Corporate Debt Standardization and The Rise of Electronic Bond Trading
[PRELIMINARY DRAFT]

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Abstract

I study the impact of standardization on secondary corporate bond markets as the industry adopts electronic trading systems. I show that covenants can reduce debt rollover costs by mitigating agency problems. However, when trading in the more liquid electronic markets is restricted to standardized securities, firms must weigh the benefits of offering credit protection against e-trading's lower transaction costs. I investigate firms' choices of leverage and debt type when creditors are not fully informed about their risk exposures nor their hedging policies. In such cases, riskier firms can have an incentive to misrepresent their types to benefit shareholders, which raises debt rollover costs for safer firms. Safer firms react by adjusting their leverage, either to signal their credit-worthiness and force separation, or to reflect the less favorable funding conditions in a pooling equilibrium. Alternatively, safe companies can signal their type by issuing bonds with debt protective covenants, leading to a separating equilibrium with a hybrid market structure, where safe bonds trade over-the-counter. I show that this is the case when the liquidity differential between over-the-counter and electronic markets is sufficiently low and risky types optimally choose not to hedge their exposure to their idiosyncratic risk in equilibrium.

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I. Introduction

Despite a slow start, the $8 trillion US corporate bond market has seen a steady increase in electronic trading (*e-trading*) in recent years. This move has been hailed as a potential solution to the growing concerns about liquidity deterioration voiced by several institutional investors since 2013. In order to be successful, *e-trading* will require the standardization of newly issued bonds to aggregate liquidity in a few securities. Such move will limit the use of covenant clauses designed to prevent firms from exploiting their private information in pursuit of financial strategies that are detrimental to debt holders. This paper studies the trade-off between external market liquidity and the informational costs associated with the structural change in corporate bond markets.

I modify He and Xiong (2012) model to study the choice of debt standardization by firms as the industry moves towards electronic trading. In the model, illiquidity is proxied by portfolio liquidation costs. Covenant-free bonds can be transacted in the more liquid electronic platforms, whereas non-standardized debt can only be traded in over-the-counter (OTC) secondary markets. I show that covenants are important for mitigating agency problems and that standardization may decrease investors' ability to more easily distinguish the credit quality of issuers.

I endogenize firms' decision to issue bonds with debt protective covenants by introducing idyosyncratic, unhedgeable shocks that affect a subset of the firms. Absent any asymmetry of information, all firms issue standardized debt because the higher liquidity of electronic markets positively affects the valuation of their debt. However, when firms' exposure to the unhedgeable shocks are not observable by bond investors, a conflict between bond holders and equity investors can arise. I show that some riskier firms may choose to deceive investors by issuing debt that is ex-ante indistinguishable from higher-quality debt. By doing so, these firms increase the rate of return to equity investors at the expanse of debt holders. In response, safer firms adjust their leverage to either discourage riskier firms' attempt to misrepresent their creditworthiness, or to accommodate a pooling equilibrium. Alternatively, safe firms might opt to issue bonds with a debt protective covenant to credibly signal their credit quality. The informational costs posed by debt standardization can thus offset the liquidity gains offered by the new trading technologies, leading to a smaller base of potential clients and reduced revenues for the new electronic markets.

The implications of the findings are two-fold. First, certain types of high-quality debt might still be traded in OTC markets, leading to differences in the composition of debt across secondary markets. Second, bond investors in electronic exchanges might have to trade liquidity for creditor protection.

A. *Electronic Trading Venues and Bond Standardization*

Since the 1940s, corporate bonds have been traded primarily over-the-counter (OTC). OTC markets are opaque and decentralized markets where dealers act as counterparties for both buyers and sellers. By setting the quotes at which they are willing to fulfill an order, they assume the trade execution risk and profit from the bid-ask spread. Liquidity in secondary corporate bond markets thus heavily depends on dealers' ability to warehouse large inventories and their willingness to absorb customer order imbalances into their own balance sheets.

The low interest rate environment that has prevailed in the post-Crisis has led to a surge in corporate debt issuance, as companies have rushed to take advantage of favorable borrowing conditions. The principal amount outstanding in corporate bond markets grew from \$5.5 bi in 2008 to \$9 bi in a decade.[1] Dealers have profited along by underwriting these new issues. Despite the burst in corporate debt, however, dealers' inventories shrunk from \$250 bi right before the Crisis to under \$50 bi in 2015.[2] This deleveraging, along with the decline in block trading and anecdotal evidence of increased price impact for larger transactions in the secondary market, suggests a decrease in intermediaries' risk appetite that has compromised their ability to provide liquidity.

Market participants have blamed the deteriorating liquidity conditions on post-crisis financial regulation.[3] Designed to curb banks' risk-taking and make the industry safer, these regulatory changes seem to have constrained dealer's market making activity. The Basel III Accord and regulations under the Dodd-Frank Act of 2010 have imposed greater capital and liquidity requirements for banks, increasing their cost of capital and hampering their ability to maintain large corporate bonds inventories. In addition, the Volker Rule, which came into effect in April 2014 and prevents banks from engaging in "risky" proprietary trading, has led to the shut down of several proprietary trading desks in Wall Street. While standard measures of liquidity based on execution costs for completed trades, such as the bid-ask spread, appear healthy, search costs have risen. Consistent with Duffie (2012) assessment that the new bank rules would hurt dealers' market making capacity, Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) find that bank-affiliated dealers have become less willing to commit capital and to accommodate block trades in recent years.

As a means of improving liquidity, some banks and large institutional investors have pushed for the modernization of secondary markets' structure, with the adoption of new technologies aimed at cutting down costs and improving the efficiency of bond trading operations. The most disruptive and controversial change has been the shift towards electronic, equity-style trading on exchanges. Proponents have argued that electronic trading ameliorates trading conditions by reducing secondary markets' dependency on intermediary capital. *E-trading* systems facilitate the direct matching of buyers and sellers because they improve the relaying and processing of information and allow customers to directly access several markets at once. In addition to partially replacing and improving upon basic broker services, these systems can help increase market transparency and lower entrance costs. More competition and improved market access then facilitate price-discovery, help restrict margins and reduce search costs by making it easier to find suitable counterparties to a trade.[4]

[1] Source: Finra, available at https://www.sifma.org/resources/research/fixed-income-chart/

[2] See BlackRock's Viewpoint publication from February 2016:
https://www.blackrock.com/corporate/literature/whitepaper/viewpoint-liquidity-bond-markets-broader-perspective-february-2016.pdf

[3] See, for instance, BlackRock's research paper from 2014:
https://www.blackrock.com/corporate/literature/whitepaper/viewpoint-corporate-bond-market-structure-september-2014.pdf
and the Financial Times, Bloomberg and WSJ coverage here https://www.ft.com/content/0d1c9b38-195a-11e3-83b9-00144feab7de?siteedition=intl, here https://www.bloomberg.com/opinion/articles/2015-06-03/people-are-worried-about-bond-market-liquidity, and here https://www.wsj.com/articles/the-new-bond-market-bigger-riskier-and-more-fragile-than-ever-1442808001.

[4] Corporate bonds were actively traded in the NYSE in the beginning of the 20th century. The exchange offered a high degree of pre- and post-trade transparency, since the book of available orders and recent trades were visible to all brokers. Trade abruptly migrated to OTC markets in the late 1940s. Biais and Green (2019) show that this migration happened as composition of secondary debt markets changed and institutional investors came to account for the majority of the trading activity. They conjecture that these large investors might have found the opacity of OTC dealer markets preferable to the transparency of limit order markets. Consistent with this, anonymity has been front and center in the debate about the modernization of corporate bond markets microstructure. Investors have favored all-to-all electronic venues, where counterparties can trade anonymously with one another (https://bloomberg.com/news/articles/2019-04-01/wall-street-is-getting-cut-out-of-bond-

E-trading has been gaining ground for the past five years.[5] According to a recent report by Greenwich Associates[6], the vast majority of the trades in secondary bond markets (over 90% of trades of \$100k or less) are now done in electronic platforms. Nonetheless, trades of larger ticket size (\$1MM or more), which make up over 80% of the notional volume traded daily, are still done over-the-counter. The main obstacle to electronic trading of corporate bonds is arguably the fragmentation of trading activity across a vast universe of securities in this asset class. Unlike in equity markets, where large companies issue at most a few dozen stocks, in corporate bond markets large issuers can have from a few hundred to thousands of bonds outstanding, the vast majority of which trades only infrequently. While multiple and varied issuances allow companies to minimize debt refinancing risks by diversifying their capital structure and debt maturity schedule, they also render each security individually more illiquid. Put differently, with such a large number of bonds available to trade, it is practically impossible for an investor to find a natural counterparty to a trade at any particular time.

To address market fragmentation, banks and large investment firms such as BlackRock have been pushing for the standardization of corporate bonds. Standardization can improve liquidity by facilitating pricing and by broadening the pool of potential investors. In addition, it allows for the development and integration of trading and settlement systems. For example, new trade protocols are being developed to enable multiple counterparties to simultaneously fulfill pieces of a single large order, increasing market depth by reducing the execution time of block trades.[7] Regulatory agencies have observed that, by promoting more centralized trading of bonds, standardization can help reduce systemic risks as trade execution risks migrate from dealers to end-investors in fixed-income markets.[8] Finally, standardization of corporate bonds makes it easier for the industry to adopt standardized index products, such as ETFs, and hedging tools, such as interest rate swaps and credit default swaps, which in turn increase the liquidity of the underlying bonds.

B. The Informational Role of Bond Covenants

Standardization can be fairly straightforward in some more homogenous asset classes. However, corporate bonds own legal and financial idiosyncrasies have always been an obstacle. Traditionally, most clients in the dealer-customer segment are buy-and-hold institutional investors, who often time look for securities specially tailored to their need for exposure to certain risks. For this reason, bonds often include contractual clauses designed to shield investors from losses. These clauses can work by precluding firms from acting in a way that is detrimental to creditors, or ensuring repayment in case of certain contingencies.

> A bond covenant is a provision, such as a limitation on the payment of dividends, which restricts the firm from engaging in specified actions after the bonds are sold. (Smith and Warner (1979))

Examples of such covenants include *make-whole* redemption compensations, which guarantee a lump sum payment to bond investors in case the debt is called off before maturity to compensate

market-it-long-dominated), and limiting post-trade reporting of large block trades to minimize the price impact of such orders (https://www.blackrock.com/corporate/literature/whitepaper/viewpoint-addressing-market-liquidity-july-2015.pdf).

[5] https://www.bloomberg.com/news/articles/2019-04-01/wall-street-is-getting-cut-out-of-bond-market-it-long-dominated

[6] https://www.greenwich.com/blog/challenge-trading-corporate-bonds-electronically

[7] See footnote 6.

[8] https://www.bloomberg.com/news/articles/2014-09-16/sec-s-gallagher-urges-standardized-bond-offerings-to-reduce-risk

them for the foregone coupon payments; and restrictive covenants preventing merger activities or the issuance of new debt. Other restrictive covenants preclude companies from paying dividends to shareholders after missing an interest payment to bond investors, or from selling the firms' assets. Such clauses are designed to protect bondholders from the payout of assets pledged as collateral.

A push to increase liquidity by homogenizing corporate bonds traded in electronic platforms might do away with creditor protection clauses, leaving investors exposed to certain firm-specific risks that are difficult to hedge. On the other hand, the benefits of electronization might be hindered if market participants deem covenants too important to be standardized or outright eliminated. In this case, a hybrid market structure might prevail, where more complex, non-standardized debt trades over-the-counter, whereas covenant-lite bonds are actively transacted in electronic venues.

The next section presents the core theoretical framework of the analysis, namely the types of investors, the micro-structure of secondary markets, the debt and equity valuation formulas, the choice of capital structure, and firms' endogenous bankruptcy decision. Section **??** introduces an asymmetry of information between creditors and shareholders and discusses the ensuing conflict of interest that can arise between these two classes of investors. The following section explains debt standardization and distinguishes between over-the-counter and electronic markets. Section **??** derives the equilibria when the only secondary market is the electronic platform, and discusses their properties. Next, the analysis is extended to accommodate both over-the-counter and electronic markets. Section III concludes.

II. The Model

To investigate the consequences of bond standardization over firms' choice of capital structure and the composition of debt across secondary markets, I propose a structural model of credit risk with asymmetric information, where debt protective covenants arise endogenously. The model features two classes of investors, bond investors (or creditors) and equity holders (shareholders), and two competing secondary markets, over-the-counter (OTC) markets and an electronic platforms (EPs.) Secondary markets differ in (i) their (external) liquidity and (ii) the types of bonds they accept. Electronic exchanges offer lower transaction costs, but intermediate only trades of standardized, covenant-free bonds, whereas OTC markets accept any type of bond. In the model, covenants constitute a costly way to produce private information and mitigate bondholder-shareholder conflicts in electronic platforms.

Equity investors observe investment opportunities (or projects), which can be either safe or risky, depending on their exposure to an idiosyncratic, unhedgeable risk. To invest in projects, these investors set up firms, which are financed with a mix of finite-maturity bonds and shares. Debt allows firms to benefit from tax-shields, but introduce the risk of costly bankruptcy. Each firm carries out one and only one project and commits to a fixed capital structure, defined by the type and measure of outstanding bonds at any given time. Finally, creditors require firms to choose their debt instruments and leverage to maximize their initial valuation.

All else constant, debt issued with standardized, covenant-free bonds is more valuable because these bonds are traded in the more liquid electronic markets. The reduced transaction costs of secondary trades in EPs render newly-issued bonds more valuable, thereby lowering firms' funding costs. Absent any asymmetry of information, therefore, all firms issue standardized bonds, so no trade takes place in OTC. When bond investors are not fully informed about the firms, however, risky firms' shareholders may be able to increase their returns by misrepresenting their firms' type. Depending on (i) the ratio of safe to risky firms, and (ii) the risky type's exposure to the unhedgeable risk, mis- representation can benefit risky-type shareholders by allowing their firms to mimic the more levered capital structure of safe firms Type misrepresentation is thus akin to an asset substitution problem, wherein creditors' valuation of a firm's debt is incommensurate with the firm's riskiness.

While bond investors may not observe firms' underlying exposure to the unhedgeable risk, their knowledge of the firm-type distribution allows them to anticipate each type's strategies. When misrepresentation occurs, creditors revise downwards their valuation of all standardized bonds, thereby raising the debt rollover costs for safe firms. Safe firms in turn adjust their debt-equity ratio, either by increasing their measure of outstanding bonds to discourage the risky type's misrepresentation, or by reducing their leverage to accommodate the increased funding costs in a pooling equilibrium Alternatively, safe firms may issue bonds with a debt protective covenant to signal their creditworthiness.

I decompose the safe type's total return differential under full and asymmetric information into a liquidity and an informational components. The liquidity component is a function of the difference in market-specific transaction costs. The informational component measures the cost of adverse selection to the absolute return of safe projects in electronic markets, and is a function of the distribution of types. More specifically, informational costs decrease with the measure of safe firms, and rise with the risky type's exposure to the unhedgeable shock.

When the unhedgeable risk differential between the two types of firms is small or the ratio of safe-to-risky firms is sufficiently high, the informational cost of adverse selection in electronic platforms is minimized by having safe firms reduce their leverage so that types pool together. When the risky differential is large or the measure of safe firms is small, however, the effect of pooling over the safe firms' debt rollover costs is so high that these firms find it preferable to increase their leverage to discourage the risky type's misrepresentation. Alternatively, when informational costs exceed the liquidity differential between over-the-counter and electronic markets, safe firms forego the liquidity gains and issue instead non-standardized bonds to signal their creditworthiness. Covenants then arise endogenously as a means of maximizing the absolute returns of safe projects by eliminating the informational asymmetry between the different types of investors. In this case, a dual-market separating equilibrium holds where only risky firms issue standardized bonds.

I now present a 2-period, structural model of credit risk that fully captures the trade-off outlined above. Absent in this formulation is the role of debt-rollover costs, which arise in a multi-period setting where firms issue finite-maturity debt. For the sake of brevity, the infinite-horizon, continuous-time version of the model is presented in a separate document.[9]

correct the link to the infinite-horizon model

A. *The Environment*

The economy lasts for two periods, $t = 0, 1$. There are two types of risk-neutral agents, *bond investors* (or *creditors*) and *equity holders* (or *shareholders*.) At the start of period 0, an equity holder may observe one, and only one, investment opportunity, which can be either *safe* with probability μ_s, or *risky* with probability $1 - \mu_s$. If she chooses to invest, the equity holder sets up a firm, to be financed with a mix of debt and equity and shares. Alternatively, she deposits her wealth at a money market account earning the risky-free rate, r_f.

All firms require the same initial capital allocation of V_0 to fund the purchase of their assets. The value of a firm's underlying assets at time 1 is log-normally distributed and depends on the type of its investment. After $t = 0$ but before period 1, risky firms experience an idiosyncratic,

[9]Please refer to the paper on my website: https://abcarvalho.github.io/uploads/abcarvalho_thesis.pdf

mean-reducing shock with probability q, so a firm's value of assets at time 1 is given by

$$V_{1,s} = V_0 e^x, \quad V_{1,r} = \begin{cases} V_0 e^x, & \text{w/ prob } 1-q \\ V_0 e^y, & \text{w/ prob } q \end{cases} \tag{1}$$

where

$$x \sim \mathcal{N}\left(r_f - \frac{1}{2}\sigma^2, \sigma\right), \quad y \sim \mathcal{N}\left(r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right), \quad s_f > 0 \tag{2}$$

The subscripts s and r correspond to *safe* and *risky*, respectively, and s_f is a scaling constant.

Firms are financed with a mix of equity and debt, issued at time 0. Debt consists of a measure $\mu_b > 0$, chosen by the firm, of coupon-less bonds with principal $p < V_0$, maturing at time 1. As with other structural models of credit risk, the issuance of bonds reduces the taxable income of the firm in proportion to its total debt outstanding, but introduces the risk of a costly bankruptcy process. Letting π denote the marginal tax benefit of debt, the tax shield at time 1 is simply $\pi\mu_b p$. A firm is declared bankrupt if, at time 1, the sum of the value of its underlying assets and tax benefits is insufficient to repay its debt:

$$V_1 + \pi\mu_b p < \mu_b p \qquad \text{(bankruptcy condition)} \tag{3}$$

In that event, its assets are liquidated at a fractional cost, and the recovery value, αV_1 for $\alpha \in (0,1)$, is then split evenly among bondholders. Shareholders are paid only if the firm is solvent at time 1, in which case they receive the value of the underlying assets and tax benefits in excess of the aggregate principal.

At the discretion of the equity investors, bonds can be issued with a covenant, or contractual clause, that distorts the time-1 payoffs to both bond investors and shareholders. More specifically, in the event of a mean-reducing shock, creditors are granted a fraction $\theta \in (0,1)$ of the equity holders' payoff at time 1. As will be shown, such covenant serves as a signaling mechanism when creditors do not observe the firm's exposure to the mean-reducing shock. I assume that a firm can issue only one type of bond, so that all of its bonds either feature the covenant or are covenant-free. Finally, since a bond's principal, p, and maturity are exogenously given, firms' only variables of choice are the measure of bonds, μ_b, and whether to include the covenant. I refer to covenant-free bonds as standardized, and bonds featuring the covenant as non-standardized. More precisely, the standardized bond is a tuple of maturity, coupon and principal, like so $\boldsymbol{b}^{SD} \equiv (m,c,p) = (1,0,p)$, while the non-standardized bond consists in $\boldsymbol{b}^{SD}$ augmented by the covenant parameter θ, $\boldsymbol{b}^{COV} \equiv (m,c,p,\theta) = (1,0,p,\theta)$.

Bonds are traded in illiquid secondary markets. Illiquidity here is understood as transactions costs, be they search costs or intermediation fees, that set a wedge between the expected payoff of a bond and the price an investor receives upon the sale of the asset. For simplicity, I do not model creditors' trading process. Following He and Xiong (2012), I assume instead bond holding periods are stochastic. When portfolio liquidation costs are proportional to bonds' expected payoff at time 1 but independent from the stochastic holding periods, illiquidity translates into a premium term in the creditors' rate of discount, r^b_{disc}, so that $r^b_{disc} > r_f$, where r_f is the risk-free rate.[10]

[10] To see this, let Θ be the bond's expected payoff at time 1. With probability $1-\xi$, the bond is held until maturity. With probability ξ, the bond investor is forced to liquidate his position at a fractional cost in the secondary market to receive $e^{-r_f}(1-\kappa)\Theta$, $\kappa \in (0,1)$. The bond's time-0 expected payoff is then

$$(1-\xi) \times e^{-r_f}\Theta + \xi \times e^{-r_f}(1-\kappa)\Theta = e^{-r_f}(1-\xi\kappa)\Theta$$

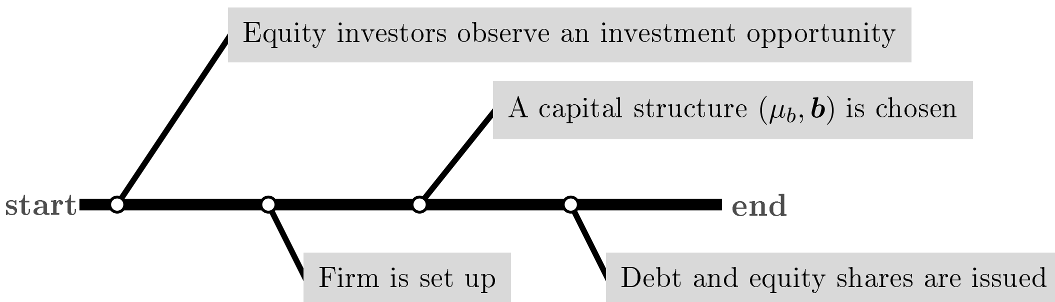
Consequently, the bond investor's effective discount factor is $e^{-r_f}(1-\xi\kappa)$. It follows that $r^b_{disc} \equiv r_f - \ln(1-\xi\kappa) > r_f$.

See the Online Appendix for a proof of this claim in a more general setting, available at https://abcarvalho.github.io/files/abcarvalho_online_app.pdf.

Secondary bond markets consist of two trading venues, an Electronic Platform (EP) and an Over-the-Counter (OTC) market. The EP is more liquid, but trades in this market are restricted to standardized bonds. The transaction costs differential between these venues leads to two distinct rates of discount, $r^{b,EP}_{disc} < r^{b,OTC}_{disc}$. Since these rates directly affect the price bond holders receive upon liquidation of their portfolio, standardized bonds are traded exclusively in EP, and command a premium over bonds whose trades are restricted to the OTC market. This premium in turn feeds into the price investors are willing to pay in primary markets, thereby affecting the firms funding costs.

The timing of actions in period-0 is summarized in figure 1 below. The economy starts with equity investors observing investment opportunities. If they so wish, shareholders set up firms and chose the type and quantity of bonds to issue. Debt and equity shares are then sold to finance the purchase of the underlying assets, V_0. After debt is purchased by credit investors, trades move to secondary markets until maturity at time 1.

Figure 1. Investors' Intra-Period 0 Timing



The figure depicts the sequence of actions within period 0. After debt and equity shares are issued, bond trading moves to secondary markets until maturity in period 1.

B. *Equilibrium when Investors are Fully Informed*

When investors are fully informed about a firm, prices depend solely on (i) that firm's measure of outstanding bonds, μ_b, (ii) the stochastic distribution of the value of its underlying assets, and (iii) the liquidity conditions in secondary markets. As mentioned above, standardized debt gets traded exclusively in the EP, due to its lower transaction costs. The time-0 price of a standardized bond issued by a safe firm, $d\left(\mu_b|s\right)$, is then given by the formula

$$d\left(\mu_b; r^b_{disc}, \mu^v_j, \sigma_j\right) = e^{-r^b_{disc}} \cdot \left\{ p \cdot N\left(-f\left(\mu_b;\mu^v_j,\sigma_j\right)\right) + \frac{\alpha V_0}{\mu_b} e^{\mu^v_j + \frac{1}{2}\sigma^2_j} \cdot N\left(f\left(\mu_b;\mu^v_j,\sigma_j\right) - \sigma_j\right)\right\} \quad (4)$$

where

$$f\left(\mu_b;\mu^v_j,\sigma_j\right) \equiv \frac{1}{\sigma_j}\log\left(\frac{(1-\pi)\,\mu_b P}{V_0 e^{\mu^v_j}}\right) \quad (5)$$

when $j = s$, with $\mu^v_s = r_f - \sigma^2/2$ and $\sigma_s = \sigma$, and the investors' rate of discount is $r^{b,EP}_{disc}$. The value of a risky firm's standardized bond prior to the arrival of a mean reducing shock is a weighted average of the pricing function in equation 4, as follows:

$$d\left(\mu_b|r\right) = (1-q) \times d\left(\mu_b; r^{b,EP}_{disc}, r_f - \frac{1}{2}\sigma^2, \sigma\right) + q \times d\left(\mu_b; r^{b,EP}_{disc}, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)$$

Since, by assumption, firms issue one type of bond only, the value of debt of a type-j firm, $D(\mu_b|j)$, is simply the price of a single bond multiplied by the measure of bonds issued by this firm: $\mu_b \times d(\mu_b|j)$, for $j \in \{s, r\}$. Finally, in this finite-horizon model, equity is but a claim to the value of the underlying assets and tax benefits in excess of the firm's debt service costs at time 1. Therefore, equity can be priced as a call option on V with strike price $(1-\pi)\mu_b P$. The time-0 value of a safe firm's equity, $E(\mu_b|s)$, is given by the formula

$$E\left(\mu_b; r^e_{disc}, \mu^v_j, \sigma_j\right) = e^{-r^e_{disc}}\left\{e^{\mu^v_j + \frac{1}{2}\sigma_j} V_0 \cdot N\left(-f\left(\mu_b; \mu^v_j, \sigma_j\right) + \sigma_j\right) - (1-\pi)\mu_b P \cdot N\left(-f\left(\mu_b; \mu^v_j, \sigma_j\right)\right)\right\} \tag{6}$$

when $j = s$, with $\mu^v_s = r_f - \sigma^2/2$ and $\sigma_s = \sigma$, and where $r^e_{disc} = r_f$ is the equity investors' rate of discount. As in the bond pricing analysis above, the value of a risky firm's equity prior to the arrival of a mean reducing shock is the the sum of the equity function in equation 6 evaluated at the two possible states (shock or no shock), weighted by the probability of each state. The derivation of these formulas can be found in Appendix C.

C. *Optimal Capital Structure and Shareholders' Return*

Shareholders' expected net gain is the difference between the value of equity and the firm's share capital, or book-value of equity, that is, the amount of money equity holders had to invest to fund the purchase of the firm's assets. This payoff can be expressed as the difference between the market and book values of the firm:

$$E(\mu_b|j) - \underbrace{(V_0 - D(\mu_b|j))}_{\text{Share Capital}} = \overbrace{(E(\mu_b|j) + D(\mu_b|j))}^{\text{Market Value of the Firm}} - V_0 \tag{7}$$

where the book value of the firm is the initial value of its assets, V_0. For convenience, hereafter I refer to the market value of the firm simply as the firm value whenever there is no confusion.

From equation 7, it follows that the optimal capital structure is determined by the measure of bonds, $\mu^\star_b$, that maximizes the total value of the firm at time 0:

$$\mu^\star_b(j) = \underset{\mu_b \geqslant 0}{\arg\max}\left\{E(\mu_b|j) + D(\mu_b|j)\right\}, \quad j \in \{s, r\} \tag{8}$$

that is, $\mu^\star_b$ is the measure of bonds that maximizes both the equity holder's expected payoff and the total economic value of the investment undertaken by the firm.

ASSUMPTION 1: *[The Optimal Capital Structure] When investors are fully informed about the firm, the optimal capital structure is that which maximizes the total (market) value of the firm.*

Shareholders will invest on a firm if, and only if, their expected payoff is positive, that is,

$$E\left(\mu^\star_b(j)|j\right) + D\left(\mu^\star_b(j)|j\right) \geqslant V_0, \quad j \in \{s, r\}$$

Equivalently, shareholders invest if, and only if, the Market-to-Book ratio of Equity (MBR) is greater than one

$$MBR\left(\mu^\star_b(j)|j\right) \equiv \frac{E\left(\mu^\star_b(j)|j\right)}{V_0 - \mu^\star_b(j)\, d\left(\mu^\star_b(j)|j\right)} \geqslant 1$$

I compute the optimal capital structure values numerically. To do so, I set the shock scaling constant s_f to 1. The initial value of the assets, V_0, the risk-free rate, r_f, the tax benefit rate, π, and the bankruptcy recovery rate, α, are taken from He and Xiong (2012) and can be found in Table I in Appendix A.

Figure 2 in Appendix B.1 shows the firm value (LHS) and market-to-book ratio of equity (RHS) by measure of bonds issued, μ_b, for both a safe firm ($q_s = 0.0$) and a risky firm when probability of the mean-reducing shock is $q_r = 0.5$. As the graph illustrates, the firm value functions are strictly concave, regardless of the likelihood of the shock. However, the optimal measure of bonds is strictly decreasing in the exposure to the shock, from which it follows that safer firms are more levered in equilibrium. Figures 3 and 4 plot the optimal firm value and the equity market-to-book ratio in a full information (FI) equilibrium for varying levels of exposure to the mean-reducing shock, q, and different measures of safe-type firms, μ_s. Since the optimal capital structure is independent from the firm-type distribution, the iso-curves are vertical. Finally, as expected, FI equilibrium payoffs are strictly decreasing in the likelihood of the mean-reducing shock.

D. *The sub-optimality of bond covenants*

The issuance of non-standardized debt is never optimal in a full information setting. For one, because bond investors are risk-neutral, a firm's pledge to transfer capital from shareholders to bond investors in case of a shock to its asset distribution commands no premium. In addition, because creditors discount cash-flows at a higher rate than equity holders, this transfer decreases the value of equity by a higher magnitude than it increases the value of debt, thus reducing the overall value of the firm. Finally, since trade of non-standardized debt is restricted to OTC markets by assumption, the introduction of a covenant further erodes the value of the firm due to the liquidity differential between the two competing trading venues.

To see this, consider first the case of a safe firm. Since such firm is not exposed to the mean-reducing shock, the covenant has no impact on its bonds' expected payoffs. In this case, the lower valuation of non-standardized debt comes solely from the higher transaction costs in the OTC market. In the case of a risky firm, all three factors mentioned above are present, however. Recall that $\theta \in (0,1)$ represents the share of the equity holders' payoff being transferred to creditors in the event of a mean-reducing shock. In Appendix C, I show that, for a given measure of outstanding bonds μ_b, the loss to the total firm value from the issuance of non-standardized debt can be decomposed into a Equity Loss (EL) term and a Liquidity Differential Loss (LDL) component, as follows:

$$EL\left(\mu_b, r^e_{disc}, r^{b,EP}_{disc}|r\right) = -q\theta\left(r^{b,EP}_{disc} - r^e_{disc}\right) E\left(\mu_b; r^e_{disc}, \mu^v_r, \sigma_r\right) < 0$$

and

$$\begin{aligned} LDL\left(\mu_b, r^e_{disc}, r^{b,EP}_{disc}, r^{b,OTC}_{disc}|r\right) &= \mu_b\left\{d\left(\mu_b, r^{b,OTC}_{disc}|r\right) - d\left(\mu_b, r^{b,EP}_{disc}|r\right)\right\} \\ &\quad - q\theta\left(r^{b,OTC}_{disc} - r^{b,EP}_{disc}\right) E\left(\mu_b; r^e_{disc}, \mu^v_r, \sigma_r\right) < 0 \end{aligned}$$

where $r^e_{disc} = r_f$, $\mu^v_r = r_f - \sigma^2/2 - s_f\sigma$ is the post-shock mean, $\sigma_r = \sigma$, and $E\left(\cdot; r^e_{disc}, \mu^v_x, \sigma_x\right)$ is the state-contingent equity value function in equation 6. The Equity Loss term captures the expected cost from the shock-contingent asset transfer to the group with lower valuation. The second term represents the loss to the value of the firm arising from the restriction that non-standardized bonds be traded in the more illiquid OTC market.

E. *Firms' Choices under Asymmetric Information*

The discussion in the previous section showed that the optimal balance between the tax incentives of debt and the risk of a costly bankruptcy is achieved by choosing a measure of bonds μ_b that maximizes the sum of the debt and equity values (total firm value). However, when creditors do not observe firms' exposures to the mean-reducing shock, they may be unable to enforce the optimal capital structure (assumption 1). Instead, risky firm's equity holders may be able to exploit their privileged information and alter their firms' leverage, in an attempt to increase their returns.

I now consider the case where only the shareholders of a firm know its exposure to the mean-reducing shock. Neither bondholders nor equity investors in other firms are privy to that information. As before, however, I assume all investors (i) know the aggregate distribution of firm types, and observe both (ii) the debt issuance by each firm and (iii) the occurrence of the mean-reducing shocks. In the presence of such asymmetry of information, bondholders must infer a firm's creditworthiness from its choice of debt instrument and capital structure.

Because changes to a firm's capital structure affect not just the value of equity but also the amount of share capital required to fund its project at time 0, the rate of return on equity alone is not necessarily a sufficient statistic for individual shareholders' payoffs. Before considering deviations from the first-best capital structures, I briefly examine equity investors' wealth process and how they relate to the total return on equity.

E.1. Revisiting Shareholders' Payoffs

An equity holder who invests a fraction $s_i^e \in [0,1]$ of her wealth W_0 on a firm is entitled to a fraction $s_i^e \cdot W_0 \left[V_0 - \mu_b d\left(\mu_b\right)\right]^{-1}$ of the equity shares. The present value of her wealth is then:

$$\underbrace{\left(\frac{E\left(\mu_b\right)}{V_0 - \mu_b d\left(\mu_b\right)}\right)}_{\equiv MBR(\mu_b)} \times s_i^e \times W_0 + \left(1 - s_i^e\right) \times W_0 \tag{9}$$

To keep the model tractable, I assume all equity investors start with the same capital and can invest in at most one firm. Additionally, I let W_0 be small enough so that no shareholder can fund an optimally levered firm alone.

$$W_0 < \min_{j \in \{s,r\}} \left\{V_0 - \mu_b^\star d\left(\mu_b^\star | j\right)\right\}$$

where $\mu_b^\star$ is the measure of bonds that maximizes the total value of the firm, and s and r stand for safe and risky types, respectively. Therefore, (i) so long as the firm's MBR is greater than one, a shareholder will invest all her wealth in the firm, $s_i^e = 1$. Moreover, (ii) once an equity investor observes an investment opportunity, she must form a coalition of shareholders wherein each member invests the same amount. The measure $\nu\left(\mu_b\right)$ of this coalition is that which matches the shareholders' capital to the value of assets in excess of the funds raised via debt issuance, as such

$$\nu\left(\mu_b\right) W_0 = V_0 - \mu_b d\left(\mu_b | j\right) \tag{10}$$

When shareholders invest all their wealth in the firm, the expected return to any single equity investor can be maximized by raising the firm's MBR, instead of the total value of the firm. But changes to the MBR can only be implemented by altering a firm's capital structure, which in turn changes the required amount of share capital invested and thus the size of the shareholder coalition. From now on, therefore, I assume variations in the equity financing requirements are entirely met by adjustments in the pool of investors: the equity investor who observes the investment opportunity choses the coalition size necessary to fund the desired capital structure.

E.2. Type Misrepresentation

In the presence of asymmetric information, the first best equilibrium allocations will typically be unattainable because the creditors' funding condition in assumption 1 can no longer be enforced. To see this, let superscript FI stand for *full information*. Suppose bondholders refused to buy bonds from a firm for which $\mu_b \notin \{\mu_{b,s}^{FI}, \mu_{b,r}^{FI}\}$ as before, and assigned probability 1 to the event a firm is safe whenever $\mu_b = \mu_{b,s}^{FI}$, and 0 otherwise. A type-j firm would still deviate from its first-best capital structure if, by issuing a measure $\mu_{b,i}^{FI}$ of bonds and being taken for a type-i firm, it could increase the return to its shareholders'. I call this strategy *type misrepresentation*. By the discussion in the previous section, this would be the case if

$$MBR^{MP}\left(\mu_{b,i}^{FI}|j \to i\right) > MBR^{FI}\left(\mu_{b,j}^{FI}|j\right) \qquad j \neq i \tag{11}$$

where superscript MP denotes *misrepresentation*. The term on the LHS of the inequality above is the equity market-to-book ratio of a type-j firm that issues a measure $\mu_{b,i}^{FI}$ of bonds, leading creditors to believe it is of type-i, and is computed from the perspective of the fully informed equity investors:

$$MBR^{MP}\left(\mu_{b,i}^{FI}|j \to i\right) = \frac{E\left(\mu_{b,i}^{FI}|j\right)}{V_0 - \mu_{b,i}^{FI} d\left(\mu_{b,i}^{FI}|i\right)}$$

Notice that, while the term in the numerator correctly reflects the expected payoff to equity holders, the share capital in the denominator coincides with that of a type-i firm. From this, it follows that the shareholder coalition size of a misrepresenting firm equals that of the firms whose type it is mimicking, that is, $\nu^{MP}\left(\mu_{b,i}^{FI}|j\right) = \nu^{FI}\left(\mu_{b,i}^{FI}|i\right)$. So long as inequality 11 holds, each shareholder in the misrepresenting coalition benefits from a higher expected payoff, albeit the size of the coalition is typically lower.

In general, risky firms' shareholders gain from misrepresentation. As figure 2 in Appendix B.1 shows, equity investors would prefer a measure of bonds greater than that which yields the optimal firm value. Misrepresentation thus gives risky firms a chance to increase the returns to their shareholders by raising their leverage beyond the optimal level. In addition, it lowers risky firms' funding costs by artificially inflating their debt price. There are thus two factors involved in type misrepresentation, namely, the change in the measure of debt, which affects both the price of the bonds and the value of equity shares, and the debt valuation differential stemming from the change in bondholders' perception of the firms' creditworthiness.

Figure 5 shows the gains to the equity market-to-book ratio obtained when a risky-firm pursues a misrepresenting strategy for varying levels of exposure to the mean-reducing shock, q, and different measures of safe-type firms, μ_s. Because, by construction, creditors assign probability 1 to the firm being safe when they observe $\mu_{b,s}^{FI}$, these iso-curves are independent of the firm-type distribution μ_s. In this example, risky firms always benefit from misrepresentation. Interestingly, though, the gains from misrepresentation are non-monotonic in the exposure to the shock. For high enough shock probability values, the required change in the firm's debt issuance $(\mu_{b,r}^{MP} - \mu_{b,r}^{FI})$ is so large that the expected costs from the service of the debt start to undermine the gains from the overvaluation of the firm's bonds.

F. Equilibrium under Asymmetric Information

I now present the concept of equilibrium under asymmetric information and analyze its properties. I start by formalizing the definition of the economy, which consists in the collection of (i) firm

creditors' belief function

types and (ii) its distribution, (iii) the risk-free and secondary-market-specific discount rates, (iv) the set of bond contracts, and (v) the set of feasible values for the measure of bonds outstanding. I then focus on the simpler case where firms issue only standardized bonds, deriving the possible equilibria in this restricted economy before expanding the set of contracts to allow for the covenant.

Since firm differ only in their exposure to the mean-reducing shock, q_j, for $j \in \{s, r\}$, I let $Q \equiv \{0, q\}$ denote the set of firm types. The associated distribution is captured by the parameter μ_s alone . I denote by $\boldsymbol{r}$ the vector of discount rates, $\boldsymbol{r} \equiv \left(r_f, r_{disc}^{b,EP}, r_{disc}^{b,OTC}\right)$. The set of bond contracts is $\boldsymbol{B} \equiv \left\{\boldsymbol{b}^{EP}, \boldsymbol{b}^{OTC}\right\}$, where the standardized contract $\boldsymbol{b}^{EP}$ is a tuple of maturity, coupon and principal, like so $\boldsymbol{b}^{EP} \equiv (m, c, p) = (1, 0, p)$, while the non-standardized bond consists in $\boldsymbol{b}^{EP}$ augmented by the covenant parameter θ, $\boldsymbol{b}^{OTC} \equiv (m, c, p, \theta) = (1, 0, p, \theta)$. As I will show, θ can take any value in $[\underline{\theta}, 1]$, where $\underline{\theta}$ is determined endogenously as the smallest covenant value satisfying the misrepresenting type's incentive compatibility constraint (more below.) Consequently, any equilibrium where firms issue non-standardized bond supports an infinity of contracts. For simplicity and without loss of generality, I assume however that firms choose one and only one value θ in equilibrium. Finally, the set of feasible measures of bonds can be restricted to a compact interval $M_b \equiv [0, \overline{\mu}_b]$, where the upper boundary $\overline{\mu}_b$ satisfies $\max_j \left\{MBR_j\left(b^{EP}, \overline{\mu}_b\right)\right\} \leqslant 1$. In other words, $\overline{\mu}_b$ is so large that it would never be chosen by shareholders, as it would imply a negative rate of return on the equity investment.[11] Any choice of capital structure must thus lie in $M_b \cup \emptyset$, where the empty set represents the decision not to invest in a project. The economy is then fully characterized by the set $E \equiv [Q, \mu_s, \boldsymbol{r}, \boldsymbol{B}, M_b]$.

Before deriving the equilibrium for the whole economy E, I consider the setting where there is only the standardized bond contract, that is, $E^{SD} \equiv \left[Q, \mu_s, \boldsymbol{r}, \boldsymbol{b}^{EP}, M_b\right]$. This is a particular case of E, where bonds trade exclusively in the EP, since $r_{disc}^{b,EP} < r_{disc}^{b,OTC}$, so the over-the-counter market is superfluous. The derivation of equilibrium in this restricted economy is made easier because the only choice available to firms is how much debt to issue. Therefore, firms' responses to the adverse selection problem discussed above takes the form of distortions to their optimal (first-best) capital structures. Under asymmetric information, firms of the same type may choose different capital structures with strictly positive probability. For each firm then, a (mixed) strategy consists in a probability function defined over M_b. That is, a type-contingent mixed strategy is a function $p_j^b \colon M_b \mapsto [0, 1]$ satisfying $p_j^b(\mu_b) \geqslant 0$ for all $\mu_b \in M_b$ and $\int_{M_b} p_j^b(x)\, dx = 1$.

F.1. Creditors' Beliefs and Bond Valuation

Since firm types are unobservable, creditors form rational beliefs about the composition of firms for each possible capital structure choice. Let $\gamma_s(\mu_b)$ denote the probability creditors assign to a firm being of the safe type when its measure of bonds is μ_b. Given $\gamma_s(\mu_b)$, creditors offer price $d_c(\mu_b|\gamma)$ for the firm's newly-issued bonds.

I call it a separating bond measure any measure believed to be chosen by one type alone, that is, any measure μ_b such that $\gamma_s(\mu_b) \in \{0, 1\}$. Conversely, a pooling measure is any measure μ_b for which $\gamma_s(\mu_b) \in (0, 1)$. Rationality of creditors' beliefs requires that no type be able to increase the return to its shareholders by choosing a separating measure played by the other type.

DEFINITION 1: *[Types' Incentive Compatibility Condition] A belief and price functions pair, $(\gamma_s(\cdot), d_c(\cdot|\gamma))$, is robust against misrepresentation if, for every separating measure $\mu_{b,i} \in M_b$ issued by type-i firms, $i \in \{s, r\}$, misrepresentation cannot increase the payoff to the shareholders of a type-j firm, that is:*

$$MBR_j\left(\mu_b'|\gamma\right) \leqslant \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma) \quad \forall \mu_b' \in M_b \ s.t.\ \gamma_i\left(\mu_b'\right) = 1$$

[11] See figure 2 in Appendix B.1.

where $MBR_j(\cdot|\gamma)$ is type-j's market-to-book ratio of equity, computed as:

$$MBR_j(\mu_b|\gamma) \equiv \frac{E_j(\mu_b)}{V_0 - \mu_b d_c(\mu_b|\gamma)}$$

Given the belief and price functions pair, $(\gamma_s(\cdot), d_c(\cdot|\gamma))$, firms are deemed optimally levered if their choice of bond issuance maximizes their total expected value. A pooling measure μ_b^{pool} is thus optimal if it solves

$$\max_{\mu_b \in M_b} \left\{ \gamma_s\left(\mu_b^{pool}\right) FV_s(\mu_b|\gamma) + \left(1 - \gamma_s\left(\mu_b^{pool}\right)\right) FV_r(\mu_b|\gamma) \right\} \tag{CFC - Pooling}$$

where

$$FV_j(\mu_b|\gamma) = E_j(\mu_b) + \mu_b d_c(\mu_b|\gamma)$$

On the other hand, a separating measure $\mu_{b,i}^{sep}$ issued by a type-i firm is optimal if, and only if, it maximizes type-i's firm value subject to type-j's incentive compatibility condition:

$$\begin{aligned} &\max_{\mu_b \in M_b \cup \emptyset} FV_i\left(\mu_{b,i}^{sep}|\gamma\right) \\ \text{s.t.} & \\ &MBR_j\left(\mu_{b,i}^{sep}|\gamma\right) \leqslant \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma) \quad \text{(IC)} \end{aligned} \tag{CFC - Separating}$$

for $j \neq i$.

I assume creditors enforce capital structure optimality given beliefs γ by withdrawing funds from firms whose choice of bonds issuance are inconsistent with the maximization of their total value.

ASSUMPTION 2: *[Creditors' Funding Condition - CFC] Any choice of capital structure μ_b must maximize the firm value given creditors' beliefs γ and offer price function $d_c(\cdot|\gamma)$, subject to type's incentive compatibility (IC) conditions.*

F.2. Weak Equilibria in E^{SD}

As defined so far, the (i) shareholders' optimality, and (ii) creditors' belief rationality, (iii) break-even and (iv) funding conditions are not sufficient to uniquely determine the equilibrium in E^{SD}. It is possible, however, to characterize the set of equilibria candidates and refine assumption 2 to ensure a single equilibrium holds. In this section, I formally define a weak equilibrium under asymmetric information and derive five results that greatly restrict its set of candidates.

DEFINITION 2: *A weak equilibrium in E^{SD} is a tuple $e \equiv \left(\left\{p_s^b(\cdot), p_r^b(\cdot)\right\}, \gamma_s(\cdot), d_c(\cdot|\gamma)\right)$ consisting in (i) a pair of type-contingent mixed-strategy functions $p_j^b(\cdot)$, $j \in \{s, r\}$, (ii) a creditors' belief function $\gamma(\cdot)$, and (iii) a creditors' pricing function $d_c(\cdot|\gamma)$ that satisfy:*

1. *[Funding] Any bond measure $\mu_b \in M_b$ such that $p_i^b(\mu_b) > 0$ for some $i \in \{s, r\}$ satisfies the creditors' funding condition;*

2. *[Shareholders' optimality] Firms maximize the payoff to their shareholders. For each μ_b such that $p_j^b(\mu_b) > 0$,*
$$MBR_j(\mu_b|\gamma) = \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma), \quad j \in \{s, r\}$$

3. *[Creditors' zero-profit condition] For any measure $\mu_b \in M_b$ such that $p_i^b(\mu_b) > 0$ for some $i \in \{s, r\}$,*

(a) *The creditors' bond pricing function $d_c(\cdot|\gamma)$ coincides with the expected bond value given $\gamma_s(\cdot)$:*

$$d_c(\mu_b|\gamma) = \gamma_s(\mu_b)\, d_s(\mu_b) + (1 - \gamma_s(\mu_b))\, d_r(\mu_b)$$

where $d_j(\cdot)$, $j \in \{s, r\}$, are the type-contingent, full-information bond pricing functions discussed in the previous section.

(b) *Creditors' beliefs are consistent with investors' strategies:*

$$\gamma_s(\mu_b) = \frac{\mu_s p_s^b(\mu_b)}{\mu_s p_s^b(\mu_b) + (1 - \mu_s)\, p_r^b(\mu_b)} \quad \forall \mu_b \text{ s.t. } p_j^b(\mu_b) > 0 \text{ for some } j \in \{s, r\}$$

The funding restriction requires that (i) the chosen measures of bonds maximize the expected firm values given creditors' beliefs, while (ii) precluding misrepresentation. The shareholders' optimality condition ensures that (i) shareholders' of a type-i firm are indifferent between any measure μ_b such that $p_i^b(\mu_b) > 0$, and (ii) no deviation $\tilde{p}_i^b(\cdot)$ yields a higher MBR, for $i \in \{s, r\}$. Finally, the third condition ensures creditors break even.

The definition above is fairly general. At first glance, it does not rule out equilibria in which, for instance, a fraction of firms of a given type opts to pool together with the other type, while the rest chooses to issue one or more separating measures of bonds. A closer inspection, however, considerably narrows down the set of equilibrium candidates. In what follows, I derive five conditions that must hold in equilibrium. Lemmas 1 and 4, and by extension lemma 5, rely on a numerical analysis wherein I calibrate the model to firms with a speculative-grade BB rating. The parameter values were partly adapted from He and Xiong (2012), and are found in Table I in Appendix A.

LEMMA 1: *No type chooses more than one separating measure with strictly positive probability in equilibrium.*

Proof. Suppose type-i firms play μ_b', μ_b'' with strictly positive probability in equilibrium, for some $i \in \{s, r\}$. Consistency of creditors' beliefs with investors' strategies requires that $\gamma_i(\mu_b') = \gamma_i(\mu_b'') = 1$, so payoffs are given by the full information (FI) formulas. By the shareholders' optimality condition, we must have $MBR_i(\mu_b'|\gamma) = MBR_i(\mu_b''|\gamma)$. However, the strict concavity of the FI firm value function (figure 2) implies that at least one of these measures violates the creditors' funding condition. Contradiction! □

LEMMA 2: *The only separating measure a risky firm can choose in equilibrium is the risky-type's first-best measure $\mu_{b,r}^{FI}$.*

Proof. Recall that no safe-type shareholder can gain from misrepresentation, that is, the risk-type's optimal measure of bonds under full-information, $\mu_{b,r}^{FI}$, already satisfies the safe-type's incentive compatibility condition. Therefore, any separating measure $\mu_{b,r}'$ such that $\mu_{b,r}' \neq \mu_{b,r}^{FI}$ violates the creditors funding condition. □

LEMMA 3: *The safe-type's separating measure in equilibrium does not depend on the distribution of types, μ_s.*

Proof. By Lemma 2 above, the incentive-compatibility constraint in CFC - Separating for the safe-type becomes:

$$MBR_r\left(\mu_{b,s}^{sep}|\gamma_s = 1\right) \leqslant MBR_r\left(\mu_{b,r}^{FI}|\gamma_s = 0\right)$$

which depends solely on the type's characteristics, but not on the ratio of safe-to-risky firms.

□

LEMMA 4: *No market equilibrium in E^{SD} can support more than one pooling measure.*

To see this, suppose, by way of contradiction, that both types chose two different measures, $\mu_b', \mu_b'' \in M_b$, with strictly positive probability, with $\mu_b'' > \mu_b'$. By the creditors funding condition, both μ_b' and μ_b'' maximize the expected firm values given creditors' beliefs γ. On the other hand, the shareholders' optimality condition requires that equity holders be indifferent between these two measures:

$$MBR_j\left(\mu_b'|\gamma\right) = MBR_j\left(\mu_b''|\gamma\right)$$

for $j \in \{s, r\}$. We have thus an overdetermined system with 4 conditions (the maximization of the expected firm value conditions and two shareholders' optimality equations) and only 2 unknowns, namely $\gamma_s\left(\mu_b'\right)$ and $\gamma_s\left(\mu_b''\right)$.

Figure 6 plots the optimal measure of bonds (upper graph) and the associated type-contingent MBRs (bottom graph) as functions of the creditors' beliefs γ_s. For each $\gamma_s' \in (0,1)$, $\mu_b^{pool}\left(\gamma_s\right)$ is the solution to

$$\max_{\mu_b > 0}\left\{\gamma_s' FV_s\left(\mu_b|\gamma_s'\right) + \left(1-\gamma_s'\right) FV_r\left(\mu_b|\gamma_s'\right)\right\}$$

where

$$\begin{aligned} FV_j\left(\mu_b|\gamma_s'\right) &= E_j\left(\mu_b\right) + \mu_b d\left(\mu_b|\gamma_s'\right) \\ d\left(\mu_b|\gamma_s'\right) &= \gamma_s' d_s\left(\mu_b\right) + \left(1-\gamma_s'\right) d_r\left(\mu_b\right) \end{aligned}$$

and the type-contingent MBRs are the equity market-to-book ratios evaluated at $\mu_b^{pool}\left(\gamma_s'\right)$ when $\gamma_s = \gamma_s'$.

The optimal bond measures and MBRs are strictly increasing in γ_s. The lower the expected risk of the investment, the higher the firm's optimal leverage; and, the higher the leverage, the greater the rate of return to shareholders. Therefore, if both μ_b' and μ_b'' satisfy the firm-value maximization restriction, then $MBR_j\left(\mu_b''\right) > MBR_j\left(\mu_b'\right)$ for $j \in \{s, r\}$, so that choosing μ_b' with strictly positive probability in equilibrium violates the shareholders' optimality condition for both type.

By the same token, there cannot be an equilibrium where $p_s^b\left(\mu_b^{pool}\right) > 0$ and $p_r^b\left(\mu_b^{pool}\right) \in (0,1)$, for some $\mu_b \in M_b$.

LEMMA 5: *There cannot be a market equilibrium in E^{SD} where risky firms choose a pooling measure with probability $p_r^b \in (0,1)$.*

Proof. If risky-types play a pooling measure with probability less than 1, by lemma 4 they must choose a separating measure with strictly positive probability. By lemma 2, this measure coincides with the risky-type's first-best measure, μ_b^{FI}. For risky firms to choose both μ_b^{FI} and μ_b^{pool} in equilibrium requires

$$MBR_r\left(\mu_b^{FI}\right) = MBR_r\left(\mu_b^{pool}|\gamma\right)$$

However, because the optimal measure μ_b is strictly increasing in the probability of safe firms γ_s, we must have $\mu_b^{pool} > \mu_b^{FI}$, which also implies $MBR_r\left(\mu_b^{pool}|\gamma\right) > MBR_r\left(\mu_b^{FI}\right)$ (see figure 6.) Contradiction! □

The lemmas above restrict the types of equilibrium candidates e to only three: (i) a weak equilibrium in which all firms pick the same pooling measure μ_b^{pool} (pooling); (ii) another where both types each choose one, and only one, separating measure $\mu_{b,j}^{sep}$, $j \in \{s, r\}$ (separating); and (iii) a weak equilibrium in which all safe firms choose to poll together with risky ones, while a fraction of the risky firms opts for issuing the first-best measure $\mu_{b,r}^{FI}$ (mixed.)

F.3. Separating, Pooling and Mixed Weak Equilibria in E^{SD}

I call it a pure pooling weak equilibrium any weak equilibrium tuple $e \equiv \left(p_s^b(\cdot), p_r^b(\cdot), \gamma_s(\cdot), d_c(\cdot|\gamma)\right)$ where both types play a single pooling measure with probability 1, that is, $p_s^b\left(\mu_b^{pool}\right) = p_s^b\left(\mu_b^{pool}\right) = 1$, for some $\mu_b^{pool} \in M_b$. Likewise, a pure separating weak equilibrium is a tuple e where both types play separating measures with probability 1. Finally, I call it a mixed weak equilibrium any tuple e where safe firms play both a pooling and a separating measure with strictly positive probabilities.

DEFINITION 3: *A pure separating weak equilibrium in E^{SD} is a tuple $e \equiv \left(p_s^b(\cdot), p_r^b(\cdot), \gamma_s(\cdot), d_c(\cdot|\gamma)\right)$ satisfying*

1. *Types play their corresponding separating measures with probability* 1*:*

$$p_j^b(\mu_b) = \mathbf{1}_{\left\{\mu_b = \mu_{b,j}^{sep} \wedge MBR_j(\mu_b|\gamma) > 1\right\}}, \quad j \in \{s, r\}$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function and $\mu_{b,j}^{sep} \in M_b \cup \emptyset$ solves the creditor funding condition problem CFC - Separating.

2. *Creditors' beliefs are consistent with type's strategies*

$$\gamma_s(\mu_b) = \begin{cases} 1, & \text{if } \mu_b = \mu_{b,s}^{sep} \\ 0, & \text{if } \mu_b = \mu_{b,r}^{sep} \\ \in [0,1) & \text{s.t. } \mu_b \text{ does not solve CFC - Pooling} \end{cases}$$

3. *Only optimally levered firms get funded and creditors break even:*

$$d_c(\mu_b|\gamma) = \begin{cases} \gamma_s(\mu_b)\, d_s(\mu_b) + (1 - \gamma_s(\mu_b))\, d_r(\mu_b) & \text{if } \mu_b \in \{\mu_{b,s}^{sep}, \mu_{b,r}^{sep}\} \\ 0 & \text{otherwise} \end{cases}$$

The first condition states that firms will choose the separating strategy corresponding to their types with probability one. By Lemma 2, the risky-type's separating measure coincides with its first-best measure $\mu_{b,r}^{sep} = \mu_{b,r}^{FI}$. The second condition requires that creditors beliefs be consistent with the type-contingent strategies. Notice there are infinite belief functions in $M_b \cup \emptyset \to [0, 1]$ that satisfy this restriction. So long as the probabilities that a firm is safe assigned by creditors to off-equilibrium measures, that is, $\gamma_s(\mu_b)$ for $\mu_b \in M_b - \{\mu_{b,s}^{sep}, \mu_{b,r}^{FI}\}$, are such that these measures do not satisfy the creditors' funding condition (CFC - Pooling), firms that deviate from the equilibrium strategies do not get funded (third condition above.) A convenient example is that of a belief function that assigns probability zero to a firm being safe whenever $\mu_b \neq \mu_{b,s}^{sep}$.

Figure 8 and 9 in Appendix B.1 show the firm values and market-to-book ratios of equity in pure separating equilibrium for multiple combinations of firm-type distribution, μ_s, and mean-reducing shock probabilities, q. The iso-curves are vertical, since the safe-type's separating eqiulibrium measures dependend on the characteristics of each type alone, and not on the ratio of safe-to-risky firms (Lemma 3.) Both the safe-type's firm value and MBR decrease with the risky-type's exposure to the shock. This should come as no surprise. Recall that, for the most part, the higher the likelihood of the shock, the more the risky type benefits from misrepresentation (figure 5.) Therefore, the larger is the adjustment in the safe type's leverage necessary to discourage pooling. Finally, since the risky firms' separating measure of bonds coincides with their first-best (full-information) choice, their payoffs in are as in figures 3 and 4.

DEFINITION 4: *A pure pooling weak equilibrium in* E^{SD} *is a tuple* $e \equiv \left(p_s^b\left(\cdot\right), p_r^b\left(\cdot\right), \gamma_s\left(\cdot\right), d_c\left(\cdot|\gamma\right)\right)$ *satisfying*

1. *Types issue the same measure of bonds* $\mu_b^{pool} \in M_b$ *with probability* 1*:*

$$p_j^b\left(\mu_b\right) = \mathbf{1}_{\left\{\mu_b = \mu_b^{pool}\right\}}, \quad j \in \{s, r\}$$

   where $\mathbf{1}_{\{\cdot\}}$ *is the indicator function and* μ_b^{pool} *solves the creditor funding condition problem CFC - Pooling.*

2. *Creditors' beliefs are consistent with type's strategies, that is, (i)* $\gamma_s\left(\mu_b^{pool}\right) = \mu_s$*, and (ii) for all* $\mu_b \in M_b - \{\mu_b^{pool}\}$*,*
   - *if* $\gamma_s\left(\mu_b\right) \in \{0, 1\}$*, then* μ_b *does not solve CFC - Separating when* $\gamma_s = \gamma_s\left(\mu_b\right)$*;*
   - *if* $\gamma_s\left(\mu_b\right) \in (0, 1)$*, then* μ_b *does not solve CFC - Pooling when* $\gamma_s = \gamma_s\left(\mu_b\right)$*;*

3. *Only optimally levered firms get funded and creditors break even:*

$$d_c\left(\mu_b|\gamma\right) = \begin{cases} \gamma_s\left(\mu_b\right) d_s\left(\mu_b\right) + \left(1 - \gamma_s\left(\mu_b\right)\right) d_r\left(\mu_b\right) & \text{if } \mu_b = \mu_b^{pool} \\ 0 & \text{otherwise} \end{cases}$$

By condition 2 above, any firm that chooses a measure $\mu_b \neq \mu_b^{pool}$ violates the creditors' funding condition, and is thus denied funds in the primary debt market (condition 3.) Once again, there are infinite belief functions consistent with equilibrium. In particular, the function that assigns probability 1 to the firm being risky ($\gamma_s\left(\mu_b\right) = 0$) for all $\mu_b \neq \mu_b^{pool}$ satisfies restriction 2 above, since μ_b^{pool} is strictly preferred to $\mu_{b,r}^{FI}$ by both types, and any other choice μ_b will result in the firm not being funded.

Just like in the separating equilibria case, figures 10 to 13 in Appendix B.1 show the firm values and market-to-book ratios of equity in a pure pooling equilibrium for multiple combinations of μ_s and q. As expected, for each type, both the firm value and MBR increase with the share of safe firms. Conversely, the higher the risky-type's exposure to the mean-reducing shock, the larger the discount creditors impose on the price of newly issued bonds, and thus the lower these payoffs are in equilibrium.

DEFINITION 5: *A mixed weak equilibrium in* E^{SD} *is a tuple* $e \equiv \left(p_s^b\left(\cdot\right), p_r^b\left(\cdot\right), \gamma_s\left(\cdot\right), d_c\left(\cdot|\gamma\right)\right)$ *satisfying*

1. *Risky firms play the pooling measure* μ_b^{pool} *with probability* 1*, while safe firms randomize between the pooling measure and their separating measure,* $\mu_{b,s}^{sep}$*, with probabilities* $p_s^{pool} \in (0, 1)$ *and* $1 - p_s^{pool}$*, respectively:*

$$p_r^b\left(\mu_b\right) = \mathbf{1}_{\left\{\mu_b = \mu_b^{pool}\right\}}$$
$$p_s^b\left(\mu_b\right) = \begin{cases} p_s^{pool} & \text{if } \mu_b = \mu_b^{pool} \\ 1 - p_s^{pool} & \text{if } \mu_b = \mu_{b,s}^{sep} \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbf{1}_{\{\cdot\}}$ *is the indicator function,* $\mu_{b,s}^{sep}$ *solves the safe-type's* CFC - Separating *problem,* μ_b^{pool} *solves* CFC - Pooling *when creditors' beliefs* γ_s *equals:*

$$\gamma_s\left(\mu_b^{pool}\right) = \frac{p_s^{pool}\mu_s}{p_s^{pool}\mu_s + (1-\mu_s)}$$

and safe-type shareholders are indifferent between pooling and separating:

$$MBR_s\left(\mu_b^{pool}|\gamma\right) = MBR_s\left(\mu_{b,s}^{sep}|\gamma\right)$$

2. *Creditors' beliefs are consistent with types' strategies:*

$$\gamma_s(\mu_b) = \begin{cases} \frac{p_s^{pool}\mu_s}{p_s^{pool}\mu_s+(1-\mu_s)} & \text{if } \mu_b = \mu_b^{pool} \\ 1 & \text{if } \mu_b = \mu_b^{sep} \\ \in [0,1] & \mu_b \text{ solves neither CFC - Separating nor CFC - Pooling} \end{cases}$$

3. *Only optimally levered firms get funded and creditors break even:*

$$d_c(\mu_b|\gamma) = \begin{cases} \gamma_s(\mu_b)\, d_s(\mu_b) + (1-\gamma_s(\mu_b))\, d_r(\mu_b) & \text{if } \mu_b = \mu_b^{pool} \\ d_s(\mu_b) & \text{if } \mu_b = \mu_{b,s}^{sep} \\ 0 & \text{otherwise} \end{cases}$$

By Lemma 5, risky firms never randomize between their separating measure, $\mu_{b,r}^{FI}$, and a pooling measure because pooling with safe firms always yields a higher MBR. Therefore, in a mixed equilibrium, only the safe type plays a mixed strategy. In addition, since (i) safe firms' equity investors must be indifferent between μ_b^{pool} and $\mu_{b,s}^{sep}$ (shareholders' optimality condition), and (ii) $\mu_{b,s}^{sep}$ does not depend on the ratio of safe-to-risky firms (lemma 3), it follows that the safe firms' MBR is determined by the solution to the CFC - Separating problem alone. Safe-type shareholders' payoffs in a mixed equilibrium thus coincide with those in a pure separating equilibrium.

If indeed such equilibrium exists, though safe-type shareholders are left indifferent between the two possible choices of capital structure, the pooling and separating measures yield different firm values. As I shall discuss in the next section, unlike equity holders, creditors will have a strict preference for one allocation over another. Moreover, it is worth noting that both types would likely be made strictly better off in a pure pooling weak equilibrium. This is because the more safe-type firms pool together with risky ones (the higher p_s^{pool}), the more they raise the valuation of the debt issued by firms whose types are otherwise unknown to bond investors, thus boosting both safe- and risky-type's firm values and MBR in a pooling allocation. Therefore, to the degree that there can be any coordination in equilibrium, mixed strategies should not played by safe firms, both because (i) creditors will prefer one pure strategy over another, and (ii) increasing the mass of safe firms pooling with risky ones will raise the payoffs to safe-type equity holders.

Finally, in the numerical exercises conducted, no combination of the measure of safe firms μ_s and a safe-type mixed strategy p_s^{pool} was found that left safe firms indifferent between pooling or separating. Indeed, in this simple two-period model, separation is achieved by drastically reducing the leverage of safe firms (decreasing $\mu_{b,s}$), which renders the separating strategy MBR strictly below that yielded by a pooling measure, for any $\gamma_s\left(\mu_b^{pool}\right)$.

G. Equilibria in the Restricted Economy E^{SD}

As discussed above, creditors will typically not be indifferent between the pooling and separating weak equilibria because these market arrangements result in distinct total firm valuations. I now refine the creditors' funding condition to characterize bond investors' preferences over competing market equilibria in E^{SD}.

DEFINITION 6 (Creditors' Preferences): *Given the pooling and separating weak equilibria in E^{SD}, bond investors prefer that which yields the highest safe firm valuation.*

Creditors' knowledge of the firm type distribution empowers them, if not to implement the first-best allocations, at least to enforce a particular type of equilibrium. For instance, a pure pooling weak equilibrium price structure as detailed in definition 4 discourages the safe type from attempting to force separation by assigning a null price to the debt from firms who issue a measure $\mu_{b,s}^{sep}$ of bonds. Conversely, a pure separating weak equilibrium price function will withhold funds from any and every firm that chooses to issue a measure of bonds consistent with a pooling equilibrium. It is still possible that safe firms achieve the same valuation in the pooling and the separating weak equilibria. When this is the case, I assume creditors enforce the equilibrium that yields the highest MBR to safe-type shareholders.

DEFINITION 7: *An equilibrium in E^{SD} is a weak equilibrium tuple e such that no other weak equilibrium $\tilde{e}^{\star}$ yields a higher total firm value for safe firms. When the safe type's firm valuations coincide in the pooling and separating weak equilibria, the prevailing equilibrium is that which maximizes the safe-type's MBR.*

Figures 14 and 15 show the safe-type's firm valuation and the risky-type's MBR in the prevailing market equilibria for a range of values of firm type distribution parameter μ_s and risky-type's exposure to the mean-reducing shock q. All else constant, the lower the measure of safe firms, the more costly it is for the safe type to pool together with the risky one, so a separating equilibrium tends to hold. Likewise, the higher the shock exposure of risky firms, the more creditors discount bonds issued in a pooling equilibrium. When this penalty is large enough, a distortion to the safe-type's capital structure so as to discourage pooling with risky firms is warranted.

Lastly, I define the cost imposed by the asymmetry of information problem as the difference in the safe-type's firm value in the equilibrium above and in the first-best (full information) of section II.C.

$$INFC\left(\mu_{b,s}^{EP}|\gamma\right) \equiv FV_s^{FI}\left(\mu_{b,s}^{FI}\right) - FV_s\left(\mu_{b,s}^{EP}|\gamma\right) \tag{12}$$

where superscript EP stands for *electronic platform*, and $\mu_{b,s}^{EP}$ is the safe-type's capital structure in equilibrium in E^{SD}.

H. Equilibria in the Dual Market Economy E

I now return to the dual-market economy E, where firms have the option to issue a non-standardized bond, which is then traded exclusively in the less liquid OTC market. As argued in section II.D, all else constant, the issuance of non-standardized debt leads to a loss in the firm value. For one, the higher transaction costs in the OTC market command a greater discount on the value of bonds traded there. Secondly, in the case of risky-firms, the wealth transfer to the risk-neutral bond investors after a shock results in a loss because creditors discount cash-flows at a higher rate than shareholders. However, the issuance of bonds with the debt-protective covenant can eliminate informational costs, thus potentially more than compensating safe-type shareholders for the losses stemming from the liquidity differential between the secondary trading venues.

In Appendix C.4, I show that, under certain conditions, perfect signaling can be achieved with a debt-protective covenant. More precisely, there exists a value $\underline{\theta} \in (0,1]$ such that any non-standardized bond with covenant parameter $\theta \in [\underline{\theta}, 1]$ effectively discourages risky-type misrepresentation (see Lemma 9.) By Corollary 6, no risky firm then ever issues a non-standardized bond. For the parameters in Table I, this condition is always satisfied. In particular, when θ is set to 1, the risky-type shareholders' misrepresentation payoff is strictly lower than their first-best MBR in the restricted economy E^{SD}, regardless of the likelihood of the mean-reducing shock q (see figure 7 in Appendix B.1.)

When perfect signaling can be achieved with a debt-protective covenant, any equilibrium in the OTC market is a full-information equilibrium: either no debt with covenant is issued, so no trade takes place in OTC, or only bonds issued by safe firms are transacted. Let $\mu_{b,s}^{OTC}$ denote the measure of bonds that maximizes the value of a safe firm should it choose to issue non-standardized debt. Likewise, let $FV_s\left(\mu_{b,s}^{OTC}\right)$ denote the safe-type's firm value in such case. The equilibrium in the dual-market economy E can then be backed-out by comparing the safe-type's firm valuation in the restricted economy E^{SD} to $FV_s^{OTC}\left(\mu_{b,s}^{OTC}\right)$.

As illustrated in Figure 14, the safe-type's firm valuation in equilibrium in E^{SD} is weakly increasing in the measure of safe firms, μ_s. Depending on the risky-type's exposure to the mean-reducing shock, there exists a value $\overline{\mu}_s$ above which a pooling equilibrium prevails, wherein creditors' debt valuation, and thus the total firm value, increases with the ratio of safe to risky firms. Denote by $\mu_j^{EP} \in [0, \mu_s]$ the measure of type-j firms that issue the standardized bond, $j \in \{s, r\}$. Consider again the restricted economy $E^{SD}\left(\mu_s^{EP}, \mu_r^{EP}\right)$, where I have made the measures of safe and risky firms explicit, and suppose there exists a value $\mu_s^{EP} < \mu_s$ such that the resulting equilibrium in $E^{SD}\left(\mu_s^{EP}, 1-\mu_s\right)$, $e^{\star}$, yields the same valuation for the safe type as that obtained via the issuance of the non-standardized bond:

$$FV_s\left(e^{\star}\right) = FV_s^{OTC}\left(\mu_{b,s}^{OTC}\right)$$

Whenever this is the case, I assume creditors enforce an equilibrium where only standardized bonds are issued by refusing to buy non-standardized debt. Conversely, if the equilibrium in $E^{SD}\left(\mu_s, 1-\mu_s\right)$ is such that the safe type valuation is strictly lower than $FV_s^{OTC}\left(\mu_{b,s}^{OTC}\right)$, bond investors then impose a separating equilibrium where only risky firms issue the standardized bond, so the safe-type debt is traded exclusively over-the-counter.

H.1. Decomposing the Inter-Market Valuation Differential under Asymmetric Information

I call *dual-market valuation differential* the difference between the safe type's initial firm valuation in the restricted economy E^{SD} equilibrium (as defined in the previous section) and its corresponding value when issuing non-standardized debt:

$$\Delta FV\left(Q, \mu_s, \boldsymbol{r}, \boldsymbol{B}\right) \equiv FV_s\left(\mu_{b,s}^{EP} | \gamma\right) - FV_s^{FI}\left(\mu_{b,s}^{OTC} | \gamma\right) \tag{13}$$

where superscript FI indicates that any equilibrium over-the-counter is a full-information equilibrium.

To study the determinants of equilibria across competing secondary markets, I decompose the valuation differential into a liquidity and an informational components. The liquidity term is defined as the gain in the safe type's initial firm valuation in a full-information equilibrium when secondary trades transition from over-the-counter markets to electronic exchanges:

$$LQD\left(Q, \boldsymbol{r}, \boldsymbol{B}\right) \equiv FV_s^{FI}\left(\mu_{b,s}^{FI}\right) - FV_s^{FI}\left(\mu_{b,s}^{OTC}\right) \tag{14}$$

This component captures the impact of the variations in transaction costs embedded in creditors' market-specific rates of discount, r_{disc}^{EP} and r_{disc}^{OTC}, over the total return of an investment in a safe project, absent any asymmetry of information between creditors and equity investors. Because it focus on valuation in full information settings, the liquidity term is independent from the measure of safe firms, μ_s.

By the definition of the information cost of adverse selection in equation 12 and the expression above, the cross-market variation in the total valuation can be expressed as the difference between the liquidity gains in EP relative OTC markets and the cost of adverse selection associated with unsecured, standardized debt.

$$
\begin{aligned}
\Delta FV\left(Q,\mu_s,\boldsymbol{r},\boldsymbol{B}\right) &= \left(FV_s\left(\mu_{b,s}^{EP}|\gamma\right) - FV_s^{FI}\left(\mu_{b,s}^{FI}\right)\right) \\
&\quad - \left(FV_s^{FI}\left(\mu_{b,s}^{OTC}|\gamma\right) - FV_s^{FI}\left(\mu_{b,s}^{FI}\right)\right) \\
&= LQD\left(Q,\boldsymbol{r},\boldsymbol{B}\right) - INFC\left(Q,\mu_s,\boldsymbol{r},\boldsymbol{B}\right)
\end{aligned}
$$

Safe firms maximize their initial firm valuation by issuing standardized bonds whenever this differential is positive. Conversely, if the informational cost in electronic exchanges is greater than the dual-market liquidity differential, creditors require safe firms to issue bonds with a debt protective covenant that fully reveals their type. In this case, a separating, dual-market equilibrium holds, in which only risky firms' bonds are traded in the exchange.

Figure 16 shows the safe-type's firm valuation in equilibrium in the dual-market economy E for varying type-distribution and shock exposure parameters. The light gray area indicates the (μ_s, q)-pairs for which a separating equilibrium with trade in the OTC market prevails. Here, the likelihood of the mean-reducing shock is so large that creditors expect safe firms to issue the non-standardized debt. Put differently, the liquidity loss stemming from the higher transaction costs in OTC is more than compensated by the informational gains supported by the type separation in equilibrium.

I. The Effect of the Measure of Safe Types over the Equilibrium Properties

The cross-market liquidity gains are driven by the market-specific transaction costs embedded in the creditors' market-specific rates of discount, r_{disc}^{EP} and r_{disc}^{OTC}, but unaffected by the measure of safe firms or the risky type's exposure to the volatility shock. Conversely, the informational costs of adverse selection are determined by the firm type distribution, and independent from the cross-market transaction costs differential. The equilibrium analysis so far has studied how informational costs, and therefore the dual-market valuation differential, vary with the size and intensity of volatility shocks, while assuming a constant measure of safe firms ($\mu_s = 0.2$). I now extend the analysis by varying both the overall shock risk and share of risky firms in an entrant cohort.

The effect of the risky type's overall exposure to the volatility shock over firms' payoffs can be reduced to a one-dimensional problem. The iso-curves in the plots analyzed in the previous sections show that, by fixing one parameter, either λ or σ_h, and varying the other, one obtains any payoff, and thus any equilibrium outcome. To see this, consider a cross-section of the safe type's firm value function along, say, $\lambda = 0.25$ in figure **??**, and notice how the resulting surface intersects the varying iso-curves and market equilibrium regions simply. Therefore, in what follows I fix the shock intensity at $\lambda = 0.3$.

Figures **??** to **??** in Appendix **??** repeat the analysis of the firm value and MBR functions in the previous plots, but this time with μ_s in the y-axis. Notice how the iso-curves in the full-information equilibrium plots are vertical, as the types' firm-value-maximizing strategies are independent from one another. Moreover, since misrepresentation payoffs are obtained by assigning probability 1 to firms being of the safe type, by construction the iso-curves in figures ?? to ?? also are vertical. Furthermore, this independence from μ_s is also observed in the safe- and risky-types' firm value and MBR functions in a separating market outcome (figures ?? to ??). The result holds because the incentive-compatibility constraint in the safe firm's best separating outcome conditional response is independent of μ_s.[12] In all of these cases, the iso-curves' levels match the values of a cross-section of the corresponding surfaces along $\lambda = 0.3$ in the plots in Appendix ??.

In contrast, the safe-type's payoffs in a pooling market outcome are directly affected by the ratio of safe to risky firms in entrant cohorts As the share of safe firms falls, the informational costs of adverse selection increase, affecting the safe firms' initial valuation and MBR (figures ?? to ??.) The lower μ_s (and the higher the exposure of the risky type to the volatility shock), the more creditors revise down their bond valuation, thereby raising the debt rollover costs for safe firms. Moreover, the concave shape of the iso-curves indicates that the sensitivity of the safe-type's initial firm valuation to μ_s is higher when the risky firms' exposure to the volatility shock is small to moderate. Interestingly, the risky-type shareholders' rate of return show little sensitivity to changes in the ratio of safe to risky firms in a pooling market outcome. The nearly vertical iso-curves of the MBR function in figures ?? to ?? suggest that the effects of μ_s over the risky firms' debt rollover costs nearly cancel each other out. On the one hand, an increase in μ_s benefits risky firms through its positive impact on creditors' bond valuation. On the other hand, the higher bond values prompt safe firms to adjust their capital structure by issuing more bonds. The increased leverage then raises the expected debt rollover costs to risky firms.

Figures ?? and ?? show the safe type's initial firm valuation and the risky type's market-to-book -ratio of equity in equilibrium, respectively, when the only secondary market is the electronic platform (EP.) As before, when the risky firms' exposure to the volatility shock is small, a pooling market outcome yields a higher initial firm valuation to the safe type. The informational costs of adverse selection in an EP-only equilibrium increase with the risky type's volatility shock exposure until the losses to the safe firms' initial value under a pooling market outcome equal those in a separating market arrangement. From this point on, a separating equilibrium prevails and the informational costs decrease with the risk exposure (σ_h), until eventually reaching zero.

The novelty here is the effect of the measure of safe firms over the risk-exposure threshold marking the transition from pooling to separating equilibrium. While informational costs in a separating market outcome are unaffected by μ_s, an increase in the measure of safe firms lowers the informational costs in a pooling market arrangement. The higher μ_s, the lower the effect of the risky type's misrepresentation over bond prices, and thus on the safe type's debt rollover costs. Consequently, the larger is the set of risky types for which a pooling equilibrium holds (and hence the concave shape of the separating equilibrium region.) By the same logic, the dual-market equilibria are more likely to hold the smaller is the measure of safe firms. The light gray area on the LHS plot of figure ?? indicates the combinations of risk exposure and μ_s for which the EP iso-curve levels are below the safe firm's valuation in OTC, $FV_s^{OTC} = 111.41$. In this region, therefore, the safe firms' best response is to issue non-standardized bonds with the debt protective covenant.

III. Conclusion

In recent years, electronic trading in corporate bond markets has seen a substantial and sustained growth. Over 90% of small ticket trades are now done in exchanges. Nonetheless, the bulk of the notional volume traded daily is concentrated in larger ticket size trades, which are still done over-the-counter. The main obstacle to the electronic trading of corporate bonds is arguably the

[12] Recall the distinction between market outcome and market equilibrium in section ??.

fragmentation of the trading activity across a vast universe of securities. In other asset classes, this *electronification* process has been accompanied by an standardization effort to facilitate pricing and expand the base of potential investors, thus improving liquidity in secondary markets. Because bonds are information-sensitive securities, however, standardization comes at a cost. Doing away with debt protective covenants can hamper firms' ability to properly signal their creditworthiness, with non-trivial effect on firms' credit spreads and debt rollover costs in primary markets.

In this paper, I analyzed the interplay between liquidity gains, arising from changes in the micro-structure of secondary markets, and the informational costs of debt standardization, from a theoretical perspective. To do so, I proposed a structural model of credit risk with asymmetric information and competing secondary markets, where debt covenants arise endogenously. The model features two classes of investors, bond investors (or creditors) and equity holders (shareholders), which invest in firms that can be either *safe* or *risky*, depending on their exposure to an idiosyncratic, unhedgeable risk. Debt is traded in competing secondary markets, which differ in (i) their (external) liquidity and (ii) the types of bonds they accept. Electronic platforms (EPs) offer lower transaction costs, but intermediate only trades of standardized, covenant-free bonds, whereas over-the-counter (OTC) markets accept any type of bond.

All else constant, debt issued with standardized, covenant-free bonds is more valuable because these bonds are traded in the more liquid electronic markets. The reduced transaction costs of secondary trades in EPs increase the price of newly-issued bonds, thereby lowering firms' debt rollover costs. When both classes of investors are fully informed about the firms' risk exposure, therefore, firms issue standardized debt and all secondary trades happen in electronic exchanges. This result, however, may not hold in the presence of information asymmetry in credit markets.

When bond investors are unable to directly observe firms' risk exposure, shareholders of riskier firms may increase their rate of return by misrepresenting their firms' creditworthiness. Type-misrepresentation is akin to an asset substitution problem, wherein creditors' valuation of a firm's debt is incommensurate with the firm's riskiness. However, so long as bond investors are knowledgeable of the distribution of firm types, misrepresentation prompts them to revise downwards the valuation of all standardized bonds. This raises the debt rollover costs for safe firms, which in turn adjust their capital structure, either by increasing their measure of outstanding bonds to discourage the risky type's misrepresentation, or by reducing their leverage to minimize the impact of the misrepresentation over their debt-rollover costs. Alternatively, safe firms may opt for issuing bonds with a debt protective covenant to signal their creditworthiness.

The direction of the safe type's leverage adjustment and choice of debt instrument in response to the risky type's misrepresentation depend on (i) the informational costs of adverse selection in electronic markets (INFC), and (ii) the liquidity differential between the competing secondary trading venues (LQD.) The lower the ratio of safe to risky firms or the higher the risky firms' exposure to the unhedgeable shock, the more the safe type's debt rollover costs are affected when pooling together with the risky firms. When the unhedgeable risk differential between the two types of firms is small or the ratio of safe-to-risky firms is sufficiently high, the INFC is minimized by having safe firms reduce their leverage so that types pool together. When the risky differential is large or the measure of safe firms is small, however, the effect of pooling over the safe firms' debt rollover costs is so high that these firms find it preferable to increase their leverage to discourage the risky type's misrepresentation. Covenants arise endogenously as a means of mitigating the informational problem. When informational costs exceed the liquidity differential between over-the-counter and electronic markets, safe firms forego the liquidity gains and issue instead non-standardized bonds to signal their creditworthiness. In this case, a dual-market separating equilibrium holds where only risky firms issue standardized bonds.

The results have implications for (i) the volume of trades in corporate bond exchanges and (ii) the composition of debt across competing secondary trading venues. When setting their clearing and trading fees, electronic platforms must consider the trade-off between liquidity and the costs of bondholder-stockholder conflicts. Debt standardization may exacerbate informational problems, partly offsetting the liquidity gains offered by the centralized trading of a reduced number of securities. In the most severe cases, the resulting informational costs may drive safer firms away from the new electronic platforms, leading to a smaller base of potential clients and reduced revenues.

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Appendix A. Tables

Table I: 2-Period Model Parameters

Firms	
caling factor	$s_f = 1.0$
volatility	$\sigma = 0.3$
initial value of assets	$V_0 = 100$

Standardized Bonds	
maturity	$m = 1$
coupon	$c = 0.0$
principal	$p = 90.0$

Bankruptcy	
bankruptcy recovery rate	$\alpha = 0.6$
debt tax benefit rate	$\pi = 27\%$

Interest Rates	
risk-free rate	$r_f = 8\%$
creditors' discount rate in EP	$r_{disc}^{b,EP} = 10\%$
creditors' discount rate in OTC	$r_{disc}^{b,OTC} = 12\%$

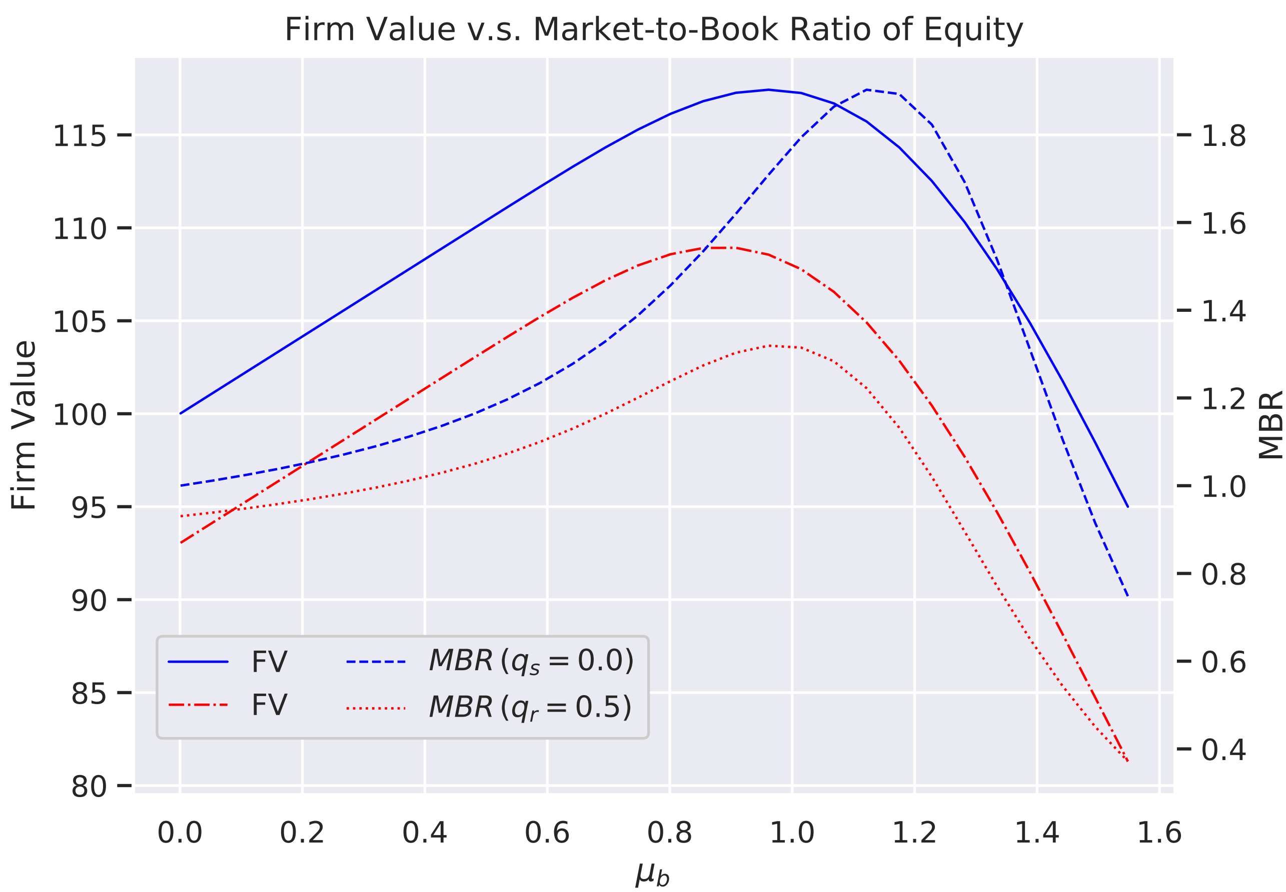
The table shows the parameter values used in the 2-period model, adapted from He and Xiong (2012) list of parameters calibrated to firms with a speculative-grade BB rating.

Appendix B. Plots

B.1 2-Period Model Plots

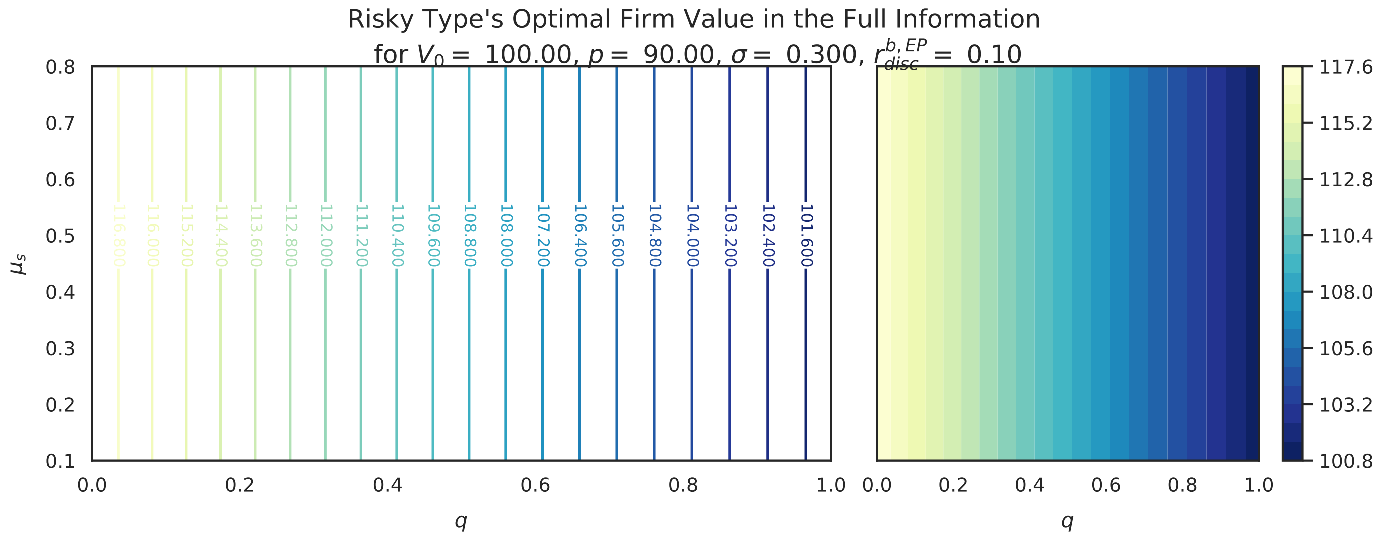
B.1.1 Full Information Equilibrium

Figure 2. Firm Value and Market-to-Book Ratio of Equity by Measure of Bonds



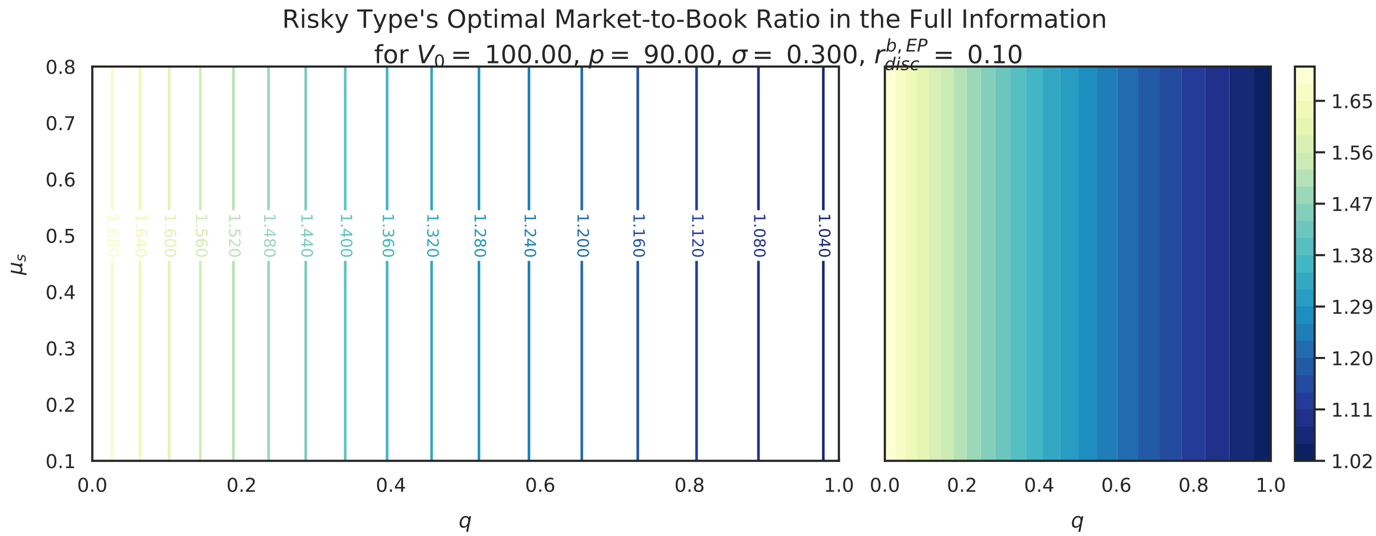
The figure shows the firm values (LHS) and market-to-book ratios (RHS) as functions of the measure of bonds issued μ_b for a safe firm ($q_s = 0.0$) and a risky type firm when the probability of the mean-reducing shock is $q_r = 0.5$. The initial value of the assets, V_0, the risk-free rate, r_f, the tax benefit rate, π, and the bankruptcy recovery rate, α, are taken from He and Xiong (2012) and can be found in Table I in Appendix A.

Figure 3. Optimal Firm Values in an Electronic Platform Full Information Equilibrium



The figure above shows the optimal firm values in an Electronic Platform full information equilibrium for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$. The horizontal iso-curves reflect the independence of the optimal firm values from the measure of safe firms in a full information setting. The values at the y-axis ($q = 0$) correspond to the safe-type payoff. As expected, the optimal firm value is strictly decreasing in the likelihood of the mean-reducing shock.

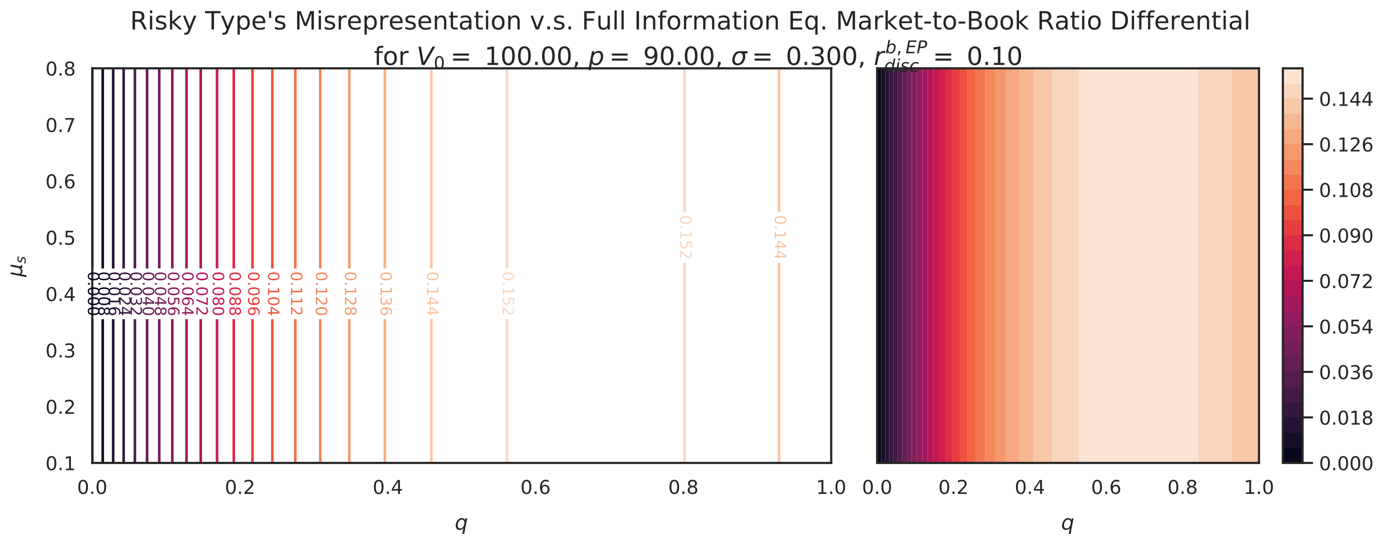
Figure 4. Equity Market-to-Book Ratio in an Electronic Platform Full Information Equilibrium



The figure above shows the market-to-book ratio of equity in an Electronic Platform full information equilibrium for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$. The horizontal iso-curves reflect the independence of the optimal capital structure from the measure of safe firms in a full information setting. The values at the y-axis $(q = 0)$ correspond to the safe-type payoff. As expected, the optimal firm value is strictly decreasing in the likelihood of the mean-reducing shock.

B.1.2 Misrepresentation

Figure 5. Risky Type's MBR Differential in EP - Misrepresentation v.s. Full Information Equilibrium Payoffs



The figure shows the difference in market-to-book ratios (MBR) yielded by a type-misrepresentation and a truth-telling (Full Information) strategies in an Electronic Platform, that is,

$$MBR_r^{MP} - MBR_r^{FI}$$

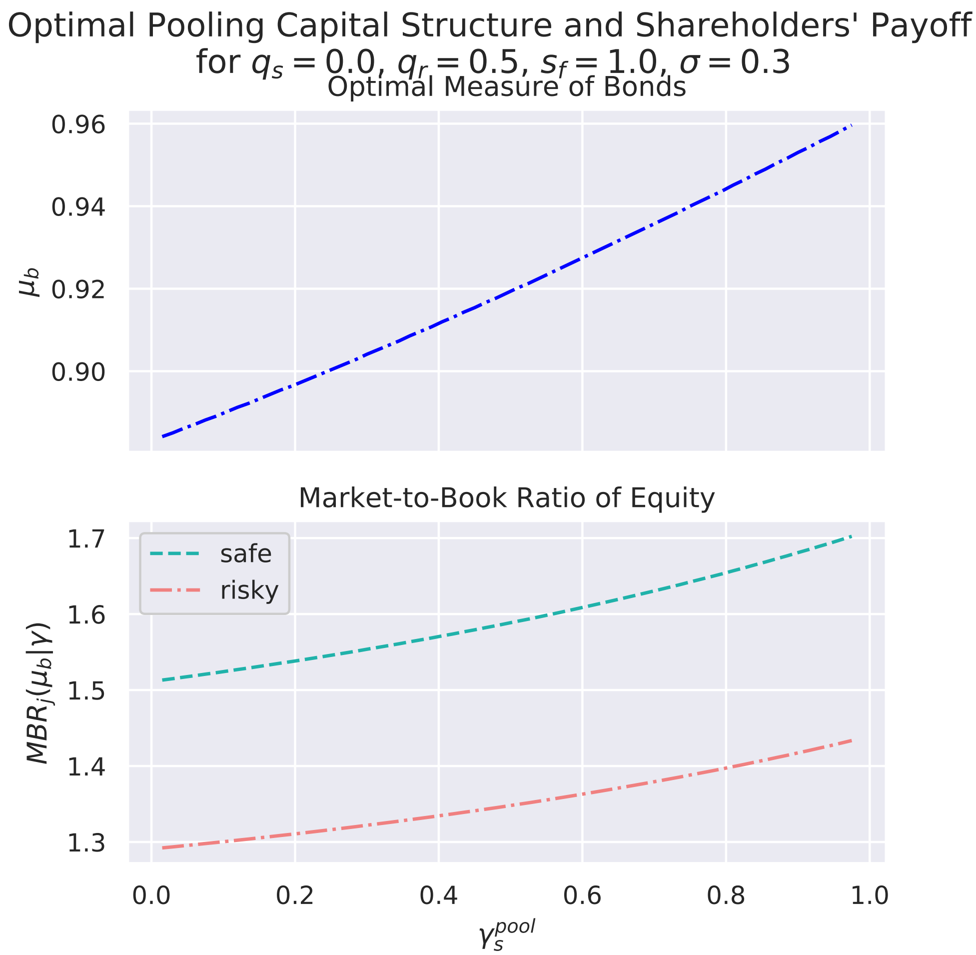
where subscript r stands for "risky", and superscripts MP and FI stand for "misrepresentation" and "full information", respectively. The differences are computed for varying levels of exposure to the mean-reducing shock, q, and different measures of safe type firms, μ_s. The type-misrepresentation strategy consists in copying the full-information capital structure of the safe type. The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

Risky types will choose to misrepresent themselves if (i) creditors cannot observe their types, and (ii) the MBR yielded by copying the capital structure of the safe type exceeds their optimal MBR.

Since, by construction, the misrepresenting payoff are computed by having creditors assign probability 1 to the event that bonds are issued by safe firms, the misrepresenting MBR's are independent from the measure of safe firms (vertical iso-curves.) Finally, notice that misrepresentation gains are non-monotonic in the exposure to the shock. For high enough shock probability values, the required change in the firm's debt issuance ($\mu_{b,r}^{MP} - \mu_{b,r}^{FI}$) is so large that the expected costs from the service of the debt start to undermine the gains from the overvaluation of the firm's bonds.

B.1.3 The Uniqueness of Pooling Equilibrium Measures

Figure 6. The Inverse Belief Function $\gamma_s^{-1}(\cdot)$



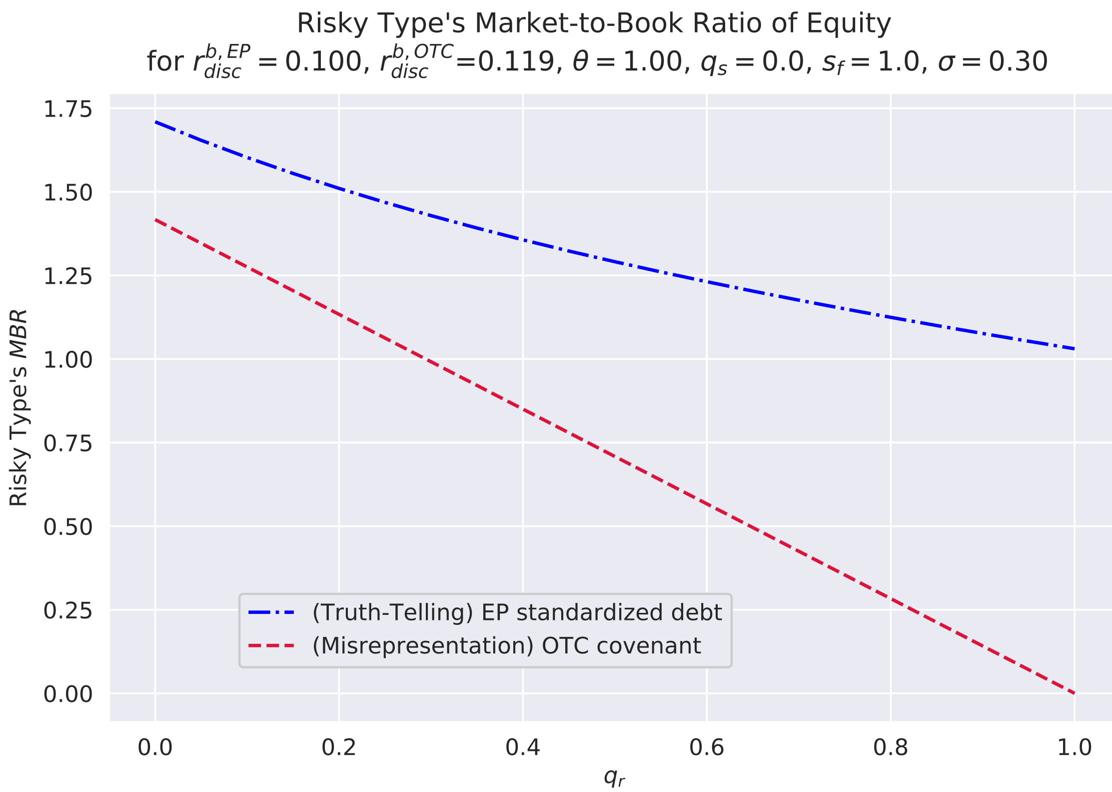
The figure considers two types of firms: safe ($q_s = 0.0$), and risky ($q_r = 0.5$). For each creditors' belief function value γ_s, the top graph shows the measure of bonds consistent with the maximization of expected firm value (CFC - Pooling.) The bottom graph shows the implied market-to-book ratios for the safe and risky firms. The higher the probability assigned by creditors to a firm being safe, the larger is the bond issuance implied by the creditors' funding condition, and the higher are the payoffs to the shareholders of both types of firms. In this example, the economy E^{SD} cannot support more than one pooling measure in equilibrium. For any pair $\mu_b', \mu_b'' \in M_b$, with $\mu_b' < \mu_b''$, consistency of creditors' beliefs with firms strategies requires:

$$\gamma_s\left(\mu_b'\right) = \frac{p_s^b\left(\mu_b'\right)\mu_s}{p_s^b\left(\mu_b'\right)\mu_s + p_r^b\left(\mu_b'\right)(1-\mu_s)} < \frac{p_s^b\left(\mu_b''\right)\mu_s}{p_s^b\left(\mu_b''\right)\mu_s + p_r^b\left(\mu_b''\right)(1-\mu_s)} = \gamma_s\left(\mu_b''\right)$$

But μ_b'' yields strictly higher MBRs and is thus strictly preferred by shareholders of both types of firms, so that $p_j^b\left(\mu_b'\right)$ must be zero, for $j \in \{s, r\}$.

B.1.4 Non-Standardized Debt and Misrepresentation

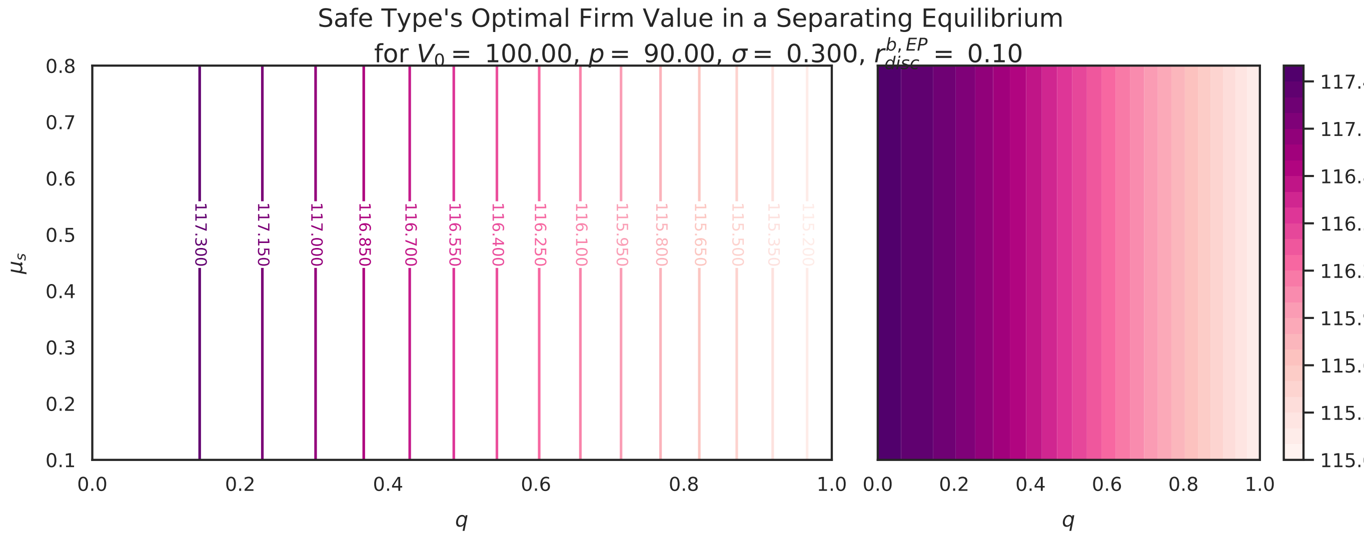
Figure 7. The Deterrence Effect of Non-Standardized Debt



The figure contrasts the risky-type shareholders' payoffs yielded by (i) a truth-telling strategy wherein their firms issue standardized debt (blue), and by (ii) a type-misrepresentation strategy in which risky firms issue the safe-type's non-standardized bond (red), for varying levels of risk-exposure, q_r. Raising funds through non-standardized debt is more costly to firms because covenant-restricted bonds trade in the more illiquid OTC market. In the example above, the liquidity differential between EP and OTC is set to 190 basis points. Moreover, when the covenant parameter θ equals 1, all equity shares are transferred to bond investors upon the occurrence of mean-reducing shock, fully erasing shareholders' payoffs in this event. These two effects render the truth-telling strategy more attractive to the risky type, regardless of their exposure to the shock. Finally, whether or not safe firms issue non-standardized debt depends on the inter-market liquidity differential and the informational costs in electronic platforms (see section II.H.1.)

B.1.5 Weak Separating Equilibrium in E^{SD}

Figure 8. Safe Type's Firm Value in a Separating Equilibrium in E^{SD}



The figure above shows the safe type's firm value iso-curves in pure separating equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q. The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

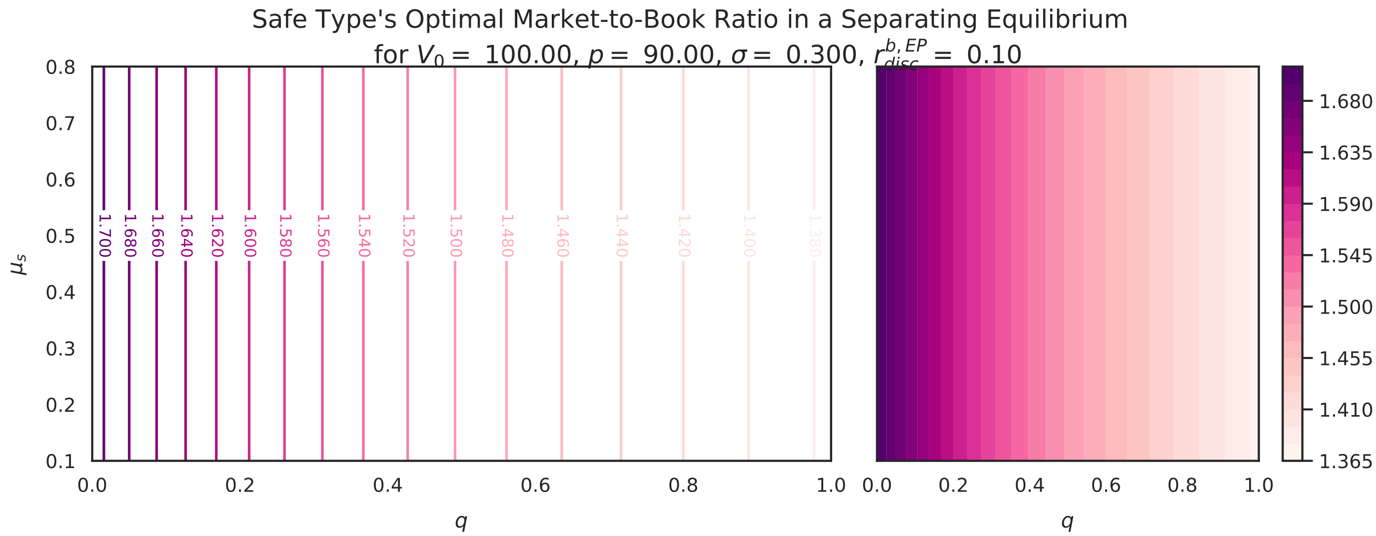
In a separating equilibrium, safe firms maximize their value subject to the risky-type's incentive compatibility condition (CFC - Separating). The measure of bonds is thus chosen among the subset of measures $\mu_b \in M_B$ that discourage misrepresentation on the part of risky firms. The strict concavity of the safe type's firm value in μ_b then implies that risky firms are left indifferent between playing their separating strategy, $\mu_{b,r}^{FI}$, and misrepresenting themselves:

$$MBR_r \left(\mu_{b,s}^{sep} | r \rightarrow s \right) = MBR_r \left(\mu_{b,r}^{FI} | \gamma \right)$$

Finally, the iso-curves are vertical because the separating measures, $\mu_{b,j}^{sep}$, $j \in \{s, r\}$, depend solely on the characteristics of each type alone, and not on the ratio of safe-to-risky firms.

Note: as explained in the text, a pure separating equilibrium exists if, and only if, $MBR_j \left(\mu_b^{sep} | \gamma \right) \geqslant 1$, for at least one $j \in \{s, r\}$. See figures 11 and 13.

Figure 9. Safe Type's Market-to-Book Ratio of Equity in a Separating Equilibrium in E^{SD}



The figure above shows the safe type's MBR iso-curves in pure separating equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q. The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

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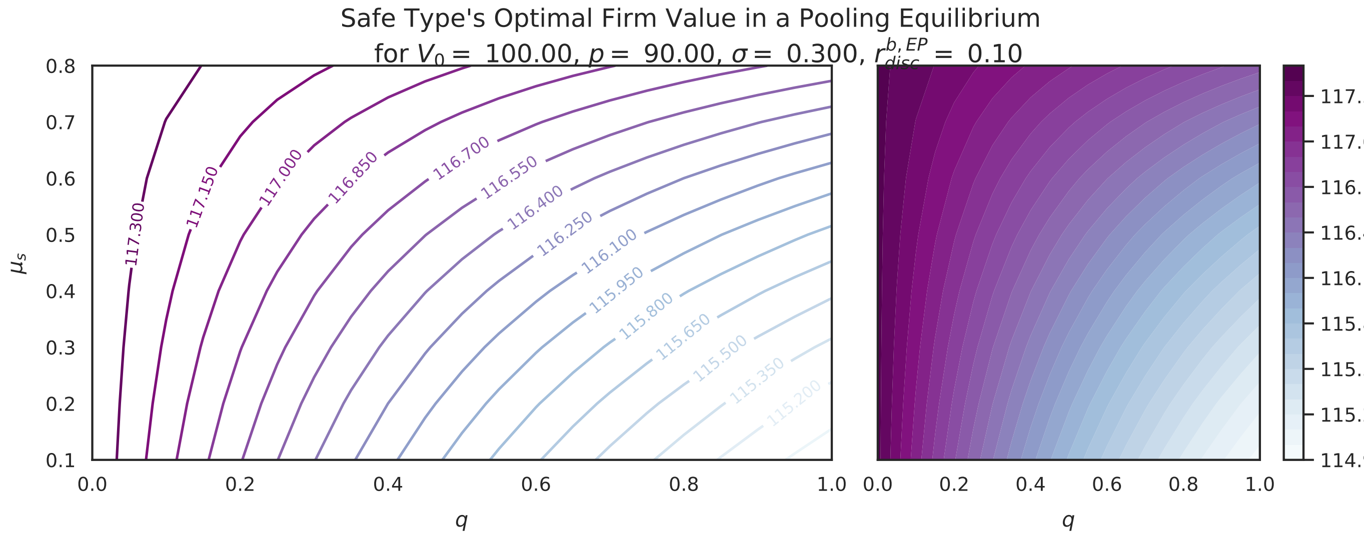
$$MBR_r\left(\mu_{b,s}^{sep}|r \to s\right) = MBR_r\left(\mu_{b,r}^{FI}|\gamma\right)$$

Finally, the iso-curves are vertical because the separating measures, $\mu_{b,j}^{sep}$, $j \in \{s, r\}$, depend solely on the characteristics of each type alone, and not on the ratio of safe-to-risky firms.

Note: as explained in the text, a pure separating equilibrium exists if, and only if, $MBR_j\left(\mu_b^{sep}|\gamma\right) \geqslant 1$, *for at least one* $j \in \{s, r\}$. *See figures 11 and 13.*

B.1.6 Weak Pooling Equilibrium in E^{SD}

Figure 10. Safe Type's Firm Value in a Pooling Equilibrium in E^{SD}



The figure above shows the safe type's firm value iso-curves in pure pooling equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q.

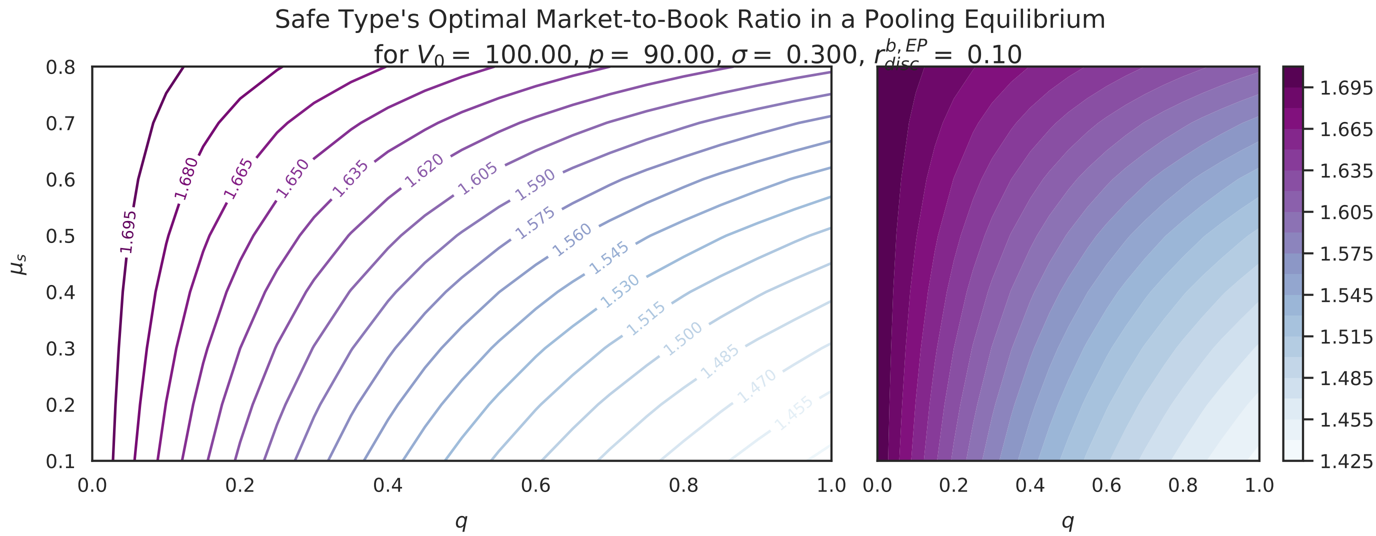
The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

In a pooling equilibrium, both types issue the same measure of bonds, μ_b^{pool}. This measure is that which maximizes the expected value of the firm given creditors' beliefs, $\gamma_s\left(\mu_b^{pool}\right) = \mu_s$. Bond prices are computed as the sum of the type-contingent, full-information bond prices, weighted by creditors' beliefs about the firm type distribution.

Note: as explained in the text, a pure pooling equilibrium exists if, and only if, $MBR_j(\mu_b^{pool}|\gamma) \geqslant 1$, *for* $j \in \{s, r\}$. *See figures 11 and 13.*

Figure 11. Safe Type's Market-to-Book Ratio of Equity in a Pooling Equilibrium in E^{SD}



The figure above shows the safe type's MBR in pure pooling equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q.

The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

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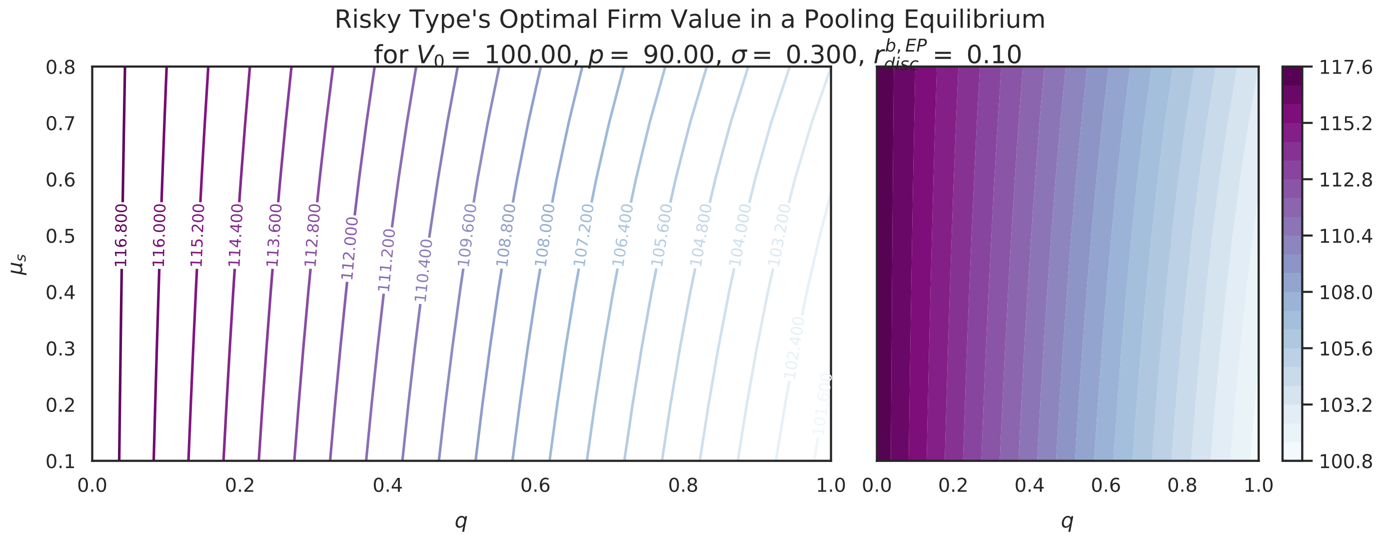
Note: as explained in the text, a pure pooling equilibrium exists if, and only if, $MBR_j(\mu_b^{pool}|\gamma) \geqslant 1$, for $j \in \{s, r\}$. See also figure 13.

Figure 12. Risky Type's Firm Value in a Pooling Equilibrium in E^{SD}



The figure above shows the risky type's firm value iso-curves in pure pooling equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q.

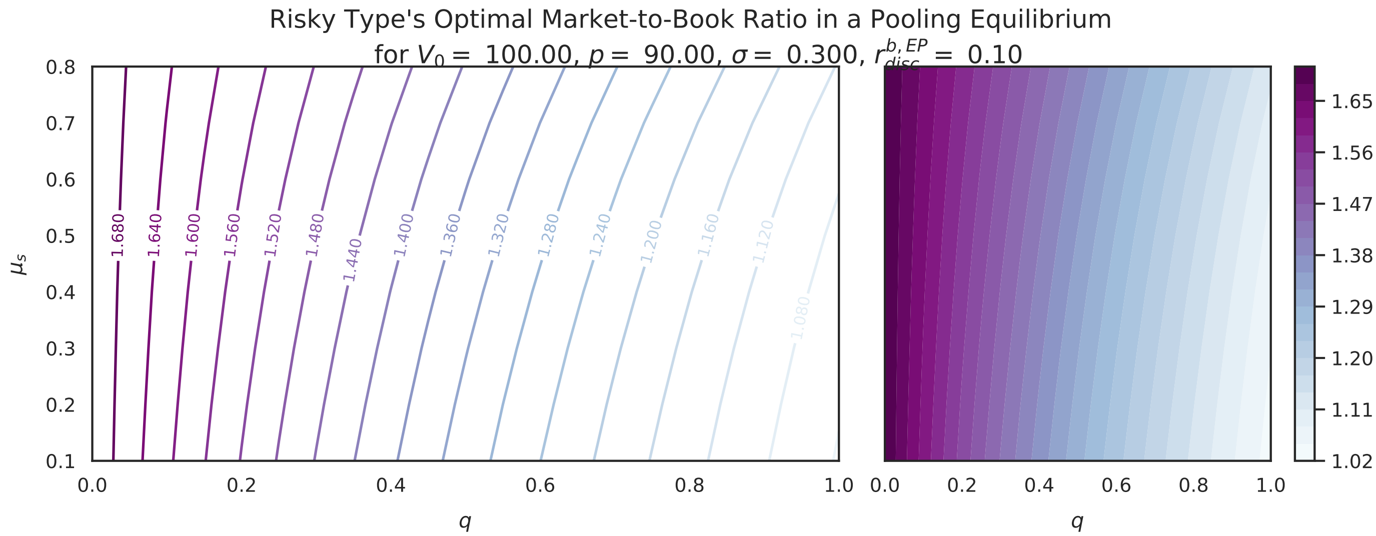
The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

In a pooling equilibrium, both types issue the same measure of bonds, μ_b^{pool}. This measure is that which maximizes the expected value of the firm given creditors' beliefs, $\gamma_s\left(\mu_b^{pool}\right) = \mu_s$. Bond prices are computed as the sum of the type-contingent, full-information bond prices, weighted by creditors' beliefs about the firm type distribution.

Note: as explained in the text, a pure pooling equilibrium exists if, and only if, $MBR_j(\mu_b^{pool}|\gamma) \geqslant 1$, *for* $j \in \{s, r\}$. *See figures 11 and 13.*

Figure 13. Risky Type's Market-to-Book Ratio of Equity in a Pooling Equilibrium in E^{SD}



The figure above shows the risky type's MBR in pure pooling equilibria in E^{SD} for varying type-distribution probabilities, μ_s, and risky-type's exposure to the mean-reducing shock values, q.

The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$.

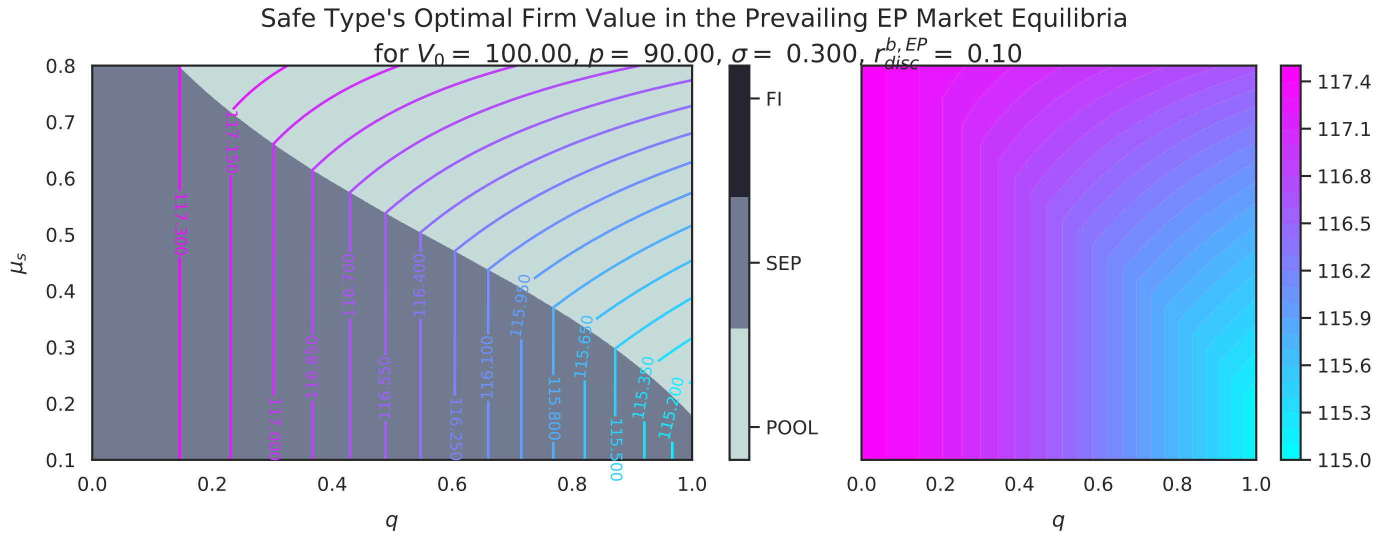
In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

In a pooling equilibrium, both types issue the same measure of bonds, μ_b^{pool}. This measure is that which maximizes the expected value of the firm given creditors' beliefs, $\gamma_s\left(\mu_b^{pool}\right) = \mu_s$. Bond prices are computed as the sum of the type-contingent, full-information bond prices, weighted by creditors' beliefs about the firm type distribution.

Note: as explained in the text, a pure pooling equilibrium exists if, and only if, $MBR_j(\mu_b^{pool}|\gamma) \geqslant 1$, for $j \in \{s, r\}$. See also figure 11.

B.1.7 Equilibria in E^{SD}

Figure 14. Safe Type's Firm Value in Equilibrium in the restricted economy E^{SD}

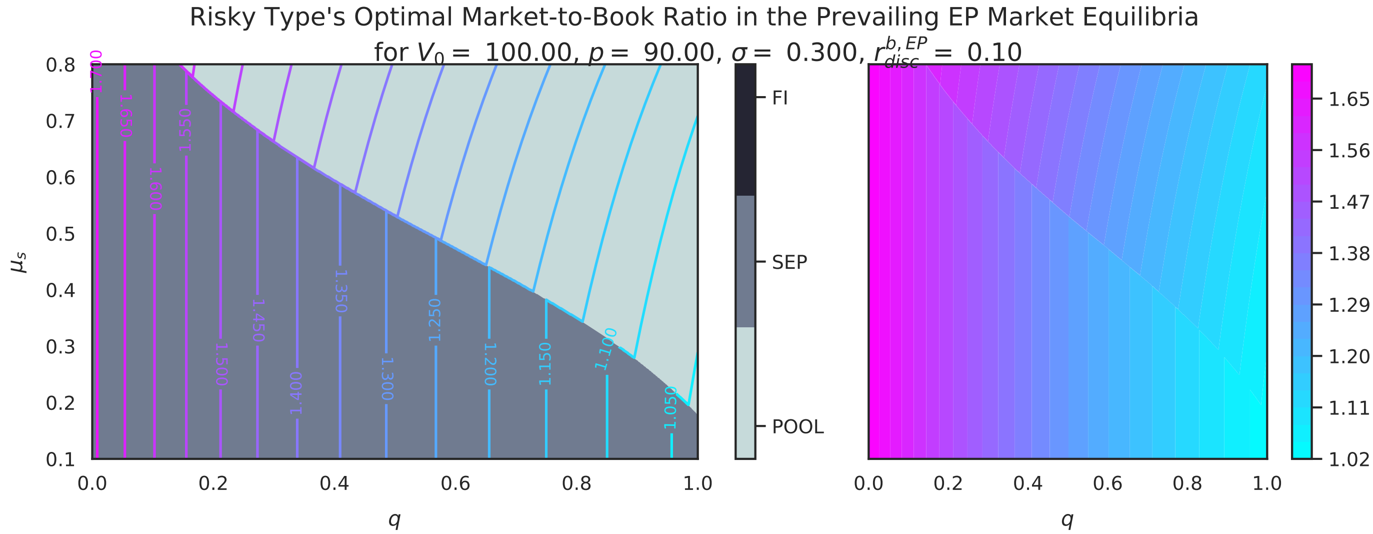


The figure above shows the safe type's firm value in equilibrium in E^{SD}. The light gray area on the LHS plot indicates the pairs of type distribution μ_s and risky firms' exposure to the mean reducing shock q for which a pooling equilibrium would prevail. Below and to the left of this area, a separating equilibrium holds. Since all risky types benefit from misrepresentation, the first best, full-information (FI) equilibrium is never attained under asymmetric information.

The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$. In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

In a pure pooling (weak) equilibrium, the safe type chooses the amount of outstanding bonds that maximizes its initial firm value, conditional on risky-types opting to pool together by issuing the same amount of debt. Bond prices are computed as a sum of the type-contingent bond prices, weighted by each types' probability. In a separating market outcome, the safe type issues the measure of bonds that maximizes its firm value, conditional on risky firms not benefiting from misrepresentation. By the creditors' funding condition, risky firms must choose their first best capital structure, $\mu_{b,r}^{FI}$. Creditors enforce the (weak) equilibrium which yields maximizes the safe-type's firm valuation. When the safe-type valuations coincide (at the boundary between the pooling and separating regions on the LHS picture), the equilibrium that prevails is that which yields the highest MBR to safe type shareholders.

Figure 15. Risky Type's Market-to-Book Ratio of Equity in Equilibrium in the restricted economy E^{SD}



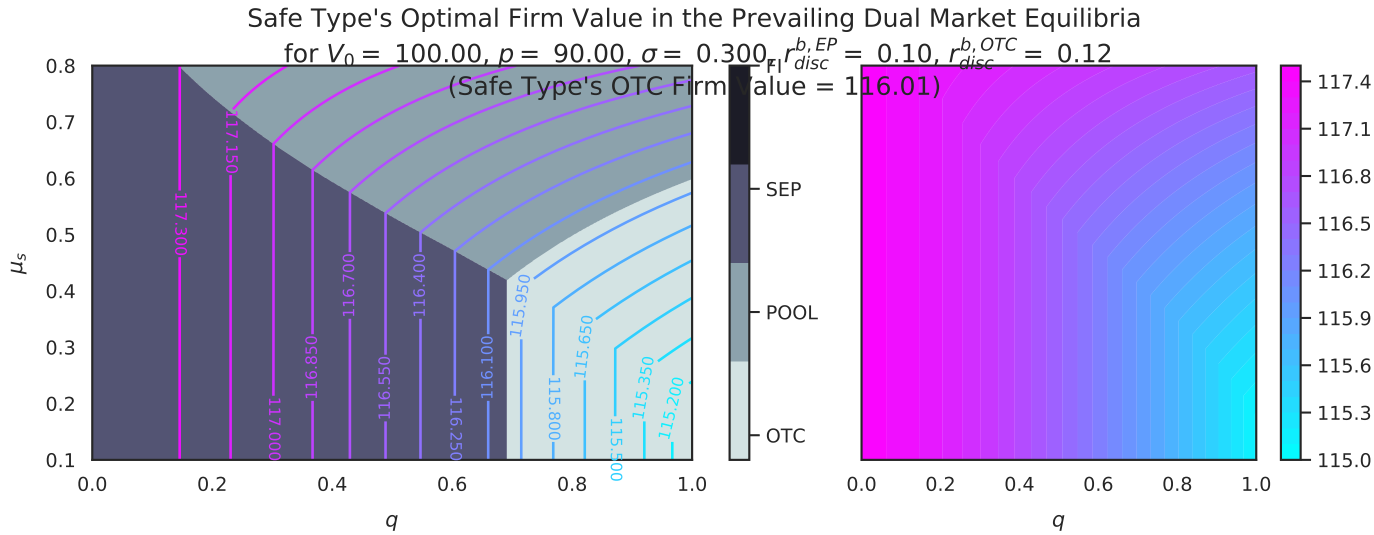
The figure above shows the risky type's MBR in equilibrium in E^{SD}. The light gray area on the LHS plot indicates the pairs of type distribution μ_s and risky firms' exposure to the mean reducing shock q for which a pooling equilibrium would prevail. Below and to the left of this area, a separating equilibrium holds. Since all risky types benefit from misrepresentation, the first best, full-information (FI) equilibrium is never attained under asymmetric information.

The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. All firms issue the same couponless bond with principal $p = 90.0$. In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors.

In a pure pooling (weak) equilibrium, the safe type chooses the amount of outstanding bonds that maximizes its initial firm value, conditional on risky-types opting to pool together by issuing the same amount of debt. Bond prices are computed as a sum of the type-contingent bond prices, weighted by each types' probability. In a separating market outcome, the safe type issues the measure of bonds that maximizes its firm value, conditional on risky firms not benefiting from misrepresentation. By the creditors' funding condition, risky firms must choose their first best capital structure, $\mu_{b,r}^{FI}$. Creditors enforce the (weak) equilibrium which yields maximizes the safe-type's firm valuation. When the safe-type valuations coincide (at the boundary between the pooling and separating regions on the LHS picture), the equilibrium that prevails is that which yields the highest MBR to safe type shareholders.

B.1.8 Equilibria in the Full Economy E

Figure 16. Safe Type's Firm Value in Equilibrium in the restricted economy E^{SD}



The figure above shows the safe type's initial valuation when firms have the option to issue the standardized bond to be traded in EP, or add a debt protective covenant that fully reveals their type, but restricts secondary trades to the less liquid OTC market. Safe firms issue standardized debt whenever the cross-market liquidity differential exceeds the informational costs of adverse selection in electronic exchanges, as defined in section II.H.

The setting is the same as in figure 3. The bond investors discount rate is set to 0.10, 200 b.p. in excess of the risk-free rate, $r_f = .08$. In computing these payoffs, only two types of firms are considered at a time: the safe type and a risky type, for which $q > 0$. Each firm's exposure to the mean-reducing shock is observable by its shareholders alone. However, creditors do know the time-invariant distribution of types, which is fully characterized by the measure of safe firms, μ_s. Because all firms start with low volatility σ, they are ex-ante identical to investors. As before, the abbreviations POOL, SEP and FI stand for pooling, separating and full-information equilibria in E^{SD}. The light gray area (OTC) on the LHS plot indicates the pairs of type-distribution and risky-type shock probability parameters, μ_s and q, respectively, that would prompt safe firms to issue non-standardized bonds, leading to a separating equilibrium in which safe debt is traded over the counter, while risky firms' bonds are transacted in electronic platforms.

In a pure pooling (weak) equilibrium, the safe type chooses the amount of outstanding bonds that maximizes its initial firm value, conditional on risky-types opting to pool together by issuing the same amount of debt. Bond prices are computed as a sum of the type-contingent bond prices, weighted by each types' probability. In a separating market outcome, the safe type issues the measure of bonds that maximizes its firm value, conditional on risky firms not benefiting from misrepresentation. By the creditors' funding condition, risky firms must choose their first best capital structure, $\mu_{b,r}^{FI}$. Creditors enforce the (weak) equilibrium which yields maximizes the safe-type's firm valuation. When the safe-type valuations coincide (at the boundary between the pooling and separating regions on the LHS picture), the equilibrium that prevails is that which yields the highest MBR to safe type shareholders. A dual market equilibrium prevails if safe firms can increase their value by issuing non-standardized bonds with a debt protective covenant that fully revails their type.

Appendix C. The 2-Period Model Derivations

C.1 Auxiliary Results

LEMMA 6: *Let the value of underlying assets at time 1 be $V_1 = V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The price of a claim that pays \$1 in the event of bankruptcy of at time 1 is*

$$p_1^d(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = e^{-r_{disc}} N(f(\mu_b; \mu_x^v, \sigma_x))$$

where $N(\cdot)$ is the cumulative normal distribution function, and

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

Proof. The firm is declared bankrupt whenever the sum of the value of assets and the tax benefits is insufficient to service the debt:

$$V_1 + \mu_b \pi p < \mu_b p$$

Define the standard normal random variable $z \equiv \frac{x-\mu_x^v}{\sigma_x}$. Substituting the expression for V_1 in the formula above and rearranging terms, we obtain

$$z < f(\mu_b; \mu_x^v, \sigma_x)$$

where

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

Let $\mathbf{1}_{\{\cdot\}}$ be the indicator function. From the result above, it follows that the price of a claim that pays \$1 in the event of bankruptcy at time 1 is

$$\begin{aligned} E\left[e^{-r_{disc}} \mathbf{1}_{\{V_1 + \mu_b \pi p < \mu_b p\}}\right] &= e^{-r_{disc}} P(z < f(\mu_b; \mu_x^v, \sigma_x)) \\ &= e^{-r_{disc}} N(f(\mu_b; \mu_x^v, \sigma_x)) \end{aligned}$$

□

COROLLARY 1: *Let the value of underlying assets at time 1 be $V_1 = V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The price of a claim that pays \$1 in the event the firm is solvent at time 1 is*

$$p_1^s(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = e^{-r_{disc}} N(-f(\mu_b; \mu_x^v, \sigma_x))$$

LEMMA 7: *Let $z \sim \mathcal{N}(0,1)$. Then,*

$$E\left[e^{\sigma_x z} \cdot \mathbf{1}_{\{z<w\}}\right] = e^{\frac{1}{2}\sigma_x} N(w - \sigma_x)$$

where $N(\cdot)$ is the cumulative normal distribution function.

Proof. We have

$$\begin{aligned} E\left[e^{\sigma_x z} \cdot \mathbf{1}_{\{z<w\}}\right] &= \int_{-\infty}^{w} e^{\sigma_x z} \left\{\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z^2\right)\right\} dz \\ &= e^{\frac{1}{2}\sigma_x} \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(z-\sigma_x)^2\right) dz \end{aligned}$$

Let $\hat{z} \equiv z - \sigma_x$, so that $z \geqslant w$ is equivalent to $\hat{z} \geqslant w - \sigma_x$. Changing variables, the integrand on the RHS becomes the normal distribution probability density function:

$$\begin{aligned} e^{\frac{1}{2}\sigma_x} \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(z-\sigma_x)^2\right) dz &= e^{\frac{1}{2}\sigma_x} \int_{-\infty}^{w-\sigma_x} \left\{ \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\hat{z}^2\right) \right\} d\hat{z} \\ &= e^{\frac{1}{2}\sigma_x} N(w-\sigma_x) \end{aligned}$$

Q.E.D. □

COROLLARY 2: *Let $z \sim \mathcal{N}(0,1)$. Then, $E\left[e^{\sigma_x z} \cdot \mathbf{1}_{\{z \geqslant w\}}\right] = e^{\frac{1}{2}\sigma_x} N(-w+\sigma_x)$.*

LEMMA 8: *Let the value of underlying assets at time 1 be $V_1 = V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The price of a claim that pays \$$V_1$ if the firm is bankrupt at time 1 is*

$$p_V^b(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = e^{-r_{disc}} V_0 e^{\mu_x^v + \frac{1}{2}\sigma_x} N(f(\mu_b; \mu_x^v, \sigma_x) - \sigma_x)$$

Proof. From the derivations in the proof of Lemma 6, the firm is bankrupt at time 1 if:

$$z < f(\mu_b; \mu_x^v, \sigma_x)$$

where $z \equiv \frac{x - \mu_x^v}{\sigma_x}$, and

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

The discounted expected payoff of the claim is thus

$$E\left[e^{-r_{disc}} V_1 \cdot \mathbf{1}_{\{V_1 + \mu_b \pi p < \mu_b p\}}\right] = e^{-r_{disc}} V_0 E\left[e^{\sigma_x z + \mu_x^v} \cdot \mathbf{1}_{\{z < f(\mu_b; \mu_x^v, \sigma_x)\}}\right]$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. A direct application of Corollary 7 gives

$$E\left[e^{-r_{disc}} V_1 \cdot \mathbf{1}_{\{V_1 + \mu_b \pi p \geqslant \mu_b p\}}\right] = e^{-r_{disc}} V_0 e^{\mu_x^v + \frac{1}{2}\sigma_x} N(f(\mu_b; \mu_x^v, \sigma_x) - \sigma_x)$$

□

COROLLARY 3: *Let the value of underlying assets at time 1 be $V_1 = V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The price of a claim that pays \$$V_1$ if the firm is solvent at time 1 is*

$$p_V^s(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = e^{-r_{disc}} V_0 e^{\mu_x^v + \frac{1}{2}\sigma_x} N(-f(\mu_b; \mu_x^v, \sigma_x) + \sigma_x)$$

C.2 Valuation

THEOREM 1: *Let the value of a firm's underlying assets at time 1 be $V_1 = V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The value of a bond issued by this firm is given by*

$$d(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = e^{-r_{disc}} \cdot \left\{ p \cdot N(-f(\mu_b; \mu_x^v, \sigma_x)) + \frac{\alpha V_0}{\mu_b} e^{\mu_x^v + \frac{1}{2}\sigma_x} \cdot N(f(\mu_b; \mu_x^v, \sigma_x) - \sigma_x) \right\}$$

where

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

and $N(\cdot)$ is the cumulative normal distribution function.

Proof. Recall that the value of the underlying assets at time 1 is given by $V_1 = V_0 e^x$, where $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The firm is declared bankrupt whenever the sum of the value of assets and the tax benefits is insufficient to service the debt:

$$V_1 + \mu_b \pi p < \mu_b p$$

The bond expected payoff is

$$\begin{aligned} d(\mu_b; r_{disc}, \mu_x^v, \sigma_x) &= e^{-r_{disc}} E\left[p \cdot \mathbf{1}_{\{V_1+\pi\mu_b p \geqslant \mu_b p\}} + \frac{\alpha V_1}{\mu_b} \cdot \mathbf{1}_{\{V_1+\pi\mu_b p<\mu_b p\}}\right] \\ &= e^{-r_{disc}} p \cdot E\left[\mathbf{1}_{\{z \geqslant f(\mu_b;\mu_x^v,\sigma_x)\}}\right] + e^{-r_{disc}} \frac{\alpha}{\mu_b} E\left[V_1 \cdot \mathbf{1}_{\{z<f(\mu_b;\mu_x^v,\sigma_x)\}}\right] \end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. A direct application of Corollary 1 and Lemma 8 gives

$$d(\mu_b; r_{disc}, \mu_x^v, \sigma_x) = p \cdot e^{-r_{disc}} N\left(-f(\mu_b; \mu_x^v, \sigma_x)\right) + \frac{\alpha}{\mu_b} e^{-r_{disc}} V_0 e^{\mu_x^v + \frac{1}{2}\sigma_x} N\left(f(\mu_b; \mu_x^v, \sigma_x) - \sigma_x\right)$$

□

COROLLARY 4: *Let the measure of outstanding bonds be μ_b. The value of a bond issued by a safe firm is*

$$d\left(\mu_b, r_{disc}^b | s\right) = d\left(\mu_b; r_{disc}^b, r_f - \frac{1}{2}\sigma^2, \sigma\right)$$

and pre-shock value of a bond issued by a risky firm is

$$d\left(\mu_b, r_{disc}^b | r\right) = (1-q)\, d\left(\mu_b; r_{disc}^b, r_f - \frac{1}{2}\sigma^2, \sigma\right) + q d\left(\mu_b; r_{disc}^b, r_f - \frac{1}{2}\sigma^2 - s_f\sigma, \sigma\right)$$

where the pricing function $d(\cdot; r_{disc}, \mu_x^v, \sigma_x)$ is defined in Theorem 1.

THEOREM 2: *Let the equity investors' rate of discount be r_{disc}^e and the measure of outstanding bonds be μ_b. Suppose the value of assets at time 1, V_1, is given by $V_0 e^x$, for $x \sim \mathcal{N}(\mu_x^v, \sigma_x)$. The price of equity is given by*

$$E\left(\mu_b; r_{disc}^e, \mu_x^v, \sigma_x\right) = e^{-r_{disc}^e}\left\{e^{\mu_x^v + \frac{1}{2}\sigma_x} V_0 \cdot N\left(-f(\mu_b; \mu_x^v, \sigma_x) + \sigma_x\right) - (1-\pi)\mu_b p \cdot N\left(-f(\mu_b; \mu_x^v, \sigma_x)\right)\right\}$$

where $N(\cdot)$ is the cumulative normal distribution function, and

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

Proof. Shareholders are paid only if the firm is solvent at time 1, in which case they receive the value of the underlying assets and tax benefits in excess of the aggregate principal, that is, $V_1 - (1-\pi)\mu_b p$. Equity in this model is thus akin to a call option with strike price $(1-\pi)\mu_b p$. From the derivations in the proof of Lemma 6, the firm is bankrupt at time 1 if:

$$z < f(\mu_b; \mu_x^v, \sigma_x)$$

where $z \equiv \frac{x - \mu_x^v}{\sigma_x}$, and

$$f(\mu_b; \mu_x^v, \sigma_x) \equiv \frac{1}{\sigma_x} \log\left(\frac{(1-\pi)\mu_b p}{V_0 e^{\mu_x^v}}\right)$$

$$\begin{aligned} E\left(\mu_b; r^e_{disc}, \mu^v_x, \sigma_x\right) &= e^{-r^e_{disc}} E\left[\left(V_1 - (1-\pi)\,\mu_b p\right)\cdot \mathbf{1}_{\{z \geqslant f(\mu_b;\mu^v_x,\sigma_x)\}}\right] \\ &= e^{-r^e_{disc}} E\left[V_1 \cdot \mathbf{1}_{\{z \geqslant f(\mu_b;\mu^v_x,\sigma_x)\}}\right] - (1-\pi)\,\mu_b p \cdot e^{-r^e_{disc}} E\left[\mathbf{1}_{\{z \geqslant f(\mu_b;\mu^v_x,\sigma_x)\}}\right] \end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. A direct application of Corollaries 1 and 3 gives

$$E\left(\mu_b; r^e_{disc}, \mu^v_x, \sigma_x\right) = e^{-r^e_{disc}} V_0 e^{\mu^v_x + \frac{1}{2}\sigma_x} N\left(-f\left(\mu_b;\mu^v_x,\sigma_x\right)+\sigma_x\right) - (1-\pi)\,\mu_b p \cdot e^{-r^e_{disc}} N\left(-f\left(\mu_b;\mu^v_x,\sigma_x\right)\right)$$

$\square$

COROLLARY 5: *Let the measure of outstanding bonds be μ_b. The safe firm's value of equity is*

$$E\left(\mu_b, r_f | s\right) = E\left(\mu_b; r_f, r_f - \frac{1}{2}\sigma^2, \sigma\right)$$

and the value of equity of a risky firm prior to the arrival of the shock is

$$E\left(\mu_b, r_f | r\right) = (1-q)\, E\left(\mu_b; r_f, r_f - \frac{1}{2}\sigma^2, \sigma\right) + qE\left(\mu_b; r_f, r_f - \frac{1}{2}\sigma^2 - s_f\sigma, \sigma\right)$$

when $r^e_{disc} = r_f$, and where the pricing function $E\left(\cdot; r^e_{disc}, \mu^v_x, \sigma_x\right)$ is defined in Theorem 2.

C.3 Decomposing the Cost of Non-Standardized Debt

Let the distribution of the value of assets at time 1, V_1, be given by

$$V_1 = V_0 e^x, \quad x \sim \mathcal{N}\left(\mu^v_r, \sigma\right) \text{ with probability } q,$$

I use a $\sim$ superscript to denote the prices and payoffs when firms issue non-standardized debt. The expected payoff of a non-standardized debt in the absence of a mean reducing shock is:

$$\tilde{d}\left(\theta, \mu_b, r^b_{disc} | \text{no shock}\right) = d\left(\mu_b; r^b_{disc}, \mu^v_s, \sigma_s\right)$$

where $\mu^v_s = r_f - \frac{1}{2}\sigma^2$ and $\sigma_s = \sigma$.

Conditional on the occurrence of a mean-reducing shock, the expected payoff of the non-standardized bond is given by:

$$\begin{aligned} \tilde{d}\left(\theta, \mu_b, r^b_{disc} | \text{shock}\right) &= e^{-r^b_{disc}} E\left[\left(p + \frac{\theta\left(V_1 - (1-\pi)\,\mu_b p\right)}{\mu_b}\right)\cdot \mathbf{1}_{\{V_1 + \pi\mu_b p \geqslant \mu_b p\}} + \frac{\alpha V_1}{\mu_b}\cdot \mathbf{1}_{\{V_1 + \pi\mu_b p < \mu_b p\}} \middle| \text{shock}\right] \\ &= \frac{\theta}{\mu_b} e^{-\left(r^b_{disc} - r^e_{disc}\right)} \overbrace{e^{-r^e_{disc}} E\left[\left(V_1 - (1-\pi)\,\mu_b p\right)\cdot \mathbf{1}_{\{V_1 + \pi\mu_b p \geqslant \mu_b p\}} \middle| \text{shock}\right]}^{=E\left(\mu_b; r^e_{disc}, \mu^v_r, \sigma_r\right)} \\ &\quad + \underbrace{e^{-r^b_{disc}} E\left[p \cdot \mathbf{1}_{\{V_1 + \pi\mu_b p \geqslant \mu_b p\}} + \frac{\alpha V_1}{\mu_b}\cdot \mathbf{1}_{\{V_1 + \pi\mu_b p < \mu_b p\}} \middle| \text{shock}\right]}_{=d\left(\mu_b;\mu^v_r,\sigma_r\right)} \\ &\Rightarrow \end{aligned}$$

$$\tilde{d}\left(\theta, \mu_b, r^b_{disc} | \text{shock}\right) = d\left(\mu_b; r^b_{disc}, \mu^v_r, \sigma_r\right) + \frac{\theta}{\mu_b} e^{-\left(r^b_{disc} - r^e_{disc}\right)} E\left(\mu_b; r^e_{disc}, \mu^v_r, \sigma_r\right)$$

Thus, the value of a non-standardized bond issued by a risky firm equals:

$$\begin{aligned}
\tilde{d}\left(\theta,\mu_b,r^b_{disc}|r\right) &= (1-q)\cdot\tilde{d}\left(\theta,\mu_b,r^b_{disc}|\text{no-shock}\right)+q\cdot\tilde{d}\left(\theta,\mu_b,r^b_{disc}|\text{shock}\right)\\
&= (1-q)\cdot d\left(\mu_b;r^b_{disc},\mu^v_s,\sigma_s\right)+q\cdot d\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)\\
&\quad +q\frac{\theta}{\mu_b}e^{-\left(r^b_{disc}-r^e_{disc}\right)}E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)\\
&\Rightarrow\\
\tilde{d}\left(\theta,\mu_b,r^b_{disc}|r\right) &= d\left(\mu_b,r^b_{disc}|r\right)+\frac{q\theta}{\mu_b}e^{-\left(r^b_{disc}-r^e_{disc}\right)}E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right) \qquad \text{(C1)}
\end{aligned}$$

The value of equity when risky firms issue bonds with covenant:

$$\begin{aligned}
\tilde{E}\left(\theta,\mu_b,r^e_{disc}|r\right) &= (1-q)\,E\left[\left(V_1-(1-\pi)\,\mu_b p\right)\cdot\mathbf{1}_{\{V_1+\pi\mu_b p\geqslant\mu_b p\}}\middle|\text{no shock}\right]\\
&\quad +qE\left[(1-\theta)\left(V_1-(1-\pi)\,\mu_b p\right)\cdot\mathbf{1}_{\{V_1+\pi\mu_b p\geqslant\mu_b p\}}\middle|\text{shock}\right]\\
&= \left\{(1-q)\,E\left(\mu_b;r^e_{disc},\mu^v_s,\sigma_s\right)+(1-\theta)\,qE\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)\right\}\\
&\Rightarrow\\
\tilde{E}\left(\theta,\mu_b,r^e_{disc}|r\right) &= E\left(\mu_b,r^e_{disc}|r\right)-q\theta E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right) \qquad \text{(C2)}
\end{aligned}$$

where $\mu^v_r=\mu^v_s-s_f\cdot\sigma$, and $\sigma_r=\sigma_s=\sigma$.

By equations C1 and C2 above, the risky-type's firm value when issuing non--standardized debt can be expressed as:

$$\begin{aligned}
\tilde{FV}\left(\theta,\mu_b,r^{b,OTC}_{disc},r^e_{disc}|r\right) &= \tilde{E}\left(\theta,\mu_b,r^e_{disc}|r\right)+\mu_b\tilde{d}\left(\theta,\mu_b,r^{b,OTC}_{disc}|r\right)\\
&= E\left(\mu_b,r^e_{disc}|r\right)-q\theta E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)\\
&\quad +\mu_b d\left(\mu_b,r^{b,OTC}_{disc}|r\right)+q\theta e^{-\left(r^{b,OTC}_{disc}-r^e_{disc}\right)}E\left(\mu_b;r^e_{disc},\mu^v_x,\sigma_x\right)\\
&= E\left(\mu_b,r^e_{disc}|r\right)+\mu_b d\left(\mu_b,r^{b,OTC}_{disc}|r\right)\\
&\quad -q\theta\left(1-e^{-\left(r^{b,OTC}_{disc}-r^e_{disc}\right)}\right)E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)\\
&\Rightarrow\\
\tilde{FV}\left(\theta,\mu_b,r^{b,OTC}_{disc},r^e_{disc}|r\right) &= FV\left(\mu_b,r^{b,OTC}_{disc},r^e_{disc}|r\right)-q\theta\left(1-e^{-\left(r^{b,OTC}_{disc}-r^e_{disc}\right)}\right)E\left(\mu_b;r^e_{disc},\mu^v_r,\sigma_r\right)
\end{aligned}$$

It follows that, for any given capital structure, the firm value differential implied by the issuance of non-standardized bonds is always negative:

$$
\begin{aligned}
\tilde{FV}\left(\theta, \mu_b, r_{disc}^{b,OTC}, r_{disc}^{e}|r\right) - FV\left(\mu_b, r_{disc}^{b,EP}, r_{disc}^{e}|r\right) &= FV\left(\mu_b, r_{disc}^{b,OTC}, r_{disc}^{e}|r\right) - FV\left(\mu_b, r_{disc}^{b,EP}, r_{disc}^{e}|r\right) \\
&\quad - q\theta\left(1 - e^{-\left(r_{disc}^{b,OTC} - r_{disc}^{e}\right)}\right) E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right) \\
&= E\left(\mu_b, r_{disc}^{e}|r\right) + \mu_b d\left(\mu_b, r_{disc}^{b,OTC}|r\right) \\
&\quad - \left\{E\left(\mu_b, r_{disc}^{e}|r\right) + \mu_b d\left(\mu_b, r_{disc}^{b,EP}|r\right)\right\} \\
&\quad - q\theta\left(1 - e^{-\left(r_{disc}^{b,OTC} - r_{disc}^{e}\right)}\right) E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right) \\
&\Rightarrow
\end{aligned}
$$

$$
\begin{aligned}
\tilde{FV}\left(\theta, \mu_b, r_{disc}^{b,OTC}, r_{disc}^{e}|r\right) - FV\left(\mu_b, r_{disc}^{b,EP}, r_{disc}^{e}|r\right) &= \overbrace{\mu_b\left\{d\left(\mu_b, r_{disc}^{b,OTC}|r\right) - d\left(\mu_b, r_{disc}^{b,EP}|r\right)\right\}}^{(-)} \\
&\quad \underbrace{- q\theta\left(1 - e^{-\left(r_{disc}^{b,OTC} - r_{disc}^{e}\right)}\right) E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right)}_{(+)} < 0
\end{aligned}
$$

Let $\Delta FV \equiv \tilde{FV}\left(\theta, \mu_b, r_{disc}^{b,OTC}, r_{disc}^{e}|r\right) - FV\left(\mu_b, r_{disc}^{b,EP}, r_{disc}^{e}|r\right)$. Approximating $e^{-\left(r_{disc}^{b,OTC} - r_{disc}^{e}\right)}$ by $1 - \left(r_{disc}^{b,OTC} - r_{disc}^{e}\right)$, we obtain

$$
\begin{aligned}
\Delta FV \approx\ & \mu_b\left\{d\left(\mu_b, r_{disc}^{b,OTC}|r\right) - d\left(\mu_b, r_{disc}^{b,EP}|r\right)\right\} \\
& - q\theta\left[\left(r_{disc}^{b,OTC} - r_{disc}^{b,EP}\right) + \left(r_{disc}^{b,EP} - r_{disc}^{e}\right)\right] E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right)
\end{aligned}
$$

From the formula above, we can decompose the firm value differential into two loss terms. The *Equity Loss* component captures the cost of the equity transfer mechanism arising from the difference in the rates at which the two classes of investors discount the firms' cash flows:

$$
EL\left(\mu_b, r_{disc}^{e}, r_{disc}^{b,EP}|r\right) = -q\theta\left(r_{disc}^{b,EP} - r_{disc}^{e}\right) E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right) < 0 \tag{C3}
$$

while the *Liquidity Differential Loss* corresponds to the loss in the firm value stemming from the higher transaction costs in the OTC market:

$$
\begin{aligned}
LDL\left(\mu_b, r_{disc}^{e}, r_{disc}^{b,EP}, r_{disc}^{b,OTC}|r\right) =\ & \mu_b\left\{d\left(\mu_b, r_{disc}^{b,OTC}|r\right) - d\left(\mu_b, r_{disc}^{b,EP}|r\right)\right\} \\
& - q\theta\left(r_{disc}^{b,OTC} - r_{disc}^{b,EP}\right) E\left(\mu_b; r_{disc}^{e}, \mu_r^{v}, \sigma_r\right) < 0
\end{aligned} \tag{C4}
$$

Finally, in the case of safe firms, since $q = 0$, the equity loss is null, while the liquidity differential loss is limited to the valuation differential between the debt traded at the two competing trading venues:

$$
LDL\left(\mu_b, r_{disc}^{e}, r_{disc}^{b,EP}, r_{disc}^{b,OTC}|s\right) = \mu_b\left\{d\left(\mu_b, r_{disc}^{b,OTC}|s\right) - d\left(\mu_b, r_{disc}^{b,EP}|s\right)\right\}
$$

C.4 Perfect Signaling Through the Issuance of Non-Standardized Debt

LEMMA 9: *Let $MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{EP}\right)$ be the risky-type's MBR in a full-information equilibrium in E^{SD}, and let $\mu_{b,s}^{OTC}$ be the safe-type's optimal measure of bonds when issuing non-standardized debt* *in a full-information equilibrium. Let $\underline{\theta}$ equal*

$$\underline{\theta} \equiv \frac{E_r\left(\mu_{b,s}^{OTC}|r\right) - MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{EP}\right)\left[V_0 - \mu_{b,s}^{OTC} d\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|s\right)\right]}{qE\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)}$$

Then, if $\underline{\theta} \leqslant 1$, the safe type can preclude risky firms' misrepresentation by issuing non-standardized bonds with $\theta \in [\underline{\theta}, 1]$, effectively signalling their type to bond investors. In the formula above, (i) $E_r\left(\mu_{b,s}^{OTC}|r\right)$ is the risky-type's value of equity when issuing $\mu_{b,s}^{OTC}$, (ii) $d\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|s\right)$ is the safe-type's full-information value of a non-standardized bond, and (iii) $E\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)$ is the value of equity conditional on the occurence of a shock and is given by the function $E(\cdot)$ in Theorem 2.

Proof. Since safe firms are not exposed to the mean-reducing shock, their payoffs when issuing non-standardized debt do not depend on the covenant parameter θ. Let then $\mu_{b,s}^{OTC}$ be the safe-type's first-best measure of bond issuance when creditors' rate of discount is r_{disc}^{b}. I now show that, by optimally choosing the value of θ, safe firms can preclude risky firms' misrepresentation, effectively signalling their type to bond investors. In the absence of any asymmetry of information, a non-standardized bond issued by a safe firm is priced as

$$d\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|s\right) = d\left(\mu_b; r_{disc}^{OTC}, r_f - \frac{1}{2}\sigma^2, \sigma\right)$$

where the function on the RHS is given in equation~\ref{eq:2pm-bond-pricing-function}. Let $MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{FI}\right)$ be the risky-type's market-to-book ratio of equity when issuing its first-best measure of standardized bonds. Likewise, denote by $MBR\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|r \to s\right)$ the MBR of a risky firm that misrepresented itself as safe by issuing non-standardized bonds:

$$MBR\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|r \to s\right) = \frac{E_r\left(\mu_{b,s}^{OTC}|r\right) - q\theta E\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)}{V_0 - \mu_{b,s}^{OTC} d\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|s\right)}$$

Notice now that $MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{EP}\right)$ is a lower-bound on the rate of return to risky-type shareholders, who benefit from a higher payoff whenever they can pool together with safe firms. Perfect signaling is achieved if

$$MBR\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|r \to s\right) < MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{FI}\right) \tag{C5}$$

Therefore, by setting θ greater than

$$\underline{\theta} \equiv \frac{E_r\left(\mu_{b,s}^{OTC}|r\right) - MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{EP}\right)\left[V_0 - \mu_{b,s}^{OTC} d\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|s\right)\right]}{qE\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)}$$

safe firms can ward off any pooling attempts by the risky type. Put differently, when properly tailored, the non-standardized debt contract ensures perfect signaling. □

In particular, when $\theta = 1$, the perfect signaling condition in equation C5 above becomes

$$MBR\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|r \to s\right) = (1-q)\, MBR_s\left(\mu_{b,s}^{OTC}; r_{disc}^{OTC}\right) \leqslant MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{EP}\right)$$

since

$$E\left(\mu_{b,s}^{OTC}|r\right) = (1-q)\,E\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2, \sigma\right) + qE\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)$$
$$= (1-q)\,E\left(\mu_{b,s}^{OTC}|s\right) + qE\left(\mu_{b,s}^{OTC}; r_f, r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right)$$

COROLLARY 6: *When $\underline{\theta} \leqslant 1$, risky firms never issue the non-standardized bond.*

Proof. The result follows from Lemma 9, which ensures $MBR\left(\mu_{b,s}^{OTC}, r_{disc}^{OTC}|r \to s\right) < MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{FI}\right)$, and the fact that $MBR_r\left(\mu_{b,r}^{FI}; r_{disc}^{FI}\right)$ is a lower bound on the risky-type's shareholders' return in E^{SD}. □