Mock exam questions for part ii paper 1 a

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Exercise 1

Consider the following normal-form game:

	L	C	R
T	-2,-2	-1,1	0,0
M	1,-1	3,5	3,4
B	0,0	4,2	2,4

A Find all Nash equilibria (including those in mixed-strategies) of the game in the table.

T and L are strictly dominated. There aren't any pure strategy Nash equilibrium. There is a unique mixed strategy Nash equilibrium: $Pr(M) = 2/3$, $Pr(B) = 1/3$, $Pr(C) = 1/2$, $Pr(R) = 1/2$.

B Now suppose that the game in the table is repeated infinitely many times and that players maximize the sum of their discounted payoffs. Find the set V^* of payoff pairs such that, for any (v_1, v_2) in V^*, there exists a discount factor $\delta > 0$ and a subgame perfect equilibrium of the repeated game for which the discounted average payoffs are (v_1, v_2) when players discount at discount factor δ.

1 minmax's 2 with T. 2 minmax's 1 with L. The minmax payoffs are 1 for each player. By Fudenberg and Maskin (1986), V^* is the set of feasible payoffs that strictly dominate (1,1), the minmax point.

C For the repeated game of part (B), sketch subgame-perfect equilibrium strategies that attain a typical point (v_1, v_2) in V^*

Take a one-shot correlated strategy profile (s_1, s_2) with payoffs (v_1, v_2). Phase (A): play s_i each period as long as (s_1, s_2) was played in the previous period. After any deviation from phase (A) go to phase (B): both players minmax for $n(\delta)$ rounds. If there are any deviations while in phase (B), then begin phase (B) again.

(Given that this result was not in the notes, it's fine if you used Friedman (1971,77) and said V^* is the set of feasible payoffs that Pareto dominate the static Nash equilibrium payoffs via a suitably described Nash-reversion strategy.)

Exercise 2

A group of athletes are competing in a multi-day triathlon. They have a running race on day one, a swimming race on day two, and a biking race on day three. You know the order in which the eligible contestants finish each of the three components. From this information, you are asked to rank them in the overall competition. You are given the following conditions:

- α The ordering of athletes should be transitive: If athlete A is ranked above athlete B, and athlete B is ranked above athlete C, then athlete A must rank above athlete C.
- β If athlete A beats athlete B in all three races, athlete A should rank higher than athlete B.
- γ The rank ordering of any two athletes should not depend on whether a third athlete drops out of the competition just before the final ranking.

There are only three ways to rank the athletes that satisfy these properties.

1 What are they? Are these desirable? Why or why not?
Ignore two of the three events. Undesirbale. Not a triathalon.

2 Can you think of a better ranking scheme? Which of the three properties above does your scheme not satisfy?
Maybe sum rank across three events. Satisfies α, β, and not a dictatorship so doesn't satisfy γ.

Exercise 3

This exercise asks you to compare the equilibria of various games with the same payoffs and different information structures. Game A is a game of perfect information: Player 1 moves first, choosing U or D. Player 2 sees player 1's move and chooses L or R. The payoff matrix is

	L	R
U	2,1	0,0
D	4,-1	1,1

a What are the strategy spaces and proper subgames in game A?

Player 1's strategies are U or D. Player 2's strategies are LL,LR,RL, RR. There are two proper subgames, one for each move of 1.

b What is the set of subgame-perfect equilibria outcomes of game A?

Via backward induction, 2,1.

c What are the Nash equilibrium outcomes of game A?

Subgame-perfect equilibrium are Nash equilibrium, so 2,1 is one. For others, draw out the normal form

	LL	LR	RL	RR
U	2,1	2,1	0,0	0,0
D	4,-1	1,1	4,-1	1,1

1,1 is the other.

d Does game A have any mixed strategy equilibria?

Yes, 2 can mix between L and R off the path of play. (By Kuhn's theorem there exists a corresponding mixed strategy.)

In game B, player 1 again moves first, choosing U or D. Player 2 does not observe player 1's move, but sees a noisy public signal of 1's play. There are two possible values of the signal, s' and s'', with $Pr(s'|U) = Pr(s''|D) = 1-\epsilon$. After observing the signal, player 2 then chooses either L or R. The payoff functions are the same as before, and in particular depend only on the actions chosen and not on the signal.

e What are the subgame-perfect equilibrium outcomes of game B?

There are no proper sub-games, so sub-game perfect and Nash equilibrium are the same.

f What are the Nash equilibrium outcomes of game B?

The normal form is given by the 2 by 2 payoff matrix. The unique Nash equilibrium outcome is 1,1.

g What happens as $\epsilon \to 0$?

We recover game A.

Exercise 4 (excluded from exam)

i Consider a two-player non-zero sum game in normal form. Player 1 gets a new strategy that can be played, that is, we add a new row to the payoff matrix. Give an example to show that player 1's equilibrium payoff can decrease as a result of this increased flexibility in strategic choice.

ii Consider a two-player zero sum game. Again assume that Player 1 gets a new strategy that can be played. What can happen to player 1's equilibrium payoff? Can it decrease? Why or why not?