CUCKOO SEARCH

IMPLEMENTATION AND PRELIMINARY TESTS

Presented by

Jorge Mario Cruz-Duarte,

Ph.D. Student

jorge.cruz@ugto.mx

CONACYT

Consejo Nacional de Ciencia y Tecnología

UNIVERSIDAD DE GUANAJUATO

DIVISIÓN DE INGENIERÍAS CAMPUS IRAPUATO-SALAMANCA

DICIS

Cuckoo Search was proposed by Yang, X. S. and Deb, S. in 2009 [1].

Cuckoo Search via Lévy Flights

Xin-She Yang
Department of Engineering,
University of Cambridge
Trumpinton Street
Cambridge CB2 1PZ, UK
Email: xy227@cam.ac.uk

Suash Deb
Department of Computer Science & Engineering
C. V. Raman College of Engineering
Bidyanagar, Mahura, Janla
Bhubaneswar 752054, INDIA
Email: suashdeb@gmail.com

***Abstract*—In this paper, we intend to formulate a new meta-heuristic algorithm, called Cuckoo Search (CS), for solving optimization problems. This algorithm is based on the obligate brood parasitic behaviour of some cuckoo species in combination with the Lévy flight behaviour of some birds and fruit flies. We validate the proposed algorithm against test functions and then compare its performance with those of genetic algorithms and particle swarm optimization. Finally, we discuss the implication of the results and suggestion for further research.**

***Index Terms*—algorithm; cuckoo search; Lévy flight; meta-heuristics; nature-inspired strategy; optimization;**

II. Cuckoo Behaviour and Lévy Flights

A. Cuckoo Breeding Behaviour

Cuckoo are fascinating birds, not only because of the beautiful sounds they can make, but also because of their aggressive reproduction strategy. Some species such as the *ani* and *Guira* cuckoos lay their eggs in communal nests, though they may remove others' eggs to increase the hatching probability of their own eggs [12]. Quite a number of species engage the obligate brood parasitism by laying their eggs in the

[1] Yang, X.-S., & Suash Deb. (2009). Cuckoo Search via Levy flights. In *2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)* (pp. 210–214). IEEE. http://doi.org/10.1109/NABIC.2009.5393690

A Brief Introduction

Image taken from: http://alpinebirding.zenfolio.com/p1056394909/h5AED9240#h5aed9240

Cuckoo birds behaviour

The brood parasitism

Images taken from: Wikipedia contributors. Brood parasite. Wikipedia, The Free Encyclopedia. April 2, 2016, 09:39 UTC. Available at: https://en.wikipedia.org/w/index.php?title=Brood_parasite&oldid=713161956. Accessed April 15, 2016. Wikipedia contributors. Striped cuckoo. Wikipedia, The Free Encyclopedia. February 11, 2016, 21:19 UTC. Available at: https://en.wikipedia.org/w/index.php?title=Striped_cuckoo&oldid=704481047. Accessed April 15, 2016.

Foraging behaviour

Survival strategies

Images taken from: http://i.kinja-img.com/gawker-media/image/upload/s--Y7LnrGO4--/18lonnm92lcl6jpg.jpg, http://cdn.instructables.com/FI5/L9MT/GE05CW88/FI5L9MTGE05CW88.LARGE.jpg

The basic idea

Three idealised rules:

1. One egg is laid in a randomly chosen nest by each cuckoo at a time

An egg represents a solution

2. High quality eggs (best nests) shall carry out to the next generation

Best nest or egg are best solution

3. There is a probability (p_D) for the host bird discover the cuckoo's egg hidden in its nest

p_D could be simplified as a proportion of nests being replaced

[1] MARICHELVAM, M.K., PRABAHARAN, T. and YANG, X.S. 2014. Improved cuckoo search algorithm for hybrid flow shop scheduling problems to minimize makespan. *Applied Soft Computing*. Vol. 19, pp. 93–101. ISSN 15684946.

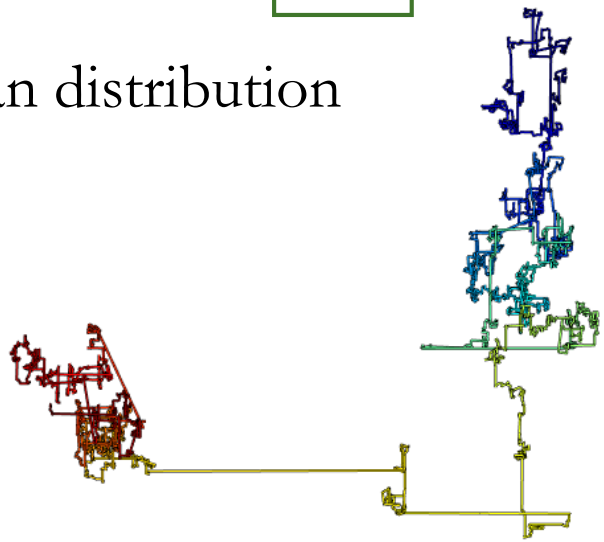
Looking for Nests via Lévy Flights

Lévy Flights have infinite mean and variance: Exploration is more efficient than performed by Gaussian processes.

$$\vec{r}(t+1) = \vec{r}(t) + \boxed{\vec{d}(t)}$$

"Independent, identically distributed (iid) random variables" in $\mathbb{R}^d$ [2].

Gaussian distribution



Lévy distribution

$$f(x;\mu,c) = \sqrt{\frac{c}{2\pi}}\ \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$$

(Lévy motion)

[1] X. Yang and S. Deb, "Cuckoo search: recent advances and applications," *Neural Comput. Appl.*, pp. 1–9, 2014.
[2] THAOKAR, R.M. 2008. *Brownian motion of a torus*. S.l. ISBN 9780521760188.

Image taken from: http://www.rspb.org.uk/discoverandenjoynature/discoverandlearn/birdguide/name/c/cuckoo/

Highlights

1. "**CS is a population-based algorithm** (like GA and PSO) **powered by some sort of elitism and/or selection** (similar to HS)."

2. "**The randomisation is more efficient** as the step length is heavy-tailed, and any large step is possible."

3. **"The number of parameters to be tuned is less than GA and PSO."**

4. "**Local and global search are controlled by the switching/discovery probability.**"

[1] X.-S. Yang and Suash Deb, "Cuckoo Search via Levy flights," in *2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)*, 2009, pp. 210–214.

Before Implement It

Image taken from: http://www.clipartqueen.com/image-files/bird-clip-art-swallow-nest.png

Definitions

Definition 1: Let $\mathfrak{X}^t = \{\vec{x}_1^t, \vec{x}_2^t, \dots, \vec{x}_{N_N}^t\}$ be a finite set of candidate solutions (or nests) of any optimisation problem in $\mathbb{R}^{N_D}$, with a cost function given by $\mathfrak{f}: \mathbb{R}^{N_D} \to \mathbb{R}$, N_N is called number of available nests and $\vec{x}_i^t \in \mathbb{R}^{N_D}$ denotes the i-th solution (or nest with an unique egg) at the time t of an iterative procedure.

Definition 2: Let $\vec{x}_*^t \in \mathfrak{X}^t$ be the best solution (or the best camouflaged cuckoo's egg in a nest) found at the t-th iteration, and determined by $\vec{x}_*^t = \operatorname{argmin}\left(\{\mathfrak{f}(\mathfrak{X}^t)\}, \mathfrak{f}(\vec{x}^*)\right)$, with $\{\mathfrak{f}(\mathfrak{X}^t)\} = \mathfrak{f}(\vec{x}_1^t), \mathfrak{f}(\vec{x}_2^t), \dots, \mathfrak{f}\left(\vec{x}_{N_N}^t\right)$.

Definition 3: $\mathfrak{X}^{t+1} = \{\vec{x}_1^{t+1}, \vec{x}_2^{t+1}, \dots, \vec{x}_{N_N}^{t+1}\}$ represents the finite set of new eggs (or candidate solutions) to be located in a different set of nests. Each new nest position $\vec{x}_i^{t+1}$ can be obtained through two stochastic strategies.

Definitions

Definition 4: Let $\lambda_{L,i}$ be a random number with a symmetric Lévy stable distribution. It can be obtained from Mantegna's algorithm, as follows,

$$\lambda_{L,i} = n_{0,\sigma}\left|n_{0,1}\right|^{-1/\xi}, \qquad \sigma^{\xi} = \frac{\sin\left(\frac{\pi\xi}{2}\right)\Gamma(\xi+1)}{\xi 2^{\frac{\xi-1}{2}}\Gamma\left(\frac{\xi+1}{2}\right)}$$

where $\xi \in [0.3, 2)$ is a distribution's parameter, $n_{0,\sigma}$ is a random number with normal distribution $\mathcal{N}(0,\sigma)$, and $\Gamma(\square)$ is the well-known Gamma function.

Definitions

Definition 5: Strategy 1—Let $\vec{x}_i^{t+1} \in \mathbb{R}^{N_D}$ a new nest position with a previous poorer solution $\vec{x}_i^t$, $\not{f}\left(\vec{x}_i^t\right) > \not{f}(\vec{x}_*^t)$, which is determined by

$$\vec{x}_i^{t+1} = \vec{x}_i^t + \delta_x \vec{\lambda}_L \odot \left(\vec{x}_i^t - \vec{x}_*^t\right),$$

where δ_x is the step size, which is usually set as 0.1 for $N_D \leq 3$ and 0.01 for $N_D > 3$; $\vec{\lambda}_L = \left(\lambda_{L,1}, \lambda_{L,2}, \ldots, \lambda_{L,N_D}\right)^T$ is a vector of i.i.d. symmetric Lévy stable random variables; and $\odot$ is the Hadamard-Schur product.

Definitions

Definition 6: Strategy 2—Let $\vec{x}_i^{t+1} \in \mathbb{R}^{N_D}$ a new nest position with an associated probability of $p_D \in [0,1]$, which is determined as

$$\vec{x}_i^{t+1} = \vec{x}_i^t + \vec{u} \odot \mathcal{H}(\vec{u} - p_D) \odot \left(\vec{x}_j^t - \vec{x}_k^t\right),$$

since $\vec{u} \in \mathbb{R}^{N_D}$ is a vector of i.i.d. uniform random variables between 0 and 1; $\mathcal{H}: \mathbb{R}^{N_D} \to \mathbb{R}^{N_D}$ is the extended Heaviside function, such as $\mathcal{H}(\vec{y}) = \left(\hbar(y_1), \hbar(y_2), \ldots, \hbar\left(y_{N_D}\right)\right)^T$, $\hbar: \mathbb{R} \to \mathbb{R}$, $\hbar(y) \coloneqq \frac{d}{dy}\max\{y, 0\}$ is the Heaviside function; and $\vec{x}_j^t, \vec{x}_k^t \in \mathfrak{X}^t$ with $j \neq k$.

CS' pseudocode

Input: Cost function $\mathbb{f}(\vec{y})$, $\vec{y} = \left(y_1, y_2, \ldots, y_{N_D}\right)^T$,

Parameters such as number of nests or population $(N_N > 2)$, step size (δ_x), switch probability (p_D), maximum number of generations (N_G) and stop criteria (if they exist).

Output: Best solution found, $\vec{x}_*^t \approx \operatorname{argmin}_{\vec{y}} \{\mathbb{f}(\vec{y})\}$.

Step 0. Set $t = 0$, initialise $\mathfrak{X}^t$ and identify $\vec{x}^*$.

Step 1. Obtain $\mathfrak{X}^{t+1}$ via $\vec{x}_i^{t+1} = \vec{x}_i^t + \delta_x \vec{\lambda}_L \odot \left(\vec{x}_i^t - \vec{x}_*^t\right)$,

Step 2. For each element of $\mathfrak{X}^{t+1}$, if $\mathbb{f}\left(\vec{x}_i^{t+1}\right) > \mathbb{f}\left(\vec{x}_i^t\right)$, make $\vec{x}_i^{t+1} = \vec{x}_i^t$, and update $\vec{x}^*$.

Step 3. Make $\mathfrak{X}^t = \mathfrak{X}^{t+1}$, choose randomly $\vec{x}_j^t$ and $\vec{x}_k^t$ from $\mathfrak{X}^t$, renew $\mathfrak{X}^{t+1}$ via $\vec{x}_i^{t+1} = \vec{x}_i^t + \vec{u} \odot \mathcal{H}(\vec{u} - p_D) \odot \left(\vec{x}_j^t - \vec{x}_k^t\right)$,

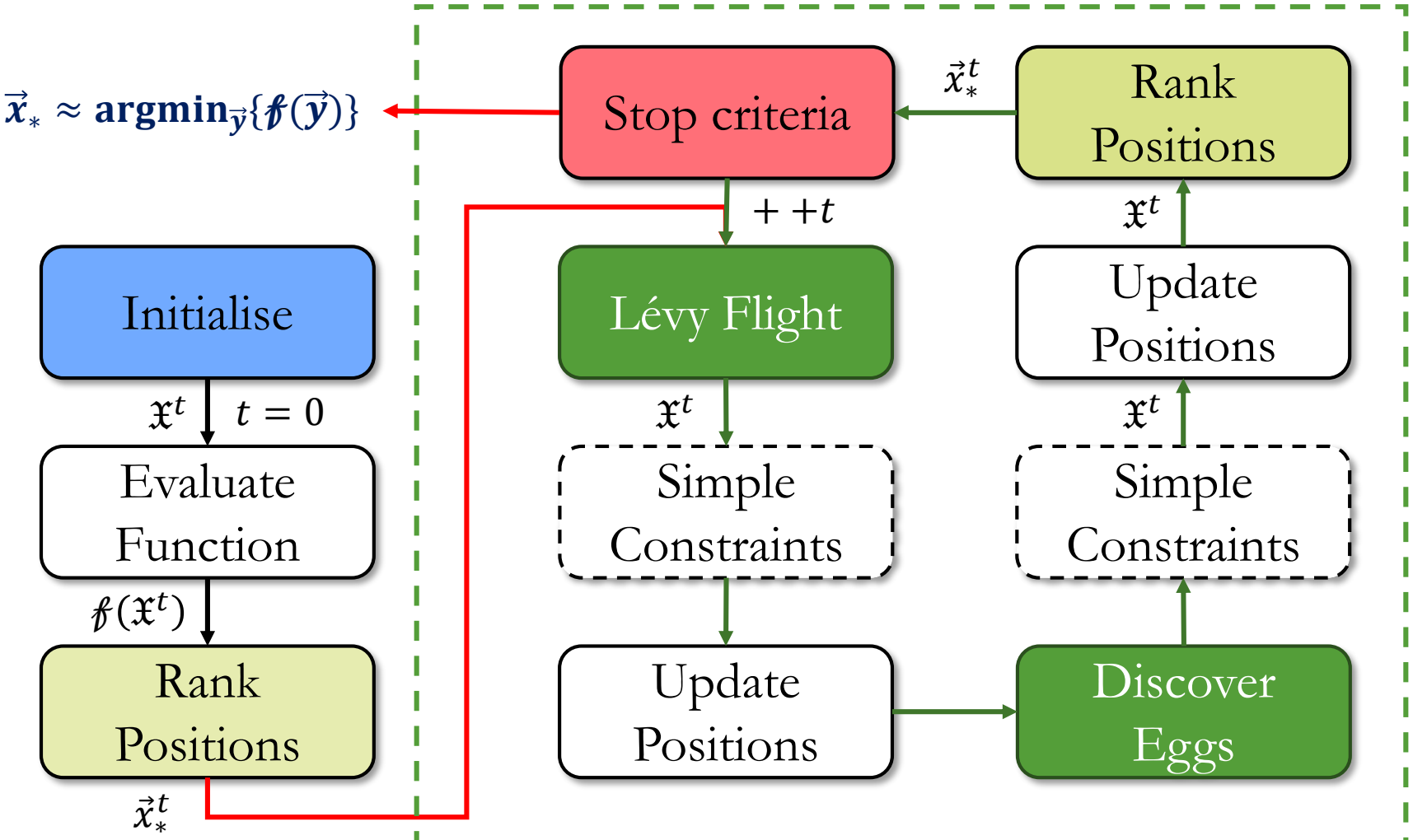
Step 4. For each element of $\mathfrak{X}^{t+1}$, if $\mathbb{f}\left(\vec{x}_i^{t+1}\right) > \mathbb{f}\left(\vec{x}_i^t\right)$, make $\vec{x}_i^{t+1} = \vec{x}_i^t$, and update $\vec{x}^*$.

Step 5. Increment $t = t + 1$ and verify if $t \leq N_G$ and stop criteria are not reached, go to Step 1. Otherwise, print $\vec{x}_*^t$.

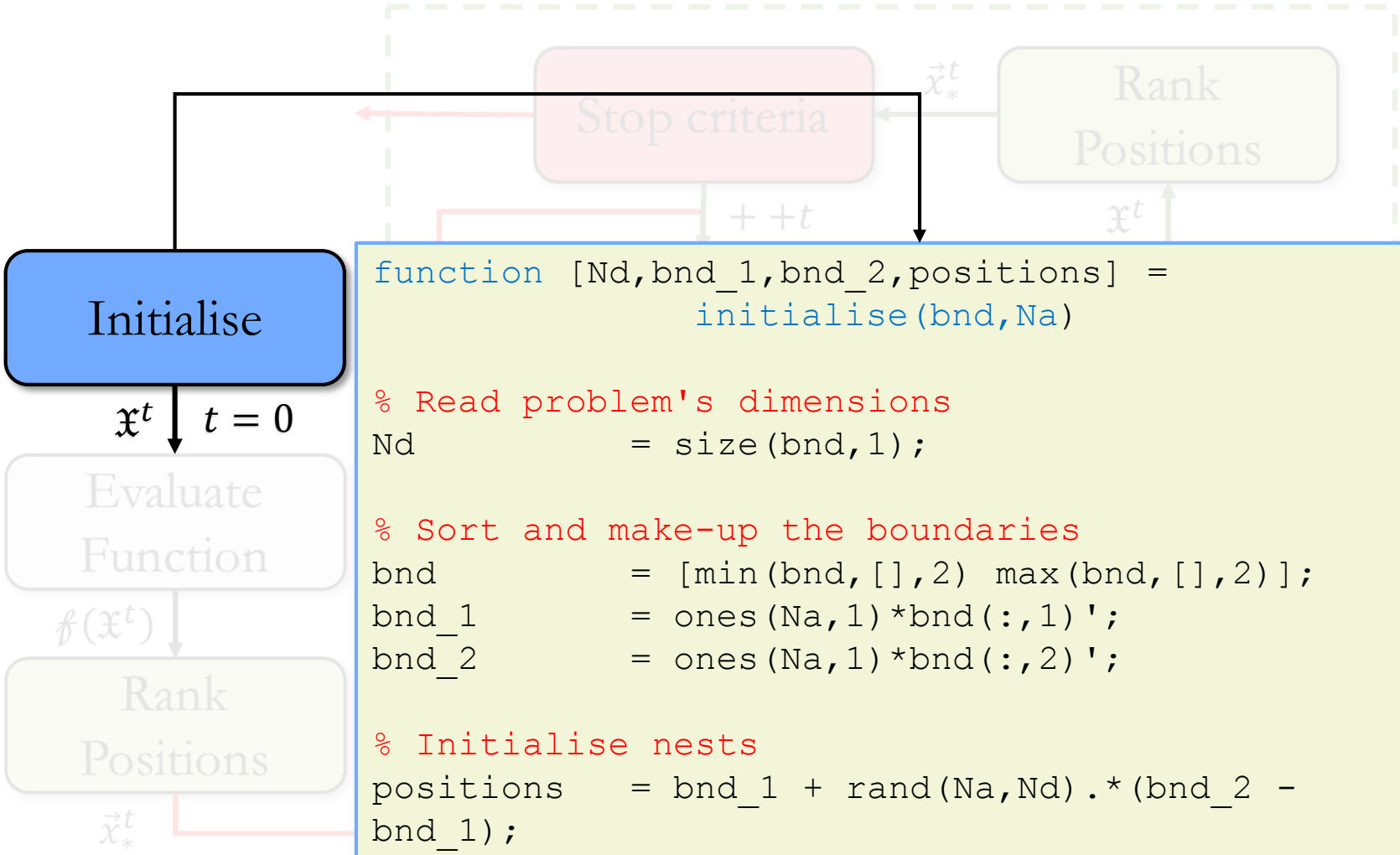
Implementing Cuckoo Search

Image taken from: http://www.planetofbirds.com/cuculiformes-cuculidae-black-billed-cuckoo-coccyzus-erythropthalmus

CS as a block diagram…

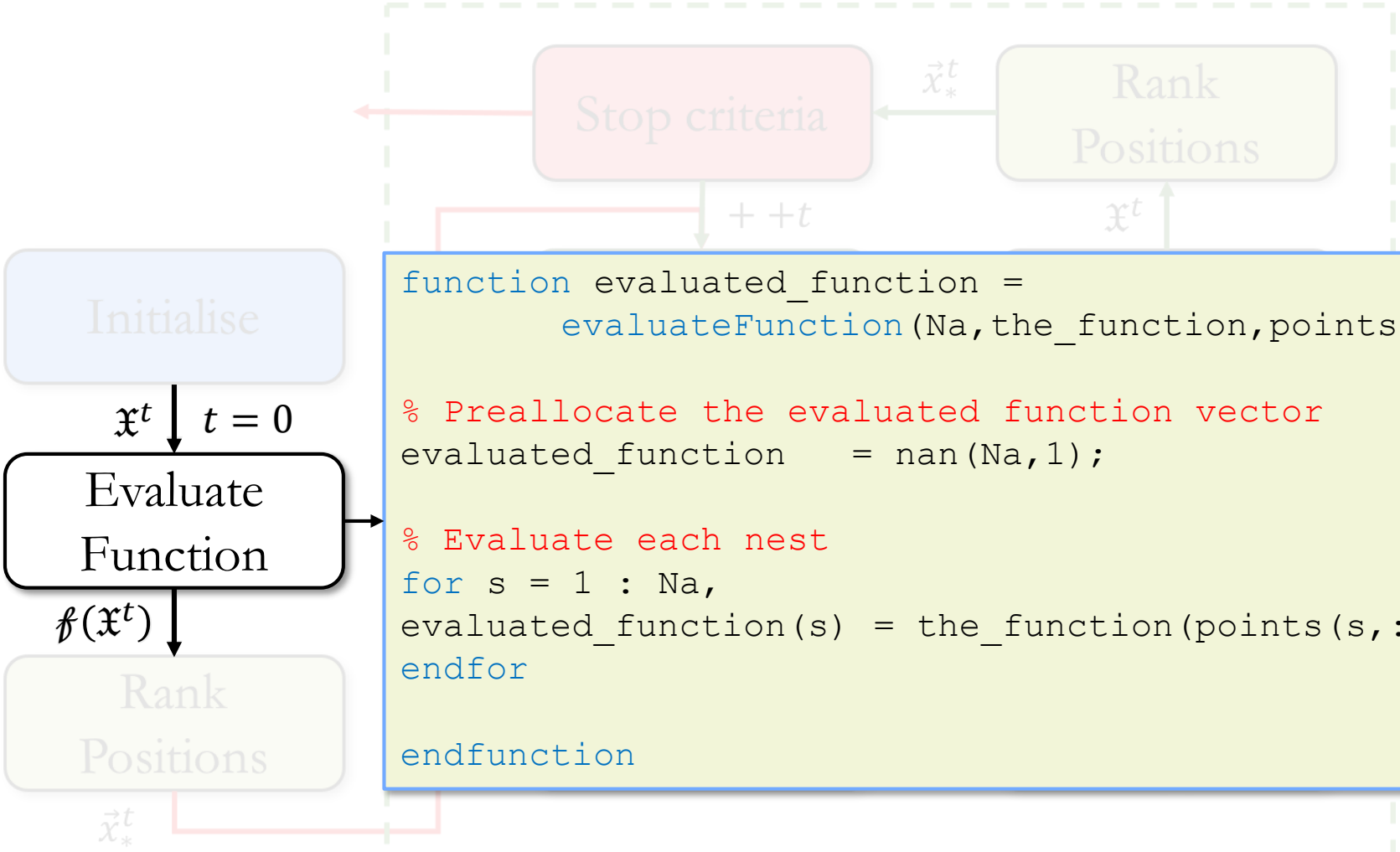


Initialise Function Block



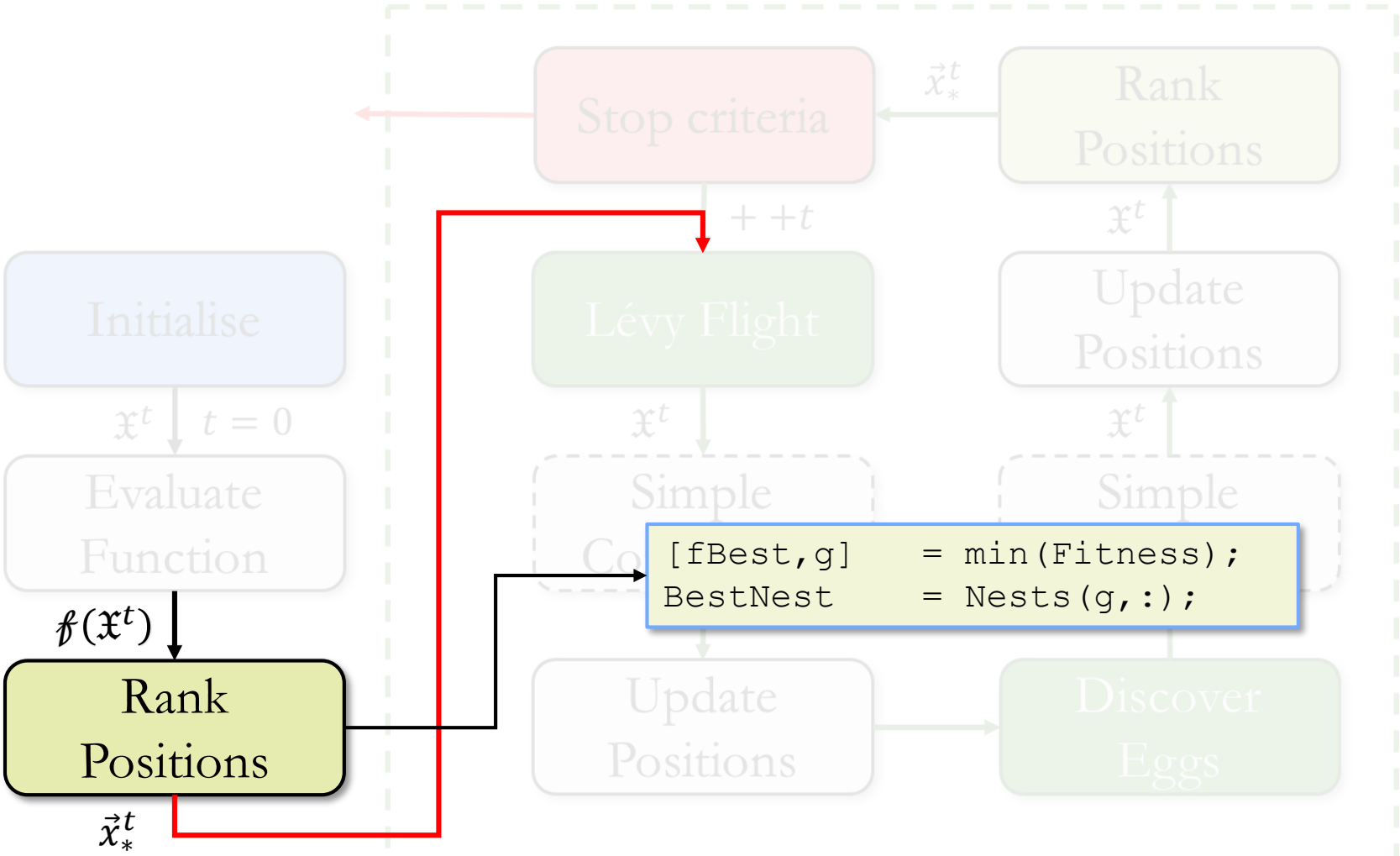

```
function [Nd,bnd_1,bnd_2,positions] =
                initialise(bnd,Na)

% Read problem's dimensions
Nd           = size(bnd,1);

% Sort and make-up the boundaries
bnd          = [min(bnd,[],2) max(bnd,[],2)];
bnd_1        = ones(Na,1)*bnd(:,1)';
bnd_2        = ones(Na,1)*bnd(:,2)';

% Initialise nests
positions    = bnd_1 + rand(Na,Nd).*(bnd_2 -
bnd_1);

endfunction
```

Evaluate Function Block



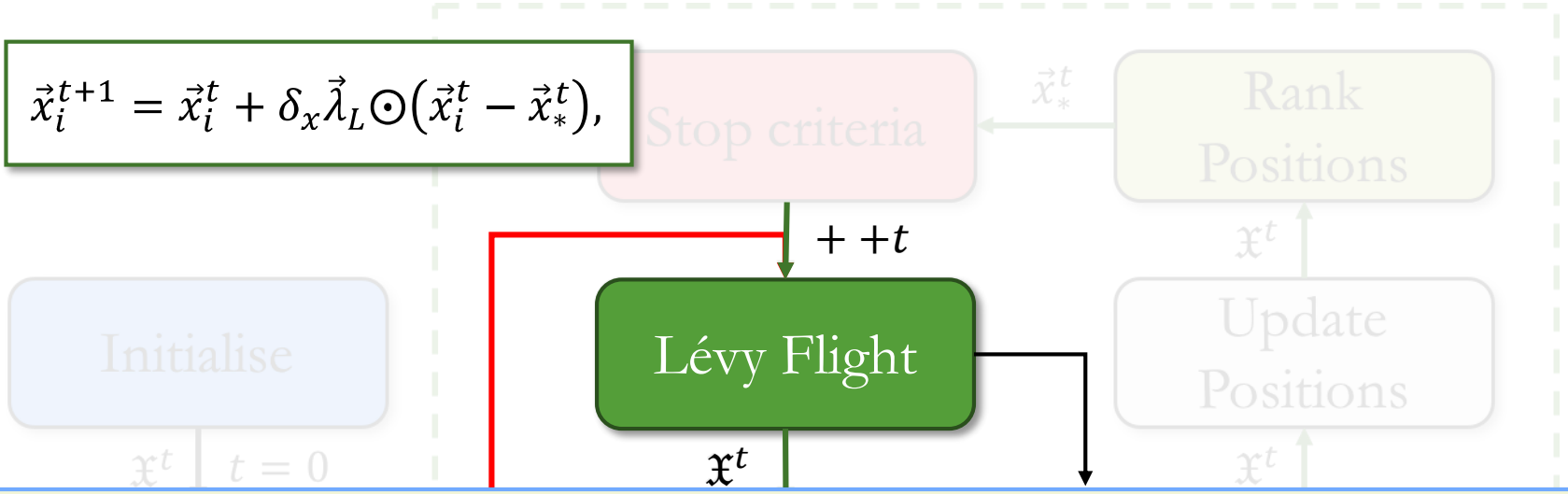

```
function evaluated_function =
        evaluateFunction(Na,the_function,points)

% Preallocate the evaluated function vector
evaluated_function    = nan(Na,1);

% Evaluate each nest
for s = 1 : Na,
evaluated_function(s) = the_function(points(s,:));
endfor

endfunction
```

Rank Positions Block



Lévy Flight Block

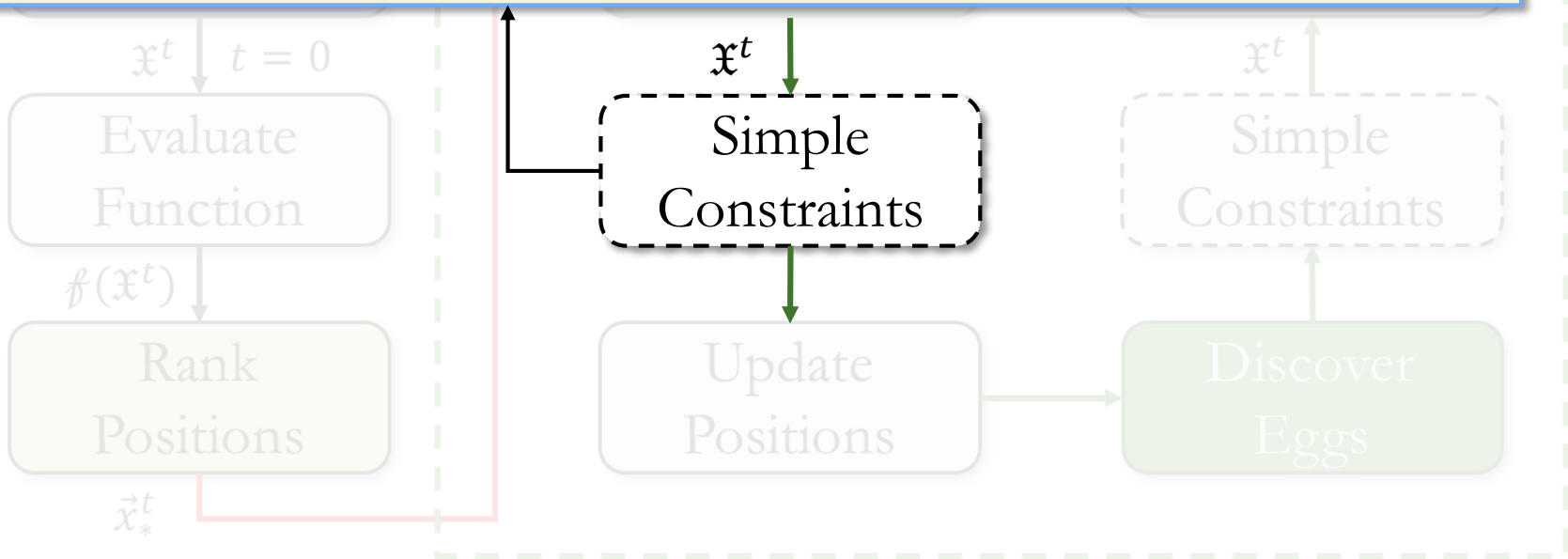
$$\vec{x}_i^{t+1} = \vec{x}_i^t + \delta_x \vec{\lambda}_L \odot \left(\vec{x}_i^t - \vec{x}_*^t\right),$$




```
function newNests = LevyFlight(Nests,Delta,Beta,BestNest)
% Read sizes
Nests   = size(Nests);

% Calculate std dev for N(0,Sigma) (Mantegna's algorithm)
Sigma   = (gamma(1 + Beta)*sin(pi*Beta/2)/(gamma((1 + Beta)/2)* ...
                Beta*2^((Beta - 1)/2)))^(1/Beta);

% Obtain Levy random variable: Lambda
Lambda      = (randn(sNests)*Sigma)./(abs(randn(sNests)).^(1/Beta));

% Update Nests' positions
newNests = Nests + Delta*randn(sNests).*Lambda.*(Nests-ones(sNests(1),1)*BestNest);
endfunction
```

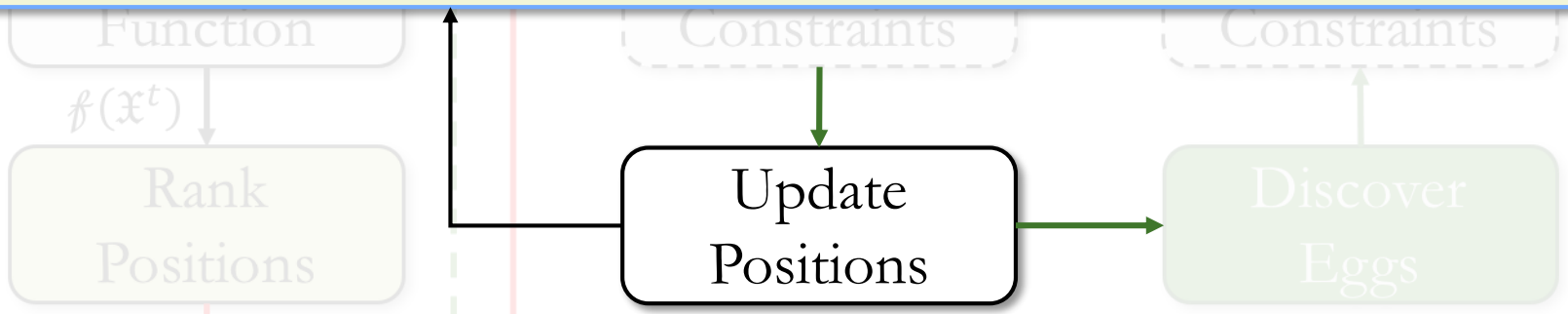
Simple Constraints Block

```
function Nests = simpleConstraints(Nests,lowerBoundaries,upperBoundaries)

% Check if solutions are inside the search space (feasible region)
check = Nests < lowerBoundaries;
Nests = ~check.*Nests + check.*lowerBoundaries;

check = Nests > upperBoundaries;
Nests = ~check.*Nests + check.*upperBoundaries;

endfunction
```



Update Positions Block

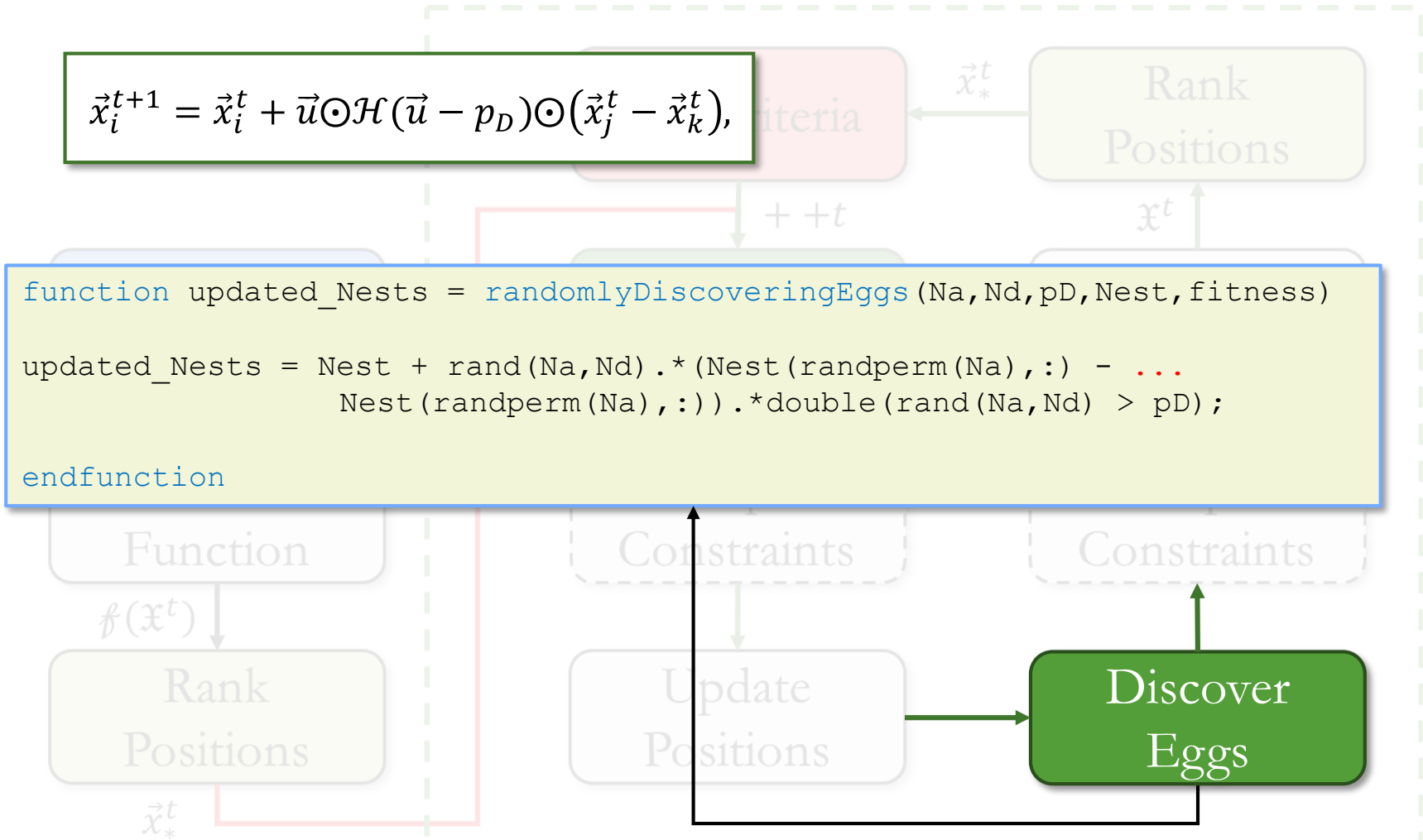
```
function [new_positions,new_fitness] =
        updatePositions(Na,Nd,the_function,fitness,positions,best_positions)

% Evaluate Function Block
evaluated_function     = evaluateFunction(Na,the_function,positions);

% Update best positions
condition      = evaluated_function < fitness;
new_positions  = (condition*ones(1,Nd)).*positions + ...
                 (!condition*ones(1,Nd)).*best_positions;
new_fitness    = min(evaluated_function,fitness);

endfunction
```



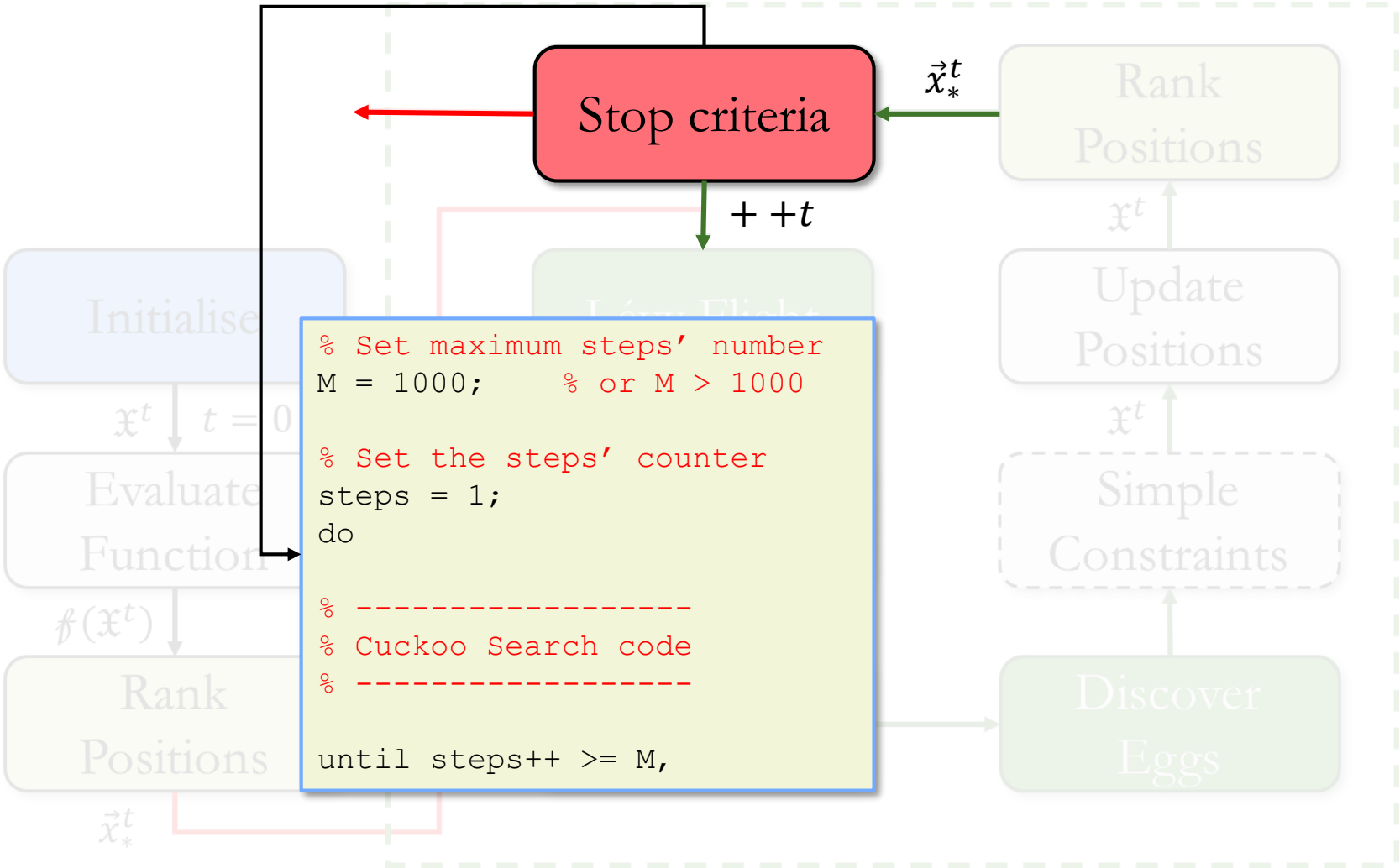
Discovering Eggs Block

$$\vec{x}_i^{t+1} = \vec{x}_i^t + \vec{u} \odot \mathcal{H}(\vec{u} - p_D) \odot (\vec{x}_j^t - \vec{x}_k^t),$$

```
function updated_Nests = randomlyDiscoveringEggs(Na,Nd,pD,Nest,fitness)

updated_Nests = Nest + rand(Na,Nd).*(Nest(randperm(Na),:) - ...
                 Nest(randperm(Na),:)).*double(rand(Na,Nd) > pD);

endfunction
```



Stop Criteria Block

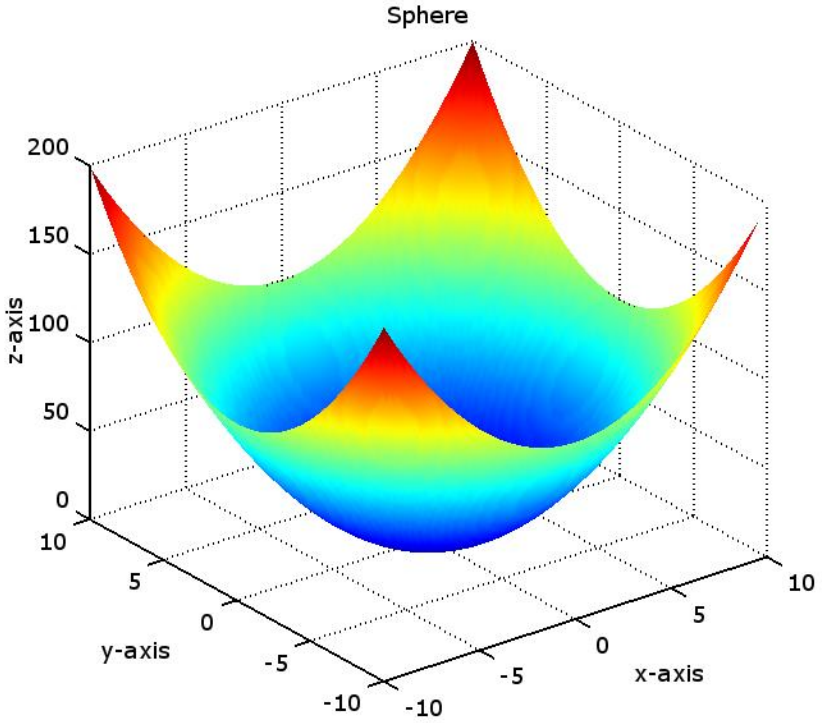


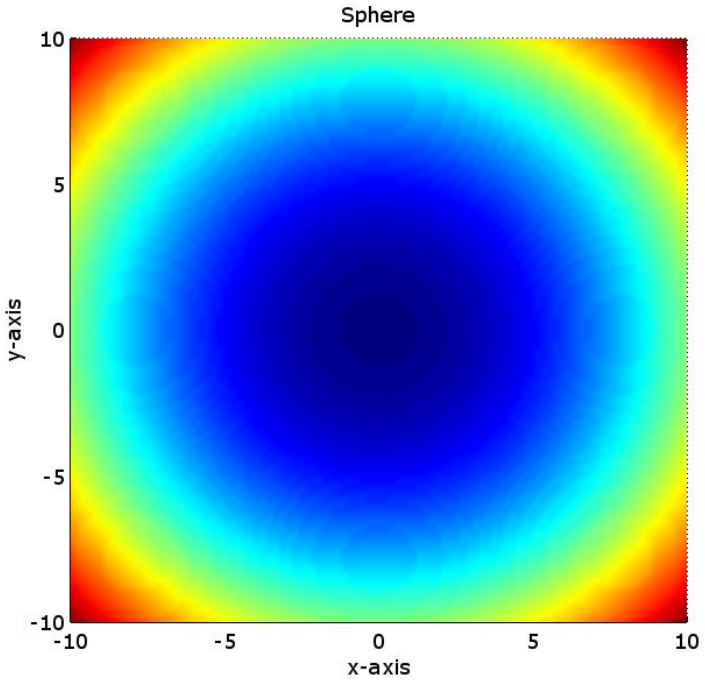
Testing Cuckoo Search

Image taken from: http://thegraphicsfairy.com/wp-content/uploads/blogger/-X-9JNfjFPJs/TkR4zWBWYMI/AAAAAAAANuU/Bv2WRiIYIok/s1600/nest%2Bvictorian%2Bimage%2Bgraphicsfairy009c.jpg

Benchmark Functions: Sphere

$$f(\vec{x}) = \sum_{i=1}^{N_D} x_i^2 \qquad -10 \leqslant \vec{x} \leqslant 10 \qquad \vec{x} = \left(x_1, x_2, \ldots, x_{N_D}\right)^T$$





$$f_{min} = \min f(\vec{x}^*) = 0 \qquad \vec{x}^* = \arg\min f(\vec{x}) = (0,0,\ldots,0)^T$$

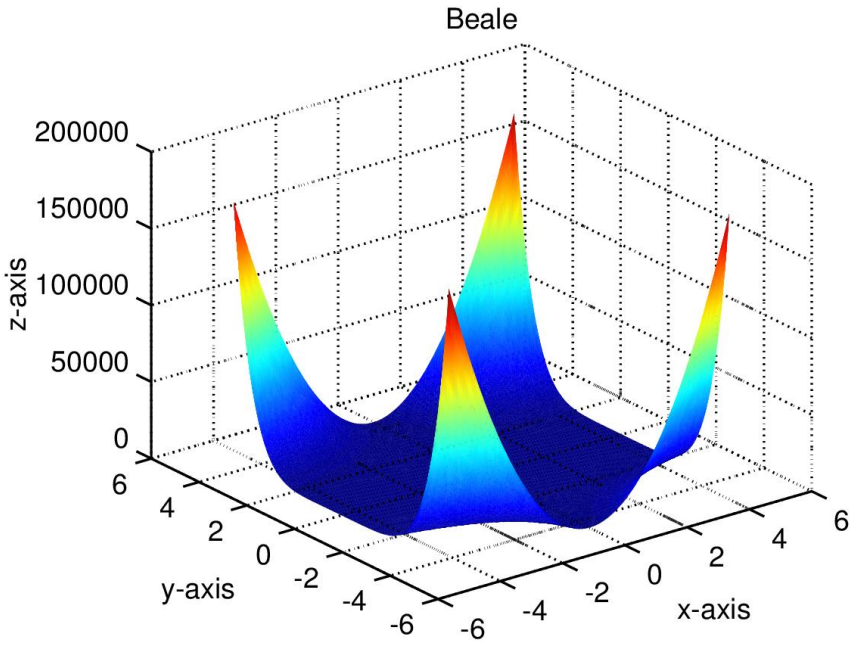
Soneji, H., & Sanghvi, R. C. (2014). Towards the improvement of Cuckoo search algorithm. *International Journal of Computer Information Systems and Industrial Management Applications*, *6*, 77–88. article.

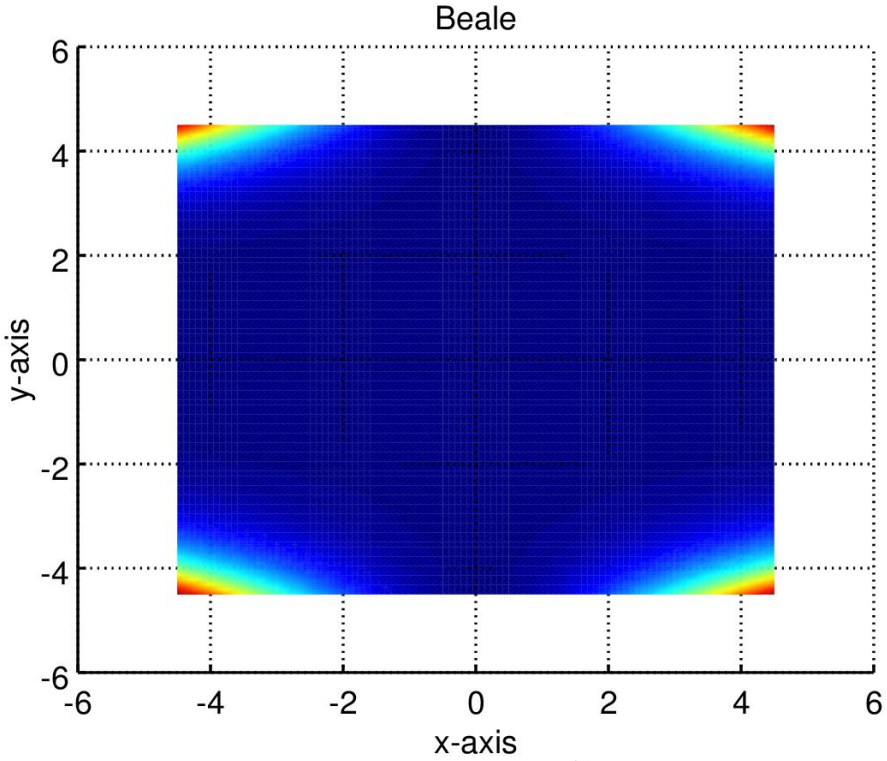
Benchmark Functions: Beale

$$f(\vec{x}) = (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$$

$$\vec{x} = (x_1, x_2)^T$$

$$-4.5 \preccurlyeq \vec{x} \preccurlyeq 4.5$$





$$\vec{x}^* = \arg\min f(\vec{x}) = (3, 0.5)^T$$

$$f_{min} = \min f(\vec{x}^*) = 0$$

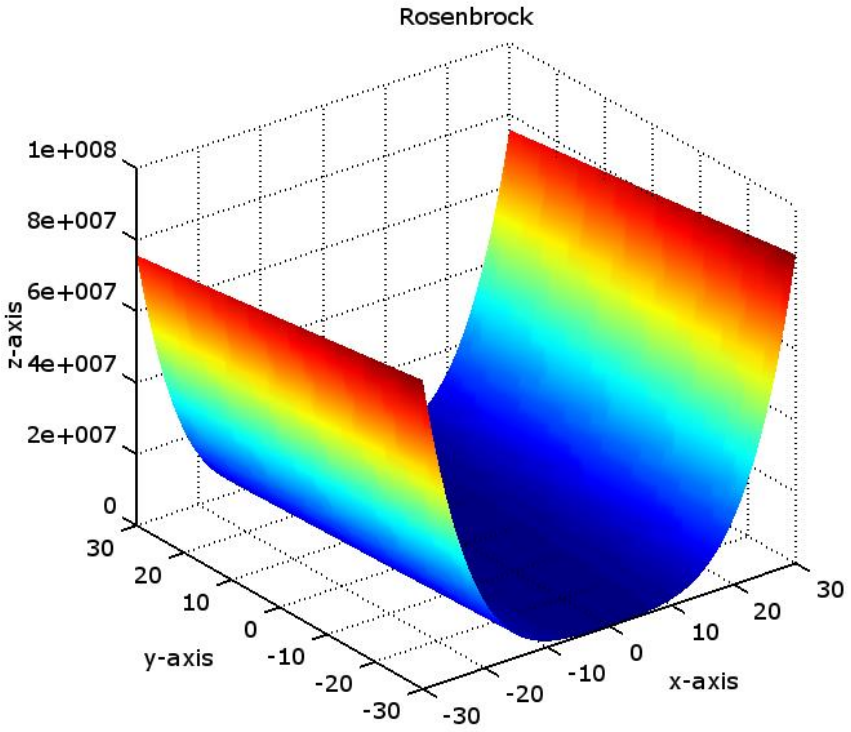
Jamil, M., & Yang, X. S. (2013). A literature survey of benchmark functions for global optimisation problems. *International Journal of Mathematical Modelling and Numerical Optimisation*, *4*(2), 150. http://doi.org/10.1504/IJMMNO.2013.055204

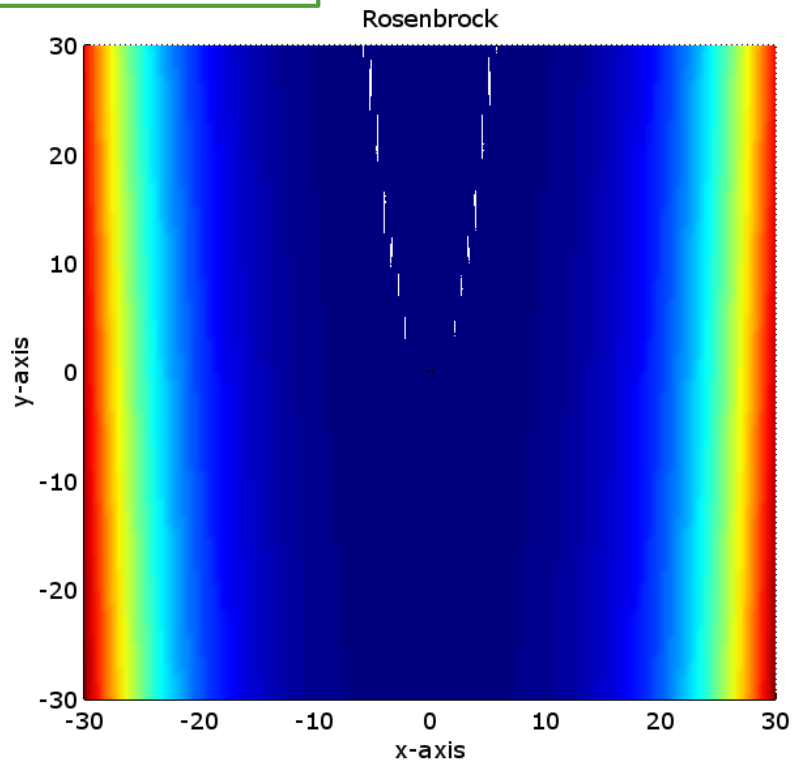
Benchmark Functions: Rosenbrock

$$f(\vec{x}) = \sum_{i=1}^{N_D-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$$

$$\vec{x} = \left(x_1, x_2, \ldots, x_{N_D}\right)^T$$

$$-30 \preccurlyeq \vec{x} \preccurlyeq 30$$



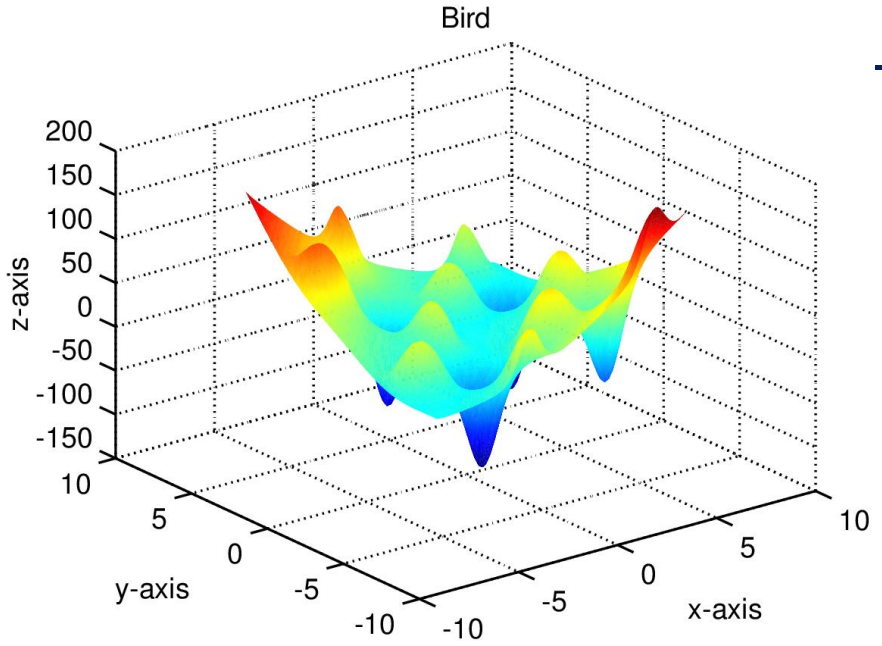


$$\vec{x}^* = \arg\min f(\vec{x}) = (1,1,\ldots,1)^T \qquad f_{min} = \min f(\vec{x}^*) = 0$$

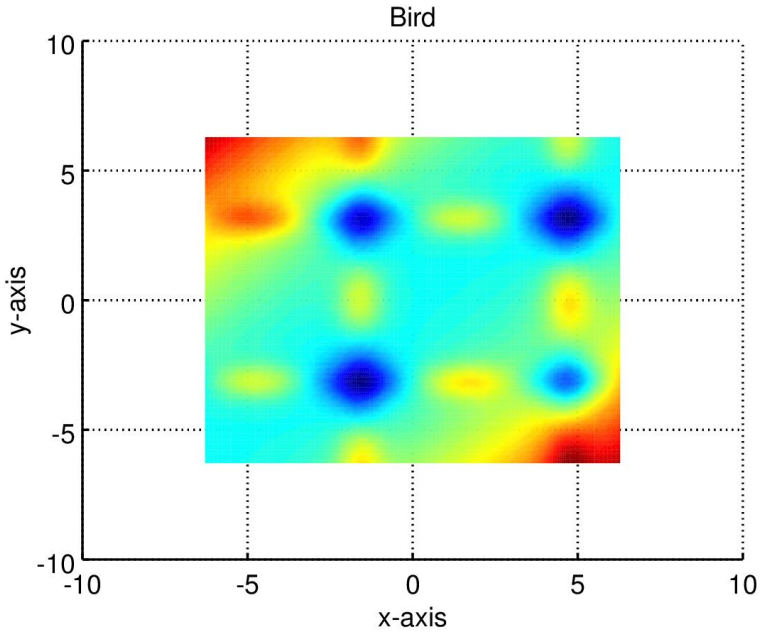
Jamil, M., & Yang, X. S. (2013). A literature survey of benchmark functions for global optimisation problems. *International Journal of Mathematical Modelling and Numerical Optimisation*, *4*(2), 150. http://doi.org/10.1504/IJMMNO.2013.055204

Benchmark Functions: Bird

$$f(\vec{x}) = \sin(x_1)\, e^{(1-\cos(x_2))^2} + \cos(x_2)\, e^{(1-\sin(x_1))^2} + (x_1 - x_2)^2$$



$$-2\pi \preccurlyeq \vec{x} \preccurlyeq 2\pi \quad \vec{x} = (x_1, x_2)^T$$



$$\vec{x}^* = \arg\min f(\vec{x})$$
$$= (4.70104, 3.15294)^T$$
$$\text{or } (-1.58214, -3.13024)^T$$

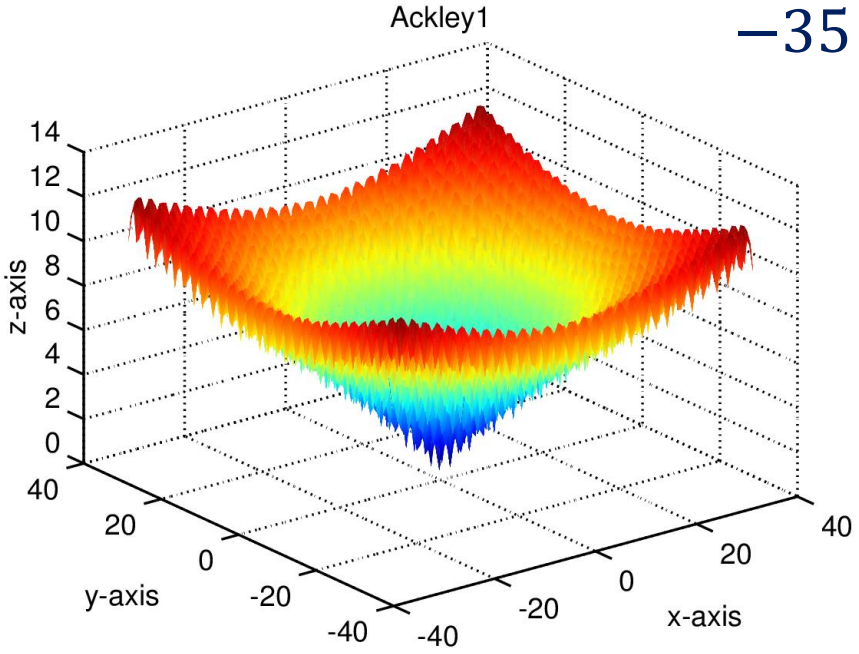
$$f_{min} = \min f(\vec{x}^*)$$
$$= -106.764537$$

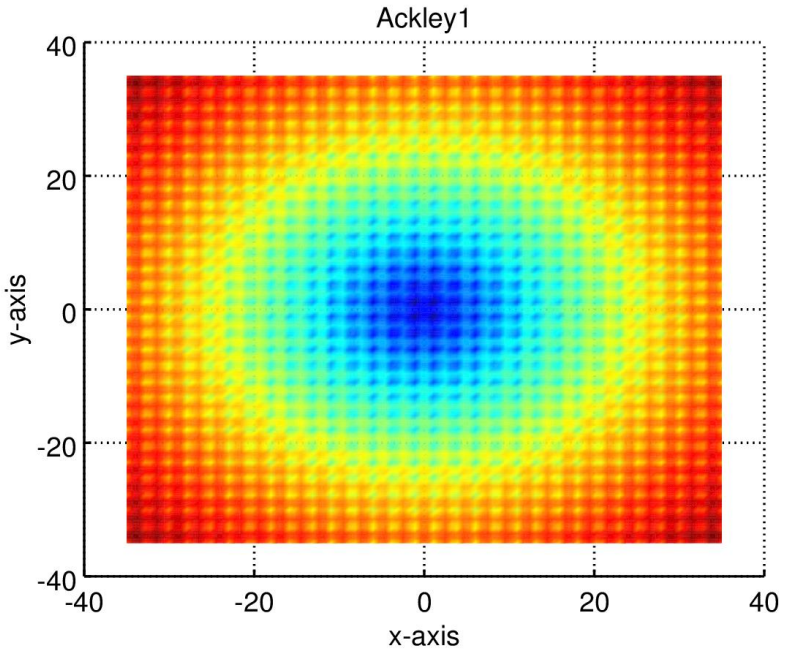
Jamil, M., & Yang, X. S. (2013). A literature survey of benchmark functions for global optimisation problems. *International Journal of Mathematical Modelling and Numerical Optimisation*, *4*(2), 150. http://doi.org/10.1504/IJMMNO.2013.055204

Benchmark Functions: Ackley1

$$f(\vec{x}) = -20e^{-0.02\sqrt{\frac{1}{N_D}\sum_{i=1}^{N_D} x_i^2}} - e^{\frac{1}{N_D}\sum_{i=1}^{N_D}\cos(2\pi x_i)} + 20 + e$$

$$-35 \leqslant \vec{x} \leqslant 35 \qquad \vec{x} = \left(x_1, x_2, \ldots, x_{N_D}\right)^T$$





$$f_{min} = \min f(\vec{x}^*) = 0 \qquad \vec{x}^* = \arg\min f(\vec{x}) = (0,0,\ldots,0)^T$$

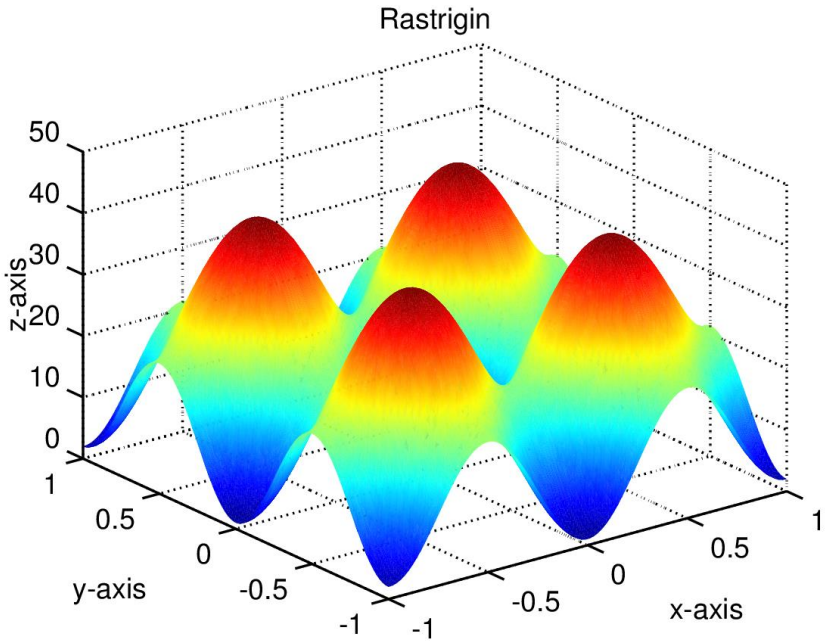
Jamil, M., & Yang, X. S. (2013). A literature survey of benchmark functions for global optimisation problems. *International Journal of Mathematical Modelling and Numerical Optimisation*, *4*(2), 150. http://doi.org/10.1504/IJMMNO.2013.055204

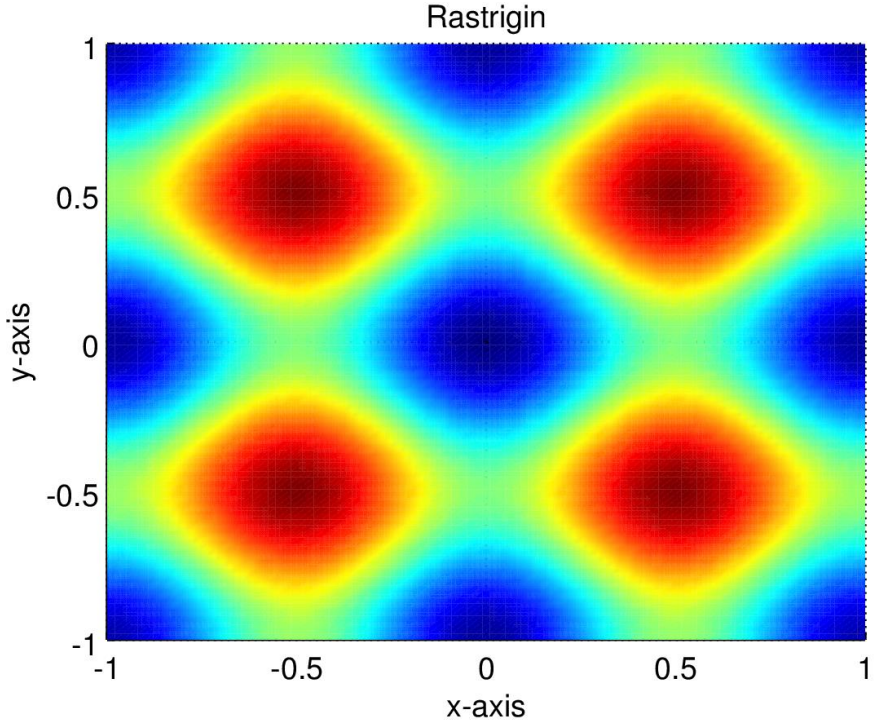
Benchmark Functions: Rastrigin

$$f(\vec{x}) = 10N_D + \sum_{i=1}^{N_D}\left(x_i^2 - 10\cos(2\pi x_i)\right)$$

$$-0.9 \leqslant \vec{x} \leqslant 0.9$$

$$\vec{x} = \left(x_1, x_2, \ldots, x_{N_D}\right)^T$$



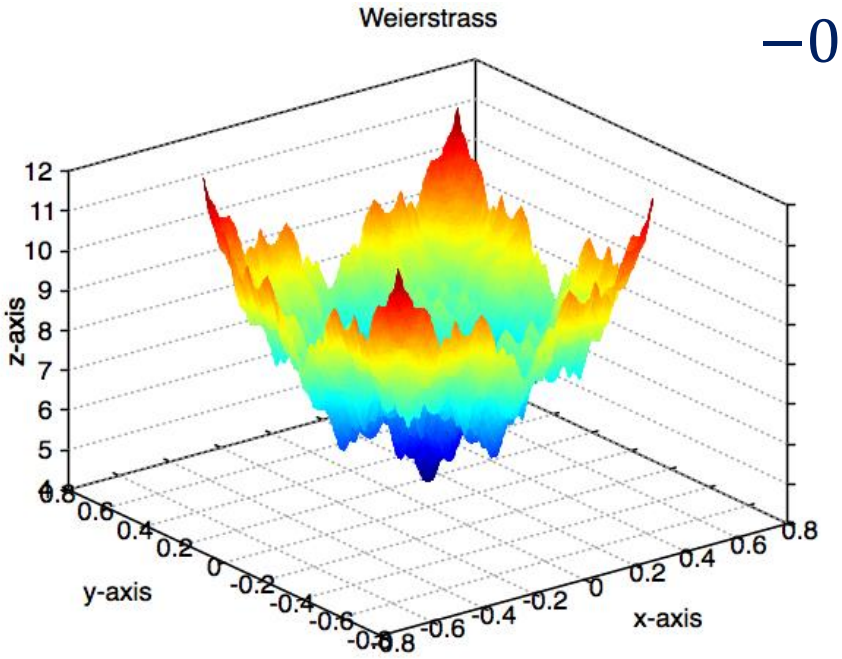


$$f_{min} = \min f(\vec{x}^*) = 0 \qquad \vec{x}^* = \arg\min f(\vec{x}) = (0,0,\ldots,0)^T$$

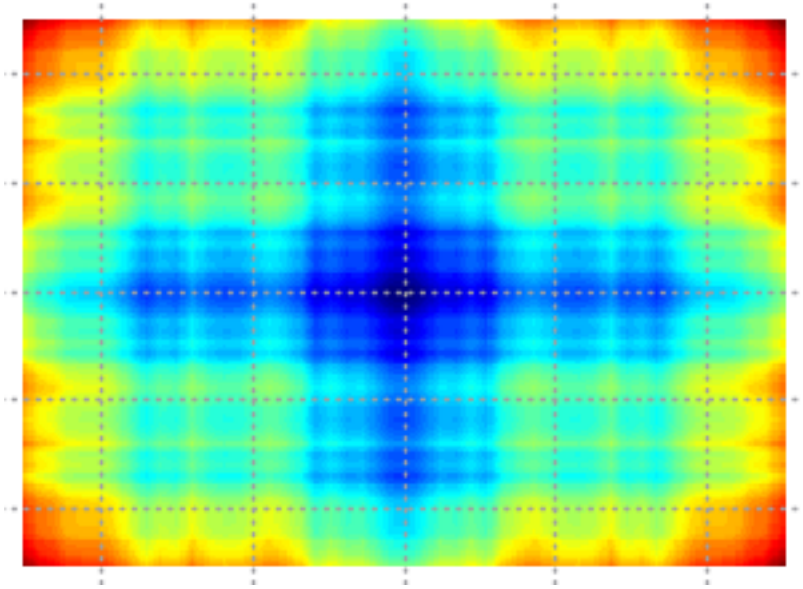
Soneji, H., & Sanghvi, R. C. (2014). Towards the improvement of Cuckoo search algorithm. *International Journal of Computer Information Systems and Industrial Management Applications*, *6*, 77–88. article.

Benchmark Functions: Weierstrass

$$f(\vec{x}) = \sum_{i=1}^{N_D} \left[\sum_{k=0}^{k_{max}} a^k \cos\left(2\pi b^k (x_i + 0.5)\right) - N_D \sum_{k=0}^{k_{max}} a^k \cos\left(\pi b^k\right) \right]$$



$$-0.5 \preccurlyeq \vec{x} \preccurlyeq 0.5 \qquad \vec{x} = \left(x_1, x_2, \ldots, x_{N_D}\right)^T$$



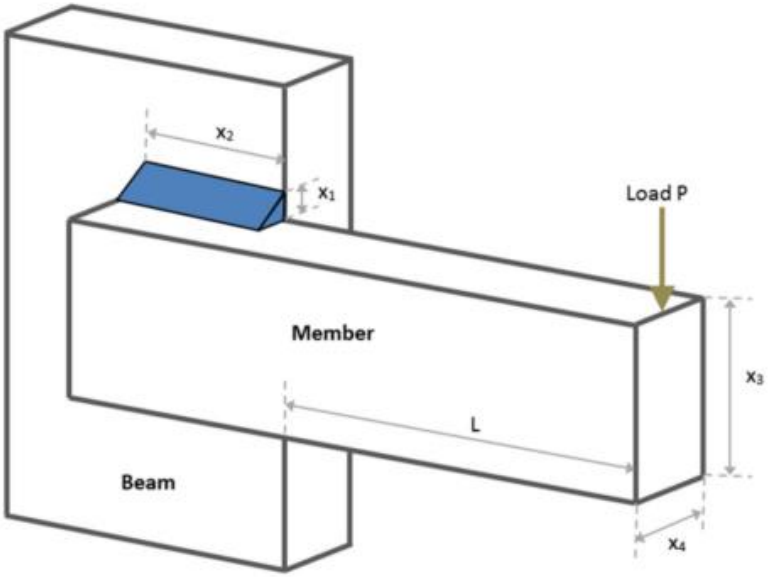
$$f_{min} = \min f(\vec{x}^*) = 4 \qquad \vec{x}^* = \arg\min f(\vec{x}) = (0,0,\ldots,0)^T$$

Soneji, H., & Sanghvi, R. C. (2014). Towards the improvement of Cuckoo search algorithm. *International Journal of Computer Information Systems and Industrial Management Applications*, *6*, 77–88. article.

Welded Beam Design Problem

Find the minimum fabrication cost considering four design variables: the width $(w = x_1)$ and length $(L = x_2)$ of the welded area, the depth $(h = x_3)$ and thickness $(d = x_4)$ of the main beam [1,2].

Shear stress (τ), bending stress in the beam (σ), buckling load on the bar (P_c) and deflection of the beam (δ).



$$0.125 \leq x_1 \leq 5$$

$$0.1 \leq x_2, x_3, x_4 \leq 10$$

x_1 and x_2 are discrete variables
and integer multiples of 0.0065 in.

$$x^* = (0.205730, 3.470489, 9.036624, 0.205729)$$

where $f(x^*) = 1.724852$.

[1] YANG, X.-S. and DEB, S. 2010. Engineering Optimisation by Cuckoo Search. *Int. J. Mathematical Modelling and Numerical Optimisation*. Vol. 1, no. 4, pp. 330–343. ISSN 2040-3607.

[2] Rakhshani, H., & Rahati, A. (2016). Intelligent Multiple Search Strategy Cuckoo Algorithm for Numerical and Engineering Optimization Problems. *Arabian Journal for Science and Engineering*. http://doi.org/10.1007/s13369-016-2270-8

Welded Beam Design Problem

$$\min_x f(x) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2)$$

Subject to:

$$g_1(x) : \tau(x) - \tau_{\max} \leq 0,$$
$$g_2(x) : \sigma(x) - \sigma_{\max} \leq 0,$$
$$g_3(x) : x_1 - x_4 \leq 0,$$
$$g_4(x) : 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \leq 0,$$
$$g_5(x) : 0.125 - x_1 \leq 0,$$
$$g_6(x) : \delta(x) - \delta_{\max} \leq 0,$$
$$g_7(x) : P - P_c(x) \leq 0,$$

$P = 6000\text{lb}$, $L = 14\,\text{in}$, $E = 30 \times 10^6\,\text{psi}$, $G = 12 \times 10^6\,\text{psi}$, $\tau_{\max} = 13{,}600\,\text{psi}$, $\sigma_{\max} = 30{,}000\,\text{psi}$, $\delta_{\max} = 0.25\,\text{in}$.

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau'' \frac{x_2}{2R} + (\tau'')^2},$$

$$\tau' = \frac{P}{\sqrt{2} x_1 x_2},$$

$$\tau'' = \frac{MR}{J},$$

$$M = P\left(l + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{\sqrt{2} x_1 x_2 \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma(x) = \frac{6PL}{x_4 x_3^2},$$

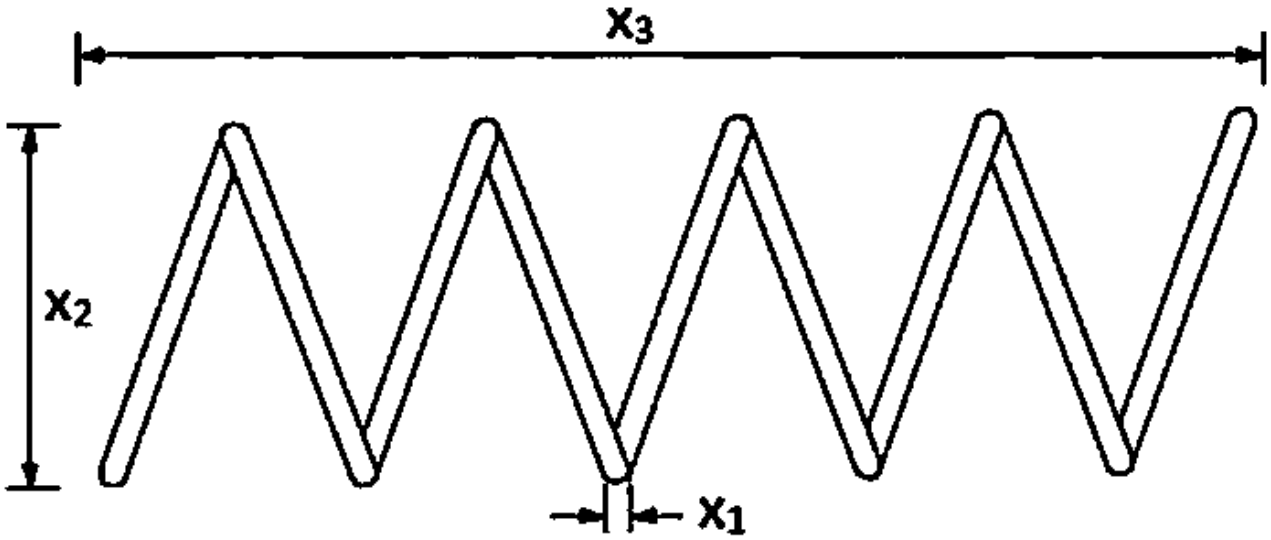
$$\delta(x) = \frac{4PL^3}{E x_3^3 x_4},$$

$$P_c(x) = \frac{4.013 E \sqrt{\frac{x_3^2 x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

[1] YANG, X.-S. and DEB, S. 2010. Engineering Optimisation by Cuckoo Search. *Int. J. Mathematical Modelling and Numerical Optimisation*. Vol. 1, no. 4, pp. 330–343. ISSN 2040-3607.

Spring Design Problem

Find the minimum weight of a spring. Its design variables are: the wire diameter (x_1), the mean coil diameter (x_2) and the number of active coils (x_3) [1].



$$0.05 \leq x_1 \leq 1$$

$$0.25 \leq x_2 \leq 1.3$$

$$2 \leq x_3 \leq 15$$

Best solution:

$$x^* = (0.051690, 0.356750, 11.287126)$$

where $f(x^*) = 0.012665$.

[1] Rakhshani, H., & Rahati, A. (2016). Intelligent Multiple Search Strategy Cuckoo Algorithm for Numerical and Engineering Optimization Problems. *Arabian Journal for Science and Engineering*. http://doi.org/10.1007/s13369-016-2270-8

[2] Cagnina, L. C., Esquivel, S. C. & Coello Coello, C. A. Solving engineering optimization problems with the simple constrained particle swarm optimizer. *Inform.* **32,** 319–326 (2008).

Spring Design Problem

$$\min_x f(x) = (x_3 + 2)\, x_2 x_1^2$$

Subject to:

$$g_1(x) : 1 - \frac{x_2^3 x_3}{71{,}785 x_1^4} \leq 0,$$

$$g_2(x) : \frac{4x_2^2 - x_1 x_2}{12{,}566\,(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) : 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0,$$

$$g_4(x) : \frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

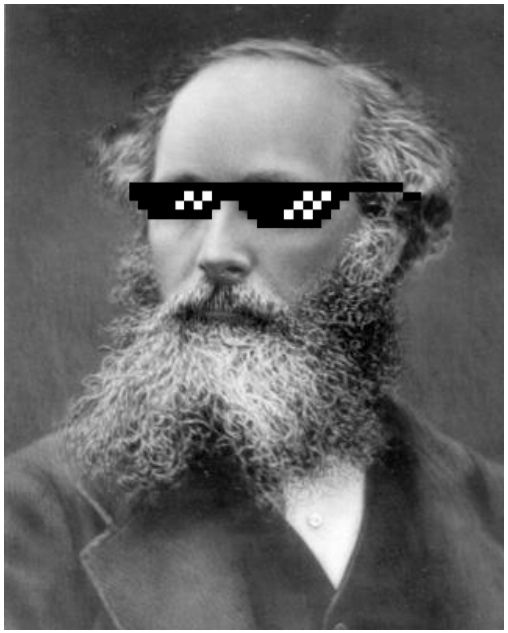
$$0.05 \leq x_1 \leq 1$$

$$0.25 \leq x_2 \leq 1.3$$

$$2 \leq x_3 \leq 15$$

[1] YANG, X.-S. and DEB, S. 2010. Engineering Optimisation by Cuckoo Search. *Int. J. Mathematical Modelling and Numerical Optimisation.* Vol. 1, no. 4, pp. 330–343. ISSN 2040-3607.

[2] Cagnina, L. C., Esquivel, S. C. & Coello Coello, C. A. Solving engineering optimization problems with the simple constrained particle swarm optimizer. *Inform.* **32,** 319–326 (2008).



Thanks!