

Contents

1 Basic	1	8.19 Minkowski Sum	20
1.1 Shell Script	1	8.20 Delaunay Triangulation	21
1.2 Default Code	1	8.21 Triangulation Voronoi	21
1.3 Increase Stack Size	1	8.22 3D Point	22
1.4 Debug Macro	1	8.23 3D Convex Hull	22
1.5 Pragma / FastIO	1		
1.6 Divide*	1	9 Else	23
2 Data Structure	2	9.1 Pbds	23
2.1 Leftist Tree	2	9.2 Bit Hack	23
2.2 Splay Tree	2	9.3 Dynamic Programming Condition	23
2.3 Link Cut Tree	2	9.3.1 Totally Monotone (Concave/Convex)	23
2.4 Treap	3	9.3.2 Monge Condition (Concave/Convex)	23
2.5 2D Segment Tree*	3	9.3.3 Optimal Split Point	23
2.6 Range Set*	4	9.4 Smawk Algorithm	23
2.7 vEB Tree*	4	9.5 Slope Trick	23
3 Flow / Matching	4	9.6 ALL LCS	24
3.1 Dinic	4	9.7 Hilbert Curve	24
3.2 Min Cost Max Flow	5	9.8 Random	24
3.3 Kuhn Munkres	5	9.9 Matroid Intersection	24
3.4 SW Min Cut	6	9.10 Python Misc	24
3.5 Gomory Hu Tree	6		
3.6 Blossom	6		
3.7 Weighted Blossom	6		
3.8 Flow Model	8		
4 Graph	8		
4.1 Heavy-Light Decomposition	8		
4.2 Centroid Decomposition	8		
4.3 Edge BCC	9		
4.4 Vertex BCC / Round Square Tree	9		
4.5 SCC / 2SAT	9		
4.6 Virtual Tree	9		
4.7 Directed MST	9		
4.8 Dominator Tree	10		
5 String	10		
5.1 Aho-Corasick Automaton	10		
5.2 KMP Algorithm	10		
5.3 Z Algorithm	10		
5.4 Manacher	11		
5.5 Suffix Array	11		
5.6 SAIS	11		
5.7 Suffix Automaton	11		
5.8 Minimum Rotation	12		
5.9 Palindrome Tree	12		
5.10 Main Lorentz	12		
6 Math	12		
6.1 Miller Rabin / Pollard Rho	12		
6.2 Ext GCD	12		
6.3 Chinese Remainder Theorem	12		
6.4 PiCount	13		
6.5 Linear Function Mod Min	13		
6.6 Determinant*	13		
6.7 Floor Sum	13		
6.8 Quadratic Residue	14		
6.9 Simplex	14		
6.10 Berlekamp Massey	14		
6.11 Linear Programming Construction	14		
6.12 Euclidean	15		
6.13 Theorem	15		
6.14 Estimation	15		
6.15 General Purpose Numbers	15		
6.16 Tips for Generating Function	16		
7 Polynomial	16		
7.1 Number Theoretic Transform	16		
7.2 Fast Fourier Transform	16		
7.3 Primes	16		
7.4 Polynomial Operations	16		
7.5 Fast Linear Recursion	17		
7.6 Fast Walsh Transform	17		
8 Geometry	18		
8.1 Basic	18		
8.2 Heart	18		
8.3 External Bisector	18		
8.4 Intersection of Segments	18		
8.5 Intersection of Circle and Line	18		
8.6 Intersection of Circles	18		
8.7 Intersection of Polygon and Circle	19		
8.8 Tangent Lines of Circle and Point	19		
8.9 Tangent Lines of Circles	19		
8.10 Point In Convex	19		
8.11 Point Segment Distance	19		
8.12 Convex Hull	19		
8.13 Convex Hull Distance	19		
8.14 Minimum Enclosing Circle	19		
8.15 Union of Circles	19		
8.16 Union of Polygons	20		
8.17 Rotating SweepLine	20		
8.18 Half Plane Intersection	20		

1 Basic

1.1 Shell Script

```
#!/usr/bin/env bash
g++ -std=c++17 -DABS -Wall -Wextra -Wshadow $1.cpp -o
    $1 && ./$1
for i in {A..J}; do cp tem.cpp $i.cpp; done;
```

1.2 Default Code

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define pb push_back
#define pii pair<int, int>
#define all(a) a.begin(), a.end()
#define sz(a) ((int)a.size())
```

1.3 Increase Stack Size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bk = (char*)rsp;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bk));
```

1.4 Debug Macro

```
void db() { cout << endl; }
template <typename T, typename ...U>
void db(T i, U ...j) { cout << i << ' ', db(j...); }
#define test(x...) db("[ " + string(x) + "]", x)
```

1.5 Pragma / FastIO

```
#pragma GCC optimize("Ofast,inline,unroll-loops")
#pragma GCC target("bmi,bmi2,lzcnt,popcnt,avx2")
#include <unistd.h>
char OB[65536]; int OP;
inline char RC() {
    static char buf[65536], *p = buf, *q = buf;
    return p == q && (q = (p = buf) + read(0, buf, 65536)
        ) == buf ? -1 : *p++;
}
inline int R() {
    static char c;
    while((c = RC()) < '0'); int a = c ^ '0';
    while((c = RC()) >= '0') a *= 10, a += c ^ '0';
    return a;
}
inline void W(int n) {
    static char buf[12], p;
    if (n == 0) OB[OP++] = '0'; p = 0;
    while (n) buf[p++] = '0' + (n % 10), n /= 10;
    for (--p; p >= 0; --p) OB[OP++] = buf[p];
    if (OP > 65520) write(1, OB, OP), OP = 0;
}
```

1.6 Divide*

```

11 floor(11 a, 11 b) {
    return a / b - (a < 0 && a % b);
}
11 ceil(11 a, 11 b) {
    return a / b + (a > 0 && a % b);
}
a / b < x -> floor(a, b) + 1 <= x
a / b <= x -> ceil(a, b) <= x
x < a / b -> x <= ceil(a, b) - 1
x <= a / b -> x <= floor(a, b)

```

2 Data Structure

2.1 Leftist Tree

```

struct node {
    11 rk, data, sz, sum;
    node *l, *r;
    node(11 k) : rk(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
11 sz(node *p) { return p ? p->sz : 0; }
11 rk(node *p) { return p ? p->rk : -1; }
11 sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (rk(a->r) > rk(a->l)) swap(a->r, a->l);
    a->rk = rk(a->r) + 1;
    a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

2.2 Splay Tree

```

struct Splay {
    int pa[N], ch[N][2], sz[N], rt, _id;
    11 v[N];
    Splay() {}
    void init() {
        rt = 0, pa[0] = ch[0][0] = ch[0][1] = -1;
        sz[0] = 1, v[0] = inf;
    }
    int newnode(int p, int x) {
        int id = _id++;
        v[id] = x, pa[id] = p;
        ch[id][0] = ch[id][1] = -1, sz[id] = 1;
        return id;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i;
        int gp = pa[p], c = ch[i][!x];
        sz[p] -= sz[i], sz[i] += sz[p];
        if (~c) sz[p] += sz[c], pa[c] = p;
        ch[p][x] = c, pa[p] = i;
        pa[i] = gp, ch[i][!x] = p;
        if (~gp) ch[gp][ch[gp][1] == p] = i;
    }
    void splay(int i) {
        while (~pa[i]) {
            int p = pa[i];
            if (~pa[p]) rotate(ch[pa[p]][1] == p ^ ch[p][1] == i ? i : p);
            rotate(i);
        }
        rt = i;
    }
    int lower_bound(int x) {
        int i = rt, last = -1;
        while (true) {
            if (v[i] == x) return splay(i), i;
            if (v[i] > x) {
                last = i;
                if (ch[i][0] == -1) break;
            }
        }
    }
}

```

```

        i = ch[i][0];
    }
    else {
        if (ch[i][1] == -1) break;
        i = ch[i][1];
    }
}
splay(i);
return last; // -1 if not found
}
void insert(int x) {
    int i = lower_bound(x);
    if (i == -1) {
        // assert(ch[rt][1] == -1);
        int id = newnode(rt, x);
        ch[rt][1] = id, ++sz[rt];
        splay(id);
    }
    else if (v[i] != x) {
        splay(i);
        int id = newnode(rt, x), c = ch[rt][0];
        ch[rt][0] = id;
        ch[id][0] = c;
        if (~c) pa[c] = id, sz[id] += sz[c];
        ++sz[rt];
        splay(id);
    }
}
}

```

2.3 Link Cut Tree

```

// weighted subtree size, weighted path max
struct LCT {
    int ch[N][2], pa[N], v[N], sz[N];
    int sz2[N], w[N], mx[N], _id;
    // sz := sum of v in splay, sz2 := sum of v in virtual subtree
    // mx := max w in splay
    bool rev[N];
    LCT() : _id(1) {}
    int newnode(int _v, int _w) {
        int x = _id++;
        ch[x][0] = ch[x][1] = pa[x] = 0;
        v[x] = sz[x] = _v;
        sz2[x] = 0;
        w[x] = mx[x] = _w;
        rev[x] = false;
        return x;
    }
    void pull(int i) {
        sz[i] = v[i] + sz2[i];
        mx[i] = w[i];
        if (ch[i][0]) {
            sz[i] += sz[ch[i][0]];
            mx[i] = max(mx[i], mx[ch[i][0]]);
        }
        if (ch[i][1]) {
            sz[i] += sz[ch[i][1]];
            mx[i] = max(mx[i], mx[ch[i][1]]);
        }
    }
    void push(int i) {
        if (rev[i]) reverse(ch[i][0]), reverse(ch[i][1]),
            rev[i] = false;
    }
    void reverse(int i) {
        if (!i) return;
        swap(ch[i][0], ch[i][1]);
        rev[i] ^= true;
    }
    bool isrt(int i) { // rt of splay
        if (!pa[i]) return true;
        return ch[pa[i]][0] != i && ch[pa[i]][1] != i;
    }
    void rotate(int i) {
        int p = pa[i], x = ch[p][1] == i;
        int c = ch[i][!x], gp = pa[p];
        if (ch[gp][0] == p) ch[gp][0] = i;
        else if (ch[gp][1] == p) ch[gp][1] = i;
        pa[i] = gp, ch[i][!x] = p, pa[p] = i;
        ch[p][x] = c, pa[c] = p;
    }
}

```

```

    pull(p), pull(i);
}
void splay(int i) {
    vector<int> anc;
    anc.push_back(i);
    while (!isrt(anc.back()))
        anc.push_back(pa[anc.back()]);
    while (!anc.empty())
        push(anc.back(), anc.pop_back());
    while (!isrt(i)) {
        int p = pa[i];
        if (!isrt(p)) rotate(ch[p][1] == i ^ ch[pa[p]][1]
                             == p ? i : p);
        rotate(i);
    }
}
void access(int i) {
    int last = 0;
    while (i) {
        splay(i);
        if (ch[i][1])
            sz2[i] += sz[ch[i][1]];
        sz2[i] -= sz[last];
        ch[i][1] = last;
        pull(i), last = i, i = pa[i];
    }
}
void makert(int i) {
    access(i), splay(i), reverse(i);
}
void link(int i, int j) {
    // assert(findrt(i) != findrt(j));
    makert(i);
    makert(j);
    pa[i] = j;
    sz2[j] += sz[i];
    pull(j);
}
void cut(int i, int j) {
    makert(i), access(j), splay(i);
    // assert(sz[i] == 2 && ch[i][1] == j);
    ch[i][1] = pa[j] = 0, pull(i);
}
int findrt(int i) {
    access(i), splay(i);
    while (ch[i][0]) push(i), i = ch[i][0];
    splay(i);
    return i;
}
};

```

2.4 Treap

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
// delete default code sz
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(), a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
}

```

```

    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *&o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
    node *&t = k < o->data ? o->l : o->r;
    return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *&o, int k) {
    node *a, *b;
    o->down(), split(o, a, b, k);
    o = merge(a, merge(new node(k), b));
    o->up();
}
void interval(node *&o, int l, int r) {
    node *a, *b, *c; // [l, r)
    o->down();
    split2(o, a, b, l), split2(b, b, c, r - 1);
    // operate
    o = merge(a, merge(b, c)), o->up();
}

```

2.5 2D Segment Tree*

```

// 2D range add, range sum in Log^2
struct seg {
    int l, r;
    ll sum, lz;
    seg *ch[2]{};
    seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
    void push() {
        if (lz) ch[0]->add(l, r, lz), ch[1]->add(l, r, lz),
            lz = 0;
    }
    void pull() { sum = ch[0]->sum + ch[1]->sum; }
    void add(int _l, int _r, ll d) {
        if (_l <= l && r <= _r) {
            sum += d * (r - l + 1), lz += d;
            return;
        }
        if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
            new seg(l + r >> 1, r);
        push();
        if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
        if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
        pull();
    }
    ll qsum(int _l, int _r) {
        if (_l <= l && r <= _r) return sum;
        if (!ch[0]) return lz * (min(r, _r) - max(l, _l) + 1);
        push();
        ll res = 0;
        if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
        if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
        return res;
    }
};
struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2]{};
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N) {}
}

```

```

    if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch
        [1] = new seg2(l + r >> 1, r);
}
void add(int _l, int _r, int _l2, int _r2, ll d) {
    v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
    if (_l <= l && r <= _r)
        return lz.add(_l2, _r2, d), void(0);
    if (_l < l + r >> 1)
        ch[0]->add(_l, _r, _l2, _r2, d);
    if (l + r >> 1 < _r)
        ch[1]->add(_l, _r, _l2, _r2, d);
}
ll qsum(int _l, int _r, int _l2, int _r2) {
    if (_l <= l && r <= _r) return v.qsum(_l2, _r2);
    ll d = min(r, _r) - max(l, _l);
    ll res = lz.qsum(_l2, _r2) * d;
    if (_l < l + r >> 1)
        res += ch[0]->qsum(_l, _r, _l2, _r2);
    if (l + r >> 1 < _r)
        res += ch[1]->qsum(_l, _r, _l2, _r2);
    return res;
}
};

```

2.6 Range Set*

```

struct RangeSet { // [L, r)
    set<pii> S;
    void cut(int x) {
        auto it = S.lower_bound({x + 1, -1});
        if (it == S.begin()) return;
        auto [l, r] = *prev(it);
        if (l >= x || x >= r) return;
        S.erase(prev(it));
        S.insert({l, x});
        S.insert({x, r});
    }
    vector<pii> split(int l, int r) {
        // remove and return ranges in [L, r)
        cut(l), cut(r);
        vector<pii> res;
        while (true) {
            auto it = S.lower_bound({l, -1});
            if (it == S.end() || r <= it->first) break;
            res.pb(*it), S.erase(it);
        }
        return res;
    }
    void insert(int l, int r) {
        // add a range [L, r), [L, r) not in S
        auto it = S.lower_bound({l, r});
        if (it != S.begin() && prev(it)->second == l)
            l = prev(it)->first, S.erase(prev(it));
        if (it != S.end() && r == it->first)
            r = it->second, S.erase(it);
        S.insert({l, r});
    }
    bool count(int x) {
        auto it = S.lower_bound({x + 1, -1});
        return it != S.begin() && prev(it)->first <= x
            && x < prev(it)->second;
    }
};

```

2.7 vEB Tree*

```

using u64=uint64_t;
constexpr int lsb(u64 x)
{ return x?__builtin_ctzll(x):1<<30; }
constexpr int msb(u64 x)
{ return x?63-__builtin_clzll(x):-1; }
template<int N, class T=void>
struct veb{
    static const int M=N>>1;
    veb<M> ch[1<<N-M];
    veb<N-M> aux;
    int mn,mx;
    veb():mn(1<<30),mx(-1){}
    constexpr int mask(int x){return x&((1<<M)-1);}
    bool empty(){return mx==-1;}
    int min(){return mn;}
    int max(){return mx;}
};

```

```

bool have(int x){
    return x==mn?true:ch[x>>M].have(mask(x));
}
void insert_in(int x){
    if(empty()) return mn=mx=x,void();
    if(x<mn) swap(x,mn);
    if(x>mx) mx=x;
    if(ch[x>>M].empty()) aux.insert_in(x>>M);
    ch[x>>M].insert_in(mask(x));
}
void erase_in(int x){
    if(mn==mx) return mn=1<<30,mx=-1,void();
    if(x==mn) mn=x=(aux.min()<<M)^ch[aux.min()].min();
    ch[x>>M].erase_in(mask(x));
    if(ch[x>>M].empty()) aux.erase_in(x>>M);
    if(x==mx){
        if(aux.empty()) mx=mn;
        else mx=(aux.max()<<M)^ch[aux.max()].max();
    }
}
void insert(int x){
    if(!have(x)) insert_in(x);
}
void erase(int x){
    if(have(x)) erase_in(x);
}
int next(int x){// >=x
    if(x>mx) return 1<<30;
    if(x<=mn) return mn;
    if(mask(x)<=ch[x>>M].max())
        return ((x>>M)<<M)^ch[x>>M].next(mask(x));
    int y=aux.next((x>>M)+1);
    return (y<<M)^ch[y].min();
}
int prev(int x){// <x
    if(x<=mn) return -1;
    if(x>mx) return mx;
    if(x<=(aux.min()<<M)+ch[aux.min()].min())
        return mn;
    if(mask(x)>ch[x>>M].min())
        return ((x>>M)<<M)^ch[x>>M].prev(mask(x));
    int y=aux.prev(x>>M);
    return (y<<M)^ch[y].max();
}
};
template<int N>
struct veb<N,typename enable_if<N<=6>::type>{
    u64 a;
    veb():a(0){}
    void insert_in(int x){a|=1ull<<x;}
    void insert(int x){a|=1ull<<x;}
    void erase_in(int x){a&=~(1ull<<x);}
    void erase(int x){a&=~(1ull<<x);}
    bool have(int x){return a>>x&1;}
    bool empty(){return a==0;}
    int min(){return lsb(a);}
    int max(){return msb(a);}
    int next(int x){return lsb(a&~((1ull<<x)-1));}
    int prev(int x){return msb(a&((1ull<<x)-1));}
};

```

3 Flow / Matching

3.1 Dinic

```

template <typename T>
struct Dinic { // 0-base
    const T INF = 1 << 30;
    struct edge {
        int to, rev;
        T cap, flow;
    };
    vector<edge> adj[N];
    int s, t, dis[N], cur[N], n;
    T dfs(int u, T cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < adj[u].size(); ++i) {
            edge &e = adj[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                T df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {

```

```

        e.flow += df;
        adj[e.to][e.rev].flow -= df;
        return df;
    }
}
dis[u] = -1;
return 0;
}
bool bfs() {
    fill_n(dis, n, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int tmp = q.front();
        q.pop();
        for (auto &u : adj[tmp])
            if (!dis[u.to] && u.flow != u.cap) {
                q.push(u.to);
                dis[u.to] = dis[tmp] + 1;
            }
    }
    return dis[t] != -1;
}
T solve(int _s, int _t) {
    s = _s, t = _t;
    T flow = 0, df;
    while (bfs()) {
        fill_n(cur, n, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].clear();
}
void reset() {
    for (int i = 0; i < n; ++i)
        for (auto &j : adj[i]) j.flow = 0;
}
void add_edge(int u, int v, T cap) {
    adj[u].pb(edge{v, (int)adj[v].size(), cap, 0});
    adj[v].pb(edge{u, (int)adj[u].size() - 1, 0, 0});
}
};

```

3.2 Min Cost Max Flow

```

template <typename T1, typename T2>
struct MCMF { // T1 -> flow, T2 -> cost, 0-based
    const T1 INF1 = 1 << 30;
    const T2 INF2 = 1 << 30;
    struct edge {
        int v; T1 f; T2 c;
    } E[M << 1];
    vector<int> adj[N];
    T2 dis[N], pot[N];
    int rt[N], vis[N], n, m, s, t;
    // bool DAG()...
    bool SPFA() {
        fill_n(rt, n, -1), fill_n(dis, n, INF2);
        fill_n(vis, n, false);
        queue<int> q;
        q.push(s), dis[s] = 0, vis[s] = true;
        while (!q.empty()) {
            int v = q.front(); q.pop();
            vis[v] = false;
            for (int id : adj[v]) {
                auto [u, f, c] = E[id];
                T2 ndis = dis[v] + c + pot[v] - pot[u];
                if (f > 0 && dis[u] > ndis) {
                    dis[u] = ndis, rt[u] = id;
                    if (!vis[u]) vis[u] = true, q.push(u);
                }
            }
        }
        return dis[t] != INF2;
    }
    bool dijkstra() {
        fill_n(rt, n, -1), fill_n(dis, n, INF2);
    }
};

```

```

priority_queue<pair<T2, int>, vector<pair<T2, int>>, greater<pair<T2, int>>> pq;
dis[s] = 0, pq.emplace(dis[s], s);
while (!pq.empty()) {
    auto [d, v] = pq.top(); pq.pop();
    if (dis[v] < d) continue;
    for (int id : adj[v]) {
        auto [u, f, c] = E[id];
        T2 ndis = dis[v] + c + pot[v] - pot[u];
        if (f > 0 && dis[u] > ndis) {
            dis[u] = ndis, rt[u] = id;
            pq.emplace(ndis, u);
        }
    }
}
return dis[t] != INF2;
}
pair<T1, T2> solve(int _s, int _t) {
    s = _s, t = _t, fill_n(pot, n, 0);
    T1 flow = 0; T2 cost = 0; bool fr = true;
    while ((fr ? SPFA() : dijkstra())) {
        for (int i = 0; i < n; ++i)
            dis[i] += pot[i] - pot[s];
        T1 add = INF1;
        for (int i = t; i != s; i = E[rt[i] ^ 1].v)
            add = min(add, E[rt[i]].f);
        for (int i = t; i != s; i = E[rt[i] ^ 1].v)
            E[rt[i]].f -= add, E[rt[i] ^ 1].f += add;
        flow += add, cost += add * dis[t], fr = false;
        for (int i = 0; i < n; ++i) swap(dis[i], pot[i]);
    }
    return make_pair(flow, cost);
}
void init(int _n) {
    n = _n, m = 0;
    for (int i = 0; i < n; ++i) adj[i].clear();
}
void reset() {
    for (int i = 0; i < m; ++i) E[i].f = 0;
}
void add_edge(int u, int v, T1 f, T2 c) {
    adj[u].pb(m), E[m++] = {v, f, c};
    adj[v].pb(m), E[m++] = {u, 0, -c};
}
};

```

3.3 Kuhn Munkres

```

template <typename T>
struct KM { // 0-based
    const T INF = 1 << 30;
    T w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], n;
    bool vl[N], vr[N];
    queue<int> q;
    KM() {}
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) w[i][j] = -INF;
    }
    void add_edge(int a, int b, T wei) { w[a][b] = wei; }
    bool check(int x) {
        if (vl[x] = 1, ~fl[x])
            return q.push(fl[x]), vr[fl[x]] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill(slk, slk + n, INF), fill(vl, vl + n, 0);
        fill(vr, vr + n, 0);
        while (!q.empty()) q.pop();
        q.push(s), vr[s] = 1;
        while (true) {
            T d;
            while (!q.empty()) {
                int y = q.front(); q.pop();
                for (int x = 0; x < n; ++x)
                    if (!vl[x] && slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!check(x)) return;
            }
        }
    }
};

```

```

    }
    d = INF;
    for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
    for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !check(x)) return;
}
}
T solve() {
    fill(fl, fl + n, -1), fill(fr, fr + n, -1);
    fill(hr, hr + n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    T res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

3.4 SW Min Cut

```

template <typename T>
struct SW { // 0-based
    const T INF = 1 << 30;
    T g[N][N], sum[N]; int n;
    bool vis[N], dead[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) fill_n(g[i], n, 0);
        fill(dead, dead + n, false);
    }
    void add_edge(int u, int v, T w) {
        g[u][v] += w, g[v][u] += w;
    }
    T solve() {
        T ans = INF;
        for (int round = 0; round + 1 < n; ++round) {
            fill(vis, vis + n, false), fill(sum, sum + n, 0);
            int num = 0, s = -1, t = -1;
            while (num < n - round) {
                int now = -1;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i] &&
                        (now == -1 || sum[now] > sum[i])) now = i;
                s = t, t = now;
                vis[now] = true, num++;
                for (int i = 0; i < n; ++i)
                    if (!vis[i] && !dead[i]) sum[i] += g[now][i];
            }
            ans = min(ans, sum[t]);
            for (int i = 0; i < n; ++i)
                g[i][s] += g[i][t], g[s][i] += g[t][i];
            dead[t] = true;
        }
        return ans;
    }
};

```

3.5 Gomory Hu Tree

```

vector <array <int, 3>> GomoryHu(Dinic <int> flow) {
    // Tree edge min = mincut (0-based)
    int n = flow.n;
    vector <array <int, 3>> ans;
    vector <int> rt(n);
    for (int i = 1; i < n; ++i) {
        int t = rt[i];
        flow.reset();
        ans.pb({i, t, flow.solve(i, t)});
        flow.bfs();
        for (int j = i + 1; j < n; ++j)
            if (rt[j] == t && flow.dis[j] != -1) rt[j] = i;
    }
    return ans;
}

```

3.6 Blossom

```

struct Matching { // 0-based
    int fa[N], pre[N], match[N], s[N], v[N], n, tk;
    vector <int> g[N];
    queue <int> q;
    Matching(int _n) : n(_n), tk(0) {
        for (int i = 0; i <= n; ++i) match[i] = pre[i] = n;
        for (int i = 0; i < n; ++i) g[i].clear();
    }
    void add_edge(int u, int v) {
        g[u].push_back(v), g[v].push_back(u);
    }
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int lca(int x, int y) {
        tk++;
        x = Find(x), y = Find(y);
        for (; ; swap(x, y)) {
            if (x != n) {
                if (v[x] == tk) return x;
                v[x] = tk;
                x = Find(pre[match[x]]);
            }
        }
    }
    void blossom(int x, int y, int l) {
        while (Find(x) != l) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            if (fa[x] == x) fa[x] = l;
            if (fa[y] == y) fa[y] = l;
            x = pre[y];
        }
    }
    bool bfs(int r) {
        for (int i = 0; i <= n; ++i) fa[i] = i, s[i] = -1;
        while (!q.empty()) q.pop();
        q.push(r);
        s[r] = 0;
        while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int u : g[x]) {
                if (s[u] == -1) {
                    pre[u] = x, s[u] = 1;
                    if (match[u] == n) {
                        for (int a = u, b = x, last; b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;
                        return true;
                    }
                    q.push(match[u]);
                    s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = lca(u, x);
                    blossom(x, u, l);
                    blossom(u, x, l);
                }
            }
        }
        return false;
    }
    int solve() {
        int res = 0;
        for (int x = 0; x < n; ++x) {
            if (match[x] == n) res += bfs(x);
        }
        return res;
    }
};

```

3.7 Weighted Blossom

```

struct WeightGraph { // 1-based
    static const int inf = INT_MAX;
    static const int maxn = 514;
    struct edge {
        int u, v, w;
        edge(){}
        edge(int u, int v, int w) : u(u), v(v), w(w) {}
    };

```



```

};
int n, n_x;
edge g[maxn * 2][maxn * 2];
int lab[maxn * 2];
int match[maxn * 2], slack[maxn * 2], st[maxn * 2],
    pa[maxn * 2];
int flo_from[maxn * 2][maxn + 1], S[maxn * 2], vis[
    maxn * 2];
vector<int> flo[maxn * 2];
queue<int> q;
int e_delta(const edge &e) { return lab[e.u] + lab[e.
    v] - g[e.u][e.v].w * 2; }
void update_slack(int u, int x) { if (!slack[x] ||
    e_delta(g[u][x]) < e_delta(g[slack[x]][x])) slack
    [x] = u; }
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) q.push(x);
    else for (size_t i = 0; i < flo[x].size(); i++)
        q_push(flo[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flo[x].size(); ++
        i) set_st(flo[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) -
        flo[b].begin();
    if (pr % 2 == 1) {
        reverse(flo[b].begin() + 1, flo[b].end());
        return (int)flo[b].size() - pr;
    }
    return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = flo_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flo[u][i],
        flo[u][i ^ 1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin() + pr, flo[u].
        end());
}
void augment(int u, int v) {
    for (; ; ) {
        int xnv = st[match[u]];
        set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    reverse(flo[b].begin() + 1, flo[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y = st[
            match[x]]), q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].
        w = 0;
    for (int x = 1; x <= n; ++x) flo_from[b][x] = 0;
    for (size_t i = 0; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        for (int x = 1; x <= n_x; ++x)
            if (g[b][x].w == 0 || e_delta(g[xs][x]) <
                e_delta(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (size_t i = 0; i < flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][g[b][pa[b]].u], pr = get_pr(b,
        xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flo[b].size(); ++i) {
        int xs = flo[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching() {
    memset(S + 1, -1, sizeof(int) * n_x);
    memset(slack + 1, 0, sizeof(int) * n_x);
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
            q_push(x);
    if (q.empty()) return false;
    for (; ; ) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta(g[u][v]) == 0) {
                        if (on_found_edge(g[u][v])) return true;
                    } else update_slack(u, st[v]);
                }
        }
        int d = inf;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1) d = min(d, lab[b]
                / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1) d = min(d, e_delta(g[slack[x]
                    ][x]));
                else if (S[x] == 0) d = min(d, e_delta(g[
                    slack[x]][x]) / 2);
            }
    }
}

```

```

    }
    for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
            if (lab[u] <= d) return 0;
            lab[u] -= d;
        } else if (S[st[u]] == 1) lab[u] += d;
    }
    for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b) {
            if (S[st[b]] == 0) lab[b] += d * 2;
            else if (S[st[b]] == 1) lab[b] -= d * 2;
        }
    q = queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && slack[x] && st[slack[x]] != x
            && e_delta(g[slack[x]][x]) == 0)
            if (on_found_edge(g[slack[x]][x])) return true;
    for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve() {
    memset(match + 1, 0, sizeof(int) * n);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flo[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flo_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u = 1; u <= n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int ui, int vi, int wi) { g[ui][vi].w = g[vi][ui].w = wi; }
void init(int _n) {
    n = _n;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge(u, v, 0);
}
};

```

3.8 Flow Model

- Maximum/Minimum flow with lower bound / Circulation problem
 - Construct super source S and sink T .
 - For each edge (x, y, l, u) , connect $x \rightarrow y$ with capacity $u - l$.
 - For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 - If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 - The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
 - Redirect every edge: $y \rightarrow x$ if $(x, y) \in M$, $x \rightarrow y$ otherwise.
 - DFS from unmatched vertices in X .
 - $x \in X$ is chosen iff x is unvisited.
 - $y \in Y$ is chosen iff y is visited.
- Maximum density induced subgraph
 - Binary search on answer, suppose we're checking answer T .
 - Construct a max flow model, let K be the sum of all weights.
 - Connect source $s \rightarrow v$, $v \in G$ with capacity K

- For each edge (u, v, w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
- For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
- T is a valid answer if the maximum flow $f < K|V|$

- Minimum weight edge cover

- For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u, v)$.
- Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
- Find the minimum weight perfect matching on G' .

- Project selection problem

- If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
- Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
- The mincut is equivalent to the maximum profit of a subset of projects.

- 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge (x, t) with capacity c_x and create edge (s, y) with capacity c_y .
- Create edge (x, y) with capacity c_{xy} .
- Create edge (x, y) and edge (x', y') with capacity $c_{xyx'y'}$.

4 Graph

4.1 Heavy-Light Decomposition

```

vector<int> g[N];
int dep[N], pa[N], sz[N], ch[N], hd[N], id[N], _id;
void dfs(int i, int p) {
    dep[i] = ~p ? dep[p] + 1 : 0;
    pa[i] = p, sz[i] = 1, ch[i] = -1;
    for (int j : g[i]) if (j != p) {
        dfs(j, i);
        if (ch[i] == -1 || sz[ch[i]] < sz[j]) ch[i] = j;
        sz[i] += sz[j];
    }
}
void hld(int i, int p, int h) {
    hd[i] = h;
    id[i] = _id++;
    if (~ch[i]) hld(ch[i], i, h);
    for (int j : g[i]) if (j != p && j != ch[i])
        hld(j, i, j);
}
void query(int i, int j) {
    // query2 -> [L, r)
    while (hd[i] != hd[j]) {
        if (dep[hd[i]] < dep[hd[j]]) swap(i, j);
        query2(id[hd[i]], id[i] + 1, i = pa[hd[i]]);
    }
    if (dep[i] < dep[j]) swap(i, j);
    query2(id[j], id[i] + 1);
}

```

4.2 Centroid Decomposition

```

vector<int> g[N];
int dis[N][logN], pa[N], sz[N], dep[N];
bool vis[N];
void dfs_sz(int i, int p) {
    sz[i] = 1;
    for (int j : g[i]) if (j != p && !vis[j])
        dfs_sz(j, i), sz[i] += sz[j];
}
int cen(int i, int p, int _n) {
    for (int j : g[i])
        if (j != p && !vis[j] && sz[j] > _n / 2)
            return cen(j, i, _n);
    return i;
}
void dfs_dis(int i, int p, int d) {
    // from i to ancestor with depth d
    dis[i][d] = ~p ? dis[p][d] + 1 : 0;
    for (int j : g[i]) if (j != p && !vis[j])

```



```

    dfs_dis(j, i, d);
}
void cd(int i, int p, int d) {
    dfs_sz(i, -1), i = cen(i, -1, sz[i]);
    vis[i] = true, pa[i] = p, dep[i] = d;
    dfs_dis(i, -1, d);
    for (int j : g[i]) if (!vis[j])
        cd(j, i, d + 1);
}

```

4.3 Edge BCC

```

vector<int> g[N], _g[N];
// Notice Multiple Edges
int pa[N], low[N], dep[N], bcc_id[N], _id;
vector<int> stk, bcc[N];
bool vis[N], is_bridge[N];
void dfs(int i, int p = -1) {
    low[i] = dep[i] = ~p ? dep[p] + 1 : 0;
    stk.pb(i), pa[i] = p, vis[i] = true;
    for (int j : g[i]) if (j != p) {
        if (!vis[j])
            dfs(j, i), low[i] = min(low[i], low[j]);
        else low[i] = min(low[i], dep[j]);
    }
    if (low[i] == dep[i]) {
        if (~p) is_bridge[i] = true; // (i, pa[i])
        int id = _id++, x;
        do {
            x = stk.back(), stk.pop_back();
            bcc_id[x] = id, bcc[id].pb(x);
        } while (x != i);
    }
}
void build(int n) {
    for (int i = 0; i < n; ++i) if (!vis[i])
        dfs(i);
    for (int i = 0; i < n; ++i) if (is_bridge[i]) {
        int u = bcc_id[i], v = bcc_id[pa[i]];
        _g[u].pb(v), _g[v].pb(u);
    }
}

```

4.4 Vertex BCC / Round Square Tree

```

vector<int> g[N], _g[N << 1];
// _g: index >= N: bcc, index < N: original vertex
int pa[N], dep[N], low[N], _id;
bool vis[N];
vector<int> stk;
void dfs(int i, int p = -1) {
    dep[i] = low[i] = ~p ? dep[p] + 1 : 0;
    stk.pb(i), pa[i] = p, vis[i] = true;
    for (int j : g[i]) if (j != p) {
        if (!vis[j]) {
            dfs(j, i), low[i] = min(low[i], low[j]);
            if (low[j] >= dep[i]) {
                int id = _id++, x;
                do {
                    x = stk.back(), stk.pop_back();
                    _g[id + N].pb(x), _g[x].pb(id + N);
                } while (x != j);
                _g[id + N].pb(i), _g[i].pb(id + N);
            }
        } else low[i] = min(low[i], dep[j]);
    }
}
bool is_cut(int x) {return _g[x].size() != 1;}
vector<int> bcc(int x) {return _g[x + N];}
int pa2[N << 1], dep2[N << 1];
void dfs2(int i, int p = -1) {
    dep2[i] = ~p ? dep2[p] + 1 : 0, pa2[i] = p;
    for (int j : _g[i]) if (j != p) {
        dfs2(j, i);
    }
}
int bcc_id(int u, int v) {
    if (dep2[u] < dep2[v]) swap(u, v);
    return pa2[u] - N;
}
void build(int n) {
    for (int i = 0; i < n; ++i) if (!vis[i])

```

```

    dfs(i), dfs2(i);
}

```

4.5 SCC / 2SAT

```

struct SAT {
    vector<int> g[N << 1], stk;
    int dep[N << 1], low[N << 1], scc_id[N << 1];
    int n, _id, _t;
    bool is[N];
    SAT() {}
    void init(int _n) {
        n = _n, _id = _t = 0;
        for (int i = 0; i < 2 * n; ++i)
            g[i].clear(), dep[i] = scc_id[i] = -1;
        stk.clear();
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
    int rev(int i) { return i < n ? i + n : i - n; }
    void add_ifthen(int x, int y)
    { add_clause(rev(x), y); }
    void add_clause(int x, int y)
    { add_edge(rev(x), y), add_edge(rev(y), x); }
    void dfs(int i) {
        dep[i] = low[i] = _t++, stk.pb(i);
        for (int j : g[i]) if (scc_id[j] == -1) {
            if (dep[j] == -1) dfs(j);
            low[i] = min(low[i], low[j]);
        }
        if (low[i] == dep[i]) {
            int id = _id++, x;
            do {
                x = stk.back(), stk.pop_back(), scc_id[x] = id;
            } while (x != i);
        }
    }
    bool solve() {
        // is[i] = true -> i, is[i] = false -> -i
        for (int i = 0; i < 2 * n; ++i) if (dep[i] == -1)
            dfs(i);
        for (int i = 0; i < n; ++i) {
            if (scc_id[i] == scc_id[i + n]) return false;
            if (scc_id[i] < scc_id[i + n]) is[i] = true;
            else is[i] = false;
        }
        return true;
    }
};

```

4.6 Virtual Tree

```

// need lca
vector<int> _g[N], stk;
int st[N], ed[N];
void solve(vector<int> v) {
    auto cmp = [&](int x, int y) {return st[x] < st[y];};
    sort(all(v), cmp);
    int sz = v.size();
    for (int i = 0; i < sz - 1; ++i)
        v.pb(lca(v[i], v[i + 1]));
    sort(all(v), cmp);
    v.resize(unique(all(v)) - v.begin());
    stk.clear(), stk.pb(v[0]);
    for (int i = 1; i < v.size(); ++i) {
        int x = v[i];
        while (ed[stk.back()] < ed[x]) stk.pop_back();
        _g[stk.back()].pb(x), stk.pb(x);
    }
    // do something
    for (int i : v) _g[i].clear();
}

```

4.7 Directed MST

```

using D = int;
struct edge {
    int u, v; D w;
};
// 0-based, return index of edges
vector<int> dmst(vector<edge> &e, int n, int root) {
    using T = pair<D, int>;
    using PQ = pair<priority_queue<T, vector<T>,
        greater<T>>, D>;

```

```

auto push = [](PQ &pq, T v) {
    pq.first.emplace(v.first - pq.second, v.second);
};
auto top = [](const PQ &pq) -> T {
    auto r = pq.first.top();
    return {r.first + pq.second, r.second};
};
auto join = [&push, &top](PQ &a, PQ &b) {
    if (a.first.size() < b.first.size()) swap(a, b);
    while (!b.first.empty())
        push(a, top(b)), b.first.pop();
};
vector<PQ> h(n * 2);
for (int i = 0; i < e.size(); ++i)
    push(h[e[i].v], {e[i].w, i});
vector<int> a(n * 2), v(n * 2, -1), pa(n * 2, -1), r(
    n * 2);
iota(all(a), 0);
auto o = [&](int x) { int y;
    for (y = x; a[y] != y; y = a[y]);
    for (int ox = x; x != y; ox = x)
        x = a[x], a[ox] = y;
    return y;
};
v[root] = n + 1;
int pc = n;
for (int i = 0; i < n; ++i) if (v[i] == -1) {
    for (int p = i; v[p] == -1 || v[p] == i; p = o(e[r[
        p]].u)) {
        if (v[p] == i) {
            int q = p; p = pc++;
            do {
                h[q].second = -h[q].first.top().first;
                join(h[pa[q] = a[q] = p], h[q]);
            } while ((q = o(e[r[q]].u)) != p);
        }
        v[p] = i;
        while (!h[p].first.empty() && o(e[top(h[p]).
            second].u) == p)
            h[p].first.pop();
        r[p] = top(h[p]).second;
    }
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--)
    if (i != root && v[i] != n) {
        for (int f = e[r[i]].v; f != -1 && v[f] != n; f =
            pa[f]) v[f] = n;
        ans.pb(r[i]);
    }
return ans;
}

```

4.8 Dominator Tree

```

struct Dominator_tree {
    int n, id, sdom[N], dom[N];
    vector<int> adj[N], radj[N], bucket[N];
    int vis[N], rev[N], pa[N], rt[N], mn[N], res[N];
    // dom[s] = s, dom[v] = -1 if s -> v not exists
    Dominator_tree () {}
    void init(int _n) {
        n = _n, id = 0;
        fill_n(dom, n, -1), fill_n(vis, n, -1);
    }
    void add_edge(int u, int v) {adj[u].pb(v);}
    int query(int v, int x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], 1);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]])
            mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : adj[v]) {
            if (vis[u] == -1) dfs(u), pa[vis[u]] = vis[v];
            radj[vis[u]].pb(vis[v]);
        }
    }
}

```

```

}
void build(int s) {
    dfs(s);
    for (int i = id - 1; ~i; --i) {
        for (int u : radj[i]) {
            sdom[i] = min(sdom[i], sdom[query(u, 0)]);
        }
        if (i) bucket[sdom[i]].pb(i);
        for (int u : bucket[i]) {
            int p = query(u, 0);
            dom[u] = sdom[p] == i ? i : p;
        }
        if (i) rt[i] = pa[i];
    }
    fill_n(res, n, -1);
    for (int i = 1; i < id; ++i) {
        if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
    }
    for (int i = 1; i < id; ++i)
        res[rev[i]] = rev[dom[i]];
    res[s] = s;
    for (int i = 0; i < n; ++i) dom[i] = res[i];
}
};

```

5 String

5.1 Aho-Corasick Automaton

```

struct AC {
    int ch[N][26], to[N][26], fail[N], sz;
    vector<int> g[N];
    int cnt[N];
    AC () {sz = 0, extend();}
    void extend() {fill(ch[sz], ch[sz] + 26, 0), sz++;}
    int nxt(int u, int v) {
        if (!ch[u][v]) ch[u][v] = sz, extend();
        return ch[u][v];
    }
    int insert(string s) {
        int now = 0;
        for (char c : s) now = nxt(now, c - 'a');
        cnt[now]++;
        return now;
    }
    void build_fail() {
        queue<int> q;
        for (int i = 0; i < 26; ++i) if (ch[0][i]) {
            q.push(ch[0][i]);
            g[0].push_back(ch[0][i]);
        }
        while (!q.empty()) {
            int v = q.front(); q.pop();
            for (int j = 0; j < 26; ++j) {
                to[v][j] = ch[v][j] ? v : to[fail[v]][j];
            }
            for (int i = 0; i < 26; ++i) if (ch[v][i]) {
                int u = ch[v][i], k = fail[v];
                while (k && !ch[k][i]) k = fail[k];
                if (ch[k][i]) k = ch[k][i];
                fail[u] = k;
                cnt[u] += cnt[k], g[k].push_back(u);
                q.push(u);
            }
        }
    }
    int match(string &s) {
        int now = 0, ans = 0;
        for (char c : s) {
            now = to[now][c - 'a'];
            if (ch[now][c - 'a']) now = ch[now][c - 'a'];
            ans += cnt[now];
        }
        return ans;
    }
};

```

5.2 KMP Algorithm

```

vector<int> build_fail(string s) {
    vector<int> f(s.length() + 1, 0);
}

```

```

int k = 0;
for (int i = 1; i < s.length(); ++i) {
    while (k && s[k] != s[i]) k = f[k];
    if (s[k] == s[i]) k++;
    f[i + 1] = k;
}
return f;
}

int match(string s, string t) {
    vector<int> f = build_fail(t);
    int k = 0, ans = 0;
    for (int i = 0; i < s.length(); ++i) {
        while (k && s[i] != t[k]) k = f[k];
        if (s[i] == t[k]) k++;
        if (k == t.length()) ans++, k = f[k];
    }
    return ans;
}

```

5.3 Z Algorithm

```

vector<int> buildZ(string s) {
    int n = s.length();
    vector<int> Z(n);
    int l = 0, r = 0;
    for (int i = 0; i < n; ++i) {
        Z[i] = max(min(Z[i - 1], r - i), 0);
        while (i + Z[i] < n && s[Z[i]] == s[i + Z[i]]) {
            l = i, r = i + Z[i], Z[i]++;
        }
    }
    return Z;
}

```

5.4 Manacher

```

// return value only consider string tmp, not s
vector<int> manacher(string tmp) {
    string s = "&";
    for (char c : tmp) s.pb(c), s.pb('%');
    int l = 0, r = 0, n = s.size();
    vector<int> Z(n);
    for (int i = 0; i < n; ++i) {
        Z[i] = r > i ? min(Z[2 * l - i], r - i) : 1;
        while (s[i + Z[i]] == s[i - Z[i]]) Z[i]++;
        if (Z[i] + i > r) l = i, r = Z[i] + i;
    }
    for (int i = 0; i < n; ++i) {
        Z[i] = (Z[i] - (i & 1)) / 2 * 2 + (i & 1);
    }
    return Z;
}

```

5.5 Suffix Array

```

int sa[N], tmp[2][N], c[N], rk[N], lcp[N];
void buildSA(string s) {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.size();
    for (int i = 0; i < m; ++i) c[i] = 0;
    for (int i = 0; i < n; ++i) c[x[i]] = s[i]++;
    for (int i = 1; i < m; ++i) c[i] += c[i - 1];
    for (int i = n - 1; ~i; --i) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        int p = 0;
        for (int i = n - k; i < n; ++i) y[p++] = i;
        for (int i = 0; i < n; ++i) if (sa[i] >= k)
            y[p++] = sa[i] - k;
        for (int i = n - 1; ~i; --i)
            sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; ++i) {
            int a = sa[i], b = sa[i - 1];
            if (!(x[a] == x[b] && a + k < n && b + k < n &&
                x[a + k] == x[b + k])) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}

```

```

}
void buildLCP(string s) {
    // lcp[i] = LCP(sa[i - 1], sa[i])
    // lcp(i, j) = query_lcp_min[rk[i] + 1, rk[j] + 1)
    int n = s.length(), val = 0;
    for (int i = 0; i < n; ++i) rk[sa[i]] = i;
    for (int i = 0; i < n; ++i) {
        if (!rk[i]) lcp[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val + p < n && s[val + i]
                == s[val + p]) val++;
            lcp[rk[i]] = val;
        }
    }
}

```

5.6 SAIS

```

int sa[N << 1], rk[N], lcp[N];
// string ASCII value need > 0
namespace sfx {
    bool _t[N << 1];
    int _s[N << 1], _c[N << 1], x[N], _p[N], _q[N << 1];
    void pre(int *sa, int *c, int n, int z) {
        fill_n(sa, n, 0), copy_n(c, z, x);
    }
    void induce(int *sa, int *c, int *s, bool *t, int n,
        int z) {
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; ++i)
            if (sa[i] && !t[sa[i] - 1])
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; --i)
            if (sa[i] && t[sa[i] - 1])
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t, int
        *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn = 0, nmxx = -1, *nsa = sa + n, *ns = s + n,
            last = -1;
        fill_n(c, z, 0);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            if (s[i] == s[i + 1]) t[i] = t[i + 1];
            else t[i] = s[i] < s[i + 1];
        pre(sa, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(sa, c, s, t, n, z);
        for (int i = 0; i < n; ++i)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                bool neq = last < 0 || !equal(s + sa[i], s + p[q[
                    sa[i]] + 1], s + last);
                ns[q[last = sa[i]]] = nmxx += neq;
            }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxx +
            1);
        pre(sa, c, n, z);
        for (int i = nn - 1; i >= 0; --i)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        induce(sa, c, s, t, n, z);
    }
    void buildSA(string s) {
        int n = s.length();
        for (int i = 0; i < n; ++i) _s[i] = s[i];
        _s[n] = 0;
        sais(_s, sa, _p, _q, _t, _c, n + 1, 256);
        for (int i = 1; i <= n; ++i) sa[i - 1] = sa[i];
    }
    // buildLCP()...
}

```

5.7 Suffix Automaton

```

struct SAM {
    int ch[N][26], len[N], link[N], pos[N], cnt[N], sz;
    // node -> strings with the same endpos set
    // length in range [len(link) + 1, len]
    // node's endpos set -> pos in the subtree of node
    // link -> longest suffix with different endpos set
    // len -> longest suffix
    // pos -> end position
    // cnt -> size of endpos set
    SAM () {len[0] = 0, link[0] = -1, pos[0] = 0, cnt[0] = 0, sz = 1;}
    void build(string s) {
        int last = 0;
        for (int i = 0; i < s.length(); ++i) {
            char c = s[i];
            int cur = sz++;
            len[cur] = len[last] + 1, pos[cur] = i + 1;
            int p = last;
            while (~p && !ch[p][c - 'a'])
                ch[p][c - 'a'] = cur, p = link[p];
            if (p == -1) link[cur] = 0;
            else {
                int q = ch[p][c - 'a'];
                if (len[p] + 1 == len[q]) {
                    link[cur] = q;
                } else {
                    int nxt = sz++;
                    len[nxt] = len[p] + 1, link[nxt] = link[q];
                    pos[nxt] = 0;
                    for (int j = 0; j < 26; ++j)
                        ch[nxt][j] = ch[q][j];
                    while (~p && ch[p][c - 'a'] == q)
                        ch[p][c - 'a'] = nxt, p = link[p];
                    link[q] = link[cur] = nxt;
                }
            }
            cnt[cur]++;
            last = cur;
        }
        vector<int> p(sz);
        iota(all(p), 0);
        sort(all(p), [&](int i, int j) {return len[i] > len[j];});
        for (int i = 0; i < sz; ++i)
            cnt[link[p[i]]] += cnt[p[i]];
    }
} sam;

```

5.8 Minimum Rotation

```

string rotate(const string &s) {
    int n = s.length();
    string t = s + s;
    int i = 0, j = 1;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && t[i + k] == t[j + k]) ++k;
        if (t[i + k] <= t[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int pos = (i < n ? i : j);
    return t.substr(pos, n);
}

```

5.9 Palindrome Tree

```

struct PAM {
    int ch[N][26], cnt[N], fail[N], len[N], sz;
    string s;
    // 0 -> even root, 1 -> odd root
    PAM () {}
    void init(string s) {
        sz = 0, extend(), extend();
        len[0] = 0, fail[0] = 1, len[1] = -1;
        int lst = 1;
        for (int i = 0; i < s.length(); ++i) {
            while (s[i - len[lst]] - 1 != s[i])
                lst = fail[lst];
            if (!ch[lst][s[i] - 'a']) {
                int idx = extend();
                len[idx] = len[lst] + 2;
            }

```

```

            int now = fail[lst];
            while (s[i - len[now]] - 1 != s[i])
                now = fail[now];
            fail[idx] = ch[now][s[i] - 'a'];
            ch[lst][s[i] - 'a'] = idx;
        }
        lst = ch[lst][s[i] - 'a'], cnt[lst]++;
    }
}
void build_count() {
    for (int i = sz - 1; i > 1; --i)
        cnt[fail[i]] += cnt[i];
}
int extend() {
    fill(ch[sz], ch[sz] + 26, 0), sz++;
    return sz - 1;
}
};

```

5.10 Main Lorentz

```

int to_left[N], to_right[N];
vector<array<int, 3>> rep; // L, r, len.
// substr( [L, r], len * 2) are tandem
void findRep(string &s, int l, int r) {
    if (r - l == 1) return;
    int m = l + r >> 1;
    findRep(s, l, m), findRep(s, m, r);
    string sl = s.substr(l, m - l);
    string sr = s.substr(m, r - m);
    vector<int> Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i)
        to_right[i] = Z[r - m + 1 + i - 1];
    reverse(all(sl));
    Z = buildZ(sl);
    for (int i = 1; i < m; ++i)
        to_left[i] = Z[m - i - 1];
    reverse(all(sl));
    for (int i = 1; i + 1 < m; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = m - i - 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(1, len - k2), tr = min(len - 1, k1);
        if (tl <= tr) rep.pb({i + 1 - tr, i + 1 - tl, len});
    }
    Z = buildZ(sr);
    for (int i = m; i < r; ++i) to_right[i] = Z[i - m];
    reverse(all(sl)), reverse(all(sr));
    Z = buildZ(sl + "#" + sr);
    for (int i = m; i < r; ++i)
        to_left[i] = Z[m - 1 + 1 + r - i - 1];
    reverse(all(sl)), reverse(all(sr));
    for (int i = m; i + 1 < r; ++i) {
        int k1 = to_left[i], k2 = to_right[i + 1];
        int len = i - m + 1;
        if (k1 < 1 || k2 < 1 || len < 2) continue;
        int tl = max(len - k2, 1), tr = min(len - 1, k1);
        if (tl <= tr)
            rep.pb({i + 1 - len - tr, i + 1 - len - tl, len});
    }
    Z = buildZ(sr + "#" + sl);
    for (int i = 1; i < m; ++i)
        if (Z[r - m + 1 + i - 1] >= m - i)
            rep.pb({i, i, m - i});
}

```

6 Math

6.1 Miller Rabin / Pollard Rho

```

ll mul(ll x, ll y, ll p) {return (x * y - (ll)((long double)x / p * y) * p + p) % p;} // __int128
vector<ll> chk = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
ll Pow(ll a, ll b, ll n) {
    ll res = 1;
    for (; b >= 1; a = mul(a, a, n))
        if (b & 1) res = mul(res, a, n);
    return res;
}
bool check(ll a, ll d, int s, ll n) {

```

```

a = Pow(a, d, n);
if (a <= 1) return 1;
for (int i = 0; i < s; ++i, a = mul(a, a, n)) {
    if (a == 1) return 0;
    if (a == n - 1) return 1;
}
return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d >>= 1, ++s;
    for (ll i : chk) if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {return (mul(a, a, n) + t) % n;};
    for (int l = 2; ; l <= 1) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(ll n) {
    if (n == 1) return;
    if (IsPrime(n)) return ++res[n], void(0);
    ll d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
}

```

6.2 Ext GCD

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    pair<ll, ll> res, tmp;
    ll f = 1, g = 1;
    if (a < 0) a *= -1, f *= -1;
    if (b < 0) b *= -1, g *= -1;
    if (b == 0) return {f, 0};
    tmp = extgcd(b, a % b);
    res.first = tmp.second * f;
    res.second = (tmp.first - tmp.second * (a / b)) * g;
    return res;
}

```

6.3 Chinese Remainder Theorem

```

ll CRT(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g, m2 /= g;
    pair<ll, ll> p = extgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}

```

6.4 PiCount

```

const int V = 10000000, N = 100, M = 100000;
vector<int> primes;
bool isp[V];
int small_pi[V], dp[N][M];
void sieve(int x){

```

```

    for(int i = 2; i < x; ++i) isp[i] = true;
    isp[0] = isp[1] = false;
    for(int i = 2; i * i < x; ++i) if(isp[i])
        for(int j = i * i; j < x; j += i) isp[j] = false;
    for(int i = 2; i < x; ++i) if(isp[i]) primes.pb(i);
}
void init(){
    sieve(V);
    small_pi[0] = 0;
    for(int i = 1; i < V; ++i)
        small_pi[i] = small_pi[i - 1] + isp[i];
    for(int i = 0; i < M; ++i) dp[0][i] = i;
    for(int i = 1; i < N; ++i) for(int j = 0; j < M; ++j)
        dp[i][j] = dp[i - 1][j] - dp[i - 1][j / primes[i - 1]];
}
ll phi(ll n, int a){
    if(!a) return n;
    if(n < M && a < N) return dp[a][n];
    if(primes[a - 1] > n) return 1;
    if(1ll * primes[a - 1] * primes[a - 1] >= n && n < V)
        return small_pi[n] - a + 1;
    return phi(n, a - 1) - phi(n / primes[a - 1], a - 1);
}
ll PiCount(ll n){
    if(n < V) return small_pi[n];
    int s = sqrt(n + 0.5), y = cbrt(n + 0.5), a =
        small_pi[y];
    ll res = phi(n, a) + a - 1;
    for(; primes[a] <= s; ++a) res -= max(PiCount(n /
        primes[a]) - PiCount(primes[a] + 1, 0ll);
    return res;
}

```

6.5 Linear Function Mod Min

```

ll topos(ll x, ll m)
{ x %= m; if (x < 0) x += m; return x; }
//min value of ax + b (mod m) for x \in [0, n - 1]. O(
    log m)
ll min_rem(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    for (ll g = __gcd(a, m); g > 1; ) return g * min_rem(n
        , m / g, a / g, b / g) + (b % g);
    for (ll nn, nm, na, nb; a; n = nn, m = nm, a = na, b
        = nb) {
        if (a <= m - a) {
            nn = (a * (n - 1) + b) / m;
            if (!nn) break;
            nn += (b < a);
            nm = a, na = topos(-m, a);
            nb = b < a ? b : topos(b - m, a);
        } else {
            ll lst = b - (n - 1) * (m - a);
            if (lst >= 0) {b = lst; break;}
            nn = -(lst / m) + (lst % m < -a) + 1;
            nm = m - a, na = m % (m - a), nb = b % (m - a);
        }
    }
    return b;
}
//min value of ax + b (mod m) for x \in [0, n - 1],
    also return min x to get the value. O(log m)
//{value, x}
pair<ll, ll> min_rem_pos(ll n, ll m, ll a, ll b) {
    a = topos(a, m), b = topos(b, m);
    ll mn = min_rem(n, m, a, b), g = __gcd(a, m);
    //ax = (mn - b) (mod m)
    ll x = (extgcd(a, m).first + m) * ((mn - b + m) / g)
        % (m / g);
    return {mn, x};
}

```

6.6 Determinant*

```

ll Det(vector<vector<ll>> a) {
    int n = a.size();
    ll det = 1;
    for (int i = 0; i < n; ++i) {
        if (!a[i][i]) {
            det = -det;
            if (det < 0) det += mod;

```

```

    for (int j = i + 1; j < n; ++j) if (a[j][i]) {
        swap(a[j], a[i]);
        break;
    }
    if (!a[i][i]) return 0;
}
det = det * a[i][i] % mod;
ll mul = mpow(a[i][i], mod - 2);
for (int j = 0; j < n; ++j)
    a[i][j] = a[i][j] * mul % mod;
for (int j = 0; j < n; ++j) if (i ^ j) {
    ll mul = a[j][i];
    for (int k = 0; k < n; ++k) {
        a[j][k] -= a[i][k] * mul % mod;
        if (a[j][k] < 0) a[j][k] += mod;
    }
}
}
return det;
}

```

6.7 Floor Sum

```

// sum^{n-1}_0 floor((a * i + b) / m) in Log(n + m + a + b)
ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m) ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m) ans += n * (b / m), b %= m;
    ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}

```

6.8 Quadratic Residue

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}
int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1ll * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    ll f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (p + 1) >> 1; e >= 1; e >>= 1) {
        if (e & 1) {
            tmp = (g0 * f0 + d * (g1 * f1 % p)) % p;
            g1 = (g0 * f1 + g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (f0 * f0 + d * (f1 * f1 % p)) % p;
        f1 = (2 * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.9 Simplex

```

struct Simplex { // 0-based
    using T = long double;
    static const int N = 410, M = 30010;
    const T eps = 1e-7;

```

```

    int n, m;
    int Left[M], Down[N];
    // Ax <= b, max c^T x
    // result : v, xi = sol[i]
    T a[M][N], b[M], c[N], v, sol[N];
    T eq(T a, T b) {return fabs(a - b) < eps;}
    bool ls(T a, T b) {return a < b && !eq(a, b);}
    void init(int _n, int _m) {
        n = _n, m = _m, v = 0;
        for (int i = 0; i < m; ++i)
            for (int j = 0; j < n; ++j) a[i][j] = 0;
        for (int i = 0; i < m; ++i) b[i] = 0;
        for (int i = 0; i < n; ++i) c[i] = sol[i] = 0;
    }
    void pivot(int x, int y) {
        swap(Left[x], Down[y]);
        T k = a[x][y]; a[x][y] = 1;
        vector<int> nz;
        for (int i = 0; i < n; ++i) {
            a[x][i] /= k;
            if (!eq(a[x][i], 0)) nz.push_back(i);
        }
        b[x] /= k;
        for (int i = 0; i < m; ++i) {
            if (i == x || eq(a[i][y], 0)) continue;
            k = a[i][y], a[i][y] = 0;
            b[i] -= k * b[x];
            for (int j : nz) a[i][j] -= k * a[x][j];
        }
        if (eq(c[y], 0)) return;
        k = c[y], c[y] = 0, v += k * b[x];
        for (int i : nz) c[i] -= k * a[x][i];
    }
    // 0: found solution, 1: no feasible solution, 2:
    // unbounded
    int solve() {
        for (int i = 0; i < n; ++i) Down[i] = i;
        for (int i = 0; i < m; ++i) Left[i] = n + i;
        while (true) {
            int x = -1, y = -1;
            for (int i = 0; i < m; ++i) if (ls(b[i], 0) && (x == -1 || b[i] < b[x])) x = i;
            if (x == -1) break;
            for (int i = 0; i < n; ++i) if (ls(a[x][i], 0) && (y == -1 || a[x][i] < a[x][y])) y = i;
            if (y == -1) return 1;
            pivot(x, y);
        }
        while (true) {
            int x = -1, y = -1;
            for (int i = 0; i < n; ++i) if (ls(0, c[i]) && (y == -1 || c[i] > c[y])) y = i;
            if (y == -1) break;
            for (int i = 0; i < m; ++i)
                if (ls(0, a[i][y]) && (x == -1 || b[i] / a[i][y] < b[x] / a[x][y])) x = i;
            if (x == -1) return 2;
            pivot(x, y);
        }
        for (int i = 0; i < m; ++i) if (Left[i] < n)
            sol[Left[i]] = b[i];
        return 0;
    }
};

```

6.10 Berlekamp Massey

```

// need add, sub, mul
vector<ll> BerlekampMassey(vector<ll> a) {
    // find min |c| such that a_n = sum c_j * a_{n-j-1}, 0-based
    // O(N^2), if |c| = k, |a| >= 2k sure correct
    auto f = [&](vector<ll> v, ll c) {
        for (ll &x : v) x = mul(x, c);
        return v;
    };
    vector<ll> c, best;
    int pos = 0, n = a.size();
    for (int i = 0; i < n; ++i) {
        ll error = a[i];
        for (int j = 0; j < c.size(); ++j)
            error = sub(error, mul(c[j], a[i - 1 - j]));
    }
}

```



```

if (error == 0) continue;
ll inv = mpow(error, mod - 2);
if (c.empty()) {
    c.resize(i + 1), pos = i, best.pb(inv);
} else {
    vector<ll> fix = f(best, error);
    fix.insert(fix.begin(), i - pos - 1, 0);
    if (fix.size() >= c.size()) {
        best = f(c, sub(0, inv));
        best.insert(best.begin(), inv);
        pos = i, c.resize(fix.size());
    }
    for (int j = 0; j < fix.size(); ++j)
        c[j] = add(c[j], fix[j]);
}
}
return c;
}

```

6.11 Linear Programming Construction

Standard form: maximize $c^T x$ subject to $Ax \leq b$ and $x \geq 0$.

Dual LP: minimize $b^T y$ subject to $A^T y \geq c$ and $y \geq 0$.

\bar{x} and \bar{y} are optimal if and only if for all $i \in [1, n]$, either $\bar{x}_i = 0$ or $\sum_{j=1}^m A_{ij} \bar{y}_j = c_i$ holds and for all $i \in [1, m]$ either $\bar{y}_i = 0$ or $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$ holds.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3. $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
 - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.12 Euclidean

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity: $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

6.13 Theorem

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each *Labeled* vertices, there are

$$\frac{(n-2)!}{(d_1-1)!(d_2-1)! \cdots (d_n-1)!}$$

spanning trees.

- Let $T_{n,k}$ be the number of *Labeled* forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős-Gallai Theorem

A sequence of non-negative integers $d_1 \geq d_2 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + d_2 + \dots + d_n$ is even and

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

holds for all $1 \leq k \leq n$.

- Burnside's Lemma

Let X be a set and G be a group that acts on X . For $g \in G$, denote by X^g the elements fixed by g :

$$X^g = \{x \in X \mid gx = x\}$$

Then

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

- Gale-Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n is bigraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and $\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- Fulkerson-Chen-Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \dots \geq a_n$ is digraphic if and only if $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ and

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Möbius inversion formula

$$\begin{aligned} - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
- Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

6.14 Estimation

- The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200000 for $n < 1e19$.
- The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for $n = 0 \sim 9$, 627 for $n = 20$, $\sim 2e5$ for $n = 50$, $\sim 2e8$ for $n = 100$.
- Total number of partitions of n distinct elements: $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \dots$

6.15 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.16 Tips for Generating Funtion

- Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x)' \Rightarrow na_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k} k!$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x) \Rightarrow na_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{n-1}{i} x^i$

7 Polynomial

7.1 Number Theoretic Transform

```
// mul, add, sub, mpow
// LL -> int if too slow
struct NTT {
    ll w[N];
    NTT() {
        ll dw = mpow(G, (mod - 1) / N);
        w[0] = 1;
        for (int i = 1; i < N; ++i)
            w[i] = mul(w[i - 1], dw);
    }
    void operator()(vector<ll>& a, bool inv = false) { //
        0 <= a[i] < P
        int x = 0, n = a.size();
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (x ^= k) < k; k >>= 1);
            if (j < x) swap(a[j], a[k]);
        }
        for (int L = 2; L <= n; L <= 1) {
            int dx = N / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx)
                {
                    ll tmp = mul(a[j + dl], w[x]);
                    a[j + dl] = sub(a[j], tmp);
                    a[j] = add(a[j], tmp);
                }
            }
        }
        if (inv) {
            reverse(a.begin() + 1, a.end());
            ll invn = mpow(n, mod - 2);
            for (int i = 0; i < n; ++i)
                a[i] = mul(a[i], invn);
        }
    }
} ntt;
```

7.2 Fast Fourier Transform

```
using T = complex<double>;
const double PI = acos(-1);
struct NTT {
    T w[N];
    FFT() {
        T dw = {cos(2 * PI / N), sin(2 * PI / N)};
        w[0] = 1;
        for (int i = 1; i < N; ++i) w[i] = w[i - 1] * dw;
    }
    void operator()(vector<T>& a, bool inv = false) {
        // see NTT, replace LL with T
        if (inv) {
            reverse(a.begin() + 1, a.end());
            T invn = 1.0 / n;
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn;
        }
    }
} ntt;
// after mul, round i.real()
```

7.3 Primes

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3
2061584302081	7	1945555039024054273	5
2748779069441	3	9223372036737335297	3

7.4 Polynomial Operations

```
vector<ll> Mul(vector<ll> a, vector<ll> b, int bound
    = N) {
    int m = a.size() + b.size() - 1, n = 1;
    while (n < m) n <= 1;
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    vector<ll> out(n);
    for (int i = 0; i < n; ++i) out[i] = mul(a[i], b[i]);
    ntt(out, true), out.resize(min(m, bound));
    return out;
}

vector<ll> Inverse(vector<ll> a) {
    // O(NlogN), a[0] != 0
    int n = a.size();
    vector<ll> res(1, mpow(a[0], mod - 2));
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        vector<ll> v1(a.begin(), a.begin() + m * 2), v2 =
            res;
        v1.resize(m * 4), v2.resize(m * 4);
        ntt(v1), ntt(v2);
        for (int i = 0; i < m * 4; ++i)
            v1[i] = mul(mul(v1[i], v2[i]), v2[i]);
        ntt(v1, true);
        res.resize(m * 2);
        for (int i = 0; i < m; ++i)
            res[i] = add(res[i], res[i]);
        for (int i = 0; i < m * 2; ++i)
            res[i] = sub(res[i], v1[i]);
    }
    res.resize(n);
    return res;
}

pair<vector<ll>, vector<ll>> Divide(vector<ll> a,
    vector<ll> b) {
    // a = bq + R, O(NlogN), b.back() != 0
    int n = a.size(), m = b.size(), k = n - m + 1;
    if (n < m) return {{0}, a};
    vector<ll> ra = a, rb = b;
    reverse(all(ra)), ra.resize(k);
    reverse(all(rb)), rb.resize(k);
    vector<ll> Q = Mul(ra, Inverse(rb), k);
    reverse(all(Q));
    vector<ll> res = Mul(b, Q), R(m - 1);
    for (int i = 0; i < m - 1; ++i)
        R[i] = sub(a[i], res[i]);
```

```

    return {Q, R};
}
vector<ll> SqrtImpl(vector<ll> a) {
    if (a.empty()) return {0};
    int z = QuadraticResidue(a[0], mod), n = a.size();
    if (z == -1) return {-1};
    vector<ll> q(1, z);
    const int inv2 = (mod + 1) / 2;
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        q.resize(m * 2);
        vector<ll> f2 = Mul(q, q, m * 2);
        for (int i = 0; i < m * 2; ++i)
            f2[i] = sub(f2[i], a[i]);
        f2 = Mul(f2, Inverse(q), m * 2);
        for (int i = 0; i < m * 2; ++i)
            q[i] = sub(q[i], mul(f2[i], inv2));
    }
    q.resize(n);
    return q;
}
vector<ll> Sqrt(vector<ll> a) {
    // O(NlogN), return {-1} if not exists
    int n = a.size(), m = 0;
    while (m < n && a[m] == 0) m++;
    if (m == n) return vector<ll>(n);
    if (m & 1) return {-1};
    vector<ll> s = SqrtImpl(vector<ll>(a.begin() + m, a.end()));
    if (s[0] == -1) return {-1};
    vector<ll> res(n);
    for (int i = 0; i < s.size(); ++i)
        res[i + m / 2] = s[i];
    return res;
}
vector<ll> Derivative(vector<ll> a) {
    int n = a.size();
    vector<ll> res(n - 1);
    for (int i = 0; i < n - 1; ++i)
        res[i] = mul(a[i + 1], i + 1);
    return res;
}
vector<ll> Integral(vector<ll> a) {
    int n = a.size();
    vector<ll> res(n + 1);
    for (int i = 0; i < n; ++i)
        res[i + 1] = mul(a[i], mpow(i + 1, mod - 2));
    return res;
}
vector<ll> Ln(vector<ll> a) {
    // O(NlogN), a[0] = 1
    int n = a.size();
    if (n == 1) return {0};
    vector<ll> d = Derivative(a);
    a.pop_back();
    return Integral(Mul(d, Inverse(a), n - 1));
}
vector<ll> Exp(vector<ll> a) {
    // O(NlogN), a[0] = 0
    int n = a.size();
    vector<ll> q(1, 1);
    a[0] = add(a[0], 1);
    for (int m = 1; m < n; m <= 1) {
        if (n < m * 2) a.resize(m * 2);
        vector<ll> g(a.begin(), a.begin() + m * 2), h(all(
            q));
        h.resize(m * 2), h = Ln(h);
        for (int i = 0; i < m * 2; ++i)
            g[i] = sub(g[i], h[i]);
        q = Mul(g, q, m * 2);
    }
    q.resize(n);
    return q;
}
vector<ll> Pow(vector<ll> a, ll k) {
    int n = a.size(), m = 0;
    vector<ll> ans(n, 0);
    while (m < n && a[m] == 0) m++;
    if (k && m && (k >= n || k * m >= n)) return ans;
    if (m == n) return ans[0] = 1, ans;
    ll lead = m * k;
    vector<ll> b(a.begin() + m, a.end());

```

```

    ll base = mpow(b[0], k), inv = mpow(b[0], mod - 2);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], inv);
    b = Ln(b);
    for (int i = 0; i < n - m; ++i)
        b[i] = mul(b[i], k % mod);
    b = Exp(b);
    for (int i = lead; i < n; ++i)
        ans[i] = mul(b[i - lead], base);
    return ans;
}
vector<ll> Evaluate(vector<ll> a, vector<ll> x) {
    if (x.empty()) return {};
    int n = x.size();
    vector<vector<ll>> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector<vector<ll>> down(n * 2);
    down[1] = Divide(a, up[1]).second;
    for (int i = 2; i < n * 2; ++i)
        down[i] = Divide(down[i >> 1], up[i]).second;
    vector<ll> y(n);
    for (int i = 0; i < n; ++i) y[i] = down[i + n][0];
    return y;
}
vector<ll> Interpolate(vector<ll> x, vector<ll> y) {
    int n = x.size();
    vector<vector<ll>> up(n * 2);
    for (int i = 0; i < n; ++i)
        up[i + n] = {sub(0, x[i]), 1};
    for (int i = n - 1; i > 0; --i)
        up[i] = Mul(up[i * 2], up[i * 2 + 1]);
    vector<ll> a = Evaluate(Derivative(up[1]), x);
    for (int i = 0; i < n; ++i)
        a[i] = mul(y[i], mpow(a[i], mod - 2));
    vector<vector<ll>> down(n * 2);
    for (int i = 0; i < n; ++i) down[i + n] = {a[i]};
    for (int i = n - 1; i > 0; --i) {
        vector<ll> lhs = Mul(down[i * 2], up[i * 2 + 1]);
        vector<ll> rhs = Mul(down[i * 2 + 1], up[i * 2]);
        down[i].resize(lhs.size());
        for (int j = 0; j < lhs.size(); ++j)
            down[i][j] = add(lhs[j], rhs[j]);
    }
    return down[1];
}

```

7.5 Fast Linear Recursion

```

ll FastLinearRecursion(vector<ll> a, vector<ll> c, ll
    k) {
    // a_n = sigma c_j * a_{n - j - 1}, 0-based
    // O(NlogNlogK), |a| = |c|
    int n = a.size();
    if (k < n) return a[k];
    vector<ll> base(n + 1, 1);
    for (int i = 0; i < n; ++i)
        base[i] = sub(0, c[n - i - 1]);
    vector<ll> poly(n);
    (n == 1 ? poly[0] = c[n - 1] : poly[1] = 1);
    auto calc = [&](vector<ll> p1, vector<ll> p2) {
        // O(n^2) brute force or O(nlogn) NTT
        return Divide(Mul(p1, p2), base).second;
    };
    vector<ll> res(n, 0); res[0] = 1;
    for (; k; k >= 1, poly = calc(poly, poly)) {
        if (k & 1) res = calc(res, poly);
    }
    ll ans = 0;
    for (int i = 0; i < n; ++i)
        (ans += res[i] * a[i]) %= mod;
    return ans;
}

```

7.6 Fast Walsh Transform

```

void fwt(vector<int> &a) {
    // and : x += y * (1, -1)
    // or  : y += x * (1, -1)
    // xor : x = (x + y) * (1, 1/2)
}

```

```
//      y = (x - y) * (1, 1/2)
int n = __lg(a.size());
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 1 << n; ++j) if (j >> i & 1) {
        int x = a[j] ^ (1 << i); y = a[j];
        // do something
    }
}
vector<int> subs_conv(vector<int> a, vector<int> b) {
    // c_i = sum_{j & k = 0, j | k = i} a_j * b_k
    int n = __lg(a.size());
    vector<vector<int>> ha(n + 1, vector<int>(1 << n));
    vector<vector<int>> hb(n + 1, vector<int>(1 << n));
    vector<vector<int>> c(n + 1, vector<int>(1 << n));
    for (int i = 0; i < 1 << n; ++i) {
        ha[__builtin_popcount(i)][i] = a[i];
        hb[__builtin_popcount(i)][i] = b[i];
    }
    for (int i = 0; i <= n; ++i)
        or_fwt(ha[i]), or_fwt(hb[i]);
    for (int i = 0; i <= n; ++i)
        for (int j = 0; i + j <= n; ++j)
            for (int k = 0; k < 1 << n; ++k)
                // mind overflow
                c[i + j][k] += ha[i][k] * hb[j][k];
    for (int i = 0; i <= n; ++i) or_fwt(c[i], true);
    vector<int> ans(1 << n);
    for (int i = 0; i < 1 << n; ++i)
        ans[i] = c[__builtin_popcount(i)][i];
    return ans;
}
```

8 Geometry

8.1 Basic

```
const double eps = 1e-8, PI = acos(-1);
int sign(double x)
{ return fabs(x) <= eps ? 0 : (x > 0 ? 1 : -1); }
double norm(double x) {
    while (x < -eps) x += PI * 2;
    while (x > PI * 2 + eps) x -= PI * 2;
    return x;
}
struct Pt {
    double x, y;
    Pt (double _x, double _y) : x(_x), y(_y) {}
    Pt operator + (Pt o) {return Pt(x + o.x, y + o.y);}
    Pt operator - (Pt o) {return Pt(x - o.x, y - o.y);}
    Pt operator * (double k) {return Pt(x * k, y * k);}
    Pt operator / (double k) {return Pt(x / k, y / k);}
    double operator * (Pt o) {return x * o.x + y * o.y;}
    double operator ^ (Pt o) {return x * o.y - y * o.x;}
};
struct Line { Pt a, b; };
struct Cir { Pt o; double r; };
double abs2(Pt o) { return o * o; }
double abs(Pt o) { return sqrt(abs2(o)); }
int ori(Pt o, Pt a, Pt b)
{ return sign((o - a) ^ (o - b)); }
bool btw(Pt a, Pt b, Pt c) // c on segment ab?
{ return ori(a, b, c) == 0 && sign((c - a) * (c - b))
    <= 0; }
int pos(Pt a)
{ return sign(a.y) == 0 ? sign(a.x) < 0 : a.y < 0; }
double area(Pt a, Pt b, Pt c)
{ return fabs((a - b) ^ (a - c)) / 2; }
double angle(Pt a, Pt b)
{ return norm(atan2(b.y - a.y, b.x - a.x)); }
Pt unit(Pt o) { return o / abs(o); }
Pt rot(Pt a, double o) { // CCW
    double c = cos(o), s = sin(o);
    return Pt(c * a.x - s * a.y, s * a.x + c * a.y);
}
Pt perp(Pt a) {return Pt(-a.y, a.x);}
Pt proj_vec(Pt a, Pt b, Pt c) { // vector ac proj to ab
    return (b - a) * ((c - a) * (b - a)) / (abs2(b - a));
}
Pt proj_pt(Pt a, Pt b, Pt c) { // point c proj to ab
    return proj_vec(a, b, c) + a;
}
```

```
}
```

8.2 Heart

```
Pt circenter(Pt p0, Pt p1, Pt p2) {
    // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
    double m = 2. * (x1 * y2 - y1 * x2);
    Pt center(0, 0);
    center.x = (x1 * x1 * y2 - x2 * x2 * y1 + y1 * y2 * (
        y1 - y2)) / m;
    center.y = (x1 * x2 * (x2 - x1) - y1 * y1 * x2 + x1 *
        y2 * y2) / m;
    return center + p0;
}
Pt incenter(Pt p1, Pt p2, Pt p3) {
    // radius = area / s * 2
    double a = abs(p2 - p3), b = abs(p1 - p3), c = abs(p1
        - p2);
    double s = a + b + c;
    return (p1 * a + p2 * b + p3 * c) / s;
}
Pt masscenter(Pt p1, Pt p2, Pt p3)
{ return (p1 + p2 + p3) / 3; }
Pt orthocenter(Pt p1, Pt p2, Pt p3)
{ return masscenter(p1, p2, p3) * 3 - circenter(p1, p2,
    p3) * 2; }
```

8.3 External Bisector

```
Pt external_bisector(Pt p1, Pt p2, Pt p3) { //213
    Pt L1 = p2 - p1, L2 = p3 - p1;
    L2 = L2 * abs(L1) / abs(L2);
    return L1 + L2;
}
```

8.4 Intersection of Segments

```
Pt LinesInter(Line a, Line b) {
    double abc = (a.b - a.a) ^ (b.a - a.a);
    double abd = (a.b - a.a) ^ (b.b - a.a);
    if (sign(abc - abd) == 0) return b.b; // no inter
    return (b.b * abc - b.a * abd) / (abc - abd);
}
vector<Pt> SegsInter(Line a, Line b) {
    if (btw(a.a, a.b, b.a)) return {b.a};
    if (btw(a.a, a.b, b.b)) return {b.b};
    if (btw(b.a, b.b, a.a)) return {a.a};
    if (btw(b.a, b.b, a.b)) return {a.b};
    if (ori(a.a, a.b, b.a) * ori(a.a, a.b, b.b) == -1 &&
        ori(b.a, b.b, a.a) * ori(b.a, b.b, a.b) == -1)
        return {LinesInter(a, b)};
    return {};
}
```

8.5 Intersection of Circle and Line

```
vector<Pt> CircleLineInter(Cir c, Line l) {
    Pt p = l.a + (l.b - l.a) * ((c.o - l.a) * (l.b - l.a)
        ) / abs2(l.b - l.a);
    double s = (l.b - l.a) ^ (c.o - l.a), h2 = c.r * c.r
        - s * s / abs2(l.b - l.a);
    if (sign(h2) == -1) return {};
    if (sign(h2) == 0) return {p};
    Pt h = (l.b - l.a) / abs(l.b - l.a) * sqrt(h2);
    return {p - h, p + h};
}
```

8.6 Intersection of Circles

```
vector<Pt> CirclesInter(Cir c1, Cir c2) {
    double d2 = abs2(c1.o - c2.o), d = sqrt(d2);
    if (d < max(c1.r, c2.r) - min(c1.r, c2.r) || d > c1.r
        + c2.r) return {};
    Pt u = (c1.o + c2.o) / 2 + (c1.o - c2.o) * ((c2.r *
        c2.r - c1.r * c1.r) / (2 * d2));
    double A = sqrt((c1.r + c2.r + d) * (c1.r - c2.r + d)
        * (c1.r + c2.r - d) * (-c1.r + c2.r + d));
    Pt v = Pt(c1.o.y - c2.o.y, -c1.o.x + c2.o.x) * A / (2
        * d2);
    if (sign(v.x) == 0 && sign(v.y) == 0) return {u};
    return {u + v, u - v};
}
```

8.7 Intersection of Polygon and Circle

```
double _area(Pt pa, Pt pb, double r){
    if (abs(pa) < abs(pb)) swap(pa, pb);
    if (abs(pb) < eps) return 0;
    double S, h, theta;
    double a = abs(pb), b = abs(pa), c = abs(pb - pa);
    double cosB = pb * (pb - pa) / a / c, B = acos(cosB);
    double cosC = (pa * pb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C / 2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < pi / 2) S -= (acos(h / r) * r * r
            - h * sqrt(r * r - h * h));
    } else if (b > r) {
        theta = pi - B - asin(sin(B) / r * a);
        S = 0.5 * a * r * sin(theta) + (C - theta) / 2 * r
            * r;
    } else S = 0.5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(vector<Pt> poly, Pt O, double r
    ) {
    double S = 0; int n = poly.size();
    for (int i = 0; i < n; ++i)
        S += _area(poly[i] - O, poly[(i + 1) % n] - O, r) *
            ori(O, poly[i], poly[(i + 1) % n]);
    return fabs(S);
}
```

8.8 Tangent Lines of Circle and Point

```
vector<Line> tangent(Cir c, Pt p) {
    vector<Line> z;
    double d = abs(p - c.o);
    if (sign(d - c.r) == 0) {
        Pt i = rot(p - c.o, pi / 2);
        z.push_back({p, p + i});
    } else if (d > c.r) {
        double o = acos(c.r / d);
        Pt i = unit(p - c.o), j = rot(i, o) * c.r, k = rot(
            i, -o) * c.r;
        z.push_back({c.o + j, p});
        z.push_back({c.o + k, p});
    }
    return z;
}
```

8.9 Tangent Lines of Circles

```
vector<Line> tangent(Cir c1, Cir c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = abs2(c1.o - c2.o);
    if (sign(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    Pt v = (c2.o - c1.o) / d;
    double c = (c1.r - sign1 * c2.r) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        Pt n = Pt(v.x * c - sign2 * h * v.y, v.y * c +
            sign2 * h * v.x);
        Pt p1 = c1.o + n * c1.r;
        Pt p2 = c2.o + n * (c2.r * sign1);
        if (sign(p1.x - p2.x) == 0 && sign(p1.y - p2.y) ==
            0)
            p2 = p1 + perp(c2.o - c1.o);
        ret.pb({p1, p2});
    }
    return ret;
}
```

8.10 Point In Convex

```
bool PointInConvex(const vector<Pt> &C, Pt p, bool
    strict = true) {
    int a = 1, b = int(C.size()) - 1, r = !strict;
    if (C.size() == 0) return false;
    if (C.size() < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(C[0], C[a], p) >= r || ori(C[0], C[b], p) <=
        -r) return false;
```

```
while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
}
return ori(C[a], C[b], p) < r;
}
```

8.11 Point Segment Distance

```
double PointSegDist(Pt q0, Pt q1, Pt p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign((q1 - q0) * (p - q0)) >= 0 && sign((q0 - q1)
        * (p - q1)) >= 0)
        return fabs(((q1 - q0) ^ (p - q0)) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}
```

8.12 Convex Hull

```
vector<Pt> ConvexHull(vector<Pt> pt) {
    int n = pt.size();
    sort(all(pt), [&](Pt a, Pt b) {return a.x == b.x ? a.
        y < b.y : a.x < b.x;});
    vector<Pt> ans = {pt[0]};
    for (int t : {0, 1}) {
        int m = ans.size();
        for (int i = 1; i < n; ++i) {
            while (ans.size() > m && ori(ans[ans.size() - 2],
                ans.back(), pt[i]) <= 0)
                ans.pop_back();
            ans.pb(pt[i]);
        }
        reverse(all(pt));
    }
    ans.pop_back();
    return ans;
}
```

8.13 Convex Hull Distance

```
double ConvexHullDist(vector<Pt> A, vector<Pt> B) {
    Pt O(0, 0);
    for (auto &p : B) p = O - p;
    auto C = Minkowski(A, B); // assert SZ(C) > 0
    if (PointInConvex(C, O)) return 0;
    double ans = PointSegDist(C.back(), C[0], O);
    for (int i = 0; i + 1 < C.size(); ++i)
        ans = min(ans, PointSegDist(C[i], C[i + 1], O));
    return ans;
}
```

8.14 Minimum Enclosing Circle

```
Cir min_enclosing(vector<Pt> &p) {
    random_shuffle(all(p));
    double r = 0.0;
    Pt cent = p[0];
    for (int i = 1; i < p.size(); ++i) {
        if (abs2(cent - p[i]) <= r) continue;
        cent = p[i], r = 0.0;
        for (int j = 0; j < i; ++j) {
            if (abs2(cent - p[j]) <= r) continue;
            cent = (p[i] + p[j]) / 2, r = abs2(p[j] - cent);
            for (int k = 0; k < j; ++k) {
                if (abs2(cent - p[k]) <= r) continue;
                cent = circcenter(p[i], p[j], p[k]);
                r = abs2(p[k] - cent);
            }
        }
    }
    return {cent, sqrt(r)};
}
```

8.15 Union of Circles

```
vector<pair<double, double>> CoverSegment(Cir a, Cir b)
    {
        double d = abs(a.o - b.o);
        vector<pair<double, double>> res;
        if (sign(a.r + b.r - d) == 0);
        else if (d <= abs(a.r - b.r) + eps) {
            if (a.r < b.r) res.emplace_back(0, 2 * pi);
```



```

} else if (d < abs(a.r + b.r) - eps) {
    double o = acos((a.r * a.r + d * d - b.r * b.r) /
        (2 * a.r * d));
    double z = norm(atan2((b.o - a.o).y, (b.o - a.o).x)
        );
    double l = norm(z - o), r = norm(z + o);
    if (l > r) res.emplace_back(l, 2 * pi), res.
        emplace_back(0, r);
    else res.emplace_back(l, r);
}
return res;
}

double CircleUnionArea(vector<Cir> c) { // circle
    should be identical
    int n = c.size();
    double a = 0, w;
    for (int i = 0; w = 0, i < n; ++i) {
        vector<pair<double, double>> s = {{2 * pi, 9}}, z;
        for (int j = 0; j < n; ++j) if (i != j) {
            z = CoverSegment(c[i], c[j]);
            for (auto &e : z) s.push_back(e);
        }
        sort(s.begin(), s.end());
        auto F = [&](double t) { return c[i].r * (c[i].r *
            t + c[i].o.x * sin(t) - c[i].o.y * cos(t)); };
        for (auto &e : s) {
            if (e.first > w) a += F(e.first) - F(w);
            w = max(w, e.second);
        }
    }
    return a * 0.5;
}

```

8.16 Union of Polygons

```

double polyUnion(vector<vector<Pt>> poly) {
    int n = poly.size();
    double ans = 0;
    auto solve = [&](Pt a, Pt b, int cid) {
        vector<pair<Pt, int>> event;
        for (int i = 0; i < n; ++i) {
            int st = 0, sz = poly[i].size();
            while (st < sz && ori(poly[i][st], a, b) != 1)
                st++;
            if (st == sz) continue;
            for (int j = 0; j < sz; ++j) {
                Pt c = poly[i][(j + st) % sz];
                Pt d = poly[i][(j + st + 1) % sz];
                if (sign((a - b) ^ (c - d)) != 0) {
                    int ok1 = ori(c, a, b) == 1;
                    int ok2 = ori(d, a, b) == 1;
                    if (ok1 ^ ok2) event.emplace_back(LinesInter
                        ({a, b}, {c, d}), ok1 ? 1 : -1);
                } else if (ori(c, a, b) == 0 && sign((a - b) *
                    (c - d)) > 0 && i <= cid) {
                    event.emplace_back(c, -1);
                    event.emplace_back(d, 1);
                }
            }
        }
        sort(all(event), [&](pair<Pt, int> i, pair<Pt,
            int> j) {
            return ((a - i.first) * (a - b)) < ((a - j.first)
                * (a - b));
        });
        int now = 0;
        Pt lst = a;
        for (auto [x, y] : event) {
            if (btw(a, b, lst) && btw(a, b, x) && !now)
                ans += lst ^ x;
            now += y, lst = x;
        }
    };
    for (int i = 0; i < n; ++i) {
        int sz = poly[i].size();
        for (int j = 0; j < sz; ++j)
            solve(poly[i][j], poly[i][(j + 1) % sz], i);
    }
    return ans / 2;
}

```

8.17 Rotating SweepLine

```

void RotatingSweepLine(vector<Pt> &pt) {
    int n = pt.size();
    vector<int> ord(n), cur(n);
    vector<pii> line;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) if (i ^ j)
            line.emplace_back(i, j);
    sort(all(line), [&](pii i, pii j) {
        Pt a = pt[i.second] - pt[i.first];
        Pt b = pt[j.second] - pt[j.first];
        if (pos(a) == pos(b)) return sign(a ^ b) > 0;
        return pos(a) < pos(b);
    });
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) {
        return (sign(pt[i].y - pt[j].y) == 0 ? pt[i].x < pt
            [j].x : pt[i].y < pt[j].y);
    });
    for (int i = 0; i < n; ++i) cur[ord[i]] = i;
    for (auto [i, j] : line) {
        // point sort by the distance to line(i, j)
        tie(cur[i], cur[j], ord[cur[i]], ord[cur[j]]) =
            make_tuple(cur[j], cur[i], j, i);
    }
}

```

8.18 Half Plane Intersection

```

vector<Pt> HalfPlaneInter(vector<Line> vec) {
    // x
    // Line.a -----> Line.b
    int n = vec.size();
    sort(all(vec), [&](Line a, Line b) {
        Pt A = a.b - a.a, B = b.b - b.a;
        if (pos(A) == pos(B)) {
            if (sign(A ^ B) == 0)
                return sign((b.a - a.a) ^ (b.b - a.a)) > 0;
            return sign(A ^ B) > 0;
        }
        return pos(A) < pos(B);
    });
    auto same = [&](Line a, Line b) {
        return sign((a.b - a.a) ^ (b.b - b.a)) == 0 &&
            sign((a.b - a.a) * (b.b - b.a)) > 0;
    };
    deque<Pt> inter;
    deque<Line> seg;
    for (int i = 0; i < n; ++i) if (!i || !same(vec[i -
        1], vec[i])) {
        while (seg.size() >= 2 && sign((vec[i].b - inter.
            back()) ^ (vec[i].a - inter.back())) == 1)
            seg.pop_back(), inter.pop_back();
        while (seg.size() >= 2 && sign((vec[i].b - inter.
            front()) ^ (vec[i].a - inter.front())) == 1)
            seg.pop_front(), inter.pop_front();
        seg.pb(vec[i]);
        if (seg.size() >= 2) inter.pb(LinesInter(seg[seg.
            size() - 2], seg.back()));
    }
    while (seg.size() >= 2 && sign((seg.front().b - inter.
        back()) ^ (seg.front().a - inter.back())) == 1)
        seg.pop_back(), inter.pop_back();
    inter.pb(LinesInter(seg.front(), seg.back()));
    return vector<Pt>(all(inter));
}

```

8.19 Minkowski Sum

```

void reorder(vector<Pt> &P) {
    rotate(P.begin(), min_element(all(P), [&](Pt a, Pt b)
        { return make_pair(a.y, a.x) < make_pair(b.y, b.
            x); }), P.end());
}

vector<Pt> Minkowski(vector<Pt> P, vector<Pt> Q) {
    // P, Q: convex polygon, CCW order
    reorder(P), reorder(Q);
    int n = P.size(), m = Q.size();
    P.pb(P[0]), P.pb(P[1]), Q.pb(Q[0]), Q.pb(Q[1]);
    vector<Pt> ans;
    for (int i = 0, j = 0; i < n || j < m; ) {
        ans.pb(P[i] + Q[j]);
        auto val = (P[i + 1] - P[i]) ^ (Q[j + 1] - Q[j]);
    }
}

```



```

    if (val >= 0) i++;
    if (val <= 0) j++;
}
return ans;
}

```

8.20 Delaunay Triangulation

```

/* Delaunay Triangulation:
   Given a sets of points on 2D plane, find a
   triangulation such that no points will strictly
   inside circumcircle of any triangle.
   find : return a triangle contain given point
   add_point : add a point into triangulation
   A Triangle is in triangulation iff. its has_chd is 0.
   Region of triangle u: iterate each u.edge[i].tri,
   each points are u.p[(i+1)%3], u.p[(i+2)%3]
   Voronoi diagram: for each triangle in triangulation,
   the bisector of all its edges will split the region.
   nearest point will belong to the triangle containing it
*/
const ll inf = MAXC * MAXC * 100; // Lower_bound
unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pll& p0, const pll& p1, const pll& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pll const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root = // Tri should at least contain all
                    points
        new(tris++) Tri(pll(-inf, -inf), pll(inf + inf, -
            inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const pll &p) { add_point(find(
        the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pll const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris++) Tri(root->p[i], root->p[(i +
                1) % 3], p);
    }
}

```

```

for (int i = 0; i < 3; ++i)
    edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
for (int i = 0; i < 3; ++i)
    edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
for (int i = 0; i < 3; ++i)
    root->chd[i] = t[i];
for (int i = 0; i < 3; ++i)
    flip(t[i], 2);
}
void flip(Tri* tri, int pi) {
    Tri* trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[
        pj])) return;
    /* flip edge between tri, trj */
    Tri* trk = new(tris++) Tri(tri->p[(pi + 1) % 3],
        trj->p[pj], tri->p[pi]);
    Tri* trl = new(tris++) Tri(trj->p[(pj + 1) % 3],
        tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->chd[0] = trk; tri->chd[1] = trl; tri->chd[2] =
        0;
    trj->chd[0] = trk; trj->chd[1] = trl; trj->chd[2] =
        0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
}
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now->has_chd())
        return triang.pb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

8.21 Triangulation Voronoi

```

vector<Line> ls[N];
pll arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate the area of point "strictly in
    the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.pb(intersect(hpi[i].X, hpi[i].Y, hpi[(i + 1) %
            SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pll, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pll(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
}

```

```

build(n, arr); // Triangulation
for (auto *t : triang) {
    vector<int> p;
    for (int i = 0; i < 3; ++i)
        if (mp.find(t->p[i]) != mp.end())
            p.pb(mp[t->p[i]]);
    for (int i = 0; i < SZ(p); ++i)
        for (int j = i + 1; j < SZ(p); ++j) {
            Line l(oarr[p[i]], oarr[p[j]]);
            ls[p[i]].pb(make_line(oarr[p[i]], l));
            ls[p[j]].pb(make_line(oarr[p[j]], l));
        }
}
}

```

8.22 3D Point

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double _y = 0, double _z = 0): x
        (_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(const Point &p1, const Point &p2)
{ return Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point cross(const Point &p1, const Point &p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x -
    p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(const Point &p1, const Point &p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(const Point &a)
{ return sqrt(dot(a, a)); }
Point cross3(const Point &a, const Point &b, const
    Point &c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}

```

8.23 3D Convex Hull

```

struct CH3D {
    struct face{int a, b, c; bool ok;} F[8 * N];
    double dblcmp(Point &p, face &f)
    {return dot(cross3(P[f.a], P[f.b], P[f.c]), p - P[f.a
        ]);}
    int g[N][N], num, n;
    Point P[N];
    void deal(int p, int a, int b) {
        int f = g[a][b];
        face add;
        if (F[f].ok) {
            if (dblcmp(P[p], F[f]) > eps) dfs(p, f);
        } else
            add.a = b, add.b = a, add.c = p, add.ok = 1, g[
                p][b] = g[a][p] = g[b][a] = num, F[num++] =
                add;
    }
    void dfs(int p, int now) {
        F[now].ok = 0;
        deal(p, F[now].b, F[now].a), deal(p, F[now].c, F[
            now].b), deal(p, F[now].a, F[now].c);
    }
    bool same(int s, int t){
        Point &a = P[F[s].a];
        Point &b = P[F[s].b];
        Point &c = P[F[s].c];
        return fabs(volume(a, b, c, P[F[t].a])) < eps &&
            fabs(volume(a, b, c, P[F[t].b])) < eps && fabs(
                volume(a, b, c, P[F[t].c])) < eps;
    }
}

```

```

}
void init(int _n){n = _n, num = 0;}
void solve() {
    face add;
    num = 0;
    if(n < 4) return;
    if([&](){
        for (int i = 1; i < n; ++i)
            if (abs(P[0] - P[i]) > eps)
                return swap(P[1], P[i]), 0;
        return 1;
    }()) || [&](){
        for (int i = 2; i < n; ++i)
            if (abs(cross3(P[i], P[0], P[1])) > eps)
                return swap(P[2], P[i]), 0;
        return 1;
    }()) || [&](){
        for (int i = 3; i < n; ++i)
            if (fabs(dot(cross3(P[0] - P[1], P[1] - P[2]), P
                [0] - P[i])) > eps)
                return swap(P[3], P[i]), 0;
        return 1;
    }())return;
    for (int i = 0; i < 4; ++i) {
        add.a = (i + 1) % 4, add.b = (i + 2) % 4, add.c =
            (i + 3) % 4, add.ok = true;
        if (dblcmp(P[i], add) > 0) swap(add.b, add.c);
        g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.
            a] = num;
        F[num++] = add;
    }
    for (int i = 4; i < n; ++i)
        for (int j = 0; j < num; ++j)
            if (F[j].ok && dblcmp(P[i], F[j]) > eps) {
                dfs(i, j);
                break;
            }
    for (int tmp = num, i = (num = 0); i < tmp; ++i)
        if (F[i].ok) F[num++] = F[i];
}
double get_area() {
    double res = 0.0;
    if (n == 3)
        return abs(cross3(P[0], P[1], P[2])) / 2.0;
    for (int i = 0; i < num; ++i)
        res += area(P[F[i].a], P[F[i].b], P[F[i].c]);
    return res / 2.0;
}
double get_volume() {
    double res = 0.0;
    for (int i = 0; i < num; ++i)
        res += volume(Point(0, 0, 0), P[F[i].a], P[F[i].b
            ], P[F[i].c]);
    return fabs(res / 6.0);
}
int triangle() {return num;}
int polygon() {
    int res = 0;
    for (int i = 0, flag = 1; i < num; ++i, res += flag
        , flag = 1)
        for (int j = 0; j < i && flag; ++j)
            flag &= !same(i, j);
    return res;
}
Point getcent(){
    Point ans(0, 0, 0), temp = P[F[0].a];
    double v = 0.0, t2;
    for (int i = 0; i < num; ++i)
        if (F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[
                i].c];
            t2 = volume(temp, p1, p2, p3) / 6.0;
            if (t2 > 0)
                ans.x += (p1.x + p2.x + p3.x + temp.x) * t2,
                ans.y += (p1.y + p2.y + p3.y + temp.y) *
                    t2, ans.z += (p1.z + p2.z + p3.z + temp.z
                        ) * t2, v += t2;
        }
    ans.x /= (4 * v), ans.y /= (4 * v), ans.z /= (4 * v
        );
    return ans;
}
}

```

```
double pointmindis(Point p) {
    double rt = 99999999;
    for(int i = 0; i < num; ++i)
        if(F[i].ok == true) {
            Point p1 = P[F[i].a], p2 = P[F[i].b], p3 = P[F[i].c];
            double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
            double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
            double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
            double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
            double temp = fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
            rt = min(rt, temp);
        }
    return rt;
}
};
```

9 Else

9.1 Pbds

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
#include <ext/rope>
using namespace __gnu_cxx;
__gnu_pbds::priority_queue<int> pq1, pq2;
pq1.join(pq2); // pq1 += pq2, pq2 = {}
cc_hash_table<int, int> m1;
tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> oset;
oset.insert(2), oset.insert(4);
*oset.find_by_order(1), oset.order_of_key(1); // 4 0
bitset<100> BS;
BS.flip(3), BS.flip(5);
BS._Find_first(), BS._Find_next(3); // 3 5
rope<int> rp1, rp2;
rp1.push_back(1), rp1.push_back(3);
rp1.insert(0, 2); // pos, num
rp1.erase(0, 2); // pos, len
rp1.substr(0, 2); // pos, len
rp2.push_back(4);
rp1 += rp2, rp2 = rp1;
rp2[0], rp2[1]; // 3 4
```

9.2 Bit Hack

```
long long next_perm(long long v) {
    long long t = v | (v - 1);
    return (t + 1) | (((~t & ~t) - 1) >> (__builtin_ctz(v) + 1));
}
void subset(long long s) {
    long long sub = s;
    while (sub) sub = (sub - 1) & s;
}
```

9.3 Dynamic Programming Condition

9.3.1 Totally Monotone (Concave/Convex)

$$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$$

$$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$$

9.3.2 Monge Condition (Concave/Convex)

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$$

$$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$$

9.3.3 Optimal Split Point

If

$$B[i][j] + B[i+1][j+1] \geq B[i][j+1] + B[i+1][j]$$

then

$$H_{i,j-1} \leq H_{i,j} \leq H_{i+1,j}$$

9.4 Smawk Algorithm

```
ll query(int l, int r) {
    // ...
}
struct SMAWK {
    // Condition:
    // If M[l][0] < M[l][1] then M[0][0] < M[0][1]
    // If M[l][0] == M[l][1] then M[0][0] <= M[0][1]
    // For all i, find r_i s.t. M[i][r_i] is maximum // minimum.
    int ans[N], tmp[N];
    void interpolate(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nl;
        for (int i = 1; i < n; i += 2) {
            nl.push_back(l[i]);
        }
        run(nl, r);
        for (int i = 1, j = 0; i < n; i += 2) {
            while (j < m && r[j] < ans[l[i]])
                j++;
            assert(j < m && ans[l[i]] == r[j]);
            tmp[l[i]] = j;
        }
        for (int i = 0; i < n; i += 2) {
            int curl = 0, curr = m - 1;
            if (i)
                curl = tmp[l[i - 1]];
            if (i + 1 < n)
                curr = tmp[l[i + 1]];
            ll res = query(l[i], r[curl]);
            ans[l[i]] = r[curl];
            for (int j = curl + 1; j <= curr; ++j) {
                ll nxt = query(l[i], r[j]);
                if (res < nxt)
                    res = nxt, ans[l[i]] = r[j];
            }
        }
    }
    void reduce(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        vector<int> nr;
        for (int j : r) {
            while (!nr.empty()) {
                int i = nr.size() - 1;
                if (query(l[i], nr.back()) <= query(l[i], j))
                    nr.pop_back();
                else
                    break;
            }
            if (nr.size() < n)
                nr.push_back(j);
        }
        run(l, nr);
    }
    void run(vector<int> l, vector<int> r) {
        int n = l.size(), m = r.size();
        if (max(n, m) <= 2) {
            for (int i : l) {
                ans[i] = r[0];
                if (m > 1) {
                    if (query(i, r[0]) < query(i, r[1]))
                        ans[i] = r[1];
                }
            }
        } else if (n >= m) {
            interpolate(l, r);
        } else {
            reduce(l, r);
        }
    }
};
```

9.5 Slope Trick

```
template<typename T>
struct slope_trick_convex {
    T minn = 0, ground_l = 0, ground_r = 0;
    priority_queue<T, vector<T>, less<T>> left;
    priority_queue<T, vector<T>, greater<T>> right;
```

```

slope_trick_convex() {left.push(numeric_limits<T>::
    min() / 2), right.push(numeric_limits<T>::max() /
    2);}
void push_left(T x) {left.push(x - ground_l);}
void push_right(T x) {right.push(x - ground_r);}
//add a line with slope 1 to the right starting from
//x
void add_right(T x) {
    T l = left.top() + ground_l;
    if (l <= x) push_right(x);
    else push_left(x), push_right(l), left.pop(), minn
        += l - x;
}
//add a line with slope -1 to the left starting from
//x
void add_left(T x) {
    T r = right.top() + ground_r;
    if (r >= x) push_left(x);
    else push_right(x), push_left(r), right.pop(), minn
        += x - r;
}
//val[i]=min(val[j]) for all i-l<=j<=i+r
void expand(T l, T r) {ground_l -= l, ground_r += r;}
void shift_up(T x) {minn += x;}
T get_val(T x) {
    T l = left.top() + ground_l, r = right.top() +
        ground_r;
    if (x >= l && x <= r) return minn;
    if (x < l) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(left.top());
            left.pop();
            if (left.top() + ground_l <= x) {
                res = cur_val + slope * (l - x);
                break;
            }
            cur_val += slope * (l - (left.top() + ground_l));
            l = left.top() + ground_l;
            slope += 1;
        }
        for (auto i : trash) left.push(i);
        return res;
    }
    if (x > r) {
        vector<T> trash;
        T cur_val = minn, slope = 1, res;
        while (1) {
            trash.push_back(right.top());
            right.pop();
            if (right.top() + ground_r >= x) {
                res = cur_val + slope * (x - r);
                break;
            }
            cur_val += slope * ((right.top() + ground_r) -
                r);
            r = right.top() + ground_r;
            slope += 1;
        }
        for (auto i : trash) right.push(i);
        return res;
    }
    assert(0);
}
};

```

9.6 ALL LCS

```

void all_lcs(string s, string t) { // 0-base
    vector<int> h(t.size());
    iota(all(h), 0);
    for (int a = 0; a < s.size(); ++a) {
        int v = -1;
        for (int c = 0; c < t.size(); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}

```

```

}

```

9.7 Hilbert Curve

```

11 hilbert(int n, int x, int y) {
    11 res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 111 * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
} // n = 2^k

```

9.8 Random

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t a) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(a + FIXED_RANDOM);
    }
};
unordered_map<int, int, custom_hash> m1;
random_device rd; mt19937 rng(rd());

```

9.9 Matroid Intersection

Start from $S = \emptyset$. In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists $x \in Y_1 \cap Y_2$, insert x into S . Otherwise for each $x \in S, y \notin S$, create edges

- $x \rightarrow y$ if $S - \{x\} \cup \{y\} \in I_1$.
- $y \rightarrow x$ if $S - \{x\} \cup \{y\} \in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2 , and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x \in S$ and $-w(x)$ if $x \notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.10 Python Misc

```

from [decimal, fractions, math, random] import *
setcontext(Context(prec=10, Emax=MAX_EMAX, rounding=
    ROUND_FLOOR))
Decimal('1.1') / Decimal('0.2')
Fraction(3, 7)
Fraction(Decimal('1.14'))
Fraction('1.2').limit_denominator(4).numerator
Fraction(cos(pi / 3)).limit_denominator()
print(*[randint(1, C) for i in range(0, N)], sep=' ')

```