Lab 3 (No Example)

Modeling of Sallen-Key Low pass filter

After modeling given in Lab manual 3

$$H(s) = \frac{V_o}{V_i} = \frac{K}{(R_1 R_2 C_1 C_2)s^2 + (R_1 C_1 + R_2 C_1 + R_1 C_2(1-K))}$$

where $k = 1 + R_4/R_3$

$R_1 = R_2 = R_3 = 10k\Omega$.

$R_4 = 10\Omega$; $C_1 = C_2 = 10nF$

After putting values.

Transfer function

$$H(s) = \frac{1.001}{1\times10^{-8} s^2 + 0.0001999 s + 1}$$

Standard form

$$\frac{1.001 \times 10^8}{s^2 + 1.999\times10^4 s + 1\times10^8}$$

Pole, zero gain

$k = 1.001$

$$P = (1\times10^3)\begin{pmatrix} -9.99 + 0.3162i \\ -9.995 - 0.3162i \end{pmatrix}$$

h(t)

$$e^{-9995t} \times \frac{1.001}{316.1882} \times \sin(\underbrace{5\sqrt{3999}\,t}_{\text{radian}})$$

Scanned by CamScanner

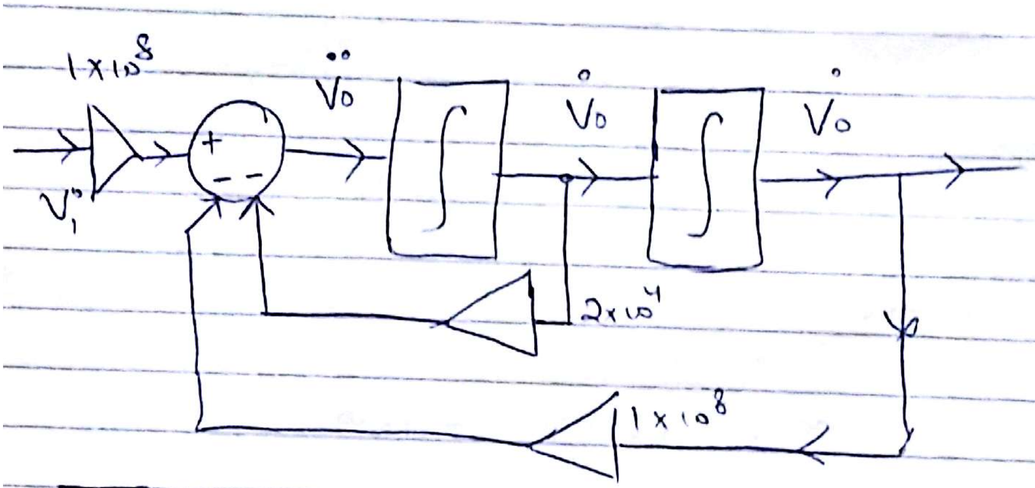
Governing equation

$$\frac{V_o}{V_i} = \frac{1\times10^8}{s^2 + 2\times10^4 s + 1\times10^8}$$

$$\ddot{V}_o + (2\times10^4)\dot{V}_o + (1\times10^8)V_o = (1\times10^8)V_i .$$

Block diagram

$$\ddot{V}_o = V_i(1\times10^8) - (2\times10^4)\dot{V}_o - (1\times10^8)V_o - (i)$$



State Space

$$x_1 = V_o \qquad x_2 = \dot{V}_o$$

$$\dot{x}_1 = \dot{V}_o = x_2$$

$$\dot{x}_2 = \ddot{V}_o = \text{eq } (i)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -1\times10^8 & -2\times10^4\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 1\times10^8\end{bmatrix}}_{B} V_i(t)$$

$$\left\{ Y = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\end{bmatrix}}_{D} V_i(t)\right.$$

let called it SS1

$$A = \begin{bmatrix} 0 & 1 \\ -1\times10^{8} & -2\times10^{4} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1\times10^{8} \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

where 1 output, 1 input, SISO, 2 states

lets suppose we have SS2.

$$A = \begin{bmatrix} -2\times10^{4} & -1\times10^{8} \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1\times10^{8} \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

SS2 → tf using $C(sI-A)^{-1}B + D$

$$(sI-A) = \begin{bmatrix} s+2\times10^{4} & +1\times10^{8} \\ -1 & s \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^2+2\times10^{4}s+1\times10^{8}} \times \begin{bmatrix} s & -1\times10^{8} \\ 1 & s+2\times10^{4} \end{bmatrix}.$$

$$(sI-A)^{-1}B = \frac{1}{s^2+2\times10^{4}s+1\times10^{8}} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$C(sI-A)^{-1}B = \frac{1\times10^{8}}{s^2+2\times10^{4}s+1\times10^{8}} = H(s)$$

$$SS1 \rightarrow H(s)$$

$$SS2 \rightarrow H(s)$$

Thus proved state space are infinetly many but T.F is "UNIQUE"

Step Response

(i) $\ddot{V}_o + (2\times10^4)\dot{V}_o + (1\times10^8)V_o = (1\times10^8)u(t)$

(ii) $V_o(t) = h(t) * u(t)$

(iii) $V_o(s) = H(s) \times \frac{1}{s}$

→ P.F and $\mathcal{L}^{-1}$

$V_o(t) =$

$$\frac{1001}{1e^{11}} - 1.001\times10^{-8} e^{-9995t} \times \cos(5\sqrt{3999}\,t) - e^{-9995t} \times \frac{0.0001}{5\sqrt{3999}} \times \sin(5\sqrt{3999}\,t)$$

where "wt" is in radian

$$V_o(t) = 1 - e^{-9995t}\left(\cos(5\sqrt{3999}\,t) + \frac{1999\sqrt{3999}\sin(5\sqrt{3999}\,t)}{3999}\right)$$

Scanned by CamScanner