Lab Assignment - 2
Signals as Vectors

February 3, 2016

This lab will discuss how to represent signals as vectors. The suggested reading from the text is Section 2.5 (Signals Versus Vectors).

Consider the following signals:

$$y_1(t) = \begin{cases} \sin(2\pi t) & \text{for } 0 \leq t \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_2(t) = \begin{cases} \sin(4\pi t) & \text{for } 0 \leq t \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_3(t) = \begin{cases} e^{-2t} & \text{for } 0 \leq t \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_4(t) = \begin{cases} \cos(2\pi t) & \text{for } 0 \leq t \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

PROBLEM 1:

(a) Compute the energy of the signal $y_1(t), y_2(t), y_3(t)$ and $y_4(t)$ on paper. Write the signals in the increasing energy value.

(b) Represent the signals $y_1(t), y_2(t)$ $y_3(t)$ and $y_4(t)$ as column vectors $\boldsymbol{y}_1$, $\boldsymbol{y}_2$, $\boldsymbol{y}_3$ and $\boldsymbol{y}_4$, respectively using a sampling rate of 16 samples/sec.

(c) Calculate the square of the length of vectors $\boldsymbol{y}_1$, $\boldsymbol{y}_2$, $\boldsymbol{y}_3$ and $\boldsymbol{y}_4$. Write the vectors in the order of increasing lengths. Is the order same as that of part (a) ?

(d) What is the relationship between energies of a signal and the square of the length of its corresponding vector for a sampling rate $F_s = 1/T_s$. (Hint:

Write down the formulas for the energy of a continuous-time signal and the square of its vector's length. Note the difference that sampling frequency produces between the two formulas.)

PROBLEM 2:

(a) Plot $\boldsymbol{y}_2$ overlayed on the plot of $\boldsymbol{y}_1$. Use proper labels on x and y axis and provide a legend to distinguish different waveforms.

(b) Take the dot product of two vectors $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$. Are the two vectors $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ orthogonal to each other?

(c) Take the dot product of two signals $y_1(t)$ and $y_2(t)$ on paper and compare it with the dot product of the vectors $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$? Are the two dot products equal? If not equal, state the reason why?

(d) Repeat the PROBLEM 2 for the $y_1(t)$ and $y_3(t)$.

PROBLEM 3:

(a) Are the two vectors $\boldsymbol{y}_1$ and $\boldsymbol{y}_4$ orthogonal to each other? Write the code below to prove that they are (are not) orthogonal to each other.

PROBLEM 4:

(a) Form two column vectors $\boldsymbol{g}, \boldsymbol{y_5}$ for waveforms $g(t)$ and $y_5(t)$, respectively using a sampling rate of 16 samples per second. Plot the two vectors.

(b) Approximate vector $\boldsymbol{g}$ using $\boldsymbol{y_5}$. Write down the approximation and the code to approximate $\boldsymbol{g}$ below.

(c) Approximate $g(t)$ using $y_5(t)$ on paper and compare the results to the previous part of the same question. Did you get the same approximation coefficient c in both cases, where $\hat{g}(t) = cy_5(t)$ and $\hat{g}(t)$ is the approximation to $g(t)$. In PROBLEM 1 (a) you noticed that the energy of a signal and the square of the length of its corresponding vector have different values. However, the approximation coefficient turns out to be approximately the same whether you calculate it using signals or their corresponding vectors. Why is it so?

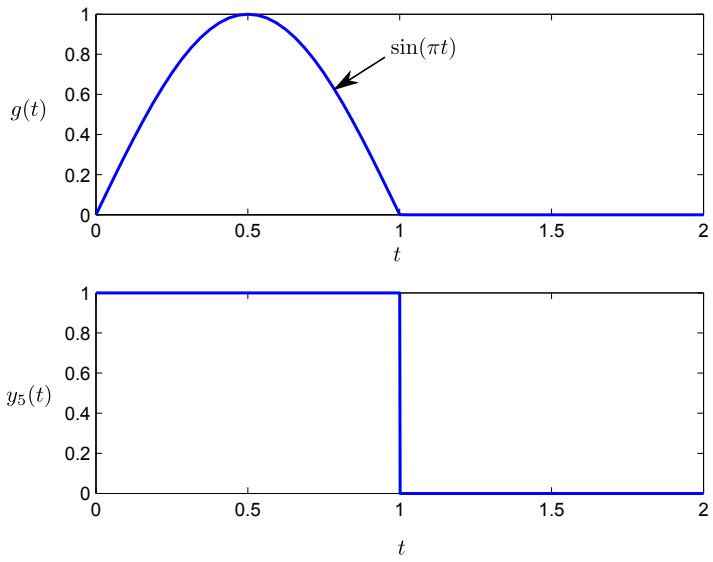

Figure 1: Signals used in PROBLEM 4.

PROBLEM 5:

Consider the following signals:

$$x(t) = \begin{cases} \sin(2\pi t) & \text{for } 0 \le t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$g_1(t) = \begin{cases} \sin(4\pi t) & \text{for } 0 \le t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$g_2(t) = \begin{cases} -\sin(2\pi t) & \text{for } 0 \le t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$g_3(t) = \begin{cases} 0.707 & \text{for } 0 \le t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$g_4(t) = \begin{cases} 0.707 & \text{for } 0 \le t \le 0.5, \\ -0.707 & \text{for } 0.5 < t \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Calculate the correlation coefficient ρ between $x(t)$ and each of the four pulses $g_1(t)$, $g_2(t)$, $g_3(t)$ and $g_4(t)$ on paper. Which of the four signals is closest to $x(t)$?

(b) Write MATLAB code to convert all the five signals to corresponding vec-

tors using a sampling rate of 16 samples/sec.

(c) Write MATLAB code to calculate the correlation coefficient of the vector $\boldsymbol{x}$ with the vectors corresponding to the four signals. Which of the four vectors is closest to $\boldsymbol{x}$? Do you get the same answer as you got in part (a). Did you get the same values of correlation coefficients? Why or why not?

Deliverable

You are supposed to bring a hand-written report of the assignment to the lab. You don't need to print plots and only need to write the code corresponding to different plots.