

Chapter 9 Decision Analysis

- 9.1 Context and basic steps (most important part)
- 9.2 Example
- 9.3 Multistage decision analysis (example)
- 9.4 Hierarchical decision analysis (example)
- 9.5 Personal vs. institutional decision analysis

Bayesian decision theory

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 - or actions a

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- Expected utility $E[U(x) | d] = \int U(x)p(x | d)dx$
- Choose decision d^* , which maximizes the expected utility

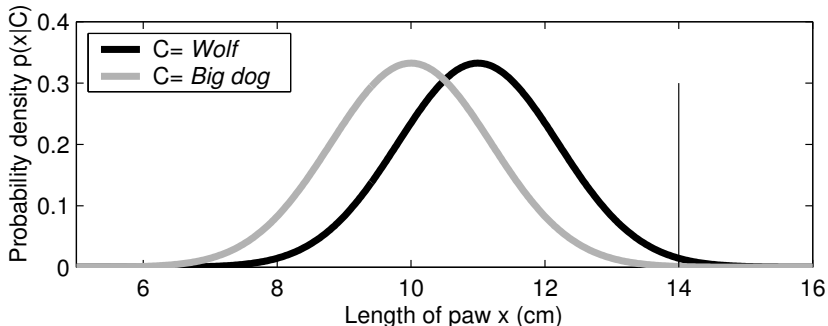
$$d^* = \arg \max_d E[U(x) | d]$$

Example of decision making: 2 choices

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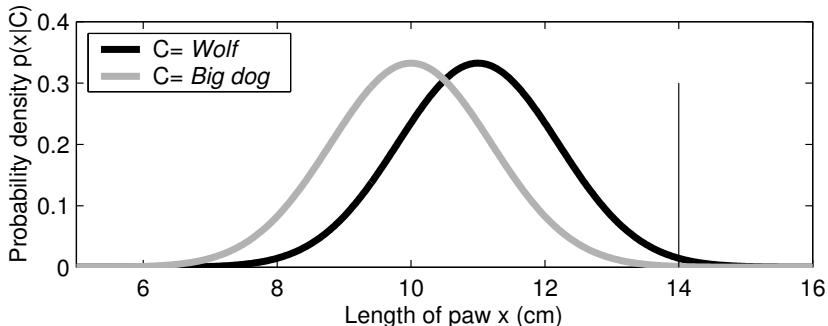
- Helen is going to pick mushrooms in a forest, while she notices a paw print which could made by a dog or a wolf
- Helen measures that the length of the paw print is 14 cm and goes home to Google how big paws dogs and wolves have, and tries then to infer which animal has made the paw print



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- Likelihood of wolf is 0.92 (alternative being dog)

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- Posterior probability of wolf is 10%

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Utility matrix $U(x)$

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Utility matrix $U(x)$

Action d	Expected utility $E[U(x) \mid d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

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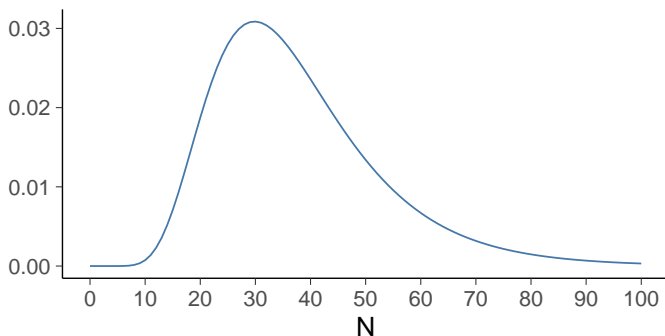
- Maximum likelihood decision would be to assume that there is a wolf
- Maximum posterior decision would be to assume that there is a dog
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- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

Example of decision making: several choices

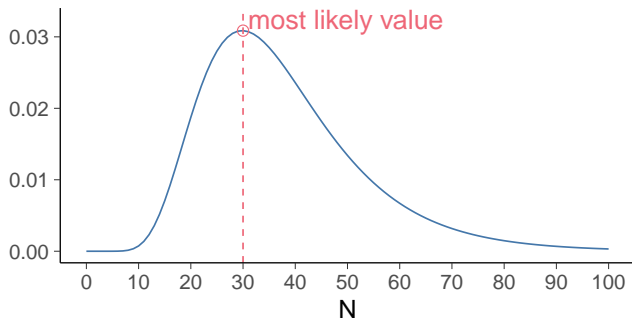
- You decide to earn money by selling a seasonal product
 - You pay 7€ per each, and sell them 10€ each
 - You need to decide how many (N) items to buy

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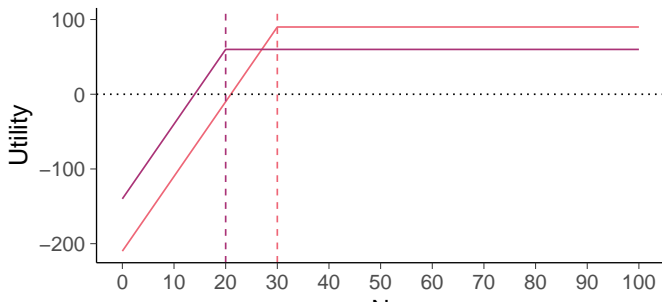
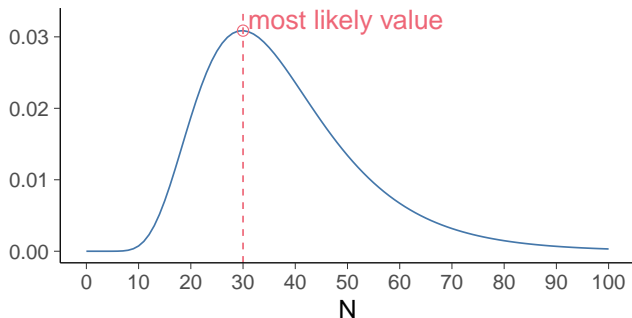
- You decide to earn money by selling a seasonal product
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 - You ask your friends how many they used to sell and estimate a distribution for how many you might sell



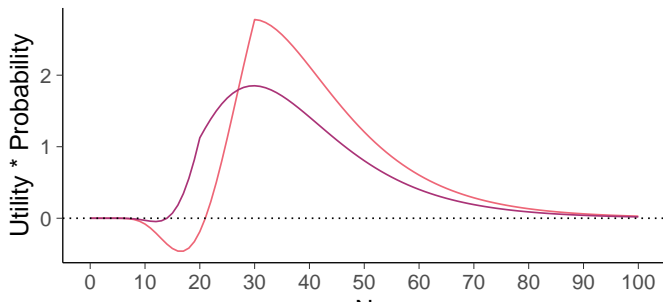
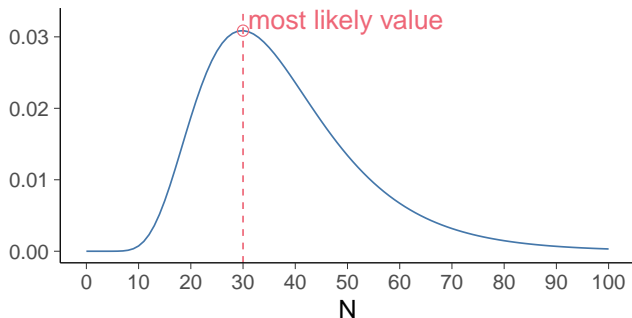
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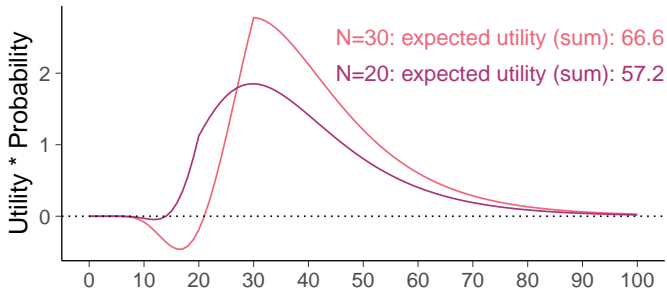
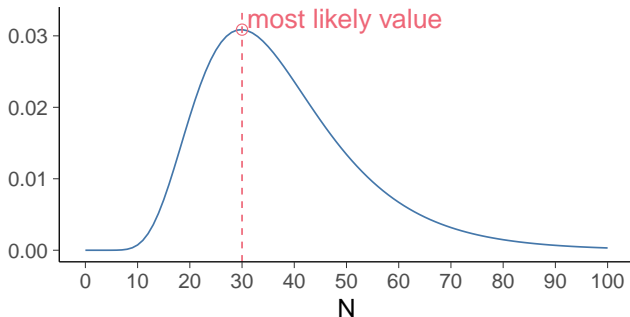
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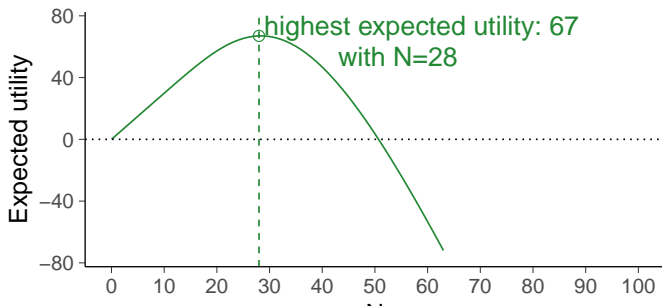
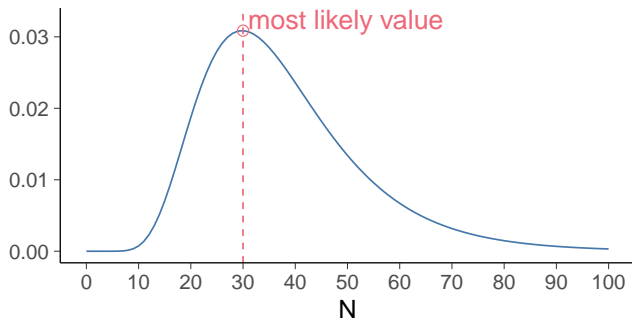
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Decision making in sales

- Common task in commerce and restaurants

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- What is the cost of human life?
- Multiple parties having different utilities

Model selection as decision problem

- Choose the model that maximizes the expected utility of using the model to make predictions / decisions in the future

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- Quality adjusted life time
 - See the book for the multi-stage decision making

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- Example 1
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- Example 2
 - Imagine that in bioassay the posterior uncertainty of LD50 is too large
 - which dose should be used in the next experiment to reduce the variance of LD50 as much as possible ?
 - this way less experiments need to be made (and less animals need to be killed)

Design of experiment

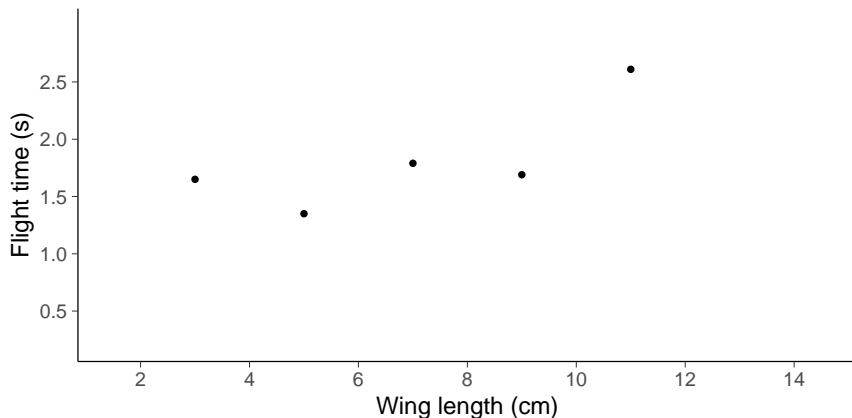
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- Example 3
 - optimal paper helicopter wing length

Bayesian optimization

- Design of experiment
- Used to optimize, for example,
 - machine learning / deep learning model structures, regularization, and learning algorithm parameters
 - material science
 - engines
 - drug testing
 - part of Bayesian inference for stochastic simulators

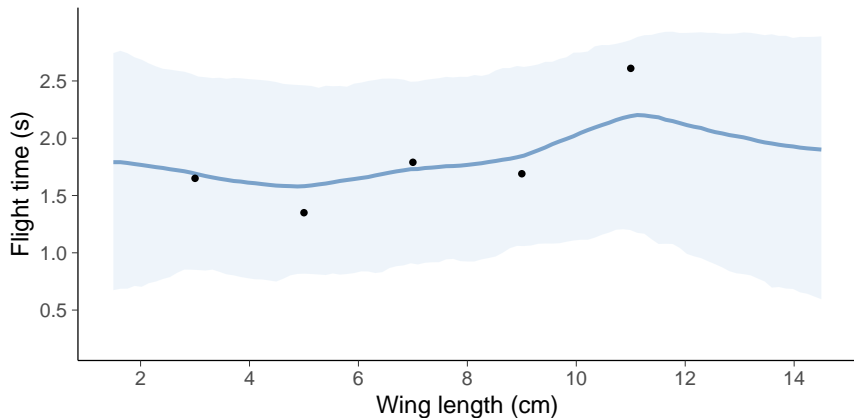
Bayesian optimization of wing length

Start with a small number of experiments



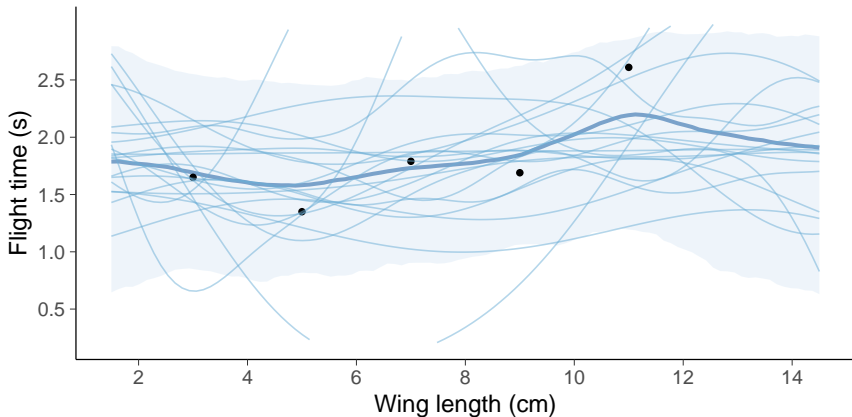
Bayesian optimization of wing length

Gaussian process model



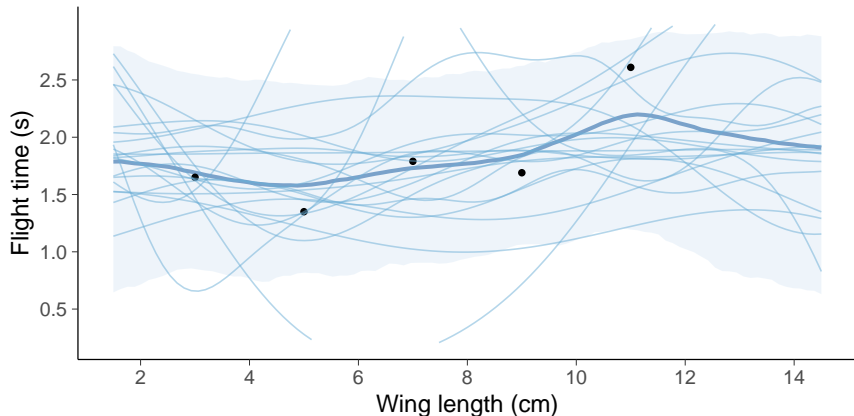
Bayesian optimization of wing length

Gaussian process model – posterior draws



Bayesian optimization of wing length

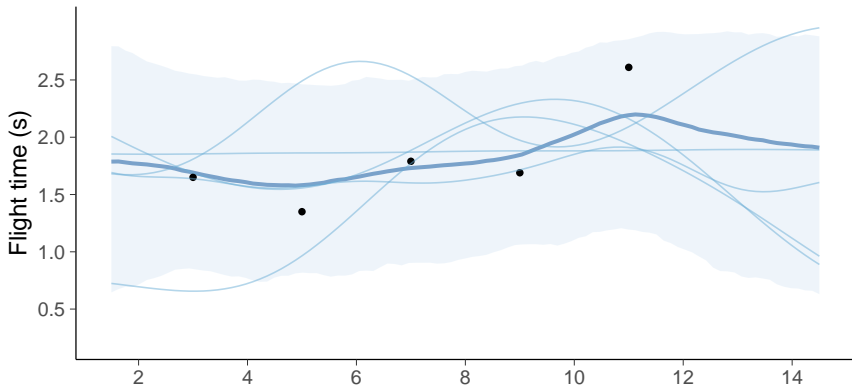
Gaussian process model – posterior draws



- Thompson sampling:
 - pick one posterior draw (function)
 - find the wing length corresponding to the max. of that draw
 - make the next observation with that wing length

Bayesian optimization of wing length

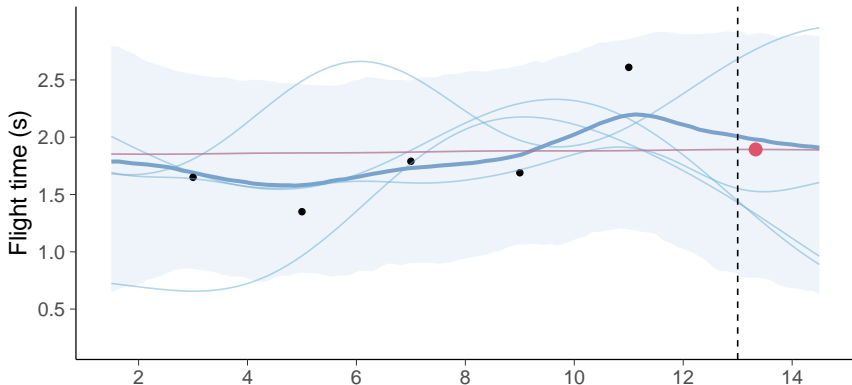
Gaussian process model – Thompson sampling



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Bayesian optimization of wing length

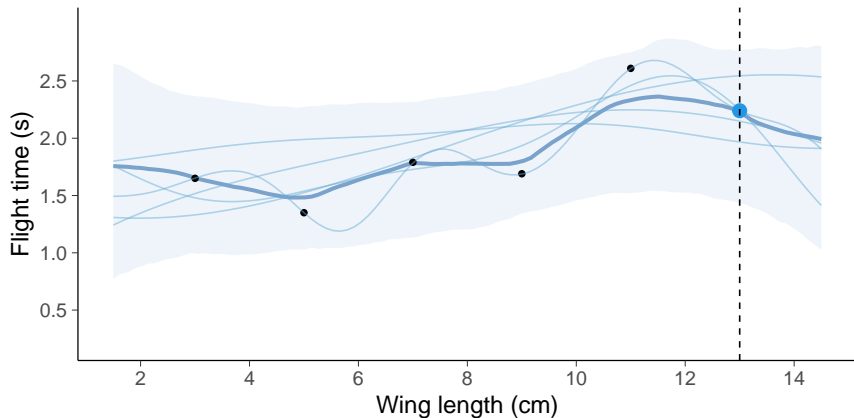
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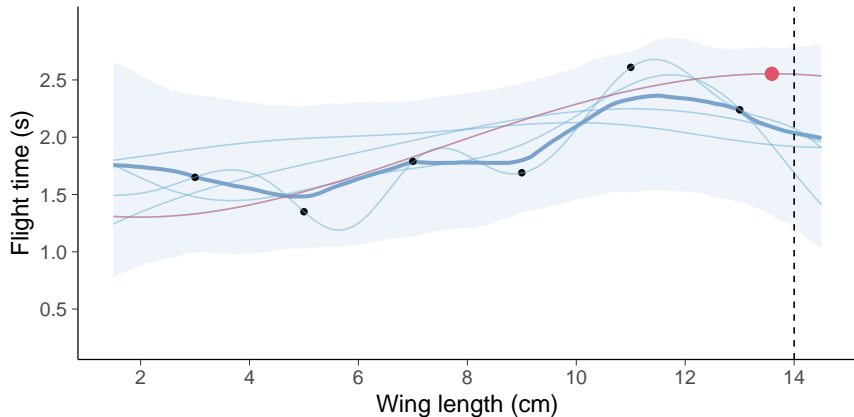
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



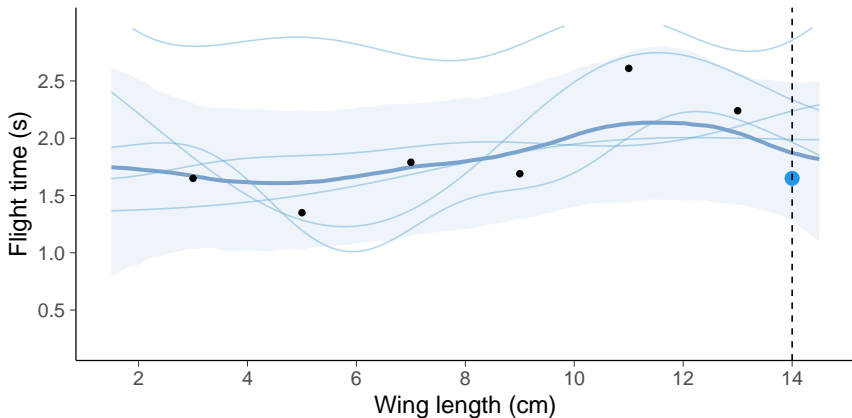
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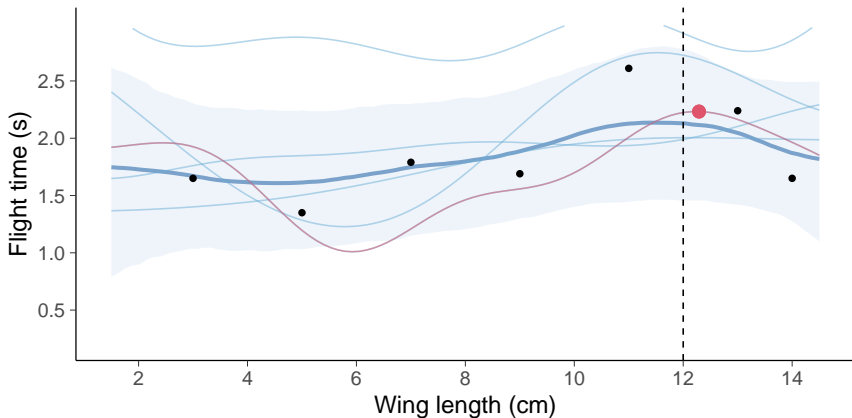
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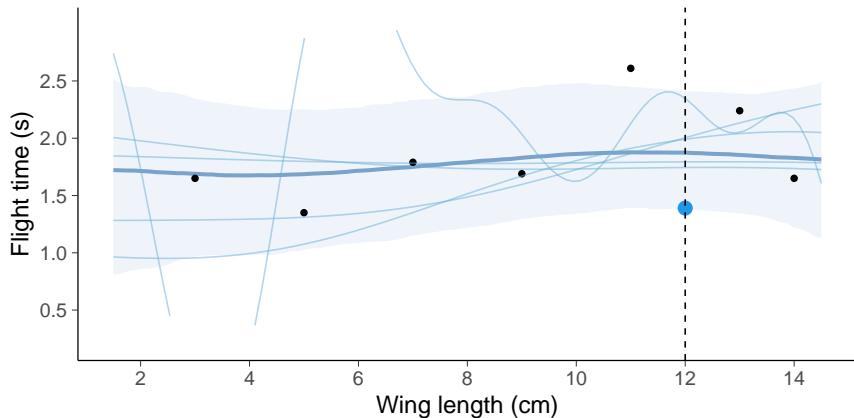
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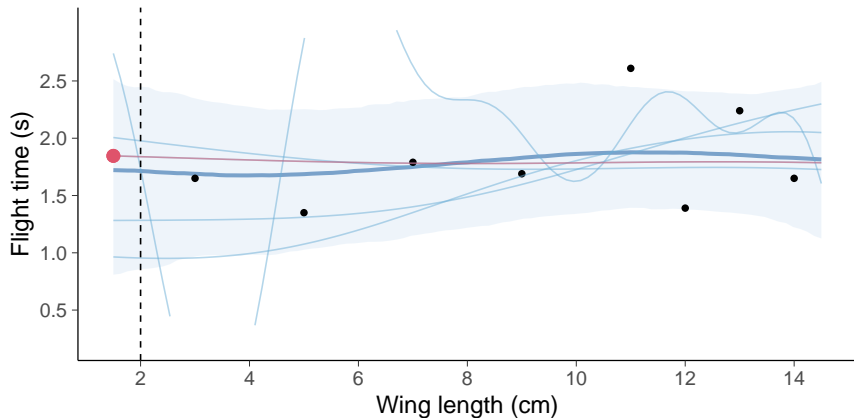
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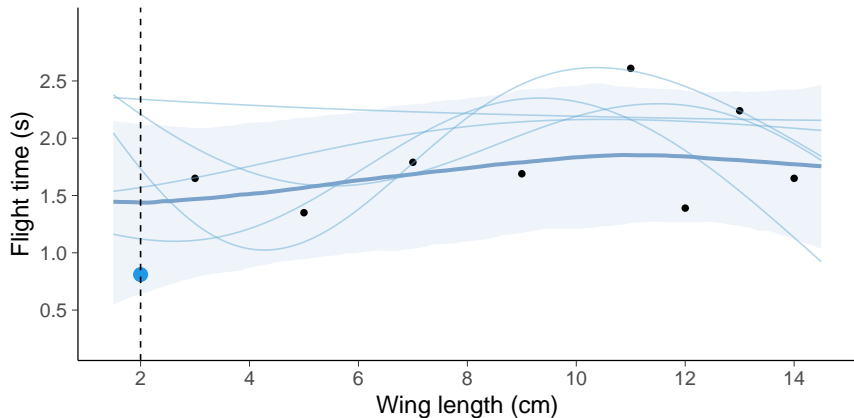
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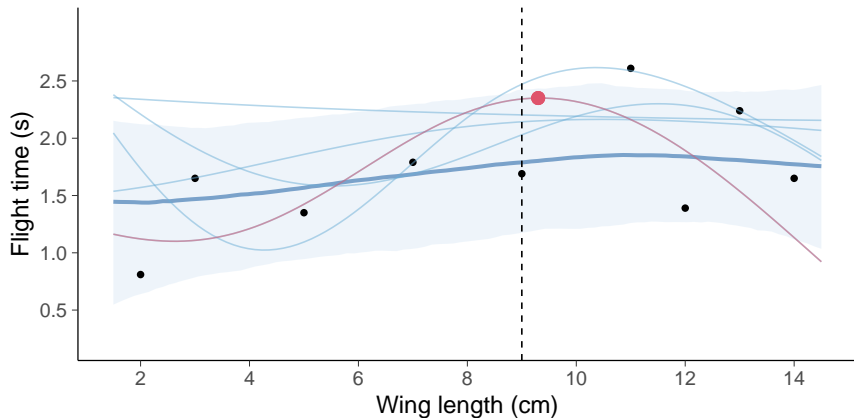
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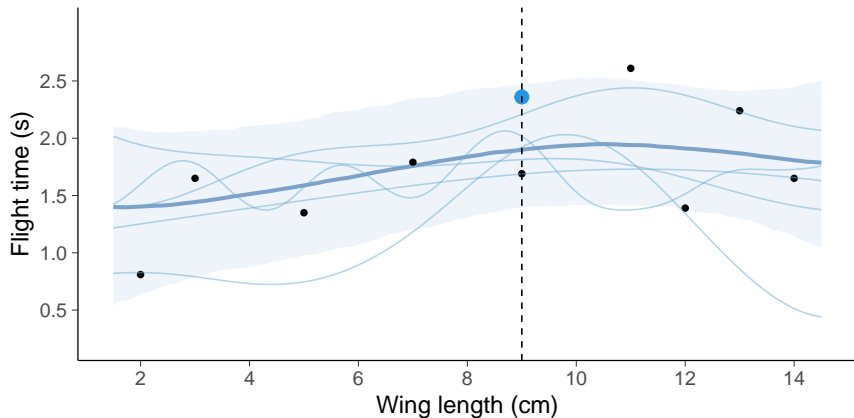
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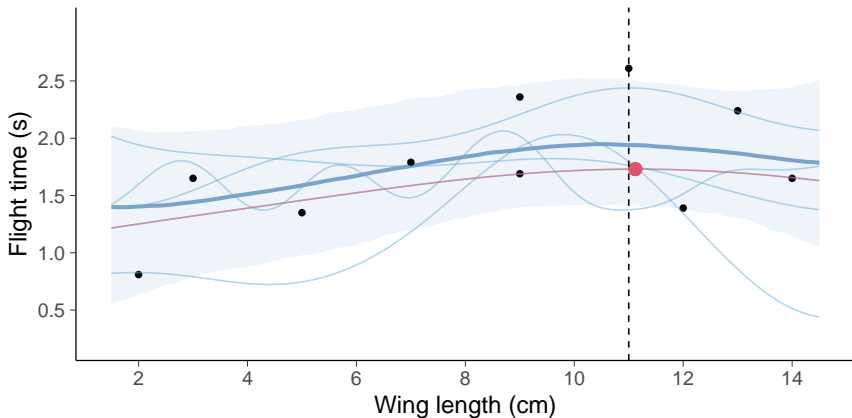
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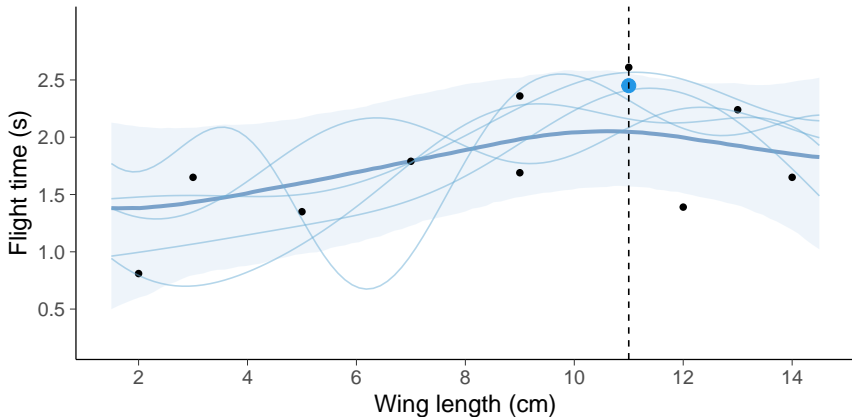
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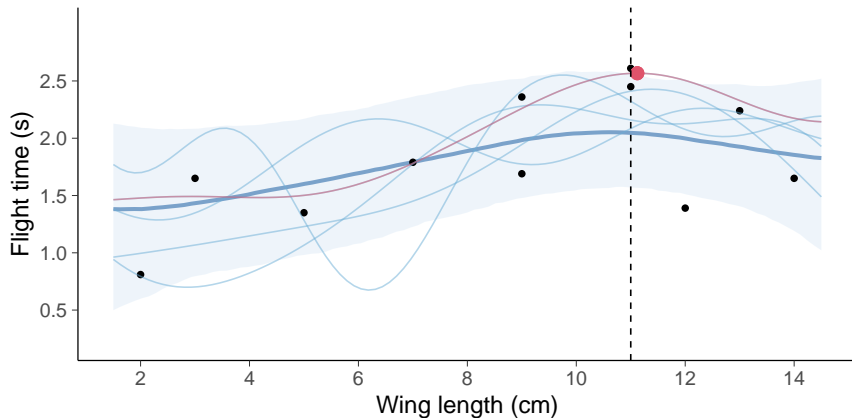
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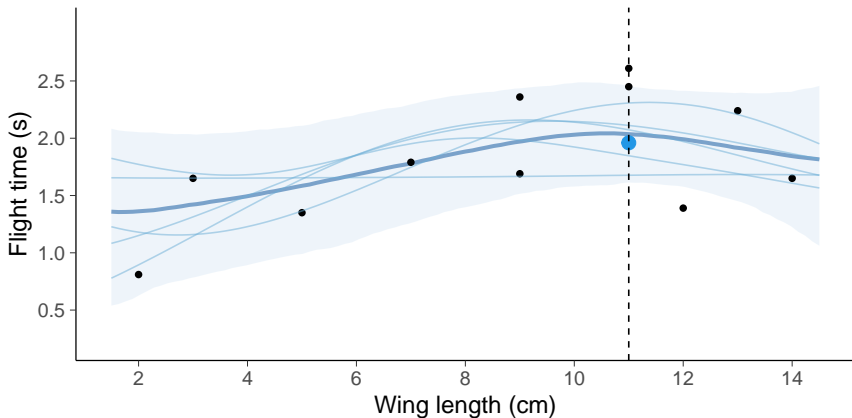
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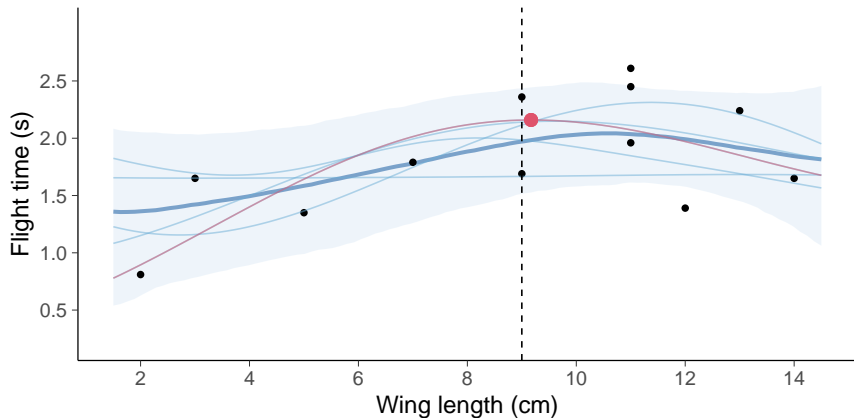
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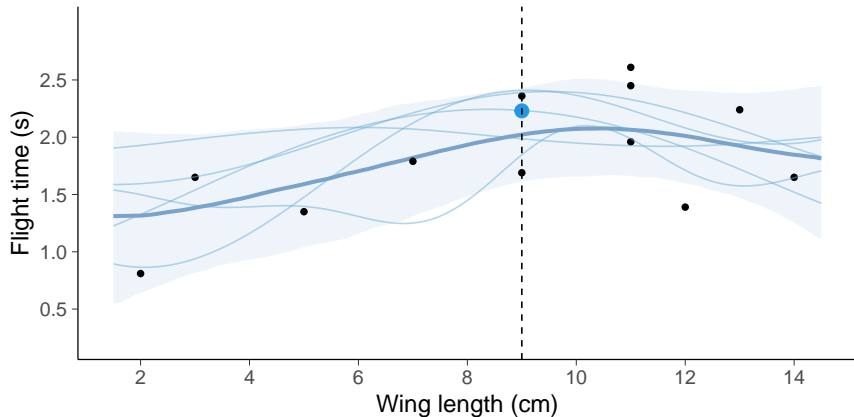
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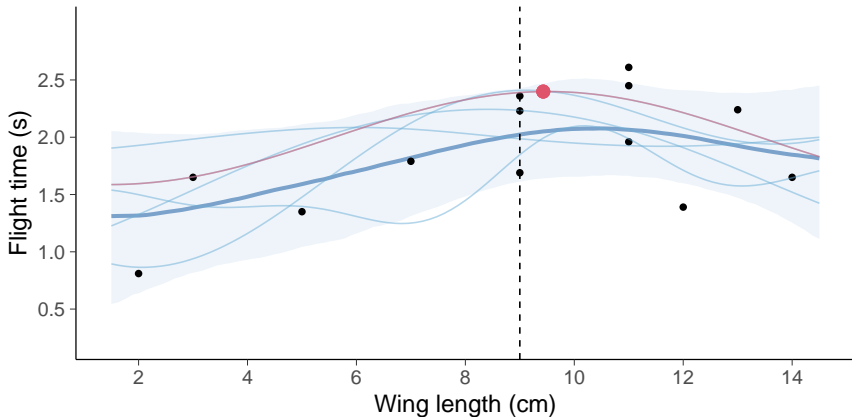
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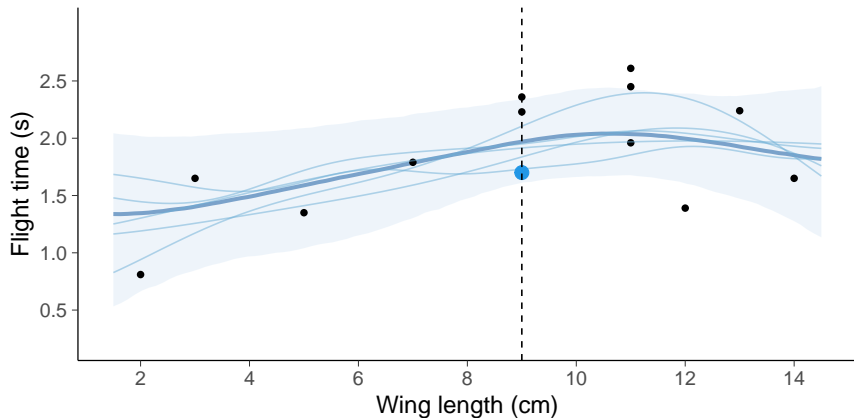
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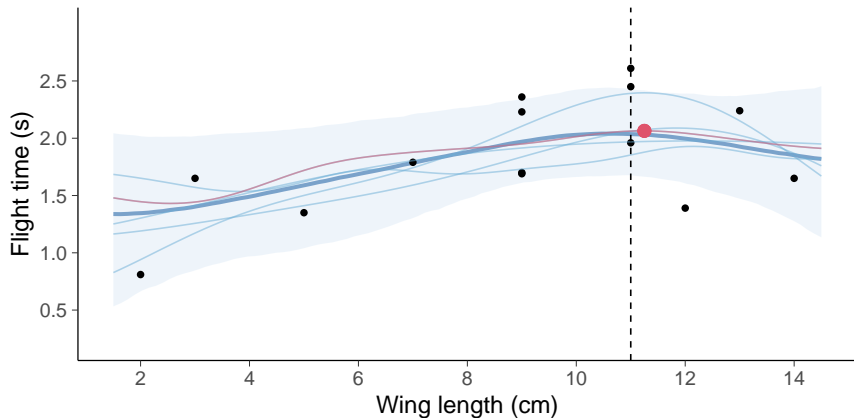
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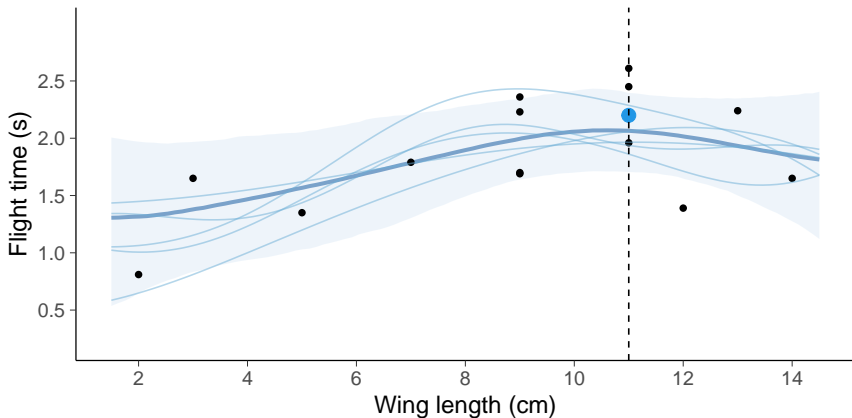
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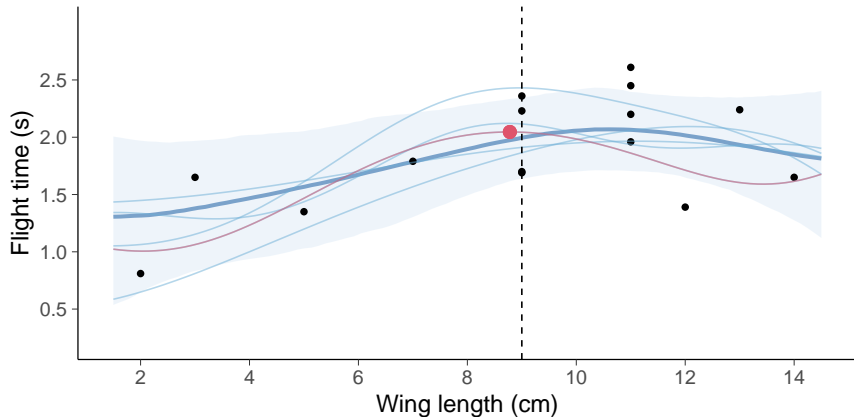
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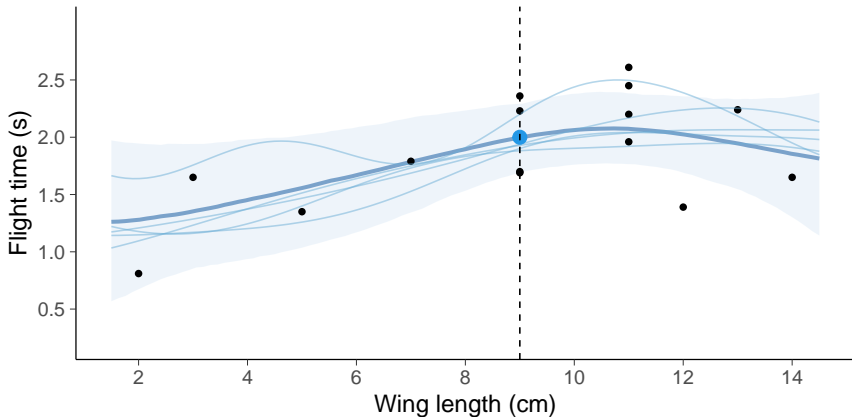
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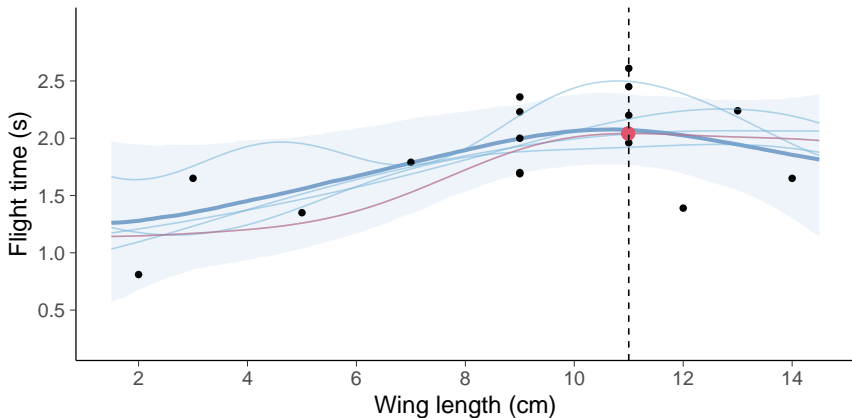
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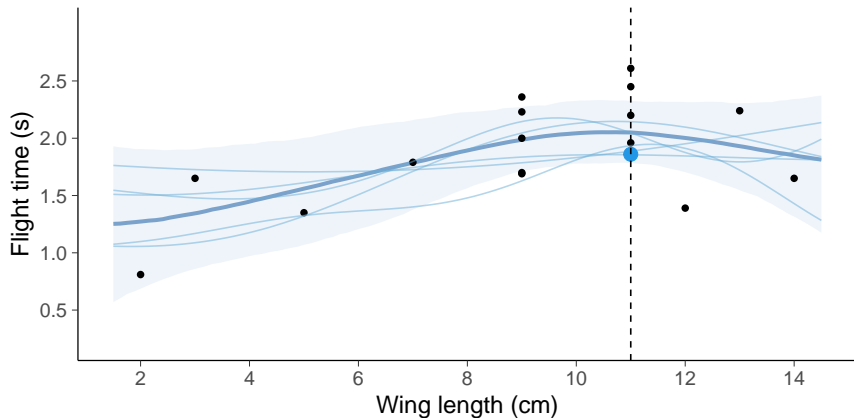
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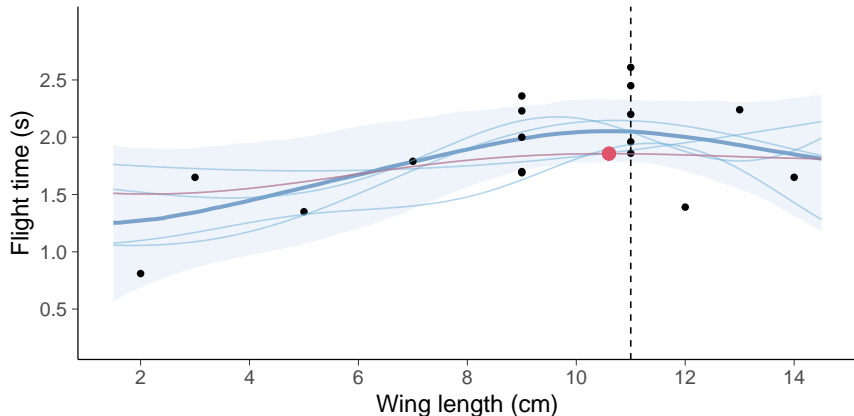
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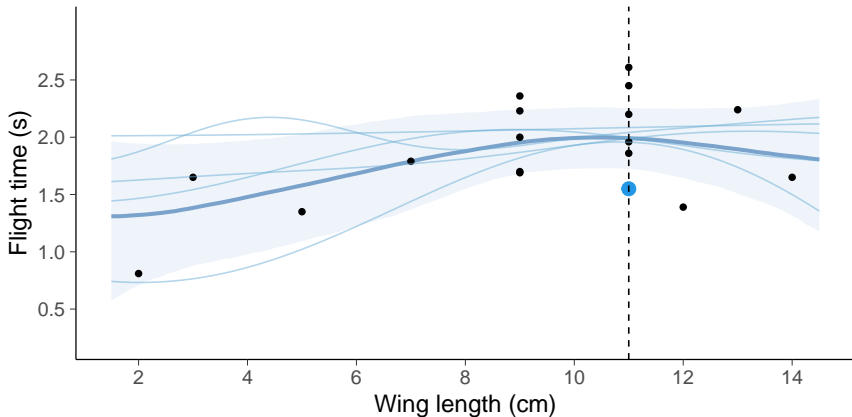
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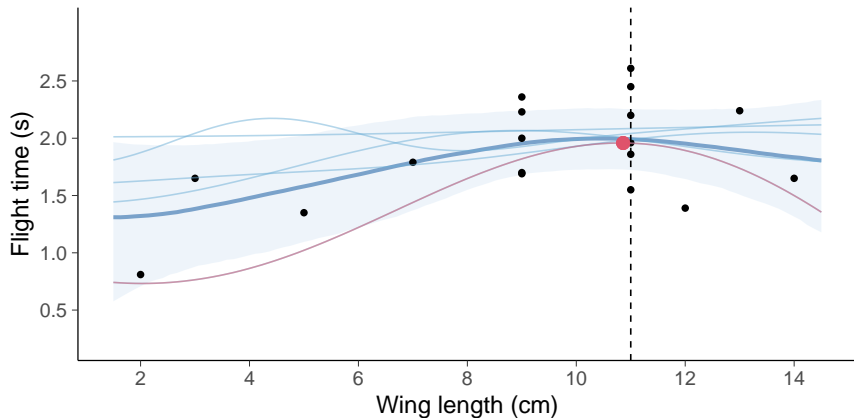
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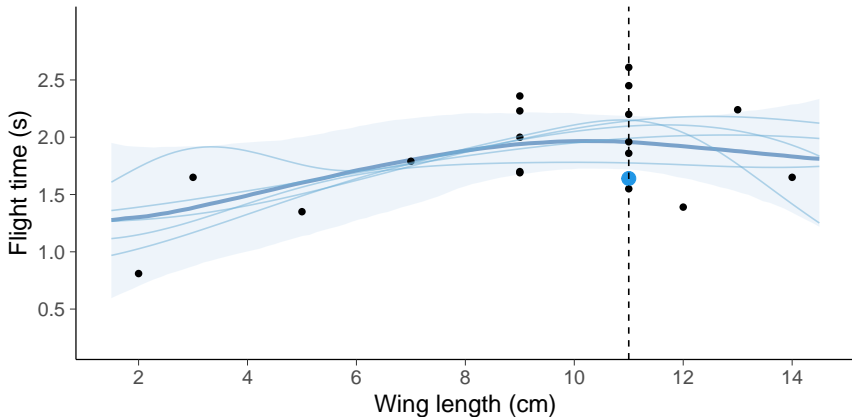
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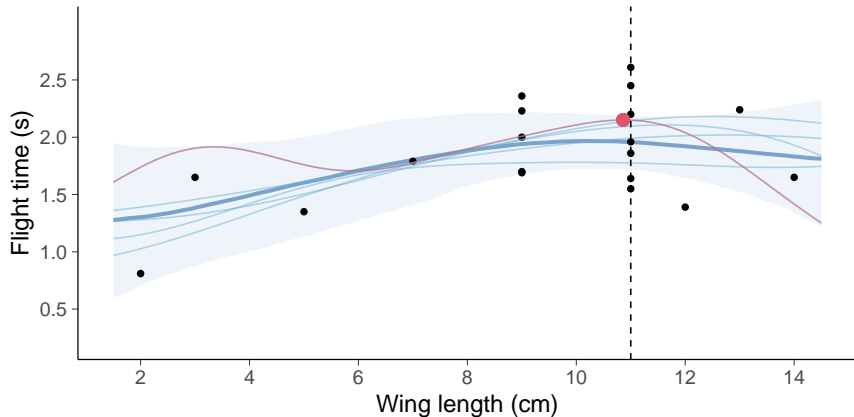
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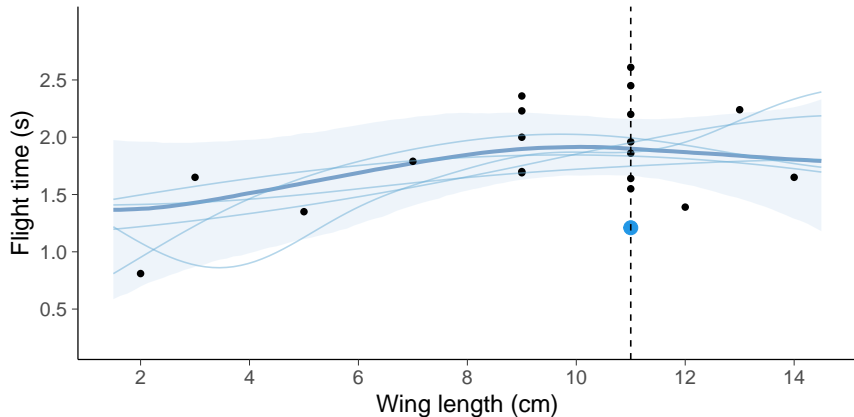
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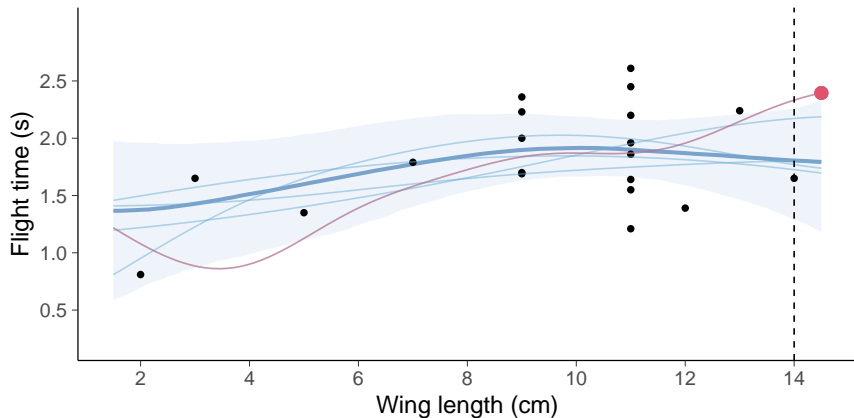
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



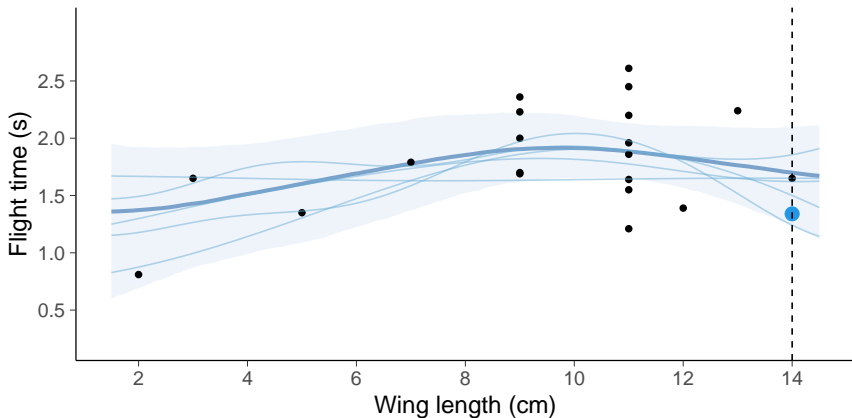
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



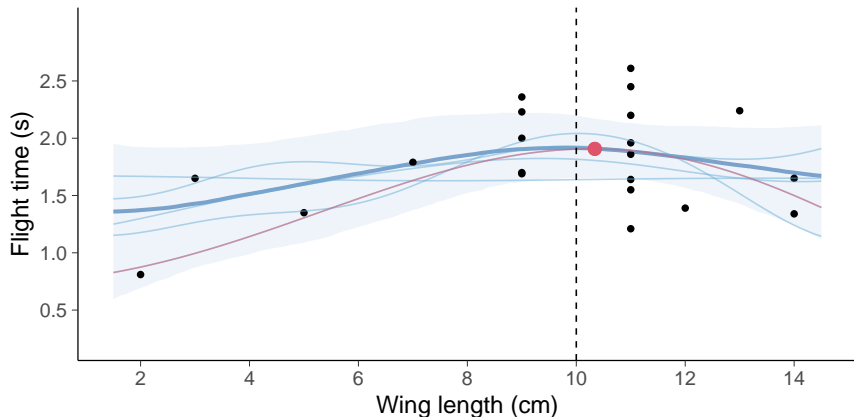
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



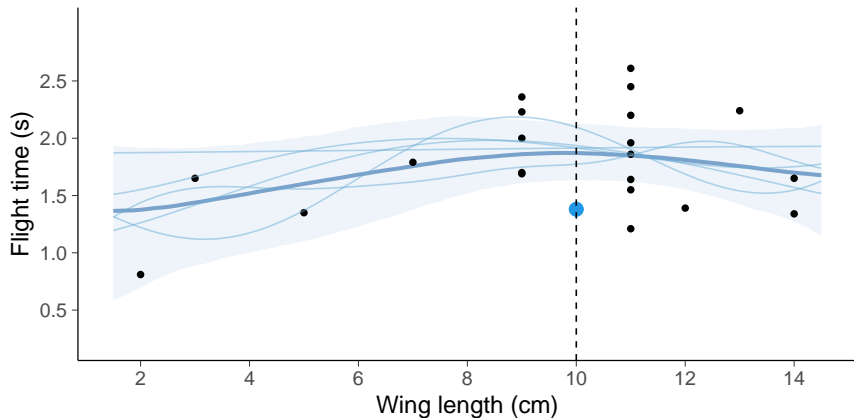
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



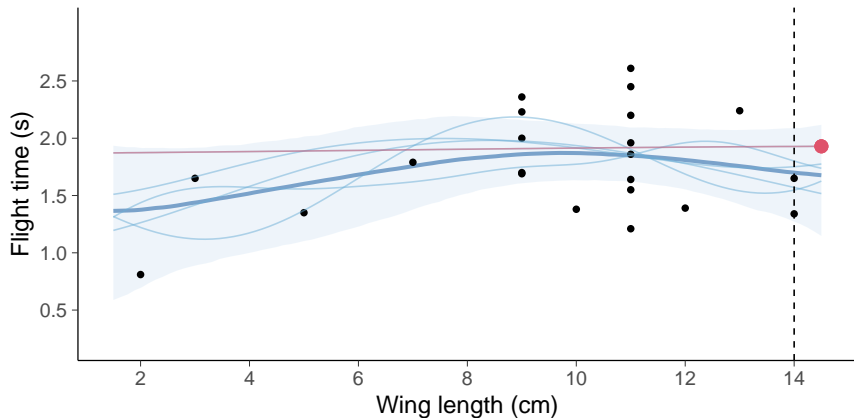
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



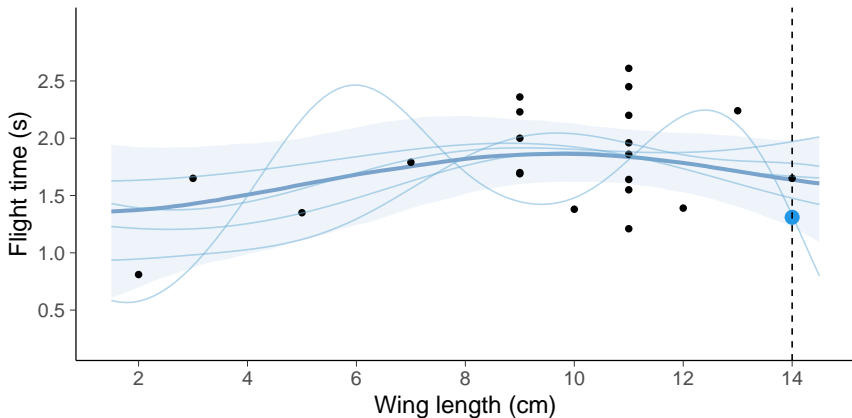
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



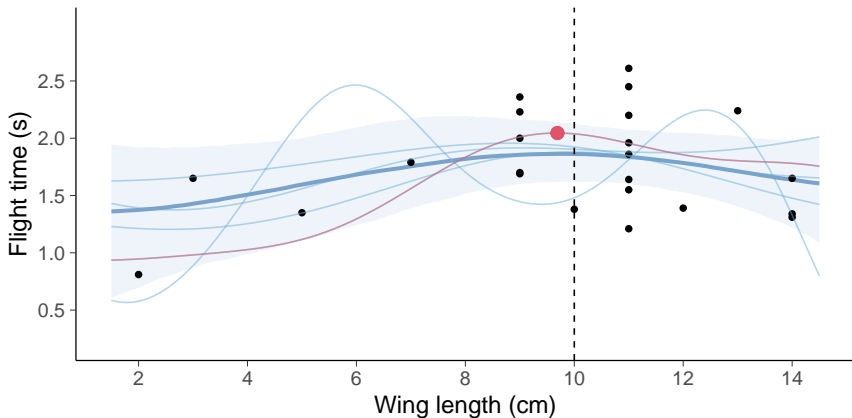
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



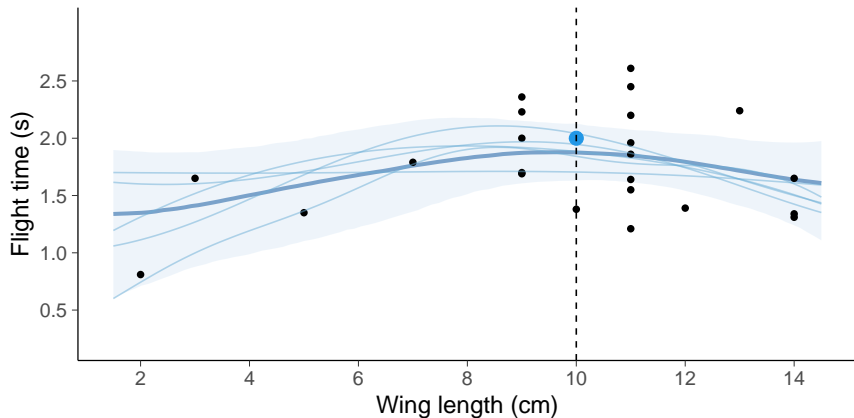
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



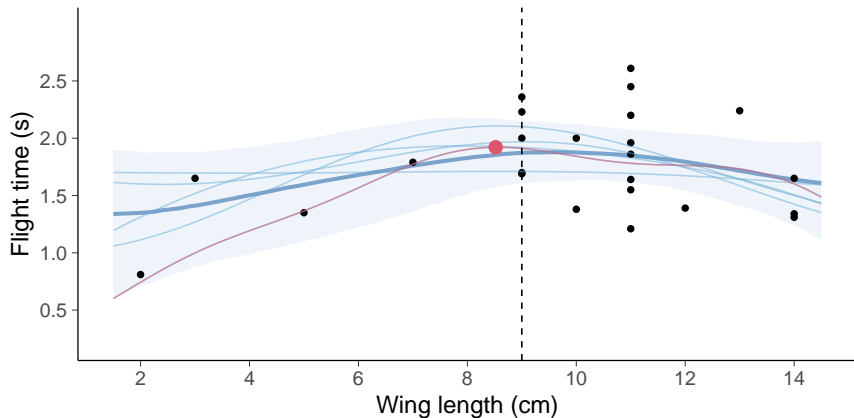
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Gaussian process model – Thompson sampling



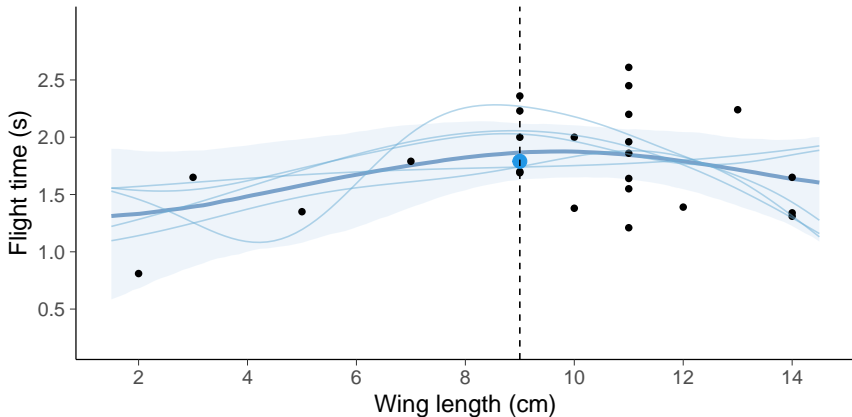
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



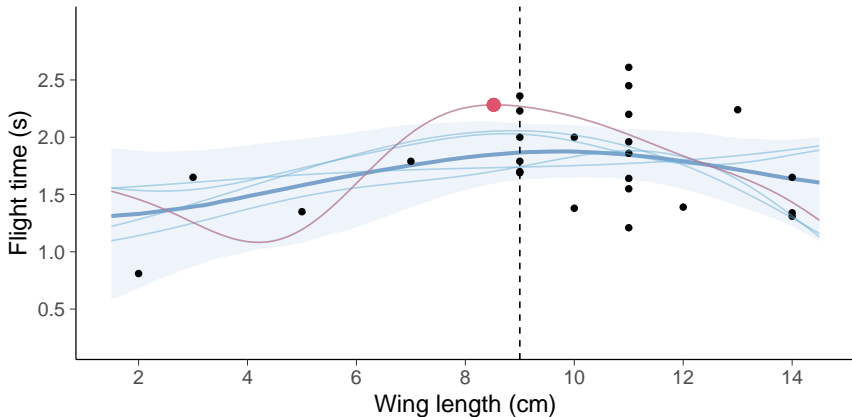
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



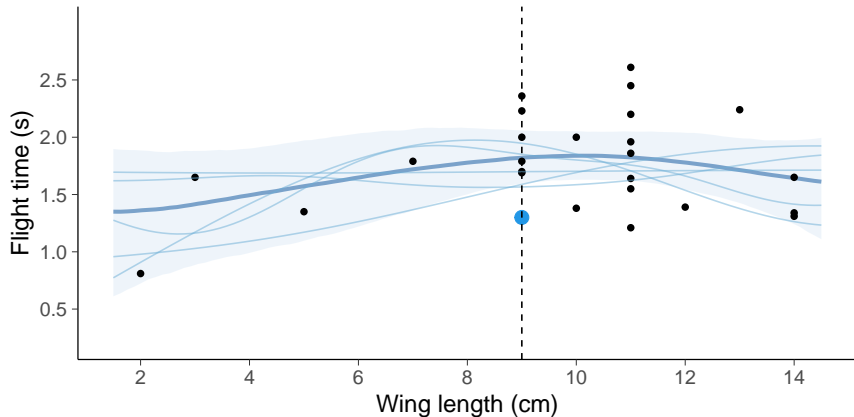
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



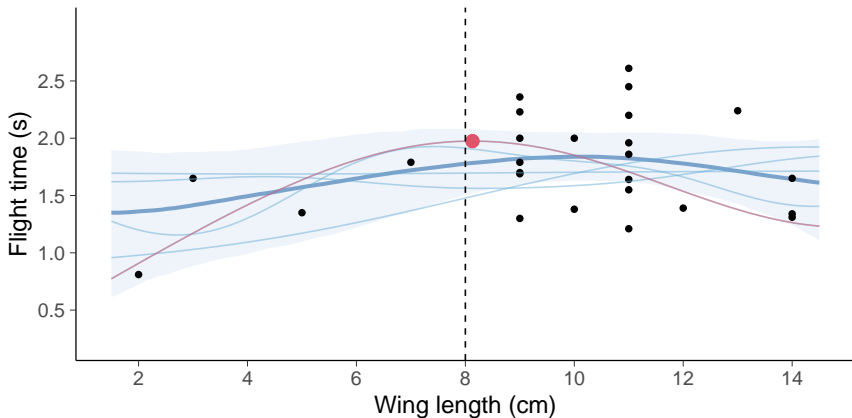
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



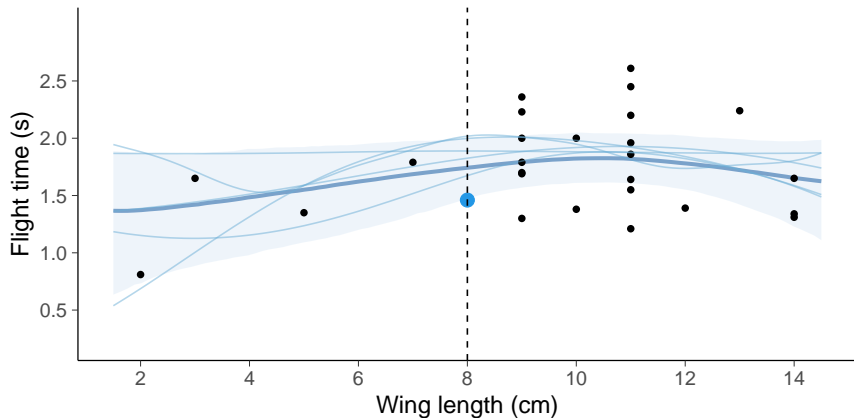
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



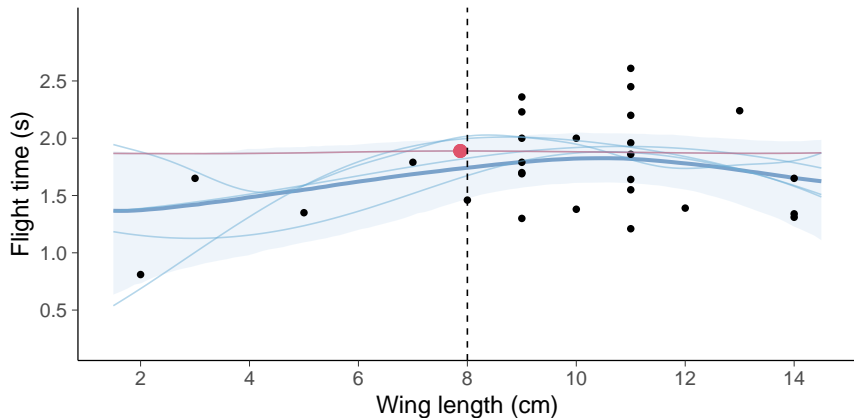
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



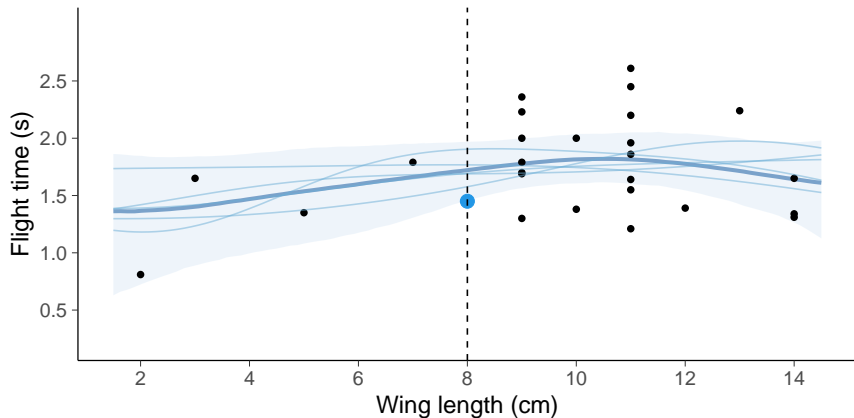
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



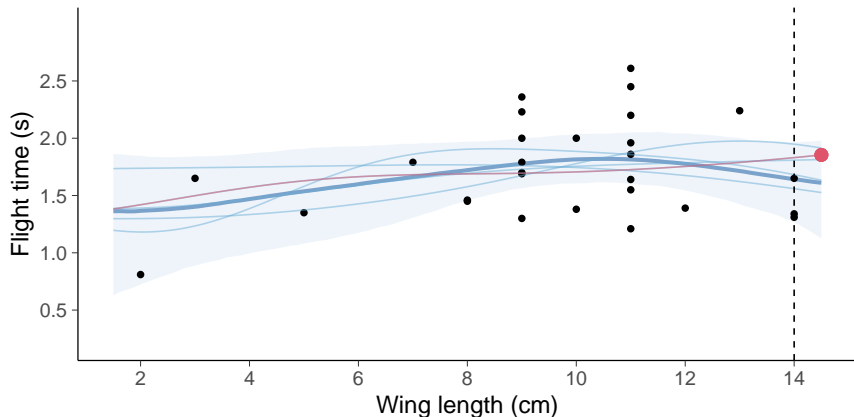
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



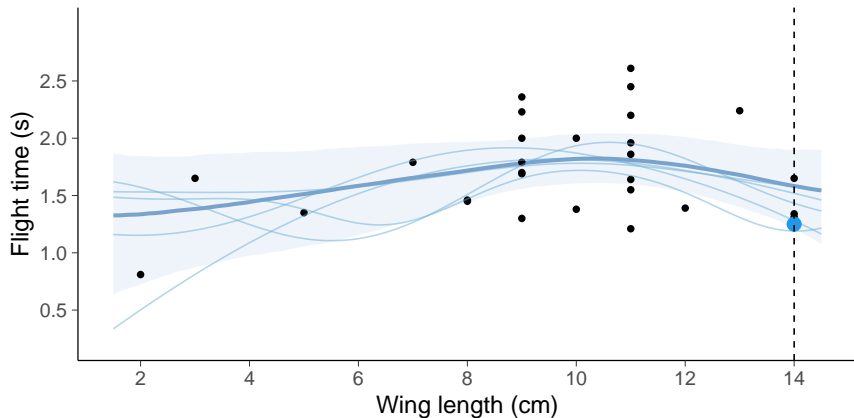
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



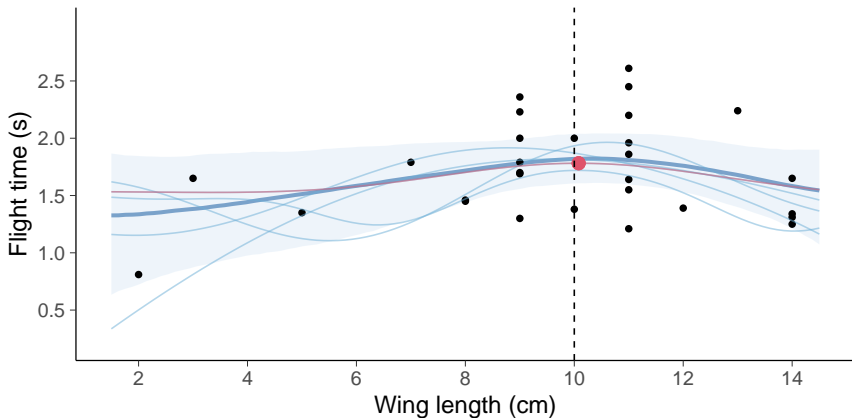
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



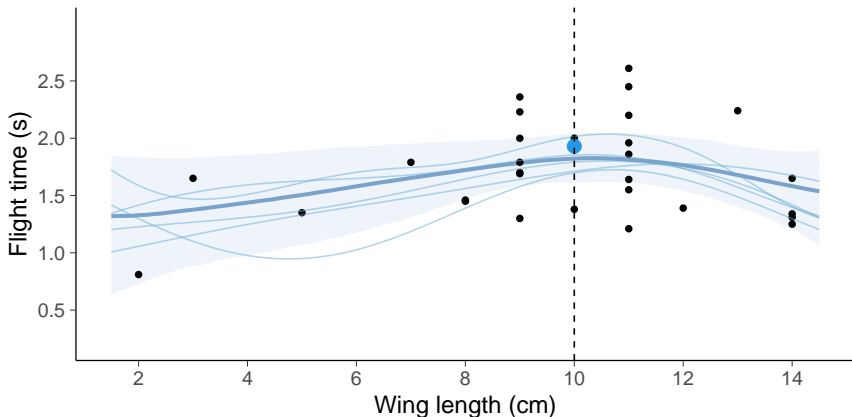
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



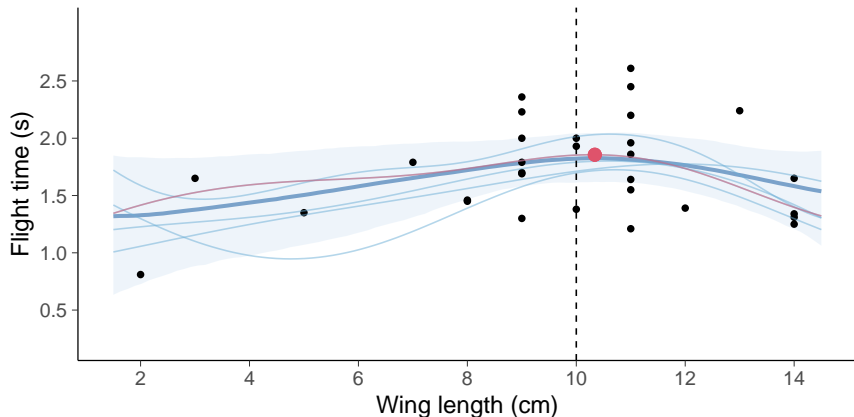
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



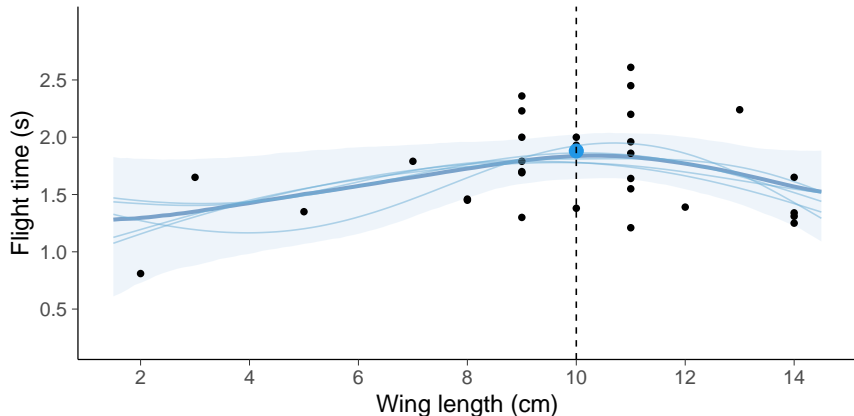
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Gaussian process model – Thompson sampling



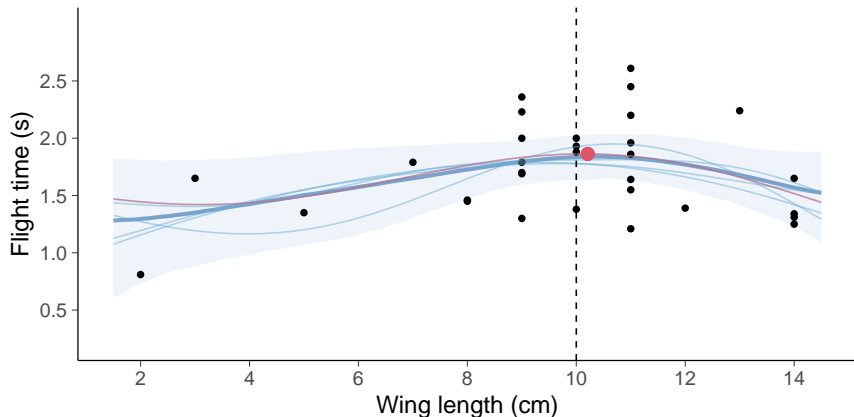
Bayesian optimization of wing length

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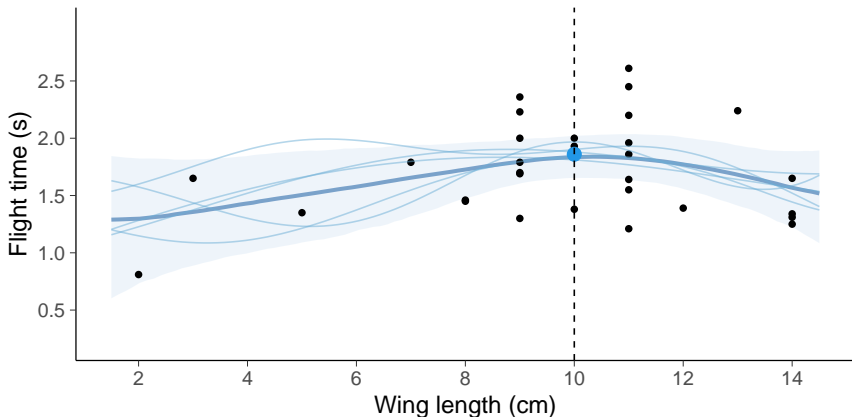
Bayesian optimization of wing length

Gaussian process model – Thompson sampling



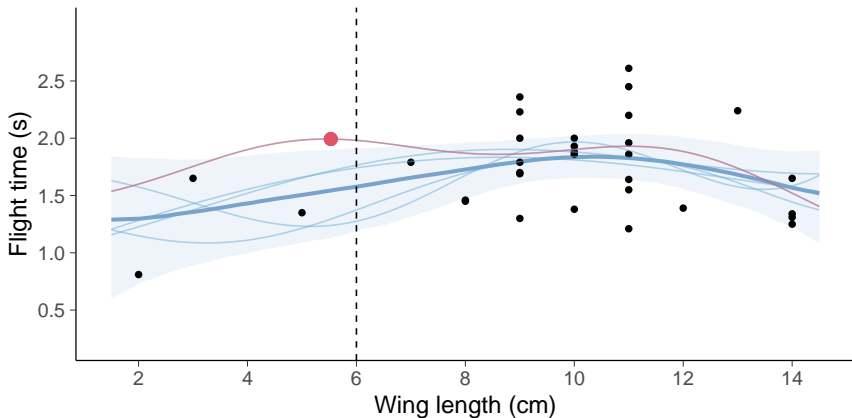
Bayesian optimization of wing length

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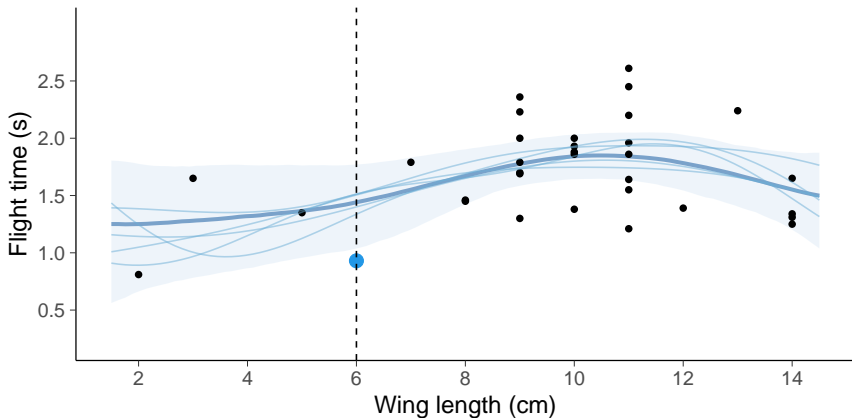
Bayesian optimization of wing length

Gaussian process model – Thompson sampling

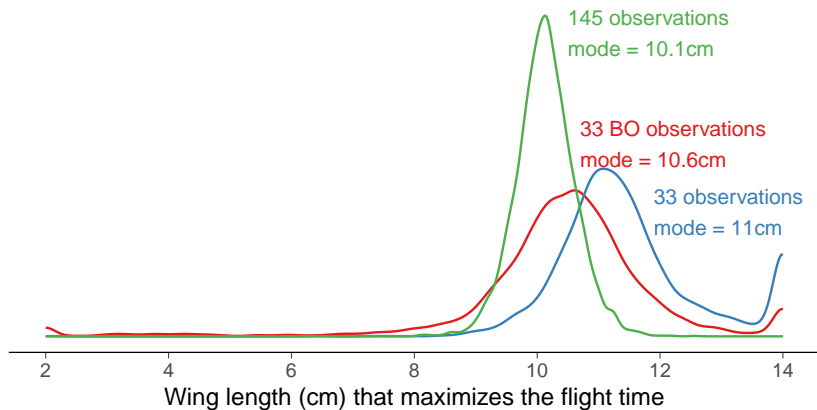


Bayesian optimization of wing length

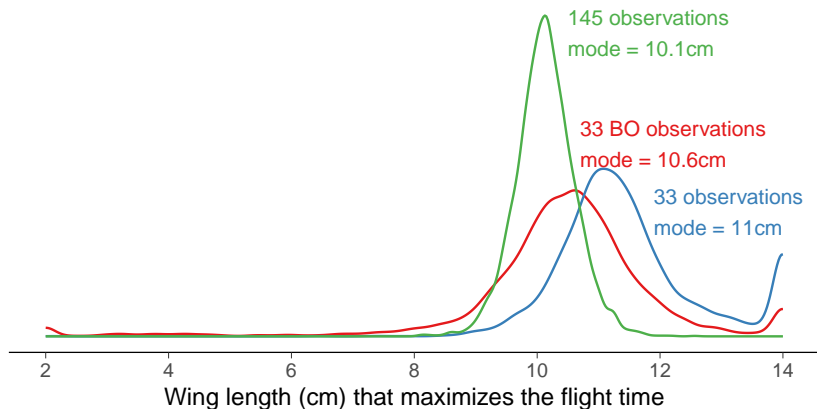
Gaussian process model – Thompson sampling



Bayesian optimization of wing length



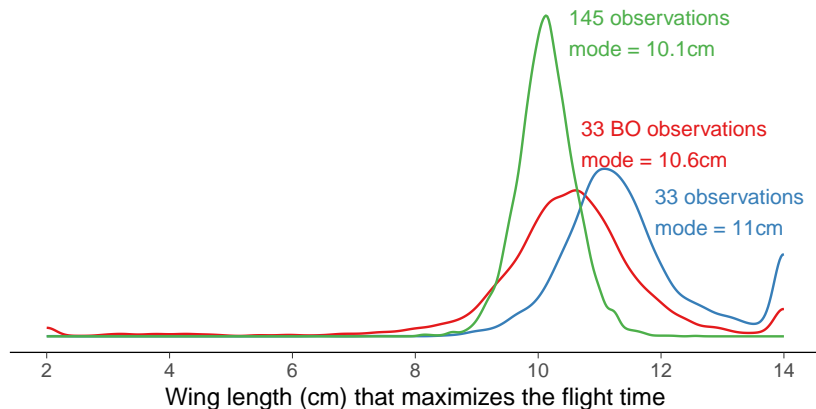
Bayesian optimization of wing length



33 BO obs. post. Wasserstein-1 distance ≈ 0.77

33 first obs. post. Wasserstein-1 distance ≈ 1.36

Bayesian optimization of wing length



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We obtain about 50% increase in efficiency

Examples of big Bayesian decision making success stories

- Bayesian optimization of ML algorithms
- Bayesian optimization of new medical molecules
- Bayesian optimization of new materials
- A/B testing
- Customer retention / satisfaction
- Marketing