Experiment No. 4

Modeling of Inverted Pendulum (SIMO Open Loop)

Objectives

- Understand system modeling.
- Understand system transfer function, state space, governing equation and block diagram.
- Understand Non-Linear model of system.
- Observe response of system (Step and Impulse).
- Observe behavior of system.

System Diagram

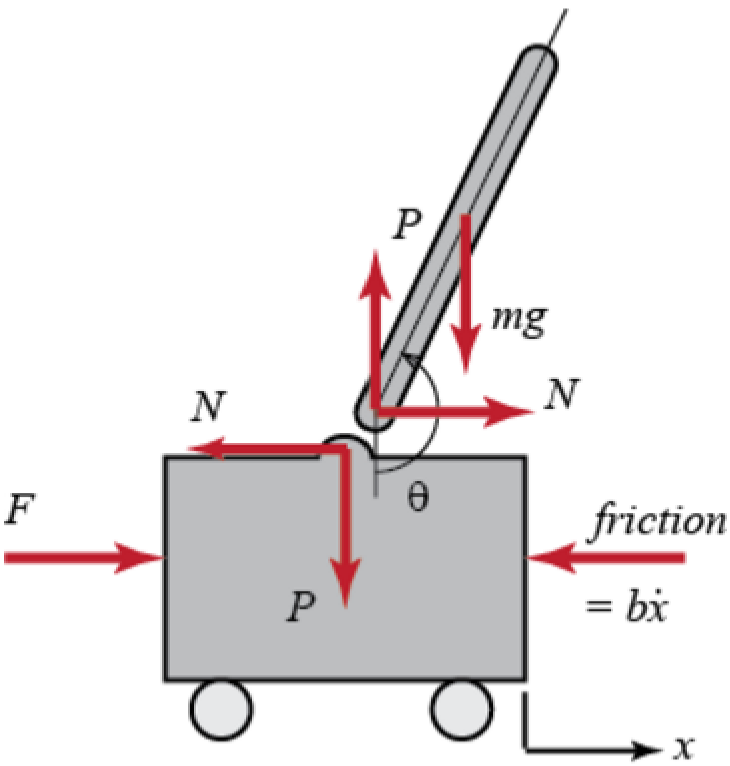

FIGURE 4.1: Forces acting on inverted pendulum

Where F is force applied on cart of mass M. P and N are perpendicular and horizontal forces respectively. b is friction coefficient and θ is angular position of pendulum of mass m. **Note: In theory, I (Inertia) and b (Friction coefficient) were assumed zero but in lab we will consider these parameters as well)**

Non-Linear Forces on system

$$\ddot{x} = \frac{1}{M}\sum_{cart} F_x = \frac{1}{M}(F - N - b\dot{x}) \tag{4.1}$$

$$\ddot{\theta} = \frac{1}{I}\sum_{pend} \tau = \frac{1}{I}(-Nlcos\theta - Plsin\theta) \tag{4.2}$$

$$N = m(\ddot{x} - l\dot{\theta}^2 sin\theta + l\ddot{\theta}cos\theta) \tag{4.3}$$

$$P = m(l\dot{\theta}^2 cos\theta + l\ddot{\theta}sin\theta) + g \tag{4.4}$$

Non-Linear governing equations of system

After substituting value of P and N in eq 4.1 and eq 4.2 and re-arranging, we get following Non-linear model of inverted pendulum

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = F \tag{4.5}$$

$$(I + ml^2)\ddot{\theta} + mglsin\theta = -ml\ddot{x}cos\theta \tag{4.6}$$

Linearization the Non-Linear Model of system

Since the analysis and control design techniques we will be employing in this example apply only to linear systems, this set of equations needs to be linearized. Specifically, we will linearize the equations about the vertically upward equillibrium position, $\theta = \pi$, and will assume that the system stays within a small neighborhood of this equillbrium. This assumption should be reasonably valid since under control we desire that the pendulum not deviate more than 20 degrees from the vertically upward position. Let ϕ represent the deviation of the pedulum's position from equilibrium, that is, $\theta = \pi + \phi$. Again presuming a small deviation (ϕ) from equilibrium, we can use the following small angle approximations of the nonlinear functions in our system equations:

$$cos\theta = cos(\pi + \phi) \approx -1 \tag{4.7}$$

$$sin\theta = sin(\pi + \phi) \approx -\phi \tag{4.8}$$

$$\dot{\theta}^2 \approx \dot{\phi}^2 \approx 0 \tag{4.9}$$

Linear governing equations of system

After substituting the above approximations into our nonlinear governing equations, we arrive at the two linearized equations of motion. Note u has been substituted for the input F.

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = F \tag{4.10}$$

$$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \tag{4.11}$$

System parameters:

No.	Parameter	Value
1	M	0.5 kg
2	m	0.2 kg
3	b	0.1 N/m/sec
4	l	0.3 m
5	I	0.006 $kg.m^2$

Section 4.1: Theoretical Calculations

- Find transfer function of system H(s) shown in Figure 4.1
- Find standard form of transfer function H(s).
- Find pole, zero and gain of H(s).
- Find h(t) from H(s).
- Find block diagram of system.
- Find state space form of system and call it SS1.
- Find number of inputs, outputs and state variables in SS1.
- Take impulse and step response of system.

Impulse Response of Pendulum:

No.	Time	Angle
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Impulse Response of Cart:

No.	Time	Position
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Step Response of Pendulum:

No.	Time	Angle
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Step Response of Cart:

No.	Time	Position
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Section 4.2: Simulation

4.2.1 Matlab Command Window

- Define Transfer function of system in Matlab using "tf".
- Find zero, pole and gain from transfer function using "tf2zp".

- Find pole zero plot of system using "pzmap".
- Find bode plot of system using "bode".
- Find state space using tf2ss.
- Find number of inputs, outputs and state variables of SS1 using "size".
- Find impulse and unit step response of system using "impulse" and "stepplot" respectively.
- Fill table below.

Impulse Response of Pendulum:

No.	Time	Angle
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Impulse Response of Cart:

No.	Time	Position
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Step Response of Pendulum:

No.	Time	Angle
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Step Response of Cart:

No.	Time	Position
1	0.1	
2	0.2	
3	0.3	
4	0.4	
5	0.5	
6	0.6	
7	0.7	
8	0.8	

Matlab Functions:

No.	Code
1	tf
2	pole
3	zero
4	zpk
5	tf2zp
6	pzmap
7	bode
8	ss
9	size
10	tf2ss
11	ss2tf
12	stepplot
13	ilaplace
14	zpkdata

4.2.2 Non-Linear Model in Simulink

We can build the inverted pendulum model in Simulink employing the Non-Linear forces equations of system.

- Use Mux and Function blocks to define functions and complete all four equations of non-Linear forces of system.
- Make subsystem.
- Find impulse and step response of system.

Step Response of Pendulum:

No.	Time	Angle
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	

Step Response of Cart:

No.	Time	Position
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	

Point to ponder:

- Product of system transfer function H(s) and unit step (1/s) IS EQUAL to convolution of system time domain h(t) and unit step u(t) and it IS EQUAL to solution of governing equation.
- System have multiple state space representations but single unique transfer function.

- Number of poles is equal to number of zeros.
- Poles in left plane of pole-zero plot make system stable while poles in right plane makes system unstable.
- System becomes marginally stable if there is at least one pole on imaginary axis in pole-zero plot.
- Two poles on origin make system unstable.
- Transfer Function us Unique while state space representation is infinitely many.