BERTRAND COMPETITION WITH COSTLY INFORMATION

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Abstract. Consider a collection of firms that produce the same good and have the same constant marginal cost. If each consumer knows the price set by each firm then this is a model of Bertrand competition. The unique pure strategy equilibrium is for each firm to set their price equal to their marginal cost. In this paper I study what happens when consumers do not know the price set by each firm but can acquire information about this at a cost. I assume this cost is proportional to the average amount the informaton reduces the entropy of the consumer's belief about the price set by each firm. My main result is that in the symmetric equilibrium each firm sets a price above marginal cost. The markup is increasing in the cost of information and decreasing in the number of firms. As the number of firms tends to infinity the markup converges to the cost of information.

Introduction

Consider a collection of n firms indexed by $j \in \{1, 2, \cdots, n\}$ and a continuum of consumers indexed by $i \in [0, 1]$. Suppose that each firm produces the same good and has a marginal cost of 1. Let x_j denote the price set by firm j. Assume that each firm sells in a different location and that these locations are far apart. Consumer i's problem is to choose which location to go to. He gets the same utility from consuming the good from any firm and once he arrives at one of the locations he will purchase one unit of the good there if and only if the price is not above his reservation price M. Complicating consumer i's problem is the fact that he is uncertain about the price at each location. This uncertainty is described by the pdf f on the set of possible price vectors $\mathbf{R}^n$. I will assume that the support of f is a subset of $[1, M]^n$.

Before deciding which location to go to the consumer can acquire information about prices. Once he feels he has acquired enough information the consumer chooses a location to go to. I will model this by assuming that each consumer chooses a *signal* which is a function $s : \{1, 2, \cdots, n\} \times [1, M]^n \to [0, 1]$ such that for each x in $[1, M]^n$ we have $s(1, x) + s(2, x) + \cdots + s(n, x) = 1$. The interpretation of $s(j, x)$ is that if the price vector set by the firms is x then the consumer's search for information leads him to choose to go to the location of firm j with probability $s(j, x)$.

I assume that the cost of a signal s is equal to a constant θ which I call *the cost of information* times the average amount the signal reduces the entropy of the consumer's belief about prices. Intuitively, a signal can be thought of as a plan to read the newspaper. What is written in the newspaper is correlated with the state of the world and so a consumer can update his belief about the state of the world based on what he reads. A more precise reading of the newspaper will deliver a signal that is more correlated with the true state of the world and so will on average change the entropy of the consumer's prior more and so be more costly. The cost of information θ may be interpreted as literacy. A larger θ means reading the newspaper to a desired level of precision is more difficult.

The simplification of this model in which each consumer knows the price set by each firm is the model of Bertrand competition. The unique pure strategy equilibrium under Bertrand competition is for each firm to set their price equal to their marginal cost. The purpose of this paper is to study what happens when consumers do not know the price set by each firm but can acquire information about this at a cost. We will characterise the solution to this problem and then show that in the symmetric equilibrium each firm sets a price above marginal cost. The markup is increasing in the cost of information and decreasing in the number of firms. As the number of firms tends to infinity the markup converges to the cost of information.

Consumers

Because the support of each consumer's prior is a subset of $[1, M]^n$ we may assume that $\{1, 2, \cdots, n\} \times [1, M]^n$ is the domain of any signal the consumer might choose. Our first task is to show that the consumer's problem of how much information to acquire and which location to go to can be represented as the consumer choosing a signal. A result in Woodford (2008) says that this is true when the consumer faces a binary decision – that is, when there are only two firms. I am going to assume that this result can be extended to the case with more than two firms.

Consumer i's problem then is to choose a signal to maximise his expected utility from consumption minus the cost of the signal. I will assume that each consumer has an expected utility function u that is an increasing function of the difference between the consumer's reservation price M and the price paid for the good. Formally, each consumer chooses a function $s : \{1, 2, \cdots, n\} \times [1, M]^n \to [0, 1]$ to maximise the objective function

$$\int \left(\sum_{j=1}^{n} s(j,x) u(M - x_j) \right) f(x) dx - \theta I(s). \tag{1}$$

Here $\theta I(s)$ denotes the cost of the signal s. The term θ is a constant and I refer to it as *the cost of information*. The term $I(s)$ denotes the average amount the signal reduces the entropy of the consumer's belief about prices. That is,

$$\begin{aligned} I(s) = & \underbrace{-\int f(x) \log(f(x)) dx}_{\text{Entropy of prior}} \\ & \underbrace{-\sum_{j=1}^{n} \bar{s}_j \left(-\int \frac{s(j,x) f(x)}{\bar{s}_j} \log \left(\frac{s(j,x) f(x)}{\bar{s}_j} \right) \right)}_{\text{Average entropy of posterior}}. \end{aligned}$$

The term $\bar{s}_j$ in this expression denotes the average probability with which the consumer goes to firm j. That is,

$$\bar{s}_j = \int s(j,x) f(x) dx.$$

Like in Woodford (2009) $I(s)$ may be written as the entropy of the average signal plus the average entropy of the signal. That is,

$$I(s) = -\sum_{j=1}^{n} \bar{s}_j \log(\bar{s}_j) + \int \left(-\sum_{j=1}^{n} s(j,x) \log(s(j,x)) \right) f(x) dx.$$

To solve for the optimal signal I will suppose that each number $s(j,x)$ is in the interior of the interval $[0,1]$. If such a solution exists then it is the only solution because the objective function is strictly concave. Given this assumption the optimal choice of $s(j,x)$ is a stationary point of the objective function and so satisfies the equation

$$u(M - x_j) f(x) - \theta D_{s(j,x)} I(s) = 0 \tag{2}$$

It is easy to make this equation show that the solution satisfies

$$s(j,x) = \left(1 + \sum_{k \neq j} \frac{\bar{s}_k}{\bar{s}_j} e^{\frac{u(M - x_k) - u(M - x_j)}{\theta}} \right)^{-1}. \tag{3}$$

From equation 3 you can see that an interesting feature of the optimal signal is that it depends on the prior distribution only through the terms $\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_n$. These are the average probabilities that the consumer will go to each of the locations. To understand this equation consider firms j and k. The probability of going to location j is decreasing in the difference between the price of firm j and the price of firm k. The sensitivity of $s(j,x)$ to this difference in price is decreasing in the cost of information θ. It is less worthwhile noticing the price difference between firm j and firm k when it is more costly to notice this difference.

Note that equation 3 only defines the consumer's optimal signal implicitly. Our goal now will be to solve for the terms $\bar{s}_1, \bar{s}_2, \cdots, \bar{s}_n$. It turns out that if each firm is viewed in the same way so that $f(x) = f(\sigma(x))$ whenever $\sigma(x)$ is a rearrangement of the coordinates of the vector x, then $\bar{s}_1 = \bar{s}_2 = \cdots = \bar{s}_n = \frac{1}{n}$. To see this note that for any firms j and k and any price vector x if $\sigma_{jk} : \mathbf{R}^n \to \mathbf{R}^n$ is the function that interchanges the j'th and k'th coordinate of its argument and leaves the other coordinates unchanged, then $s(j,x) = s(k, \sigma_{jk}(x))$. Also note that if E is a measurable subset of $\mathbf{R}^n$, then the Lebesgue measure of E is equal to the Lebesgue measure of $\sigma_{jk}(E)$. Therefore

$$\bar{s}_j = \int s(j,x) f(x) dx = \int s(k, \sigma_{jk}(x)) f(\sigma_{jk}(x)) dx = \int s(k,y) f(y) dy = \bar{s}_k.$$

It follows that $\bar{s}_1 = \bar{s}_2 = \cdots = \bar{s}_n = \frac{1}{n}$. The optimal signal is then given by the expression

$$s(j,x) = \left(1 + \sum_{k \neq j} e^{\frac{u(M-x_k) - u(M-x_j)}{\theta}}\right)^{-1}$$

and this is an explicit solution to the consumer's problem.

I suspect that the class of priors satisfying the condition $f(x) = f(\sigma(x))$ for all x and for all σ is the largest class such that $\bar{s}_1 = \bar{s}_2 = \cdots = \bar{s}_n = \frac{1}{n}$. In any case it is easy to construct examples in which this conclusion does not hold.

For instance consider the case of two firms and suppose that each consumer is risk neutral such that their utility function is given by the formula $u(t) = t$. Suppose that each consumer's prior distribution assigns probability $\frac{1}{2}$ to the price vector $(M - \frac{1}{r}, M)$ and probability $\frac{1}{2}$ to the price vector $(M, 1)$. Note that each consumer assigns a probability of $\frac{1}{2}$ to the event that firm 1 has the lower price but whenever $\theta > 0$ it is not true that the optimal signal sends each consumer to firm 1 with probability $\frac{1}{2}$.

To see this suppose that the cost of thinking is infinite, that is $\theta = \infty$. The optimal signal is then such that the consumer's posterior is equal to his prior. And the optimal action under the prior distribution is to always go to firm 2. So the optimal signal is $s(2, (x_1, x_2)) = 1$ for all (x_1, x_2) and this signal sends the consumer to firm 2 all the time. Now consider the case when there is no cost of thinking, that is when $\theta = 0$. In this case the optimal signal tells the consumer to go to the location with the cheaper price. That is, $s(1, (x_1, x_2)) = 1$ if $x_1 \leq x_2$ and $s(1, (x_1, x_2)) = 0$ if $x_1 > x_2$. Here the probability that the signal sends the consumer to firm 1 is $\frac{1}{2}$. Now consider any positive and finite cost of thinking, $0 < \theta < \infty$. Suppose that under the optimal signal the probability the consumer goes to firm 1 is $\frac{1}{2}$. Then the consumer's expected utility is

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{r}\right)+\frac{1}{2}(0)\right)+\frac{1}{2}\left(\frac{1}{2}(0)+\frac{1}{2}(M-1)\right)=\frac{1}{4}\left(\frac{1}{r}+M-1\right)$$

and their information cost is not negative. If the consumer chose instead to go to firm 2 all the time (that is, chose a signal $s(1,(x_1,x_2))=0$ for all (x_1,x_2)) then their utility would instead be

$$\frac{1}{2}(M-1)$$

and their information cost would be zero. Assuming $M>1$ we can choose r large enough so that the payoff from choosing the signal that always sends the consumer to firm 2 is larger than the payoff from choosing the signal that sends the consumer to firm 1 with a probability of $\frac{1}{2}$.

Firms

When each firm sets their price such that the price vector is x and $x\in[1,M]^n$ then firm j expects that $s(j,x)$ is the fraction of consumers that will demand its good. If a firm sets a price larger than M then the demand for their good is zero. I will ignore this possibility. I will also ignore the case where a firm sets a price below its marginal cost. These possibilities will not happen in equilibrium and ignoring them does not change the equilibrium.

Suppose that each firm $k\neq j$ sets a price x_k. Then firm j will choose its price x_j to maximise its profit

$$s(j,x)(x_j-1).$$

The solution x_j to this problem is a stationary point of this function so it solves the equation

$$D_{x_j}[s(j,x)](x_j-1)+s(j,x)=0.$$

This expression implies that

$$x_j=1-\frac{s(j,x)}{D_{x_j}[s(j,x)]}.$$

Since

$$D_{x_j}[s(j,x)]=\frac{-u'(M-x_j)}{\theta(1-s(j,x))s(j,x)}$$

we have that

$$x_j=1+\frac{\theta}{u'(M-x_j)(1-s(j,x))}. \tag{4}$$

Equilibrium

Denote the solution x_j to equation 4 by $x_j^*(x)$. An *equilibrium* is a price vector x such that $(x_1^*(x),x_2^*(x),\cdots,x_n^*(x))=x$.

To solve for the equilibrium let's assume that $f(x) = f(\sigma(x))$ for all x whenever $\sigma(x)$ is rearrangement of the coordinates of the vector x. As we have shown this implies that $\bar{s}_1 = \bar{s}_2 = \cdots = \bar{s}_n = \frac{1}{n}$. In the symmetric equilibrum of this game we have that all firms set the same price. It can be seen from equation 3 that if x is a price vector in which each firm sets the same price then $s(j, x) = \frac{1}{n}$ for each firm j. Therefore from equation 4 we have that the symmetric equilibrium prices $x_1, x_2, \cdots, x_n$ which are all equal to some number z solve the equation

$$z = 1 + \frac{\theta}{u'(M - z)} \frac{n}{n - 1}.$$

If each consumer is risk neutral with utility function $u(t) = t$ then this becomes

$$z = 1 + \theta \frac{n}{n - 1}.$$

You can see from this equation that in the symmetric equilibrium each firm sets a price above marginal cost. The markup is increasing in the cost of information and decreasing in the number of firms. As the number of firms tends to infinity the price converges to $1 + \theta$ and the markup converges to the cost of information.

References

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