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1.6 Pragma

```
#pragma GCC optimize("Ofast,inline,unroll-loops")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")
```

1.7 Fast IO

```
#include<unistd.h>
char OB[65536]; int OP;
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)
        ) == buf ? -1 : *p++;
}
inline int R() {
    static char c;
    while((c = RC()) < '0'); int a = c ^ '0';
    while((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

2 Data Structure

2.1 Leftist Tree

```
struct node {
    ll rk, data, sz, sum;
    node *l, *r;
    node(ll k) : rk(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }
ll rk(node *p) { return p ? p->rk : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}
```

2.2 Splay Tree

```
struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    ll v[N];
    Splay() {}
    void init() {
        rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
        sz[0] = 1, v[0] = inf;
    }
    int newnode(int p, int x) {
        int id = _id++;
        v[id] = x, pa[id] = p;
        ch[id][0] = ch[id][1] = -1, sz[id] = 1;
        return id;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i, gp = pa[p], c =
            ch[i][!x];
        sz[p] -= sz[i], sz[i] += sz[p];
        if (~c) sz[p] += sz[c], pa[c] = p;
        ch[p][x] = c, pa[p] = i;
        pa[i] = gp, ch[i][!x] = p;
        if (~gp) ch[gp][ch[gp][1] == p] = i;
    }
}
```

```

}
void splay(int i) {
    while (~pa[i]) {
        int p = pa[i];
        if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1]
            == i ? i : p);
        rotate(i);
    }
    rt = i;
}
int lower_bound(int x) {
    int i = rt, last = -1;
    while (true) {
        if (v[i] == x) return splay(i), i;
        if (v[i] > x) {
            last = i;
            if (ch[i][0] == -1) break;
            i = ch[i][0];
        }
        else {
            if (ch[i][1] == -1) break;
            i = ch[i][1];
        }
    }
    splay(i);
    return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
};
```

2.3 Link Cut Tree

```
// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N], sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in
    // virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0])
            sz[i] += sz[ch[i][0]], mx[i] = max(mx[i], mx[ch[i][0]]);
        if (ch[i][1])
            sz[i] += sz[ch[i][1]], mx[i] = max(mx[i], mx[ch[i][1]]);
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
            rev[i] = false;
    }
    void reverse(int i) {
```

```

    if (!i) return;
    swap(ch[i][0], ch[i][1]);
    rev[i] ^= true;
}
bool isrt(int i) { // rt of splay
    if (!pa[i]) return true;
    return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
}
void rotate(int i) {
    int p = pa[i], x = ch[p][1] == i, c = ch[i][!x], gp = pa[p];
    if (ch[gp][0] == p) ch[gp][0] = i;
    else if (ch[gp][1] == p) ch[gp][1] = i;
    pa[i] = gp, ch[i][!x] = p, pa[p] = i;
    ch[p][x] = c, pa[c] = p;
    pull(p), pull(i);
}
void splay(int i) {
    vector<int> anc;
    anc.push_back(i);
    while (!isrt(anc.back())) anc.push_back(pa[anc.back()]);
    while (!anc.empty()) push(anc.back()), anc.pop_back();
    while (!isrt(i)) {
        int p = pa[i];
        if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1] == p ? i : p);
        rotate(i);
    }
}
void access(int i) {
    int last = 0;
    while (i) {
        splay(i);
        if (ch[i][1])
            sz2[i] += sz[ch[i][1]];
        sz2[i] -= sz[last];
        ch[i][1] = last;
        pull(i), last = i, i = pa[i];
    }
}
void makert(int i) {
    access(i), splay(i), reverse(i);
}
void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
}
void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
}
int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
}
};

```

2.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
}

```

```

    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}

```

2.5 Persistent Segment Tree

```

struct Seg {
    // Persistent Segment Tree, single point modify,
    // range query sum
    // 0-indexed, [l, r)
    static Seg mem[M], *pt;
    int l, r, m, val;
    Seg* ch[2];
    Seg() = default;
    Seg(int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
        val(0) {
        if (r - l > 1) {
            ch[0] = new (pt++) Seg(l, m);
            ch[1] = new (pt++) Seg(m, r);
        }
    }
    void pull() { val = ch[0]->val + ch[1]->val; }
    Seg* modify(int p, int v) {
        Seg *now = new (pt++) Seg(*this);
        if (r - l == 1) {
            now->val = v;
        } else {
            now->ch[p >= m] = ch[p >= m]->modify(p, v);
            now->pull();
        }
        return now;
    }
}

```

```

int query(int a, int b) {
    if (a <= 1 && r <= b) return val;
    int ans = 0;
    if (a < m) ans += ch[0]->query(a, b);
    if (m < b) ans += ch[1]->query(a, b);
    return ans;
}
} Seg::mem[M], *Seg::pt = mem;
// Init Tree
Seg *root = new (Seg::pt++) Seg(0, n);

```

2.6 2D Segment Tree

```

// 2D range add, range sum in Log^2
struct seg {
    int l, r;
    ll sum, lz;
    seg *ch[2]{};
    seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
    void push() {
        if (lz) ch[0]->add(l, r, lz), ch[1]->modify(l, r,
            lz), lz = 0;
    }
    void pull() {sum = ch[0]->sum + ch[1]->sum;}
    void add(int _l, int _r, ll d) {
        if (_l <= 1 && r <= _r) {
            sum += d * (r - l);
            lz += d;
            return;
        }
        if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
            new seg(l + r >> 1, r);
        push();
        if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
        if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
        pull();
    }
    ll qsum(int _l, int _r) {
        if (_l <= 1 && r <= _r) return sum;
        if (!ch[0]) return lz * (min(r, _r) - max(l, _l));
        push();
        ll res = 0;
        if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
        if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
        return res;
    }
};
struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2]{};
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N) {
        if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch[1] =
            new seg2(l + r >> 1, r);
    }
    void add(int _l, int _r, int _l2, int _r2, ll d) {
        v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
        if (_l <= 1 && r <= _r) {
            lz.add(_l2, _r2, d);
            return;
        }
        if (_l < l + r >> 1) ch[0]->add(_l, _r, _l2, _r2, d);
        if (l + r >> 1 < _r) ch[1]->add(_l, _r, _l2, _r2, d);
    }
    ll qsum(int _l, int _r, int _l2, int _r2) {
        ll res = v.qsum(_l2, _r2);
        if (_l <= 1 && r <= _r) return res;
        res += lz.qsum(_l2, _r2) * (min(r, _r) - max(l, _l));
        if (_l < l + r >> 1) res += ch[0]->query(_l, _r,
            _l2, _r2);
        if (l + r >> 1 < _r) res += ch[1]->query(_l, _r,
            _l2, _r2);
        return res;
    }
};

```

2.7 Zkw

```

ll mx[N << 1], sum[N << 1], lz[N << 1];
void add(int l, int r, ll d) { // [L, r), 0-based
    int len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <= 1) {
        sum[l] += cntl * d, sum[r] += cnt[r] * d;
        if (len > 1) {
            mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
            mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
        }
        if (~l & 1)
            sum[l ^ 1] += d * len, mx[l ^ 1] += d, lz[l ^ 1]
                += d, cntl += len;
        if (r & 1)
            sum[r ^ 1] += d * len, mx[r ^ 1] += d, lz[r ^ 1]
                += d, cntr += len;
    }
    sum[l] += cntl * d, sum[r] += cntr * d;
    if (len > 1) {
        mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
        mx[r] = max(mx[r << 1], mx[r << 1 | 1]) + lz[r];
    }
    cntl += cntr;
    for (l >>= 1; l; l >>= 1) {
        sum[l] += cntl * d;
        mx[l] = max(mx[l << 1], mx[l << 1 | 1]) + lz[l];
    }
}
ll qsum(int l, int r) {
    ll res = 0, len = 1, cntl = 0, cntr = 0;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1,
        len <= 1) {
        res += cntl * lz[l] + cntr * lz[r];
        if (~l & 1) res += sum[l ^ 1], cntl += len;
        if (r & 1) res += sum[r ^ 1], cntr += len;
    }
    res += cntl * lz[l] + cntr * lz[r];
    cntl += cntr;
    for (l >>= 1; l; l >>= 1) res += cntl * lz[l];
    return res;
}
ll qmax(int l, int r) {
    ll maxl = -INF, maxr = -INF;
    for (l += N, r += N + 1; l ^ r ^ 1; l >>= 1, r >>= 1)
        {
            maxl += lz[l], max[r] += lz[r];
            if (~l & 1) maxl = max(maxl, mx[l ^ 1]);
            if (r & 1) maxr = max(maxr, mx[r ^ 1]);
        }
    maxl = max(maxl + lz[l], maxr + lz[r]);
    for (l >>= 1; l; l >>= 1) maxl += lz[l];
    return maxl;
}

```

2.8 Chtholly Tree

```

struct ChthollyTree {
    struct interval {
        int l, r;
        ll v;
        interval(int _l, int _r, ll _v) : l(_l), r(_r), v(_v) {}
    };
    struct cmp {
        bool operator () (const interval &a, const interval
            &b) const {
            return a.l < b.l;
        }
    };
    set<interval, cmp> s;
    vector<interval> split(int l, int r) {
        // split into [0, l), [l, r), [r, n) and return [L,
            r)
        vector<interval> del, ans, re;
        auto it = s.lower_bound(interval(l, -1, 0));
        if (it != s.begin() && (it == s.end() || l < it->l))
            {
                --it;
                del.pb(*it);
                if (r < it->r) {
                    re.pb(interval(it->l, l, it->v));
                    ans.pb(interval(l, r, it->v));
                }
            }
    }
}

```

```

    re.pb(interval(r, it->r, it->v));
} else {
    re.pb(interval(it->l, l, it->v));
    ans.pb(interval(l, it->r, it->v));
}
++it;
}
for (; it != s.end() && it->r <= r; ++it) {
    ans.pb(*it);
    del.pb(*it);
}
if (it != s.end() && it->l < r) {
    del.pb(*it);
    ans.pb(interval(it->l, r, it->v));
    re.pb(interval(r, it->r, it->v));
}
for (interval &i : del)
    s.erase(i);
for (interval &i : re)
    s.insert(i);
return ans;
}
void merge(vector<interval> a) {
    for (interval &i : a)
        s.insert(i);
}
};

```

2.9 Incremental Min Sum

```

struct IncrementalMinSum {
    multiset<int, greater<int>> in;
    multiset<int> out;
    ll sum; int cap;
    DS () : sum(0), cap(0) {}
    void enlarge() {
        if (!out.empty()) {
            int mx = *out.begin();
            sum += mx, in.insert(mx), out.erase(out.begin());
        }
        cap++;
    }
    void insert(int x) {
        if (!cap) {
            out.insert(x);
            return;
        }
        if (in.size() < cap) {
            in.insert(x), sum += x;
            return;
        }
        int mx = *in.begin();
        if (x < mx) {
            sum -= mx, out.insert(mx), in.erase(in.begin());
            sum += x, in.insert(x);
        } else {
            out.insert(x);
        }
    }
    void erase(int x) {
        if (out.find(x) != out.end()) {
            out.erase(out.lower_bound(x));
        } else {
            in.erase(in.lower_bound(x)), sum -= x;
            if (!out.empty()) {
                int mx = *out.begin();
                sum += mx, out.erase(out.begin()), in.insert(mx);
            }
        }
    }
};

```

3 Flow / Matching

3.1 Dinic

```

struct Dinic { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
};

```

```

vector<edge> adj[N];
int s, t, dis[N], cur[N], n;
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < (int)adj[u].size(); ++i)
    {
        edge &e = adj[u][i];
        if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df;
                adj[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int tmp = q.front();
        q.pop();
        for (auto &u : adj[tmp])
            if (!dis[u.to] && u.flow != u.cap) {
                q.push(u.to);
                dis[u.to] = dis[tmp] + 1;
            }
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].clear();
}
void reset() {
    for (int i = 0; i < n; ++i)
        for (auto &j : adj[i]) j.flow = 0;
}
void add_edge(int u, int v, int cap) {
    adj[u].pb(edge{v, cap, 0, (int)adj[v].size()});
    adj[v].pb(edge{u, 0, 0, (int)adj[u].size() - 1});
}
};

```

3.2 Min Cost Max Flow

```

template<typename T>
struct MCMF {
    const T INF = 111 << 60;
    struct edge {
        int v;
        T f, c;
        edge(int _v, T _f, T _c) : v(_v), f(_f), c(_c) {}
    };
    vector<edge> E;
    vector<vector<int>> adj;
    vector<T> dis, pot;
    vector<int> rt;
    int n, s, t;
    MCMF(int _n, int _s, int _t) : n(_n), s(_s), t(_t) {
        adj.resize(n);
    }
    void add_edge(int u, int v, T f, T c) {
        adj[u].pb(E.size()), E.pb(edge(v, f, c));
        adj[v].pb(E.size()), E.pb(edge(u, 0, -c));
    }
    bool SPFA() {
        rt.assign(n, -1), dis.assign(n, INF);
        vector<bool> vis(n, false);
    }
};

```

```

queue <int> q;
q.push(s), dis[s] = 0, vis[s] = true;
while (!q.empty()) {
    int v = q.front(); q.pop();
    vis[v] = false;
    for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
        dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v];
        rt[E[id].v] = id;
        if (!vis[E[id].v]) vis[E[id].v] = true, q.push(E[id].v);
    }
}
return dis[t] != INF;
}
bool dijkstra() {
    rt.assign(n, -1), dis.assign(n, INF);
    priority_queue <pair <T, int>, vector <pair <T, int>, greater <pair <T, int>>> pq;
    dis[s] = 0, pq.emplace(dis[s], s);
    while (!pq.empty()) {
        int d, v; tie(d, v) = pq.top(); pq.pop();
        if (dis[v] < d) continue;
        for (int id : adj[v]) if (E[id].f > 0 && dis[E[id].v] > dis[v] + E[id].c + pot[v] - pot[E[id].v]) {
            dis[E[id].v] = dis[v] + E[id].c + pot[v] - pot[E[id].v];
            rt[E[id].v] = id;
            pq.emplace(dis[E[id].v], E[id].v);
        }
    }
    return dis[t] != INF;
}
pair <T, T> solve() {
    pot.assign(n, 0);
    T cost = 0, flow = 0;
    bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; ++i) {
            dis[i] += pot[i] - pot[s];
        }
        T add = INF;
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            add = min(add, E[rt[i]].f);
        }
        for (int i = t; i != s; i = E[rt[i] ^ 1].v) {
            E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
        }
        flow += add, cost += add * dis[t];
        fr = false;
        swap(dis, pot);
    }
    return make_pair(flow, cost);
}
};

```

3.3 Kuhn Munkres

```

template <typename T>
struct KM { // 0-based
    T w[N][N], hl[N], hr[N], slk[N];
    T fl[N], fr[N], pre[N]; int n;
    bool vl[N], vr[N];
    const T INF = 1e9;
    queue <int> q;
    KM(int _n) : n(_n) {
        for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)
            w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x]) return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {

```

```

        fill(slk, slk + n, INF), fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        q.push(s), vr[s] = 1;
        while (1) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
            }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x) if (!vl[x] && !slk[x] && !check(x)) return;
        }
    }
    T solve() {
        fill(fl, fl + n, -1), fill(fr, fr + n, -1), fill(hr, hr + n, 0);
        for (int i = 0; i < n; ++i) hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) bfs(i);
        T res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

3.4 SW Min Cut

```

template <typename T>
struct SW { // 0-based
    T g[N][N], sum[N]; int n;
    bool vis[N], dead[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) fill(g[i], g[i] + n, 0);
        fill(dead, dead + n, false);
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w;
    }
    T solve() {
        T ans = 1 << 30;
        for (int round = 0; round + 1 < n; ++round) {
            fill(vis, vis + n, false), fill(sum, sum + n, 0);
            int num = 0, s = -1, t = -1;
            while (num < n - round) {
                int now = -1;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    if (now == -1 || sum[now] < sum[i]) now = i;
                }
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i) if (!vis[i] && !dead[i]) {
                    sum[i] += g[now][i];
                }
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i) {
                g[i][s] += g[i][t];
                g[s][i] += g[t][i];
            }
            dead[t] = true;
        }
        return ans;
    }
};

```


3.5 Gomory Hu Tree

```
vector <array <int, 3>> GomoryHu(vector <vector <pii>>
    adj, int n) {
    // Tree edge min -> mincut (0-based)
    Dinic flow(n);
    for (int i = 0; i < n; ++i) for (auto [j, w] : adj[i])
        flow.add_edge(i, j, w);
    flow.record();
    vector <array <int, 3>> ans;
    vector <int> rt(n);
    for (int i = 0; i < n; ++i) rt[i] = 0;
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset(); // clear flows on all edge
        ans.push_back({i, t, flow.solve(i, t)});
        flow.runbfs(i);
        for (int j = i + 1; j < n; ++j) if (rt[j] == t &&
            flow.vis[j]) {
            rt[j] = i;
        }
    }
    return ans;
}
```

3.6 Blossom

```
struct Matching { // 0-based
    int fa[N], pre[N], match[N], s[N], v[N], n, tk;
    vector <int> g[N];
    queue <int> q;
    Matching(int _n) : n(_n), tk(0) {
        for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
        for (int i = 0; i < n; ++i) g[i].clear();
    }
    void add_edge(int u, int v) {
        g[u].push_back(v), g[v].push_back(u);
    }
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (v[x] == tk) return x;
                v[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
    void blossom(int x, int y, int l) {
        while (Find(x) != l) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            if (fa[x] == x) fa[x] = 1;
            if (fa[y] == y) fa[y] = 1;
            x = pre[y];
        }
    }
    bool bfs(int r) {
        for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
        while (!q.empty()) q.pop();
        q.push(r);
        s[r] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int u : g[x]) {
                if (s[u] == -1) {
                    pre[u] = x, s[u] = 1;
                    if (match[u] == n) {
                        for (int a = u, b = x, last; b != n; a =
                            last, b = pre[a])
                            last = match[b], match[b] = a, match[a] =
                                b;
                        return true;
                    }
                }
                q.push(match[u]);
                s[match[u]] = 0;
            }
        }
        return false;
    }
};
```

```
int l = lca(u, x);
blossom(x, u, l);
blossom(u, x, l);
}
}
return false;
}
int solve() {
    int res = 0;
    for (int x = 0; x < n; ++x) {
        if (match[x] == n) res += bfs(x);
    }
    return res;
}
};
```

3.7 Weighted Blossom

```
struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w): u(u), v(v), w(w) {}
    };
    int n, n_x;
    edge g[maxn * 2][maxn * 2];
    int lab[maxn * 2];
    int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
        pa[maxn * 2];
    int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
        maxn * 2];
    vector<int> flo[maxn * 2];
    queue<int> q;
    int e_delta(const edge &e) { return lab[e.u] + lab[e.
        v] - g[e.u][e.v].w * 2; }
    void update_slack(int u, int x) { if (!slack[x] ||
        e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
        [x] = u; }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (size_t i = 0; i < flo[x].size(); i++)
            q_push(flo[x][i]);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (size_t i = 0; i < flo[x].size(); ++
            i) set_st(flo[x][i], b);
    }
    int get_pr(int b, int xr) {
        int pr = find(flo[b].begin(), flo[b].end(), xr) -
            flo[b].begin();
        if (pr % 2 == 1) {
            reverse(flo[b].begin() + 1, flo[b].end());
            return (int)flo[b].size() - pr;
        }
        return pr;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        edge e = g[u][v];
        int xr = flo_from[u][e.u], pr = get_pr(u, xr);
        for (int i = 0; i < pr; ++i) set_match(flo[u][i],
            flo[u][i ^ 1]);
        set_match(xr, v);
        rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
            end());
    }
    void augment(int u, int v) {
        for (; ; ) {
            int xnv = st[match[u]];
            set_match(u, v);
            if (!xnv) return;
        }
    }
};
```

```

    set_match(xnv, st[pa[xnv]]);
    u = st[pa[xnv]], v = xnv;
}
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
        w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        for (int x = 1; x <= n_x; ++x)
            if (g[b][x].w == 0 || e_delta(g[xs][x]) <
                e_delta(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);

```

```

    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
            q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b]
                / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1) d = min(d, e_delta(g[slack[x]
                    ][x]));
                else if (S[x] == 0) d = min(d, e_delta(g[
                    slack[x]][x]) / 2);
            }
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d) return 0;
                lab[u] -= d;
            } else if (S[st[u]] == 1) lab[u] += d;
        }
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b) {
                if (S[st[b]] == 0) lab[b] += d * 2;
                else if (S[st[b]] == 1) lab[b] -= d * 2;
            }
        q = queue<int>();
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x] && st[slack[x]] != x
                && e_delta(g[slack[x]][x]) == 0)
                if (on_found_edge(g[slack[x]][x])) return
                    true;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)
                expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].
        clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int ui, int vi, int wi) { g[ui][vi].w =
    g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
};

```

3.8 Flow Model

• Maximum/Minimum flow with lower bound / Circulation problem

1. Construct super source S and sink T .
 2. For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 3. For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 4. If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 1. Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 2. DFS from unmatched vertices in X .
 3. $x \in X$ is chosen iff x is unvisited.
 4. $y \in Y$ is chosen iff y is visited.
 - Maximum density induced subgraph
 1. Binary search on answer, suppose we're checking answer T .
 2. Construct a max flow model, let K be the sum of all weights.
 3. Connect source $s \rightarrow v$, $v \in G$ with capacity K .
 4. For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w .
 5. For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$.
 6. T is a valid answer if the maximum flow $f < K|V|$.
 - Minimum weight edge cover
 1. For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
 2. Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 3. Find the minimum weight perfect matching on G' .
 - Project selection problem
 1. If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 3. The mincut is equivalent to the maximum profit of a subset of projects.
 - 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

1. Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
2. Create edge (x, y) with capacity c_{xy} .
3. Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4 Graph

4.1 Heavy-Light Decomposition

```
vector<int> dep, pa, sz, ch, hd, id;
int _id;
void dfs(int i, int p) {
    dep[i] = ~p ? dep[p] + 1 : 0;
    pa[i] = p, sz[i] = 1, ch[i] = -1;
    for (int j : g[i])
        if (j != p) {
            dfs(j, i);
            if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
            sz[i] += sz[j];
        }
}
void hld(int i, int p, int h) {
    hd[i] = h;
    id[i] = _id++;
    if (~ch[i]) hld(ch[i], i, h);
    for (int j : g[i]) if (j != p && j != ch[i])
        hld(j, i, j);
}
void query(int i, int j) {
    while (hd[i] != hd[j]) {
        if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
        query2(id[hd[i]], id[i] + 1, i = pa[hd[i]]);
    }
    if (dep[i] < dep[j]) swap(i, j);
    query2(id[j], id[i] + 1);
}
```

4.2 Centroid Decomposition

```
vector<vector<int>> dis;
vector<int> pa, sz;
vector<bool> vis;
void dfs_sz(int i, int p) {
    sz[i] = 1;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_sz(j, i), sz[i] += sz[j];
}
void cen(int i, int p, int _n) {
    for (int j : g[i]) if (j != p && !vis[j] && sz[j] > _n / 2)
        return cen(j, i, _n);
    return i;
}
void dfs_dis(int i, int p, int d) { // from i to
    ancestor with depth d
    dis[i][d] = ~p ? dis[p][d] + 1 : 0;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
    dfs_sz(i), i = cen(i, -1, sz[i]);
    vis[i] = true, pa[i] = p;
    dfs_dis(i, -1, d);
    for (int j : g[i]) if (!vis[j])
        cd(j, i, d + 1);
}
```

4.3 Edge BCC

```
vector<int> low, dep, bcc_id, stk;
vector<bool> vis;
int _id;
void dfs(int i, int p) {
    low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    vis[i] = true;
    for (int j : g[i])
        if (j != p) {
            if (!vis[j])
                dfs(j, i), low[i] = min(low[i], low[j]);
            else
                low[i] = min(low[i], dep[j]);
        }
    if (low[i] == dep[i]) {
        int id = _id++;
        while (stk.back() != i) {
            int x = stk.back();
            stk.pop_back();
            bcc_id[x] = id;
        }
        stk.pop_back();
        bcc_id[i] = id;
    }
}
```

4.4 Block Cut Tree

```
vector<vector<int>> g, _g;
vector<int> dep, low, stk;
void dfs(int i, int p) {
    dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
    stk.push_back(i);
    for (int j : g[i]) if (j != p) {
        if (dep[j] == -1) {
            dfs(j, i), low[i] = min(low[i], low[j]);
            if (low[j] >= dep[i]) {
                int id = _g.size();
                _g.emplace_back();
                while (stk.back() != j) {
                    int x = stk.back();
                    stk.pop_back();
                    _g[x].push_back(id), _g[id].push_back(x);
                }
                stk.pop_back();
                _g[j].push_back(id), _g[id].push_back(j);
                _g[i].push_back(id), _g[id].push_back(i);
            }
        } else low[i] = min(low[i], dep[j]);
    }
}
```

4.5 SCC / 2SAT

```

struct SAT {
    vector<vector<int>> g;
    vector<int> dep, low, scc_id;
    vector<bool> is;
    vector<int> stk;
    int n, _id, _t;
    SAT() {}
    void init(int _n) {
        n = _n, _id = _t = 0;
        g.assign(2 * n, vector<int>());
        dep.assign(2 * n, -1), low.assign(2 * n, -1);
        scc_id.assign(2 * n, -1), is.assign(2 * n, false);
        stk.clear();
    }
    void add_edge(int x, int y) {g[x].push_back(y);}
    int rev(int i) {return i < n ? i + n : i - n;}
    void add_ifthen(int x, int y) {add_clause(rev(x), y);}
    void add_clause(int x, int y) {
        add_edge(rev(x), y);
        add_edge(rev(y), x);
    }
    void dfs(int i) {
        dep[i] = low[i] = _t++;
        stk.push_back(i);
        for (int j : g[i])
            if (scc_id[j] == -1) {
                if (dep[j] == -1)
                    dfs(j);
                low[i] = min(low[i], low[j]);
            }
        if (low[i] == dep[i]) {
            int id = _id++;
            while (stk.back() != i) {
                int x = stk.back();
                stk.pop_back();
                scc_id[x] = id;
            }
            stk.pop_back();
            scc_id[i] = id;
        }
    }
    bool solve() {
        for (int i = 0; i < 2 * n; ++i)
            if (dep[i] == -1)
                dfs(i);
        for (int i = 0; i < n; ++i) {
            if (scc_id[i] == scc_id[i + n]) return false;
            if (scc_id[i] < scc_id[i + n])
                is[i] = true;
            else
                is[i + n] = true;
        }
        return true;
    }
};

```

4.6 Negative Cycle

```

vector<pair<int, long long>> adj[N];
template<typename T>
struct NegativeCycle {
    vector<T> dis;
    vector<int> rt;
    int n; T INF;
    vector<int> cycle;
    NegativeCycle() = default;
    NegativeCycle(int _n) : n(_n), INF(numeric_limits<T>
        ::max()) {
        dis.assign(n, 0), rt.assign(n, -1);
        int relax = -1;
        for (int t = 0; t < n; ++t) {
            relax = -1;
            for (int i = 0; i < n; ++i) {
                for (auto [j, w] : adj[i]) if (dis[j] > dis[i]
                    + w) {
                    dis[j] = dis[i] + w, rt[j] = i;
                    relax = j;
                }
            }
        }
    }
};

```

```

}
if (relax != -1) {
    int s = relax;
    for (int i = 0; i < n; ++i) s = rt[s];
    vector<bool> vis(n, false);
    while (!vis[s]) {
        cycle.push_back(s), vis[s] = true;
        s = rt[s];
    }
    reverse(cycle.begin(), cycle.end());
}
};

```

4.7 Virtual Tree

```

vector<vector<int>> _g;
vector<int> st, ed, stk;
void solve(vector<int> v) {
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    int sz = v.size();
    for (int i = 0; i < sz - 1; ++i)
        v.push_back(lca(v[i], v[i + 1]));
    sort(all(v), [&](int x, int y) {return st[x] < st[y];});
    v.resize(unique(all(v)) - v.begin());
    stk.clear(); stk.push_back(v[0]);
    for (int i = 1; i < v.size(); ++i) {
        int x = v[i];
        while (ed[stk.back()] < ed[x]) stk.pop_back();
        _g[stk.back()].push_back(x), stk.push_back(x);
    }
    // do something
    for (int i : v) _g[i].clear();
}

```

4.8 Directed MST

```

template<typename T> struct DMST { // 1-based
    T g[maxn][maxn], fw[maxn];
    int n, fr[maxn];
    bool vis[maxn], inc[maxn];
    void clear() {
        for (int i = 0; i < maxn; ++i) {
            for (int j = 0; j < maxn; ++j) g[i][j] = inf;
            vis[i] = inc[i] = false;
        }
    }
    void addedge(int u, int v, T w) {
        g[u][v] = min(g[u][v], w);
    }
    T query(int root, int _n) {
        n = _n;
        if (dfs(root) != n) return -1;
        T ans = 0;
        while (true) {
            for (int i = 1; i <= n; ++i) fw[i] = inf, fr[i] = i;
            for (int i = 1; i <= n; ++i) if (!inc[i]) {
                for (int j = 1; j <= n; ++j) {
                    if (!inc[j] && i != j && g[j][i] < fw[i]) {
                        fw[i] = g[j][i];
                        fr[i] = j;
                    }
                }
            }
            int x = -1;
            for (int i = 1; i <= n; ++i) if (i != root && !
                inc[i]) {
                int j = i, c = 0;
                while (j != root && fr[j] != i && c <= n) ++c,
                    j = fr[j];
                if (j == root || c > n) continue;
                else { x = i; break; }
            }
            if (!~x) {
                for (int i = 1; i <= n; ++i) if (i != root && !
                    inc[i]) ans += fw[i];
                return ans;
            }
            int y = x;
        }
    }
};

```

```

    for (int i = 1; i <= n; ++i) vis[i] = false;
    do { ans += fw[y]; y = fr[y]; vis[y] = inc[y] =
        true; } while (y != x);
    inc[x] = false;
    for (int k = 1; k <= n; ++k) if (vis[k]) {
        for (int j = 1; j <= n; ++j) if (!vis[j]) {
            if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
            if (g[j][k] < inf && g[j][k] - fw[k] < g[
                j][x]) g[j][x] = g[j][k] - fw[k];
        }
    }
}
return ans;
}
int dfs(int now) {
    int r = 1;
    vis[now] = true;
    for (int i = 1; i <= n; ++i) if (g[now][i] < inf &&
        !vis[i]) r += dfs(i);
    return r;
}
};

```

4.9 Dominator Tree

```

struct Dominator_tree {
    int n, id;
    vector<vector<int>> adj, radj, bucket;
    vector<int> sdom, dom, vis, rev, par, rt, mn;
    Dominator_tree(int _n) : n(_n), id(0) {
        adj.resize(n), radj.resize(n), bucket.resize(n);
        sdom.resize(n), dom.resize(n, -1), vis.resize(n,
            -1);
        rev.resize(n), rt.resize(n), mn.resize(n), par.
            resize(n);
    }
    void add_edge(int u, int v) {adj[u].pb(v);}
    int query(int v, bool x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], true);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]]) mn[v] = mn[rt[v]
            ];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : adj[v]) {
            if (vis[u] == -1) dfs(u), par[vis[u]] = vis[v];
            radj[vis[u]].pb(vis[v]);
        }
    }
    void build(int s) {
        dfs(s);
        for (int i = id - 1; ~i; --i) {
            for (int u : radj[i]) {
                sdom[i] = min(sdom[i], sdom[query(u, false)]);
            }
            if (i) bucket[sdom[i]].pb(i);
            for (int u : bucket[i]) {
                int p = query(u, false);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = par[i];
        }
        vector<int> res(n, -1);
        for (int i = 1; i < id; ++i) {
            if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
        }
        for (int i = 1; i < id; ++i) res[rev[i]] = rev[dom[
            i]];
        res[s] = s;
        dom = res;
    }
};

```

5 String

5.1 Aho-Corasick Automaton

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    vector<int> g[N];
    int cnt[N];
    AC() {sz = 0, extend();}
    void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
        return ch[u][v];
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) now = nxt(now, c - 'a');
        cnt[now]++;
        return now;
    }
    void build_fail() {
        queue<int> q;
        for (int i = 0; i < 26; ++i) if (ch[0][i]) {
            to[0][i] = ch[0][i];
            q.push(ch[0][i]);
            g[0].push_back(ch[0][i]);
        }
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int j = 0; j < 26; ++j) {
                to[v][j] = ch[v][j] ? ch[v][j] : to[fail[v]][j
                    ];
            }
            for (int i = 0; i < 26; ++i) if (ch[v][i]) {
                int u = ch[v][i], k = fail[v];
                while (k && !ch[k][i]) k = fail[k];
                if (ch[k][i]) k = ch[k][i];
                fail[u] = k;
                cnt[u] += cnt[k], g[k].push_back(u);
                q.push(u);
            }
        }
    }
    int match(string &s) {
        int now = 0, ans = 0;
        for (char c : s) {
            now = to[now][c - 'a'];
            if (ch[now][c - 'a']) now = ch[now][c - 'a'];
            ans += cnt[now];
        }
        return ans;
    }
};

```

5.2 KMP Algorithm

```

vector<int> build_fail(string s) {
    vector<int> f(s.length() + 1, 0);
    int k = 0;
    for (int i = 1; i < s.length(); ++i) {
        while (k && s[k] != s[i]) k = f[k];
        if (s[k] == s[i]) k++;
        f[i + 1] = k;
    }
    return f;
}
int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < s.length(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == t.length()) ans++, k = f[k];
    }
    return ans;
}

```

5.3 Z Algorithm

```

vector<int> build(string s) {
    int n = s.length();
    vector<int> Z(n);
    int l = 0, r = 0;
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < s.size() && s[Z[i]] == s[i + Z[i]
            ]) {

```

```

    l = i, r = i + Z[i], Z[i]++;
}
}
return Z;
}

```

5.4 Manacher

```

vector<int> manacher(string &s) {
    string t = "^#";
    for (char c : s) t += c, t += '#';
    t += '&';
    int n = t.length();
    vector<int> r(n, 0);
    int C = 0, R = 0;
    for (int i = 1; i < n - 1; ++i) {
        int mirror = 2 * C - i;
        r[i] = (i < R ? min(r[mirror], R - i) : 0);
        while (t[i - 1 - r[i]] == t[i + 1 + r[i]]) r[i]++;
        if (i + r[i] > R) R = i + r[i], C = i;
    }
    return r;
}

```

5.5 Suffix Array

```

int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.length();
    ;
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]] = s[i]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; i >= 0; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++] = sa[i] - k;
        for (int i = n - 1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!(x[a] == x[b] && a + k < n && b + k < n && x[a + k] == x[b + k])) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}
void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = min(lcp[rk[i] + 1], lcp[rk[i] + 2], ..., lcp[rk[j]])
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i] == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}

```

5.6 SAIS

```

namespace sfx {
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
void pre(int *sa, int *c, int n, int z) {
    fill_n(sa, n, 0), copy_n(c, z, x);
}
}

```

```

void induce(int *sa, int *c, int *s, bool *t, int n, int z) {
    copy_n(c, z - 1, x + 1);
    for (int i = 0; i < n; ++i) if (sa[i] && !t[sa[i] - 1]) sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    for (int i = n - 1; i >= 0; --i) if (sa[i] && t[sa[i] - 1]) sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || !equal(s + sa[i], s + p[q[sa[i]] + 1], s + last);
            ns[q[last = sa[i]]] = nmzx += neq;
        }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
vector<int> build(int *s, int n) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    vector<int> sa(n);
    for (int i = 0; i < n; ++i)
        sa[i] = SA[i + 1];
    return sa;
}
}

```

5.7 Suffix Automaton

```

struct SAM {
    int ch[N][26], len[N], link[N], cnt[N], sz;
    // Link -> suffix endpos
    SAM() {len[0] = 0, link[0] = -1, sz = 1;}
    void build(string s) {
        int last = 0;
        for (char c : s) {
            int cur = sz++;
            len[cur] = len[last] + 1;
            int p = last;
            while (~p && !ch[p][c - 'a']) ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) {
                link[cur] = 0;
            } else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q];
                    for (int j = 0; j < 26; ++j) ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q) ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
        }
    }
}

```

```

    cnt[cur]++;
    last = cur;
}
vector<int> p(sz);
iota(all(p), 0);
sort(all(p), [&](int i, int j) {return len[i] > len[j]});
for (int i = 0; i < sz; ++i) cnt[link[p[i]]] += cnt[p[i]];
}
};

```

5.8 Minimum Rotation

```

string rotate(const string &s) {
    int n = s.length();
    string t = s + s;
    int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

5.9 Palindrome Tree

```

struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM(string _s) : s(_s) {
        sz = 0;
        extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst] - 1] != s[i]) lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
                int now = fail[lst];
                while (s[i - len[now] - 1] != s[i]) now = fail[now];
                fail[idx] = ch[now][s[i] - 'a'];
                ch[lst][s[i] - 'a'] = idx;
            }
            lst = ch[lst][s[i] - 'a'], cnt[lst]++;
        }
        void build_count() {
            for (int i = sz - 1; i > 1; --i)
                cnt[fail[i]] += cnt[i];
        }
        int extend() {
            fill(ch[sz], ch[sz] + 26, 0), sz++;
            return sz - 1;
        }
    }
};

```

5.10 Main Lorentz

```

int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, Len.
// substr(L ~ r, Len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l), sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i) to_right[i] = Z[r - m + 1 + i - 1];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = 1; i < m; ++i) to_left[i] = Z[m - i - 1];
}

```

```

reverse(all(sl));
for (int i = 1; i + 1 < m; ++i) {
    int k1 = to_left[i], k2 = to_right[i + 1], len = m - i - 1;
    if (k1 < 1 || k2 < 1 || len < 2) continue;
    int tl = max(1, len - k2), tr = min(len - 1, k1);
    if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
}
Z = buildZ(sr);
for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
reverse(all(sl)), reverse(all(sr));
Z = buildZ(sl + "#" + sr);
for (int i = m; i < r; ++i) to_left[i] = Z[m - 1 + 1 + r - i - 1];
reverse(all(sl)), reverse(all(sr));
for (int i = m; i + 1 < r; ++i) {
    int k1 = to_left[i], k2 = to_right[i + 1], len = i - m + 1;
    if (k1 < 1 || k2 < 1 || len < 2) continue;
    int tl = max(len - k2, 1), tr = min(len - 1, k1);
    if (tl <= tr) rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
}
Z = buildZ(sr + "#" + sl);
for (int i = 1; i < m; ++i) {
    if (Z[r - m + 1 + i - 1] >= m - i) {
        rep.pb({i, i, m - i});
    }
}
}

```

6 Math

6.1 Fraction

```

struct fraction {
    ll n, d;
    fraction(const ll _n=0, const ll _d=1): n(_n), d(_d) {
        ll t = gcd(n, d);
        n /= t, d /= t;
        if (d < 0) n = -n, d = -d;
    }
    fraction operator-() const { return fraction(-n, d); }
    fraction operator+(const fraction &b) const { return fraction(n * b.d + b.n * d, d * b.d); }
    fraction operator-(const fraction &b) const { return fraction(n * b.d - b.n * d, d * b.d); }
    fraction operator*(const fraction &b) const { return fraction(n * b.n, d * b.d); }
    fraction operator/(const fraction &b) const { return fraction(n * b.d, d * b.n); }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};

```

6.2 Miller Rabin / Pollard Rho

```

ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long double)x / p * y) * p + p) % p;}
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
ll Pow(ll a, ll b, ll n) {ll res = 1; for (; b; b >>= 1, a = mul(a, a, n)) if (b & 1) res = mul(res, a, n); return res;}
bool check(ll a, ll d, int s, ll n) {
    a = Pow(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
}

```

```

    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> PollardRho(ll n) {
    map<ll, int> res;
    if (n == 1) return res;
    if (IsPrime(n)) return ++res[n], res;
    ll d = FindFactor(n);
    res = PollardRho(n / d);
    auto res2 = PollardRho(d);
    for (auto [x, y] : res2) res[x] += y;
    return res;
}

```

6.3 Ext GCD

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    pair<ll, ll> res;
    if (a < 0) {
        res = extgcd(-a, b);
        res.first *= -1;
        return res;
    }
    if (b < 0) {
        res = extgcd(a, -b);
        res.second *= -1;
        return res;
    }
    if (b == 0) return {1, 0};
    res = extgcd(b, a % b);
    return {res.second, res.first - res.second * (a / b)};
}

```

6.4 PiCount

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){
    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i]) for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.push_back(i);
}
void init(){
    sieve(V);
    small_pi[0] = 0;
    for(int i = 1; i < V; ++i) small_pi[i] = small_pi[i - 1] + isp[i];
    for(int i = 0; i < M; ++i) dp[0][i] = i;
    for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
        dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i] - 1];
}

```

```

}
ll phi(ll n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(((ll)primes[a - 1]) * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    ll de = phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
    return de;
}
ll PiCount(ll n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a = small_pi[y];
    ll res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n / primes[a]) - PiCount(primes[a] + 1, 0), 0);
    return res;
}

```

6.5 Linear Function Mod Min

```

ll topos(ll x, ll m) {x %= m; if (x < 0) x += m; return x;}
//min value of ax + b (mod m) for x \in [0, n - 1]. O(Log m)
ll min_rem(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    for (ll g = __gcd(a, m); g > 1; ) return g * min_rem(n / g, m / g, a / g, b / g) + (b % g);
    for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            ll lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1], also return min x to get the value. O(Log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g) % (m / g);
    return {mn, x};
}

```

6.6 Floor Sum

```

// sum^{n-1}_0 floor((a * i + b) / m) in Log(n + m + a + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

6.7 Quadratic Residue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
    }
}

```



```

    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
}
return s;
}
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL * g0 * f0 + 1LL * d * (1LL * g1 * f1 %
                p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p
            )) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
}

```

6.8 Simplex

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;
    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]. 1 based
    T a[M][N], b[M], c[N], v, sol[N];
    bool eq(T a, T b) {return fabs(a - b) < eps;}
    bool ls(T a, T b) {return a < b && !eq(a, b);}
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;
        for (int i = 0; i < m; ++i) for (int j = 0; j < n;
            ++j) a[i][j] = 0;
        for (int i = 0; i < m; ++i) b[i] = 0;
        for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
    }
    void pivot(int x, int y) {
        swap(Left[x], Down[y]);
        T k = a[x][y]; a[x][y] = 1;
        vector<int> nz;
        for (int i = 0; i < n; ++i) {
            a[x][i] /= k;
            if (!eq(a[x][i], 0)) nz.push_back(i);
        }
        b[x] /= k;
        for (int i = 0; i < m; ++i) {
            if (i == x || eq(a[i][y], 0)) continue;
            k = a[i][y], a[i][y] = 0;
            b[i] -= k * b[x];
            for (int j : nz) a[i][j] -= k * a[x][j];
        }
        if (eq(c[y], 0)) return;
        k = c[y], c[y] = 0, v += k * b[x];
        for (int i : nz) c[i] -= k * a[x][i];
    }
    // 0: found solution, 1: no feasible solution, 2:
    // unbounded
    int solve() {
        for (int i = 0; i < n; ++i) Down[i] = i;
        for (int i = 0; i < m; ++i) Left[i] = n + i;
        while (1) {
            int x = -1, y = -1;

```

```

            for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x
                == -1 || b[i] < b[x])) x = i;
            if (x == -1) break;
            for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) &&
                (y == -1 || a[x][i] < a[x][y])) y = i;
            if (y == -1) return 1;
            pivot(x, y);
        }
        while (1) {
            int x = -1, y = -1;
            for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y
                == -1 || c[i] > c[y])) y = i;
            if (y == -1) break;
            for (int i = 0; i < m; ++i) if (ls(0, a[i][y]) &&
                (x == -1 || b[i] / a[i][y] < b[x] / a[x][y]))
                x = i;
            if (x == -1) return 2;
            pivot(x, y);
        }
        for (int i = 0; i < m; ++i) if (Left[i] < n) sol[
            Left[i]] = b[i];
        return 0;
    }
};

```

6.9 Linear Programming Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.
 Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.
 \bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.10 Theorem

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there are

$$\frac{(n-2)!}{(d_1-1)!(d_2-1)! \cdots (d_n-1)!}$$

spanning trees.

- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős-Gallai Theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

- Burnside's Lemma

Let X be a set and G be a group that acts on X . For $g \in G$, denote by X^g the elements fixed by g :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- Möbius inversion formula

$$\begin{aligned} - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$.
- Area $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$.

- Chinese Remainder Theorem

- $x \equiv a_i \pmod{m_i}$
- $M = \prod m_i, M_i = M/m_i$
- $t_i M_i \equiv 1 \pmod{m_i}$
- $x = \sum a_i t_i M_i \pmod{M}$

7 Geometry

7.1 Basic

```
int sign(double x) {return abs(x) <= eps ? 0 : (x > 0 ? 1 : -1);}
struct Pt {
    double x, y;
    Pt (double _x, double _y) : x(_x), y(_y) {}
    Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
    Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
    Pt operator * (double k) {return Pt(x * k, y * k);}
    Pt operator / (double k) {return Pt(x / k, y / k);}
    double operator * (Pt o) {return x * o.x + y * o.y;}
    double operator ^ (Pt o) {return x * o.y - y * o.x;}
};
double abs2(Pt o) {return o.x * o.x + o.y * o.y;}
double abs(Pt o) {return sqrt(abs2(o));}
int ori(Pt o, Pt a, Pt b) {return sign((o - a) ^ (o - b));}
bool btw(Pt a, Pt b, Pt c) { // c on segment ab?
    return ori(a, b, c) == 0 && sign((c - a) * (c - b)) <= 0;
}
double area(Pt a, Pt b, Pt c) {return abs((a - b) ^ (a - c)) / 2;}
Pt proj_vector(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / ((b - a) * (b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vector(a, b, c) + a;
}
struct Line {
    Pt a, b;
};
struct Cir {
    Pt o; double r;
};
```

7.2 Heart

```
Pt circenter(Pt p0, Pt p1, Pt p2) { // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    double m = 2. * (x1 * y2 - y1 * x2);
    Pt center(0, 0);
    center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) { // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3) {
    return (p1 + p2 + p3) / 3;
}
Pt orthocenter(Pt p1, Pt p2, Pt p3) {
    return masscenter(p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2;
}
```

7.3 External Bisector

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
    Pt L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}
```

7.4 Intersection of Segments

```
Pt LinesInter(Line a, Line b) {
    double abc = (a.b - a.a) ^ (b.a - a.a);
    double abd = (a.b - a.a) ^ (b.b - a.a);
    if (sign(abc - abd) == 0) return b.b; // no inter
    return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
    if (btw(a.a, a.b, b.a)) return {b.a};
    if (btw(a.a, a.b, b.b)) return {b.b};
    if (btw(b.a, b.b, a.a)) return {a.a};
    if (btw(b.a, b.b, a.b)) return {a.b};
    if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 && ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
        return {LinesInter(a, b)};
    return {};
}
```

7.5 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)) / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
}
```

7.6 Intersection of Circles

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r * c2.r - c1.r * c1.r) / (2 * d2));
    double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d) * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
    Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2 * d2);
    if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
    return {u + v, u - v};
}
```

7.7 Intersection of Polygon and Circle

```
double _area(Pt pa, Pt pb, double r){
    if(abs(pa) < abs(pb)) swap(pa, pb);
    if(abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
    double cosC = (pa * pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < pi / 2) S -= (acos(h / r) * r * r
            - h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = pi - B - asin(sin(B) / r * a);
        S = .5 * a * r * sin(theta) + (C - theta) / 2 * r *
            r;
    } else
        S = .5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
    double S = 0; int n = poly.size();
    for(int i = 0; i < n; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
            ori(O, poly[i], poly[(i + 1) % n]);
    return fabs(S);
}
```

7.8 Tangent Lines of Circle and Point

```
vector<Line> tangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sign(d - c.r) == 0) {
        Pt i = rot(p - c.o, pi / 2);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
            i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}
```

7.9 Tangent Lines of Circles

```
vector<Line> tangent(Cir a, Cir b) {
#define Pij \
    Pt i = unit(b.o - a.o) * a.r, j = Pt(i.y, -i.x);\
    z.push_back({a.o + i, a.o + i + j});
#define deo(I,J) \
    double d = abs(a.o - b.o), e = a.r I b.r, o = acos(e
        / d);\
    Pt i = unit(b.o - a.o), j = rot(i, o), k = rot(i, -o)
        ;\
    z.push_back({a.o + j * a.r, b.o J j * b.r});\
    z.push_back({a.o + k * a.r, b.o J k * b.r});
    if (a.r < b.r) swap(a, b);
    vector<Line> z;
    if (abs(a.o - b.o) + b.r < a.r) return z;
    else if (sign(abs(a.o - b.o) + b.r - a.r) == 0) { Pij
        ; }
    else {
        deo(-,+); // inter
        // outer
        if (sign(d - a.r - b.r) == 0) { Pij; }
        else if (d > a.r + b.r) { deo(+,-); }
    }
    return z;
}
```

7.10 Point In Convex

```
bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
    int a = 1, b = int(C.size()) - 1, r = !strict;
    if (C.size() == 0) return false;
```

```
    if (C.size() < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}
```

7.11 Point Segment Distance

```
double PointSegDist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
        * (p - q1)) >= 0)
        return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}
```

7.12 Convex Hull

```
vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.
        y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2],
                ans.back(), pt[i]) <= 0)
                ans.pop_back();
            ans.push_back(pt[i]);
        }
        reverse(all(pt));
    }
    ans.pop_back();
    return ans;
}
```

7.13 Convex Hull Distance

```
double ConvexHullDist(vector<Pt> A, vector<Pt> B) {
    for (auto &p : B) p = Pt(0, 0) - p;
    auto C = Minkowski(A, B); // assert SZ(C) > 0
    if (PointInConvex(C, Pt(0, 0))) return 0;
    double ans = PointSegDist(C.back(), C[0], Pt(0, 0))
        ;
    for (int i = 0; i + 1 < C.size(); ++i) {
        ans = min(ans, PointSegDist(C[i], C[i + 1], Pt
            (0, 0)));
    }
    return ans;
}
```

7.14 Minimum Enclosing Circle

```
Cir min_enclosing(vector<Pt> &p) {
    random_shuffle(p.begin(), p.end());
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < p.size(); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i];
        r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2;
            r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}
```

7.15 Union of Circles

```
vector<pair<double, double>> CoverSegment(Cir a, Cir b)
{
    double d = abs(a.o - b.o);
    vector<pair<double, double>> res;
    if (sign(a.r + b.r - d) == 0);
    else if (d <= abs(a.r - b.r) + eps) {
        if (a.r < b.r) res.emplace_back(0, 2 * pi);
    } else if (d < abs(a.r + b.r) - eps) {
        double o = acos((sqrt(a.r) + sqrt(d) - sqrt(b.r)) /
            (2 * a.r * d)), z = atan2((b.o - a.o).y, (b.o
            - a.o).x);
        if (z < 0) z += 2 * pi;
        double l = z - o, r = z + o;
        if (l < 0) l += 2 * pi;
        if (r > 2 * pi) r -= 2 * pi;
        if (l > r) res.emplace_back(l, 2 * pi), res.
            emplace_back(0, r);
        else res.emplace_back(l, r);
    }
    return res;
}

double CircleUnionArea(vector<Cir> c) { // circle
    should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; w = 0, i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
            for (auto &e : z) s.push_back(e);
        }
        sort(s.begin(), s.end());
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
        for (auto &e : s) {
            if (e.first > w) a += F(e.first) - F(w);
            w = max(w, e.second);
        }
    }
    return a * 0.5;
}
```

7.16 Polar Angle Sort

```
void PolarAngleSort(vector<Pt> &pts) {
    auto pos = [&](Pt a) {return sign(a.y) == 0 ? sign(a
        .x) < 0 : sign(a.y) > 0;};
    sort(all(pts), [&](Pt a, Pt b) {return pos(a) == pos(
        b) ? sign(a ^ b) > 0 : pos(a) < pos(b);});
}
```

7.17 Rotating Caliper

```
void RotatingCaliper(vector<Pt> &pts) {
    int n = pts.size();
    for (int i = 0, j = 2; i < n; ++i) {
        int ni = (i + 1) % n;
        while (true) {
            int nj = (j + 1) % n;
            if (area(pts[j], pts[i], pts[ni]) < area(pts[nj],
                pts[i], pts[ni])) {
                j = nj;
            } else {
                break;
            }
        }
        // do something
    }
}
```

7.18 Rotating SweepLine

```
void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> id(n), pos(n);
    vector<pair<int, int>> line;
    for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++
        j) if (i ^ j) line.emplace_back(i, j);
    sort(line.begin(), line.end(), [&](pair<int, int> i,
        pair<int, int> j) {
```

```
    Pt a = pt[i.second] - pt[i.first], b = pt[j.second]
        - pt[j.first];
    return (a.pos() == b.pos() ? sign(a ^ b) > 0 : a.
        pos() < b.pos());
});
iota(id.begin(), id.end(), 0);
sort(id.begin(), id.end(), [&](int i, int j) {
    return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt
        [j].x : pt[i].y < pt[j].y);
});
for (int i = 0; i < n; ++i)
    pos[id[i]] = i;
for (auto [i, j] : line) {
    // point sort by the distance to line(i, j)
    // do something.
    tie(pos[i], pos[j], id[pos[i]], id[pos[j]]) =
        make_tuple(pos[j], pos[i], j, i);
}
}
```

7.19 Half Plane Intersection

```
vector<Pt> HalfPlaneInter(vector<pair<Pt, Pt>> vec)
{
    // x
    // first -----> second
    auto pos = [&](Pt a) {return sign(a.y) == 0 ? sign(a
        .x) < 0 : sign(a.y) > 0;};
    sort(all(vec), [&](pair<Pt, Pt> a, pair<Pt, Pt> b)
        {
            Pt A = a.second - a.first, B = b.second - b.first;
            if (pos(A) == pos(B)) {
                if (sign(A ^ B) == 0) return sign((b.first - a.
                    first) * (b.second - a.first)) > 0;
                return sign(A ^ B) > 0;
            }
            return pos(A) < pos(B);
        });
    deque<Pt> inter;
    deque<pair<Pt, Pt>> seg;
    int n = vec.size();
    auto get = [&](pair<Pt, Pt> a, pair<Pt, Pt> b) {
        return intersect(a.first, a.second, b.first, b.
            second);
    };
    for (int i = 0; i < n; ++i) if (!i || vec[i] != vec[i
        - 1]) {
        while (seg.size() >= 2 && sign((vec[i].second -
            inter.back()) ^ (vec[i].first - inter.back()))
            == 1) seg.pop_back(), inter.pop_back();
        while (seg.size() >= 2 && sign((vec[i].second -
            inter.front()) ^ (vec[i].first - inter.front()))
            == 1) seg.pop_front(), inter.pop_front();
        seg.push_back(vec[i]);
        if (seg.size() >= 2) inter.pb(get(seg[seg.size() -
            2], seg.back()));
    }
    while (seg.size() >= 2 && sign((seg.front().second -
        inter.back()) ^ (seg.front().first - inter.back()))
        == 1) seg.pop_back(), inter.pop_back();
    inter.push_back(get(seg.front(), seg.back()));
    return vector<Pt>(all(inter));
}
```

7.20 Minkowski Sum

```
vector<Pt> Minkowski(vector<Pt> a, vector<Pt> b) {
    a = ConvexHull(a), b = ConvexHull(b);
    int n = a.size(), m = b.size();
    vector<Pt> c = {a[0] + b[0]}, s1, s2;
    for (int i = 0; i < n; ++i)
        s1.pb(a[(i + 1) % n] - a[i]);
    for (int i = 0; i < m; ++i)
        s2.pb(b[(i + 1) % m] - b[i]);
    for (int p1 = 0, p2 = 0; p1 < n || p2 < m;)
        if (p2 == m || (p1 < n && sign(s1[p1] ^ s2[p2]) >=
            0))
            c.pb(c.back() + s1[p1++]);
        else
            c.pb(c.back() + s2[p2++]);
    return ConvexHull(c);
}
```

8 Polynomial

8.1 Number Theoretic Transform

```
const int N = 1 << 18, mod = 998244353, G = 3;
struct NTT {
    ll w[N];
    NTT() {
        ll dw = mpow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw %
            mod;
    }
    void bitrev(vector<ll>& a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(vector<ll>& a, int n, bool inv =
        false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            ll invn = mpow(n, mod - 2);
            for (int i = 0; i < n; ++i) a[i] = mul(a[i], invn);
        }
    }
};
vector<ll> mul(vector<ll> a, vector<ll> b, int M = N /
    2){
    int m = a.size() + b.size() - 1, n = 1;
    while(n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a, n), ntt(b, n);
    for(int i = 0; i < n; ++i) a[i] = mul(a[i], b[i]);
    ntt(a, n, 1);
    a.resize(min(m, M));
    return a;
}
```

8.2 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

8.3 Inverse of Polynomial

```
vector<ll> inv(vector<ll> a){
    int m = a.size();
    vector<ll> res(1, modpow(a[0], mod - 2));
    for(int n = 2; n / 2 < m; n <= 1){
        if(a.size() < n) a.resize(n);
        vector<ll> v1(a.begin(), a.begin() + n), v2 = res;
        v1.resize(n * 2), v2.resize(n * 2);
        ntt(v1, n * 2), ntt(v2, n * 2);
        for(int i = 0; i < n * 2; ++i) v1[i] = mul(mul(v1[i],
            v2[i]), v2[i]);
        ntt(v1, n * 2, 1);
        vector<ll> nres(n);
        for(int i = 0; i < n / 2; ++i) nres[i] = add(res[i],
            res[i]);
    }
}
```

```
for(int i = 0; i < n; ++i) nres[i] = sub(nres[i],
    v1[i]);
res = nres;
}
res.resize(m);
return res;
}
```

8.4 Fast Walsh Transform

```
void fwt(vector<int> &a) {
    // and : a[j] += x;
    //       : a[j] -= x;
    // or  : a[j ^ (1 << i)] += y;
    //       : a[j ^ (1 << i)] -= y;
    // xor : a[j] = x - y, a[j ^ (1 << i)] = x + y;
    //       : a[j] = (x - y) / 2, a[j ^ (1 << i)] = (x + y)
    //         / 2;
    int n = __lg(a.size());
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
            int x = a[j ^ (1 << i)], y = a[j];
            // do something
        }
    }
}
```

9 Else

9.1 Bit Hack

```
long long next_perm(long long v) {
    long long t = v | (v - 1);
    return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(
        v) + 1));
}
void subset(long long s) {
    long long sub = s;
    while (sub) sub = (sub - 1) & s;
}
```

9.2 Dynamic Programming Condition

9.2.1 Totally Monotone (Concave/Convex)

$$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$$

$$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$$

9.2.2 Monge Condition (Concave/Convex)

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$$

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$$

9.2.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$$

9.3 Slope Trick

```
template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
    slope_trick_convex() {left.push(numeric_limits<T>::
        min() / 2), right.push(numeric_limits<T>::max() /
        2);}
    void push_left(T x) {left.push(x - ground_l);}
    void push_right(T x) {right.push(x - ground_r);}
    //add a line with slope 1 to the right starting from
    //x
    void add_right(T x) {
        T l = left.top() + ground_l;
        if (l <= x) push_right(x);
        else push_left(x), push_right(l), left.pop(), minn
            += 1 - x;
    }
    //add a line with slope -1 to the left starting from
    //x
    void add_left(T x) {
```



```

    T r = right.top() + ground_r;
    if (r >= x) push_left(x);
    else push_right(x), push_left(r), right.pop(), minn
        += x - r;
}
//val[i]=min(val[j]) for all i-l<=j<=i+r
void expand(T l, T r) {ground_l -= l, ground_r += r;}
void shift_up(T x) {minn += x;}
T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() +
        ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(left.top());
            left.pop();
            if (left.top() + ground_l <= x) {
                res = cur_val + slope * (l - x);
                break;
            }
            cur_val += slope * (l - (left.top() + ground_l));
            l = left.top() + ground_l;
            slope += 1;
        }
        for (auto i : trash) left.push(i);
        return res;
    }
    if (x > r) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(right.top());
            right.pop();
            if (right.top() + ground_r >= x) {
                res = cur_val + slope * (x - r);
                break;
            }
            cur_val += slope * ((right.top() + ground_r) -
                r);
            r = right.top() + ground_r;
            slope += 1;
        }
        for (auto i : trash) right.push(i);
        return res;
    }
    assert(0);
}
};

```

9.4 Manhattan MST

```

void solve(int n) {
    init();
    vector<int> v(n), ds;
    for (int i = 0; i < n; ++i) {
        v[i] = i;
        ds.push_back(x[i] - y[i]);
    }
    sort(ds.begin(), ds.end());
    ds.resize(unique(ds.begin(), ds.end()) - ds.begin());
    sort(v.begin(), v.end(), [&](int i, int j) { return x
        [i] == x[j] ? y[i] > y[j] : x[i] > x[j]; });
    int j = 0;
    for (int i = 0; i < n; ++i) {
        int p = lower_bound(ds.begin(), ds.end(), x[v[i]] -
            y[v[i]]) - ds.begin() + 1;
        pair<int, int> q = query(p);
        // query return prefix minimum
        if (~q.second) add_edge(v[i], q.second);
        add(p, make_pair(x[v[i]] + y[v[i]], v[i]));
    }
}

void make_graph() {
    solve(n);
    for (int i = 0; i < n; ++i) swap(x[i], y[i]);
    solve(n);
    for (int i = 0; i < n; ++i) x[i] = -x[i];
    solve(n);
    for (int i = 0; i < n; ++i) swap(x[i], y[i]);
}

```

```

    solve(n);
}

```

9.5 Dynamic MST

```

int cnt[maxn], cost[maxn], st[maxn], ed[maxn];
pair<int, int> qr[maxn];
// qr[i].first = id of edge to be changed, qr[i].second
// = weight after operation
// cnt[i] = number of operation on edge i
// call solve(0, q - 1, v, 0), where v contains edges i
// such that cnt[i] == 0

void contract(int l, int r, vector<int> v, vector<int>
    &x, vector<int> &y) {
    sort(v.begin(), v.end(), [&](int i, int j) {
        if (cost[i] == cost[j]) return i < j;
        return cost[i] < cost[j];
    });
    djs.save();
    for (int i = l; i <= r; ++i) djs.merge(st[qr[i].first],
        ed[qr[i].first]);
    for (int i = 0; i < (int)v.size(); ++i) {
        if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
            x.push_back(v[i]);
            djs.merge(st[v[i]], ed[v[i]]);
        }
    }
    djs.undo();
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) djs.merge(st[
        x[i]], ed[x[i]]);
    for (int i = 0; i < (int)v.size(); ++i) {
        if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
            y.push_back(v[i]);
            djs.merge(st[v[i]], ed[v[i]]);
        }
    }
    djs.undo();
}

void solve(int l, int r, vector<int> v, long long c) {
    if (l == r) {
        cost[qr[l].first] = qr[l].second;
        if (st[qr[l].first] == ed[qr[l].first]) {
            printf("%lld\n", c);
            return;
        }
        int minv = qr[l].second;
        for (int i = 0; i < (int)v.size(); ++i) minv = min(
            minv, cost[v[i]]);
        printf("%lld\n", c + minv);
        return;
    }
    int m = (l + r) >> 1;
    vector<int> lv = v, rv = v;
    vector<int> x, y;
    for (int i = m + 1; i <= r; ++i) {
        cnt[qr[i].first]--;
        if (cnt[qr[i].first] == 0) lv.push_back(qr[i].first);
    }
    contract(l, m, lv, x, y);
    long long lc = c, rc = c;
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        lc += cost[x[i]];
        djs.merge(st[x[i]], ed[x[i]]);
    }
    solve(l, m, y, lc);
    djs.undo();
    x.clear(), y.clear();
    for (int i = m + 1; i <= r; ++i) cnt[qr[i].first]++;
    for (int i = l; i <= m; ++i) {
        cnt[qr[i].first]--;
        if (cnt[qr[i].first] == 0) rv.push_back(qr[i].first);
    }
    contract(m + 1, r, rv, x, y);
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        rc += cost[x[i]];
    }
}

```



```

    djs.merge(st[x[i]], ed[x[i]]);
}
solve(m + 1, r, y, rc);
djs.undo();
for (int i = 1; i <= m; ++i) cnt[qr[i].first]++;
}

```

9.6 ALL LCS

```

void all_lcs(string s, string t) { // 0-base
    vector<int> h(t.size());
    iota(all(h), 0);
    for (int a = 0; a < s.size(); ++a) {
        int v = -1;
        for (int c = 0; c < t.size(); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}

```

9.7 Hilbert Curve

```

long long hilbertOrder(int x, int y, int pow, int
    rotate) {
    if (pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y <
        hpow) ? 1 : 2);
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    long long subSquareSize = 1ll << (pow * 2 - 2);
    long long ans = seg * subSquareSize;
    long long add = hilbertOrder(nx, ny, pow - 1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}

```

9.8 Pbds

```

#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
int main () {
    __gnu_pbds::priority_queue<int> pq1, pq2;
    pq1.join(pq2); // pq1 += pq2, pq2 = {}
    cc_hash_table<int, int> m1;
    tree<int, null_type, less<int>, rb_tree_tag,
        tree_order_statistics_node_update> oset;
    oset.insert(2), oset.insert(4);
    cout << *oset.find_by_order(1) << ' ' << oset.
        order_of_key(1) << '\n'; // 4 0
    bitset<100> BS;
    BS.flip(3), BS.flip(5);
    cout << BS._Find_first() << ' ' << BS._Find_next(3)
        << '\n'; // 3 5
    rope<int> rp1, rp2;
    rp1.push_back(1), rp1.push_back(3);
    rp1.insert(0, 2); // pos, num
    rp1.erase(0, 2); // pos, Len
    rp1.substr(0, 2); // pos, Len
    rp2.push_back(4);
    rp1 += rp2, rp2 = rp1;
    cout << rp2[0] << ' ' << rp2[1] << '\n'; // 3 4
}

```

9.9 Random

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
}

```

```

size_t operator()(uint64_t a) const {
    static const uint64_t FIXED_RANDOM = chrono::
        steady_clock::now().time_since_epoch().count();
    return splitmix64(i + FIXED_RANDOM);
};
unordered_map<int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());

```

9.10 Smawk Algorithm

```

ll query(int l, int r) {
    // ...
}
struct SMAWK {
    // Condition:
    // If M[l][0] < M[l][1] then M[0][0] < M[0][1]
    // If M[l][0] == M[l][1] then M[0][0] <= M[0][1]
    // For all i, find r_i s.t. M[i][r_i] is maximum //
        minimum.
    int ans[N], tmp[N];
    void interpolate(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nl;
        for (int i = 1; i < n; i += 2) {
            nl.push_back(l[i]);
        }
        run(nl, r);
        for (int i = 1, j = 0; i < n; i += 2) {
            while (j < m && r[j] < ans[l[i]])
                j++;
            assert(j < m && ans[l[i]] == r[j]);
            tmp[l[i]] = j;
        }
        for (int i = 0; i < n; i += 2) {
            int curl = 0, curr = m - 1;
            if (i)
                curl = tmp[l[i - 1]];
            if (i + 1 < n)
                curr = tmp[l[i + 1]];
            ll res = query(l[i], r[curl]);
            ans[l[i]] = r[curl];
            for (int j = curl + 1; j <= curr; ++j) {
                ll nxt = query(l[i], r[j]);
                if (res < nxt)
                    res = nxt, ans[l[i]] = r[j];
            }
        }
    }
    void reduce(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nr;
        for (int j : r) {
            while (!nr.empty()) {
                int i = nr.size() - 1;
                if (query(l[i], nr.back()) <= query(l[i], j))
                    nr.pop_back();
                else
                    break;
            }
            if (nr.size() < n)
                nr.push_back(j);
        }
        run(l, nr);
    }
    void run(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        if (max(n, m) <= 2) {
            for (int i : l) {
                ans[i] = r[0];
                if (m > 1) {
                    if (query(i, r[0]) < query(i, r[1]))
                        ans[i] = r[1];
                }
            }
        } else if (n >= m) {
            interpolate(l, r);
        } else {
            reduce(l, r);
        }
    }
};

```

9.11 Two Dimension Add Sum

```

struct TwoDimensionAddAndSum {
    // 0-index, [l, r)
    struct Seg {
        int l, r, m;
        ll vala, valb, lza, lzb;
        Seg* ch[2];
        Seg (int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
            vala(0), valb(0), lza(0), lzb(0) {
            if (r - l > 1) {
                ch[0] = new Seg(l, m);
                ch[1] = new Seg(m, r);
            }
        }
        void pull() { vala = ch[0]->vala + ch[1]->vala, valb =
            ch[0]->valb + ch[1]->valb; }
        void give(ll a, ll b) {
            lza += a, lzb += b;
            vala += a * (r - l), valb += b * (r - l);
        }
        void push() {
            ch[0]->give(lza, lzb), ch[1]->give(lza, lzb), lza =
                lzb = 0;
        }
        void add(int a, int b, ll va, ll vb) {
            if (a <= l && r <= b)
                give(va, vb);
            else {
                push();
                if (a < m) ch[0]->add(a, b, va, vb);
                if (m < b) ch[1]->add(a, b, va, vb);
                pull();
            }
        }
        long long query(int a, int b, int v) {
            if (a <= l && r <= b) return vala * v + valb;
            push();
            long long ans = 0;
            if (a < m) ans += ch[0]->query(a, b, v);
            if (m < b) ans += ch[1]->query(a, b, v);
            return ans;
        }
    };
    // note integer overflow.
    vector<array<int, 4>> E[N];
    vector<array<int, 4>> Q[N];
    vector<ll> ans;
    void add_event(int x1, int y1, int x2, int y2, ll v) {
        {
            E[x1].pb({y1, y2, v, -v * x1});
            E[x2].pb({y1, y2, -v, v * x2});
        }
        void add_query(int x1, int y1, int x2, int y2, int id) {
            {
                Q[x1].pb({y1, y2, -1, id});
                Q[x2].pb({y1, y2, 1, id});
            }
            ans.pb(0);
        }
        void solve(int n) {
            Seg root(0, n);
            for (int i = 0; i <= n; ++i) {
                for (auto j : E[i]) root.add(j[0], j[1], j[2], j[3]);
                for (auto j : Q[i]) ans[j[3]] += j[2] * root.query(j[0], j[1], i);
            }
        }
    };
};

```

9.12 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each

iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.13 Water

```

const int mod = 1e9 + 7, N = 1e5 + 4, logN = 18, C = 1;
typedef pair<ll, bool> OP;
OP operator + (OP i, OP j) {
    OP tmp;
    tmp.X = i.X + j.X;
    tmp.Y = i.Y | j.Y;
    return tmp;
}
OP operator += (OP &i, OP j) {
    return i = i + j;
}
bool dead(OP i) {
    return !i.X && !i.Y;
}
vector<int> adj[N];
int n;
struct TreeQuery {
    int dep[N], jump[N][logN], in[N], out[N], euler[N << 1], _t;
    void dfs(int v, int pa) {
        euler[_t] = v;
        in[v] = _t++;
        dep[v] = ~pa ? dep[pa] + 1 : 0;
        jump[v][0] = pa;
        for (int i = 1; i < logN; ++i) {
            int k = jump[v][i - 1];
            jump[v][i] = ~k ? jump[k][i - 1] : -1;
        }
        for (int u : adj[v]) if (u != pa) {
            dfs(u, v);
        }
        euler[_t] = v;
        out[v] = _t++;
    }
    void build() {
        _t = 0;
        dfs(0, -1);
    }
    bool anc(int u, int v) {
        // whether u is an ancestor of v
        return in[u] <= in[v] && out[u] >= out[v];
    }
    int lca(int u, int v) {
        if (anc(u, v))
            return u;
        if (anc(v, u))
            return v;
        for (int i = logN - 1; ~i; --i) {
            int k = jump[u][i];
            if (~k && !anc(k, v))
                u = k;
        }
        return jump[u][0];
    }
    int dis(int u, int v) {
        return dep[u] + dep[v] - dep[lca(u, v)] * 2;
    }
    bool btw(int a, int b, int c) {
        // check whether a lie in bc and a != b and a != c
        return a != b && a != c && dis(a, b) + dis(a, c) ==
            dis(b, c);
    }
    int lift(int v, int d) {
        for (int i = logN - 1; ~i; --i) if (d >> i & 1)
            v = jump[v][i];
        return v;
    }
    pii gen_pair(int pos) {
        return mp(dep[euler[pos]], euler[pos]);
    }
} query_solver;

struct Nearestvertex {
    bool black[N];
    struct Seg {
        pii val[N << 2];
    };
};

```

```

void build() {
    for (int i = 0; i < n * 2; ++i)
        val[i + N * 2] = query_solver.gen_pair(i);
    for (int i = N * 2 - 1; ~i; --i)
        val[i] = min(val[i << 1], val[i << 1 | 1]);
}
void modify(int p, bool empty) {
    val[p + N * 2] = empty ? mp(1 << 30, -1) :
        query_solver.gen_pair(p);
    for (int i = p + N * 2; i > 1; i >= 1) {
        val[i >> 1] = min(val[i], val[i ^ 1]);
    }
}
pii query(int l, int r) {
    pii res(1 << 30, -1);
    for (l += N * 2, r += N * 2; l < r; l >= 1, r
        >= 1) {
        if (l & 1)
            res = min(res, val[l++]);
        if (r & 1)
            res = min(res, val[--r]);
    }
    return res;
}
} root;
void build() {
    for (int i = 0; i < n; ++i)
        black[i] = true;
    root.build();
}
void chg(int v) {
    if (black[v]) {
        black[v] = false;
        root.modify(query_solver.in[v], true);
        root.modify(query_solver.out[v], true);
    } else {
        black[v] = true;
        root.modify(query_solver.in[v], false);
        root.modify(query_solver.out[v], false);
    }
}
}
pii query(int v) {
    {
        pii res = root.query(query_solver.in[v],
            query_solver.out[v] + 1);
        if (res.Y != -1)
            return mp(res.Y, query_solver.jump[res.Y][0]);
    }
    {
        pii res = root.query(0, 2 * n);
        if (res.Y == -1)
            return mp(-1, -1);
    }
    for (int i = logN - 1; ~i; --i) {
        int k = query_solver.jump[v][i];
        if (k == -1)
            continue;
        pii res = root.query(query_solver.in[k],
            query_solver.out[k] + 1);
        if (res.Y == -1)
            v = k;
    }
    // one step from goal
    if (black[query_solver.jump[v][0]]) {
        return mp(query_solver.jump[v][0], v);
    }
    v = query_solver.jump[v][0];
    pii res = root.query(query_solver.in[v],
        query_solver.out[v] + 1);
    return mp(res.Y, query_solver.jump[res.Y][0]);
}
} finder;
struct Treap {
    int pri;
    OP mn, mx, val;
    int cnt, sz;
    Treap *tl, *tr;
    Treap (OP x) : mn(x), mx(x), val(x), cnt(1), sz(1),
        pri(rand()), tl(NULL), tr(NULL) {}
    void pull() {
        cnt = 1, sz = 1, mn = mx = val;
        if (tl) {

```

```

            mx = max(mx, tl->mx);
            if (mn > tl->mn)
                mn = tl->mn, cnt = tl->cnt;
            else if (mn == tl->mn)
                mn = tl->mn, cnt += tl->cnt;
            sz += tl->sz;
        }
        if (tr) {
            mx = max(mx, tr->mx);
            if (mn > tr->mn)
                mn = tr->mn, cnt = tr->cnt;
            else if (mn == tr->mn)
                mn = tr->mn, cnt += tr->cnt;
            sz += tr->sz;
        }
    }
};
void debug(Treap *t) {
    if (t->tl)
        debug(t->tl);
    cout << t->val << ' ';
    if (t->tr)
        debug(t->tr);
}
Treap* merge(Treap *a, Treap *b) {
    if (!a || !b)
        return a ? a : b;
    if (a->pri < b->pri) {
        a->tr = merge(a->tr, b);
        a->pull();
        return a;
    } else {
        b->tl = merge(a, b->tl);
        b->pull();
        return b;
    }
}
void split(Treap* t, int k, Treap* &a, Treap* &b) {
    if (!t) a = b = NULL;
    else if ((t->tl ? t->tl->sz : 0) + 1 <= k) {
        a = t;
        split(t->tr, k - (t->tl ? t->tl->sz : 0) - 1, a->tr
            , b);
        a->pull();
    } else {
        b = t;
        split(t->tl, k, a, b->tl);
        b->pull();
    }
}
int find_first_alive(Treap *t) {
    if (t->tl && !dead(t->tl->mx))
        return find_first_alive(t->tl);
    if (!dead(t->val))
        return t->tl ? t->tl->sz : 0;
    return (t->tl ? t->tl->sz : 0) + 1 + find_first_alive
        (t->tr);
}
int find_last_alive(Treap *t) {
    if (t->tr && !dead(t->tr->mx))
        return (t->tl ? t->tl->sz : 0) + 1 +
            find_last_alive(t->tr);
    if (!dead(t->val))
        return t->tl ? t->tl->sz : 0;
    return find_last_alive(t->tl);
}
struct Chain {
    Treap *t;
    int top, bottom;
    void build(int u, int v) {
        if (query_solver.dep[u] > query_solver.dep[v])
            swap(u, v);
        top = u, bottom = v;
    }
    void move(int p) {
        if (dead(t->mx))
            return;
        int fr = find_first_alive(t), se = find_last_alive(
            t);
        if (p == fr && p == se) {
            Treap *a, *b, *c, *d;
            split(t, p, a, b);

```

```

    split(b, 1, c, d);
    c->val = c->mn = c->mx = mp(0, false);
    t = merge(a, merge(c, d));
    chg(p);
    return;
}
if (p < fr) {
    Treap *a, *b;
    if (fr)
        chg(fr - 1);
    chg(se);
    split(t, 1, a, b);
    t = merge(b, new Treap(mp(0, false)));
} else if (se < p) {
    Treap *a, *b;
    if (se + 1 < t->sz)
        chg(se + 1);
    chg(fr);
    split(t, t->sz - 1, a, b);
    t = merge(new Treap(mp(0, false)), a);
} else {
    Treap *a, *b, *c, *d, *e, *f, *g, *h;
    split(t, p, a, b);
    split(b, 1, c, d);
    if (a)
        split(a, a->sz - 1, e, f);
    else
        e = f = NULL;
    if (d)
        split(d, 1, g, h);
    else
        g = h = NULL;
    int pre = dead(c->val);
    c->val = c->mn = c->mx = (f ? f->val : mp(0, false)) + (g ? g->val : mp(0, false));
    pre ^= dead(c->val);
    t = merge(e, merge(c, h));
    if (f)
        t = merge(new Treap(mp(0, false)), t);
    if (g)
        t = merge(t, new Treap(mp(0, false)));
    if (f && fr != p)
        chg(fr);
    if (g && se != p)
        chg(se);
    if (pre)
        chg(p);
}
}
void chg(int p) {
    int v = query_solver.lift(bottom, t->sz - p);
    finder.chg(v);
}
void add(int p, OP i) {
    Treap *a, *b, *c, *d;
    split(t, p, a, b);
    split(b, 1, c, d);
    int pre = dead(c->val);
    c->val += i;
    c->mx = c->mn = c->val;
    pre ^= dead(c->val);
    t = merge(a, merge(c, d));
    if (pre)
        chg(p);
}
}
OP query(int p) {
    Treap *a, *b, *c, *d;
    split(t, p, a, b);
    split(b, 1, c, d);
    OP res = c->val;
    t = merge(a, merge(c, d));
    return res;
}
bool empty() {
    return !t || (dead(t->mn) && t->cnt == t->sz);
}
}
chains[N];
OP imp_val[N], imp_tmp_val[N];
set<int> imp_node;
struct ChainMaintainer {
    set<int> unused_index, used_index;
    void build() {

```

```

        for (int i = 0; i < N; ++i)
            unused_index.insert(i);
    }
    int create_chain(int u, int v) {
        int x = *unused_index.begin();
        unused_index.erase(unused_index.begin());
        used_index.insert(x);
        chains[x].build(u, v);
        return x;
    }
    void merge_chain(int u, int v) {
        // merge chain index u and v (chain number become u)
        // chain u top, v bottom
        int x = chains[u].bottom;
        chains[u].t = merge(chains[u].t, merge(new Treap(
            imp_val[x]), chains[v].t));
        chains[u].bottom = chains[v].bottom;
        delete_chain(v);
    }
    int split_chain(int u, int k) {
        // split chain index u into size k, (n - k). Return
        // chain index.
        int x = *unused_index.begin();
        unused_index.erase(unused_index.begin());
        used_index.insert(x);
        int split_vertex = query_solver.lift(chains[u].
            bottom, chains[u].t->sz - k);
        chains[x].top = split_vertex, chains[x].bottom =
            chains[u].bottom, chains[u].bottom =
            split_vertex;
        Treap *a, *b, *c, *d;
        split(chains[u].t, k, a, b);
        split(b, 1, c, d);
        chains[u].t = a, chains[x].t = d;
        return x;
    }
    bool delete_chain(int u) {
        // delete chain index u
        if (!used_index.count(u))
            return false;
        used_index.erase(u), unused_index.insert(u);
        chains[u].t = NULL;
        return true;
    }
} chain_maintainer;
void build() {
    query_solver.build();
    finder.build();
    chain_maintainer.build();
    for (int i = 0; i < n; ++i) {
        for (int j : adj[i]) if (i < j) {
            chain_maintainer.create_chain(i, j);
        }
    }
}
vector<int> adj2[N];
int dis_tmp[N];
void pull(int v, int avoid) {
    vector<int> order(chain_maintainer.used_index.begin(),
        chain_maintainer.used_index.end());
    for (int i : order) {
        dis_tmp[i] = min(query_solver.dis(v, chains[i].top),
            query_solver.dis(v, chains[i].bottom));
    }
    sort(all(order), [&](int i, int j) {
        return dis_tmp[i] < dis_tmp[j];
    });
    for (int id : order) if (id ^ avoid) {
        int top = chains[id].top, bottom = chains[id].
            bottom;
        int mode = query_solver.dis(top, v) <
            query_solver.dis(bottom, v);
        if (mode) {
            // up
            if (!chains[id].t) {
                imp_tmp_val[top] += imp_val[bottom], imp_val[
                    bottom] = mp(0, false);
            } else {
                imp_tmp_val[top] += chains[id].query(0);
                chains[id].move(0);
                chains[id].add(chains[id].t->sz - 1, imp_val[

```

```

        bottom]), imp_val[bottom] = mp(0, false);
    }
    else {
        // down
        if (!chains[id].t) {
            imp_tmp_val[bottom] += imp_val[top], imp_val[
                top] = mp(0, false);
        }
        else {
            imp_tmp_val[bottom] += chains[id].query(
                chains[id].t->sz - 1);
            chains[id].move(chains[id].t->sz - 1);
            chains[id].add(0, imp_val[top]), imp_val[top]
                = mp(0, false);
        }
    }
}
for (int i : imp_node)
    imp_val[i] += imp_tmp_val[i], imp_tmp_val[i] = mp
        (0, false);
}
OP solve(int v) {
    OP res;
    if (imp_node.empty()) {
        return mp(0, false);
    }
    if (imp_node.count(v)) {
        // v is imp_node
        res = imp_val[v], imp_val[v] = mp(0, false);
        pull(v, -1);
    }
    else {
        // find a chain
        int btw_id = -1;
        for (int id : chain_maintainer.used_index) {
            if (query_solver.btw(v, chains[id].top, chains[id]
                .bottom)) {
                btw_id = id;
                break;
            }
        }
        int x;
        if (btw_id == -1) {
            // need to find the nearest vertex
            res = mp(0, false);
            int u, pa; tie(u, pa) = finder.query(v);
            if (imp_node.count(u)) {
                imp_val[pa] += imp_val[u]; imp_val[u] = mp(0,
                    false);
                if (!imp_node.count(pa))
                    imp_node.insert(pa), finder.chg(pa);
                x = chain_maintainer.create_chain(u, pa);
            }
            else {
                int btw_id = -1;
                for (int id : chain_maintainer.used_index) {
                    if (query_solver.btw(u, chains[id].top,
                        chains[id].bottom)) {
                        btw_id = id;
                        break;
                    }
                }
                int p = query_solver.dis(chains[btw_id].top, u)
                    - 1;
                imp_val[pa] += chains[btw_id].query(p);
                if (!imp_node.count(pa)) {
                    imp_node.insert(pa);
                    if (!finder.black[pa])
                        finder.chg(pa);
                }
                if (!imp_node.count(u)) {
                    imp_node.insert(u);
                    if (!finder.black[pa])
                        finder.chg(pa);
                }
                chain_maintainer.split_chain(btw_id, p);
                x = chain_maintainer.create_chain(u, pa);
            }
            pull(u, x);
        }
        else {
            int pos = query_solver.dis(chains[btw_id].top, v)
                - 1;
            res = chains[btw_id].query(pos);
            chains[btw_id].move(pos);
            chains[btw_id].add(0, imp_val[chains[btw_id].top

```

```

                ]), imp_val[chains[btw_id].top] = mp(0, false
                );
            chains[btw_id].add(chains[btw_id].t->sz - 1,
                imp_val[chains[btw_id].bottom]), imp_val[
                chains[btw_id].bottom] = mp(0, false);
            pull(v, btw_id);
        }
    }
    // build new graph
    for (int id : chain_maintainer.used_index) {
        adj2[chains[id].top].pb(id), adj2[chains[id].bottom
            ].pb(id);
    }
    // remove chain
    vector<int> remove;
    for (int center : imp_node) if (adj2[center].size()
        <= 1 && dead(imp_val[center])) {
        int id = adj2[center][0];
        if (chains[id].empty())
            remove.pb(center), chain_maintainer.
                delete_chain(id);
        adj2[center].pop_back();
    }
    while (!remove.empty())
        imp_node.erase(remove.back()), finder.chg(remove.
            back()), imp_val[remove.back()] = mp(0, false),
            remove.pop_back();
    // clear graph
    for (int v : imp_node)
        adj2[v].clear();
    // build new graph
    for (int id : chain_maintainer.used_index) {
        adj2[chains[id].top].pb(id), adj2[chains[id].bottom
            ].pb(id);
    }
    // merge chain
    vector<int> order(imp_node.begin(), imp_node.end());
    sort(all(order), [&](int i, int j) {
        return query_solver.dep[i] > query_solver.dep[j];
    });
    for (int center : order) if (adj2[center].size() ==
        2) {
        int id1 = adj2[center][0], id2 = adj2[center][1];
        int u = chains[id1].top ^ chains[id1].bottom ^
            center;
        int v = chains[id2].top ^ chains[id2].bottom ^
            center;
        if (query_solver.anc(v, u))
            swap(v, u), swap(id1, id2);
        if (!query_solver.anc(u, v))
            continue;
        chain_maintainer.merge_chain(id1, id2);
        adj2[center].clear(), remove.pb(center);
    }
    while (!remove.empty())
        imp_node.erase(remove.back()), imp_val[remove.back
            ()] = mp(0, false), remove.pop_back();
    // clear graph
    for (int v : imp_node)
        adj2[v].clear();
    return res;
}
int main () {
    owo;
    cin >> n;
    for (int i = 0, u, v; i < n - 1; ++i) {
        cin >> u >> v, --u, --v;
        adj[u].pb(v), adj[v].pb(u);
    }
    for (int i = 0; i < n; ++i) {
        cin >> imp_val[i].X, imp_val[i].Y = true;
    }
    for (int i = 0; i < n; ++i)
        imp_node.insert(i);
    build();
    int q; cin >> q;
    while (q--) {
        int v; cin >> v, --v;
        cout << solve(v).X << " \n"[q == 0];
    }
}

```