Communication Systems Lab
Lab Manual 12
Random Processes

Pre-Lab Assignment

Familiarize yourself with MATLAB's **rand**, **repmat** and **sum** functions. Also practice matrix indexing using colons. You will have to use these commands in this lab session.

Theoretical Background

The understanding of random processes is important because communication signals, noise, etc., are well-characterized by random processes. In this lab, we will look at the ensemble statistics of a random process and prove that it is a wide-sense stationary (WSS) process.

For a random process $x(t, \zeta)$, we can compute two types of averages:

- **Time average:** Averaging over t for a constant ζ.
- **Ensemble average:** Averaging over ζ for a constant t.

Wide-sense stationary processes are a class of random processes which are characterized by the following properties:

- WSS processes have a constant ensemble mean.
- The ensemble autocorrelation function of a WSS process is a function of the time difference.

Laboratory Tasks

You need to develop a random process $x(t) = \cos(2\pi f_c t + \Theta)$, where $f_c = 100$ Hz and $0 \leq t \leq 1$sec. Θ is a uniformly distributed random variable in the range $(0, 2\pi)$. Let N be the number of signals in the ensemble. Take the

value of N as at least 5000. To develop the random process, you will form matrices t and Θ.

Task 1: Generate a row matrix t_{row} containing the sampling instants between 0 and 1 sec. Use MATLAB's **repmat** command to form a new matrix t having N rows, with each row having the same entries as t_{row}.

Task 2: MATLAB's **rand** function generates uniformly distributed random variables distributed between 0 and 1. Generate a column matrix Θ_{col} containing N rows of uniformly distributed random variable in the range $[0, 2\pi]$. Use MATLAB's **repmat** command to form a new matrix Θ , having the same dimensions as t by replicating the column matrix Θ_{col} in every column of Θ.

Task 3: Form the random process $x(t)$ using the following relation:

$$x(t) = \cos(2\pi f_c t + \Theta).$$

Note that the approach described in the above three steps is the only approach you are allowed to use to form the random process. You are **not allowed** to use a loop to form the matrix of the random process $x(t)$.

Task 4: Take the ensemble average at 5 different time instants by extracting corresponding columns from the matrix of the random process and computing their mean.

Task 5: Recall that the ensemble autocorrelation function is the expected value of the product of two random variables at two time instants of a random process. Choose any three time instants t_1, t_2 and t_3 such that $\tau = t_2 - t_1 = t_3 - t_2$. Extract the columns in the random process corresponding to these time instants. Form a new column matrix by the multiplication of the entries in column t_2 and column t_1 and take its mean. Repeat the process for columns t_2 and t_3.

Task 6: Repeat Task 5 for t_4, t_5 and t_6, where $\tau = t_5 - t_4 = t_6 - t_5$.

Task 7: Based on the results of Task 4, 5 and 6, comment on whether the process is a WSS process.

Task 8: Using MATLAB's **repmat** command, calculate the ensemble autocorrelation function of the process and plot it.