

DM HOMEWORK 3 (12 февраля 2016 г.)*Tropin Andrew**e-mail: andrewtropin@gmail.com**github: [abcdw](#)***Problem 1.**

Write negations for each of the following statements:

A - John is six feet tall

B - John weights at least 200 pounds

 $A \wedge B$ - John is six feet tall and he weighs at least 200 pounds $\sim (A \wedge B) = \sim A \vee \sim B$

John is not six feet tall or he don't weighs at least 200 pounds.

A - The bus was late

B - Tom's watch was slow

 $A \vee B$ - The bus was late or Tom's watch was slow $\sim (A \vee B) = \sim A \wedge \sim B$

The bus was not late and Tom's watch was not slow

Problem 2.

p	q	$\sim p$	$\sim p \vee q$	$\sim q$	$(\sim p \vee q) \rightarrow \sim q$
1	1	0	1	0	0
1	0	0	0	1	1
0	1	1	1	0	0
0	0	1	1	1	1

 $(\sim p \vee q) \rightarrow \sim q$

p	q	r	$\sim p$	$\sim p \wedge q$	$\sim r$	$(\sim p \wedge q) \rightarrow \sim r$
1	1	1	0	0	0	1
1	1	0	0	0	1	1
1	0	1	0	0	0	1
1	0	0	0	0	1	1
0	1	1	1	1	0	0
0	1	0	1	1	1	1
0	0	1	1	0	0	1
0	0	0	1	0	1	1

 $(\sim p \wedge q) \rightarrow \sim r$ **Problem 3.**

p: Grizzly bears have been seen in the area.

q: Hiking is safe on the trail.

r: Berries are ripe along the trail.

- Berries are ripe along the trail, but grizzly bears have not been seen in the area.
 $r \wedge \sim p$
- Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail. $\sim p \wedge r \wedge r$
- If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area. $r \rightarrow (q \leftrightarrow p)$

Problem 4.

- 101 1110, 010 0001
 $1011110 \vee 0100001 = 1111111$
 $1011110 \wedge 0100001 = 0000000$
 $1011110 \oplus 0100001 = 1111111$
- 1111 0000, 1010 1010
 $11110000 \vee 10101010 = 11111010$
 $11110000 \wedge 10101010 = 10100000$
 $11110000 \oplus 10101010 = 01011010$

Problem 5.

Given any statement form, is it possible to find a logically equivalent form that uses only \sim and \wedge ?

Any logical formula can be represented in CNF with \sim , \wedge , \vee operators only. Using De Morgan's law you can remove all \vee operators.

Problem 6.

Disjunction is commutative.

p	q	$p \vee q$	p	q	$q \vee p$
1	1	1	1	1	1
1	0	1	1	0	1
0	1	1	0	1	1
0	0	0	0	0	0

Disjunction is associative.

p	q	r	$p \vee q$	$(p \vee q) \vee r$	p	q	r	$q \vee r$	$p \vee (q \vee r)$
1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	0	1	1
1	0	1	1	1	1	0	1	1	1
1	0	0	1	1	1	0	0	1	1
0	1	1	1	1	0	1	1	1	1
0	1	0	1	1	0	1	0	1	1
0	0	1	0	1	0	0	1	1	1
0	0	0	0	0	0	0	0	0	0

Problem 7.

$\sim(p \wedge q) \neq \sim p \wedge \sim q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
1	1	1	0	1	1	0	0	0
1	0	0	1	1	0	0	1	0
0	1	0	1	0	1	1	0	0
0	0	0	1	0	0	1	1	1

	p	q	$p \wedge q$	$\sim (p \wedge q)$		p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
	1	1	1	0		1	1	0	0	0
$\sim (p \wedge q) = \sim p \vee \sim q$	1	0	0	1		1	0	0	1	1
	0	1	0	1		0	1	1	0	1
	0	0	0	1		0	0	1	1	1

Problem 8.

Implication is logically equivalent to its contrapositive.

p	q	$p \rightarrow q$	p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
1	1	1	1	1	0	0	1
1	0	0	1	0	1	0	0
0	1	1	0	1	0	1	1
0	0	1	0	0	1	1	1

Converse is logically equivalent to the inverse.

p	q	$q \rightarrow p$	p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
1	1	1	1	1	0	0	1
1	0	1	1	0	0	1	1
0	1	0	0	1	1	0	0
0	0	1	0	0	1	1	1