solution 1

Take the prisoner's dilemma, and impose a large punishment for finking (so it becomes strictly dominated)

	quiet	fink
quiet	3,3	0,4-10
fink	4-10,0	1-10,1-10

Nash: quiet,quiet

— common mistake is to call (3,3) the Nash equilibrium—this is the *outcome*. obviously that would be bad terminology: might be same outcome induced by non-Nash strategy profile

solution 2

(a)

$$sit, stand >_1 sit, sit >_1 stand, sit >_1 stand, stand$$

	sit	stand
sit	2,2	3,1
stand	1,3	0,0

Nash: sit,sit

(b)

$$stand, sit >_1 sit, sit >_1 sit, stand >_1 stand, stand$$

	sit	stand
sit	2,2	1,3
stand	3,1	0,0

Nash: stand, stand

Both players now less comfortable

— prisoner's dilemma shows how selfish behavior can be Pareto inefficient; here, (in (a)) it is efficient

solution 3

$$aggressive, passive >_1 passive, passive >_1 passive, aggressive >_1 aggressive, aggressive$$

	aggressive	passive
aggressive	0,0	3,1
passive	1,3	2,2

Nash: aggressive, passive,
passive, aggressive

— there can be multiple Nash equilibria

solution 4

Straightforward.

- — point is to gain familiarity with best response correspondence and their characterization of Nash equilibrium

- — i saw two methods used for computing Nash: drawing arrows to indicate that a player has a profitable deviation, and drawing stars under a player's payoff when they are best responding

- — "star" method corresponds to best response description of Nash: all stars, all best responding

solution 5

Straightforward.

- — there are no strictly dominated strategies here, but when presented with a large game try to eliminate strictly dominated strategies: obviously, they are never part of a Nash equilibrium

- — this problem illustrates that weakly dominated strategies may be played in a Nash equilibrium, so don't flippantly eliminate those when looking for Nash

solution 6

Observe that it is always a Nash equilibrium for both players to put in zero effort. On the other hand, if both players exert effort 1, each gets $\alpha/2 - 1 > 0$. So there always exists (for any $\alpha > 2$) an inefficient Nash equilibrium.
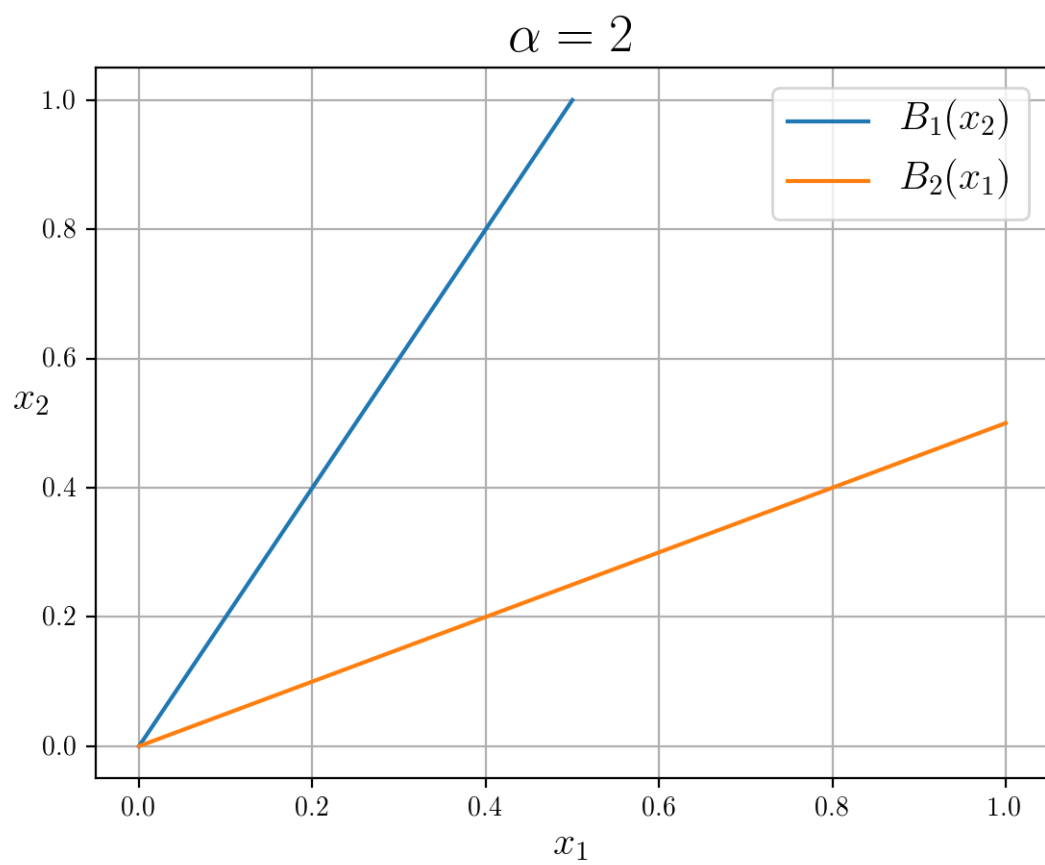
For remainder, try to solve for all Nash:
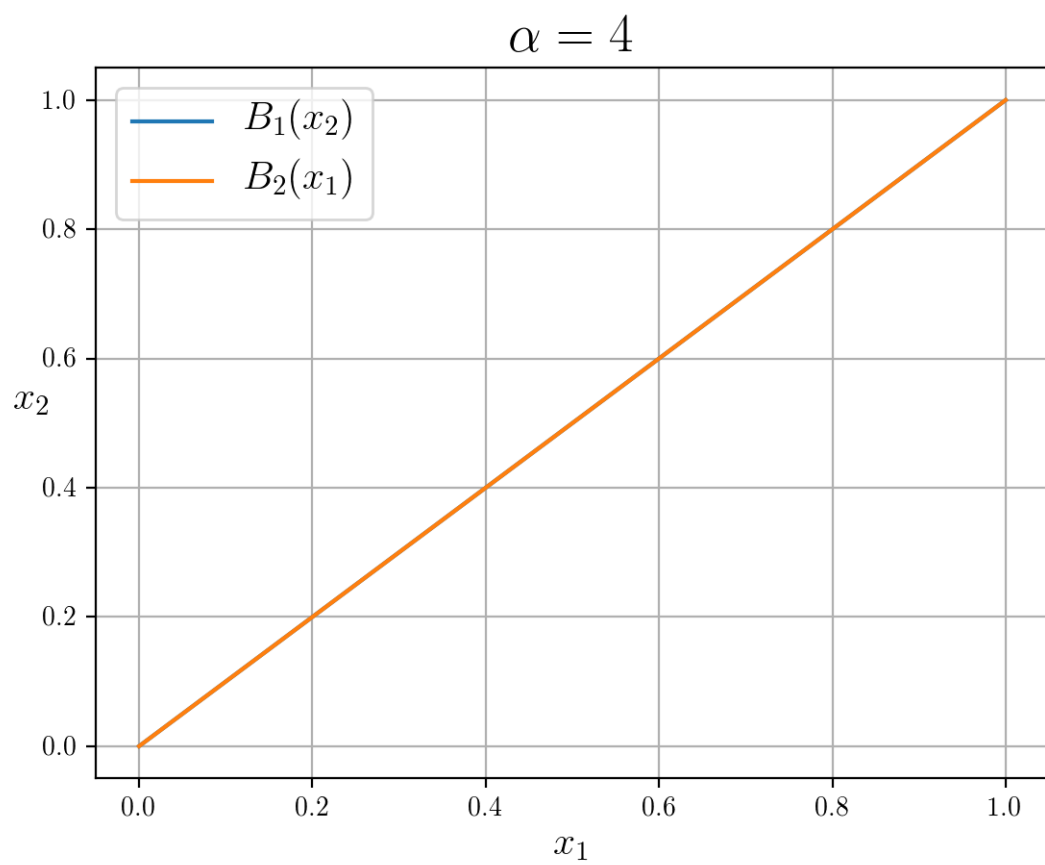
$$u_i(x_1, x_2) = \frac{\alpha x_1 x_2}{2} - x_i^2$$

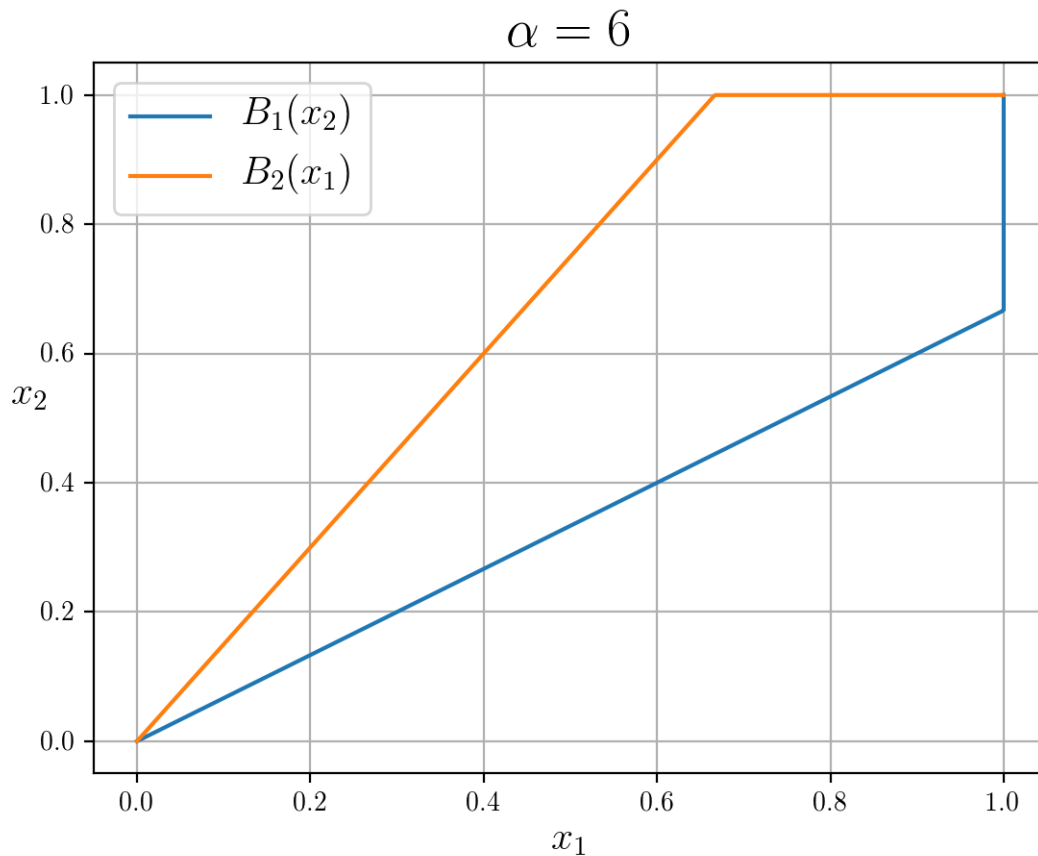$$\frac{\partial u_1(x_1, x_2)}{\partial x_1} = 0 \implies x_1 = \frac{\alpha}{4} x_2$$

$$B_1(x_2) = \min\{\frac{\alpha}{4} x_2, 1\}$$

Three cases: (1) $\alpha < 4$ only $(0,0)$ Nash. (2) $\alpha = 4$ any (x,x), $x \in [0,1]$ Nash, $\alpha > 4$ only $(1,1)$ Nash

- — always good to graph best response correspondences. here are plots for $\alpha = 2$, 4, and 6 (for $\alpha = 4$, the graphs overlap exactly)

$$\alpha = 6$$

solution 7

(a) there are no strictly dominated strategies: if $x_2 \geq 100$, player 1 is indifferent between all strategies

(b) bids of 0, and greater than 100 are weakly dominated—they pay zero no matter what the other player's bid; any other bid x is the uniquely optimal response to the opponent bidding $100 - x$, so can't be weakly dominated

(c) as some of you pointed out, graphing the best responses, and thus solving for the Nash equilibria, is quick and painless:
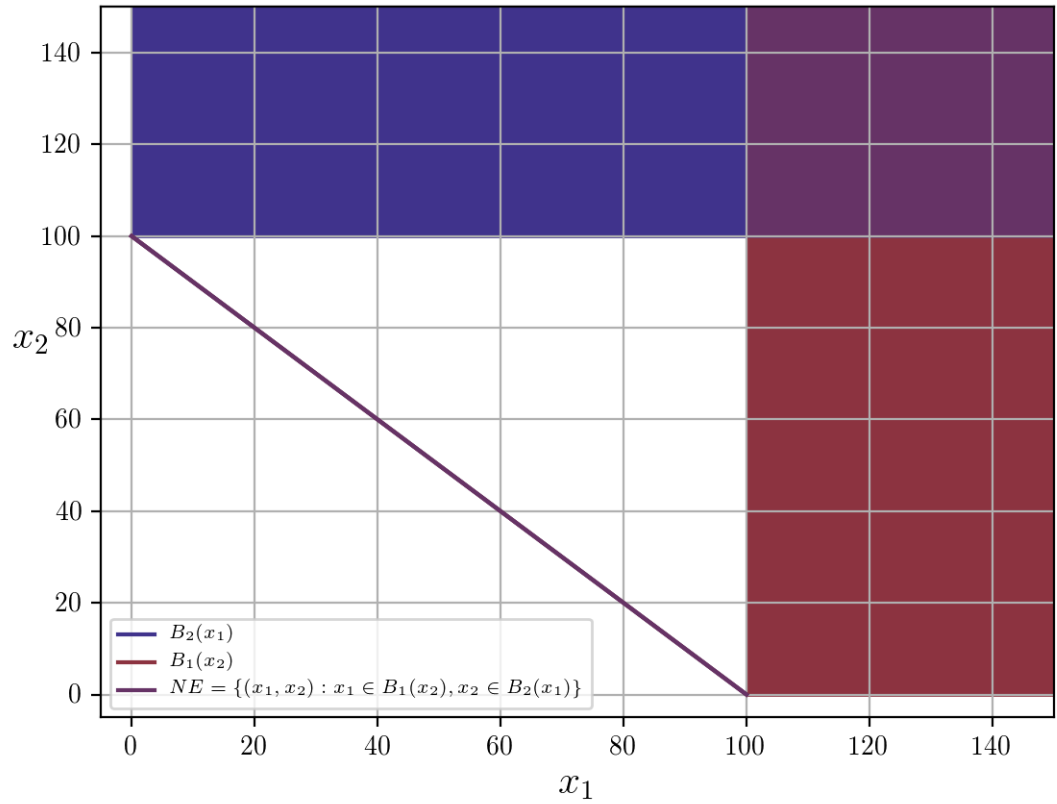
solution 8

(a) see graph below

    (b) equilibrium traffic patterns at $x = 0$, 0.1, and 0.8

    by considering small peturbation to equilibrium, easy to show $x = 0, 0.8$ stable, $x = 0.1$ unstable.

    — note though that if a single agent controlled a positive fraction of traffic, then $x = 0.1$ would not be an equilibrium to begin with

    (c) we maximize total benefit to the popultation $x(1+9x-10x^2)+(1-x)1.8$. the maximum cannot occur at $x = 0$ or $x = 1$, since there are obviously better options. (in fact the shape of the graphs immediatly imply that the solution will be where the blue curve dominates the green, and to the right of the blue's maximizer)
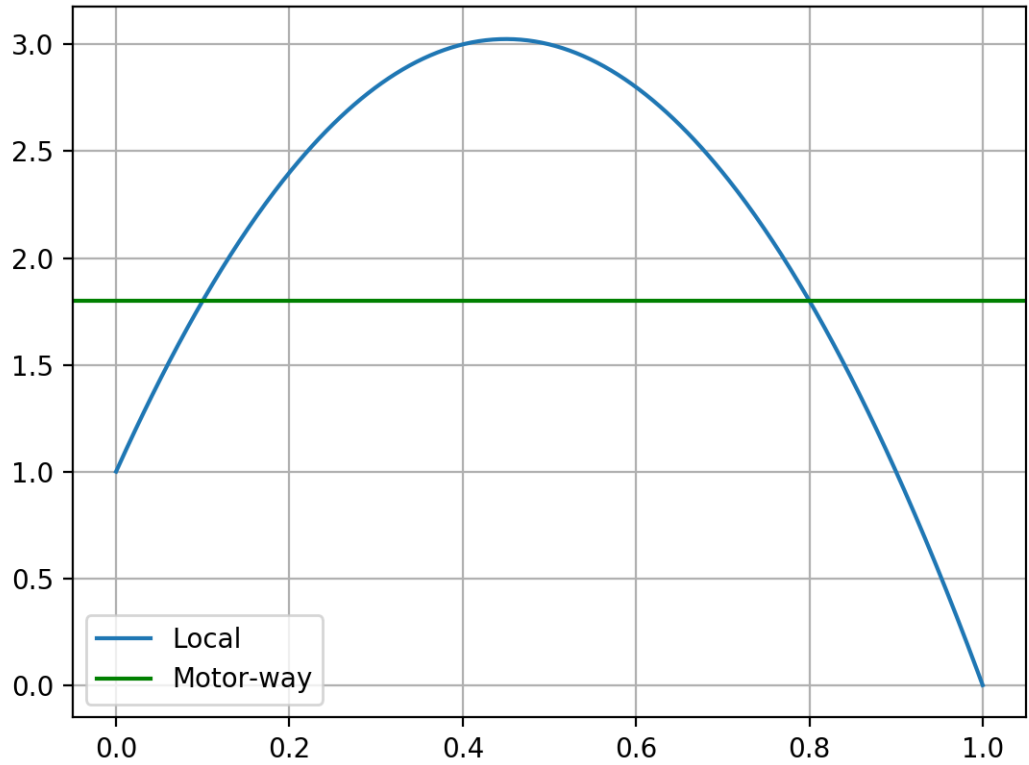
solving for the critical points of the benefit function and choosing the one that yields most benefits gives $x^* = 0.5516$

this could be implemented as an equilibrium by subsidizing those on the motor-way so that the green and blue curves intersect at x^*

it is also desirable to eliminate the two other equilibria that remain after such a subsidy

a few of you made the ingenious suggestion that this could be achieved by inducing congestion on the motor-way by closing some of its lanes whenever x is less than some $\hat{x} < x^*$.