;
  dfs_dis(i, -1, d);
  for (int j : g[i]) if (!vis[j])
    cd(j, i, d + 1);
}
```

4.3 Edge BCC

```
vector<int> low, dep, bcc_id, stk;
vector<bool> vis;
int _id;
void dfs(int i, int p) {
  low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
  stk.push_back(i);
  vis[i] = true;
  for (int j : g[i])
    if (j != p) {
      if (!vis[j])
        dfs(j, i), low[i] = min(low[i], low[j]);
      else
        low[i] = min(low[i], dep[j]);
    }
  if (low[i] == dep[i]) {
    int id = _id++;
    while (stk.back() != i) {
      int x = stk.back();
      stk.pop_back();
      bcc_id[x] = id;
    }
    stk.pop_back();
    bcc_id[i] = id;
  }
}
```

4.4 Block Cut Tree

```
vector<vector<int>> g, _g;
vector<int> dep, low, stk;
void dfs(int i, int p) {
  dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
  stk.push_back(i);
  for (int j : g[i]) if (j != p) {
    if (dep[j] == -1) {
      dfs(j, i), low[i] = min(low[i], low[j]);
      if (low[j] >= dep[i]) {
        int id = _g.size();
        _g.emplace_back();
        while (stk.back() != j) {
          int x = stk.back();
          stk.pop_back();
          _g[x].push_back(id), _g[id].push_back(x);
        }
        stk.pop_back();
        _g[j].push_back(id), _g[id].push_back(j);
        _g[i].push_back(id), _g[id].push_back(i);
      }
    } else low[i] = min(low[i], dep[j]);
  }
}
```

4.5 SCC / 2SAT

```
struct SAT {
  vector<vector<int>> g;
  vector<int> dep, low, scc_id;
  vector<bool> is;
  vector<int> stk;
  int n, _id;
  SAT() {}
  void init(int _n) {
    n = _n, _id = 0;
    g.assign(2 * n, vector<int>());
    dep.assign(2 * n, -1), low.assign(2 * n, -1);
    scc_id.assign(2 * n, -1), is.assign(2 * n, false);
    stk.clear();
  }
  void add_edge(int x, int y) {g[x].push_back(y);}
  int rev(int i) {return i < n ? i + n : i - n;}
  void add_ifthen(int x, int y) {add_clause(rev(x), y)
    ;}
  void add_clause(int x, int y) {
    add_edge(rev(x), y);
    add_edge(rev(y), x);
  }
  void dfs(int i, int p) {
    dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    for (int j : g[i])
      if (j != p && scc_id[j] == -1) {
        if (dep[j] == -1)
          dfs(j, i);
        low[i] = min(low[i], low[j]);
      }
    if (low[i] == dep[i]) {
      int id = _id++;
      while (stk.back() != i) {
        int x = stk.back();
        stk.pop_back();
        scc_id[x] = id;
      }
      stk.pop_back();
      scc_id[i] = id;
    }
  }
  bool solve() {
    for (int i = 0; i < 2 * n; ++i)
      if (dep[i] == -1)
        dfs(i, -1);
    for (int i = 0; i < n; ++i) {
      if (scc_id[i] == scc_id[i + n]) return false;
      if (scc_id[i] < scc_id[i + n])
        is[i] = true;
      else
        is[i + n] = true;
    }
    return true;
  }
};
```

4.6 Negative Cycle

```
vector <pair <int, long long>> adj[N];
template <typename T>
struct NegativeCycle {
  vector <T> dis;
  vector <int> rt;
  int n; T INF;
  vector <int> cycle;
  NegativeCycle () = default;
  NegativeCycle (int _n) : n(_n), INF(numeric_limits<T
      >::max()) {
    dis.assign(n, 0), rt.assign(n, -1);
    int relax = -1;
    for (int t = 0; t < n; ++t) {
      relax = -1;
      for (int i = 0; i < n; ++i) {
        for (auto [j, w] : adj[i]) if (dis[j] > dis[i]
            + w) {
          dis[j] = dis[i] + w, rt[j] = i;
          relax = j;
        }
      }
    }
    if (relax != -1) {
      int s = relax;
      for (int i = 0; i < n; ++i) s = rt[s];
      vector <bool> vis(n, false);
      while (!vis[s]) {
        cycle.push_back(s), vis[s] = true;
        s = rt[s];
      }
      reverse(cycle.begin(), cycle.end());
    }
  }
};
```

4.7 Virtual Tree

```
vector<vector<int>> _g;
vector<int> st, ed, stk;
void solve(vector<int> v) {
  sort(all(v), [&](int x, int y) {return st[x] < st[y
      ];});
  int sz = v.size();
  for (int i = 0; i < sz - 1; ++i)
    v.push_back(lca(v[i], v[i + 1]));
  sort(all(v), [&](int x, int y) {return st[x] < st[y
      ];});
  v.resize(unique(all(v)) - v.begin());
  stk.clear(); stk.push_back(v[0]);
  for (int i = 1; i < v.size(); ++i) {
    int x = v[i];
    while (ed[stk.back()] < ed[x]) stk.pop_back();
    _g[stk.back()].push_back(x), stk.push_back(x);
  }
  // do something
  for (int i : v) _g[i].clear();
}
```

4.8 Directed MST

```
template <typename T> struct DMST { // 1-based
  T g[maxn][maxn], fw[maxn];
  int n, fr[maxn];
  bool vis[maxn], inc[maxn];
  void clear() {
    for (int i = 0; i < maxn; ++i) {
      for (int j = 0; j < maxn; ++j) g[i][j] = inf;
      vis[i] = inc[i] = false;
    }
  }
  void addedge(int u, int v, T w) {
    g[u][v] = min(g[u][v], w);
  }
  T query(int root, int _n) {
    n = _n;
    if (dfs(root) != n) return -1;
    T ans = 0;
    while (true) {
      for (int i = 1; i <= n; ++i) fw[i] = inf, fr[i] =
        i;
      for (int i = 1; i <= n; ++i) if (!inc[i]) {
          for (int j = 1; j <= n; ++j) {
            if (!inc[j] && i != j && g[j][i] < fw[i]) {
              fw[i] = g[j][i];
              fr[i] = j;
            }
          }
        }
      int x = -1;
      for (int i = 1; i <= n; ++i) if (i != root && !
        inc[i]) {
          int j = i, c = 0;
          while (j != root && fr[j] != i && c <= n) ++c
            , j = fr[j];
          if (j == root || c > n) continue;
          else { x = i; break; }
        }
      if (!~x) {
        for (int i = 1; i <= n; ++i) if (i != root && !
          inc[i]) ans += fw[i];
        return ans;
      }
      int y = x;
      for (int i = 1; i <= n; ++i) vis[i] = false;
      do { ans += fw[y]; y = fr[y]; vis[y] = inc[y] =
        true; } while (y != x);
      inc[x] = false;
      for (int k = 1; k <= n; ++k) if (vis[k]) {
          for (int j = 1; j <= n; ++j) if (!vis[j]) {
              if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
              if (g[j][k] < inf && g[j][k] - fw[k] < g[
                j][x]) g[j][x] = g[j][k] - fw[k];
            }
        }
    }
    return ans;
  }
  int dfs(int now) {
    int r = 1;
    vis[now] = true;
    for (int i = 1; i <= n; ++i) if (g[now][i] < inf &&
       !vis[i]) r += dfs(i);
    return r;
  }
};
```

4.9 Dominator Tree

```
struct Dominator_tree {
  int n, id;
  vector <vector <int>> adj, radj, bucket;
  vector <int> sdom, dom, vis, rev, par, rt, mn;
  Dominator_tree (int _n) : n(_n), id(0) {
    adj.resize(n), radj.resize(n), bucket.resize(n);
    sdom.resize(n), dom.resize(n, -1), vis.resize(n,
        -1);
    rev.resize(n), rt.resize(n), mn.resize(n), par.
        resize(n);
  }
  void add_edge(int u, int v) {adj[u].pb(v);}
  int query(int v, bool x) {
    if (rt[v] == v) return x ? -1 : v;
    int p = query(rt[v], true);
    if (p == -1) return x ? rt[v] : mn[v];
    if (sdom[mn[v]] > sdom[mn[rt[v]]]) mn[v] = mn[rt[v
        ]];
    rt[v] = p;
    return x ? p : mn[v];
  }
  void dfs(int v) {
    vis[v] = id, rev[id] = v;
    rt[id] = mn[id] = sdom[id] = id, id++;
    for (int u : adj[v]) {
      if (vis[u] == -1) dfs(u), par[vis[u]] = vis[v];
      radj[vis[u]].pb(vis[v]);
    }
  }
  void build(int s) {
    dfs(s);
    for (int i = id - 1; ~i; --i) {
      for (int u : radj[i]) {
        sdom[i] = min(sdom[i], sdom[query(u, false)]);
      }
      if (i) bucket[sdom[i]].pb(i);
      for (int u : bucket[i]) {
        int p = query(u, false);
        dom[u] = sdom[p] == i ? i : p;
      }
      if (i) rt[i] = par[i];
    }
    vector <int> res(n, -1);
    for (int i = 1; i < id; ++i) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    }
    for (int i = 1; i < id; ++i) res[rev[i]] = rev[dom[
        i]];
    res[s] = s;
    dom = res;
  }
};
```

5 String

5.1 Aho-Corasick Automaton

```
struct AC {
  int ch[N][26], to[N][26], fail[N], sz;
  vector <int> g[N];
  int cnt[N];
  AC () {sz = 0, extend();}
  void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
  int nxt(int u, int v) {
    if (!ch[u][v]) ch[u][v] = sz, extend();
    return ch[u][v];
  }
  int insert(string s) {
    int now = 0;
    for (char c : s) now = nxt(now, c - 'a');
    cnt[now]++;
    return now;
  }
  void build_fail() {
    queue <int> q;
    for (int i = 0; i < 26; ++i) if (ch[0][i]) {
        to[0][i] = ch[0][i];
        q.push(ch[0][i]);
        g[0].push_back(ch[0][i]);
      }
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (int j = 0; j < 26; ++j) {
        to[v][j] = ch[v][j] ? ch[v][j] : to[fail[v]][j
            ];
      }
      for (int i = 0; i < 26; ++i) if (ch[v][i]) {
        int u = ch[v][i], k = fail[v];
        while (k && !ch[k][i]) k = fail[k];
        if (ch[k][i]) k = ch[k][i];
        fail[u] = k;
        cnt[u] += cnt[k], g[k].push_back(u);
        q.push(u);
      }
    }
  }
  int match(string &s) {
    int now = 0, ans = 0;
    for (char c : s) {
      now = to[now][c - 'a'];
      if (ch[now][c - 'a']) now = ch[now][c - 'a'];
      ans += cnt[now];
    }
    return ans;
  }
};
```

5.2 KMP Algorithm

```
vector <int> build_fail(string s) {
  vector <int> f(s.length() + 1, 0);
  int k = 0;
  for (int i = 1; i < s.length(); ++i) {
    while (k && s[k] != s[i]) k = f[k];
    if (s[k] == s[i]) k++;
    f[i + 1] = k;
  }
  return f;
}
int match(string s, string t) {
  vector <int> f = build_fail(t);
  int k = 0, ans = 0;
  for (int i = 0; i < s.length(); ++i) {
    while (k && s[i] != t[k]) k = f[k];
    if (s[i] == t[k]) k++;
    if (k == t.length()) ans++, k = f[k];
  }
  return ans;
}
```

5.3 Z Algorithm

```
vector <int> build(string s) {
  int n = s.length();
  vector <int> Z(n);
  int l = 0, r = 0;
  for (int i = 0; i < n; ++i) {
    Z[i] = max(min(Z[i - l], r - i), 0);
    while (i + Z[i] < s.size() && s[Z[i]] == s[i + Z[i
        ]]) {
      l = i, r = i + Z[i], Z[i]++;
    }
  }
  return Z;
}
```

5.4 Manacher

```
vector <int> manacher(string &s) {
  string t = "^#";
  for (char c : s) t += c, t += '#';
  t += '&';
  int n = t.length();
  vector <int> r(n, 0);
  int C = 0, R = 0;
  for (int i = 1; i < n - 1; ++i) {
    int mirror = 2 * C - i;
    r[i] = (i < R ? min(r[mirror], R - i) : 0);
    while (t[i - 1 - r[i]] == t[i + 1 + r[i]]) r[i]++;
    if (i + r[i] > R) R = i + r[i], C = i;
  }
  return r;
}
```

5.5 Suffix Array

```
int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
  int *x = tmp[0], *y = tmp[1], m = 256, n = s.length()
      ;
  for (int i = 0; i < m; ++i) c[i] = 0;
  for (int i = 0; i < n; ++i) c[x[i] = s[i]]++;
  for (int i = 1; i < m; ++i) c[i] += c[i - 1];
  for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
  for (int k = 1; k < n; k <<= 1) {
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    int p = 0;
    for (int i = n - k; i < n; ++i) y[p++] = i;
    for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++]
        = sa[i] - k;
    for (int i = n - 1; ~i; --i) sa[--c[x[y[i]]]] = y[i
        ];
    y[sa[0]] = p = 0;
    for (int i = 1; i < n; ++i) {
      int a = sa[i], b = sa[i - 1];
      if (!(x[a] == x[b] && a + k < n && b + k < n && x
          [a + k] == x[b + k])) p++;
      y[sa[i]] = p;
    }
    if (n == p + 1) break;
    swap(x, y), m = p + 1;
  }
}
void buildLCP(string s) {
  // lcp[i] = LCP(sa[i - 1], sa[i])
  // lcp(i, j) = min(lcp[rk[i] + 1], lcp[rk[i] + 2],
      ..., lcp[rk[j]])
  int n = s.length(), val = 0;
  for (int i = 0; i < n; ++i) rk[sa[i]] = i;
  for (int i = 0; i < n; ++i) {
    if (!rk[i]) lcp[rk[i]] = 0;
    else {
      if (val) val--;
      int p = sa[rk[i] - 1];
      while (val + i < n && val + p < n && s[val + i]
          == s[val + p]) val++;
      lcp[rk[i]] = val;
    }
  }
}
```

5.6 Suffix Automaton

```
struct SAM {
  int ch[N][26], len[N], link[N], cnt[N], sz;
  SAM () {len[0] = 0, link[0] = -1, sz = 1;}
  void build(string s) {
    int last = 0;
    for (char c : s) {
      int cur = sz++;
      len[cur] = len[last] + 1;
      int p = last;
      while (~p && !ch[p][c - 'a']) ch[p][c - 'a'] =
          cur, p = link[p];
      if (p == -1) {
        link[cur] = 0;
      } else {
        int q = ch[p][c - 'a'];
        if (len[p] + 1 == len[q]) {
          link[cur] = q;
        } else {
          int nxt = sz++;
          len[nxt] = len[p] + 1, link[nxt] = link[q];
          for (int j = 0; j < 26; ++j) ch[nxt][j] = ch[
              q][j];
          while (~p && ch[p][c - 'a'] == q) ch[p][c - '
              a'] = nxt, p = link[p];
          link[q] = link[cur] = nxt;
        }
      }
      cnt[cur]++;
      last = cur;
    }
    vector <int> p(sz);
    iota(all(p), 0);
    sort(all(p), [&](int i, int j) {return len[i] > len
        [j];});
    for (int i = 0; i < sz; ++i) cnt[link[p[i]]] += cnt
        [p[i]];
  }
};
```

5.7 Minimum Rotation

```
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
  int k = 0;
  while (k < n && t[i + k] == t[j + k]) ++k;
    if (t[i + k] <= t[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return t.substr(pos, n);
}
```

5.8 Palindrome Tree

```
struct PAM {
  int ch[N][26], cnt[N], fail[N], len[N], sz;
  string s;
  // 0 -> even root, 1 -> odd root
  PAM (string _s) : s(_s) {
    sz = 0;
    extend(), extend();
    len[0] = 0, fail[0] = 1, len[1] = -1;
    int lst = 1;
    for (int i = 0; i < s.length(); ++i) {
      while (s[i - len[lst] - 1] != s[i]) lst = fail[
          lst];
      if (!ch[lst][s[i] - 'a'])  {
        int idx = extend();
        len[idx] = len[lst] + 2;
        int now = fail[lst];
        while (s[i - len[now] - 1] != s[i]) now = fail[
            now];
        fail[idx] = ch[now][s[i] - 'a'];
        ch[lst][s[i] - 'a'] = idx;
      }
      lst = ch[lst][s[i] - 'a'], cnt[lst]++;
    }
  }
  void build_count() {
    for (int i = sz - 1; i > 1; --i)
      cnt[fail[i]] += cnt[i];
  }
  int extend() {
    fill(ch[sz], ch[sz] + 26, 0), sz++;
    return sz - 1;
  }
};
```

6 Math

6.1 Fraction

```
struct fraction {
  ll n, d;
  fraction(const ll _n=0, const ll _d=1): n(_n), d(_d)
      {
    ll t = gcd(n, d);
    n /= t, d /= t;
    if (d < 0) n = -n, d = -d;
  }
  fraction operator-() const
  { return fraction(-n, d); }
  fraction operator+(const fraction &b) const
  { return fraction(n * b.d + b.n * d, d * b.d); }
  fraction operator-(const fraction &b) const
  { return fraction(n * b.d - b.n * d, d * b.d); }
  fraction operator*(const fraction &b) const
  { return fraction(n * b.n, d * b.d); }
  fraction operator/(const fraction &b) const
  { return fraction(n * b.d, d * b.n); }
  void print() {
    cout << n;
    if (d != 1) cout << "/" << d;
  }
};
```

6.2 Miller Rabin / Pollard Rho

```
ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long
    double)x / p * y) * p + p) % p;}
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
     1795265022};
ll Pow(ll a, ll b, ll n) {ll res = 1; for (; b; b >>=
    1, a = mul(a, a, n)) if (b & 1) res = mul(res, a, n
    ); return res;}
bool check(ll a, ll d, int s, ll n) {
  a = Pow(a, d, n);
  if (a <= 1) return 1;
  for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
    if (a == 1) return 0;
    if (a == n - 1) return 1;
  }
  return 0;
}
bool IsPrime(ll n) {
  if (n < 2) return 0;
  if (n % 2 == 0) return n == 2;
  ll d = n - 1, s = 0;
  while (d % 2 == 0) d >>= 1, ++s;
  for (ll i : chk) if (!check(i, d, s, n)) return 0;
  return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
  if (IsPrime(n)) return 1;
  for (ll p : small) if (n % p == 0) return p;
  ll x, y = 2, d, t = 1;
  auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
  for (int l = 2; ; l <<= 1) {
    x = y;
    int m = min(l, 32);
    for (int i = 0; i < l; i += m) {
      d = 1;
      for (int j = 0; j < m; ++j) {
        y = f(y), d = mul(d, abs(x - y), n);
      }
      ll g = __gcd(d, n);
      if (g == n) {
        l = 1, y = 2, ++t;
        break;
      }
      if (g != 1) return g;
    }
  }
}
map<ll, int> PollardRho(ll n) {
  map<ll, int> res;
  if (n == 1) return res;
  if (IsPrime(n)) return ++res[n], res;
  ll d = FindFactor(n);
  res = PollardRho(n / d);
  auto res2 = PollardRho(d);
  for (auto [x, y] : res2) res[x] += y;
  return res;
}
```

6.3 Ext GCD

```
//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
  pair<ll, ll> res;
  if (a < 0) {
    res = extgcd(-a, b);
    res.first *= -1;
    return res;
  }
  if (b < 0) {
    res = extgcd(a, -b);
    res.second *= -1;
    return res;
  }
  if (b == 0) return {1, 0};
  res = extgcd(b, a % b);
  return {res.second, res.first - res.second * (a / b)
      };
}
```

6.4 Linear Function Mod Min

```
ll topos(ll x, ll m) {x %= m; if (x < 0) x += m; return
     x;}
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    log m)
ll min_rem(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  for (ll g = __gcd(a, m); g > 1;) return g * min_rem(n
      , m / g, a / g, b / g) + (b % g);
  for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
      = nb) {
    if (a <= m - a) {
      nn = (a * (n - 1) + b) / m;
      if (!nn) break;
      nn += (b < a);
      nm = a, na = topos(-m, a);
      nb = b < a ? b : topos(b - m, a);
    } else {
      ll lst = b - (n - 1) * (m - a);
      if (lst >= 0) {b = lst; break;}
      nn = -(lst / m) + (lst % m < -a) + 1;
      nm = m - a, na = m % (m - a), nb = b % (m - a);
    }
  }
  return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
  //ax = (mn - b) (mod m)
  ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
      % (m / g);
  return {mn, x};
}
```

6.5 Floor Sum

```
// sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a
    + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
  ll ans = 0;
  if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
  if (b >= m) ans += n * (b / m), b %= m;
  ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
  if (y_max == 0) return ans;
  ans += (n - (x_max + a - 1) / a) * y_max;
  ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
  return ans;
}
```

6.6 Simplex

```
struct Simplex { // 0-based
  using T = long double;
  static const int N = 410, M = 30010;
  const T eps = 1e-7;
  int n, m;
  int Left[M], Down[N];
  // Ax <= b, max c^T x
  // result : v, xi = sol[i]. 1 based
  T a[M][N], b[M], c[N], v, sol[N];
  bool eq(T a, T b) {return fabs(a - b) < eps;}
  bool ls(T a, T b) {return a < b && !eq(a, b);}
  void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
    for (int i = 0; i < m; ++i) for (int j = 0; j < n;
        ++j) a[i][j] = 0;
    for (int i = 0; i < m; ++i) b[i] = 0;
    for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
  }
  void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector <int> nz;
    for (int i = 0; i < n; ++i) {
      a[x][i] /= k;
      if (!eq(a[x][i], 0)) nz.push_back(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
      if (i == x || eq(a[i][y], 0)) continue;
      k = a[i][y], a[i][y] = 0;
      b[i] -= k * b[x];
      for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
  }
  // 0: found solution, 1: no feasible solution, 2:
      unbounded
  int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (1) {
      int x = -1, y = -1;
      for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x
           == -1 || b[i] < b[x])) x = i;
      if (x == -1) break;
      for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) &&
           (y == -1 || a[x][i] < a[x][y])) y = i;
      if (y == -1) return 1;
      pivot(x, y);
    }
    while (1) {
      int x = -1, y = -1;
      for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y
           == -1 || c[i] > c[y])) y = i;
      if (y == -1) break;
      for (int i = 0; i < m; ++i) if (ls(0, a[i][y]) &&
           (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])
          ) x = i;
      if (x == -1) return 2;
      pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n) sol[
        Left[i]] = b[i];
    return 0;
  }
};
```

6.7 Linear Programming Construction

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y} \ge \mathbf{c}$ and $\mathbf{y} \ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^{m} A_{ji}\bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^{n} A_{ij}\bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.8 Theorem

- Kirchhoff's Theorem

  Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

  Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Cayley's Formula

  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each *labeled* vertices, there are
  $$\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$$
  spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős–Gallai Theorem

  A sequence of non-negative integers $d_1 \ge d_2 \ge \ldots \ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and
  $$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$
  holds for all $1 \le k \le n$.

- Burnside's Lemma

  Let X be a set and G be a group that acts on X. For $g \in G$, denote by X^g the elements fixed by g:
  $$X^g = \{x \in X \mid gx \in X\}$$
  Then
  $$|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|.$$

- Gale-Ryser theorem

  A pair of sequences of nonnegative integers $a_1 \ge \cdots \ge a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$.

- Fulkerson-Chen-Anstee theorem

  A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \ge \cdots \ge a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$.

- Möbius inversion formula

  - $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

- Spherical cap

  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

- Chinese Remainder Theorem

  - $x \equiv a_i \pmod{m_i}$
  - $M = \prod m_i, M_i = M/m_i$
  - $t_i M_i \equiv 1 \pmod{m_i}$
  - $x = \sum a_i t_i M_i \pmod{M}$

7 Geometry

7.1 Basic

```
int sign(double x) {return abs(x) <= eps ? 0 : (x > 0 ?
     1 : -1);}
struct Pt {
  double x, y;
  Pt (double _x, double _y) : x(_x), y(_y) {}
  Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
  Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
  Pt operator * (double k) {return Pt(x * k, y * k);}
  Pt operator / (double k) {return Pt (x / k, y / k);}
  double operator * (Pt o) {return x * o.x + y * o.y;}
  double operator ^ (Pt o) {return x * o.y - y * o.x;}
  double abs() {return hypot(x, y);}
};
int ori(Pt o, Pt a, Pt b) {return sign((o - a) ^ (o - b
    ));}
bool btw(Pt a, Pt b, Pt c) { // c on segment ab?
  return ori(a, b, c) == 0 && sign((c - a) * (c - b))
      <= 0;
}
double area(Pt a, Pt b, Pt c) {return abs((a - b) ^ (a
    - c)) / 2;}
Pt proj_vector(Pt a, Pt b, Pt c) { // vector ac proj to
     ab
  return (b - a) * ((c - a) * (b - a)) / ((b - a) * (b
      - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
  return proj_vector(a, b, c) + a;
}
```

7.2 Segment Intersection

```
bool banana(Pt a, Pt b, Pt c, Pt d) { // segment ab and
     cd
  if (btw(a, b, c) || btw(a, b, d) || btw(c, d, a) ||
      btw(c, d, b)) return true;
  return ori(a, b, c) * ori(a, b, d) == -1 && ori(c, d,
       a) * ori(c, d, b) == -1;
}
Pt intersect(Pt a, Pt b, Pt c, Pt d) { // segment ab
    and cd
  double abc = (b - a) ^ (c - a);
  double abd = (b - a) ^ (d - a);
  if (sign(abc - abd) == 0) return d;
  return (d * abc - c * abd) / (abc - abd);
}
```

7.3 Convex Hull

```
vector <Pt> ConvexHull(vector <Pt> pt) {
  int n = pt.size();
  sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.
      y < b.y : a.x < b.x;});
  vector <Pt> ans = {pt[0]};
  for (int t : {0, 1}) {
    int m = ans.size();
    for (int i = 1; i < n; ++i) {
      while (ans.size() > m && ori(ans[ans.size() - 2],
           ans.back(), pt[i]) <= 0)
        ans.pop_back();
      ans.push_back(pt[i]);
    }
    reverse(all(pt));
  }
  ans.pop_back();
  return ans;
}
```

7.4 PolarAngle Sort

```
void PolarAngleSort(vector <Pt> &pts) {
  auto pos = [&](Pt a) {return sign(a.y) == 0 ?  sign(a
      .x) < 0 : sign(a.y) > 0;};
  sort(all(pts), [&](Pt a, Pt b) {return pos(a) == pos(
      b) ? sign(a ^ b) > 0 : pos(a) < pos(b);});
}
```

7.5 Rotating Caliper

```
void RotatingCaliper(vector <Pt> &pts) {
  int n = pts.size();
  for (int i = 0, j = 2; i < n; ++i) {
    int ni = (i + 1) % n;
    while (true) {
      int nj = (j + 1) % n;
      if (area(pts[j], pts[i], pts[ni]) < area(pts[nj],
           pts[i], pts[ni])) {
        j = nj;
      } else {
        break;
      }
    }
    // do something
  }
}
```

7.6 Rotating SweepLine

```
void RotatingSweepLine(vector <Pt> &pt) {
  int n = pt.size();
  vector <int> id(n), pos(n);
  vector <pair <int, int>> line;
  for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++
      j) if (i ^ j) line.emplace_back(i, j);
  sort(line.begin(), line.end(), [&](pair <int, int> i,
       pair <int, int> j) {
    Pt a = pt[i.second] - pt[i.first], b = pt[j.second]
         - pt[j.first];
    return (a.pos() == b.pos() ? sign(a ^ b) > 0 : a.
        pos() < b.pos());
  });
  iota(id.begin(), id.end(), 0);
  sort(id.begin(), id.end(), [&](int i, int j) {
    return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt
        [j].x : pt[i].y < pt[j].y);
  });
  for (int i = 0; i < n; ++i)
    pos[id[i]] = i;
  for (auto [i, j] : line) {
    // point sort by the distance to line(i, j)
    // do something.
    tie(pos[i], pos[j], id[pos[i]], id[pos[j]]) =
        make_tuple(pos[j], pos[i], j, i);
  }
}
```

7.7 Half Plane Intersection

```
vector <Pt> HalfPlaneInter(vector <pair <Pt, Pt>> vec)
    {
  //          x
  // first -------> second
  auto pos = [&](Pt a) {return sign(a.y) == 0 ?  sign(a
      .x) < 0 : sign(a.y) > 0;};
  sort(all(vec), [&](pair <Pt, Pt> a, pair <Pt, Pt> b)
      {
    Pt A = a.second - a.first, B = b.second - b.first;
    if (pos(A) == pos(B)) {
      if (sign(A ^ B) == 0) return sign((b.first - a.
          first) * (b.second - a.first)) > 0;
      return sign(A ^ B) > 0;
    }
    return pos(A) < pos(B);
  });
  deque <Pt> inter;
  deque <pair <Pt, Pt>> seg;
  int n = vec.size();
  auto get = [&](pair <Pt, Pt> a, pair <Pt, Pt> b) {
      return intersect(a.first, a.second, b.first, b.
      second);};
  for (int i = 0; i < n; ++i) if (!i || vec[i] != vec[i
       - 1]) {
    while (seg.size() >= 2 && sign((vec[i].second -
        inter.back()) ^ (vec[i].first - inter.back()))
        == 1) seg.pop_back(), inter.pop_back();
    while (seg.size() >= 2 && sign((vec[i].second -
        inter.front()) ^ (vec[i].first - inter.front())
        ) == 1) seg.pop_front(), inter.pop_front();
    seg.push_back(vec[i]);
    if (seg.size() >= 2) inter.pb(get(seg[seg.size() -
        2], seg.back()));
  }
  while (seg.size() >= 2 && sign((seg.front().second -
      inter.back()) ^ (seg.front().first - inter.back()
      )) == 1) seg.pop_back(), inter.pop_back();
  inter.push_back(get(seg.front(), seg.back()));
  return vector <Pt>(all(inter));
}
```

7.8 Minkowski Sum

```
vector <Pt> Minkowski(vector <Pt> a, vector <Pt> b) {
  a = ConvexHull(a), b = ConvexHull(b);
  int n = a.size(), m = b.size();
  vector <Pt> c = {a[0] + b[0]}, s1, s2;
  for(int i = 0; i < n; ++i)
    s1.pb(a[(i + 1) % n] - a[i]);
  for(int i = 0; i < m; i++)
    s2.pb(b[(i + 1) % m] - b[i]);
  for(int p1 = 0, p2 = 0; p1 < n || p2 < m;)
    if (p2 == m || (p1 < n && sign(s1[p1] ^ s2[p2]) >=
        0))
      c.pb(c.back() + s1[p1++]);
    else
      c.pb(c.back() + s2[p2++]);
  return ConvexHull(c);
}
```

8 Polynomial

8.1 Number Theoretic Transform

```
const int N = 1 << 20, mod = 998244353, G = 3;
void run (vector <ll> &P, bool inv = false) {
    int N = P.size();
    const ll w = modpow(G, (mod - 1) / N);
    int lg = __lg(N);
    vector <int> rev(N);
    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (lg -
             1));
        if (i < rev[i])
            swap(P[i], P[rev[i]]);
    }
    vector <ll> ws = {inv ? modpow(w, mod - 2) : w};
    for (int i = 1; i < lg; ++i) ws.push_back(ws.back()
         * ws.back() % mod);
    reverse(ws.begin(), ws.end());
    for (int i = 0; i < lg; ++i) {
        for (int k = 0; k < N; k += (2 << i)) {
            ll base = 1;
            for (int j = k; j < k + (1 << i); ++j, base
                 = base * ws[i] % mod) {
                ll t = base * P[j + (1 << i)] % mod, u
                     = P[j];
                P[j] = u + t, P[j + (1 << i)] = u - t;
                if (P[j] >= mod) P[j] -= mod;
                if (P[j + (1 << i)] < 0) P[j + (1 << i)
                    ] += mod;
            }
        }
    }
    if (inv) {
        ll ninv = modpow(N, mod - 2);
        for (int i = 0; i < N; ++i) {
            P[i] = P[i] * ninv % mod;
        }
    }
}
```

8.2 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

8.3 Fast Walsh Transform

```
void fwt(vector <int> &a) {
  // and : a[j] += x;
  //     : a[j] -= x;
  // or  : a[j ^ (1 << i)] += y;
  //     : a[j ^ (1 << i)] -= y;
  // xor : a[j] = x - y, a[j ^ (1 << i)] = x + y;
  //     : a[j] = (x - y) / 2, a[j ^ (1 << i)] = (x + y
      ) / 2;
  int n = __lg(a.size());
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
      int x = a[j ^ (1 << i)], y = a[j];
      // do something
    }
  }
}
```

9 Else

9.1 Bit Hack

```
long long next_perm(long long v) {
  long long t = v | (v - 1);
  return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(
       v) + 1));
}
void subset(long long s) {
  long long sub = s;
  while (sub) sub = (sub - 1) & s;
}
```

9.2 Hilbert Curve

```
long long hilbertOrder(int x, int y, int pow, int
    rotate) {
  if (pow == 0) return 0;
  int hpow = 1 << (pow-1);
  int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y <
       hpow) ? 1 : 2);
  seg = (seg + rotate) & 3;
  const int rotateDelta[4] = {3, 0, 0, 1};
  int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
  int nrot = (rotate + rotateDelta[seg]) & 3;
  long long subSquareSize = 1ll << (pow * 2 - 2);
  long long ans = seg * subSquareSize;
  long long add = hilbertOrder(nx, ny, pow - 1, nrot);
  ans += (seg == 1 || seg == 2) ? add : (subSquareSize
       - add - 1);
  return ans;
}
```

9.3 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
int main () {
   __gnu_pbds::priority_queue <int> pq1, pq2;
  pq1.join(pq2); // pq1 += pq2, pq2 = {}
  cc_hash_table<int, int> m1;
  tree<int, null_type, less<int>, rb_tree_tag,
       tree_order_statistics_node_update> oset;
  oset.insert(2), oset.insert(4);
  cout << *oset.find_by_order(1) << ' ' << oset.
       order_of_key(1) << '\n'; // 4 0
  bitset <100> BS;
  BS.flip(3), BS.flip(5);
  cout << BS._Find_first() << ' ' << BS._Find_next(3)
       << '\n'; // 3 5
  rope <int> rp1, rp2;
  rp1.push_back(1), rp1.push_back(3);
  rp1.insert(0, 2); // pos, num
  rp1.erase(0, 2); // pos, len
  rp1.substr(0, 2); // pos, len
  rp2.push_back(4);
  rp1 += rp2, rp2 = rp1;
  cout << rp2[0] << ' ' << rp2[1] << '\n'; // 3 4
}
```

9.4 Random

```
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t a) const {
    static const uint64_t FIXED_RANDOM = chrono::
        steady_clock::now().time_since_epoch().count();
    return splitmix64(i + FIXED_RANDOM);
  }
};
unordered_map <int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());
```

9.5 Mo's Algorithm

```
struct MoSolver {
  struct query {
    int l, r, id;
    bool operator < (const query &o) {
      if (l / C == o.l / C) return (l / C) & 1 ? r > o.
          r : r < o.r;
      return l / C < o.l / C;
    }
  };
  int cur_ans;
  vector <int> ans;
  void add(int x) {
    // do something
  }
  void sub(int x) {
    // do something
  }
  vector <query> Q;
  void add_query(int l, int r, int id) {
    // [l, r)
    Q.push_back({l, r, id});
    ans.push_back(0);
  }
  void run() {
    sort(Q.begin(), Q.end());
    int pl = 0, pr = 0;
    cur_ans = 0;
    for (query &i : Q) {
      while (pl > i.l)
        add(a[--pl]);
      while (pr < i.r)
        add(a[pr++]);
      while (pl < i.l)
        sub(a[pl++]);
      while (pr > i.r)
        sub(a[--pr]);
      ans[i.id] = cur;
    }
  }
};
```

9.6 Smawk Algorithm

```
ll query(int l, int r) {
  // ...
}
struct SMAWK {
  // Condition:
  // If M[1][0] < M[1][1] then M[0][0] < M[0][1]
  // If M[1][0] == M[1][1] then M[0][0] <= M[0][1]
  // For all i, find r_i s.t. M[i][r_i] is maximum ||
      minimum.
  int ans[N], tmp[N];
  void interpolate(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    vector <int> nl;
    for (int i = 1; i < n; i += 2) {
      nl.push_back(l[i]);
    }
    run(nl, r);
    for (int i = 1, j = 0; i < n; i += 2) {
      while (j < m && r[j] < ans[l[i]])
        j++;
      assert(j < m && ans[l[i]] == r[j]);
      tmp[l[i]] = j;
    }
    for (int i = 0; i < n; i += 2) {
      int curl = 0, curr = m - 1;
      if (i)
        curl = tmp[l[i - 1]];
      if (i + 1 < n)
        curr = tmp[l[i + 1]];
      ll res = query(l[i], r[curl]);
      ans[l[i]] = r[curl];
      for (int j = curl + 1; j <= curr; ++j) {
        ll nxt = query(l[i], r[j]);
        if (res < nxt)
          res = nxt, ans[l[i]] = r[j];
      }
    }
  }
  void reduce(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    vector <int> nr;
    for (int j : r) {
      while (!nr.empty()) {
        int i = nr.size() - 1;
        if (query(l[i], nr.back()) <= query(l[i], j))
          nr.pop_back();
        else
          break;
      }
      if (nr.size() < n)
        nr.push_back(j);
    }
    run(l, nr);
  }
  void run(vector <int> l, vector <int> r) {
    int n = l.size(), m = r.size();
    if (max(n, m) <= 2) {
      for (int i : l) {
        ans[i] = r[0];
        if (m > 1) {
          if (query(i, r[0]) < query(i, r[1]))
            ans[i] = r[1];
        }
      }
    } else if (n >= m) {
      interpolate(l, r);
    } else {
      reduce(l, r);
    }
  }
};
```

9.7 Two Dimension Add Sum

```
struct TwoDimensionAddAndSum {
  // 0-index, [l, r)
  struct Seg {
    int l, r, m;
    ll vala, valb, lza, lzb;
    Seg* ch[2];
    Seg (int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
         vala(0), valb(0), lza(0), lzb(0) {
      if (r - l > 1) {
        ch[0] = new Seg(l, m);
        ch[1] = new Seg(m, r);
      }
    }
    void pull() {vala = ch[0]->vala + ch[1]->vala, valb
        = ch[0]->valb + ch[1]->valb;}
    void give(ll a, ll b) {
      lza += a, lzb += b;
      vala += a * (r - l), valb += b * (r - l);
    }
    void push() {
      ch[0]->give(lza, lzb), ch[1]->give(lza, lzb), lza
          = lzb = 0;
    }
    void add(int a, int b, ll va, ll vb) {
      if (a <= l && r <= b)
        give(va, vb);
      else {
        push();
        if (a < m) ch[0]->add(a, b, va, vb);
        if (m < b) ch[1]->add(a, b, va, vb);
        pull();
      }
    }
    long long query(int a, int b, int v) {
      if (a <= l && r <= b) return vala * v + valb;
      push();
      long long ans = 0;
      if (a < m) ans += ch[0]->query(a, b, v);
      if (m < b) ans += ch[1]->query(a, b, v);
      return ans;
    }
  };
  // note integer overflow.
  vector <array <int, 4>> E[N];
  vector <array <int, 4>> Q[N];
  vector <ll> ans;
  void add_event(int x1, int y1, int x2, int y2, ll v)
      {
    E[x1].pb({y1, y2,  v, -v * x1});
    E[x2].pb({y1, y2, -v, v * x2});
  }
  void add_query(int x1, int y1, int x2, int y2, int id
      ) {
    Q[x1].pb({y1, y2, -1, id});
    Q[x2].pb({y1, y2, 1, id});
    ans.pb(0);
  }
  void solve(int n) {
    Seg root(0, n);
    for (int i = 0; i <= n; ++i) {
      for (auto j : E[i]) root.add(j[0], j[1], j[2], j
          [3]);
      for (auto j : Q[i]) ans[j[3]] += j[2] * root.
          query(j[0], j[1], i);
    }
  }
};
```

9.8 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S. Otherwise for each $x \in S, y \notin S$, create edges

- $x \to y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \to x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2, and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.