Contents

1 Basic

1.1 Shell Script

```
cpp hash.cpp -dD -P -fpreprocessed | tr -d "[:space:]"
    | md5sum | cut -c -6
```

1.2 Debug Macro* [2e0e48]

```
#ifdef ABS
template <typename T>
ostream& operator << (ostream &o, vector <T> vec) {
    o << "{"; int f = 0;
    for (T i : vec) o << (f++ ? " " : "") << i;
    return o << "}"; }
void bug__(int c, auto ...a) {
    cerr << "\e[1;" << c << "m";
    (..., (cerr << a << " "));
    cerr << "\e[0m" << endl; }
#define bug_(c, x...) bug__(c, __LINE__, "[" + string(#
    x) + "]", x)
#define bug(x...) bug_(32, x)
#define bugv(x...) bug_(36, vector(x))
#define safe bug_(33, "safe")
#else
#define bug(x...) void(0)
#define bugv(x...) void(0)
#define safe void(0)
#endif
```

1.3 Pragma / FastIO

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr()|0x8040)
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
  static char buf[65536], *p = buf, *q = buf;
  return p == q && (q = (p = buf) + read(0, buf, 65536)
      ) == buf ? -1 : *p++;
}
inline int R() {
  static char c;
  while((c = RC()) < '0'); int a = c ^ '0';
  while((c = RC()) >= '0') a *= 10, a += c ^ '0';
  return a;
}
inline void W(int n) {
  static char buf[12], p;
  if (n == 0) OB[OP++]='0'; p = 0;
  while (n) buf[p++] = '0' + (n % 10), n /= 10;
  for (--p; p >= 0; --p) OB[OP++] = buf[p];
  if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.4 Divide

```
ll floor(ll a, ll b) {return a / b - (a < 0 && a % b);}
ll ceil(ll a, ll b)  {return a / b + (a > 0 && a % b);}
a / b <  x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x <  a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)
```

2 Data Structure

2.1 Leftist Tree [75d338]

```
// max heap
struct node {
  ll rk, data, size, sum;
  node *l, *r;
  node(ll k) : rk(0), data(k), size(1), sum(k), l(0), r
      (0) {}
};
#undef sz
ll sz(node *p) { return p ? p->size : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (a->data < b->data) swap(a, b);
  a->r = merge(a->r, b);
  if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
  a->rk = rk(a->r) + 1;
  a->size = sz(a->l) + sz(a->r) + 1;
  a->sum = sum(a->l) + sum(a->r) + a->data;
  return a;
}
void pop(node *&o) {
  node *tmp = o;
  o = merge(o->l, o->r);
  delete tmp;
}
```

2.2 Splay Tree [21142b]

```
struct Splay {
  int pa[N], ch[N][2], sz[N], rt, _id;
  ll v[N];
  Splay() {}
  void init() {
```

```cpp
    rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
    sz[0] = 1, v[0] = inf;
  }
  int newnode(int p, int x) {
    int id = _id++;
    v[id] = x, pa[id] = p;
    ch[id][0] = ch[id][1] = -1, sz[id] = 1;
    return id;
  }
  void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int gp = pa[p], c = ch[i][!x];
    sz[p] -= sz[i], sz[i] += sz[p];
    if (~c) sz[p] += sz[c], pa[c] = p;
    ch[p][x] = c, pa[p] = i;
    pa[i] = gp, ch[i][!x] = p;
    if (~gp) ch[gp][ch[gp][1] == p] = i;
  }
  void splay(int i) {
    while (~pa[i]) {
      int p = pa[i];
      if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
          == i ? i : p);
      rotate(i);
    }
    rt = i;
  }
  int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
      if (v[i] == x) return splay(i), i;
      if (v[i] > x) {
        last = i;
        if (ch[i][0] == -1) break;
        i = ch[i][0];
      }
      else {
        if (ch[i][1] == -1) break;
        i = ch[i][1];
      }
    }
    splay(i);
    return last; // -1 if not found
  }
  void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
      // assert(ch[rt][1] == -1);
      int id = newnode(rt, x);
      ch[rt][1] = id, ++sz[rt];
      splay(id);
    }
    else if (v[i] != x) {
      splay(i);
      int id = newnode(rt, x), c = ch[rt][0];
      ch[rt][0] = id;
      ch[id][0] = c;
      if (~c) pa[c] = id, sz[id] += sz[c];
      ++sz[rt];
      splay(id);
    }
  }
};
```

2.3 Link Cut Tree [d01a7d]

```cpp
// weighted subtree size, weighted path max
struct LCT {
  int ch[N][2], pa[N], v[N], sz[N];
  int sz2[N], w[N], mx[N], _id;
  // sz := sum of v in splay, sz2 := sum of v in
      virtual subtree
  // mx := max w in splay
  bool rev[N];
  LCT() : _id(1) {}
  int newnode(int _v, int _w) {
    int x = _id++;
    ch[x][0] = ch[x][1] = pa[x] = 0;
    v[x] = sz[x] = _v;
    sz2[x] = 0;
    w[x] = mx[x] = _w;
    rev[x] = false;
    return x;
  }
  void pull(int i) {
    sz[i] = v[i] + sz2[i];
    mx[i] = w[i];
    if (ch[i][0]) {
      sz[i] += sz[ch[i][0]];
      mx[i] = max(mx[i], mx[ch[i][0]]);
    }
    if (ch[i][1]) {
      sz[i] += sz[ch[i][1]];
      mx[i] = max(mx[i], mx[ch[i][1]]);
    }
  }
  void push(int i) {
    if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
        rev[i] = false;
  }
  void reverse(int i) {
    if (!i) return;
    swap(ch[i][0], ch[i][1]);
    rev[i] ^= true;
  }
  bool isrt(int i) {// rt of splay
    if (!pa[i]) return true;
    return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
  }
  void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i;
    int c = ch[i][!x], gp = pa[p];
    if (ch[gp][0] == p) ch[gp][0] = i;
    else if (ch[gp][1] == p) ch[gp][1] = i;
    pa[i] = gp, ch[i][!x] = p, pa[p] = i;
    ch[p][x] = c, pa[c] = p;
    pull(p), pull(i);
  }
  void splay(int i) {
    vector<int> anc;
    anc.pb(i);
    while (!isrt(anc.back()))
      anc.pb(pa[anc.back()]);
    while (!anc.empty())
      push(anc.back()), anc.pop_back();
    while (!isrt(i)) {
      int p = pa[i];
      if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
          == p ? i : p);
      rotate(i);
    }
  }
  void access(int i) {
    int last = 0;
    while (i) {
      splay(i);
      if (ch[i][1])
        sz2[i] += sz[ch[i][1]];
      sz2[i] -= sz[last];
      ch[i][1] = last;
      pull(i), last = i, i = pa[i];
    }
  }
  void makert(int i) {
    access(i), splay(i), reverse(i);
  }
  void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
  }
  void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
  }
  int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
```

```
  }
};
```

2.4 Treap [fbf3b7]

```
struct node {
  int data, size;
  node *l, *r;
  node(int k) : data(k), size(1), l(0), r(0) {}
  void up() {
    size = 1;
    if (l) size += l->size;
    if (r) size += r->size;
  }
  void down() {}
};
#undef sz
int sz(node *a) { return a ? a->size : 0; }
node *merge(node *a, node *b) {
  if (!a || !b) return a ? a : b;
  if (rand() % (sz(a) + sz(b)) < sz(a))
    return a->down(), a->r = merge(a->r, b), a->up(),a;
  return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
  if (!o) return a = b = 0, void();
  o->down();
  if (o->data <= k)
    a = o, split(o->r, a->r, b, k), a->up();
  else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
  if (sz(o) <= k) return a = o, b = 0, void();
  o->down();
  if (sz(o->l) + 1 <= k)
    a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
  else b = o, split2(o->l, a, b->l, k);
  o->up();
}
node *kth(node *o, int k) {
  if (k <= sz(o->l)) return kth(o->l, k);
  if (k == sz(o->l) + 1) return o;
  return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
  if (!o) return 0;
  if (o->data < key)
    return sz(o->l) + 1 + Rank(o->r, key);
  else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
  if (!o) return 0;
  if (o->data == k) {
    node *t = o;
    o->down(), o = merge(o->l, o->r);
    delete t;
    return 1;
  }
  node *&t = k < o->data ? o->l : o->r;
  return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
  node *a, *b;
  o->down(), split(o, a, b, k),
  o = merge(a, merge(new node(k), b));
  o->up();
}
void interval(node *&o, int l, int r) {
  node *a, *b, *c; // [l, r)
  o->down();
  split2(o, a, b, l), split2(b, b, c, r - l);
  // operate
  o = merge(a, merge(b, c)), o->up();
}
```

2.5 vEB Tree [087d11]

```
using u64 = uint64_t;
constexpr int lsb(u64 x)
{ return x ? __builtin_ctzll(x) : 1 << 30; }
constexpr int msb(u64 x)
{ return x ? 63 - __builtin_clzll(x) : -1; }
template<int N, class T = void>
struct veb {
  static const int M = N >> 1;
  veb<M> ch[1 << N - M];
  veb<N - M> aux;
  int mn, mx;
  veb() : mn(1 << 30), mx(-1) {}
  constexpr int mask(int x) { return x & ((1 << M) - 1)
      ; }
  bool empty() { return mx == -1; }
  int min() { return mn; }
  int max() { return mx; }
  bool have(int x)
  { return x == mn ? true : ch[x >> M].have(mask(x)); }
  void insert_in(int x) {
    if (empty()) return mn = mx = x, void();
    if (x < mn) swap(x, mn);
    if (x > mx) mx = x;
    if (ch[x >> M].empty()) aux.insert_in(x >> M);
    ch[x >> M].insert_in(mask(x));
  }
  void erase_in(int x) {
    if (mn == mx) return mn = 1 << 30, mx = -1, void();
    if (x == mn)
      mn = x = (aux.min() << M) ^ ch[aux.min()].min();
    ch[x >> M].erase_in(mask(x));
    if (ch[x >> M].empty()) aux.erase_in(x >> M);
    if (x == mx) {
      if (aux.empty()) mx = mn;
      else mx = (aux.max() << M) ^ ch[aux.max()].max();
    }
  } // 06a669
  void insert(int x) {
    if (!have(x)) insert_in(x); }
  void erase(int x) {
      if (have(x)) erase_in(x); }
  int next(int x) { // >= x
    if (x > mx) return 1 << 30;
    if (x <= mn) return mn;
    if (mask(x) <= ch[x >> M].max()) return ((x >> M)
        << M) ^ ch[x >> M].next(mask(x));
    int y = aux.next((x >> M) + 1);
    return (y << M) ^ ch[y].min();
  }
  int prev(int x) { // < x
    if (x <= mn) return -1;
    if (x > mx) return mx;
    if (x <= (aux.min() << M) + ch[aux.min()].min())
      return mn;
    if (mask(x) > ch[x >> M].min()) return ((x >> M) <<
         M) ^ ch[x >> M].prev(mask(x));
    int y = aux.prev(x >> M);
    return (y << M) ^ ch[y].max();
  }
};
template <int N>
struct veb <N, typename enable_if<N <= 6>::type> {
  u64 a;
  veb() : a(0) {}
  void insert_in(int x) { a |= 1ull << x; }
  void insert(int x) { a |= 1ull << x; }
  void erase_in(int x) { a &= ~(1ull << x); }
  void erase(int x) { a &= ~(1ull << x); }
  bool have(int x) { return a >> x & 1; }
  bool empty() { return a == 0; }
  int min() { return lsb(a); }
  int max() { return msb(a); }
  int next(int x) {return lsb(a & ~((1ull << x) - 1));}
  int prev(int x) {return msb(a & ((1ull << x) - 1));}
}; // e36c96
```

3 Flow / Matching

3.1 Dinic [b68676]

```
template <typename T>
struct Dinic { // 0-based
  const T INF = numeric_limits<T>::max() / 2;
  struct edge { int to, rev; T cap, flow; };
  int n, s, t;
  vector <vector <edge>> g;
  vector <int> dis, cur;
```

```
  T dfs(int u, T cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < sz(g[u]); ++i) {
      edge &e = g[u][i];
      if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
        T df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df;
          g[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    dis.assign(n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (auto &u : g[v])
        if (dis[u.to] == -1 && u.flow != u.cap) {
          q.push(u.to);
          dis[u.to] = dis[v] + 1;
        }
    }
    return dis[t] != -1;
  }
  T solve(int _s, int _t) {
    s = _s, t = _t;
    T flow = 0, df;
    while (bfs()) {
      cur.assign(n, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  void add_edge(int u, int v, T cap) {
    g[u].pb(edge{v, sz(g[v]), cap, 0});
    g[v].pb(edge{u, sz(g[u]) - 1, 0, 0});
  }
  Dinic (int _n) : n(_n), g(n) {}
//void reset() {
//  for (int i = 0; i < n; ++i)
//    for (auto &j : g[i]) j.flow = 0;
//}
};
```

3.2 Min Cost Max Flow [92f08e]

```
template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
  const T1 INF1 = numeric_limits<T1>::max() / 2;
  const T2 INF2 = numeric_limits<T2>::max() / 2;
  struct edge { int v; T1 f; T2 c; };
  int n, s, t;
  vector <vector <int>> g;
  vector <edge> e;
  vector <T2> dis, pot;
  vector <int> rt, vis;
  // bool DAG()...
  bool SPFA() {
    rt.assign(n, -1), dis.assign(n, INF2);
    vis.assign(n, false);
    queue <int> q;
    q.push(s), dis[s] = 0, vis[s] = true;
    while (!q.empty()) {
      int v = q.front(); q.pop();
      vis[v] = false;
      for (int id : g[v]) {
        auto [u, f, c] = e[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
          dis[u] = ndis, rt[u] = id;
          if (!vis[u]) vis[u] = true, q.push(u);
        }
      }
    }
    return dis[t] != INF2;
  } // df1862
  bool dijkstra() {
    rt.assign(n, -1), dis.assign(n, INF2);
    priority_queue <pair <T2, int>, vector <pair <T2,
      int>>, greater <pair <T2, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
      auto [d, v] = pq.top(); pq.pop();
      if (dis[v] < d) continue;
      for (int id : g[v]) {
        auto [u, f, c] = e[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
          dis[u] = ndis, rt[u] = id;
          pq.emplace(ndis, u);
        }
      }
    }
    return dis[t] != INF2;
  } // d46baf
  vector <pair <T1, T2>> solve(int _s, int _t) {
    s = _s, t = _t, pot.assign(n, 0);
    vector <pair <T1, T2>> ans; bool fr = true;
    while ((fr ? SPFA() : SPFA())) {
      for (int i = 0; i < n; ++i)
        dis[i] += pot[i] - pot[s];
      T1 add = INF1;
      for (int i = t; i != s; i = e[rt[i] ^ 1].v)
        add = min(add, e[rt[i]].f);
      for (int i = t; i != s; i = e[rt[i] ^ 1].v)
        e[rt[i]].f -= add, e[rt[i] ^ 1].f += add;
      ans.emplace_back(add, dis[t]), fr = false;
      for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return ans;
  }
  void add_edge(int u, int v, T1 f, T2 c) {
    g[u].pb(sz(e)), e.pb({v, f, c});
    g[v].pb(sz(e)), e.pb({u, 0, -c});
  }
  MCMF (int _n) : n(_n), g(n), e() {}
//void reset() {
//  for (int i = 0; i < sz(e); ++i) e[i].f = 0;
//}
}; // 383274
```

3.3 Kuhn Munkres [7f3209]

```
template <typename T> // maximum perfect matching
struct KM { // 0-based, remember to init edge weight
  const T INF = numeric_limits<T>::max() / 2;
  int n; vector <vector <T>> w;
  vector <T> hl, hr, slk;
  vector <int> fl, fr, vl, vr, pre;
  queue <int> q;
  bool check(int x) {
    if (vl[x] = 1, ~fl[x])
      return q.push(fl[x]), vr[fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    vl.assign(n, 0), vr.assign(n, 0);
    slk.assign(n, INF), pre.assign(n, 0);
    while (!q.empty()) q.pop();
    q.push(s), vr[s] = 1;
    while (true) {
      T d;
      while (!q.empty()) {
        int y = q.front(); q.pop();
        for (int x = 0; x < n; ++x) {
          d = hl[x] + hr[y] - w[x][y];
          if (!vl[x] && slk[x] >= d) {
            if (pre[x] = y, d) slk[x] = d;
            else if (!check(x)) return;
          }
        }
      }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
```

```
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
    }
  }
  T solve() {
    fl.assign(n, -1), fr.assign(n, -1);
    hl.assign(n, 0), hr.assign(n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(all(w[i]));
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
  void add_edge(int a, int b, T wei) { w[a][b] = wei; }
  KM (int _n) : n(_n), w(n, vector<T>(n, -INF)) {}
};
```

3.4 Hopcroft Karp [372c8b]

```
struct HopcroftKarp { // 0-based
  int n, m;
  vector <vector <int>> g;
  vector <int> l, r, d;
  bool dfs(int x) {
    for (int y : g[x]) if (r[y] == -1 ||
      (d[r[y]] == d[x] + 1 && dfs(r[y])))
      return l[x] = y, r[y] = x, d[x] = -1, true;
    return d[x] = -1, false;
  }
  bool bfs() {
    d.assign(n, -1);
    queue <int> q;
    for (int x = 0; x < n; ++x) if (l[x] == -1)
      d[x] = 0, q.push(x);
    bool good = false;
    while (!q.empty()) {
      int x = q.front(); q.pop();
      for (int y : g[x])
        if (r[y] == -1) good = true;
        else if (d[r[y]] == -1)
          d[r[y]] = d[x] + 1, q.push(r[y]);
    }
    return good;
  }
  int solve() {
    int res = 0;
    l.assign(n, -1), r.assign(m, -1);
    while (bfs())
      for (int x = 0; x < n; ++x) if (l[x] == -1)
        res += dfs(x);
    return res;
  }
  void add_edge(int x, int y) { g[x].pb(y); }
  HopcroftKarp (int _n, int _m) : n(_n), m(_m), g(n) {}
};
```

3.5 SW Min Cut [f7fc17]

```
template <typename T>
struct SW { // 0-based
  const T INF = numeric_limits<T>::max() / 2;
  vector <vector <T>> g;
  vector <T> sum;
  vector <bool> vis, dead;
  int n;
  T solve() {
    T ans = INF;
    for (int r = 0; r + 1 < n; ++r) {
      vis.assign(n, 0), sum.assign(n, 0);
      int num = 0, s = -1, t = -1;
      while (num < n - r) {
        int now = -1;
        for (int i = 0; i < n; ++i)
          if (!vis[i] && !dead[i] &&
            (now == -1 || sum[now] > sum[i])) now = i;
        s = t, t = now;
        vis[now] = true, num++;
        for (int i = 0; i < n; ++i)
          if (!vis[i] && !dead[i]) sum[i] += g[now][i];
      }
      ans = min(ans, sum[t]);
      for (int i = 0; i < n; ++i)
        g[i][s] += g[i][t], g[s][i] += g[t][i];
      dead[t] = true;
    }
    return ans;
  }
  void add_edge(int u, int v, T w) {
    g[u][v] += w, g[v][u] += w; }
  SW (int _n) : n(_n), g(n, vector <T>(n)), dead(n) {}
};
```

3.6 Gomory Hu Tree [90ead2]

```
vector <array <int, 3>> GomoryHu(Dinic <int> flow) {
  // Tree edge min = mincut (0-based)
  int n = flow.n;
  vector <array <int, 3>> ans;
  vector <int> rt(n);
  for (int i = 1; i < n; ++i) {
    int t = rt[i];
    flow.reset();
    ans.pb({i, t, flow.solve(i, t)});
    flow.bfs();
    for (int j = i + 1; j < n; ++j)
      if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
  }
  return ans;
}
```

3.7 Blossom [cbc9d3]

```
struct Matching { // 0-based
  int n, tk;
  vector <vector <int>> g;
  vector <int> fa, pre, match, s, t;
  queue <int> q;
  int Find(int u) {
    return u == fa[u] ? u : fa[u] = Find(fa[u]); }
  int lca(int x, int y) {
    tk++, x = Find(x), y = Find(y);
    for (; ; swap(x, y)) if (x != n) {
      if (t[x] == tk) return x;
      t[x] = tk;
      x = Find(pre[match[x]]);
    }
  }
  void blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
    }
  }
  bool bfs(int r) {
    iota(all(fa), 0), fill(all(s), -1);
    while (!q.empty()) q.pop();
    q.push(r), s[r] = 0;
    while (!q.empty()) {
      int x = q.front(); q.pop();
      for (int u : g[x]) {
        if (s[u] == -1) {
          pre[u] = x, s[u] = 1;
          if (match[u] == n) {
            for (int a = u, b = x, last; b != n; a =
                last, b = pre[a])
              last = match[b], match[b] = a, match[a] =
                  b;
            return true;
          }
          q.push(match[u]);
          s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = lca(u, x);
          blossom(x, u, l), blossom(u, x, l);
        }
      }
    }
    return false;
  }
```

```
  int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) if (match[x] == n)
      res += bfs(x);
    return res;
  }
  void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
  Matching (int _n) : n(_n), tk(0), g(n), fa(n + 1),
    pre(n + 1, n), match(n + 1, n), s(n + 1), t(n) {}
};
```

3.8 Min Cost Circulation [53a447]

```
struct MinCostCirculation { // 0-base
  struct Edge {
    ll from, to, cap, fcap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  ll dis[N], inq[N], n;
  void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, Edge *e) {
      if (dis[u] > d) {
        dis[u] = d, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u])
        if (e.cap > e.flow)
          relax(e.to, dis[u] + e.cost, &e);
    }
  }
  void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return cur.cap++, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
      cur.flow++, G[cur.to][cur.rev].flow--;
      for (int i = cur.from; past[i]; i = past[i]->from
          ) {
        auto &e = *past[i];
        e.flow++, G[e.to][e.rev].flow--;
      }
    }
    cur.cap++;
  }
  void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          e.cap *= 2, e.flow *= 2;
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          if (e.fcap >> b & 1)
            try_edge(e);
    }
  }
  void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge{a, b, 0, cap, 0, cost, sz(G[b]) + (a
        == b)});
    G[b].pb(Edge{b, a, 0, 0, 0, -cost, sz(G[a]) - 1});
  }
} mcmf; // O(VE * ElogC)
```

3.9 Weighted Blossom [dc42e4]

```
#define pb emplace_back
#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
  static const int inf = INT_MAX;
  struct edge { int u, v, w; }; int n, nx;
  vector<int> lab; vector<vector<edge>> g;
  vector<int> slack, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from; queue<int> q;
  WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
    g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
    flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
  }
  int ED(edge e) {
    return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
  void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u; }
  void set_slack(int x) {
    slack[x] = 0;
    REP(u, 1, n)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x, slack[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (int y : flo[x]) set_st(y, b);
  } // ae3b3a
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(all(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
      reverse(1 + all(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    REP(i, 0, int(z.size())-1) set_match(z[i], z[i ^
        1]);
    set_match(xr, v); f.insert(f.end(), all(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]]; set_match(u, v);
      if (!xnv) return;
      set_match(v = xnv, u = st[pa[xnv]]);
    }
  }
  int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u) {
      if (vis[u] == t) return u;
      vis[u] = t; u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  } // 0569c4
  void add_blossom(int u, int o, int v) {
    int b = int(find(n + 1 + all(st), 0) - begin(st));
    lab[b] = 0, S[b] = 0; match[b] = match[o];
    vector<int> f = {o};
    for (int x : {u, v}) {
      for (int y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y = st[match[x]]), q_push(y);
      reverse(1 + all(f));
    }
    flo[b] = f; set_st(b, b);
    REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
    REP(x, 1, n) flo_from[b][x] = 0;
    for (int xs : flo[b]) {
      REP(x, 1, nx)
        if (g[b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x
            ]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
      REP(x, 1, n)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
```

```cpp
      if (xs == -1) { xs = x; continue; }
      pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
      slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
    }
    for (int x : flo[b])
      if (x == xr) S[x] = 1, pa[x] = pa[b];
      else S[x] = -1, set_slack(x);
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
      int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
      slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
      if (int o = lca(u, v)) add_blossom(u, o, v);
      else return augment(u, v), augment(v, u), true;
    }
    return false;
  } // 61368c
  bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    REP(x, 1, nx) if (st[x] == x && !match[x])
      pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (q.size()) {
        int u = q.front(); q.pop();
        if (S[st[u]] == 1) continue;
        REP(v, 1, n)
          if (g[u][v].w > 0 && st[u] != st[v]) {
            if (ED(g[u][v]) != 0)
              update_slack(u, st[v], slack[st[v]]);
            else if (on_found_edge(g[u][v])) return
                true;
          }
      }
      int d = inf;
      REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
        d = min(d, lab[b] / 2);
      REP(x, 1, nx)
        if (int s = slack[x]; st[x] == x && s && S[x]
            <= 0)
          d = min(d, ED(g[s][x]) / (S[x] + 2));
      REP(u, 1, n)
        if (S[st[u]] == 1) lab[u] += d;
        else if (S[st[u]] == 0) {
          if (lab[u] <= d) return false;
          lab[u] -= d;
        }
      REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
      REP(x, 1, nx)
        if (int s = slack[x]; st[x] == x &&
            s && st[s] != x && ED(g[s][x]) == 0)
          if (on_found_edge(g[s][x])) return true;
      REP(b, n + 1, nx)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  } // 61b100
  pair<ll, int> solve() {
    fill(all(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
      flo_from[u][v] = (u == v ? u : 0);
      w_max = max(w_max, g[u][v].w);
    }
    REP(u, 1, n) lab[u] = w_max;
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
      tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
  }
  void set_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w; }
}; // f1e757
```

3.10 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of **incoming** lower bounds and the sum of **outgoing** lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge: $y \to x$ if $(x, y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C + K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e\in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. Change the weight of each edge to $\mu(u) + \mu(v) - w(u, v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  2. Let the maximum weight matching of the graph be x, the answer will be $\sum \mu(v) - x$.

4 Graph

4.1 Heavy-Light Decomposition [9ec77f]

```cpp
struct HLD { // 0-based, remember to build
  int n, _id;
  vector <vector <int>> g;
  vector <int> dep, pa, tsz, ch, hd, id;
  void dfs(int v, int p) {
    dep[v] = ~p ? dep[p] + 1 : 0;
    pa[v] = p, tsz[v] = 1, ch[v] = -1;
    for (int u : g[v]) if (u != p) {
      dfs(u, v);
      if (ch[v] == -1 || tsz[ch[v]] < tsz[u])
        ch[v] = u;
      tsz[v] += tsz[u];
    }
  }
  void hld(int v, int p, int h) {
    hd[v] = h, id[v] = _id++;
    if (~ch[v]) hld(ch[v], v, h);
    for (int u : g[v]) if (u != p && u != ch[v])
      hld(u, v, u);
  }
  vector <pii> query(int u, int v) {
    vector <pii> ans;
    while (hd[u] != hd[v]) {
      if (dep[hd[u]] > dep[hd[v]]) swap(u, v);
      ans.emplace_back(id[hd[v]], id[v] + 1);
      v = pa[hd[v]];
    }
    if (dep[u] > dep[v]) swap(u, v);
    ans.emplace_back(id[u], id[v] + 1);
    return ans;
```

```
  }
  void build() {
    for (int i = 0; i < n; ++i) if (id[i] == -1)
      dfs(i, -1), hld(i, -1, i);
  }
  void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
  HLD (int _n) : n(_n), _id(0), g(n), dep(n), pa(n),
    tsz(n), ch(n), hd(n), id(n, -1) {}
};
```

4.2 Centroid Decomposition [28b80a]

```
struct CD { // 0-based, remember to build
  int n, lg; // pa, dep are centroid tree attributes
  vector <vector <int>> g, dis;
  vector <int> pa, tsz, dep, vis;
  void dfs1(int v, int p) {
    tsz[v] = 1;
    for (int u : g[v]) if (u != p && !vis[u])
      dfs1(u, v), tsz[v] += tsz[u];
  }
  int dfs2(int v, int p, int _n) {
    for (int u : g[v])
      if (u != p && !vis[u] && tsz[u] > _n / 2)
        return dfs2(u, v, _n);
    return v;
  }
  void dfs3(int v, int p, int d) {
    dis[v][d] = ~p ? dis[p][d] + 1 : 0;
    for (int u : g[v]) if (u != p && !vis[u])
      dfs3(u, v, d);
  }
  void cd(int v, int p, int d) {
    dfs1(v, -1), v = dfs2(v, -1, tsz[v]);
    vis[v] = true, pa[v] = p, dep[v] = d;
    dfs3(v, -1, d);
    for (int u : g[v]) if (!vis[u])
      cd(u, v, d + 1);
  }
  void build() { cd(0, -1, 0); }
  void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
  CD (int _n) : n(_n), lg(__lg(n) + 1), g(n),
    dis(n, vector <int>(lg)), pa(n), tsz(n),
    dep(n), vis(n) {}
};
```

4.3 Edge BCC [cf5e55]

```
struct EBCC { // 0-based, remember to build
  int n, m, nbcc;
  vector <vector <pii>> g;
  vector <int> pa, low, dep, bcc_id, stk, is_bridge;
  void dfs(int v, int p, int f) {
    low[v] = dep[v] = ~p ? dep[p] + 1 : 0;
    stk.pb(v), pa[v] = p;
    for (auto [u, e] : g[v]) {
      if (low[u] == -1)
        dfs(u, v, e), low[v] = min(low[v], low[u]);
      else if (e != f)
        low[v] = min(low[v], dep[u]);
    }
    if (low[v] == dep[v]) {
      if (~f) is_bridge[f] = true;
      int id = nbcc++, x;
      do {
        x = stk.back(), stk.pop_back();
        bcc_id[x] = id;
      } while (x != v);
    }
  }
  void build() {
    is_bridge.assign(m, 0);
    for (int i = 0; i < n; ++i) if (low[i] == -1)
      dfs(i, -1, -1);
  }
  void add_edge(int u, int v) {
    g[u].emplace_back(v, m), g[v].emplace_back(u, m++);
  }
  EBCC (int _n) : n(_n), m(0), nbcc(0), g(n), pa(n),
    low(n, -1), dep(n), bcc_id(n), stk() {}
};
```

4.4 Vertex BCC / Round Square Tree [66d85d]

```
struct BCC { // 0-based, remember to build
  int n, nbcc; // note for isolated point
  vector <vector <int>> g, _g; // id >= n: bcc
  vector <int> pa, dep, low, stk, pa2, dep2;
  void dfs(int v, int p) {
    dep[v] = low[v] = ~p ? dep[p] + 1 : 0;
    stk.pb(v), pa[v] = p;
    for (int u : g[v]) if (u != p) {
      if (low[u] == -1) {
        dfs(u, v), low[v] = min(low[v], low[u]);
        if (low[u] >= dep[v]) {
          int id = nbcc++, x;
          do {
            x = stk.back(), stk.pop_back();
            _g[id + n].pb(x), _g[x].pb(id + n);
          } while (x != u);
          _g[id + n].pb(v), _g[v].pb(id + n);
        }
      } else low[v] = min(low[v], dep[u]);
    }
  }
  bool is_cut(int x) { return sz(_g[x]) != 1; }
  vector <int> bcc(int id) { return _g[id + n]; }
  int bcc_id(int u, int v) {
    return pa2[dep2[u] < dep2[v] ? v : u] - n; }
  void dfs2(int v, int p) {
    dep2[v] = ~p ? dep2[p] + 1 : 0, pa2[v] = p;
    for (int u : _g[v]) if (u != p) dfs2(u, v);
  }
  void build() {
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) if (low[i] == -1)
      dfs(i, -1), dfs2(i, -1);
  }
  void add_edge(int u, int v) {
    g[u].pb(v), g[v].pb(u); }
  BCC (int _n) : n(_n), nbcc(0), g(n), _g(2 * n),
    pa(n), dep(n), low(n), stk(), pa2(n * 2),
    dep2(n * 2) {}
};
```

4.5 SCC [9bee8c]

```
struct SCC {
  int n, nscc, _id;
  vector <vector <int>> g;
  vector <int> dep, low, scc_id, stk;
  void dfs(int v) {
    dep[v] = low[v] = _id++, stk.pb(v);
    for (int u : g[v]) if (scc_id[u] == -1) {
      if (low[u] == -1) dfs(u);
      low[v] = min(low[v], low[u]);
    }
    if (low[v] == dep[v]) {
      int id = nscc++, x;
      do {
        x = stk.back(), stk.pop_back(), scc_id[x] = id;
      } while (x != v);
    }
  }
  void build() {
    for (int i = 0; i < n; ++i) if (low[i] == -1)
      dfs(i);
  }
  void add_edge(int u, int v) { g[u].pb(v); }
  SCC (int _n) : n(_n), nscc(0), _id(0), g(n), dep(n),
    low(n, -1), scc_id(n, -1), stk() {}
};
```

4.6 2SAT [938072]

```
struct SAT { // 0-based, need SCC
  int n; vector <pii> edge; vector <int> is;
  int rev(int x) { return x < n ? x + n : x - n; }
  void add_ifthen(int x, int y) {
    add_clause(rev(x), y); }
  void add_clause(int x, int y) {
    edge.emplace_back(rev(x), y);
    edge.emplace_back(rev(y), x); }
  bool solve() {
    // is[i] = true -> i, is[i] = false -> -i
```

```
    SCC scc(2 * n);
    for (auto [u, v] : edge) scc.add_edge(u, v);
    scc.build();
    for (int i = 0; i < n; ++i) {
      if (scc.scc_id[i] == scc.scc_id[i + n])
        return false;
      is[i] = scc.scc_id[i] < scc.scc_id[i + n];
    }
    return true;
  }
  SAT (int _n) : n(_n), edge(), is(n) {}
};
```

4.7 Virtual Tree [f7650b]

```
// need lca, in, out
vector <pii> virtual_tree(vector <int> &v) {
  auto cmp = [&](int x, int y) {return in[x] < in[y];};
  sort(all(v), cmp);
  for (int i = 1; i < sz(v); ++i)
    v.pb(lca(v[i - 1], v[i]));
  sort(all(v), cmp);
  v.resize(unique(all(v)) - v.begin());
  vector <int> stk(1, v[0]);
  vector <pii> res;
  for (int i = 1; i < sz(v); ++i) {
    int x = v[i];
    while (out[stk.back()] < out[x]) stk.pop_back();
    res.emplace_back(stk.back(), x), stk.pb(x);
  }
  return res;
}
```

4.8 Directed MST [a2498b]

```
using D = int;
struct edge { int u, v; D w; };
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
  using T = pair <D, int>;
  using PQ = pair <priority_queue <T, vector <T>,
      greater <T>>, D>;
  auto push = [](PQ &pq, T v) {
    pq.first.emplace(v.first - pq.second, v.second);
  };
  auto top = [](const PQ &pq) -> T {
    auto r = pq.first.top();
    return {r.first + pq.second, r.second};
  };
  auto join = [&push, &top](PQ &a, PQ &b) {
    if (sz(a.first) < sz(b.first)) swap(a, b);
    while (!b.first.empty())
      push(a, top(b)), b.first.pop();
  };
  vector<PQ> h(n * 2);
  for (int i = 0; i < sz(e); ++i)
    push(h[e[i].v], {e[i].w, i});
  vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
      n * 2);
  iota(all(a), 0);
  auto o = [&](int x) { int y;
    for (y = x; a[y] != y; y = a[y]);
    for (int ox = x; x != y; ox = x)
      x = a[x], a[ox] = y;
    return y;
  };
  v[root] = n + 1;
  int pc = n;
  for (int i = 0; i < n; ++i) if (v[i] == -1) {
    for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
        p]].u)) {
      if (v[p] == i) {
        int q = p; p = pc++;
        do {
          h[q].second = -h[q].first.top().first;
          join(h[pa[q] = a[q] = p], h[q]);
        } while ((q = o(e[r[q]].u)) != p);
      }
      v[p] = i;
      while (!h[p].first.empty() && o(e[top(h[p]).
          second].u) == p)
        h[p].first.pop();
      r[p] = top(h[p]).second;
    }
  }
  vector<int> ans;
  for (int i = pc - 1; i >= 0; i--)
    if (i != root && v[i] != n) {
      for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
          pa[f]) v[f] = n;
      ans.pb(r[i]);
    }
  return ans;
}
```

4.9 Dominator Tree [7eadea]

```
struct DominatorTree {
  int n, id;
  vector <vector <int>> g, rg, bucket;
  vector <int> sdom, dom, vis, rev, pa, rt, mn, res;
  // dom[s] = s, dom[v] = -1 if s -> v not exists
  int query(int v, int x) {
    if (rt[v] == v) return x ? -1 : v;
    int p = query(rt[v], 1);
    if (p == -1) return x ? rt[v] : mn[v];
    if (sdom[mn[v]] > sdom[mn[rt[v]]])
      mn[v] = mn[rt[v]];
    rt[v] = p;
    return x ? p : mn[v];
  }
  void dfs(int v) {
    vis[v] = id, rev[id] = v;
    rt[id] = mn[id] = sdom[id] = id, id++;
    for (int u : g[v]) {
      if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];
      rg[vis[u]].pb(vis[v]);
    }
  }
  void build(int s) {
    dfs(s);
    for (int i = id - 1; ~i; --i) {
      for (int u : rg[i])
        sdom[i] = min(sdom[i], sdom[query(u, 0)]);
      if (i) bucket[sdom[i]].pb(i);
      for (int u : bucket[i]) {
        int p = query(u, 0);
        dom[u] = sdom[p] == i ? i : p;
      }
      if (i) rt[i] = pa[i];
    }
    fill(all(res), -1);
    for (int i = 1; i < id; ++i) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    }
    for (int i = 1; i < id; ++i)
      res[rev[i]] = rev[dom[i]];
    res[s] = s;
    for (int i = 0; i < n; ++i) dom[i] = res[i];
  }
  void add_edge(int u, int v) { g[u].pb(v); }
  DominatorTree (int _n) : n(_n), id(0), g(n), rg(n),
    bucket(n), sdom(n), dom(n, -1), vis(n, -1),
    rev(n), pa(n), rt(n), mn(n), res(n) {}
};
```

4.10 Bipartite Edge Coloring [a22d96]

```
struct BipartiteEdgeColoring { // 1-based
  // returns edge coloring in adjacent matrix G
  int n, m;
  vector <vector <int>> col, G;
  int find_col(int x) {
    int c = 1;
    while (col[x][c]) c++;
    return c;
  }
  void dfs(int v, int c1, int c2) {
    if (!col[v][c1]) return col[v][c2] = 0, void(0);
    int u = col[v][c1];
    dfs(u, c2, c1);
    col[v][c1] = 0, col[v][c2] = u, col[u][c2] = v;
  }
  void solve() {
```

```
    for (int i = 1; i <= n + m; ++i)
      for (int j = 1; j <= max(n, m); ++j)
        if (col[i][j])
          G[i][col[i][j]] = G[col[i][j]][i] = j;
  } // u = left index, v = right index
  void add_edge(int u, int v) {
    int c1 = find_col(u), c2 = find_col(v + n);
    dfs(u, c2, c1);
    col[u][c2] = v + n, col[v + n][c2] = u;
  }
  BipartiteEdgeColoring (int _n, int _m) : n(_n),
    m(_m), col(n + m + 1, vector <int>(max(n, m) + 1)),
    G(n + m + 1, vector <int>(n + m + 1)) {}
};
```

4.11 Edge Coloring [5b1e8f]

```
struct Vizing { // 1-based
  // returns edge coloring in adjacent matrix G
  int n;
  vector <vector <int>> C, G;
  vector <int> X, vst;
  vector <pii> E;
  void solve() {
    auto update = [&](int u)
    { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
    auto color = [&](int u, int v, int c) {
      int p = G[u][v];
      G[u][v] = G[v][u] = c;
      C[u][c] = v, C[v][c] = u;
      C[u][p] = C[v][p] = 0;
      if (p) X[u] = X[v] = p;
      else update(u), update(v);
      return p;
    };
    auto flip = [&](int u, int c1, int c2) {
      int p = C[u][c1];
      swap(C[u][c1], C[u][c2]);
      if (p) G[u][p] = G[p][u] = c2;
      if (!C[u][c1]) X[u] = c1;
      if (!C[u][c2]) X[u] = c2;
      return p;
    };
    fill(1 + all(X), 1);
    for (int t = 0; t < sz(E); ++t) {
      auto [u, v0] = E[t];
      int v = v0, c0 = X[u], c = c0, d;
      vector<pii> L;
      fill(1 + all(vst), 0);
      while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) {
          for (int a = sz(L) - 1; a >= 0; --a)
            c = color(u, L[a].first, c);
        } else if (!C[u][d]) {
          for (int a = sz(L) - 1; a >= 0; --a)
            color(u, L[a].first, L[a].second);
        } else if (vst[d]) break;
        else vst[d] = 1, v = C[u][d];
      }
      if (!G[u][v0]) {
        for (; v; v = flip(v, c, d), swap(c, d));
        if (int a; C[u][c0]) {
          for (a = sz(L) - 2;
            a >= 0 && L[a].second != c; --a);
          for (; a >= 0; --a)
            color(u, L[a].first, L[a].second);
        }
        else --t;
      }
    }
  }
  void add_edge(int u, int v) { E.emplace_back(u, v); }
  Vizing(int _n) : n(_n), C(n + 1, vector<int>(n + 1)),
  G(n + 1, vector<int>(n + 1)), X(n + 1), vst(n + 1) {}
};
```

4.12 Maximum Clique [5ed877]

```
struct MaxClique { // Maximum Clique
  bitset<N> a[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) a[i].reset();
  }
  void add_edge(int u, int v) { a[u][v] = a[v][u] = 1;
    }
  void csort(vector<int> &r, vector<int> &c) {
    int mx = 1, km = max(ans - q + 1, 1), t = 0, m = sz
      (r);
    cs[1].reset(), cs[2].reset();
    for (int i = 0; i < m; ++i) {
      int p = r[i], k = 1;
      while ((cs[k] & a[p]).count()) k++;
      if (k > mx) mx++, cs[mx + 1].reset();
      cs[k][p] = 1;
      if (k < km) r[t++] = p;
    }
    c.resize(m);
    if (t) c[t - 1] = 0;
    for (int k = km; k <= mx; ++k)
      for (int p = cs[k]._Find_first(); p < N;
           p = cs[k]._Find_next(p))
        r[t] = p, c[t] = k, t++;
  }
  void dfs(vector<int> &r, vector<int> &c, int l,
    bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr, nc;
      bitset<N> nmask = mask & a[p];
      for (int i : r)
        if (a[p][i]) nr.pb(i);
      if (!nr.empty()) {
        if (l < 4) {
          for (int i : nr)
            d[i] = (a[i] & nmask).count();
          sort(nr.begin(), nr.end(),
            [&](int x, int y) { return d[x] > d[y]; });
        }
        csort(nr, nc), dfs(nr, nc, l + 1, nmask);
      } else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), q--;
    }
  }
  int solve(bitset<N> mask = bitset<N>(
              string(N, '1'))) { // vertex mask
    vector<int> r, c;
    ans = q = 0;
    for (int i = 0; i < n; ++i)
      if (mask[i]) r.pb(i);
    for (int i = 0; i < n; ++i)
      d[i] = (a[i] & mask).count();
    sort(r.begin(), r.end(),
      [&](int i, int j) { return d[i] > d[j]; });
    csort(r, c), dfs(r, c, 1, mask);
    return ans; // sol[0 ~ ans-1]
  }
};
```

5 String

5.1 Aho-Corasick Automaton [77096b]

```
struct AC { // remember to build_fail!!!
  int ch[N][C], to[N][C], fail[N], cnt[N], _id;
  // fail link tree: fail[i] -> i
  AC () { reset(); }
  int newnode() {
    fill_n(ch[_id], C, 0), fill_n(to[_id], C, 0);
    fail[_id] = cnt[_id] = 0; return _id++; }
  int insert(string s) {
    int now = 0;
    for (char c : s) {
      if (!ch[now][c - 'a'])
        ch[now][c - 'a'] = newnode();
      now = ch[now][c - 'a'];
    }
    cnt[now]++; return now;
  }
```

```
  void build_fail() {
    queue <int> q;
    for (int i = 0; i < C; ++i) if (ch[0][i])
      q.push(ch[0][i]), to[0][i] = ch[0][i];
    while (!q.empty()) {
      int v = q.front(); q.pop();
      for (int i = 0; i < C; ++i) {
        if (!ch[v][i]) to[v][i] = to[fail[v]][i];
        else {
          int u = ch[v][i], k = fail[v];
          while (k && !ch[k][i]) k = fail[k];
          if (ch[k][i]) k = ch[k][i];
          fail[u] = k, cnt[u] += cnt[k], to[v][i] = u;
          q.push(u);
        }
      }
    }
  }
  // int match(string &s) {
  //   int now = 0, ans = 0;
  //   for (char c : s) {
  //     now = to[now][c - 'a'];
  //     ans += cnt[now];
  //   }
  //   return ans;
  // }
  void reset() { _id = 0, newnode(); }
} ac;
```

5.2 KMP Algorithm [9f8819]

```
auto build_fail(auto s) {
  vector <int> f(sz(s) + 1, 0);
  int k = 0;
  for (int i = 1; i < sz(s); ++i) {
    while (k && s[k] != s[i]) k = f[k];
    if (s[k] == s[i]) k++;
    f[i + 1] = k;
  }
  return f;
}
int match(auto s, auto t) {
  vector <int> f = build_fail(t);
  int k = 0, ans = 0;
  for (int i = 0; i < sz(s); ++i) {
    while (k && s[i] != t[k]) k = f[k];
    if (s[i] == t[k]) k++;
    if (k == sz(t)) ans++, k = f[k];
  }
  return ans;
}
```

5.3 Z Algorithm [e028f9]

```
auto buildZ(auto s) {
  int n = sz(s), l = 0, r = 0;
  vector <int> Z(n);
  for (int i = 0; i < n; ++i) {
    Z[i] = max(min(Z[i - l], r - i), 0);
    while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]])
      l = i, r = i + Z[i], Z[i]++;
  }
  return Z;
}
```

5.4 Manacher [4e2fd6]

```
// return value only consider string tmp, not s
// return array length = 2N - 1
auto manacher(string tmp) {
  string s = "&";
  for (char c : tmp) s.pb(c), s.pb('%');
  int l = 0, r = 0, n = sz(s);
  vector <int> Z(n);
  for (int i = 0; i < n; ++i) {
    Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
    while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
    if (Z[i] + i > r) l = i, r = Z[i] + i;
  }
  for (int i = 0; i < n; ++i)
    Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
  return vector<int>(1 + all(Z) - 1);
}
```

5.5 Suffix Array [58ed43]

```
auto sais(const auto &s) {
  const int n = sz(s), z = ranges::max(s) + 1;
  if (n == 1) return vector{0};
  vector<int> c(z); for (int x : s) c[x]++;
  partial_sum(all(c), c.begin());
  vector<int> sa(n); auto I = views::iota(0, n);
  vector<bool> t(n, true);
  for (int i = n - 2; i >= 0; --i)
    t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i +
        1]);
  auto is_lms = views::filter([&t](int x) {
    return x && t[x] && !t[x - 1];
  });
  auto induce = [&] {
    for (auto x = c; int y : sa)
      if (y--) if (!t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
      if (y--) if (t[y]) sa[--x[s[y]]] = y;
  };
  vector<int> lms, q(n); lms.reserve(n);
  for (auto x = c; int i : I | is_lms)
    q[i] = sz(lms), lms.pb(sa[--x[s[i]]] = i);
  induce(); vector<int> ns(sz(lms));
  for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
      int len = min({n - i, n - j, lms[q[i] + 1] - i});
      ns[q[i]] = nz += lexicographical_compare(
          s.begin() + j, s.begin() + j + len,
          s.begin() + i, s.begin() + i + len);
    }
    j = i;
  }
  fill(all(sa), 0); auto nsa = sais(ns);
  for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
  return induce(), sa;
} // 0eb2d2
struct Suffix {
  // lcp[i] = LCP(sa[i - 1], sa[i])
  int n; vector<int> sa, lcp, rk;
  Suffix(auto _s) : n(sz(_s)), lcp(n), rk(n) {
    vector<int> s(n + 1); // s[n] = 0;
    for (int i = 0; i < n; ++i) s[i] = _s[i];
    // _s shouldn't contain 0
    sa = sais(s), sa.erase(sa.begin());
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0, h = 0; i < n; ++i) {
      if (!rk[i]) { h = 0; continue; }
      for (int j = sa[rk[i] - 1]; max(i, j) + h < n &&
          s[i + h] == s[j + h];) ++h;
      lcp[rk[i]] = h ? h-- : 0;
    }
  }
//int queryLCP(int i, int j) {
//  auto [l, r] = minmax({rk[i] + 1, rk[j] + 1});
//  return lcp_range_min(l, r);
//}
}; // 4422fa
```

5.6 Suffix Automaton [12d8e9]

```
struct SAM {
  int ch[2 * N][C], len[2 * N], link[2 * N], pos[2 * N
      ], cnt[2 * N], _id;
  // node -> strings with the same endpos set
  // length in range [len(link) + 1, len]
  // node's endpos set -> pos in the subtree of node
  // link -> longest suffix with different endpos set
  // len  -> longest suffix
  // pos  -> end position
  // cnt  -> size of endpos set
  SAM () { reset(); }
  int newnode() {
    fill_n(ch[_id], C, 0);
    len[_id] = link[_id] = pos[_id] = cnt[_id] = 0;
    return _id++;
  }
  void build(string s) {
    int lst = 0;
    for (int i = 0; i < sz(s); ++i) {
      char c = s[i];
```

```
      int cur = newnode();
      len[cur] = len[lst] + 1, pos[cur] = i + 1;
      int p = lst;
      while (~p && !ch[p][c - 'a'])
        ch[p][c - 'a'] = cur, p = link[p];
      if (p == -1) link[cur] = 0;
      else {
        int q = ch[p][c - 'a'];
        if (len[p] + 1 == len[q]) {
          link[cur] = q;
        } else {
          int nxt = newnode();
          len[nxt] = len[p] + 1, link[nxt] = link[q];
          pos[nxt] = 0;
          for (int j = 0; j < C; ++j)
            ch[nxt][j] = ch[q][j];
          while (~p && ch[p][c - 'a'] == q)
            ch[p][c - 'a'] = nxt, p = link[p];
          link[q] = link[cur] = nxt;
        }
      }
      cnt[cur]++, lst = cur;
    }
  }
  // void build_count() {
  //   vector <int> p(_id);
  //   iota(all(p), 0);
  //   sort(all(p),
  //     [&](int i, int j) {return len[i] > len[j];});
  //   for (int i = 0; i < _id; ++i) if (~link[p[i]])
  //     cnt[link[p[i]]] += cnt[p[i]];
  // }
  void reset() { _id = 0, newnode(), link[0] = -1; }
} sam;
```

5.7 Minimum Rotation [561109]

```
string rotate(const string &s) {
  int n = sz(s), i = 0, j = 1;
  string t = s + s;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && t[i + k] == t[j + k]) ++k;
    if (t[i + k] <= t[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return t.substr(pos, n);
}
```

5.8 Palindrome Tree [f67ae4]

```
struct PAM {
  int ch[N][C], cnt[N], fail[N], len[N], _id;
  // 0 -> even root, 1 -> odd root
  PAM () { reset(); }
  int newnode() {
    fill_n(ch[_id], C, 0);
    cnt[_id] = fail[_id] = len[_id] = 0;
    return _id++;
  }
  void build(string s) {
    int lst = 1;
    for (int i = 0; i < sz(s); ++i) {
      while (s[i - len[lst] - 1] != s[i])
        lst = fail[lst];
      if (!ch[lst][s[i] - 'a'])  {
        int idx = newnode();
        len[idx] = len[lst] + 2;
        int now = fail[lst];
        while (s[i - len[now] - 1] != s[i])
          now = fail[now];
        fail[idx] = ch[now][s[i] - 'a'];
        ch[lst][s[i] - 'a'] = idx;
      }
      lst = ch[lst][s[i] - 'a'], cnt[lst]++;
    }
  }
  void build_count() {
    for (int i = _id - 1; i > 1; --i)
      cnt[fail[i]] += cnt[i];
  }
  void reset() { _id = 0, newnode(), newnode(),
    len[0] = 0, fail[0] = 1, len[1] = -1; }
} pam;
```

5.9 Lyndon Factorization [5e52cf]

```
// partition s = w[0] + w[1] + ... + w[k-1],
// w[0] >= w[1] >= ... >= w[k-1]
// each w[i] strictly smaller than all its suffix
vector <string> duval(const string &s) {
  vector <string> ans;
  for (int n = sz(s), i = 0, j, k; i < n; ) {
    for (j = i + 1, k = i; j < n && s[k] <= s[j]; j++)
      k = (s[k] < s[j] ? i : k + 1);
    for (; i <= k; i += j - k)
      ans.pb(s.substr(i, j - k)); // s.substr(l, len)
  }
  return ans;
}
```

5.10 Main Lorentz [b38f07]

```
// [l, r, len]: p in [l, r] => s[p, p + len * 2] tandem
// you might need to compress manually
auto main_lorentz(string _s) {
  vector <array <int, 3>> rep;
  auto dfs = [&](auto self, string s, int sft) -> void
      {
    int n = sz(s);
    if (n == 1) return;
    int nu = n / 2, nv = n - nu;
    string u = s.substr(0, nu), v = s.substr(nu),
          ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.
              rend());
    self(self, u, sft), self(self, v, sft + nu);
    auto z1 = buildZ(ru), z2 = buildZ(v + '#' + u),
              z3 = buildZ(ru + '#' + rv), z4 = buildZ(v
                  );
    auto get_z = [](vector<int> &z, int i) {
      return 0 <= i && i < sz(z) ? z[i] : 0; };
    auto add_rep = [&](bool left, int c, int l, int k1,
         int k2) {
      int L = max(1, l - k2), R = min(l - left, k1);
      if (L > R) return;
      if (left) rep.pb({sft + c - R, sft + c - L, l});
      else rep.pb({sft + c - R - l + 1, sft + c - L - l
          + 1, l});
    };
    for (int cntr = 0; cntr < n; cntr++) {
      int l, k1, k2;
      if (cntr < nu) {
        l = nu - cntr;
        k1 = get_z(z1, nu - cntr);
        k2 = get_z(z2, nv + 1 + cntr);
      } else {
        l = cntr - nu + 1;
        k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
        k2 = get_z(z4, (cntr - nu) + 1);
      }
      if (k1 + k2 >= l)
        add_rep(cntr < nu, cntr, l, k1, k2);
    }
  };
  dfs(dfs, _s, 0);
  return rep;
}
```

6 Math

6.1 Miller Rabin / Pollard Rho [f7e2bb]

```
ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long
    double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504,
     1795265022};
ll Pow(ll a, ll b, ll n) {
  ll res = 1;
  for (; b; b >>= 1, a = mul(a, a, n))
    if (b & 1) res = mul(res, a, n);
  return res;
}
bool check(ll a, ll d, int s, ll n) {
```

```
  a = Pow(a, d, n);
  if (a <= 1) return 1;
  for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
    if (a == 1) return 0;
    if (a == n - 1) return 1;
  }
  return 0;
}
bool IsPrime(ll n) {
  if (n < 2) return 0;
  if (n % 2 == 0) return n == 2;
  ll d = n - 1, s = 0;
  while (d % 2 == 0) d >>= 1, s++;
  for (ll i : chk) if (!check(i, d, s, n)) return 0;
  return 1;
} // 5761f3
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
  if (IsPrime(n)) return 1;
  for (ll p : small) if (n % p == 0) return p;
  ll x, y = 2, d, t = 1;
  auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
  for (int l = 2; ; l <<= 1) {
    x = y;
    int m = min(l, 32);
    for (int i = 0; i < l; i += m) {
      d = 1;
      for (int j = 0; j < m; ++j) {
        y = f(y), d = mul(d, abs(x - y), n);
      }
      ll g = __gcd(d, n);
      if (g == n) {
        l = 1, y = 2, t++;
        break;
      }
      if (g != 1) return g;
    }
  }
}
map <ll, int> res;
void PollardRho(ll n) {
  if (n == 1) return;
  if (IsPrime(n)) return res[n]++, void(0);
  ll d = FindFactor(n);
  PollardRho(n / d), PollardRho(d);
} // 57e9e3
```

6.2 Ext GCD [a4b22d]

```
//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
  if (b == 0) return {1, 0};
  auto [y, x] = extgcd(b, a % b);
  return pair<ll, ll>(x, y - (a / b) * x);
}
```

6.3 Chinese Remainder Theorem [90d2ce]

```
pair<ll, ll> CRT(ll x1, ll m1, ll x2, ll m2) {
  ll g = gcd(m1, m2);
  if ((x2 - x1) % g) return make_pair(-1, -1);// no sol
  m1 /= g, m2 /= g;
  pair<ll, ll> p = extgcd(m1, m2);
  ll lcm = m1 * m2 * g;
  ll res = p.first * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return make_pair((res % lcm + lcm) % lcm, lcm);
}
```

6.4 PiCount [1db46f]

```
const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
  for(int i = 2; i < x; ++i) isp[i] = true;
  isp[0] = isp[1] = false;
  for(int i = 2; i * i < x; ++i) if(isp[i])
    for(int j = i * i; j < x; j += i) isp[j] = false;
  for(int i = 2; i < x; ++i) if(isp[i]) primes.pb(i);
}
void init(){
  sieve(V);
  small_pi[0] = 0;
  for(int i = 1; i < V; ++i)
    small_pi[i] = small_pi[i - 1] + isp[i];
  for(int i = 0; i < M; ++i) dp[0][i] = i;
  for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
    dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i -
        1]];
}
ll phi(ll n, int a){
  if(!a) return n;
  if(n < M && a < N) return dp[a][n];
  if(primes[a - 1] > n) return 1;
  if(1ll * primes[a - 1] * primes[a - 1] >= n && n < V)
    return small_pi[n] - a + 1;
  return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
ll PiCount(ll n){
  if(n < V) return small_pi[n];
  int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
      small_pi[y];
  ll res = phi(n, a) + a - 1;
  for(; primes[a] <= s; ++a) res -= max(PiCount(n /
      primes[a]) - PiCount(primes[a]) + 1, 0ll);
  return res;
}
```

6.5 Linear Function Mod Min [5552e3]

```
ll topos(ll x, ll m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    log m)
ll min_rem(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  for (ll g = __gcd(a, m); g > 1;) return g * min_rem(n
      , m / g, a / g, b / g) + (b % g);
  for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
      = nb) {
    if (a <= m - a) {
      nn = (a * (n - 1) + b) / m;
      if (!nn) break;
      nn += (b < a);
      nm = a, na = topos(-m, a);
      nb = b < a ? b : topos(b - m, a);
    } else {
      ll lst = b - (n - 1) * (m - a);
      if (lst >= 0) {b = lst; break;}
      nn = -(lst / m) + (lst % m < -a) + 1;
      nm = m - a, na = m % (m - a), nb = b % (m - a);
    }
  }
  return b;
} // ab2d19
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
  a = topos(a, m), b = topos(b, m);
  ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
  //ax = (mn - b) (mod m)
  ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
      % (m / g);
  return {mn, x};
} // 017ca5
```

6.6 Gauss Elimination [41dc4e]

```
auto gauss(vector<vector<int>> a, vector<int> b) {
  // solve ax = b
  int n = sz(a), m = sz(a[0]), rk = 0;
  vector<int> depv, free(m, true);
  for (int i = 0; i < m; ++i) {
    int p = -1;
    for (int j = rk; j < n; ++j)
      if (p == -1 || abs(a[j][i]) > abs(a[p][i]))
        p = j;
    if (p == -1 || a[p][i] == 0) continue;
    swap(a[p], a[rk]), swap(b[p], b[rk]);
    int inv = Pow(a[rk][i], mod - 2);
    for (int &x : a[rk]) x = mul(x, inv);
    b[rk] = mul(b[rk], inv);
```

```
    for (int j = 0; j < n; ++j) if (j ^ rk) {
      int x = a[j][i];
      for (int k = 0; k < m; ++k)
        a[j][k] = sub(a[j][k], mul(x, a[rk][k]));
      b[j] = sub(b[j], mul(x, b[rk]));
    }
    depv.pb(i), free[i] = false, rk++;
  }
  vector<int> x; vector<vector <int>> h;
  for (int i = rk; i < n; ++i) if (b[i] != 0)
    return make_pair(x, h); // not consistent
  x.resize(m);
  for (int i = 0; i < rk; ++i) x[depv[i]] = b[i];
  for (int i = 0; i < m; ++i) if (free[i]) {
    h.emplace_back(m), h.back()[i] = 1;
    for (int j = 0; j < rk; ++j)
      h.back()[depv[j]] = sub(0, a[j][i]);
  }
  return make_pair(x, h); // solution = x + span(h[i])
}
```

6.7 Floor Sum [49de67]

```
// sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a
    + b)
// only works for a, b >= 0!!!
ll floor_sum(ll n, ll m, ll a, ll b) {
  ll ans = 0;
  if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
  if (b >= m) ans += n * (b / m), b %= m;
  ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
  if (y_max == 0) return ans;
  ans += (n - (x_max + a - 1) / a) * y_max;
  ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
  return ans;
}
```

6.8 Quadratic Residue [51ec55]

```
int Jacobi(int a, int m) {
  int s = 1;
  for (; m > 1; ) {
    a %= m;
    if (a == 0) return 0;
    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
  }
  return s;
}
int QuadraticResidue(int a, int p) {
  if (p == 2) return a & 1;
  const int jc = Jacobi(a, p);
  if (jc == 0) return 0;
  if (jc == -1) return -1;
  int b, d;
  for (; ; ) {
    b = rand() % p;
    d = (1ll * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
  }
  ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
  for (int e = (p + 1) >> 1; e; e >>= 1) {
    if (e & 1) {
      tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
      g1 = (g0 * f1 + g1 * f0) % p;
      g0 = tmp;
    }
    tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
    f1 = (2 * f0 * f1) % p;
    f0 = tmp;
  }
  return g0;
}
```

6.9 Discrete Log [cf360f]

```
ll DiscreteLog(ll a, ll b, ll m) { // a^x = b (mod m)
  const int B = 35000;
  ll k = 1 % m, ans = 0, g;
  while ((g = gcd(a, m)) > 1) {
    if (b == k) return ans;
    if (b % g) return -1;
    b /= g, m /= g, ans++, k = (k * a / g) % m;
  }
  if (b == k) return ans;
  unordered_map <ll, int> m1;
  ll tot = 1;
  for (int i = 0; i < B; ++i)
    m1[tot * b % m] = i, tot = tot * a % m;
  ll cur = k * tot % m;
  for (int i = 1; i <= B; ++i, cur = cur * tot % m)
    if (m1.count(cur))
      return 1ll * i * B - m1[cur] + ans;
  return -1;
}
```

6.10 Factorial without Prime Factor [c324f3]

```
// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
  prod[0] = 1;
  for (int i = 1; i <= pk; ++i)
    if (i % p) prod[i] = prod[i - 1] * i % pk;
    else prod[i] = prod[i - 1];
  ll rt = 1;
  for (; n; n /= p) {
    rt = rt * mpow(prod[pk], n / pk, pk) % pk;
    rt = rt * prod[n % pk] % pk;
  }
  return rt;
} // (n! without factor p) % p^k
```

6.11 Berlekamp Massey [3a60a6]

```
// need add, sub, mul
// find min |c| such that a_n = sum c_j * a_{n - j -
    1}, 0-based
// O(N^2), if |c| = k, |a| >= 2k sure correct
vector <int> BerlekampMassey(vector <int> a) {
  auto f = [&](vector<int> v, ll c)
  { for (int &x : v) x = mul(x, c); return v; };
  vector <int> c, best;
  int pos = 0, n = sz(a);
  for (int i = 0; i < n; ++i) {
    int error = a[i];
    for (int j = 0; j < sz(c); ++j)
      error = sub(error, mul(c[j], a[i - 1 - j]));
    if (error == 0) continue;
    int inv = Pow(error, mod - 2);
    if (c.empty()) {
      c.resize(i + 1), pos = i, best.pb(inv);
    } else {
      vector <int> fix = f(best, error);
      fix.insert(fix.begin(), i - pos - 1, 0);
      if (sz(fix) >= sz(c)) {
        best = f(c, sub(0, inv));
        best.insert(best.begin(), inv);
        pos = i, c.resize(sz(fix));
      }
      for (int j = 0; j < sz(fix); ++j)
        c[j] = add(c[j], fix[j]);
    }
  }
  return c;
}
```

6.12 Simplex [e34964]

```
struct Simplex { // 0-based
  using T = long double;
  static const int N = 410, M = 30010;
  const T eps = 1e-7;
  int n, m;
  int Left[M], Down[N];
  // Ax <= b, max c^T x
  // result : v, xi = sol[i]
  T a[M][N], b[M], c[N], v, sol[N];
  bool eq(T a, T b) {return fabs(a - b) < eps;}
  bool ls(T a, T b) {return a < b && !eq(a, b);}
  void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
```

```
    for (int i = 0; i < m; ++i)
      for (int j = 0; j < n; ++j) a[i][j] = 0;
    for (int i = 0; i < m; ++i) b[i] = 0;
    for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
  }
  void pivot(int x, int y) {
    swap(Left[x], Down[y]);
    T k = a[x][y]; a[x][y] = 1;
    vector <int> nz;
    for (int i = 0; i < n; ++i) {
      a[x][i] /= k;
      if (!eq(a[x][i], 0)) nz.pb(i);
    }
    b[x] /= k;
    for (int i = 0; i < m; ++i) {
      if (i == x || eq(a[i][y], 0)) continue;
      k = a[i][y], a[i][y] = 0;
      b[i] -= k * b[x];
      for (int j : nz) a[i][j] -= k * a[x][j];
    }
    if (eq(c[y], 0)) return;
    k = c[y], c[y] = 0, v += k * b[x];
    for (int i : nz) c[i] -= k * a[x][i];
  }
  // 0: found solution, 1: no feasible solution, 2:
      unbounded
  int solve() {
    for (int i = 0; i < n; ++i) Down[i] = i;
    for (int i = 0; i < m; ++i) Left[i] = n + i;
    while (true) {
      int x = -1, y = -1;
      for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x
          == -1 || b[i] < b[x])) x = i;
      if (x == -1) break;
      for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) &&
          (y == -1 || a[x][i] < a[x][y])) y = i;
      if (y == -1) return 1;
      pivot(x, y);
    }
    while (true) {
      int x = -1, y = -1;
      for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y
          == -1 || c[i] > c[y])) y = i;
      if (y == -1) break;
      for (int i = 0; i < m; ++i)
        if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y
            ] < b[x] / a[x][y])) x = i;
      if (x == -1) return 2;
      pivot(x, y);
    }
    for (int i = 0; i < m; ++i) if (Left[i] < n)
      sol[Left[i]] = b[i];
    return 0;
  }
};
```

6.13 Euclidean

$m = \lfloor \frac{an+b}{c} \rfloor$

$$g(a,b,c,n) = \sum_{i=0}^{n} i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) & \\ -h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) & \\ +\lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) & \\ -2f(c, c-b-1, a, m-1) - f(a,b,c,n), & \text{otherwise} \end{cases}$$

6.14 Linear Programming Construction

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y} \ge \mathbf{c}$ and $\mathbf{y} \ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i \in [1,n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^{m} A_{ji}\bar{y}_j = c_i$ holds and for all $i \in [1,m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^{n} A_{ij}\bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.15 Theorem

- Kirchhoff's Theorem

  Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i,j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

  Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i,j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Erdős–Gallai Theorem

  A sequence of non-negative integers $d_1 \ge d_2 \ge \ldots \ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \ldots + d_n$ is even and

  $$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$$

  holds for all $1 \le k \le n$.

- Burnside's Lemma

  Let X be a set and G be a group that acts on X. For $g \in G$, denote by X^g the elements fixed by g:

  $$X^g = \{x \in X \mid gx \in X\}$$

  Then

  $$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

  A pair of sequences of nonnegative integers $a_1 \ge \cdots \ge a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$. Sequences a and b called bigraphic if there is a labeled simple bipartite graph such that a and b is the degree sequence of this bipartite graph.

- Fulkerson-Chen-Anstee theorem

  A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \ge \cdots \ge a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \le \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \le k \le n$. Sequences a and b called digraphic if there is a labeled simple directed graph such that each vertex v_i has indegree a_i and outdegree b_i.

- Pick's theorem

  For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$

- Spherical cap

  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

6.16 Estimation

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8

n	100	1e3	1e6	1e9	1e12	1e15	1e18
$d(i)$	12	32	240	1344	6720	26880	103680
arg	60	840	720720	735134400	963761198400	866421317361600	8976124847866

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8

n	2	3	4	5	6	7	8	9	10	11	12	13
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7

6.17 General Purpose Numbers

- Bernoulli numbers

  $B_0 = 1, B_1^{\pm} = \pm\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$

  $$\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

  $$S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k^{+} n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

  $S(n,k) = S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1$

  $S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$

  $x^n = \sum_{i=0}^{n} S(n,i)(x)_i$

- Pentagonal number theorem

  $$\prod_{n=1}^{\infty}(1-x^n) = 1 + \sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2} + x^{k(3k-1)/2}\right)$$

- Catalan numbers

  $$C_n^{(k)} = \frac{1}{(k-1)n+1}\binom{kn}{n}$$

  $C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$

- Eulerian numbers

  Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

  $E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$

  $E(n,0) = E(n,n-1) = 1$

  $E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

6.18 Calculus

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\text{erf}(x) \qquad \int xe^{ax} = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int \sin^2(x) = \frac{x}{2} - \frac{1}{4}\sin 2x \qquad \int \sin^3 x = \frac{1}{12}\cos 3x - \frac{3}{4}\cos x$$

$$\int \cos^2(x) = \frac{x}{2} + \frac{1}{4}\sin 2x \qquad \int \cos^3 x = \frac{1}{12}\sin 3x + \frac{3}{4}\sin x$$

$$\int \sin ax\cos ax = \frac{1}{2a}\sin^2(ax) \qquad \int x\sin x\cos x = -\frac{x}{4}\cos 2x + \frac{1}{8}\sin 2x$$

$$\int x\sin x = \sin x - x\cos x \qquad \int x\cos x = \cos x + x\sin x$$

$$\int xe^x = e^x(x-1) \qquad \int x^2e^x = e^x(x^2-2x+2)$$

$$\int x^2\sin x = 2x\sin x - (x^2-2)\cos x$$

$$\int x^2\cos x = 2x\cos x + (x^2-2)\sin x$$

$$\int e^x\sin x = \frac{1}{2}e^x(\sin x - \cos x)$$

$$\int e^x\cos x = \frac{1}{2}e^x(\sin x + \cos x)$$

$$\int xe^x\sin x = \frac{1}{2}e^x(x\sin x - x\cos x + \cos x)$$

$$\int xe^x\cos x = \frac{1}{2}e^x(x\sin x + x\cos x - \sin x)$$

7 Polynomial

7.1 Number Theoretic Transform [36b2ce]

```cpp
// mul, add, sub, Pow
struct NTT {
  int w[N];
  NTT() {
    int dw = Pow(G, (mod - 1) / N);
    w[0] = 1;
    for (int i = 1; i < N; ++i)
      w[i] = mul(w[i - 1], dw);
  } // 0 <= a[i] < P
  void operator()(vector<int>& a, bool inv = false) {
    int n = sz(a);
    for (int j = 1, x = 0; j < n - 1; ++j) {
      for (int k = n >> 1; (x ^= k) < k; k >>= 1);
      if (j < x) swap(a[x], a[j]);
    }
    for (int L = 2; L <= n; L <<= 1) {
      int dx = N / L, dl = L >> 1;
      for (int i = 0; i < n; i += L) {
        for (int j = i, x = 0; j < i + dl; ++j, x += dx
            ) {
          int tmp = mul(a[j + dl], w[x]);
          a[j + dl] = sub(a[j], tmp);
          a[j] = add(a[j], tmp);
        }
      }
    }
    if (inv) {
      reverse(1 + all(a));
      int invn = Pow(n, mod - 2);
      for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], invn);
    }
  }
} ntt;
```

7.2 Fast Fourier Transform [026694]

```cpp
using T = complex<double>;
const double PI = acos(-1);
struct FFT {
  T w[N];
  FFT() {
    T dw = {cos(2 * PI / N), sin(2 * PI / N)};
    w[0] = 1;
    for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
  }
  void operator()(vector<T>& a, bool inv = false) {
    // see NTT, replace LL with T
    if (inv) {
      reverse(1 + all(a));
      T invn = 1.0 / n;
      for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
    }
  }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	9223372036737335297	3

7.4 Polynomial Operations [e39ccf]

```cpp
typedef vector<int> Poly;
Poly Mul(Poly a, Poly b, int bound = N) {
  int m = sz(a) + sz(b) - 1, n = 1;
  while (n < m) n <<= 1;
  a.resize(n), b.resize(n);
  ntt(a), ntt(b);
  for (int i = 0; i < n; ++i) a[i] = mul(a[i], b[i]);
  ntt(a, true), a.resize(min(m, bound));
```

```cpp
  return a;
} // 8e2e8b
Poly Inverse(Poly a) {
  // O(NLogN), a[0] != 0
  int n = sz(a);
  Poly res(1, Pow(a[0], mod - 2));
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    Poly v1(a.begin(), a.begin() + m * 2), v2 = res;
    v1.resize(m * 4), v2.resize(m * 4);
    ntt(v1), ntt(v2);
    for (int i = 0; i < m * 4; ++i)
      v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
    ntt(v1, true);
    res.resize(m * 2);
    for (int i = 0; i < m; ++i)
      res[i] = add(res[i], res[i]);
    for (int i = 0; i < m * 2; ++i)
      res[i] = sub(res[i], v1[i]);
  }
  res.resize(n);
  return res;
} // 4d79c8
pair <Poly, Poly> Divide(Poly a, Poly b) {
  // a = bQ + R, O(NLogN), b.back() != 0
  int n = sz(a), m = sz(b), k = n - m + 1;
  if (n < m) return {{0}, a};
  Poly ra = a, rb = b;
  reverse(all(ra)), ra.resize(k);
  reverse(all(rb)), rb.resize(k);
  Poly Q = Mul(ra, Inverse(rb), k);
  reverse(all(Q));
  Poly res = Mul(b, Q), R(m - 1);
  for (int i = 0; i < m - 1; ++i)
    R[i] = sub(a[i], res[i]);
  return {Q, R};
} // 7d15e3
Poly SqrtImpl(Poly a) {
  if (a.empty()) return {0};
  int z = QuadraticResidue(a[0], mod), n = sz(a);
  if (z == -1) return {-1};
  Poly q(1, z);
  const int inv2 = (mod + 1) / 2;
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    q.resize(m * 2);
    Poly f2 = Mul(q, q, m * 2);
    for (int i = 0; i < m * 2; ++i)
      f2[i] = sub(f2[i], a[i]);
    f2 = Mul(f2, Inverse(q), m * 2);
    for (int i = 0; i < m * 2; ++i)
      q[i] = sub(q[i], mul(f2[i], inv2));
  }
  q.resize(n);
  return q;
} // 984549
Poly Sqrt(Poly a) {
  // O(NLogN), return {-1} if not exists
  int n = sz(a), m = 0;
  while (m < n && a[m] == 0) m++;
  if (m == n) return Poly(n);
  if (m & 1) return {-1};
  Poly s = SqrtImpl(Poly(a.begin() + m, a.end()));
  if (s[0] == -1) return {-1};
  Poly res(n);
  for (int i = 0; i < sz(s); ++i)
    res[i + m / 2] = s[i];
  return res;
} // d1acd7
Poly Derivative(Poly a) {
  int n = sz(a);
  Poly res(n - 1);
  for (int i = 0; i < n - 1; ++i)
    res[i] = mul(a[i + 1], i + 1);
  return res;
} // 001be0
Poly Integral(Poly a) {
  int n = sz(a);
  Poly res(n + 1);
  for (int i = 0; i < n; ++i)
    res[i + 1] = mul(a[i], Pow(i + 1, mod - 2));
  return res;
} // 6fc53d
Poly Ln(Poly a) {
  // O(NLogN), a[0] = 1
  int n = sz(a);
  if (n == 1) return {0};
  Poly d = Derivative(a);
  a.pop_back();
  return Integral(Mul(d, Inverse(a), n - 1));
} // 377d20
Poly Exp(Poly a) {
  // O(NLogN), a[0] = 0
  int n = sz(a);
  Poly q(1, 1);
  a[0] = add(a[0], 1);
  for (int m = 1; m < n; m <<= 1) {
    if (n < m * 2) a.resize(m * 2);
    Poly g(a.begin(), a.begin() + m * 2), h(all(q));
    h.resize(m * 2), h = Ln(h);
    for (int i = 0; i < m * 2; ++i)
      g[i] = sub(g[i], h[i]);
    q = Mul(g, q, m * 2);
  }
  q.resize(n);
  return q;
} // 525e8f
Poly PolyPow(Poly a, ll k) {
  int n = sz(a), m = 0;
  Poly ans(n, 0);
  while (m < n && a[m] == 0) m++;
  if (k && m && (k >= n || k * m >= n)) return ans;
  if (m == n) return ans[0] = 1, ans;
  int lead = m * k;
  Poly b(a.begin() + m, a.end());
  int base = Pow(b[0], k), inv = Pow(b[0], mod - 2);
  for (int i = 0; i < n - m; ++i)
    b[i] = mul(b[i], inv);
  b = Ln(b);
  for (int i = 0; i < n - m; ++i)
    b[i] = mul(b[i], k % mod);
  b = Exp(b);
  for (int i = lead; i < n; ++i)
    ans[i] = mul(b[i - lead], base);
  return ans;
} // 7d695a
vector <int> Evaluate(Poly a, vector <int> x) {
  if (x.empty()) return {};
  int n = sz(x);
  vector <Poly> up(n * 2);
  for (int i = 0; i < n; ++i)
    up[i + n] = {sub(0, x[i]), 1};
  for (int i = n - 1; i > 0; --i)
    up[i] = Mul(up[i * 2], up[i * 2 + 1]);
  vector <Poly> down(n * 2);
  down[1] = Divide(a, up[1]).second;
  for (int i = 2; i < n * 2; ++i)
    down[i] = Divide(down[i >> 1], up[i]).second;
  Poly y(n);
  for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
  return y;
} // bff354
Poly Interpolate(vector <int> x, vector <int> y) {
  int n = sz(x);
  vector <Poly> up(n * 2);
  for (int i = 0; i < n; ++i)
    up[i + n] = {sub(0, x[i]), 1};
  for (int i = n - 1; i > 0; --i)
    up[i] = Mul(up[i * 2], up[i * 2 + 1]);
  Poly a = Evaluate(Derivative(up[1]), x);
  for (int i = 0; i < n; ++i)
    a[i] = mul(y[i], Pow(a[i], mod - 2));
  vector <Poly> down(n * 2);
  for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
  for (int i = n - 1; i > 0; --i) {
    Poly lhs = Mul(down[i * 2], up[i * 2 + 1]);
    Poly rhs = Mul(down[i * 2 + 1], up[i * 2]);
    down[i].resize(sz(lhs));
    for (int j = 0; j < sz(lhs); ++j)
      down[i][j] = add(lhs[j], rhs[j]);
  }
  return down[1];
} // af80e7
Poly TaylorShift(Poly a, int c) {
```

```cpp
  // return sum a_i(x + c)^i;
  // fac[i] = i!, facp[i] = inv(i!)
  int n = sz(a);
  for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
  reverse(all(a));
  Poly b(n);
  int w = 1;
  for (int i = 0; i < n; ++i)
    b[i] = mul(facp[i], w), w = mul(w, c);
  a = Mul(a, b, n), reverse(all(a));
  for (int i = 0; i < n; ++i) a[i] = mul(a[i],facp[i]);
  return a;
} // 3a3763
vector<int> SamplingShift(vector<int> a, int c, int m){
  // given f(0), f(1), ..., f(n - 1)
  // return f(c), f(c + 1), ..., f(c + m - 1)
  int n = sz(a); // 4d649d
  for (int i = 0; i < n; ++i) a[i] = mul(a[i],facp[i]);
  Poly b(n);
  for (int i = 0; i < n; ++i) {
    b[i] = facp[i];
    if (i & 1) b[i] = sub(0, b[i]);
  }
  a = Mul(a, b, n);
  for (int i = 0; i < n; ++i) a[i] = mul(a[i], fac[i]);
  reverse(all(a));
  int w = 1;
  for (int i = 0; i < n; ++i)
    b[i] = mul(facp[i], w), w = mul(w, sub(c, i));
  a = Mul(a, b, n);
  reverse(all(a));
  for (int i = 0; i < n; ++i) a[i] = mul(a[i],facp[i]);
  a.resize(m), b.resize(m);
  for (int i = 0; i < m; ++i) b[i] = facp[i];
  a = Mul(a, b, m);
  for (int i = 0; i < m; ++i) a[i] = mul(a[i], fac[i]);
  return a;
} // 2e52c1
```

7.5 Fast Linear Recursion [8b69ed]

```cpp
int FastLinearRecursion(vector <int> a, vector <int> c,
     ll k) {
  // a_n = sigma c_j * a_{n - j - 1}, 0-based
  // O(NlogNlogK), |a| = |c|
  int n = sz(a);
  if (k < n) return a[k];
  vector <int> base(n + 1, 1);
  for (int i = 0; i < n; ++i)
    base[i] = sub(0, c[n - i - 1]);
  vector <int> poly(n);
  (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
  auto calc = [&](vector <int> p1, vector <int> p2) {
    // O(n^2) bruteforce or O(nlogn) NTT
    return Divide(Mul(p1, p2), base).second; };
  vector <int> res(n, 0); res[0] = 1;
  for (; k; k >>= 1, poly = calc(poly, poly)) {
    if (k & 1) res = calc(res, poly);
  }
  int ans = 0;
  for (int i = 0; i < n; ++i)
    ans = add(ans, mul(res[i], a[i]));
  return ans;
}
```

7.6 Fast Walsh Transform

```cpp
void fwt(vector <int> &a, bool inv = false) {
  // and : x += y * (1, -1)
  // or  : y += x * (1, -1)
  // xor : x = (x + y) * (1, 1/2)
  //       y = (x - y) * (1, 1/2)
  int n = __lg(sz(a));
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
      int x = a[j ^ (1 << i)], y = a[j];
      // do something
    }
  }
}
vector<int> subs_conv(vector<int> a, vector<int> b) {
  // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
  int n = __lg(sz(a));
  vector ha(n + 1, vector<int>(1 << n));
  vector hb(n + 1, vector<int>(1 << n));
  vector c(n + 1, vector<int>(1 << n));
  for (int i = 0; i < 1 << n; ++i) {
    ha[__builtin_popcount(i)][i] = a[i];
    hb[__builtin_popcount(i)][i] = b[i];
  }
  for (int i = 0; i <= n; ++i)
    or_fwt(ha[i]), or_fwt(hb[i]);
  for (int i = 0; i <= n; ++i)
    for (int j = 0; i + j <= n; ++j)
      for (int k = 0; k < 1 << n; ++k)
        c[i + j][k] = add(c[i + j][k],
          mul(ha[i][k], hb[j][k]));
  for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
  vector <int> ans(1 << n);
  for (int i = 0; i < 1 << n; ++i)
    ans[i] = c[__builtin_popcount(i)][i];
  return ans;
}
```

8 Geometry

8.1 Basic

```cpp
template <typename T> struct P {};
using Pt = P<ll>;
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
ll abs2(Pt a) { return a * a; }
double abs(Pt a) { return sqrt(abs2(a)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 &&
         sign((c - a) * (c - b)) <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
bool cmp(Pt a, Pt b)
{ return pos(a) == pos(b) ? sign(a ^ b) > 0 :
         pos(a) < pos(b); }
bool same_vec(Pt a, Pt b, int d) // d = 1: check dir
{ return sign(a ^ b) == 0 && sign(a * b) > d * 2 - 2; }
bool same_vec(Line a, Line b, int d)
{ return same_vec(a.b - a.a, b.b - b.a, d); }
Pt perp(Pt a) { return Pt(-a.y, a.x); } // CCW 90 deg
Pt ref(Pt a) {return pos(a) == 1 ? Pt(-a.x, -a.y) : a;}
// double part
double theta(Pt a)
{ return normalize(atan2(a.y, a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) // CCW
{ double c = cos(o), s = sin(o);
  return Pt(c * a.x - s * a.y, s * a.x + c * a.y); }
Pt proj_vec(Pt a, Pt b, Pt c) // vector ac proj to ab
{return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));}
Pt proj_pt(Pt a, Pt b, Pt c) // point c proj to ab
{ return proj_vec(a, b, c) + a; }
```

8.2 SVG Writer

```cpp
#ifdef ABS
class SVG { // SVG("test.svg", 0, 0, 10, 10)
  void p(string_view s) { o << s; }
  void p(string_view s, auto v, auto... vs) {
    auto i = s.find('$');
    o << s.substr(0, i) << v, p(s.substr(i + 1), vs...)
        ;
  }
  ofstream o; string c = "red";
public:
  SVG(auto f,auto x1,auto y1,auto x2,auto y2) : o(f) {
    p("<svg xmlns='http://www.w3.org/2000/svg' "
      "viewBox='$ $ $ $'>\n"
      "<style>*{stroke-width:0.5%;}</style>\n",
      x1, -y2, x2 - x1, y2 - y1); }
  ~SVG() { p("</svg>\n"); }
  void color(string nc) { c = nc; }
  void line(auto x1, auto y1, auto x2, auto y2) {
    p("<line x1='$' y1='$' x2='$' y2='$' stroke='$'/>\n
      ",
```

```
      x1, -y1, x2, -y2, c); }
  void circle(auto x, auto y, auto r) {
    p("<circle cx='$' cy='$' r='$' stroke='$' "
      "fill='none'/>\n", x, -y, r, c); }
  void text(auto x, auto y, string s, int w = 12) {
    p("<text x='$' y='$' font-size='$px'>$</text>\n",
      x, -y, w, s); }
}; // write wrapper for complex if use complex
#else
struct SVG { SVG(auto ...) {} }; // you know how to
#endif
```

8.3 Sort

```
// cmp in Basic: polar angle sort
// all points are on line ab. closer to a: front
bool cmp_line(Pt s, Pt t, Pt a, Pt b) {
  Pt v = a - b;
  if (sign(v.x)) return sign(s.x - t.x) == sign(v.x);
  else return sign(s.y - t.y) == sign(v.y);
} // 3dc688
// intersect points polar angle sort, deno: positive
bool cmp_fraction_polar(pair<Pt, ll> o, pair<Pt, ll> s,
     pair<Pt, ll> t) { // C^3 / C^2
  Pt u = s.first * o.second - o.first * s.second; //C^5
  Pt v = t.first * o.second - o.first * t.second; //C^5
  // u /= gcd(u.x, u.y) might lower the range to C
  return cmp(u, v);
} // 2d4450
struct Polar_Seg {
  Pt a, b; // ori(Pt(0, 0), a, b) > 0
  bool operator < (const Polar_Seg &o) const {
    if (btwangle(Pt(0, 0), a, b, o.a, 0)) {
      if (a == o.a) return ori(o.b, a, b) == -1;
      if (b == o.a) return false;
      return ori(o.a, a, b) == -1;
    }
    if (b == o.a) return false;
    return ori(a, o.a, o.b) == 1;
  }
};
struct Arc {
  Cir c; int s; // 0 -> up, 1 -> down
  bool operator < (const Arc &b) const {
    if (c.id == b.c.id) return s < b.s;
    if (contain(c, b.c)) return b.s == 1;
    else if (contain(b.c, c)) return s == 0;
    else if (c.o.y == b.c.o.y) return c.id < b.c.id;
    else return c.o.y > b.c.o.y;
  }
};
```

8.4 Intersections

```
// m=0: segment, m=1: ray from l.a to l.b, m=2: line
bool lines_intersect_check(Line l1, int m1, Line l2,
    int m2, int strict) {
  auto on = [&](Line l, int m, Pt p) {
    if (ori(l.a, l.b, p) != 0) return false;
    if (m && abs2(l.a - p) > abs2(l.b - p)) return true
        ;
    return m == 2 || sign((p - l.a) * (p - l.b)) <= -
        strict;
  };
  if (same_vec(l1, l2, 0)) {
    return on(l1, m1, l2.a) || on(l1, m1, l2.b) ||
           on(l2, m2, l1.a) || on(l2, m2, l1.b);
  }
  auto good = [&](Line l, int m, Line o) {
    if (m && abs((l.a - o.a) ^ (l.a - o.b)) > abs((l.b
      - o.a) ^ (l.b - o.b))) return true;
    return m == 2 || ori(l.a, o.a, o.b) * ori(l.b, o.a,
        o.b) == -1;
  };
  if (good(l1, m1, l2) && good(l2, m2, l1)) return 1;
  if (!strict) {
    if (m2 != 2 && on(l1, m1, l2.a)) return 1;
    if (m2 == 0 && on(l1, m1, l2.b)) return 1;
    if (m1 != 2 && on(l2, m2, l1.a)) return 1;
    if (m1 == 0 && on(l2, m2, l1.b)) return 1;
  }
  return 0;
} // 56cc8d
// notice two lines are parallel
auto lines_intersect(Line a, Line b) {
  auto abc = (a.b - a.a) ^ (b.a - a.a);
  auto abd = (a.b - a.a) ^ (b.b - a.a);
  return make_pair((b.b * abc - b.a * abd), abc - abd);
} // 726acc
// res[0] -> res[1] and l.a -> l.b: same direction
vector<Pt> circle_line_intersect(Cir c, Line l) {
  Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
      ) / abs2(l.b - l.a);
  double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
      - s * s / abs2(l.b - l.a);
  if (sign(h2) == -1) return {};
  if (sign(h2) == 0) return {p};
  Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
  return {p - h, p + h};
} // b7bdce
// covered area of c1: arc from res[0] to res[1], CCW
vector<Pt> circles_intersect(Cir c1, Cir c2) {
  double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
  if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
       + c2.r) return {};
  Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
      c2.r - c1.r * c1.r) / (2 * d2));
  double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
       * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
  Pt v = perp(c2.o - c1.o) * A / (2 * d2);
  if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
  return {u - v, u + v};
} // 0acf68
// return edge endpoint, at point -> ids are the same
vector<pii> convex_line_intersect(vector <Pt> &C, Line
    la) {
  auto dis = [&](int p)
  { return (la.b - la.a) ^ (C[p] - la.a); };
  auto gao = [&](int s) {
    return cyc_tsearch(sz(C), [&](int i, int j)
    { return sign(dis(i) - dis(j)) == s; });
  };
  int x = gao(1), y = gao(-1), n = sz(C);
  if (sign(dis(x)) < 0 || sign(dis(y)) > 0) return {};
  if (sign(dis(x)) == 0 || sign(dis(y)) == 0) {
    int v = ((sign(dis(x)) == 0 ? x : y) + n - 1) % n;
    vector <pii> vec;
    for (int i = 0; i < 3; ++i, v = (v + 1) % n)
      if (sign(dis(v)) == 0) vec.emplace_back(v, v);
    return vec;
  }
  auto get = [&](int l, int r, int s) {
    while ((l + 1) % n != r) {
      int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
      (sign(dis(m)) == s ? l : r) = m;
    }
    if (sign(dis(r)) == 0) return pii(r, r);
    return pii(l, r);
  };
  return {get(x, y, 1), get(y, x, -1)};
} // 5ddd35
```

8.5 Point Inside Check

```
// get edge index: check (0, a), (0, b) first
// then after binary search, check (a, b)
bool point_in_convex(vector<Pt> &C, Pt p, bool strict =
     true) {
  // only works when no three points are collinear
  int a = 1, b = sz(C) - 1, r = !strict;
  if (sz(C) == 0) return false;
  if (sz(C) < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
      -r) return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
} // 722991
// -1: out, 0: edge, 1: in
int point_in_poly(vector <Pt> poly, Pt o, int strict) {
  int cnt = 0;
```

```cpp
  for (int i = 0; i < sz(poly); ++i) {
    Pt a = poly[i], b = poly[(i + 1) % sz(poly)];
    if (btw(o, a, b)) return !strict;
    cnt ^= ((o.y < a.y) - (o.y - b.y)) * ori(o, a, b) >
        0;
  }
  return cnt ? 1 : -1;
} // 94b56b
// return q's relation with circumcircle of tri(p[0],p
    [1],p[2])
bool point_in_cc(array<Pt, 3> p, Pt q) {
  __int128 det = 0;
  for (int i = 0; i < 3; ++i)
    det += __int128(abs2(p[i]) - abs2(q)) * ((p[(i + 1)
        % 3] - q) ^ (p[(i + 2) % 3] - q));
  return det > 0; // in: >0, on: =0, out: <0
} // cc76d3
```

8.6 Convex Hull [d490c0]

```cpp
auto convex_hull(vector<Pt> pts) {
  sort(all(pts), [&](Pt a, Pt b)
    {return a.x == b.x ? a.y < b.y : a.x < b.x;});
  vector<Pt> ans = {pts[0]};
  for (int t = 0; t < 2; ++t, reverse(all(pts))) {
    for (int i = 1, m = sz(ans); i < sz(pts); ++i) {
      while (sz(ans) > m && ori(ans[sz(ans) - 2],
        ans.back(), pts[i]) <= 0) ans.pop_back();
      ans.pb(pts[i]);
    }
  }
  if (sz(ans) > 1) ans.pop_back();
  return ans;
}
```

8.7 Point Segment Distance [4249fd]

```cpp
double point_segment_dist(Pt q0, Pt q1, Pt p) {
  if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
  if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
      * (p - q1)) >= 0)
    return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

8.8 Vector In Polygon [6dac08]

```cpp
// ori(a, b, c) >= 0, valid: "strict" angle from a-b to
     a-c
bool btwangle(Pt a, Pt b, Pt c, Pt p, int strict) {
  return ori(a, b, p) >= strict && ori(a, p, c) >=
      strict;
}
// whether vector{cur, p} in counter-clockwise order
    prv, cur, nxt
bool inside(Pt prv, Pt cur, Pt nxt, Pt p, int strict) {
  if (ori(cur, nxt, prv) >= 0)
    return btwangle(cur, nxt, prv, p, strict);
  return !btwangle(cur, prv, nxt, p, !strict);
}
// call "inside" not btwangle
```

8.9 Minkowski Sum [2ff069]

```cpp
void reorder(vector<Pt> &P) {
  rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
    { return make_pair(a.y, a.x) < make_pair(b.y, b.x);
  }), P.end());
}
auto minkowski(vector<Pt> P, vector<Pt> Q) {
  // P, Q: convex polygon, CCW order
  reorder(P), reorder(Q); int n = sz(P), m = sz(Q);
  P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
  vector<Pt> ans;
  for (int i = 0, j = 0; i < n || j < m; ) {
    ans.pb(P[i] + Q[j]);
    auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
    if (val >= 0) i++;
    if (val <= 0) j++;
  }
  return ans;
}
```

8.10 Rotating SweepLine [56f0e2]

```cpp
struct Event {
  Pt d; int u, v;
  bool operator < (const Event &o) {
    return sign(d ^ o.d) > 0; }
};
void rotating_sweepline(vector<Pt> pts) {
  int n = sz(pts);
  vector<int> ord(n), pos(n);
  vector<Event> e;
  for (int i = 0; i < n; ++i)
    for (int j = i + 1; j < n; ++j)
      e.pb({ref(pts[i] - pts[j]), i, j});
  sort(all(e));
  iota(all(ord), 0);
  sort(all(ord), [&](int i, int j) {
    return (sign(pts[i].y - pts[j].y) == 0 ?
        pts[i].x < pts[j].x : pts[i].y < pts[j].y); });
  for (int i = 0; i < n; ++i) pos[ord[i]] = i;
  auto makeReverse = [](auto v) {
    sort(all(v)), v.resize(unique(all(v)) - v.begin());
    vector<pii> segs;
    for (int i = 0, j = 0; i < sz(v); i = j) {
      for (;j < sz(v) && v[j] - v[i] <= j - i; ++j);
      segs.emplace_back(v[i], v[j - 1] + 1 + 1);
    }
    return segs;
  };
  for (int i = 0, j = 0; i < sz(e); i = j) {
    vector<int> tmp;
    for (; j < sz(e) && !(e[i] < e[j]); j++)
      tmp.pb(min(pos[e[j].u], pos[e[j].v]));
    for (auto [l, r] : makeReverse(tmp)) {
      reverse(ord.begin() + l, ord.begin() + r);
      for (int t = l; t < r; ++t) pos[ord[t]] = t;
      // update value here
    }
  }
}
```

8.11 Half Plane Intersection [f6c2b0]

```cpp
/* Having solution, check size > 2 */
/* --^-- Line.a --^-- Line.b --^-- */
auto halfplane_intersection(vector<Line> arr) {
  auto area_pair = [&](Line a, Line b) {
    return make_pair((a.b - a.a) ^ (b.a - a.a),
                     (a.b - a.a) ^ (b.b - a.a)); };
  auto isin = [&](Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return (__int128)a02Y * a12X -
           (__int128)a02X * a12Y > 0; // C^4
  };
  sort(all(arr), [&](Line a, Line b) {
    if (same_vec(a, b, 1))
      return ori(a.a, a.b, b.b) < 0;
    return cmp(a.b - a.a, b.b - b.a); });
  deque<Line> dq(1, arr[0]);
  auto pop_back = [&](int t, Line p) {
    while (sz(dq) >= t && !isin(p, dq[sz(dq) - 2], dq.
        back()))
      dq.pop_back(); };
  auto pop_front = [&](int t, Line p) {
    while (sz(dq) >= t && !isin(p, dq[0], dq[1]))
      dq.pop_front(); };
  for (auto p : arr)
    if (!same_vec(dq.back(), p, 1))
      pop_back(2, p), pop_front(2, p), dq.pb(p);
  pop_back(3, dq[0]), pop_front(3, dq.back());
  return vector<Line>(all(dq));
}
```

8.12 Minimum Enclosing Circle [2db817]

```cpp
Cir min_enclosing(vector<Pt> p) {
  random_shuffle(all(p));
  double r = 0.0;
  Pt cent = p[0];
  for (int i = 1; i < sz(p); ++i) {
```

```
    if (abs2(cent - p[i]) <= r) continue;
    cent = p[i], r = 0.0;
    for (int j = 0; j < i; ++j) {
      if (abs2(cent - p[j]) <= r) continue;
      cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
      for (int k = 0; k < j; ++k) {
        if (abs2(cent - p[k]) <= r) continue;
        cent = circenter(p[i], p[j], p[k]);
        r = abs2(p[k] - cent);
      }
    }
  }
  return {cent, sqrt(r)};
}
```

8.13 Point Inside Triangle

```
// number of points p with a < p < b such that ori(p, a
    , b) < 0
int under(Pt a, Pt b) {  }
// number of points with a < p < b and ori(p, a, b) = 0
int edge(Pt a, Pt b) {  }
// check if this number is calculated
bool check(Pt p) {  }
// number of points that strictly inside the triangle
int in_tri(array <Pt, 3> arr) {
  sort(all(arr), [&](Pt i, Pt j) {
    return i.x == j.x ? i.y < j.y : i.x < j.x; });
  auto [a, b, c] = arr;
  int x = ori(b, a, c);
  if (x == 0) return 0;
  if (x == 1) return under(a, b) + under(b, c) - under(
      a, c) - edge(a, c);
  return under(a, c) - under(a, b) - under(b, c) - edge
      (a, b) - edge(b, c) - check(b);
}
```

8.14 Heart [043c0d]

```
Pt circenter(Pt p0, Pt p1, Pt p2) {
  // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
  double m = 2. * (x1 * y2 - y1 * x2);
  Pt center(0, 0);
  center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
      y1 - y2)) / m;
  center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
      y2 * y2) / m;
  return center + p0;
} // 24710a
Pt incenter(Pt p1, Pt p2, Pt p3) {
  // radius = area / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
       - p2);
  double s = a + b + c;
  return (p1 * a + p2 * b + p3 * c) / s;
} // 342b59
Pt masscenter(Pt p1, Pt p2, Pt p3)
{ return (p1 + p2 + p3) / 3; }
Pt orthocenter(Pt p1, Pt p2, Pt p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
     p3) * 2; }
```

8.15 Tangents [277413]

```
auto circle_point_tangent(Cir c, Pt p) {
  vector<Line> res;
  double d_sq = abs2(p - c.o);
  if (sign(d_sq - c.r * c.r) == 0) {
    res.pb({p, p + perp(p - c.o)});
  } else if (d_sq > c.r * c.r) {
    double s = d_sq - c.r * c.r;
    Pt v = p + (c.o - p) * s / d_sq;
    Pt u = perp(c.o - p) * sqrt(s) * c.r / d_sq;
    res.pb({p, v + u});
    res.pb({p, v - u});
  }
  return res;
} // 6af9a8
auto circles_tangent(Cir c1, Cir c2, int sign1) {
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> res;
  double d_sq = abs2(c1.o - c2.o);
  if (sign(d_sq) == 0) return res;
  double d = sqrt(d_sq);
  Pt v = (c2.o - c1.o) / d;
  double c = (c1.r - sign1 * c2.r) / d;
  if (c * c > 1) return res;
  double h = sqrt(max((double)0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
        sign2 * h * v.x);
    Pt p1 = c1.o + n * c1.r;
    Pt p2 = c2.o + n * (c2.r * sign1);
    if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
        0)
      p2 = p1 + perp(c2.o - c1.o);
    res.pb({p1, p2});
  }
  return res;
} // 956705
/* The point should be strictly out of hull
  return arbitrary point on the tangent line */
pii point_convex_tengent(vector<Pt> &C, Pt p) {
  auto gao = [&](int s) {
    return cyc_tsearch(sz(C), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0, 63a82a
```

8.16 Convex Cut [1f8a27]

```
vector<Pt> cut(vector<Pt> poly, Pt s, Pt e) {
  vector<Pt> res;
  for (int i = 0; i < sz(poly); ++i) {
    Pt cur = poly[i], prv = i ? poly[i - 1] : poly.back
        ();
    bool side = ori(s, e, cur) < 0;
    if (side != (ori(s, e, prv) < 0))
      res.pb(lines_intersect({s, e}, {cur, prv}));
    if (side) res.pb(cur);
  }
  return res;
}
```

8.17 Union of Circles [e5c3ee]

```
// notice identical circles, compare cross -> x if the
    precision is bad
auto circles_border(vector<Cir> c, int id) {
  vector<pair<Pt, int>> vec;
  int base = 0;
  for (int i = 0; i < sz(c); ++i) if (id != i) {
    if (sign(c[id].r - c[i].r) < 0 && abs2(c[id].o - c[
        i].o) <= (c[id].r - c[i].r) * (c[id].r - c[i].r
        )) base++;
    auto tmp = circles_intersect(c[id], c[i]);
    if (sz(tmp) == 2) {
      Pt l = tmp[0] - c[id].o, r = tmp[1] - c[id].o;
      vec.emplace_back(l, 1);
      vec.emplace_back(r, -1);
      if (cmp(r, l)) base++;
    }
  }
  vec.emplace_back(Pt(-c[id].r, 0), 0);
  sort(all(vec), [&](auto i, auto j) {
    return cmp(i.first, j.first);
  });
  vector<pair<Pt, Pt>> seg;
  Pt v = Pt(c[id].r, 0), lst = v;
  for (auto [cur, val] : vec) {
    if (base == 0) seg.emplace_back(lst, cur);
    lst = cur, base += val;
  }
  if (base == 0) seg.emplace_back(lst, v);
  for (auto &[l, r] : seg)
    l = l + c[id].o, r = r + c[id].o;
  return seg;
} // 95d3c6
double circles_union_area(vector<Cir> c) {
  double res = 0;
  for (int i = 0; i < sz(c); ++i) {
    auto seg = circles_border(c, i);
```

```
    auto F = [&] (double t) { return c[i].r * (c[i].r *
         t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
    for (auto [l, r] : seg) {
      double tl = theta(l - c[i].o), tr = theta(r - c[i
          ].o);
      if (sign(tl - tr) > 0) tr += PI * 2;
      res += F(tr) - F(tl);
    }
  }
  return res / 2;
} // 22d249
```

8.18 Union of Polygons [c7ddf6]

```
// in CCW order, use index as tiebreaker when collinear
auto polys_border(vector<vector<Pt>> poly, int id) {
  auto get = [&](auto &p, int i) {
    return make_pair(p[i], p[(i + 1) % sz(p)]); };
  vector<pair<Pt, Pt>> seg;
  for (int e = 0; e < sz(poly[id]); ++e) {
    auto [s, t] = get(poly[id], e);
    vector<pair<Pt, int>> vec;
    vec.emplace_back(s, -1 << 30);
    vec.emplace_back(t, 1 << 30);
    for (int i = 0; i < sz(poly); ++i) {
      int st = find_if(all(poly[i]), [&](Pt p) {
        return ori(p, s, t) == 1; }) - poly[i].begin();
      if (st == sz(poly[i])) continue;
      for (int j = st; j < st + sz(poly[i]); ++j) {
        auto [a, b] = get(poly[i], j % sz(poly[i]));
        if (same_vec(a - b, s - t, -1)) {
          if (ori(a, b, s) == 0 && same_vec(a - b, s -
              t, 1) && i <= id) {
            vec.emplace_back(a, -1);
            vec.emplace_back(b, 1);
          }
        } else {
          int s1 = ori(a, s, t) == 1, s2 = ori(b, s, t)
              == 1;
          if (s1 ^ s2) {
            auto p = lines_intersect({a, b}, {s, t});
            vec.emplace_back(p, s1 ? 1 : -1);
          }
        }
      }
    }
    sort(all(vec), [&](auto i, auto j) {
      return cmp_line(i.first, j.first, s, t); });
    int base = 1 << 30; Pt lst(0, 0);
    for (auto [cur, val] : vec) {
      if (!base) seg.emplace_back(lst, cur);
      lst = cur, base += val;
    }
  }
  return seg;
} // 704477
double polys_union_area(vector<vector<Pt>> poly) {
  double res = 0;
  for (int i = 0; i < sz(poly); ++i) {
    auto seg = polys_border(poly, i);
    for (auto [l, r] : seg) res += l ^ r;
  }
  return res / 2;
} // d055fb
```

8.19 Delaunay Triangulation [953c88]

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle. */
struct Edge {
  int id; // oidx[id]
  list<Edge>::iterator twin;
  Edge (int _id = 0) : id(_id) {}
};
struct Delaunay { // 0-base
  int n;
  vector<int> oidx;
  vector<list<Edge>> head; // result udir. graph
  vector<Pt> p;
  Delaunay (vector<Pt> _p) : n(sz(_p)), oidx(n), head(n
      ), p(_p) {
    iota(all(oidx), 0);
    sort(all(oidx), [&](int a, int b) {
      return make_pair(_p[a].x, _p[a].y) < make_pair(_p
          [b].x, _p[b].y); });
    for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
    divide(0, n - 1);
  }
  void add_edge(int u, int v) {
    head[u].push_front(Edge(v));
    head[v].push_front(Edge(u));
    head[u].begin()->twin = head[v].begin();
    head[v].begin()->twin = head[u].begin();
  }
  void divide(int l, int r) {
    if (l == r) return;
    if (l + 1 == r) return add_edge(l, l + 1);
    int mid = (l + r) >> 1, nw[2] = {l, r};
    divide(l, mid), divide(mid + 1, r);
    auto gao = [&](int t) {
      Pt pts[2] = {p[nw[0]], p[nw[1]]};
      for (auto it : head[nw[t]]) {
        int v = ori(pts[1], pts[0], p[it.id]);
        if (v > 0 || (v == 0 && abs2(pts[t ^ 1] - p[it.
            id]) < abs2(pts[1] - pts[0])))
          return nw[t] = it.id, true;
      }
      return false;
    };
    while (gao(0) || gao(1));
    add_edge(nw[0], nw[1]); // add tangent
    while (true) {
      Pt pts[2] = {p[nw[0]], p[nw[1]]};
      int ch = -1, sd = 0;
      for (int t = 0; t < 2; ++t)
        for (auto it : head[nw[t]])
          if (ori(pts[0], pts[1], p[it.id]) > 0 && (ch
              == -1 || point_in_cc({pts[0], pts[1], p[
              ch]}, p[it.id])))
            ch = it.id, sd = t;
      if (ch == -1) break; // upper common tangent
      for (auto it = head[nw[sd]].begin(); it != head[
          nw[sd]].end(); )
        if (lines_intersect_check({pts[sd], p[it->id]},
             0, {pts[sd ^ 1], p[ch]}, 0, 1))
          head[it->id].erase(it->twin), head[nw[sd]].
              erase(it++);
        else ++it;
      nw[sd] = ch, add_edge(nw[0], nw[1]);
    }
  }
};
```

8.20 Triangulation Vonoroi [46f248]

```
// all coord. is even, half plane intersection
auto build_voronoi_line(vector<Pt> arr) {
  int n = sz(arr);
  Delaunay tool(arr);
  vector<vector<Line>> vec(n);
  for (int i = 0; i < n; ++i)
    for (auto e : tool.head[i]) {
      int u = tool.oidx[i], v = tool.oidx[e.id];
      Pt m = (arr[v] + arr[u]) / 2, d = perp(arr[v] -
          arr[u]);
      vec[u].pb(Line{m, m + d});
    }
  return vec;
}
```

8.21 External Bisector [cafb92]

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
  Pt L1 = p2 - p1, L2 = p3 - p1;
  L2 = L2 * abs(L1) / abs(L2);
  return L1 + L2;
}
```

8.22 Intersection Area of Polygon and Circle [000043]

```
double _area(Pt pa, Pt pb, double r){
  if (abs(pa) < abs(pb)) swap(pa, pb);
```

```
  if (abs(pb) < eps) return 0;
  double S, h, theta;
  double a = abs(pb), b = abs(pa), c = abs(pb - pa);
  double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
  double cosC = (pa * pb) / a / b, C = acos(cosC);
  if (a > r) {
    S = (C / 2) * r * r;
    h = a * b * sin(C) / c;
    if (h < r && B < PI / 2) S -= (acos(h / r) * r * r
        - h * sqrt(r * r - h * h));
  } else if (b > r) {
    theta = PI - B - asin(sin(B) / r * a);
    S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
        * r;
  } else S = 0.5 * sin(C) * a * b;
  return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
  double S = 0; int n = sz(poly);
  for (int i = 0; i < n; ++i)
    S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
         ori(O, poly[i], poly[(i + 1) % n]);
  return fabs(S);
}
```

8.23 3D Point

```
struct Pt {
  double x, y, z;
  Pt(double _x = 0, double _y = 0, double _z = 0): x(_x
      ), y(_y), z(_z){}
  Pt operator + (const Pt &o) const
  { return Pt(x + o.x, y + o.y, z + o.z); }
  Pt operator - (const Pt &o) const
  { return Pt(x - o.x, y - o.y, z - o.z); }
  Pt operator * (const double &k) const
  { return Pt(x * k, y * k, z * k); }
  Pt operator / (const double &k) const
  { return Pt(x / k, y / k, z / k); }
  double operator * (const Pt &o) const
  { return x * o.x + y * o.y + z * o.z; }
  Pt operator ^ (const Pt &o) const
  { return {Pt(y * o.z - z * o.y, z * o.x - x * o.z, x
      * o.y - y * o.x)}; }
};
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
Pt cross3(Pt a, Pt b, Pt c)
{ return (b - a) ^ (c - a); }
double area(Pt a, Pt b, Pt c)
{ return abs(cross3(a, b, c)); }
double volume(Pt a, Pt b, Pt c, Pt d)
{ return cross3(a, b, c) * (d - a); }
bool coplaner(Pt a, Pt b, Pt c, Pt d)
{ return sign(volume(a, b, c, d)) == 0; }
Pt proj(Pt o, Pt a, Pt b, Pt c) // o proj to plane abc
{ Pt n = cross3(a, b, c);
  return o - n * ((o - a) * (n / abs2(n)));}
Pt line_plane_intersect(Pt u, Pt v, Pt a, Pt b, Pt c) {
  // intersection of line uv and plane abc
  Pt n = cross3(a, b, c);
  double s = n * (u - v);
  if (sign(s) == 0) return {-1, -1, -1}; // not found
  return v + (u - v) * ((n * (a - v)) / s); }
```

8.24 3D Convex Hull [cc038d]

```
struct Face {
  int a, b, c;
  Face(int _a, int _b, int _c) : a(_a), b(_b), c(_c) {}
};
auto preprocess(auto pts) {
  auto G = pts.begin();
  vector<int> id;
  auto fail = tuple{-1, -1, -1, id};
  int a = find_if(all(pts), [&](Pt z) {
    return z != *G; }) - G;
  if (a == sz(pts)) return fail;
  int b = find_if(all(pts), [&](Pt z) {
    return cross3(*G, pts[a], z) != Pt(0, 0, 0); }) -G;
  if (b == sz(pts)) return fail;
  int c = find_if(all(pts), [&](Pt z) {
    return sign(volume(*G, pts[a], pts[b], z)) != 0; })
      - G;
  if (c == sz(pts)) return fail;
  for (int i = 0; i < sz(pts); i++)
    if (i != a && i != b && i != c) id.pb(i);
  return tuple{a, b, c, id};
}
// return the faces with pts indexes
vector<Face> convex_hull_3D(vector<Pt> pts) {
  int n = sz(pts);
  if (n <= 3) return {}; // be careful about edge case
  vector<Face> now;
  vector<vector<int>> z(n, vector<int>(n));
  auto [a, b, c, ord] = preprocess(pts);
  if (a == -1) return {};
  now.emplace_back(a, b, c); now.emplace_back(c, b, a);
  for (auto i : ord) {
    vector<Face> nxt;
    for (auto &f : now) {
      auto v = volume(pts[f.a], pts[f.b], pts[f.c], pts
          [i]);
      if (sign(v) <= 0) nxt.pb(f);
      z[f.a][f.b] = z[f.b][f.c] = z[f.c][f.a] = sign(v)
          ;
    }
    auto F = [&](int x, int y) {
      if (z[x][y] > 0 && z[y][x] <= 0)
        nxt.emplace_back(x, y, i);
    };
    for (auto &f : now)
      F(f.a, f.b), F(f.b, f.c), F(f.c, f.a);
    now = nxt;
  }
  return now;
}
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case
// test @ SPOJ CH3D
// double area = 0, vol = 0; // surface area / volume
// for (auto [a, b, c]: faces)
//   area += abs(ver(p[a], p[b], p[c]))/2.0,
//   vol += volume(P3(0, 0, 0), p[a], p[b], p[c])/6.0;
```

9 Else

9.1 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue <int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1);// 4 0
bitset <100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope <int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```
ll next_perm(ll v) { ll t = v | (v - 1);
  return (t + 1) |
    (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1)); }
```

9.3 Dynamic Programming Condition

9.3.1 Totally Monotone (Concave/Convex)

$\forall i < i', j < j',\ B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
$\forall i < i', j < j',\ B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

9.3.2 Monge Condition (Concave/Convex)

$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

9.3.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$$

9.4 Smawk Algorithm [6edb6c]

```
ll f(int l, int r) { }
bool select(int r, int u, int v) {
  // if f(r, v) is better than f(r, u), return true
  return f(r, u) < f(r, v);
}
// For all 2x2 submatrix: (x < y => y is better than x)
// If M[1][0] < M[1][1], M[0][0] < M[0][1]
// If M[1][0] == M[1][1], M[0][0] <= M[0][1]
// M[i][ans_i] is the best value in the i-th row
vector<int> solve(vector<int> &r, vector<int> &c) {
  const int n = sz(r);
  if (n == 0) return {};
  vector <int> c2;
  for (const int &i : c) {
    while (!c2.empty() && select(r[sz(c2) - 1], c2.back(), i)) c2.pop_back();
    if (sz(c2) < n) c2.pb(i);
  }
  vector <int> r2;
  for (int i = 1; i < n; i += 2) r2.pb(r[i]);
  const auto a2 = solve(r2, c2);
  vector <int> ans(n);
  for (int i = 0; i < sz(a2); i++)
    ans[i * 2 + 1] = a2[i];
  int j = 0;
  for (int i = 0; i < n; i += 2) {
    ans[i] = c2[j];
    const int end = i + 1 == n ? c2.back() : ans[i + 1];
    while (c2[j] != end) {
      j++;
      if (select(r[i], ans[i], c2[j])) ans[i] = c2[j];
    }
  }
  return ans;
}
vector<int> smawk(int n, int m) {
  vector<int> row(n), col(m);
  iota(all(row), 0), iota(all(col), 0);
  return solve(row, col);
}
```

9.5 Slope Trick [4969a5]

```
template<typename T>
struct slope_trick_convex {
  T minn = 0, ground_l = 0, ground_r = 0;
  priority_queue<T, vector<T>, less<T>> left;
  priority_queue<T, vector<T>, greater<T>> right;
  slope_trick_convex() {left.push(numeric_limits<T>::min() / 2), right.push(numeric_limits<T>::max() / 2);}
  void push_left(T x) {left.push(x - ground_l);}
  void push_right(T x) {right.push(x - ground_r);}
  //add a line with slope 1 to the right starting from x
  void add_right(T x) {
    T l = left.top() + ground_l;
    if (l <= x) push_right(x);
    else push_left(x), push_right(l), left.pop(), minn += l - x;
  }
  //add a line with slope -1 to the left starting from x
  void add_left(T x) {
    T r = right.top() + ground_r;
    if (r >= x) push_left(x);
    else push_right(x), push_left(r), right.pop(), minn += x - r;
  }
  //val[i]=min(val[j]) for all i-l<=j<=i+r
  void expand(T l, T r) {ground_l -= l, ground_r += r;}
  void shift_up(T x) {minn += x;}
  T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() + ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
      vector<T> trash;
      T cur_val = minn, slope = 1, res;
      while (1) {
        trash.pb(left.top());
        left.pop();
        if (left.top() + ground_l <= x) {
          res = cur_val + slope * (l - x);
          break;
        }
        cur_val += slope * (l - (left.top() + ground_l));
        l = left.top() + ground_l;
        slope += 1;
      }
      for (auto i : trash) left.push(i);
      return res;
    }
    if (x > r) {
      vector<T> trash;
      T cur_val = minn, slope = 1, res;
      while (1) {
        trash.pb(right.top());
        right.pop();
        if (right.top() + ground_r >= x) {
          res = cur_val + slope * (x - r);
          break;
        }
        cur_val += slope * ((right.top() + ground_r) - r);
        r = right.top() + ground_r;
        slope += 1;
      }
      for (auto i : trash) right.push(i);
      return res;
    }
    assert(0);
  }
};
```

9.6 ALL LCS [ba9cc9]

```
void all_lcs(string s, string t) { // 0-base
  vector<int> h(sz(t));
  iota(all(h), 0);
  for (int a = 0; a < sz(s); ++a) {
    int v = -1;
    for (int c = 0; c < sz(t); ++c)
      if (s[a] == t[c] || h[c] < v)
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

9.7 Hilbert Curve [1274a3]

```
ll hilbert(int n, int x, int y) {
  ll res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
} // n = 2^k
```

9.8 Line Container [673ffd]

```cpp
// only works for integer coordinates!! maintain max
struct Line {
  mutable ll a, b, p;
  bool operator<(const Line &rhs) const { return a <
      rhs.a; }
  bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
  static const ll kInf = 1e18;
  ll Div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a
       % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = kInf; return 0; }
    if (x->a == y->a) x->p = x->b > y->b ? kInf : -kInf
        ;
    else x->p = Div(y->b - x->b, x->a - y->a);
    return x->p >= y->p;
  }
  void addline(ll a, ll b) { // ax + b
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
        isect(x, erase(y));
  }
  ll query(ll x) {
    auto l = *lower_bound(x);
    return l.a * x + l.b;
  }
};
```

9.9 Min Plus Convolution [57ecb0]

```cpp
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution(vector<int> &a, vector
    <int> &b) {
  int n = sz(a), m = sz(b);
  vector<int> c(n + m - 1, INF);
  auto dc = [&](auto Y, int l, int r, int jl, int jr) {
    if (l > r) return;
    int mid = (l + r) / 2, from = -1, &best = c[mid];
    for (int j = jl; j <= jr; ++j)
      if (int i = mid - j; i >= 0 && i < n)
        if (best > a[i] + b[j])
          best = a[i] + b[j], from = j;
    Y(Y, l, mid - 1, jl, from);
    Y(Y, mid + 1, r, from, jr);
  };
  return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

9.10 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S. Otherwise for each $x \in S, y \notin S$, create edges

- $x \to y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \to x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2, and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.11 Simulated Annealing

```cpp
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
  // ans: answer, nw: current value, rnd(): mt19937 rnd
      ()
  if (exp(-(nw - ans) / factor) >= (double)(rnd() %
      base) / base)
    ans = nw;
  factor *= 0.99995;
}
```

9.12 Bitset LCS

```cpp
cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
  cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
  cin >> x, (g = f) |= p[x];
  f.shiftLeftByOne(), f.set(0);
  ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';
```

9.13 Binary Search On Fraction [765c5a]

```cpp
struct Q {
  ll p, q;
  Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(ll N) {
  Q lo{0, 1}, hi{1, 0};
  if (pred(lo)) return lo;
  assert(pred(hi));
  bool dir = 1, L = 1, H = 1;
  for (; L || H; dir = !dir) {
    ll len = 0, step = 1;
    for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
      if (Q mid = hi.go(lo, len + step);
          mid.p > N || mid.q > N || dir ^ pred(mid))
        t++;
      else len += step;
    swap(lo, hi = hi.go(lo, len));
    (dir ? L : H) = !!len;
  }
  return dir ? hi : lo;
}
```

9.14 Cyclic Ternary Search [9017cc]

```cpp
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
  if (n == 1) return 0;
  int l = 0, r = n; bool rv = pred(1, 0);
  while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
    else l = m;
  }
  return pred(l, r % n) ? l : r % n;
}
```

9.15 Tree Hash [34aae5]

```cpp
ull seed;
ull shift(ull x) { x ^= x << 13; x ^= x >> 7;
  x ^= x << 17; return x; }
ull dfs(int u, int f) {
  ull sum = seed;
  for (int i : G[u]) if (i != f)
    sum += shift(dfs(i, u));
  return sum;
}
```

9.16 Python Misc

```python
from [decimal, fractions, math, random] import *
arr = list(map(int, input().split())) # input
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
    ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
S = set(), S.add((a, b)), S.remove((a, b)) # set
if not (a, b) in S:
D = dict(), D[(a, b)] = 1, del D[(a, b)] # dict
for (a, b) in D.items():
arr = [randint(1, C) for i in range(N)]
choice([8, 6, 4, 1]) # random pick one
```