1

In the prisoner's dilemma

$$fink, quiet >_1 fink, fink >_1 quiet, quiet >_1 quiet, fink$$

With a punishment to finking worse than any amount of time in prison

$$quiet, quiet >_1 quiet, fink >_1 fink, quiet >_1 fink, fink$$

Symmetrically for player 2.

The game is

	quiet	fink
quiet	3,3	2,1
fink	1,2	0,0

Alternatively, start with the prisoner's dilemma and punish finking until it becomes strictly dominated:

	quiet	fink
quiet	2,2	0,3-x
fink	3-x,0	1-x,1-x

for some $x > 3$

Nash: quiet,quiet

— common mistake is to call (3,3) the Nash equilibrium—this is the *outcome*. obviously that would be bad terminology as in general the same outcome might be induced by a strategy profile that is not a Nash equilibrium

2

(a)

$$sit, stand >_1 sit, sit >_1 stand, sit >_1 stand, stand$$

	sit	stand
sit	2,2	3,1
stand	1,3	0,0

Nash: sit,sit

(b)

$$stand, sit >_1 sit, sit >_1 sit, stand >_1 stand, stand$$

	sit	stand
sit	2,2	1,3
stand	3,1	0,0

Nash: stand, stand

Both players now less comfortable

— prisoner's dilemma shows how selfish behavior can be Pareto inefficient; here, (in (a)) it is efficient

3

$$aggressive, passive >_1 passive, passive$$

$$>_1 passive, aggressive >_1 aggressive, aggressive$$

	aggressive	passive
aggressive	0,0	3,1
passive	1,3	2,2

Nash: aggressive, passive, and passive, aggressive

4

Straightforward.

— Point is to gain familiarity with best response correspondence and their characterization of Nash equilibrium

— I saw two methods used for computing Nash: drawing arrows to indicate that a player has a profitable deviation, and drawing stars under a player's payoff when they are best responding

— "star" method corresponds to best response description of Nash: all stars, all best responding

5

Straightforward.

— There are no strictly dominated strategies here, but when presented with a large game try to eliminate strictly dominated strategies: obviously, they are never part of a Nash equilibrium

— This problem illustrates that weakly dominated strategies may be played in a Nash equilibrium, so don't flippantly eliminate those when looking for Nash

6

Observe that it is always a Nash equilibrium for both players to put in zero effort. On the other hand, if both players exert effort 1, each gets $\alpha/2 - 1 > 0$, so there always exists (for any $\alpha > 2$) an inefficient Nash equilibrium.

For remainder, try to solve for all Nash:

$$u_i(x_1, x_2) = \frac{\alpha x_1 x_2}{2} - x_i^2$$

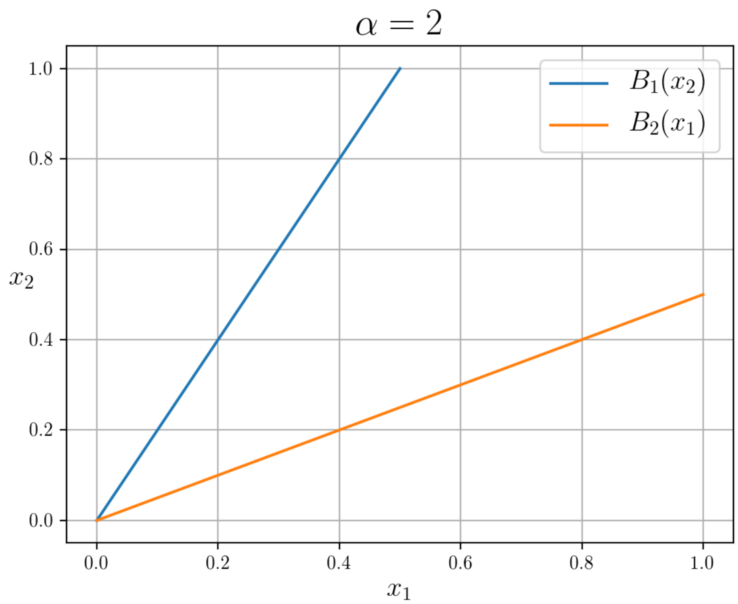
$$\frac{\partial u_1(x_1, x_2)}{\partial x_1} = 0 \implies x_1 = \frac{\alpha}{4} x_2$$

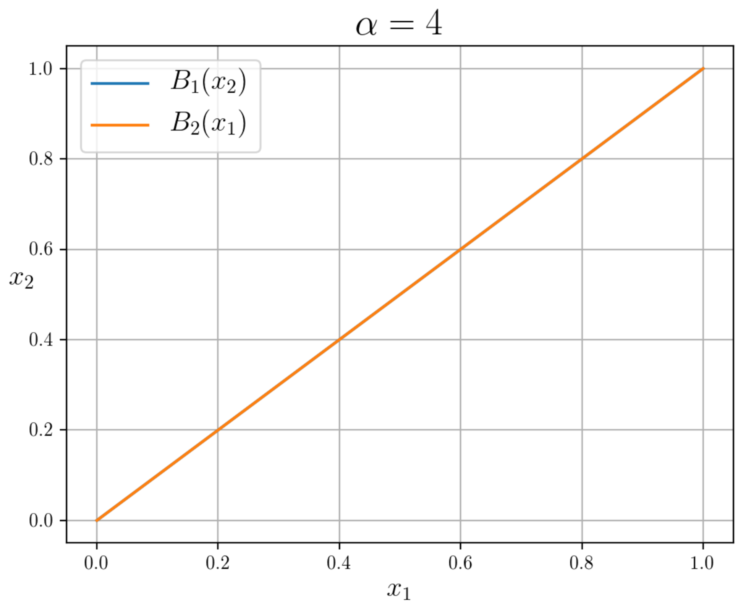
$$B_1(x_2) = \min\{\frac{\alpha}{4} x_2, 1\}$$

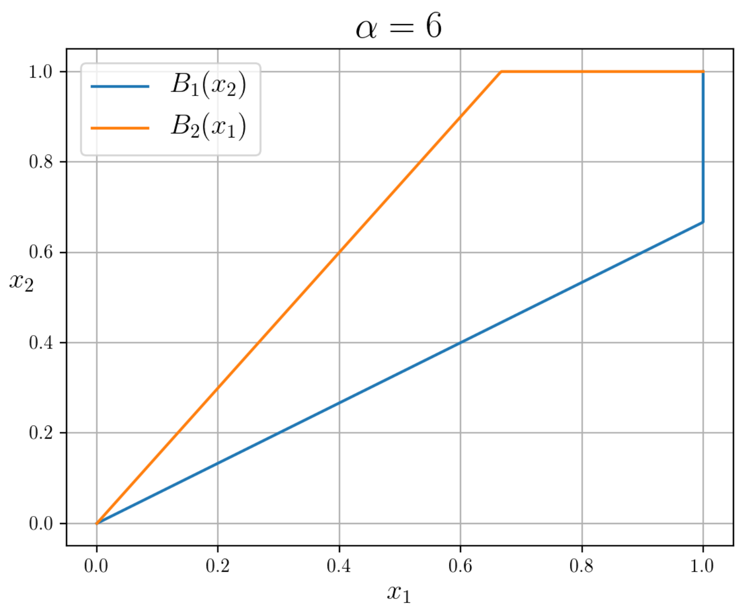
Three cases: (1) $\alpha < 4$ only $(0, 0)$ Nash. (2) $\alpha = 4$ any (x, x), $x \in [0, 1]$ Nash, $\alpha > 4$ only $(1, 1)$ Nash

To wrap up: all Nash equilibria are Pareto dominated by both player's exerting maximal effort, except obviously when this is itself Nash, which it is once $\alpha \geq 4$. That is, all Nash equilibria are inefficient if and only if $2 < \alpha < 4$.

— Always good to graph best response correspondences. here are plots for $\alpha = 2$, 4, and 6 (for $\alpha = 4$, the graphs overlap exactly)




$\alpha = 4$
$B_1(x_2)$
$B_2(x_1)$
x_1
x_2

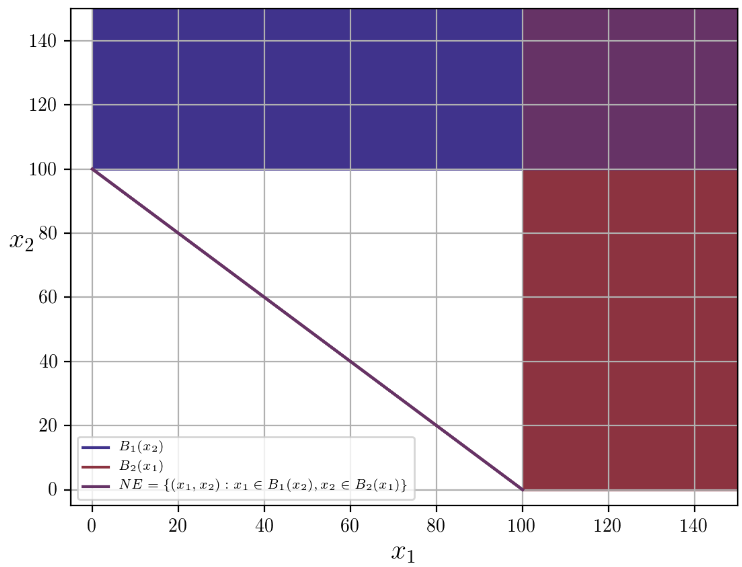

$\alpha = 6$
$B_1(x_2)$
$B_2(x_1)$
x_1
x_2

7

(a) There are no strictly dominated strategies: if $x_2 \geq 100$, player 1 is indifferent between all strategies

(b) Bids of 0, and greater than 100 are weakly dominated—they pay zero no matter what the other player's bid; any other bid x is the uniquely optimal response to the opponent bidding $100 - x$, so can't be weakly dominated

(c) As some of you pointed out, graphing the best responses, and thus solving for the Nash equilibria, is quick and painless:

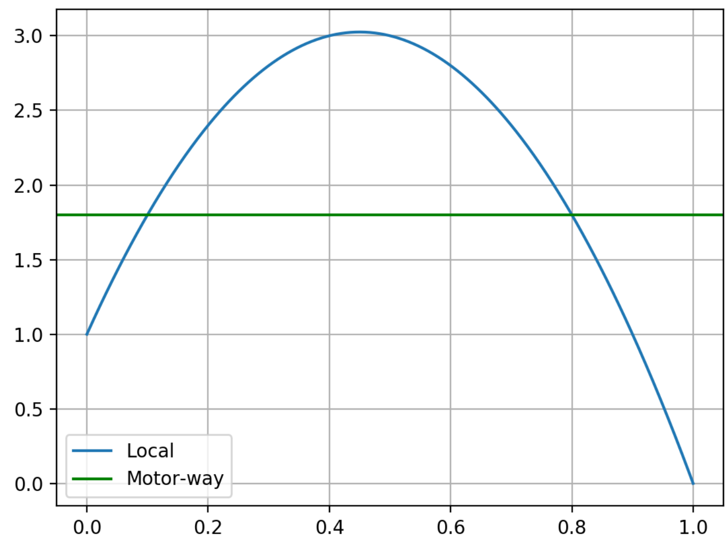


Nash equilibria at $(x, 100 - x)$ for $0 \leq x \leq 100$ and any two bids (x_1, x_2) both at least equal to 100

8

(a) See graph below

(b) Equilibrium traffic patterns at $x = 0$, 0.1, and 0.8. By considering small perturbation to equilibrium, easy to show $x = 0, 0.8$ stable, $x = 0.1$ unstable.



— Note though that if a single agent controlled a positive fraction of traffic, then $x = 0.1$ would not be an equilibrium to begin with.

(c) We maximize total benefit to the population $x(1+9x-10x^2)+(1-x)1.8$. the maximum cannot occur at $x = 0$ or $x = 1$, since there are obviously better options. (in fact the shape of the graphs immediatly imply that the solution will be where the blue curve dominates the green, and to the right of the blue's maximizer).

Solving for the critical points of the benefit function and choosing the one that yields most benefits gives $x^* = 0.5516$.

One possibility for implementing x^* is capping motorway usage at $1 - x^*$ and road usage at x^*.

This could be implemented as an equilibrium by subsidizing those on the motor-way, or taxing those on the local road so that the green and blue curves intersect at x^*.

It is also desirable to eliminate the two other equilibria that remain after such a subsidy/tax.

The caps achieve this.

With the subsidy/tax, inducing congestion on the motor-way by closing lanes (say) when x is less than some $\hat{x} < x^*$ could do this. Better policing on local roads (paid for from the revenue of the tax) could work, too.