Experiment No. 3

Modeling of Sallen-Key Low Pass Filter

Objectives

- Understand system modeling.
- Understand system transfer function, state space, governing equation and block diagram.
- Observe response of system.
- Observe behavior of system.

Equipment

The following equipments are used in this laboratory:

- DC voltage source, Oscilloscope and Function signal generator.
- Op-Amp LM358.
- Resistors: 10, 10K, 100K
- Capacitors: 10nF (103)

Introduction to Sallen-Key Topology

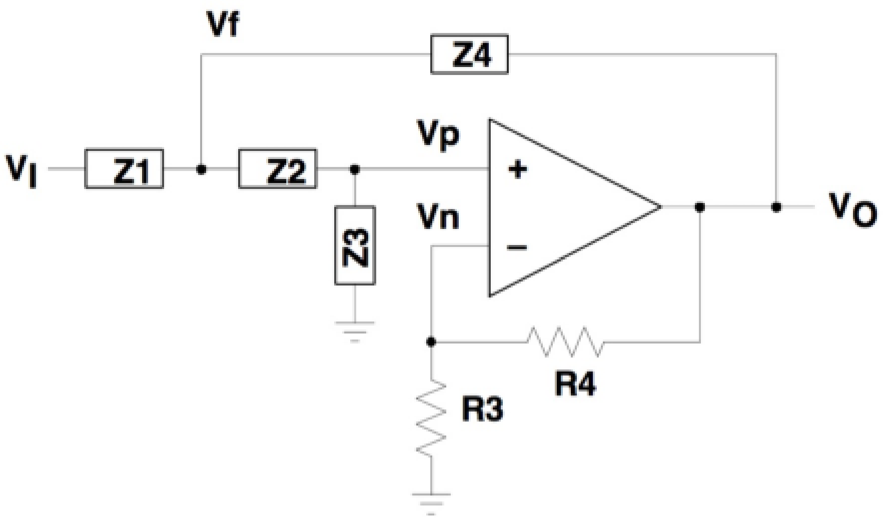

FIGURE 3.1: A generic Sallen-Key filter topology Karki (1999)

The Sallen-Key topology is an electronic filter topology used to implement second-order active filters that is particularly valued for its simplicity.The Sallen-Key topology was introduced by R. P. Sallen and E. L. Key of MIT Lincoln Laboratory in 1955.

Note that

$$V_p = V_f \frac{Z_3}{Z_2 + Z_3} \tag{3.1}$$

$$V_n = V_O \frac{R_3}{R_3 + R_4} \tag{3.2}$$

It follows by the KCL that

$$\frac{V_i - V_f}{Z_1} = \frac{V_f - V_O}{Z_4} + \frac{V_f}{Z_2 + Z_3} \tag{3.3}$$

Combing the above equations yields

$$\frac{V_O}{V_i} = \frac{K}{\frac{Z_1 \times Z_2}{Z_3 \times Z_4} + \frac{Z_1}{Z_3} + \frac{Z_2}{Z_3} + \frac{Z_1(1-K)}{Z_4} + 1} \tag{3.4}$$

$$\frac{V_O}{V_i} = \frac{K Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_4 + Z_1 Z_3 (1-K) + Z_3 Z_4} \tag{3.5}$$

where K $= 1 + \frac{R_4}{R_3}$

Second Order Low Pass Sallen-Key Filter

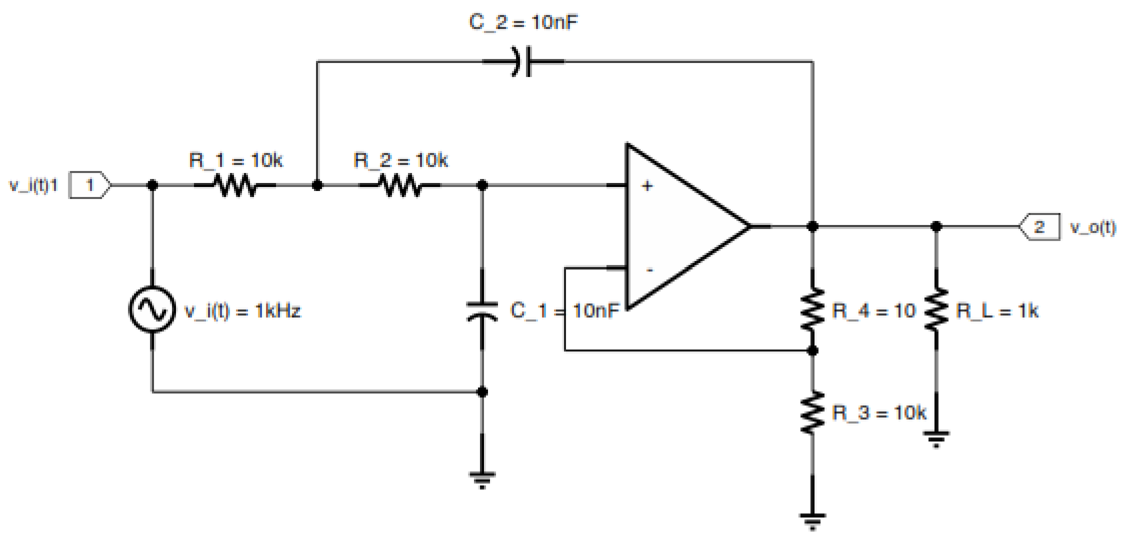

FIGURE 3.2: Low-pass Sallen-Key circuit

Considering the filter in Fig 3.2, We let $Z_1 = R_1, Z_2 = R_2, Z_3 = \frac{1}{sC_1}$ and $Z_4 = \frac{1}{sC_2}$.

$$\frac{V_O}{V_i} = \frac{K}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_1 + R_1C_2(1-K))s + 1} \tag{3.6}$$

Section 3.1: Theoretical Calculations

- Find transfer function of system H(s) shown in Figure 3.2
- Find standard form of transfer function H(s).
- Find pole, zero and gain of H(s).
- Find h(t) from H(s).
- Find governing equation of system.
- Find block diagram of system.
- Find state space form of system. We called this stated space form as SS1
- Find number of inputs, outputs and state variables in SS1.
- Let us suppose there is another state space form SS2.

$$A = \begin{bmatrix} -2\times 10^4 & -1\times 10^8 \\ 1 & 0 \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \mathrm{C} = \begin{bmatrix} 0 & 1\times 10^8 \end{bmatrix} \qquad \mathrm{D} = \begin{bmatrix} 0 \end{bmatrix}$$

- Calculate transfer function from SS2 using $C(sI - A)^{-1}B + D$.
- Conclusion: Transfer function will be same whether it has been calculated from SS1 or SS2. Thus proved, state space of system are infinitely many but transfer function is unique.
- Find step response of system and fill following table

Observation Set:

No.	Time	V_o (dc)
1	0.1 ms	
2	0.2 ms	
3	0.3 ms	
4	0.4 ms	
5	0.5 ms	
6	0.6 ms	
7	0.7 ms	
8	0.8 ms	

Section 3.2: Simulation

3.2.1 Matlab Command Window

- Define Transfer function of system in Matlab using ”tf”.
- Find zero, pole and gain from transfer function using ”tf2zp”.
- Find pole zero plot of system using ”pzmap”.
- Find bode plot of system using ”bode”.
- Find state space using tf2ss you will observe it will be equal to SS2.
- Find number of inputs, outputs and state variables of SS2 using ”size”.
- Now define state space SS1 using ”ss”.
- Find transfer function from state space SS1 using ”ss2tf”.
- Conclusion: Transfer function will be same whether it has been calculated from SS1 or SS2. Thus proved, state space of system are infinitely many but transfer function is unique.
- Find unit step response of system using ”stepplot”.
- Fill table below.

Observation Set:

No.	Time	V_o (dc)
1	0.1 ms	
2	0.2 ms	
3	0.3 ms	
4	0.4 ms	
5	0.5 ms	
6	0.6 ms	
7	0.7 ms	
8	0.8 ms	

Matlab Functions:

No.	Code
1	tf
2	pole
3	zero
4	zpk
5	tf2zp
6	pzmap
7	bode
8	ss
9	size
10	tf2ss
11	ss2tf
12	stepplot
13	ilaplace

3.2.2 Matlab Simulink

- Make Transfer function block of system.
- Make Block diagram of system.
- Find unit step response of system.
- Fill table below.

Observation Set:

No.	Time	V_o (dc)
1	0.1 ms	
2	0.2 ms	
3	0.3 ms	
4	0.4 ms	
5	0.5 ms	
6	0.6 ms	
7	0.7 ms	
8	0.8 ms	

Section 3.3: Hardware Results

- Patch circuit on breadboard.
- Generate 250 Hz Square wave with Vp-p 1V and offset 0.5V and use as input.

- Observe output using oscilloscope.
- HINT: You can use Proteus or Multisim for hardware simulation as well.
- Fill table below.

Observation Set:

No.	Time	V_o (dc)
1	0.1 ms	
2	0.2 ms	
3	0.3 ms	
4	0.4 ms	
5	0.5 ms	
6	0.6 ms	
7	0.7 ms	
8	0.8 ms	

Section 3.4: References

- Karki, J. (1999). Analysis of the Sallen Key architecture. Technical report, Texas Instrucments, Dallas, Texas, USA.

Point to ponder:

- Product of system transfer function H(s) and unit step (1/s) IS EQUAL to convolution of system time domain h(t) and unit step u(t) and it IS EQUAL to solution of governing equation.
- System have multiple state space representations but single unique transfer function.
- Number of poles is equal to number of zeros.
- Poles in left plane of pole-zero plot make system stable while poles in right plane makes system unstable.
- System becomes marginally stable if there is at least one pole on imaginary axis in pole-zero plot.
- Two poles on origin make system unstable.
- Transfer Function us Unique while state space representation is infinitely many.