Corporate Debt Standardization and The Rise of Electronic Bond Trading (in progress)

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The Model in a Nutshell

- Illiquid secondary bond markets + asymmetric information;
- Electronic (standardized debt only) v.s. OTC (less liquid);
- Equity investors exploit their private information about firms;
- Covenants arise endogenously $\Rightarrow$ signaling mechanism.

Informational v.s. Liquidity Costs

- Private information affects firms' funding costs $\Rightarrow$ Δ leverage;
- For high enough informational costs, safer firms may forego liquidity gains to signal their creditworthiness.

A 2-Period Model

Capital Structure

- The economy lasts for two periods: $t = 0, 1$;
- Risk-neutral investors: *bond investors* and *equity holders*;
- Firm types: *safe* (prob. μ_s) or *risky* (prob. $1 - \mu_s$).
  - After $t = 0$ but before period 1, risky firms experience an idiosyncratic, mean-reducing shock with probability q:

$$V_{1,s} = V_0 e^x, \quad V_{1,r} = \begin{cases} V_0 e^x, & \text{w/ prob } 1 - q \\ V_0 e^y, & \text{w/ prob } q \end{cases}$$

where

$$x \sim \mathcal{N}\left(r_f - \frac{1}{2}\sigma^2, \sigma\right), \quad y \sim \mathcal{N}\left(r_f - \frac{1}{2}\sigma^2 - s_f \cdot \sigma, \sigma\right), \quad s_f > 0$$

A 2-Period Model

Capital Structure

- Financed with a mix of debt and equity, issued at time 0;
- Debt: measure μ_b of coupon-less bonds with principal $p < V_0$;

Tax benefits v.s. bankruptcy costs

- tax shield: $\pi \mu_b p$
- risk of a costly bankruptcy: lost tax shields and fractional recovery value αV_1.

A 2-Period Model

Secondary Bond Markets

- Bonds are traded in illiquid secondary markets: $r^b_{disc} > r_f$ ▷
  - $\downarrow$ value of newly-issued bonds in primary markets;
  - $\uparrow$ firms' funding costs.
- Electronic Platforms (EP) v.s. Over-the-Counter (OTC) markets
  - EPs are more liquid: $r^{b,EP}_{disc} < r^{b,OTC}_{disc}$
  - But accept only covenant-free bonds.

A 2-Period Model

Payoffs, Prices & The Optimal Capital Structure

- Bankruptcy condition: $V_1 + \pi \mu_b p < \mu_b p$
- Debt: $D(\mu_b) = e^{-r^b_{disc}} E\left[\mu_b p + (\mu_b p - \alpha V_1)\,\mathbf{1}_{\{V_1 + \pi \mu_b p < \mu_b p\}}\right]$
- Equity: $E(\mu_b) = e^{-r_f} E\left[\max\left\{V_1 + \pi \mu_b p - \mu_b p, 0\right\}\right]$
- Expected Equity Return (ER):

$$E(\mu_b) - \underbrace{(V_0 - D(\mu_b))}_{\text{Cash Infusion}} = \overbrace{(E(\mu_b) + D(\mu_b))}^{\text{Firm Value}} - V_0$$

Optimal Capital Structure

μ_b that maximizes the total initial valuation of the firm.

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Assumption (*Creditors' Information Set*)

Creditor's know the distribution of types and observe V, but not firms' exposure to the mean-reducing shock.

- Firms are ex-ante indistinguishable to debt holders;
- Misrepresentation raises the return to risky-type shareholders':

$$\frac{E_r\left(\mu^\star_{b,s}\right)}{V_0 - D_s\left(\mu^\star_{b,s}\right)} > \frac{E_r\left(\mu^\star_{b,r}\right)}{V_0 - D_r\left(\mu^\star_{b,r}\right)}$$

- But also $\uparrow$ safe firm's funding costs;

$$D^{POOL}\left(\mu_b\right) = \mu_s D_s\left(\mu_b\right) + \left(1 - \mu_s\right) D_r\left(\mu_b\right)$$

Characterizing the Economy

The Dual Market Economy

The economy is fully characterized by $E \equiv [Q, \mu_s, \boldsymbol{r}, \boldsymbol{B}, M_b]$

- Set of types: $Q \equiv \{0, q\}$
- Interest rates: $\boldsymbol{r} \equiv \left(r_f, r_{disc}^{b,EP}, r_{disc}^{b,OTC}\right)$
- Bond contracts: $\boldsymbol{B} \equiv \left\{\boldsymbol{b}^{EP}, \boldsymbol{b}^{OTC}\right\}$
  - $\boldsymbol{b}^{EP} \equiv (m, c, p) = (1, 0, p)$, $\boldsymbol{b}^{OTC} \equiv \left(\boldsymbol{b}^{EP}, \theta\right)$
- Measure of bonds: $M_b \equiv [0, \overline{\mu}_b]$

Focus first on a Restricted Economy: $E^{SD} \equiv \left[Q, \mu_s, \boldsymbol{r}, \boldsymbol{b}^{EP}, M_b\right]$

- Trades happen exclusively in EP, since $r_{disc}^{b,EP} < r_{disc}^{b,OTC}$.

Game Setup

Type-Contingent Strategies, Creditors' Beliefs and Offer Price Function

- Types play mixed strategies $p_j^b : M_b \mapsto [0, 1]$ s.t.
  - $p_j^b(\mu_b) \geqslant 0$ for all $\mu_b \in M_b$, and $\int_{M_b} p_j^b(x)\, dx = 1$.
- Creditors form rational beliefs about firms' types: $\gamma_s(\mu_b)$
- Creditors' offer price function: $d_c(\cdot|\gamma)$

Pooling v.s. Separating Bond Measures

- Separating measure: μ_b s.t. $\gamma_s(\mu_b) \in \{0, 1\}$.
- Pooling measure: μ_b for which $\gamma_s(\mu_b) \in (0, 1)$.

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Types' Incentive Compatibility Condition (IC)

A belief and price functions pair, $(\gamma_s(\cdot), d_c(\cdot|\gamma))$, is robust against misrepresentation iff

$$MBR_j(\mu'_b|\gamma) \leqslant \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma) \quad \forall \mu'_b \in M_b \text{ s.t. } \gamma_i(\mu'_b) = 1$$

where $i, j \in \{s, r\}$, $j \neq i$.

Assumption (Creditors' Funding Condition - CFC)

Any choice of capital structure μ_b must maximize the firm value given creditors' beliefs γ and offer price function $d_c(\cdot|\gamma)$, subject to type's incentive compatibility (IC) conditions.

Game Setup

Creditors' Funding Condition - Cont'd

Optimal pooling measure μ_b^{pool}

$$\max_{\mu_b \in M_b} \left\{ \gamma_s FV_s(\mu_b|\gamma) + (1-\gamma_s) FV_r(\mu_b|\gamma) \right\} \qquad \text{(CFC - Pooling)}$$

where $FV_j(\mu_b|\gamma) = E_j(\mu_b) + \mu_b d_c(\mu_b|\gamma)$.

Optimal separating measure $\mu_{b,i}^{sep}$

$$\max_{\mu_b \in M_b \cup \emptyset} FV_i\left(\mu_{b,i}^{sep}|\gamma\right)$$

s.t. (CFC - Separating)

$$MBR_j\left(\mu_{b,i}^{sep}|\gamma\right) \leqslant \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma) \qquad \text{(IC)}$$

for $j \neq i$.

Weak Equilibrium in E^{SD}

A weak equilibrium in E^{SD} is a tuple $e \equiv \left(\left\{p_s^b(\cdot), p_r^b(\cdot)\right\}, \gamma_s(\cdot), d_c(\cdot|\gamma)\right)$ satisfying:

1. [Funding] μ_b satisfies CFC, $\forall \mu_b \in M_b$ s.t. $p_i^b(\mu_b) > 0$, $i \in \{s, r\}$;
2. [Shareholders' optimality] For each μ_b such that $p_j^b(\mu_b) > 0$,
$$MBR_j(\mu_b|\gamma) = \max_{\mu_b \in M_b \cup \emptyset} MBR_j(\mu_b|\gamma), \quad j \in \{s, r\}$$
3. [Creditors' zero-profit condition]
   3.1 For $\mu_b \in M_b$ s.t. $p_i^b(\mu_b) > 0$, $i \in \{s, r\}$,
   $$d_c(\mu_b|\gamma) = \gamma_s(\mu_b) d_s(\mu_b) + (1 - \gamma_s(\mu_b)) d_r(\mu_b)$$
   3.2 Creditors' beliefs are rational:
   $$\gamma_s(\mu_b) = \frac{\mu_s p_s^b(\mu_b)}{\mu_s p_s^b(\mu_b) + (1 - \mu_s) p_r^b(\mu_b)}$$
   for all μ_b s.t. $p_j(\mu_b) > 0$, some $j \in \{s, r\}$.

Characterizing the Weak Equilibria in E^{SD}

- Lemma. 1: No type chooses more than one separating measure with strictly positive probability in equilibrium. ▷
- Lemma. 2: The only separating measure a risky firm can choose in equilibrium is the risky-type's first-best measure $\mu_{b,r}^{FI}$. ▷
- Lemma. 3: The safe-type's separating measure in equilibrium does not depend on the measure of safe types, μ_s. ▷

Characterizing the Weak Equilibria in E^{SD}

- Lemma. 4: No market equilibrium in E^{SD} can support more than one pooling measure. ▷
- Lemma. 5: There cannot be a market equilibrium in E^{SD} where risky firms choose a pooling measure with probability $p_r^b \in (0, 1)$ ▷

Types of Weak Equilibria

- Pure Separating ▷
- Pure Pooling ▷
- Mixed ▷
  - $MBR_s\left(\mu_b^{pool}|\gamma\right) = MBR_s\left(\mu_{b,s}^{sep}|\gamma\right)$
  - $\mu_{b,s}^{sep}$ does not depend on μ_s (Lemma 3)
  - back out μ_b^{pool} from $MBR_s\left(\mu_{b,s}^{sep}|\gamma\right) \Rightarrow \gamma_s^{pool}$ (CFC)
  - solve for $p_s^b\left(\mu_b^{pool}\right) \in (0,1)$ that solves:

$$\gamma_s^{pool} = \frac{p_s^b\left(\mu_b^{pool}\right)\mu_s}{p_s^b\left(\mu_b^{pool}\right)\mu_s + (1-\mu_s)}$$

Mixed equilibrium shouldn't hold: $\uparrow p_s^{pool} \Rightarrow \uparrow MBR_s^{pool}$

Equilibria in E^{SD}

Creditors' Preferences

Given the pooling and separating weak equilibria in E^{SD}, bond investors prefer that which yields the highest safe firm valuation.

Equilibrium in E^{SD}

An equilibrium in E^{SD} is a weak equilibrium tuple $e^{\star}$ such that no other weak equilibrium $\tilde{e}^{\star}$ yields a higher total firm value for safe firms.

When the safe type's firm valuations coincide in the pooling and separating weak equilibria, the prevailing equilibrium is that which maximizes the safe-type's MBR. ▷

Equilibria in the Dual Market Economy E

The equilibrium in the dual-market economy E can then be backed-out by comparing the safe-type's firm valuation in the restricted economy E^{SD} to $FV_s^{OTC}\left(\mu_{b,s}^{OTC}\right)$.

Mixed equilibrium where safe firms randomize between standardized and non-standardized debt shouldn't hold:

- $MBR_s\left(\mu_{b,s}^{EP}\right) = MBR_s\left(\mu_{b,s}^{OTC}\right)$;
- $\mu_{b,s}^{OTC}$ does not depend on μ_s;
- $FV_s\left(\mu_{b,s}^{EP}\right)$ likely different than $FV_s\left(\mu_{b,s}^{OTC}\right)$:
  - $\Rightarrow$ creditors not indifferent between $\mu_{b,s}^{EP}$ and $\mu_{b,s}^{OTC}$;
- Moreover, MBR_s^{EP} is weakly increasing in the measure of safe firms in issuing standardized debt, μ_s^{EP}.

Data Analysis

- Merge TRACE and Mergent FISD;
- Restrict sample to U.S. Corporate Bonds w/ covenant data available;
- Use ATS indicator as proxy for electronic trades;
- Group covenant variables by Billett, King, and Mauer (2007) covenant categories;
- Use number of covenant categories as proxy for bond contract complexity;
- Compare number of covenant categories by secondary market and credit rating.

Statistics

Table: Weighted Trade Count and Volume Statistics by Credit Rating

IG		ATS	OTCS	Δ
Trade Count	mean	5.53801	5.43625	0.101759
	std	2.2011	2.19756	0.00354002
Trade Volume	mean	5.25326	5.35874	-0.105477
	std	2.15628	2.18164	-0.0253664

HY		ATS	OTCS	Δ
Trade Count	mean	7.35734	7.0484	0.308941
	std	2.06867	2.27364	-0.204966
Trade Volume	mean	6.67689	6.70326	-0.0263694
	std	2.19783	2.48021	-0.282371

- Trade Count Weights ▷
- Trade Volume Weights ▷

Bond Investor's Discount Rate

- Creditors are subject to i.i.d. liquidity shocks before time 1;
- Shocks force portfolio liquidation at a fractional cost in secondary markets.

Equity Return and Misrepresentation

- Shareholders' investment: $W_0 < V_0$
- Measure of shareholders: $\nu(\mu_b)$

$$\nu(\mu_b) W_0 = \underbrace{V_0 - D(\mu_b)}_{\text{Book Value of Equity}}$$

- Individual shareholder's return:

$$\begin{aligned} \frac{E(\mu_b)}{\nu(\mu_b)} - W_0 &= \left(\frac{E(\mu_b)}{\nu(\mu_b) W_0} - 1\right) W_0 \\ &= \left(\frac{E(\mu_b)}{V_0 - D(\mu_b)} - 1\right) W_0 \\ &= (MBR(\mu_b) - 1) W_0 \end{aligned}$$

Proof. of Lemma 1

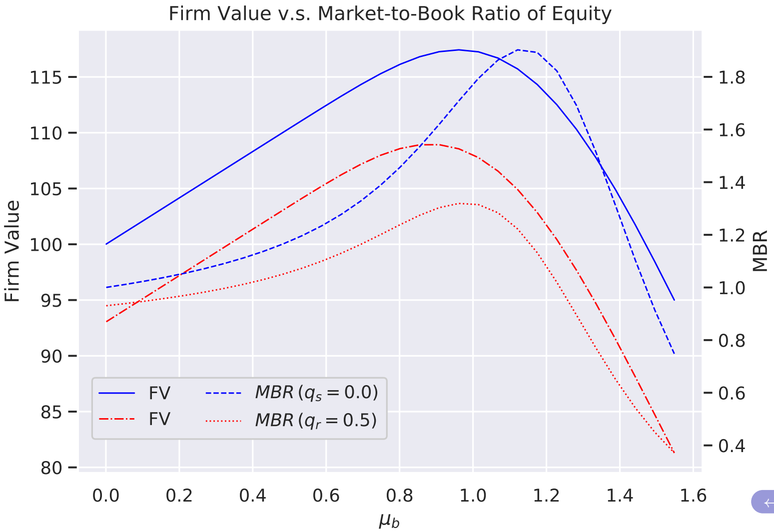
Suppose type-i firms play μ_b', μ_b'' with strictly positive probability in equilibrium, for some $i \in \{s, r\}$.

- Consistency of creditors' beliefs with investors' strategies requires that $\gamma_i(\mu_b') = \gamma_i(\mu_b'') = 1$, so payoffs are given by the full information (FI) formulas.
- By the shareholders' optimality condition, we must have

$$MBR_i(\mu_b'|\gamma) = MBR_i(\mu_b''|\gamma)$$

- However, the strict concavity of the FI firm value function (figure 23) implies that at least one of these measures violates the creditors' funding condition. Contradiction!

Proof. of Lemma 1 - Cont'd



Proof. of Lemma 2

- $\mu_{b,r}^{FI}$ already satisfies the safe-type's IC condition;
- Any separating measure $\mu'_{b,r}$ s.t. $\mu'_{b,r} \neq \mu_{b,r}^{FI}$ violates *CFC*.

Proof. of Lemma 3

By Lemma 2, the IC constraint for the safe-type becomes:

$$MBR_r\left(\mu_{b,s}^{sep}|\gamma_s = 1\right) \leqslant MBR_r\left(\mu_{b,r}^{FI}|\gamma_s = 0\right)$$

which depends solely on the type's characteristics, but not on the ratio of safe-to-risky firms.

Proof. of Lemma 4

Let $\mu_b', \mu_b'' \in M_b$ be pooling measures.

- Shareholders' Optimality:

$$MBR_j\left(\mu_b'|\gamma\right) = MBR_j\left(\mu_b''|\gamma\right)$$

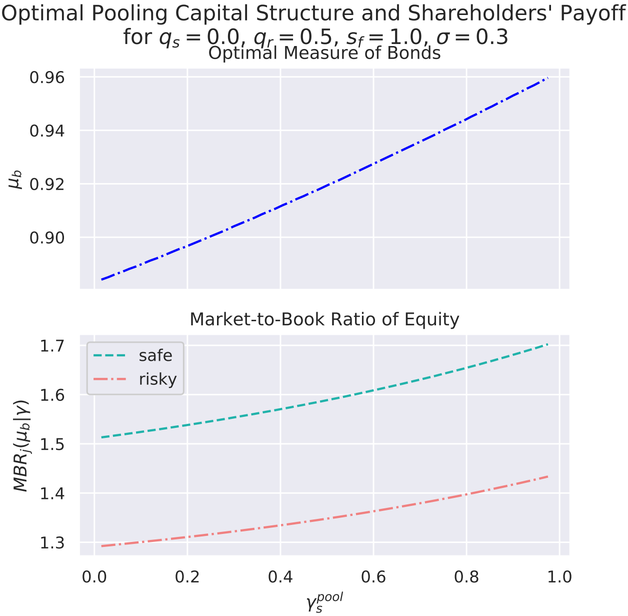
- Creditors' Funding Condition:

$$\max_{\mu_b > 0}\left\{\gamma_s' FV_s\left(\mu_b|\gamma_s'\right) + \left(1-\gamma_s'\right) FV_r\left(\mu_b|\gamma_s'\right)\right\}$$

where

$$\begin{aligned} FV_j\left(\mu_b|\gamma_s'\right) &= E_j\left(\mu_b\right) + \mu_b d\left(\mu_b|\gamma_s'\right) \\ d\left(\mu_b|\gamma_s'\right) &= \gamma_s' d_s\left(\mu_b\right) + \left(1-\gamma_s'\right) d_r\left(\mu_b\right) \end{aligned}$$

Proof. of Lemma 4 - Cont'd



Proof. of Lemma 5

If $p_r\left(\mu_b^{pool}\right) < 1$, then

- By lemma 4 they must risky firms choose a separating measure $\mu_{b,r}^{sep}$ with strictly positive probability;
- By lemma 2, $\mu_{b,r}^{sep} = \mu_b^{FI}$;
- Shareholders' optimality condition requires

$$MBR_r\left(\mu_b^{FI}\right) = MBR_r\left(\mu_b^{pool}|\gamma\right)$$

- However, optimal μ_b is strictly increasing in γ_s, so that $\mu_b^{pool} > \mu_b^{FI}$,
- Therefore, $MBR_r\left(\mu_b^{pool}|\gamma\right) > MBR_r\left(\mu_b^{FI}\right)$ (figure 27 ▷) Contradiction!

Separating Weak Equilibrium

A pure separating weak equilibrium in E^{SD} is a tuple e satisfying:

1. $p_j(\mu_b) = \mathbf{1}_{\{\mu_b = \mu_{b,j}^{sep}\}}$, for $j \in \{s, r\}$, where $\mu_{b,j}^{sep} \in M_b \cup \emptyset$ solves the creditor funding condition problem CFC - Separating.
2. Creditors' beliefs are consistent with type's strategies

$$\gamma_s(\mu_b) = \begin{cases} 1, & \text{if } \mu_b = \mu_{b,s}^{sep} \\ 0, & \text{if } \mu_b = \mu_{b,r}^{sep} \\ \in [0,1) & \text{s.t. } \mu_b \text{ does not solve CFC - Pooling} \end{cases}$$

3. Only optimally levered firms get funded and creditors break even:

$$d_c(\mu_b|\gamma) = \begin{cases} \gamma_s(\mu_b)\, d_s(\mu_b) + (1-\gamma_s(\mu_b))\, d_r(\mu_b) & \text{if } \mu_b \in \{\mu_{b,s}^{sep}, \mu_{b,r}^{sep}\} \\ 0 & \text{otherwise} \end{cases}$$

Pooling Weak Equilibrium

A pure pooling weak equilibrium in E^{SD} is a tuple e satisfying:

1. $p_j(\mu_b) = \mathbf{1}_{\{\mu_b=\mu_b^{pool}\}}$, for $j \in \{s, r\}$, where μ_b^{pool} solves the creditor funding condition problem CFC - Pooling.
2. Creditors' beliefs are consistent with type's strategies, that is, (i) $\gamma_s\left(\mu_b^{pool}\right) = \mu_s$, and (ii) for all $\mu_b \in M_b - \{\mu_b^{pool}\}$,
   - if $\gamma_s(\mu_b) \in \{0, 1\}$, then μ_b does not solve CFC - Separating when $\gamma_s = \gamma_s(\mu_b)$;
   - if $\gamma_s(\mu_b) \in (0, 1)$, then μ_b does not solve CFC - Pooling when $\gamma_s = \gamma_s(\mu_b)$;
3. Only optimally levered firms get funded and creditors break even:

$$d_c(\mu_b|\gamma) = \begin{cases} \gamma_s(\mu_b)\, d_s(\mu_b) + (1-\gamma_s(\mu_b))\, d_r(\mu_b) & \text{if } \mu_b = \mu_b^{pool} \\ 0 & \text{otherwise} \end{cases}$$

Mixed Weak Equilibrium

A mixed weak equilibrium in E^{SD} is a tuple e satisfying

1. Type-contingent strategies: $p_r^b(\mu_b) = \mathbf{1}_{\left\{\mu_b = \mu_b^{pool}\right\}}$ and

$$p_s^b(\mu_b) = p_s^{pool} \cdot \mathbf{1}_{\left\{\mu_b = \mu_b^{pool}\right\}} + \left(1 - p_s^{pool}\right) \cdot \mathbf{1}_{\left\{\mu_b = \mu_{b,s}^{sep}\right\}}$$

   and $MBR_s\left(\mu_b^{pool}|\gamma\right) = MBR_s\left(\mu_{b,s}^{sep}|\gamma\right)$

2. Creditors' beliefs are consistent with types' strategies:

$$\gamma_s(\mu_b) = \begin{cases} \frac{p_s^{pool}\mu_s}{p_s^{pool}\mu_s + (1-\mu_s)} & \text{if } \mu_b = \mu_b^{pool} \\ 1 & \text{if } \mu_b = \mu_b^{sep} \\ \in [0,1] & \mu_b \text{ does not satisfy CFC} \end{cases}$$

3. Only optimally levered firms get funded and creditors break even:

$$d_c(\mu_b|\gamma) = \begin{cases} \gamma_s(\mu_b)\, d_s(\mu_b) + (1 - \gamma_s(\mu_b))\, d_r(\mu_b) & \text{if } \mu_b = \mu_b^{pool} \\ d_s(\mu_b) & \text{if } \mu_b = \mu_{b,s}^{sep} \\ 0 & \text{otherwise} \end{cases}$$

Electronic Market Equilibria

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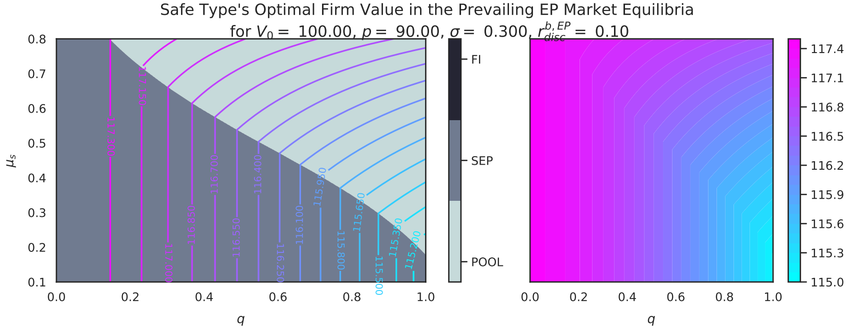
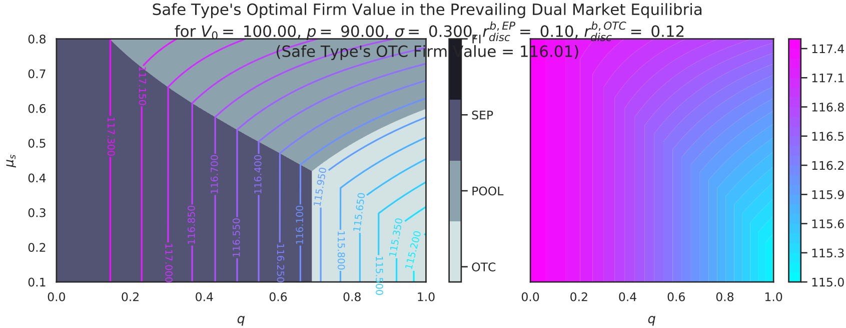
Safe Type's Firm Value

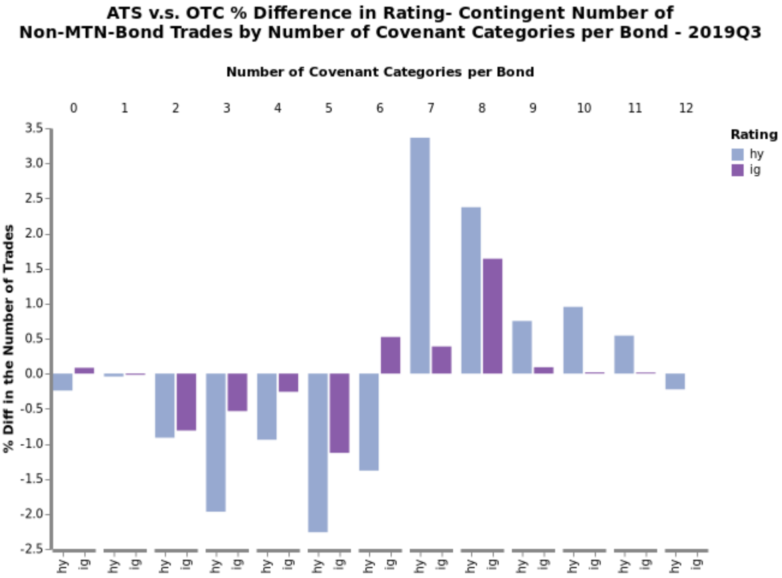


Dual Market Equilibria

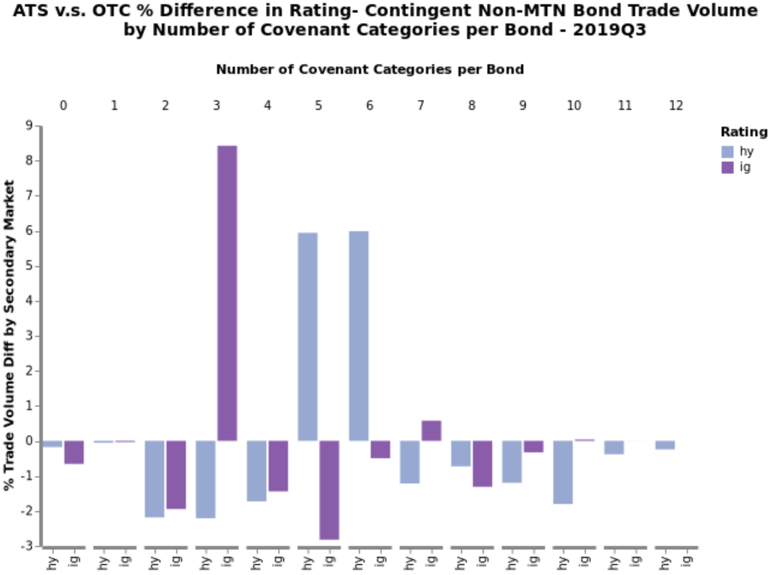
Safe Type's Firm Value



Trade Count Weight Difference



Trade Volume Weight Difference



Bibliography

Billett, Matthew T., Tao-Hsien Dolly King, and David C. Mauer, 2007, Growth Opportunities and the Choice of Leverage, Debt Maturity, and Covenants, *The Journal of Finance* 62, 697–730.