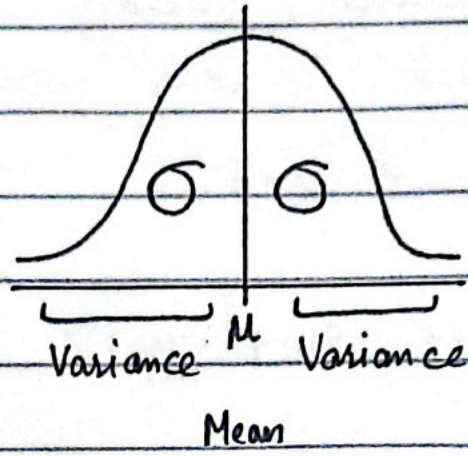
→ Normal Distribution:



→ Area under the curve is 1 which is total sum of probabilities.

→ We can find the probability between specific interval by using the formula:

$$n(x, \mu, \sigma) = \int_a^b \frac{1 . e^{-1/2 \left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} dx$$

$$-\infty < X < +\infty$$

$$\sigma > 0$$

→ In this case of normal distribution, solving the integral is very complicated and difficult.

→ So we simplify it

Let, $Z = \frac{X - \mu}{\sigma}$

Here, $E(Z) = 0$, $STD(Z) = 1$

→ All questions of normal distribution are done using z-table.

→ Negative Binomial Distribution:

The negative binomial distribution is the distribution of the number of trials needed to get the r^{th} success.
The negative binomial distribution is the distribution of the number of trials needed to get a fixed number of successes.

$$P(X = x) = \binom{x-1}{r-1} p^{r} (1-p)^{x-r}$$

$$\text{Mean} = \frac{r}{p}$$

$$\text{Variance} = \frac{r(1-p)}{p^2}$$

→ Poisson Distribution:

Poisson distribution is a probability distribution used to model the number of events that occur in a fixed interval of time or space, given the average rate of occurence, assuming that the events happen independently and at a constant rate.

$$P(X=x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

=> λ is the average number of times an event occurs

=> x is the number of times an event occurs

=> e is Euler's constant (≈ 2.718)

Mean $= \lambda = n.p$, Variance $= \lambda$

→ Hyper-Geometric Distribution:

The hypergeometric distribution describes the probability of choosing 'x' objects with a certain feature in 'n' draws without replacement, from a finite population of size 'N' that contains 'K' objects with that feature.

$$P(X=x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

→ N : Population Size
→ K : Total successors in the population
→ n : Sample Size
→ x : No. of successes in sample
→ N-K : Total no. of failures
→ n-x : Obtained no. of failures

Continuous Probability Distribution:

→ Probability Density Function (PDF):

The function $f(x)$ is a probability density function for a continuous random variable X, defined on the set of real numbers, if:

1) $f(x) \geq 0 \quad \forall x \in R$

2) $\int_{-\infty}^{+\infty} f(x)\, dx = 1$

3) $P(a \leq X \leq b) = \int_{a}^{b} f(x)\, dx$

→ Cumulative Distribution Function (CDF):

The CDF, $F(x)$, of a continuous random variable X with probability

$$\text{Mean} = n \cdot \frac{K}{N}$$

$$\text{Variance} = n \cdot \frac{K}{N}\left(1 - \frac{K}{N}\right)\left(\frac{N-n}{N-1}\right)$$

→ Mathematical Expectation:

For discrete random variable, the expected value or mean, usually denoted as μ or $E(X)$, is calculated as:

$$E(X) = \sum_{i=1}^{k} x_i f(x_i)$$

The expected value of X is the sum of each outcome multiplied by its corresponding probability:

$$E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

→ Variance:

The variance of a discrete random variable is given by:

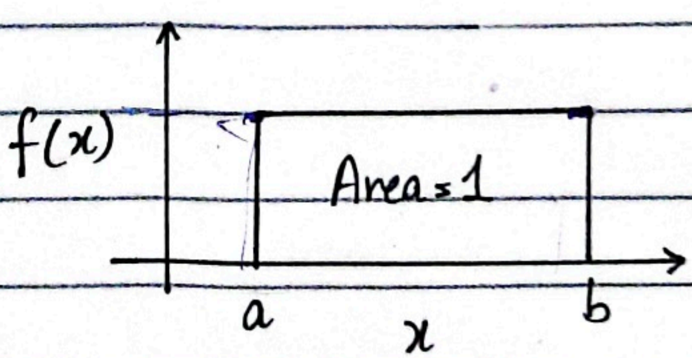
$$\sigma^2 = Var(X) = \sum x_i^2 f(x_i) - [E(X)]^2$$
$$= E(X^2) - [E(X)]^2$$

→ Important Continuous Random Variables:

→ Uniform Distribution

A continuous probability distribution is a Uniform distribution and is related to the events which are equally likely to occur.

For the uniform distribution, $f(x)$ is constant over the possible values of x.



Area = base × height

$= (b-a) f(x)$

$\Rightarrow (b-a)\, f(x) = 1$

$$f(x) = \frac{1}{b-a}$$

Hence, $f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$

Mean $= E(X) = \dfrac{a+b}{2}$

Variance $= Var(X) = \dfrac{(b-a)^2}{12}$

$P(x < X < x) =$ base $\times$ height

$\frac{1}{2}$

density function $f(x)$ is given by:

$$F(X) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$$

for $-\infty < x < \infty$

For Probability:

$$P(\text{Lowebound} \leq X \leq \text{Upperbound})$$

$$= F(\text{upperbound}) - F(\text{lowerbound})$$

→ **Mathematical Expectation:**

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx$$

→ **Variance:**

$$Var(X) = E(X^2) - [E(X)]^2$$

Important Discrete Random Variables:

→ Bernoulli Distribution:

The Bernoulli distribution is a discrete probability distribution that models a binary outcome for one trial.

Suppose we have a single trial. The trial can result in one of two possible outcomes, labelled success and failure.

$P(\text{Success}) = p$

$P(\text{Failure}) = 1 - p$

Let $X = 1$ if a success occurs, and $X = 0$ if a failure occurs. Then X has a Bernoulli distribution:

$$P(X = x) = p^x (1-p)^{1-x}$$

μ = Mean = p
σ^2 = Variance = $p(1-p)$

→ Geometric Distribution:

Geometric distribution defines the probability that first success occurs after k number of trials.

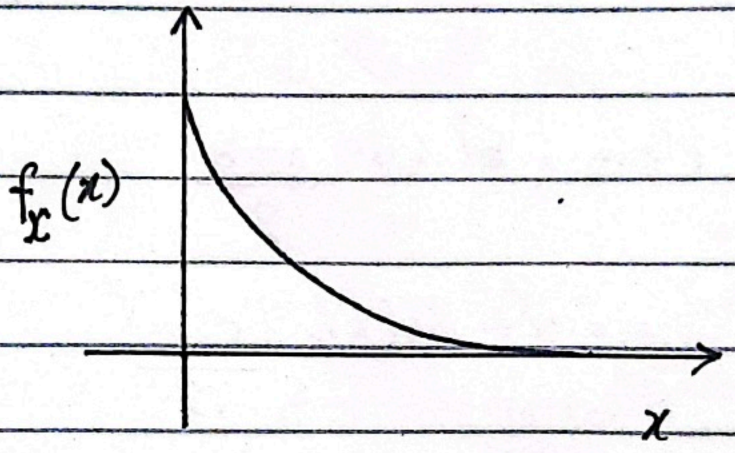
$$P(X=k) = (1-p)^{k-1}\,p$$
$$= (q)^{k-1}\,p$$

Mean = $\frac{1}{p}$

Variance = $\frac{1-p}{p^2}$

→ Exponential Distribution:

The exponential distribution is often concerned with the amount of time until some specific event occurs.



$$f(x) = \begin{cases} \lambda e^{-\lambda x} & 0 \leq x \\ 0 & \text{otherwise} \end{cases}$$

$$P(X > x) = 1 - (1 - e^{-\lambda x})$$
$$= e^{-\lambda x}$$

$$P(X < x) = 1 - e^{-\lambda x}$$

$$E(X) = 1/\lambda \qquad Var(X) = 1/\lambda^2$$

→ Binomial Distribution:

Binomial Distribution for a random variable $X = 0, 1, 2, \ldots, n$ is defined as the probability distribution of two outcomes success or failure in a series of events. Binomial Distribution in statistics uses one of the two independent variables in each trial where the outcome of each trial is independent of the other trials.

$$P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x} \quad , x = 0, 1, 2, \ldots$$

Where, p is success
q is failure, $q = 1 - p$
n is total number of trials

Mean $= n.p$

Variance $= n.p.q$