2. Polynomial

$p(x) = a_0 x^0 + a_1 x + a_2 x^2 + \ldots\ldots + a_n x^n$ where $a_0, a_1, \ldots a_n$ are real numbers.

Degree of Polynomial :- The highest power of variable in a polynomial

Ex: (i) $u^3 + 7u + 4 \Rightarrow$ Degree $= 3$

(ii) $\frac{7}{6} y^6 + y^8 - 4y^2 \Rightarrow$ Degree $= 8$ $\boxed{u^0 = 1}$

(iii) $x^2 + 2x^{1/2} + 4 \rightarrow$ Not a Polynomial

(iv) $y^2 + \frac{2}{y} \rightarrow 2 \cdot y^{-1} \rightarrow$ $\qquad \frac{1}{y} = y^{-1} = \frac{1}{y}$

$y^2 + y^{-1} \rightarrow$ Not a Polynomial

Based on Degree, types of Polynomial

1. Linear Polynomial → Degree of polynomial is 1.

Ex: $2x+1$, y, etc.

2. Quadratic Polynomial → Degree = 2

Ex: $7x^2+4x$, $7y^2+3y-2$

3. Cubic Polynomial → Degree = 3

Ex: $7x^3+4x^2+2x$, $7y^3$, etc.

Zeroes of Polynomial

$$P(x) = 5x^2 + 2x$$

$\boxed{x = a} \rightarrow P(a) = 0$

$\Downarrow$

Zeroes of polynomial.

Eg: $P(x) = x - 1$

$x = 1$ $\quad P(1) = 1 - 1 = 0$

$$\boxed{P(1) = 0}$$

$x = 1 \leftarrow$ zeros of polynomial

$$P(x) = x - 1$$

✡

Number of zeroes in any polynomial = Degree of that polynomial

Linear Polynomial $\rightarrow$ 1 zeroes

Quadratic " $\rightarrow$ 2 zeroes.

Eg:- $7x^5 + 3x^3 + 2x$ $\longrightarrow$ 5 zeroes.

Quadratic Polynomial

General form :- $p(x) = ax^2 + bx + c$

where a, b, c are coefficients & real number.

Real = R + IR

Natura Integer Decimal

$1 \rightarrow$ Rational

$-5 \rightarrow$ Rational

$2.2 \rightarrow \frac{22}{10} = \frac{11}{5}$

Quadratic polynomial has 2 zeroes.

The zeroes are also called roots of polynomials

So, In quadratic polynomial

α and β are roots of

Alpha Beta

quadratic polynomial.

It means :-

$$P(x) = ax^2 + bx + c$$

$P(\alpha) = 0$ (because α is root)

$$\Rightarrow a \cdot \alpha^2 + b \cdot \alpha + c = 0$$

$p(\beta) = 0$ (because β is root

$\Rightarrow a \cdot \beta^2 + b \cdot \beta + c = 0$

Relation between zeros & coefficient of Quadratic Polynomial

$p(x) = ax^2 + bx + c$

✓ Sum of zeroes $= \frac{-b}{a}$

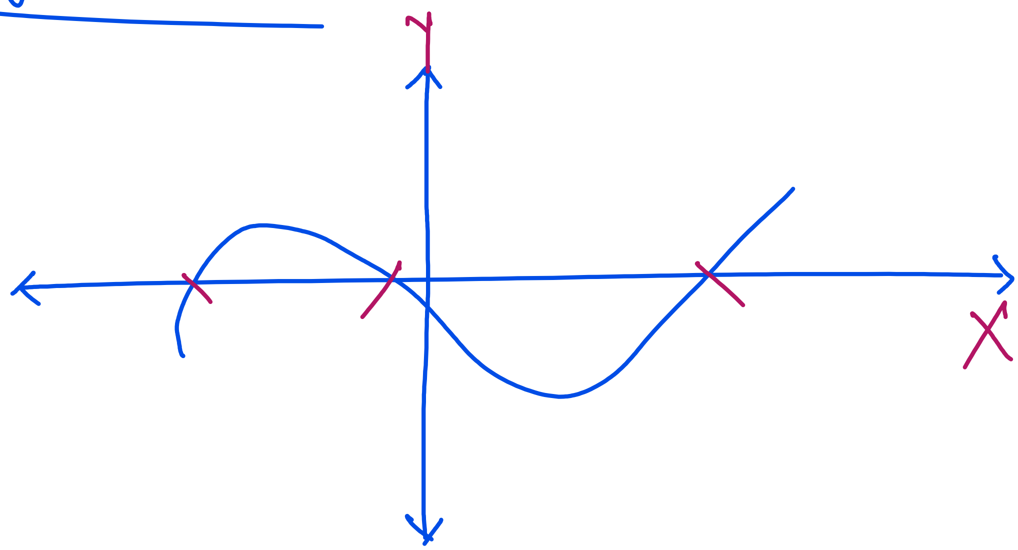
$\boxed{\alpha + \beta = \frac{-b}{a}}$

Product of zeroes $= \frac{c}{a}$

$$\boxed{\alpha \cdot \beta = \frac{c}{a}}$$

Geometrical meaning of Zeros of Polynomial

$P(x)$

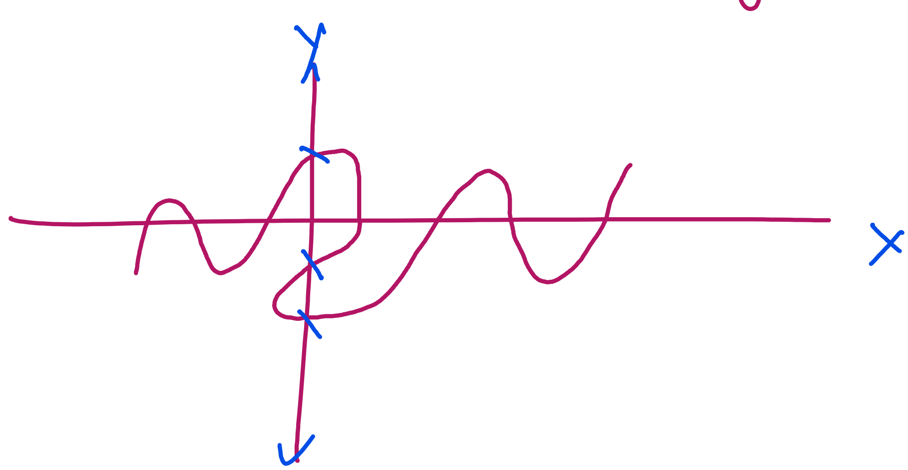


No. of intersecting point = No. of zeros

$P(y) = 3$

(i) 3

(ii) 7

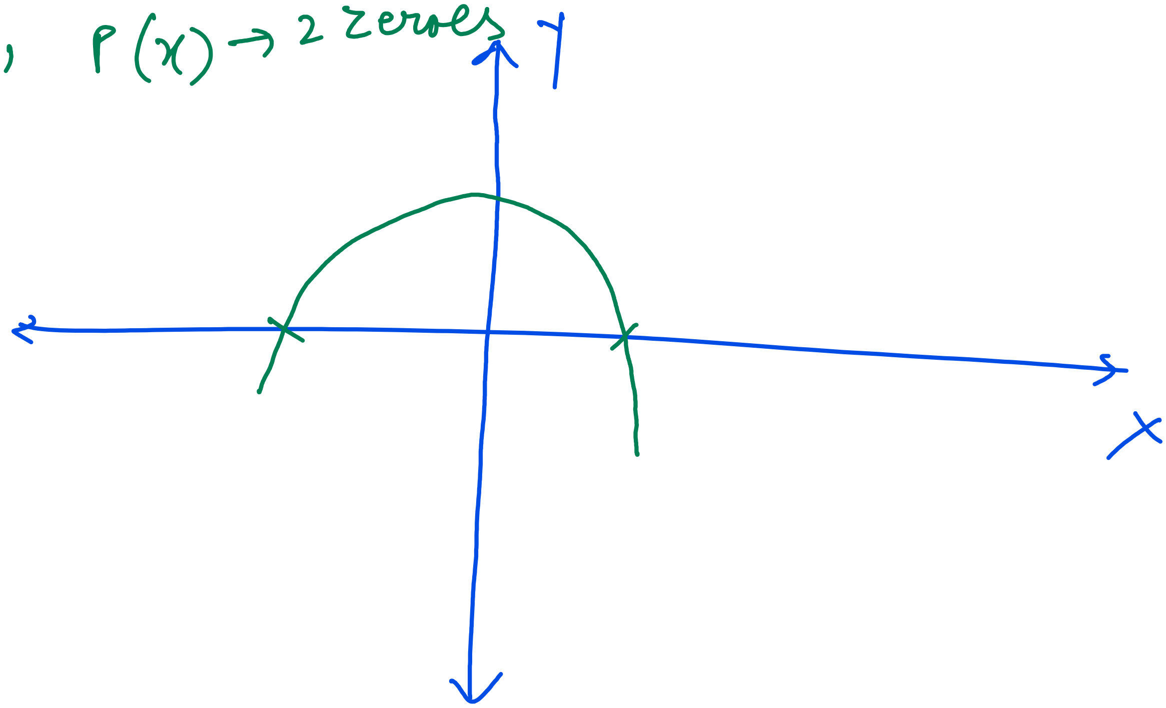


⇒ If polynomial contains only variable 'x' then we check how many points graph of that polynomial is intersecting on x-axis.

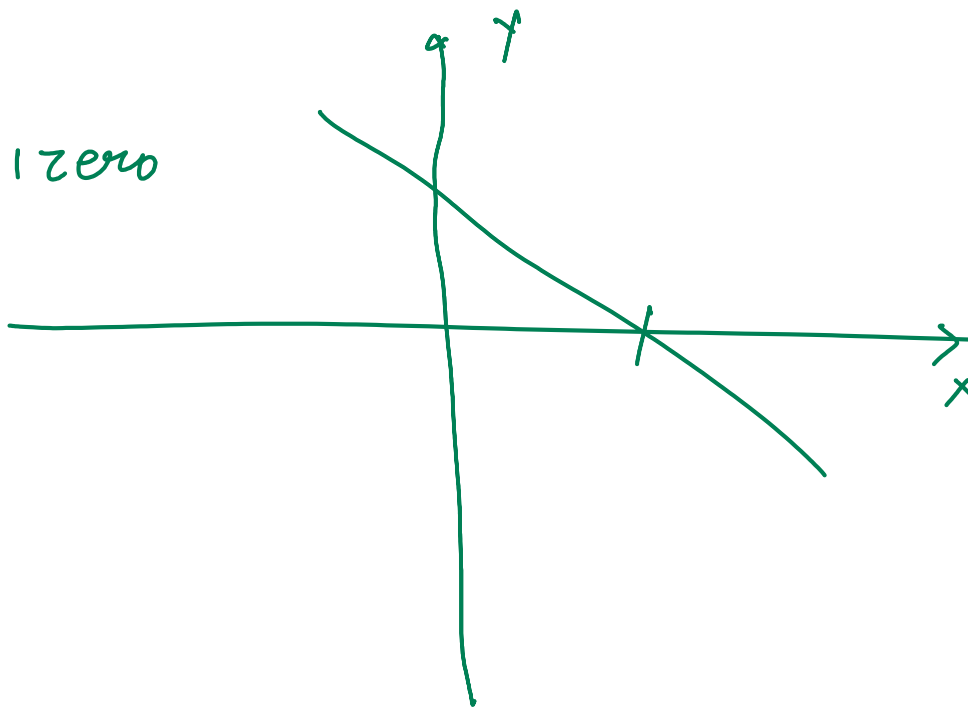
⇒ If polynomial contains only variable 'y' then we check how many points the graph is intersecting on y-axis.

Eg:-

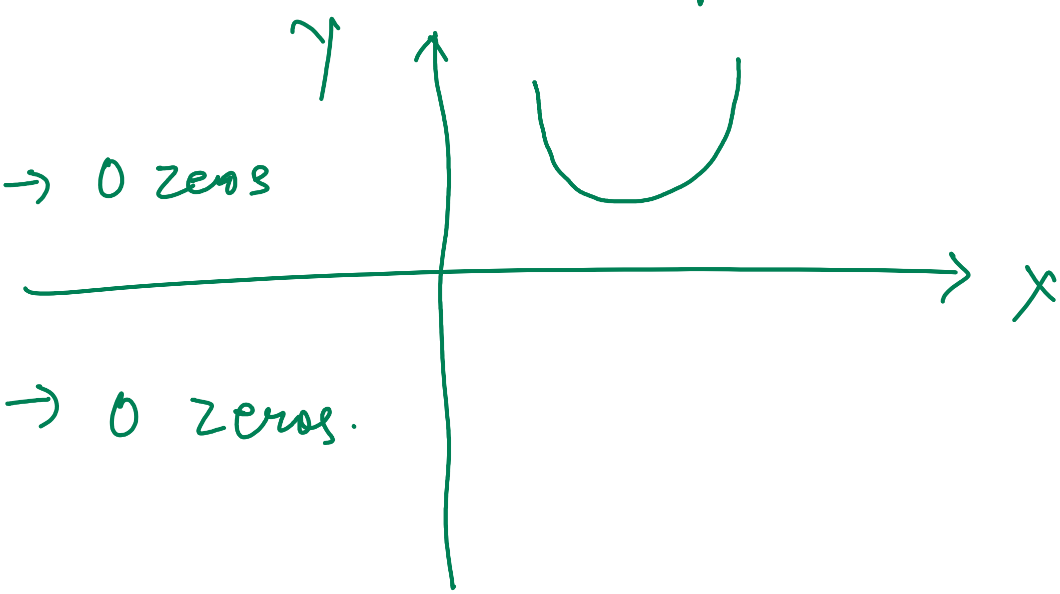
(i) $P(x) \to 2$ zeroes



(ii) $P(x) \to 1$ zero



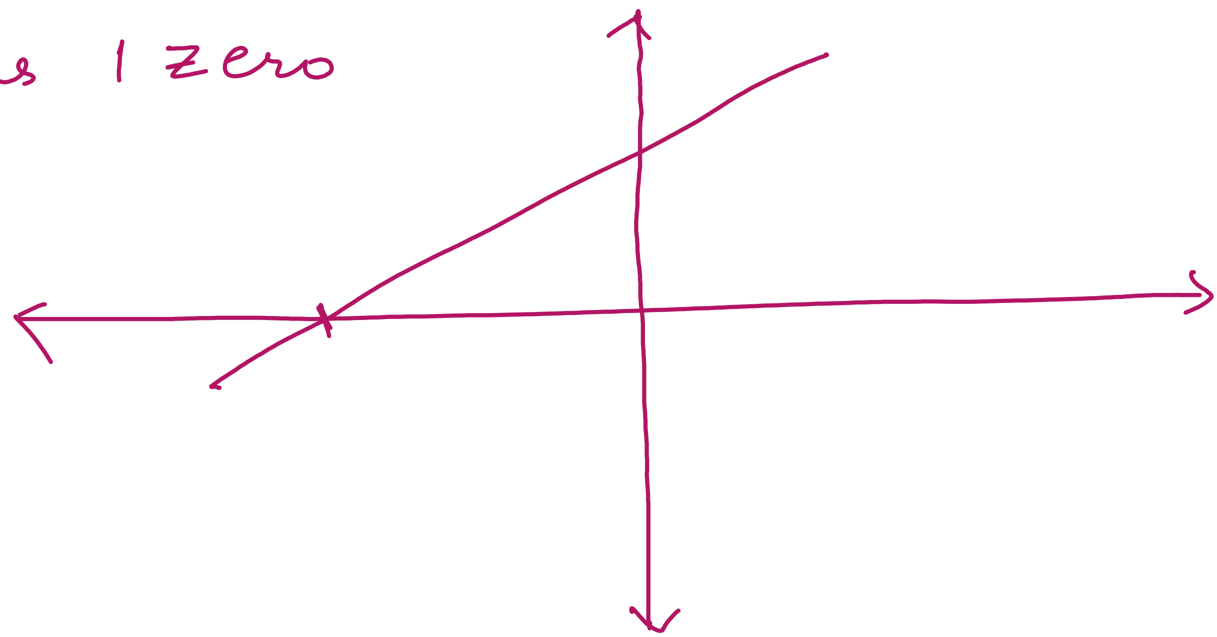
(iii) $P(x) \to 0$ zeros



$P(y) \to 0$ zeros.

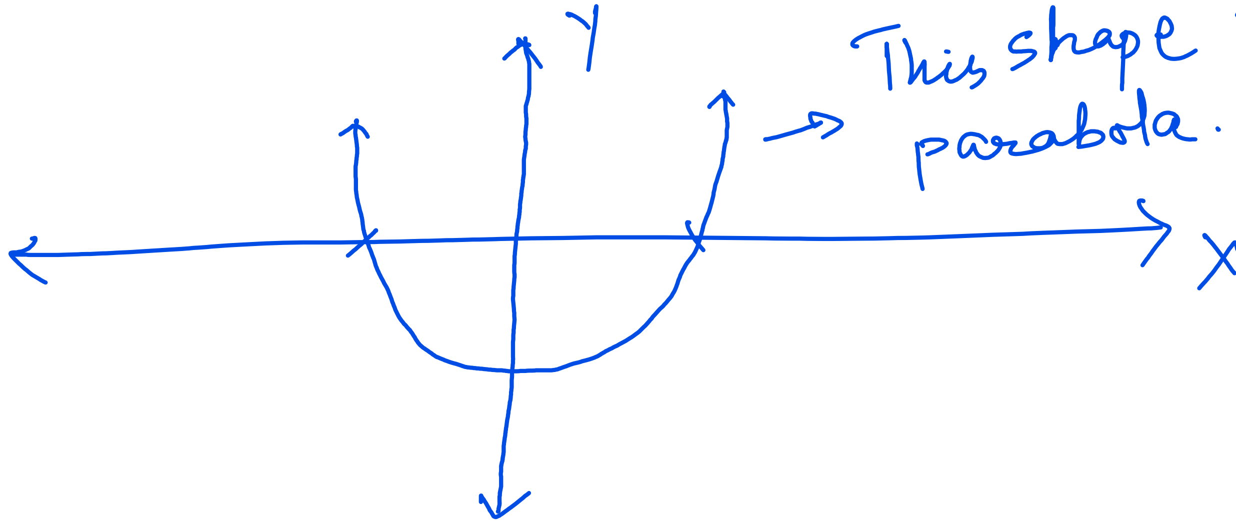
Graph of linear polynomial, $P(x)$

It has 1 zero



Graph of Quadratic Polynomial

$$p(x) = ax^2 + bx + c$$

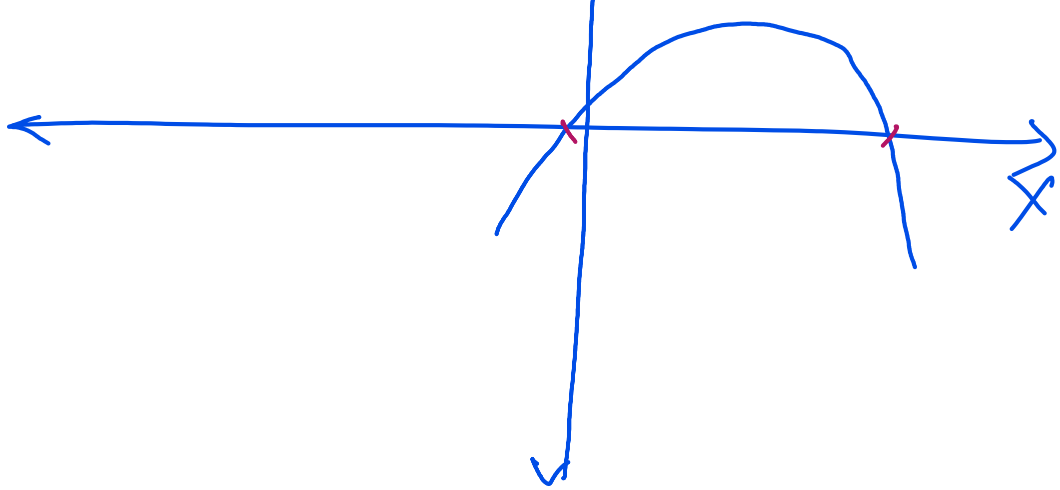


This shape is parabola.

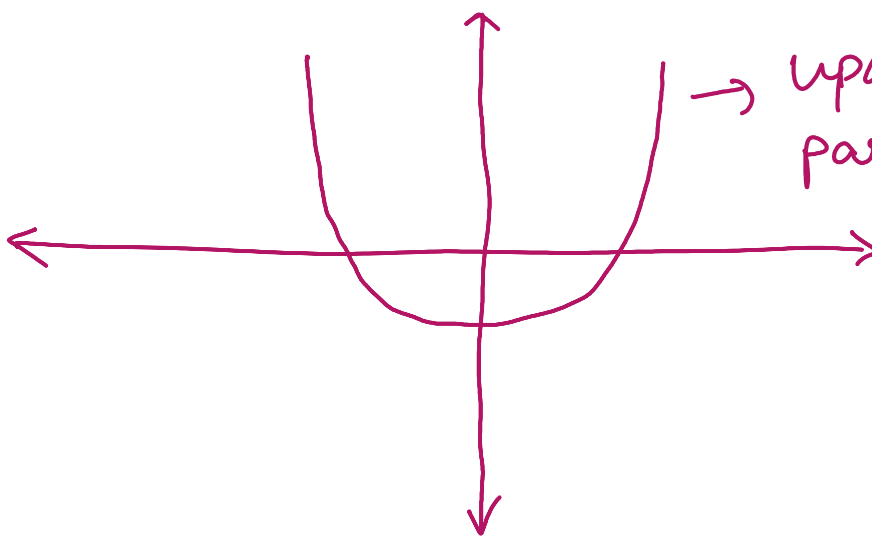
$p(x) = ax^2 + bx + c$

Case I :

Downward Parabola

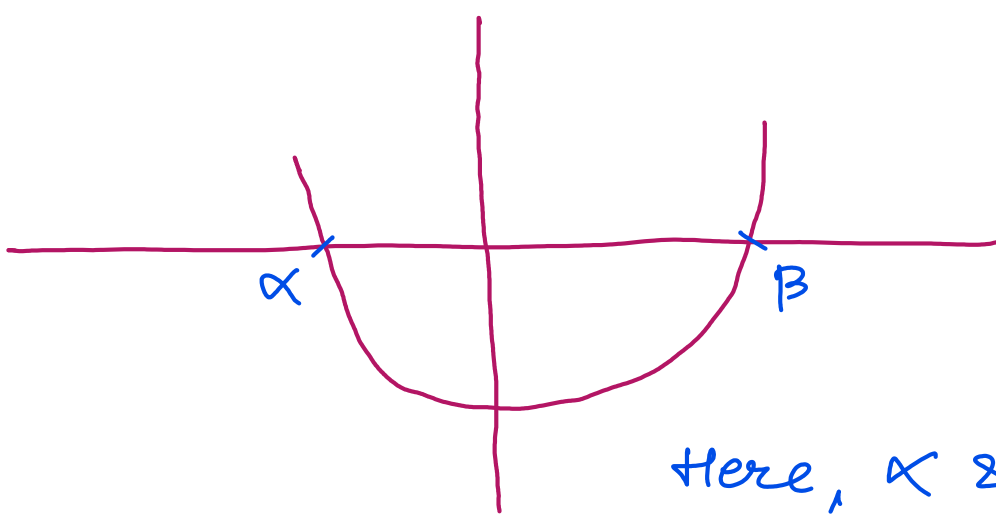


Case II :



→ upward parabola

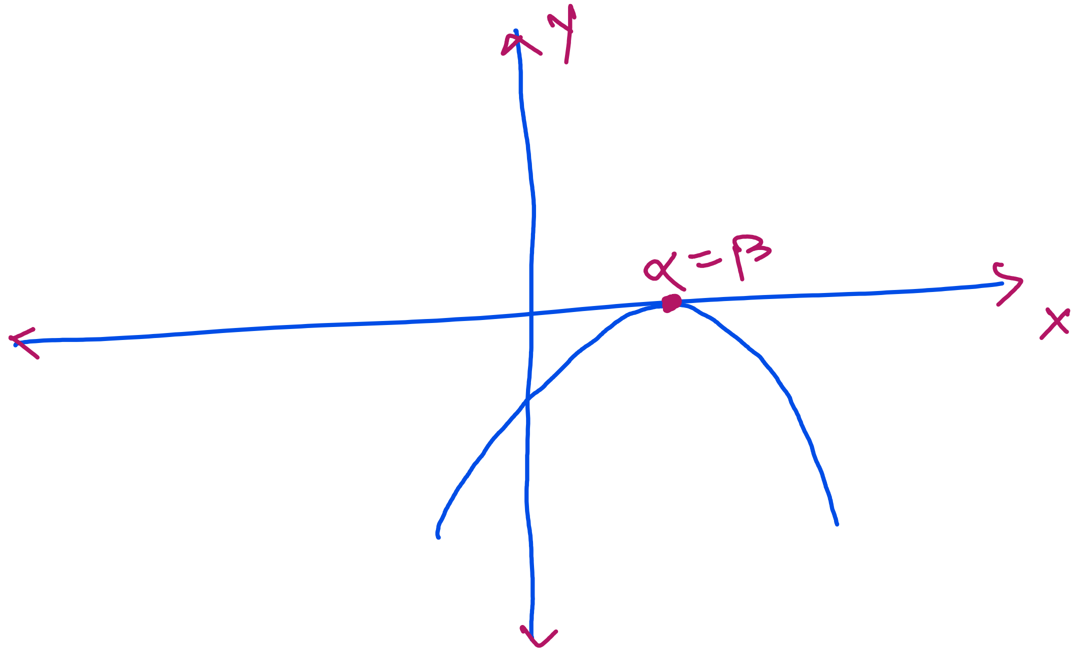
Q.



$\left.\begin{array}{l} \alpha = 1 \\ \beta = -1 \end{array}\right\}$

Here, α & β are at diff points which means the values of α & β are distinct.

Q.

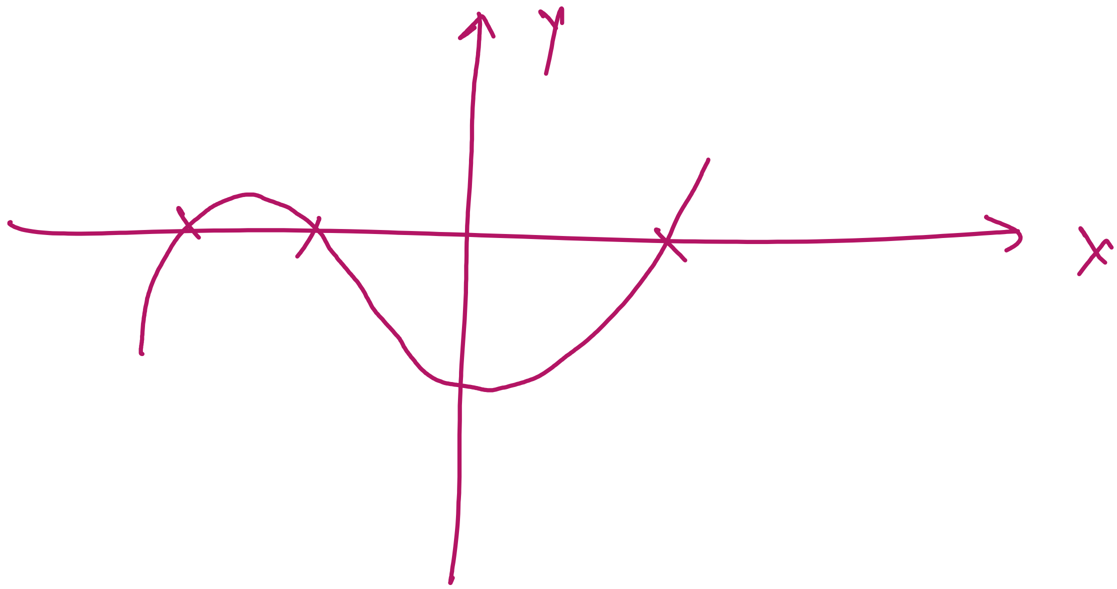


$\alpha = 1$ ✓

$\beta = 1$ ✓

Here, both zeros of quadratic polynomial are same. So, α & β coincide at one point

Graph of Cubic Polynomial



Q. $p(x) = cx^2 + ax + b$

Sum of zero $= \frac{-a}{c}$

Product $= \frac{b}{c}$

Sol. ~~General~~ form: $p(x) = ax^2 + bx + c$

Given: $p(x) = cx^2 + ax + b$

$a = c$, $b = a$, $c = b$

Sum of zeros $= -\frac{b}{a} = \frac{-a}{c}$

Product of zeros $= \frac{c}{a} = \frac{b}{c}$

Finding Roots / zeros of Quadratic Polynomial

Q. Find zeros of $p(x) = x^2 + 7x + 10$ and verify the relationship.

Sol.

$$p(x) = x^2 + 7x + 10$$

$$= x^2 + 2x + 5x + 10$$

$$= x(x+2) + 5(x+2)$$

$$= (x+2)(x+5)$$

Sum = 7 ✓

Prod. = 10 ✓

$a + b = 7$

$a \times b = 10$

$\frac{x^2}{x} = x$

$\frac{2\cancel{x}}{\cancel{x}} = 2$

$2 \,|\, 10$, 5

$10 = 2 \times 5$

$x + 2 = 0$	$x + 5 = 0$
$\Rightarrow x = -2$	$\Rightarrow x = -5$

$\therefore$ -2 & -5 are zeros/roots of given polynomial.

Verify :-

$ax^2 + bx + c$

$x^2 + 7x + 10$

$a = 1, \quad b = 7, \quad c = 10$

(1) Sum of zeros $= \dfrac{-b}{a}$

LHS : Sum of zeroes $= -2 + (-5)$

$= -2 - 5$

$= -7$

RHS : $\dfrac{-b}{a} = \dfrac{-7}{1} = -7$

$\therefore$ LHS = RHS

(ii) Product of zeroes $= \dfrac{c}{a}$

LHS :- Product of zeros $= (-2)\cdot(-5)$

$= 10$

$- \times - = +$

$+ \times - = -$

$+ \times + = +$

RHS :- $\dfrac{c}{a} = \dfrac{10}{1} = 10$

$\therefore$ LHS = RHS

Hence, the relationship is verified.

Find zeros & verify relationship

Q.1 $4s^2 - 4s + 1$

Sol. $4s^2 - 4s + 1$

$\underline{4s^2 - 2s} - \underline{2s + 1}$

$\Rightarrow 2s(2s-1) - 1(2s-1)$

Sum $= -4$

$P = 4$

$4 = 2 \times 2$

$2 + 2 = 4$

$(-2) + (-2) = -4$

Q.2. $x^2 - 2x - 8$

Sol. $x^2 - 2x - 8$

$x^2 - 4x + 2x - 8$

$\Rightarrow x(x-4) + 2(x-4)$

$S = -2$

$P = -8$

$-4 \times 2 = -8$

$-4 + 2 = -2$

$(2s-1)(2s-1)$

$s = \frac{1}{2}, \frac{1}{2}$

$\boxed{-2 \cdot -2 = 4}$

$\Rightarrow (x-4)(x+2)$

$x-4=0$ | $x+2=0$

$\Rightarrow x=4$ | $\Rightarrow x=-2$

Q. If one zero of $p(x) = 6x^2+37x-(k-2)$ is reciprocal of other, find value of k.

Sol. The zeroes are α & $\frac{1}{\alpha}$

$$p(x) = 6x^2+37x-(k-2)$$

$$\alpha \cdot \beta = \frac{c}{a}$$

$$\Rightarrow \cancel{\alpha} \cdot \frac{1}{\cancel{\alpha}} = \frac{-(k-2)}{6}$$

$$\Rightarrow 1 \times 6 = -(k-2)$$

$\Rightarrow \quad 6 = -k + 2$

$\Rightarrow \quad 6 - 2 = -k$

$\Rightarrow \quad -k = 4$

$\Rightarrow \quad \boxed{k = -4}$

2021

Q. If one zero of $p(x) = x^2 + 3x + k$ is 2 then find k.

Sol.
$$p(x) = x^2 + 3x + k$$
$$p(2) = 0$$
$$\Rightarrow 2^2 + 3 \cdot 2 + k = 0$$
$$\Rightarrow 4 + 6 + k = 0$$
$$\Rightarrow 10 + k = 0$$

$\Rightarrow \quad k = -10$ Ans

2019

Q. Find K such that $p(x) = x^2 - (K+6)x + 2(2K-1)$ has sum of it's zeroes equal to half of their product.

Sol. $p(x) = x^2 - (K+6)x + 2(2K-1)$

General form: $p(x) = ax^2 + bx + c$

$\Rightarrow \quad a = 1, \quad b = -(K+6), \quad c = 2(2K-1)$

ATQ,

Sum of zeroes $= \frac{1}{2} \times$ Product of zeroes

$\Rightarrow \quad -\frac{b}{a} = \frac{1}{2} \times \frac{c}{a}$

$$\Rightarrow \frac{-(-(k+6))}{1} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}(2k-1)}{1}$$

$$\Rightarrow k+6 = 2k-1$$

$$\Rightarrow k-2k = -1-6$$

$$\Rightarrow +k = +7$$

$$\Rightarrow \boxed{k = 7}$$ Ans

Assign.

Q.8.

$$p(x) = 3x^2 - 8x + 2k+1$$

$a = 3$, $b = -8$, $c = 2k+1$

Zeroes are α & 7α

$$\text{Sum of zeros} = -\frac{b}{a}$$

$$\Rightarrow \alpha + 7\alpha = \frac{-(-8)}{3}$$

$$\Rightarrow \cancel{8}\alpha = \frac{\cancel{8}}{3}$$

$$\Rightarrow \alpha = \frac{1}{3}$$

Now,

Product of zeroes $= \frac{c}{a}$

$$\Rightarrow \alpha \cdot 7\alpha = \frac{2k+1}{3}$$

$$\Rightarrow \frac{1}{3} \cdot 7 \cdot \frac{1}{\cancel{3}} = \frac{2k+1}{\cancel{3}}$$

$$\Rightarrow \frac{7}{3} - 1 = 2k$$

$$\Rightarrow 2k = \frac{7-3}{3}$$

$$\Rightarrow \cancel{2}k = \frac{\cancel{4}^{2}}{3}$$

$$\Rightarrow k = 2/3$$

Polynomials

$p(x) = ax^2 + bx + c$

Zeroes/Roots: $-\alpha$ & β

Relation

① Sum $= \frac{-b}{a}$

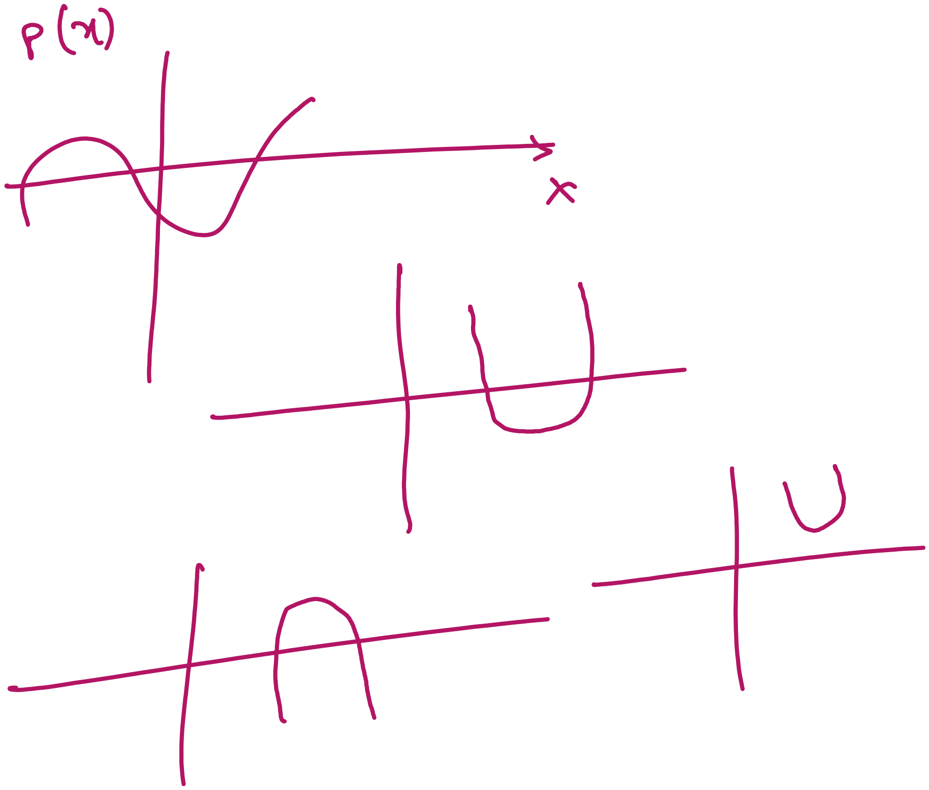
$$\alpha + \beta = -\frac{b}{a}$$

② Product $= \frac{c}{a}$

$$\alpha \cdot \beta = \frac{c}{a}$$

③ finding roots & verify.

Geometrical



Given: $p(x) = 3x^2 - 4x + 5$

Ask: Zeroes/Roots.

Finding quadratic polynomial when sum of zeroes / product of zeroes / roots are known

$P(x) = x^2 - (\text{Sum of zeros})x + \text{Product}$

$$\boxed{P(x) = x^2 - (\alpha+\beta)x + \alpha\beta}$$

Q. Find quadratic polynomial whose sum & product are $\frac{1}{4}$, -1

Sol. $\alpha+\beta = \frac{1}{4}$, $\alpha\beta = -1$

$$x^2 - (\alpha+\beta)x + \alpha\beta$$

$$\Rightarrow k\left(x^2 - \frac{1}{4}x + (-1)\right)$$

$$\Rightarrow k\left(x^2 - \frac{x}{4} - 1\right)$$

$$k\left(\frac{4x^2 - x - 4}{4}\right)$$

If $k = 4$

$$\Rightarrow P(x) = 4x^2 - x - 4$$

Q. Find a quadratic polynomial whose zeroes are $5-3\sqrt{2}$ and $5+3\sqrt{2}$.

Sol. $\alpha = 5-3\sqrt{2}$, $\beta = 5+3\sqrt{2}$

$$p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$$= x^2 - (5-\cancel{3\sqrt{2}}+5+\cancel{3\sqrt{2}})x + (5-3\sqrt{2})(5+3\sqrt{2})$$

$$= x^2 - 10x + (5^2-(3\sqrt{2})^2)$$

$$= x^2 - 10x + (25-18)$$

$$p(x) = x^2 - 10x + 7$$ Ans

Q. If α & β are zeroes of $p(x) = x^2 - x - 2$ then find a polynomial whose zeroes are $2\alpha + 1$ & $2\beta + 1$

Sol. Let α & β are zeroes of

$$p(x) = x^2 - x - 2$$

$$\Rightarrow \alpha + \beta = +1, \quad \alpha\beta = -2$$

New zeroes are $2\alpha + 1$ & $2\beta + 1$

Sum of zeroes $= 2\alpha + 1 + 2\beta + 1$

$= 2\alpha + 2\beta + 2$

$= 2(\alpha + \beta) + 2$

$= 2(+1) + 2$

$= +2 + 2$

$= 4$

Product of zeroes $= (2\alpha+1)(2\beta+1)$

$= 4\alpha\beta + 2\alpha + 2\beta + 1$

$= 4.(-2) + 2(\alpha+\beta) + 1$

$= -8 + 2(1) + 1$

$= -8 + 2 + 1$

$= -5$

Required Polynomial

$= x^2 - (\text{Sum of zeros})x + \text{Product}$

$= x^2 - 4x - 5$

Q. If α, β are zeroes of $p(x) = x^2 - x - 1$

then find $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol.

$$p(x) = x^2 - x - 1$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{1} = 1$$

$$\alpha\beta = \frac{c}{a} = -1$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{-1}{1}$$

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = -1 \quad \text{Ans.}$$

Q. If α & β are roots of $p(x) = x^2 - 7x + 10$ then find polynomial whose zeroes are α^2 and β^2.

Sol. $p(x) = x^2 - 7x + 10$

let α & β are zeroes.

$$\alpha + \beta = \frac{-b}{a} = +7$$

$$\alpha\beta = \frac{c}{a} = 10$$

New zeroes are α^2 & β^2

Sum of zeroes $= \alpha^2 + \beta^2$

$$(\alpha+\beta)^2 = \underline{\alpha^2 + \beta^2} + 2\alpha\beta$$

$$\Rightarrow \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$= (7)^2 - 2 \cdot 10$

$= 49 - 20$

$\Rightarrow \alpha^2 + \beta^2 = 29$

Product of zeroes $= \alpha^2 \cdot \beta^2$

$= (\alpha\beta)^2$

$= (10)^2$

$= 100$

Required Polynomial

$= x^2 - (\text{sum of zeroes})x + \text{Product}$

$= x^2 - 29x + 100$

Q. If α & β are zeroes of $P(x) = x^2 - 6x + k$ then find k such that $\alpha^2 + \beta^2 = 40$

Q. If α and β are zeros of $p(x) = x^2 - x - 4$ then find value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Sol.

$$p(x) = x^2 - x - 4$$

$$\boxed{\frac{1}{\alpha} + \frac{1}{\beta}} - \alpha\beta$$

$$\frac{\beta + \alpha}{\alpha\beta} - \alpha\beta$$

$$\Rightarrow \frac{-(-1)}{-4} - (-4)$$

$$= \frac{1}{-4} + 4$$

$$= \frac{-1+16}{4} = 15/4$$

Q. find zeroes and verify relationship

$$q(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$$

Sol. $q(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

$$= \frac{21y^2 - 11y - 2}{3} = 0$$

$$\Rightarrow 21y^2 - 11y - 2 = 0$$

$$\Rightarrow 21y^2 - 14y + 3y - 2$$

$$= 7y(3y-2) + 1(3y-2)$$

$= (3y-2)(7y+1)$

$\Rightarrow 3y-2 = 0$ | $7y+1 = 0$

$\Rightarrow y = \frac{2}{3}$ | $y = -\frac{1}{7}$

$\therefore \frac{2}{3}$ & $-\frac{1}{7}$ are zeroes of polynomial.

(2023)

Q. The number of polynomials having -2 and 5 as zeroes is

a) 1 b) 3 c) 2 d) Infinite

$$K\left[x^2-(\alpha+\beta)x+\alpha\beta\right] \rightarrow \infty \text{ polynomial.}$$

$K = 1$, $K = -1$ | $K = 2$, $K = -2$ | $K = \infty$

8. A polynomial with zeroes -3 & 4 whose graph is parabola opening upward is :

$a>0$

a) ~~$x^2 - x - 12$~~

b) $x^2 - x + 7$

~~c)~~ $-x^2 + x + 12$

$a<0$

~~d)~~ $-x^2 - x + 12$

$a<0$

$$x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - (-3+4)x + (-3)\cdot 4$$

$$x^2 - x - 12$$