Experiment No. 5

Time Response of First and Second Order Systems

Objectives

- Understand characteristics of step response of first and second order systems.

Section 5.1: First Order System

5.1.1 Introduction

System Order

The order of a dynamic system is the order of the highest derivative of its governing differential equation. Equivalently, it is the highest power of s in the denominator of its transfer function. The important properties of first and second will be reviewed in this section.

First Order Systems

First order systems are the simplest dynamic systems to analyze. Some common examples include cruise control systems and RC circuits.

The general form of the first order differential equation is as follows

$$\dot{y} + ay = bu \quad \text{or} \quad \tau\dot{y} + y = k_{dc}u \tag{5.1}$$

The first order transfer function is

$$G(s) = \frac{b}{s+a} = \frac{k_{dc}}{\tau s + 1} \tag{5.2}$$

DC Gain

The DC gain, k_{dc}, is the ratio of the magnitude of the steady-state step response to the magnitude of the step input. From the Final Value Theorem, for stable transfer functions the DC gain is the value of the transfer function when s=0. For first order systems equal to $k_{dc} = b/a$.

Time Constant

The time constant

$$T_c = \tau = \frac{1}{a} \tag{5.3}$$

is the time it takes for the system to reach 63 % of the steady-state value for a step response or to decrease to 37 % of the inital value for an impulse response. More generally, it represents the time scale for which the dynamics of the system are significant.

Poles/Zeros

There is a single real pole at $s = -a$. Therefore, the system is stable if a is positive and unstable if a is negative. There are no zeros.

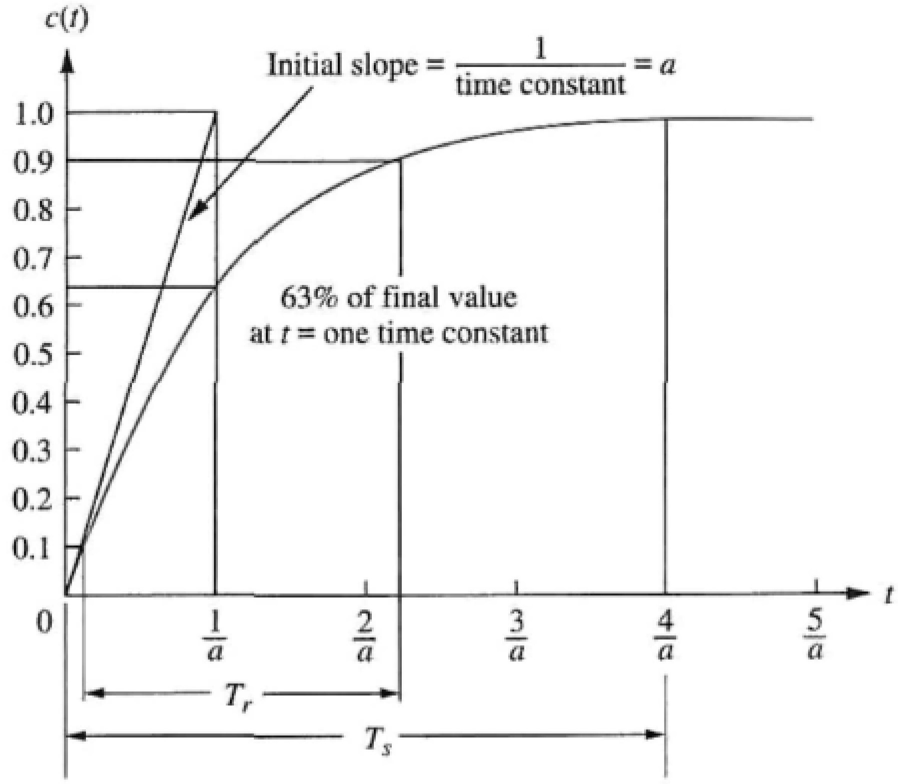

FIGURE 5.1: First Order System Response to a Unit Step

Settling Time

The settling time, T_s, is the time required for the system ouput to fall within a certain percentage (i.e. 2%) of the steady state value for a step input or equivalently to decrease to a certain percentage of the initial value for an impulse input. The settling times for first order system for the most common tolerances are provided in the table below. Note that the tighter the tolerance, the longer the system response takes to settle to within this tolerance, as expected.

10%	5%	2%	1%
$Ts = \frac{2.3}{a} = 2.3 \times Tc$	$Ts = \frac{3}{a} = 3 \times Tc$	$Ts = \frac{3.9}{a} = 3.9 \times Tc$	$Ts = \frac{4.6}{a} = 4.6 \times Tc$

Rise Time

The rise time, T_r, is the time required for the system output to rise from some lower level x% to some higher level y% of the final steady-state value. For first order systems, the typical range is 10% - 90%.

$$T_r = \frac{2.2}{a} \tag{5.4}$$

5.1.2 Single Case:

System Diagram

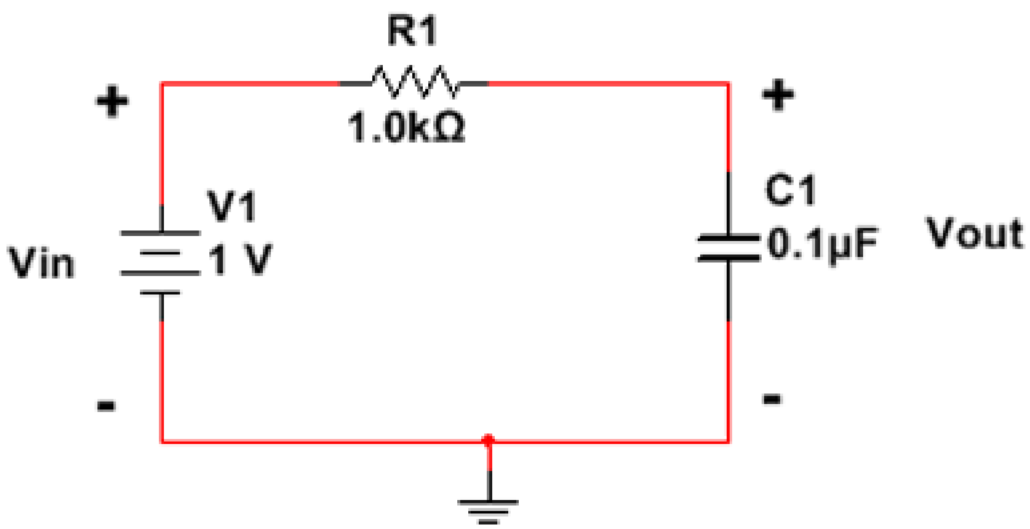

FIGURE 5.2: RC Circuit

- Find transfer function of system $H_1(s)$ shown in Figure 5.2 (Theoretically)
- Find pole and sketch pzmap (Matlab).
- Sketch Bode Plot (Matlab).
- Find step response equation (Matlab).
- Sketch Waveform of step response(Matlab).
- Find Rise time T_r, Settling time T_s and time constant T_c and show in waveform. (Theoretically, Matlab and Hardware).

5.1.3 Questions

- What is generic form of $H(s)$ for first order system.

- Plot poles of $H_1(s)$ in pzmap and find out whether poles are always at real axis or not?.
- Generic form of time domain equation of step response of first order system.

Section 5.2: Second Order System

5.2.1 Introduction

Second order systems are commonly encountered in practice, and are the simplest type of dynamic system to exhibit oscillations. In fact many real higher order systems are modeled as second order to facilitate analysis. Examples include mass-spring-damper systems and RLC circuits.

The general form of the first order differential equation is as follows

$$m\ddot{y} + b\dot{y} + ky = f(t) \quad \text{or} \quad \ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = k_{dc}\omega_n^2 u \tag{5.5}$$

The second order transfer function is

$$G(s) = \frac{1}{ms^2 + bs + k} = \frac{k_{dc}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5.6}$$

DC Gain

The DC gain, k_{dc}, again is the ratio of the magnitude of the steady-state step response to the magnitude of the step input, and for stable systems it is the value of the transfer function when $s = 0$. For second order systems,

$$k_{dc} = \frac{1}{k} \tag{5.7}$$

Damping Ratio

The damping ratio is a dimensionless quantity charaterizing the energy losses in the system due to such effects as viscous friction or electrical resistance. From the above definitions,

$$\zeta = \frac{b}{2\sqrt{k * m}} \tag{5.8}$$

Natural Frequency

The natural frequency is the frequency (in rad/s) that the system will oscillate at when there is no damping, $\zeta = 0$.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{5.9}$$

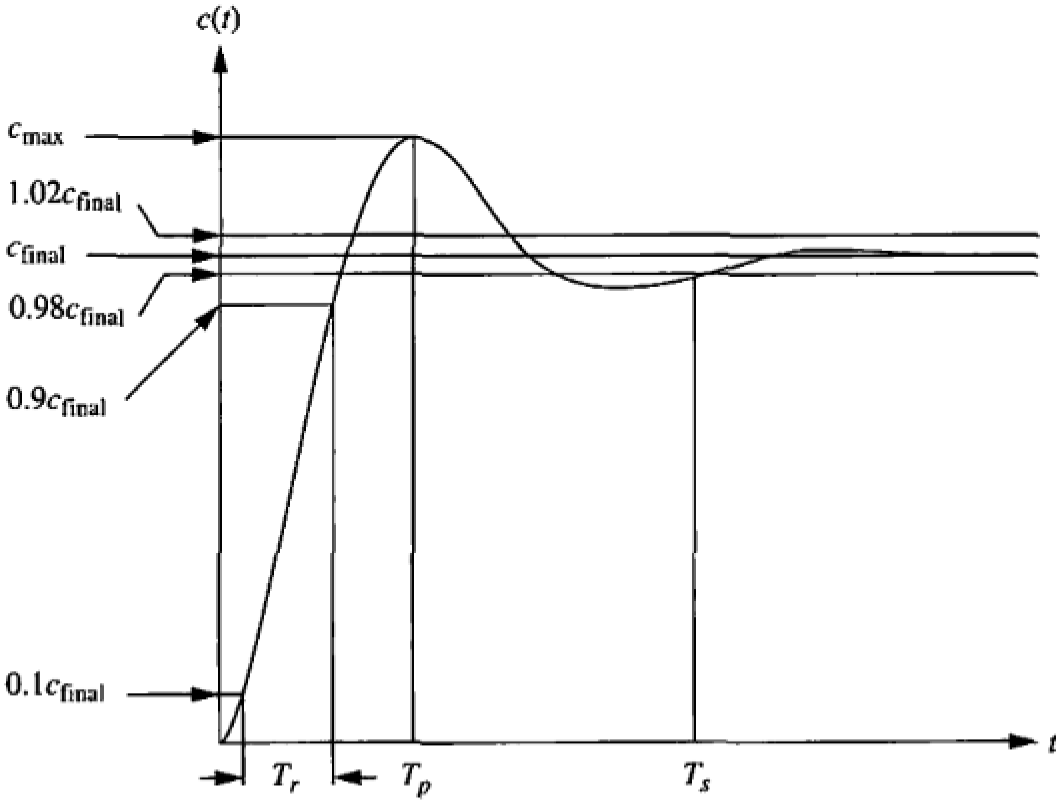

FIGURE 5.3: Second Order System Response to a Unit Step

Damping Frequency (For Under Damped Case Only)
System oscillations will damp out with frequency:

$$w_d = \omega_n \sqrt{1 - \zeta^2} \tag{5.10}$$

Poles/Zeros
The second order transfer function has two poles at:

$$s_p = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \tag{5.11}$$

Settling Time
The settling time, T_s, is the time required for the system ouput to fall within a certain percentage of the steady state value for a step input or equivalently to decrease to a certain percentage of the initial value for an impulse input. For a second order, underdamped system, the settling time can be approximated by the following equation:

$$T_s = \frac{-\ln(\text{tolerance fraction})}{\zeta\omega_n} \tag{5.12}$$

The settling times for the most common tolerances are presented in the following table:

10%	5%	2%	1%
$Ts = 2.3/(zeta * wn)$	$Ts = 3/(zeta * w_n)$	$Ts = 3.9/(zeta * w_n)$	$Ts = 4.6/(zeta * w_n)$

Percent Overshoot

The percent overshoot is the percent by which a system exceeds its final steady-state value. For a second order under damped system, the percent overshoot is directly related to the damping ratio by the following equation:
(For Under Damped Case Only)

$$\%OS = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \cdot 100\% \tag{5.13}$$

For second order under damped systems, the 2% settling time, T_s, rise time, T_r, and percent overshoot, %OS, are related to the damping and natural frequency as shown below.

$$T_s \approx \frac{3.9}{\zeta\omega_n} \tag{5.14}$$

$$T_r \approx \frac{2.2}{\zeta\omega_n} \tag{5.15}$$

(For Under Damped Case Only)

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{5.16}$$

5.2.2 Case 1: Under Damped System $0 < \zeta < 1$

System Diagram

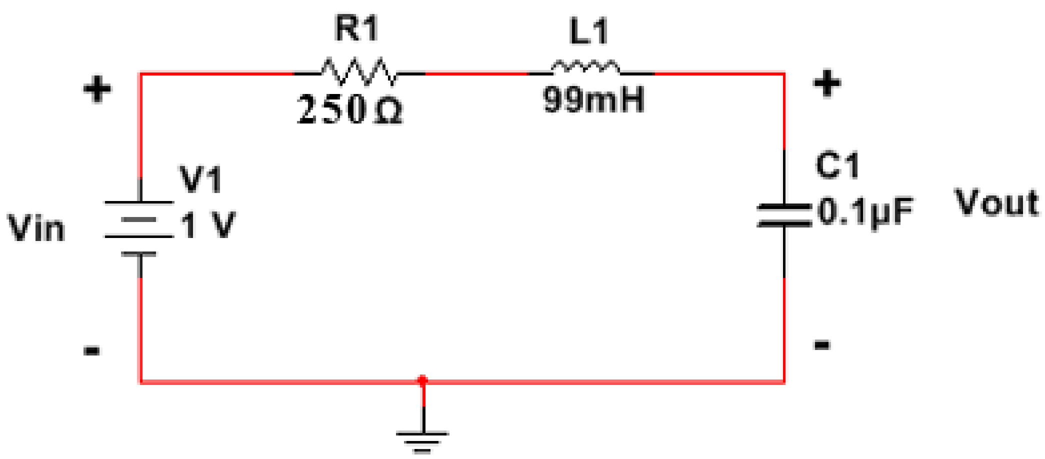

FIGURE 5.4: RLC Circuit

- Find transfer function of system $H_3(s)$ shown in Figure 5.4 (Theoretically)
- Find pole and sketch pzmap (Matlab).
- Sketch Bode Plot (Matlab).
- Find step response equation (Matlab).
- Sketch Waveform of step response(Matlab).
- Find Natural frequency w_n , Damping Coefficient ζ, Damping frequency w_d, Rise Time T_r, Peak Time T_p, Percent Overshoot O.S % and Settling Time T_s and show in waveform. (Theoretically, Matlab and Hardware).

5.2.3 Case 2: Critically Damped System $\zeta = 1$

System Diagram

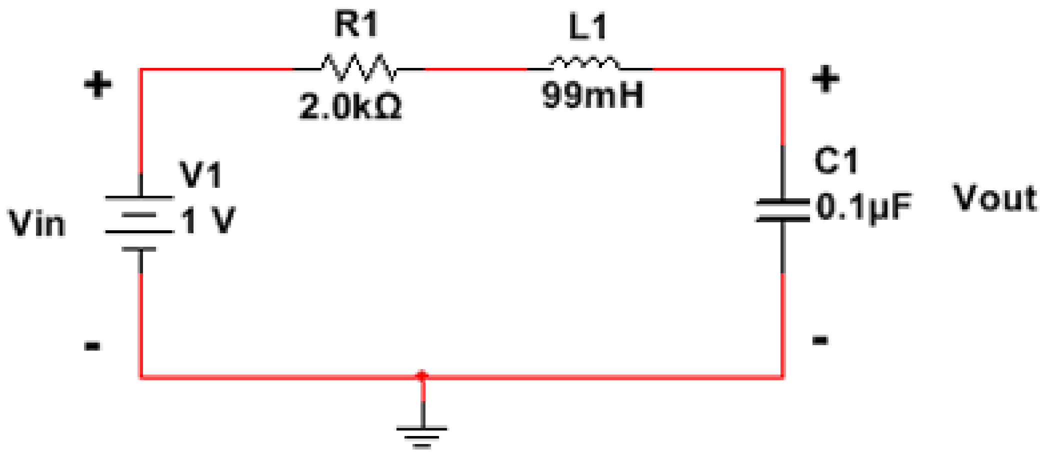

FIGURE 5.5: RLC Circuit

- Find transfer function of system $H_4(s)$ shown in Figure 5.5 (Theoretically)
- Find pole and sketch pzmap (Matlab).
- Sketch Bode Plot (Matlab).
- Find step response equation (Matlab).
- Sketch Waveform of step response(Matlab).
- Find Natural frequency w_n, attenuation Coefficient α , Damping Coefficient ξ, Damping frequency w_d, Rise Time T_r, Peak Time T_p, Percent Overshoot O.S % and Settling Time T_s and show in waveform. (Theoretically, Matlab and Hardware).

5.2.4 Case 3: Over Damped System $\zeta > 1$

System Diagram

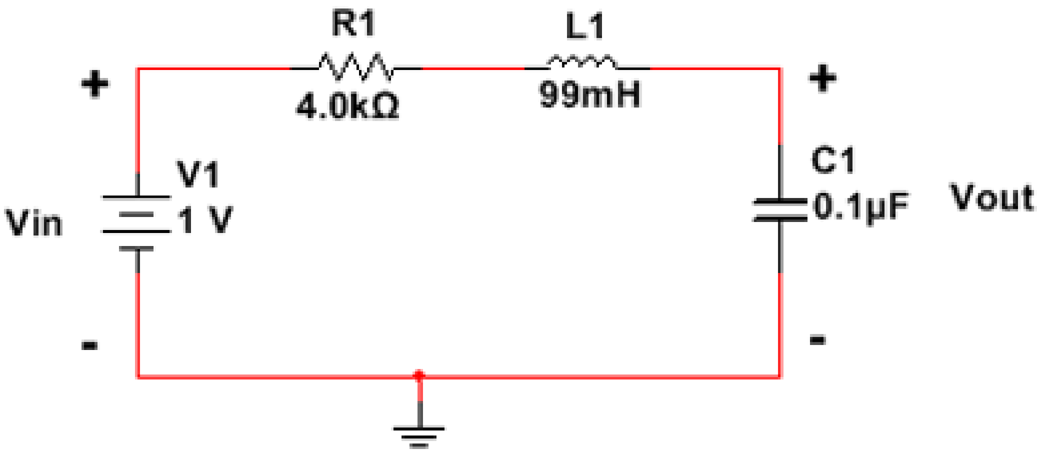

FIGURE 5.6: RLC Circuit

- Find transfer function of system $H_5(s)$ shown in Figure 5.6 (Theoretically)
- Find pole and sketch pzmap (Matlab).
- Sketch Bode Plot (Matlab).
- Find step response equation (Matlab).
- Sketch Waveform of step response(Matlab).
- Find Natural frequency w_n, attenuation Coefficient α , Damping Coefficient ξ, Damping frequency w_d, Rise Time T_r, Peak Time T_p, Percent Overshoot O.S % and Settling Time T_s and show in waveform. (Theoretically, Matlab and Hardware).

Matlab Function:

No.	Code
1	stepinfo

5.2.5 Questions

- What is generic form of $H(s)$ for each case in second order system.
- Plot poles of $H_3(s)$, $H_4(s)$, $H_5(s)$ in same pzmap and find out whether poles are always at real axis or not?.

- Shape of bode plot is same or not?.
- Generic form of time domain equation of step response of second order system in each case.
- Is shape of time waveform of step response of first order system remain same or not?.