DM HOMEWORK 1 (21 января 2016 г.)

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Problem 1.
Prove or disprove:

1. if $a * b = a$: prove that $b = 1$

$$\begin{aligned} a^{-1} * a * b &= a^{-1} * a \\ 1 * b &= 1;\ 1 * b = b \\ b &= 1 \end{aligned}$$

2. The difference of any two odd integers is even.

$$\begin{aligned} n_1 &= 2 * k + 1 \\ n_2 &= 2 * m + 1 \\ n_1 - n_2 &= 2 * k + 1 - (2 * m + 1) \\ n_1 - n_2 &= 2 * (k - m) \end{aligned}$$

Problem 2.
Prime numbers:

1. Is $n^k - 1$ prime for any integers n and k?
   Nope, 1 isn't integer($n = 2,\ k = 1$).

2. Is expression $n^2 - n + 41$ a prime number?
   Nope, if we take $n = k * 41$ $(k \neq 0)$, that expression won't be equal to 41 and will be divisible at least by 41.

Problem 3.
Divisibility:

1. Prove that sum of $2 * n + 1$ consecutive numbers is divisible by $2 * n + 1$

$$a = k,\ b = k + 2n + 1 - 1$$

$$\sum_{x=a}^{b} x = \frac{(a + b)}{2} * (b - a + 1) = \frac{(k + k + 2n)(2n + 1)}{2} = (k + n)(2n + 1)$$

2. Find quotient and divisor of:

- $n^3 + 2n - 1$ divided by n

$$
\begin{array}{r|l}
n^3 + 2n - 12 & n \\ \cline{2-2}
n^3 \phantom{{}+ 2n - 12} & n^2 + 2 \\ \cline{1-1}
2n - 12 & \\
2n \phantom{{}- 12} & \\ \cline{1-1}
-12 &
\end{array}
$$

divisor: $n^2 + 2$, quotient: -12

- $12n^5 + 10n^4 + 2$ divided by $2n + 1$

$$
\begin{array}{r|l}
12n^5 + 10n^4 \quad + 2 & 2n + 1 \\ \cline{2-2}
12n^5 + 6n^4 \quad\quad & 6n^4 + 2n^3 - n^2 + \frac{1}{2}n - \frac{1}{4} \\ \cline{1-1}
4n^4 \quad + 2 & \\
4n^4 + 2n^3 & \\ \cline{1-1}
-2n^3 \quad + 2 & \\
-2n^3 \quad - n^2 & \\ \cline{1-1}
n^2 \quad + 2 & \\
n^2 \quad + \frac{n}{2} & \\ \cline{1-1}
-\frac{n}{2} \quad + 2 & \\
-\frac{n}{2} \quad - \frac{1}{4} & \\ \cline{1-1}
\frac{9}{4} &
\end{array}
$$

divisor: $6n^4 + 2n^3 - n^2 + \frac{1}{2}n - \frac{1}{4}$, quotient: $\frac{9}{4}$

Problem 4.

Write each rational number as a ratio of two integers:

- $a = 0.4\overline{6271}$

$$
\begin{aligned}
a * 10000 - a &= 4627.1... - 0.4... \\
9999a &= 4626.7 \\
a &= \frac{46267}{99990}
\end{aligned}
$$

- $b = 12.1\overline{121}$

$$
\begin{aligned}
b * 1000 - b &= 12112.1... - 12.1... \\
999b &= 12100 \\
b &= \frac{12100}{999}
\end{aligned}
$$

Problem 5.

1. If r is any rational number, then $3r^2 - 2r + 4$ is rational.

$$
\begin{aligned}
r &= \frac{p}{q},\ p \in \mathbb{Z},\ q \in \mathbb{N} \backslash \{0\} \\
\frac{3p^2}{q^2} - 2\frac{p}{q} + 4 &= \frac{3p^2 - 2pq + 4q^2}{q^2} = \frac{a}{b},\ a \in \mathbb{Z},\ b \in \mathbb{N}
\end{aligned}
$$

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2. Product and sum of two rational numbers is rational.

$$a = \frac{n}{m},\ b = \frac{p}{q},\ n, p \in \mathbb{Z},\ m, q \in \mathbb{N}$$
$$a * b = \frac{nm}{pq},\ nm \in \mathbb{Z},\ pq \in \mathbb{N}$$
$$a + b = \frac{nq + mp}{pq},\ (nq + mp) \in \mathbb{Z},\ pq \in \mathbb{N}$$

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