Number System

Rational Number :- Any number in the form of $\frac{p}{q}$ where $q \neq 0$ is called rational number. Ex: $\frac{1}{2}, \frac{5}{7}$ etc.

Finding rational number between two given number

<u>Formula</u>: let a and b be two given numbers

$\frac{a+b}{2}$ → It will give first rational number.

Q. Find six rational numbers between <u>3 and 4</u>.

Sol.

1st Rational Number = $\frac{7}{2}$

IInd Rational Number =

$$\frac{3+\frac{7}{2}}{2}$$

$$= \frac{6+7}{2} \div 2$$

$$= \frac{13}{2} \times \frac{1}{2}$$

$$= \frac{13}{4}$$

$$\frac{4+\frac{13}{4}}{2} = \frac{16+13}{4} \div 2$$

$$= \frac{29}{4} \times \frac{1}{2}$$

$$= \frac{29}{8}$$

$\underline{8 \div 2 = \underline{4 \times 7}}$

$$\frac{\frac{29}{8} + \frac{7}{2}}{2} = \frac{29 + 28}{8} \div 2$$

$$= \frac{57}{8} \times \frac{1}{2}$$

$$= \frac{57}{16}$$

Q: Find two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

$$\frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{7}{5} \div 2$$

$$= \frac{7}{5} \times \frac{1}{2}$$

$$= \frac{7}{10}$$

$$\frac{\frac{3}{5}+\frac{7}{10}}{2} = \frac{6+7}{10} \div 2$$

$$= \frac{13}{10} \times \frac{1}{2}$$

$$= \frac{13}{20} \text{ Ans.}$$

A: 0 is a rational number.

$\frac{p}{q} = \frac{0}{1} = 0 \quad q \neq 0 \qquad 2 = \frac{2}{1} \qquad 5 = \frac{5}{1}$

R: Any number in the form of $\frac{p}{q}$, $q \neq 0$ is a rational number.

(I) Both A and R are true and R is correct explan-ation of A.

(II) Both A and R are true but R is not correct explanation.

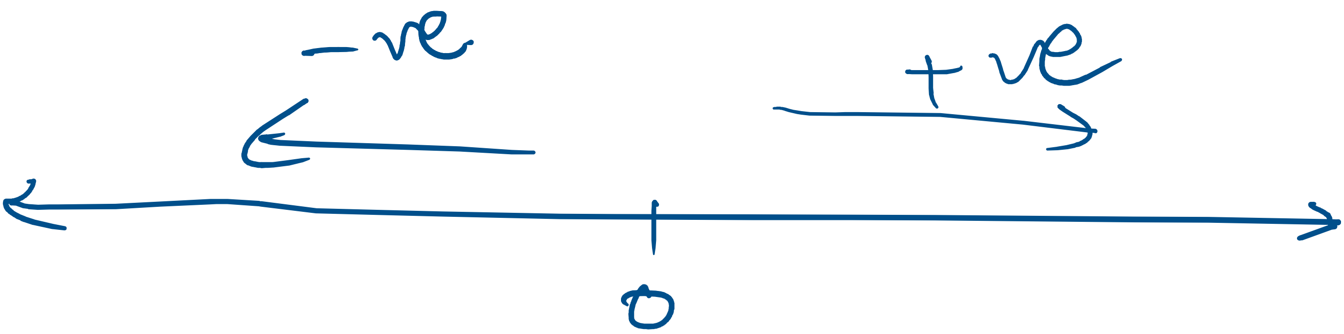
(III) A is true but R is false

(IV) A is false but R is true

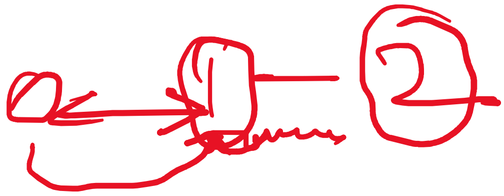
Irrational Number :- Any number which cannot be written in form of $\frac{p}{q}$ where $q \neq 0$.

Ex:- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$, etc.

$\sqrt{}$ all the prime numbers



Q. Locate $\sqrt{2}$ on number line.



sol.

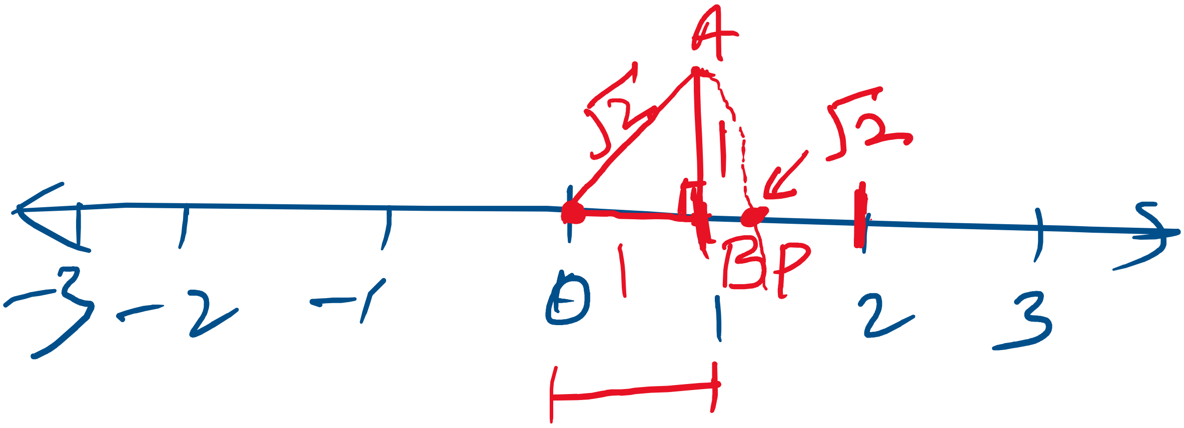
$\sqrt{2}$

$\sqrt{1} = 1$

$\sqrt{4} = 2$

50%

$$OA = \sqrt{OB^2 + AB^2} = \sqrt{1+1} = \sqrt{2}$$



Q. Locate $\sqrt{5}$ on number line.

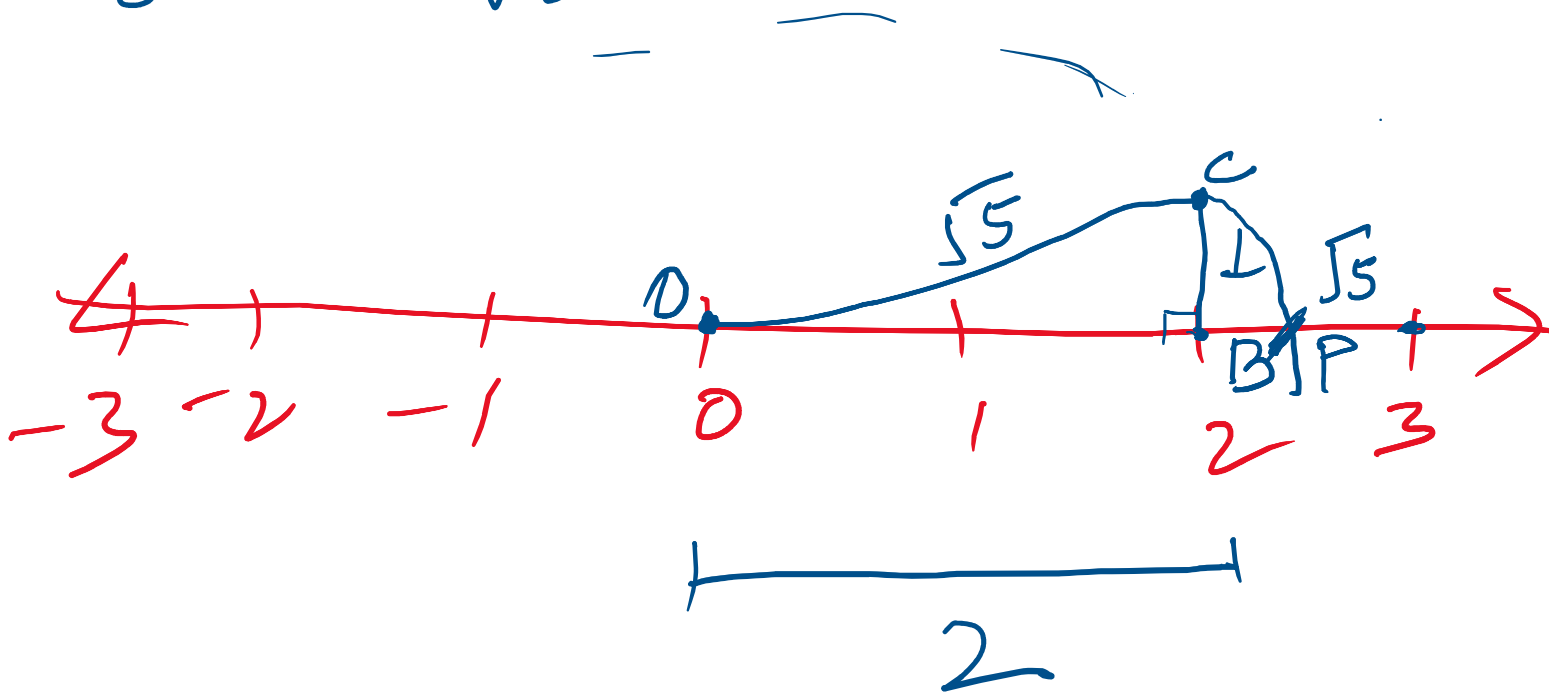
Sol.

$$\sqrt{\cancel{4}} = 2 \quad (\text{back})$$

$$\sqrt{9} = \sqrt{3 \times 3} = 3$$

$$OC^2 = 2^2 + 1^2 = 5$$

$$OC = \sqrt{5}$$



Q. Find decimal expansion of

(i) $\frac{10}{3}$ (ii) $\frac{5}{2}$

Sol.

$$\begin{array}{r|l} & 3.333\ldots \\ \hline 3 & 10 \\ & -9 \\ \hline & 10 \\ & -9 \\ \hline & 10 \\ & -9 \\ \hline & 10 \\ & -9 \\ \hline & 1 \end{array}$$

Sol.

$$\begin{array}{r|l} & 2.5 \\ \hline 2 & 5 \\ & -4 \\ \hline & 10 \\ & -10 \\ \hline & 0 \end{array}$$

$\frac{10}{3} = 3.333\ldots \rightarrow$

$\frac{5}{2} = 2.5$

(a) Remainder becomes zero. Ex!- $\frac{5}{2} = 2.5$,

$\frac{1}{2} = 0.5$

It is called terminating.

(b) Remainder doesn't becomes zero.

Eg:-

$$\frac{10}{3} = 3.333\ldots\ldots$$

$$\frac{1}{3} = 0.333\ldots\ldots$$

These are called non-terminating rational.

$$\frac{1}{3} = 0.333\ldots\ldots$$

$$= 0.\overline{3}$$

Numbers repeat that's why

they are also called recurring decimal expansion

Ex:- $1.272727\ldots\ldots$

$= 1.\overline{27}$

$\Rightarrow 0.\overline{142857}$

Q. check the type of rational number for $\frac{1}{11}$.

$\frac{1}{11} = 0.\overline{09}$

$$\begin{array}{r} 0.0909 \\ 11 \overline{)\,100} \\ -99 \\ \hline 100 \\ -99 \\ \hline 1 \end{array}$$

↓

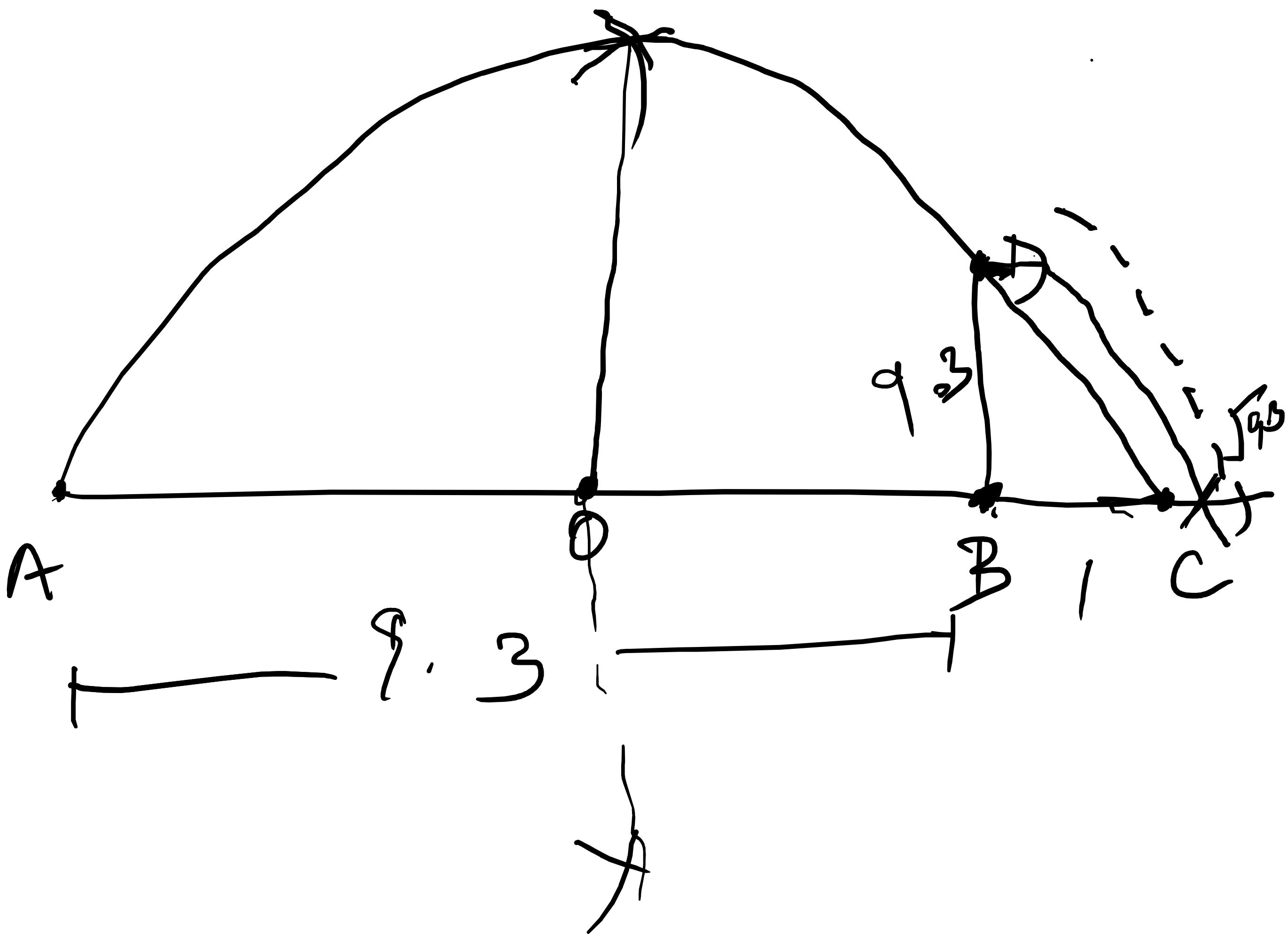
Non-terminating

Recurring

Ex: $1.2, \; 1.3 - 1, 2, 3$

Number System

$\sqrt{9.3}$



$$2^3 \times 2^2 = 2^{3+2}$$

$$2^3 \times 3^2 = 2^3 \times 3^2$$

$$2^3 \div 2^2 = 2^{3-2} = 2^1$$

$$\cancel{2^3 \div 3^2}$$

Laws of exponents

(I) $a^m \cdot a^n = a^{m+n}$

(II) $(a^m)^n = a^{mn}$

Ex: $(2^3)^2 = 2^6$

(III) $\dfrac{a^m}{a^n} = a^{m-n}$

(IV) $a^m \cdot b^m = (ab)^m$

Ex: $2^3 \cdot 3^3 = (6)^3$ ✓

$\boxed{2^3 \cdot 3^2} = \times\!\!\times$

Q. <u>Simplify :-</u>

(1) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}$

<u>Sol.</u> $2^{\frac{2}{3}+\frac{1}{3}}$ $\left(\because a^m \cdot a^n = a^{m+n}\right)$

$2^{\frac{2+1}{3}} = 2^{\cancel{3}/\cancel{3}} = 2^1$

$= 2$ Ans.

Q. $\left(3^{\frac{1}{5}}\right)^{4} = 3^{4/5}$

Q. $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}} = 7^{\frac{1}{5}-\frac{1}{3}}$

$= 7^{\frac{3-5}{15}}$

$= 7^{-2/15}$

$= 7$ Ans

Q. $13^{\frac{1}{5}} \cdot 17^{\frac{1}{5}} =$

$a^{m} \cdot b^{m} = (ab)^{m}$

$= (13\times 17)^{\frac{1}{5}}$

$= (221)^{\frac{1}{5}}$ Ans. $3^{1/5}$ ✓

221

$8 = 2 \times 2 \times 2$

$= 2^3$

2	8
2	4
	2

Q. $(64)^{1/2}$

Sol.

$64 = \boxed{2}^6 = 2^6$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$(64)^{1/2}$

$= (2^6)^{1/2}$

$= 2^{\cancel{36} \times \frac{1}{\cancel{2}}}$

$= 2^3$ Ans.

1. $a^m \cdot a^n = a^{m+n}$

2. $(a^m)^n = a^{mn}$

$= 8$ Ans.

Q. $(8)^4 = (p^m)^4$

1, 2, 4, 8

~~$8 = 2^3$~~

2	8
2	4
2	2
	1 ←

stop

$8 = 2 \times 2 \times 2$

$= 2^3$

$(8)^4 = (2^3)^4 = 2^{12}$ Ans.

Q. $(27)^{\frac{2}{3}}$

$(3^3)^{2/3}$

$= 3^{\frac{3}{1} \times \frac{2}{3}}$

$\frac{3}{1} \times \frac{2}{3} = \frac{\cancel{6}^{2}}{\cancel{3}_{1}} = 2$

$= 3^{2}$

$= 9$ **Ans.**

Q: $(125)^{1/3}$

$5^{3 \times \frac{1}{3}} = 5^{1} = 5$ **Ans**

Imp:

Q. $\boxed{x^{p-q} \cdot x^{q-r}} \cdot x^{r-p}$

Simplify.

$a^{m} \times a^{n} = a^{m+n}$

Sol: $x^{p-q+q-r} \cdot x^{r-p}$

$$= x^{p-r} \cdot x^{r-p}$$

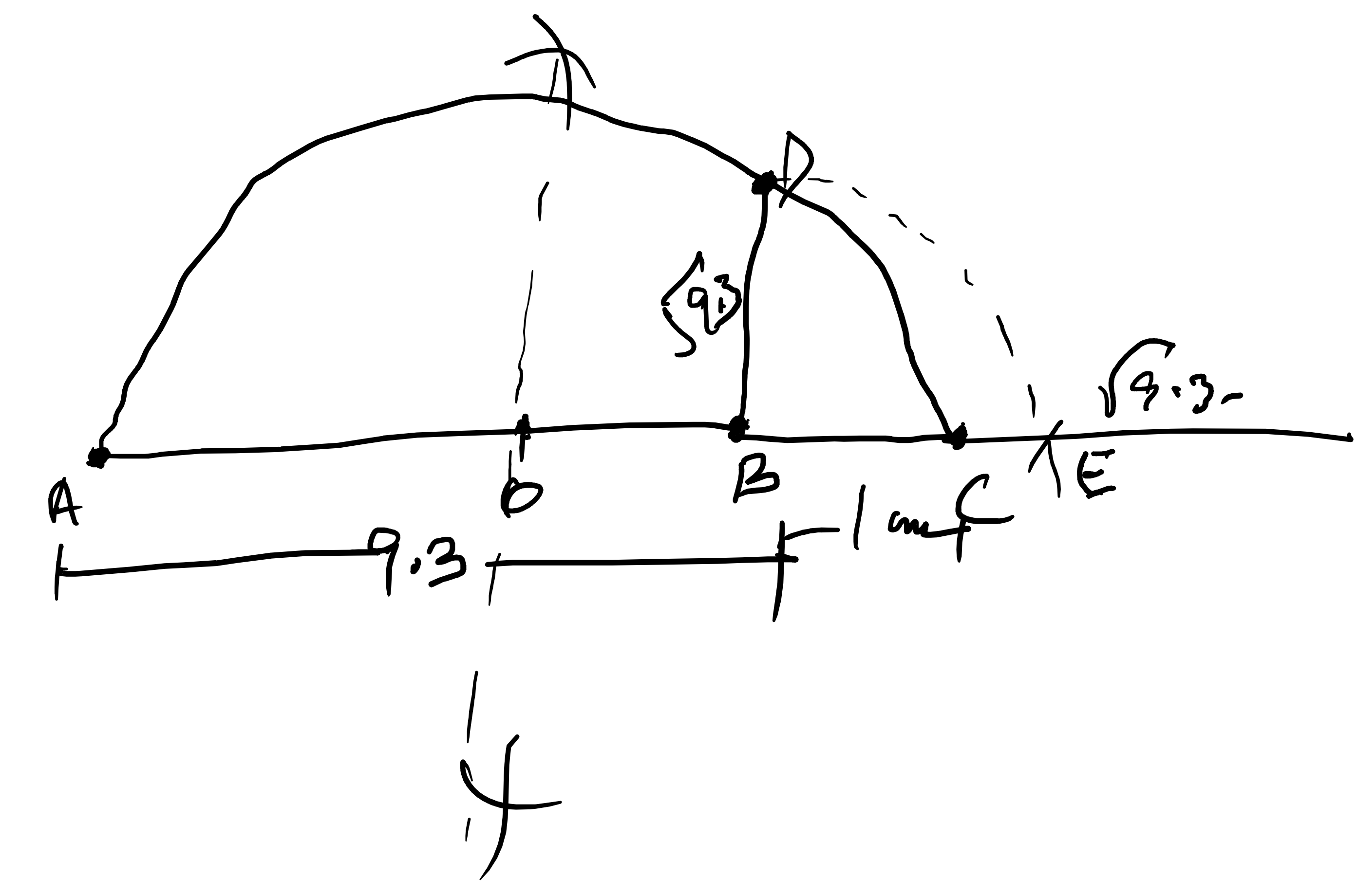
$$= x^{p-r+r-p}$$

$$= x^{0+0} = x^{0}$$

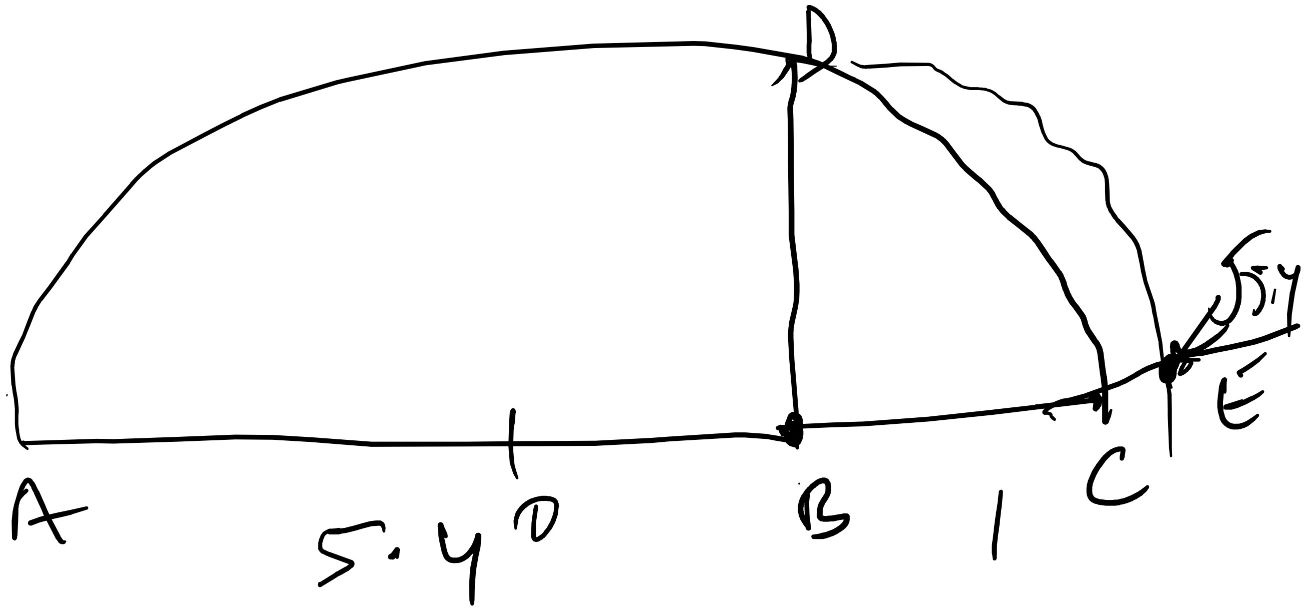
$$= 1 \text{ Ans.}$$

$$\boxed{\begin{aligned} a^0 &= 1 \\ a^1 &= a \end{aligned}}$$

$\sqrt{9.3}$



Q. Draw $\sqrt{5.4}$ on number line.



Imp.

(3 marks)

Q. Determine rational numbers p and q if

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = p - 7\sqrt{5}q$$

Sol.

LHS :-

$$\boxed{\frac{7+\sqrt{5}}{7-\sqrt{5}}} - \boxed{\frac{7-\sqrt{5}}{7+\sqrt{5}}}$$

$$\frac{7+\sqrt{5}}{\boxed{7-\sqrt{5}}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}}$$

$(a+b)(a-b)$
$= a^2-b^2$

$$\frac{(7+\sqrt{5})(7+\sqrt{5})}{7^2-(\sqrt{5})^2}$$

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} = \frac{(7+\sqrt{5})^2}{49-5} = \boxed{\frac{(7+\sqrt{5})^2}{44}}$$

LHS :

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}}$$

$$= \frac{(7+\sqrt{5})^2}{44} - \frac{(7-\sqrt{5})^2}{44}$$

$$= \frac{(7+\sqrt{5})^2 - (7-\sqrt{5})^2}{44}$$

$$= \frac{(7^2+(\sqrt{5})^2+2\cdot 7\cdot \sqrt{5}) - (7^2+(\sqrt{5})^2 - 2\cdot 7\cdot \sqrt{5})}{44}$$

$-, - = +$

$-, + = -$

$+, + = +$

$- \times - = +$

$- \times + = -$

$$= \frac{(49+5+14\sqrt{5}) - (49+5-14\sqrt{5})}{44}$$

$$= \frac{\cancel{49}+\cancel{5}+14\sqrt{5}-\cancel{49}-\cancel{5}+14\sqrt{5}}{44}$$

$$= \frac{\overset{7}{\cancel{\overset{14}{\cancel{28}}}}\sqrt{5}}{\cancel{44}\,\underset{11}{\cancel{22}}} \qquad + \frac{\frac{14\sqrt{5}}{14\sqrt{5}}}{28\sqrt{5}}$$

$$= \frac{7\sqrt{5}}{11} = \underline{LHS}$$

RHS :- $p - 7\sqrt{5}q$

$$LHS = RHS$$

$$\frac{7\sqrt{5}}{11} = p - 7\sqrt{5}q$$

On comparing both sides

$$\frac{7\sqrt{5}}{11} = -7\sqrt{5}q$$

$$\Rightarrow q = \frac{\cancel{7\sqrt{5}}}{11 \times (-\cancel{7\sqrt{5}})}$$

$$\Rightarrow q = -\frac{1}{11}$$

$$p = 0$$

$x = 1 + \sqrt{2}$

1) $x^2 + \dfrac{1}{x^2}$

$(a+b)^2 = a^2 + b^2 + 2ab$

Soln

$x^2 = (x)^2$

$x^2 = (1+\sqrt{2})^2$

$x^2 = 1^2 + (\sqrt{2})^2 + 2.1.\sqrt{2}$

$x^2 = 1 + 2 + 2\sqrt{2}$

$\boxed{x^2 = 3 + 2\sqrt{2}}$

$\dfrac{1}{x^2} = \dfrac{1}{3+2\sqrt{2}} \times \dfrac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$$\boxed{\frac{1}{x^2} = 3-2\sqrt{2}}$$

$$x^2 + \frac{1}{x^2} = 3+\cancel{2\sqrt{2}} + 3-\cancel{2\sqrt{2}}$$

$$\boxed{x^2 + \frac{1}{x^2} = 6}$$ Ans.

11) $x^3 - \dfrac{1}{x^3}$

$(1+\sqrt{2})^2$

$(a+b)^2$

Sol:

$$x^3 = (x)^3$$

$$= (1+\sqrt{2})^3$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$x^3 = (1+\sqrt{2})^3$$

$$x^3 = 1^3 + (\sqrt{2})^3 + 3.1.\sqrt{2}(1+\sqrt{2})$$

$$x^3 = 1 + 2\sqrt{2} + 3\sqrt{2}(1+\sqrt{2})$$

$$x^3 = 1 + \underline{2\sqrt{2} + 3\sqrt{2}} + 6$$

$$\boxed{x^3 = 7 + 5\sqrt{2}}$$ ✓

$$\frac{1}{x^3} = \frac{1}{7+5\sqrt{2}}$$

$$\frac{1}{x^3} = \frac{7-5\sqrt{2}}{}$$

$$= -(7-5\sqrt{2})$$

$$\boxed{\frac{1}{x^3} = -7+5\sqrt{2}}$$

$$x^3 - \frac{1}{x^3} = 7+5\sqrt{2} - (-7+5\sqrt{2})$$

$$x^3 - \frac{1}{x^3} = 7+\cancel{5\sqrt{2}}+7 - \cancel{5\sqrt{2}}$$

$$x^3 - \frac{1}{x^3} = 14 \text{ Ans.}$$

Q. Rationalise $\frac{1}{\sqrt{3}+\sqrt{2}}$

and evaluate by

taking $\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

$\sqrt{3} \times \sqrt{3} = 3$

$$\boxed{\begin{aligned} \sqrt{a} \times \sqrt{a} &= a \\ (\sqrt{a})^2 &= a \end{aligned}}$$

Q. ~~The~~ What is coefficient of x^2 in $3x^3+2x^2-x+1$

Ans. 2

Q. Write a binomial of degree 20?

Ans. $x^{20}+y$ $\quad$ ~~$x^{20}\times y$~~

$y x^{20}$ ~~op~~ $x^{20}-1$ $\quad + \quad -$

$x^{20}+1$

Q. what is value of

$$f(x) = 5x - 4x^2 + 3$$

when $x = -1$?

$$f(-1) = 5(-1) - 4(-1)^2 + 3$$

$$= -5 - 4 \times 1 + 3$$

$$= -9 + 3$$

$$= -6 \text{ Ans.}$$

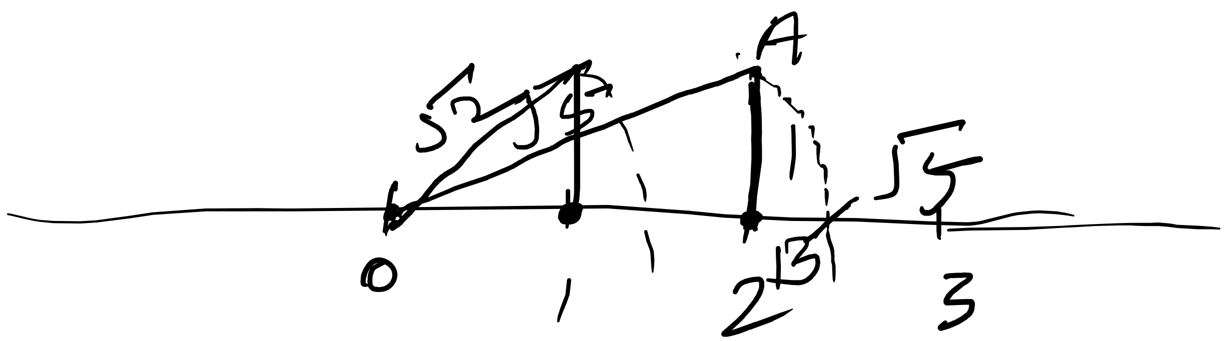
11:30 to 1:30

10 to 12

Your time

$\sqrt{4} = 2$ is a rational.

$\sqrt{2} \to 1 - 2 \qquad 2 - 3$



$$\begin{array}{r} 0.23076 \\ 13\,\overline{)\,3\qquad} \end{array}$$

0.23076

$$13 \overline{\big) \; 30} \quad 0.230769230\ldots$$

$$0.\overline{230769}$$

$$
\begin{array}{r}
30 \\
-26 \\
\hline
40 \\
-39 \\
\hline
100 \\
-91 \\
\hline
90 \\
-78 \\
\hline
120 \\
-117 \\
\hline
30 \\
-26 \\
\hline
4
\end{array}
$$

$$400 \overline{)\quad} \quad 0.82225$$

$0.4\bar{7}$

7777

$x = 0.477\ldots$

$100x = 47.777 \quad - \; 0.477$

$90x = 4.300$

$x = \frac{43}{90}$

$x = 0.\bar{9}999\ldots$

$10x = 9.999\ldots$

$9x = 9$

$x = 1$

$\frac{1}{1} = \frac{p}{q}$

$\sqrt{225} = \sqrt{15 \times 15} = \underline{15}$

$\underline{2+2} \rightarrow \underline{2x + 2x} =$

Operations on Real Numbers

Rational (R)

Irrational (IR)

① $\left.\begin{array}{l} R + R \\ R - R \\ R \times R \\ R \div R \end{array}\right\} = R$

(2) IR + IR = IR

IR − IR = Rational / IR

$\sqrt{2} - \sqrt{2} = 0$ → Rational; $\sqrt{3} - \sqrt{2}$ → IR

IR × IR = 2 → Rational

$\sqrt{2} \times \sqrt{2}$

IR ÷ IR = 1 → Rational

$\sqrt{2} / \sqrt{2}$

1. The sum, difference, product and division of two rational number will be rational.

2. If we add, subtract a rational with an

irrational then the outcome will be irrational.

3. If we multiply or divide any irrational number with a rational then the result will be irrational.

$\sqrt{5}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{1}$,

$\sqrt{3.6}$, $\sqrt{1.2}$, $\sqrt{2.2}$

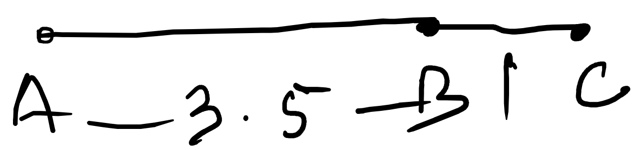
Finding roots of positive real number on number line.

$\sqrt{x}$ where x is decimal

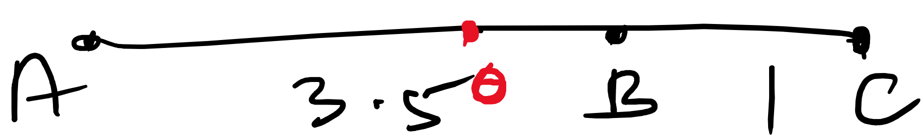
Q. Represent $\sqrt{3.5}$ on number line.

Step 1: On a line measure distance = 3.5 such that
AB = 3.5

Step 2: From B mark a point C such that BC = 1



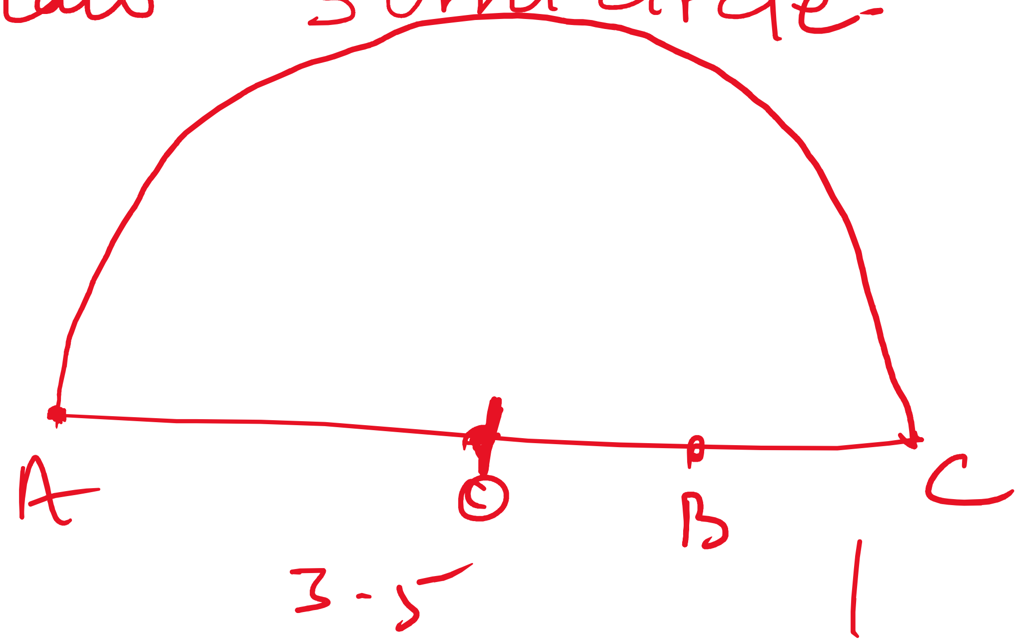
Step 3: - take midpoint of AC and mark it as O.



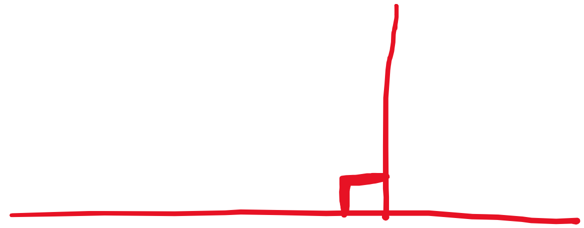
O is midpoint of AC

It means AO = OC

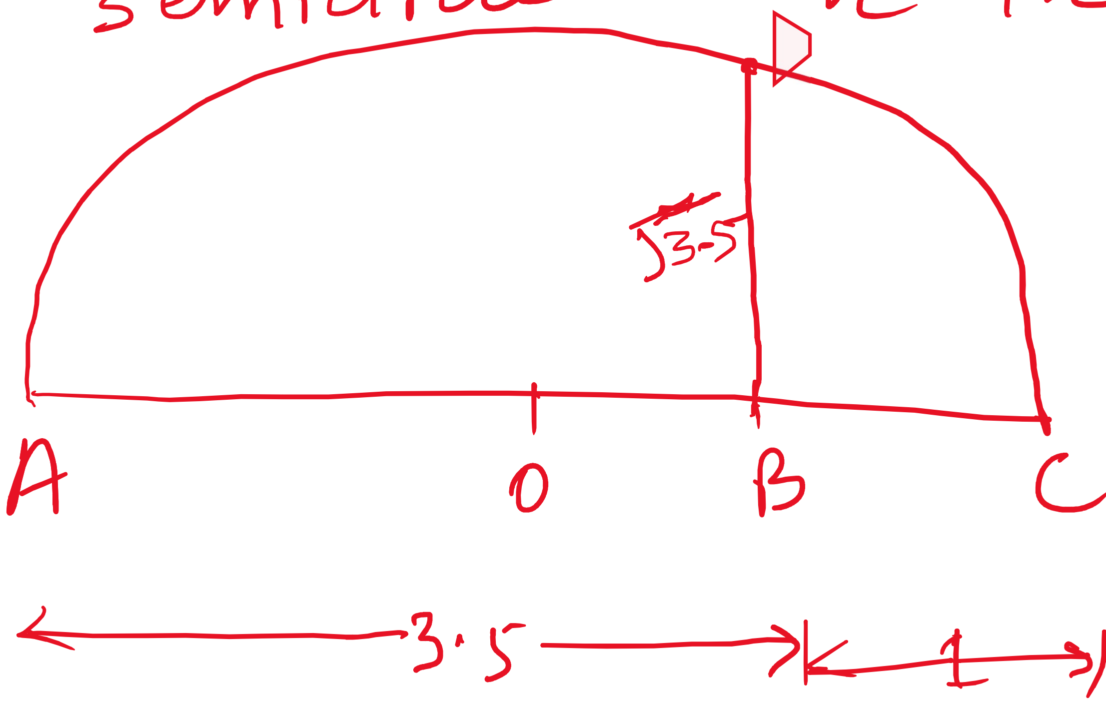
<u>Step 4:</u> Take radius as OC on compass and draw semicircle.



Steps 5 :



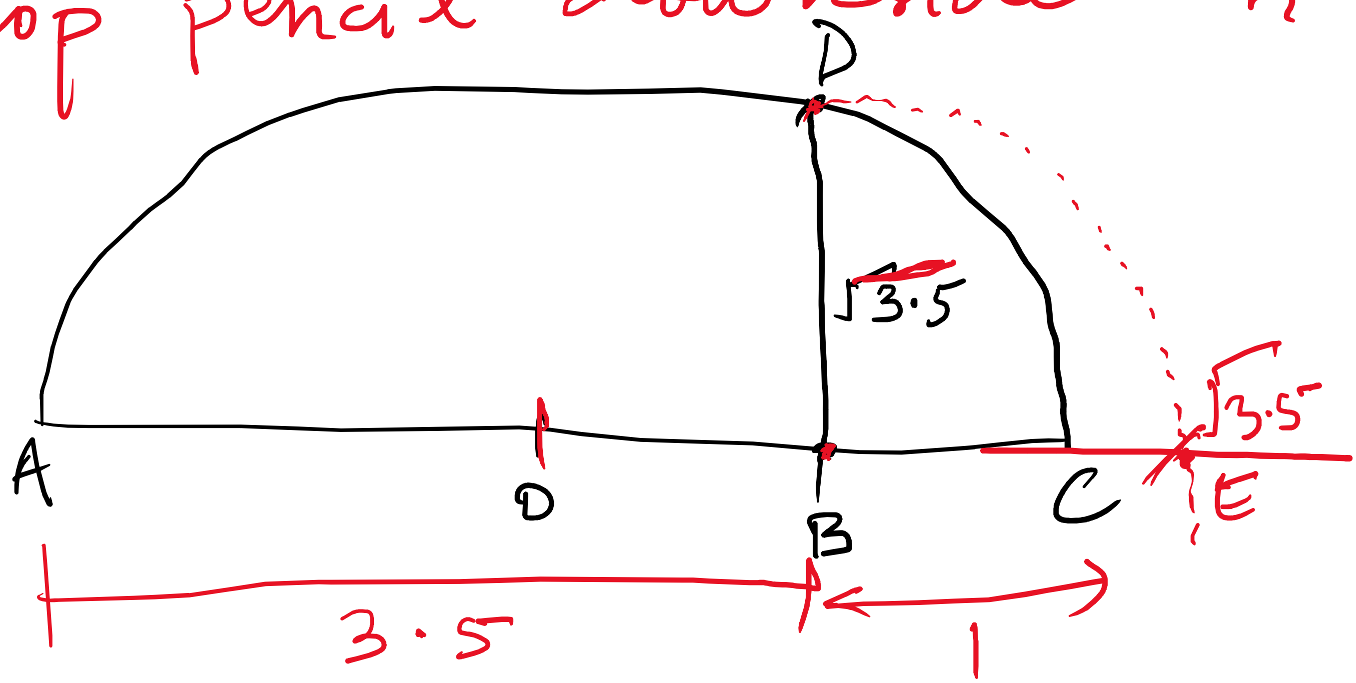
From B draw perpendicular on semicircle such that it



intersect semicircle at point D. So, $BD = \sqrt{3.5}$.

Step 6 :-

We will keep needle of compass on point B and pencil on point D and drop pencil downside on



the number line such that it cut at point E. Point E is $\sqrt{3.5}$.

Q. Represent $\sqrt{9.3}$ on the number line.