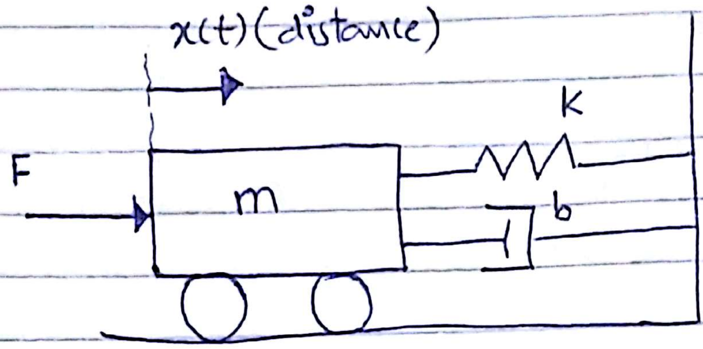
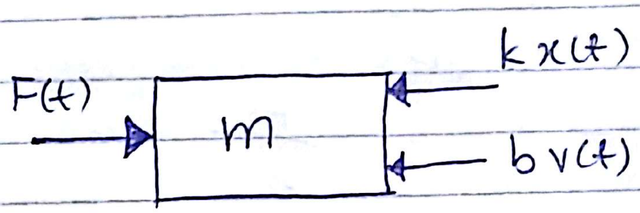
Lab 2:

Modeling of Mass-Spring-Damper System



forces on System



Physical System to Governing Equation

$$F(t) - k\,x(t) - b\,v(t) = m\,a(t)$$

$$F(t) - k\,x(t) - b\,\dot{x}(t) = m\,\ddot{x}(t)$$

$$m\,\ddot{x}(t) + b\,\dot{x}(t) + k\,x(t) = F(t) \quad \text{— (i)}$$

Governing Equation

State Space

Governing equation → S.S

State variables $x_1 = x$, $x_2 = \dot{x}$ ⇒ $\dot{x}_1 = x_2 = \dot{x}$, $\dot{x}_2 =$ from eq(i)

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = \ddot{x} = -\frac{b}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}F$$

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = \ddot{x} = -\frac{b}{m} x_2 - \frac{k}{m} x_1 + \frac{1}{m} F$$

S.S

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + B\, F(t)$$

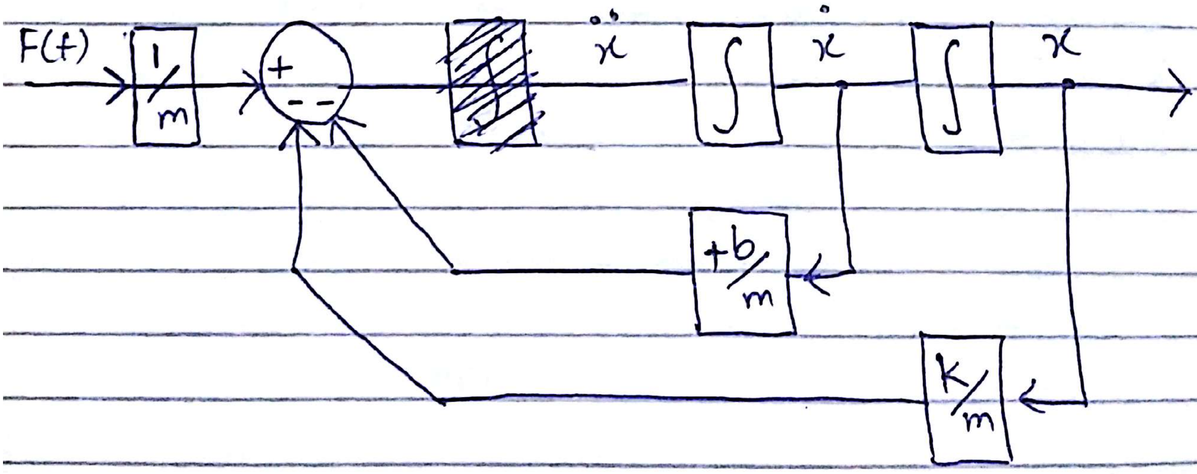
$$y = C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D\, F(t).$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}}_{(A)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{(B)} F(t)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{(C)} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{(D)} F(t)$$

$\boxed{\text{Block diagram}}$ Governing eq $\rightarrow$ B.D

$$m\ddot{x} + b\dot{x} + kx = F$$



Transfer Function

S.S → T.F $\quad C(sI - A)^{-1} B + D$

Governing eq → T.F ✓

$$m\ddot{x} + b\dot{x} + kx = F$$

$$\Downarrow \mathcal{L} \quad \text{(initial conditions} = 0)$$

$$ms^2 X(s) + bs\,X(s) + k\,X(s) = F(s)$$

$$X(s)\left(ms^2 + bs + k\right) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}.$$

$m = 1\,kg \qquad b = 0.2\,Ns/m$

$k = 1\,N/m \qquad \cancel{F = 1\,N}.$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 0.2s + 1}$$

z(zeros) : Two zeros at infinity

K(gain) : 1

P(poles) :

$$P_1 = -0.1 + 0.995\,i$$

$$P_2 = -0.1 - 0.995\,i$$

System in "S" domain

$$H(s) = \frac{1}{s^2 + 0.2s + 1}$$

System in "Time" domain

$$h(t) = e^{-0.1t} \times 1.005 \times \sin(0.9949t)$$

* "wt" is in radian

$\boxed{\text{Behavior}}$

✓ Bode plot

✓ pole zero plot

$\boxed{\text{Response}}$

Unit step

$$\mathcal{L}u(t) = \frac{1}{s} \longrightarrow \boxed{\frac{1}{s^2 + 0.2s + 1}} \longrightarrow X(s)$$

$$X(s) = \frac{1}{s} \times \frac{1}{(s^2 + 0.2s + 1)}$$

Partial fraction & Laplace inverse

$$x(t) = -e^{-0.1t} \cos(0.9949t) - 0.1e^{-0.1t} \sin(0.9949t) + 1$$

* "wt" = in radian

Made by

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