Thanks [zzq](https://www.cnblogs.com/zzqsblog/p/9904271.html) for his improvement on my original observation.

Given a [multiplicative function](https://en.wikipedia.org/wiki/Multiplicative_function) , we’d like to calculate its prefix sum .

For any multiplicative function , there exists another multiplicative function satisfying that where denotes the [Dirichlet convolution](https://en.wikipedia.org/wiki/Dirichlet_convolution). Since they are multiplicative, it is enough to expand and check their values at prime powers: where is prime. Specifically, . Denote the prefix sum of as , i.e., . We have . If there is a multiplicative function such that the corresponding satisfies that at primes (or, equivalently, ), to calculate we only need to consider those which is a [powerful number](https://en.wikipedia.org/wiki/Powerful_number). (Explanation: if is not powerful, is zero, thus they do not contribute to the sum and we can safely ignore them). Since there are powerful numbers, if can be calculated in time complexity , can be calculated in time complexity where is the Riemann zeta function. In practice, it runs very fast.

A few examples:

1. : we can let . The final time complexity is .
2. : we can let , the divisor counting function. The final time complexity is .
3. : we can let , the Euler’s totient function. The final time complexity is .

Note that if we let the multiplicative function , the above method converts the problem “summing a multiplicative function” into a problem “summing a completely multiplicative function”. We can just focus on the latter one to solve the class of these problems.