Lab Assignment - 3
Fourier Series

February 15, 2016

Consider the following signal periodic signal with time period $T_0 = 1$ sec:

$$y(t) = e^{-2t} \text{ for } 0 \leq t \leq T_0.$$

PROBLEM 1:

(a) Suppose we sample the signal at a sampling frequency of $F_s = 16$ samples per second. Write MATLAB code to declare the vector $\boldsymbol{y}$ corresponding to the signal $y(t)$.

(b) We know from Fourier series that we can write

$$y(t) = a_0 + \sum_n \left(a_n \cos(2\pi f_0 n t) + b_n \sin(2\pi f_0 n t)\right).$$

Write MATLAB code to declare vectors corresponding to the basis signals

$$1, \cos(2\pi t n_1/T_0) \text{ and } \sin(2\pi t n_2/T_0),$$

where $n_1 \in \{1, 2, \ldots, N_0/2\}$, $n_2 \in \{1, 2, \ldots, N_0/2 - 1\}$ and $N_0 = T_0 F_s$. In our example, we will have 16 vectors corresponding to 16 basis signals.

(c) Write MATLAB code to compute the approximation coefficient a_0 for basis signal 1, a_n for $\cos(2\pi t n/T_0)$ and b_n for $\sin(2\pi t n/T_0)$. Write down all the coefficients on the paper.

(d) Write MATLAB code to multiply the approximation coefficient to respective basis vectors and add them up to form $\hat{\boldsymbol{y}}$. Plot $\boldsymbol{y}$ and $\hat{\boldsymbol{y}}$ overlayed on each other on the same plot. Do they look similar?

(e) Write MATLAB code to compute the energy of the error vector.

(f) Compute the energy of the reconstructed vector on paper using Parseval's theorem.

PROBLEM 2:

Repeat problem 1 but instead of taking 16 basis signals, use only 8 basis signals corresponding to $n_1 \in \{1, 2, \ldots, T_0 \frac{F_s}{4}\}$ and $n_2 \in \{1, 2, \ldots, T_0 \frac{F_s}{4} - 1\}$.

PROBLEM 3:

Repeat problem 1 using exponential Fourier series for the following signal:

$$y(t) = t \text{ for } 0 \leq t \leq T_0.$$

PROBLEM 4:

Repeat problem 1 for a sampling rate of $F_s = 32$ samples per second.

MATLAB Challenge

You may approach problem 1 by declaring separate vectors for signal $y(t)$ as well as all the basis functions. The approach has the problem that you may need to rewrite the code if you increase or decrease the sampling rate. I have written the code for problem 1 that can cater for changing sampling rate using 11 statements, excluding plotting routines. Anyone who can write the same code in less than or equal to 11 statements will be titled 'the Mastero' until the next challenge and will be given consideration for a grade raise if he happens to loose grade later in some lab.

Hint: Try writing a phase matrix which has rows corresponding to the discrete time instants for basis signals of different frequencies. In other words, write the code of the phase matrix with kth row having sequence $2\pi f_0 k n T_s$, where $n \in \{0, 1, 2, ..., N_0 - 1\}$ and $k \in \{0, 1, 2, ..., N_0/2\}$ without using `for` loops. You can write the phase matrix as an outer product of a time and a frequency vector. The phase matrix will help you define a F matrix whose rows correspond to the vectors for different basis signals.

Deliverable

You are supposed to bring a hand-written report of the assignment to the lab. You don't need to print plots and only need to write the code corresponding to different plots.