Formula:

$$f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

=> λ is the average number of times an event occurs

=> x is the number of times an event occurs

=> e is Euler's constant (≈ 2.718)

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Example: Suppose there is a bakery on the corner of the street and on average 10 customers arrive at the bakery per hour. For this case, we can calculate the probabilities of different numbers of customers arriving at the bakery at an hour using the Poisson distribution. As probability mass function or Poisson Distribution formula is given as:

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Probability of having exactly 5 customers arrive in an hour:

$$P(X=5) = \frac{10^5 \times e^{-10}}{5!} \approx 0.037$$

Binomial Distribution:

Binomial Distribution for a random variable $X = 0, 1, 2, \ldots, n$ is defined as the probability distribution of two outcomes success or failure in a series of events. Binomial Distribution in statistics uses one of the two independent variables in each trial where the outcome of each trial is independent of the other trials.

Formula:

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots$$

Where, p is success

q is failure and $q = 1 - p$

$p, q > 0$ such that $p + q = 1$

Example: Let's say we toss a coin twice, and getting head is a success we have to calculate the probability of success and failure then, in this case, we will calculate the probility distribution as follows:

In each trial getting a head that is a success, its probability is given as $p = 1/2$

$n = 2$ as we throw coin twice

$x = 0$ for no success, $x = 1$ for getting head once, and $x = 2$ for getting head twice

$$q = 1 - p = 1 - 1/2 = 1/2$$

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$P(X=1) = {}^2C_1 (1/2)^1 (1/2)^1$

$= 2 \times 1/2 \times 1/2 = 1/2$

$P(X=2) = {}^2C_2 (1/2)^2 (1/2)^0 = 1/4$

$P(X=0) = {}^2C_0 (1/2)^0 (1/2)^2 = 1/4$

Binomial Distribution Mean:

The mean of Binomial Distribution is the measurement of average that would be obtained in 'n' number of trials.

$$\mu = n \cdot p$$

=> 'n' is total number of trials

=> 'p' p is the probability of success in each trial

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Example: If we toss a coin 20 times and getting head is the success then what is the mean of success?

$n = 20$

$p = 1/2 = 0.5$

$\text{Mean} = n.p = 20 \times 0.5$

$= 10$

It means on average we would get head 10 times on tossing a coin 20 times.

Binomial Distribution Variance:

The variance of Binomial Distribution tells about the dispersion or spread of the distribution.

$$\sigma^2 = n.p.q$$

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=> n is total number of trials
=> p is the probability of success in each trial
=> q is the probability of failure in each trial

From Previous Example:

$p = 0.5$
$q = 1 - p = 0.5$
$n = 20$

$\sigma^2 = 20 \times 0.5 \times 0.5 = 5$

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→ All questions of normal distribution are done using z_table.

Example 1: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

The z values corresponding to $x_1 = 45$ and $x_2 = 62$ are

$$z_1 = \frac{45-50}{10} = -0.5$$

$$z_2 = \frac{62-50}{10} = 1.2$$

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Therefore,

$$P(45 < X < 62) = P(-0.5 < Z < 1.2)$$
$$= P(Z < 1.2) - P(Z < -0.5)$$
$$= 0.8849 - 0.3085$$
$$= 0.5764$$

Example 2: Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362

$$Z = \frac{362 - 300}{50}$$

$$= 1.24$$

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$$P(X > 362) = P(Z > 1.24)$$
$$= 1 - P(Z < 1.24)$$
$$= 1 - 0.8925$$
$$= 0.1075$$