2. Polynomials

$\boxed{x^2 + 1}$

$x + 1$

$x + y + 1$

$(x^1 + 1) \rightarrow$ Polynomial

$x + 1$

$\frac{1}{2}, -1, -2$

$1, 2,$

$-1, 0, 1, 2-$

Integers

Exponent can only be
$\boxed{\text{whole number.}}$

Polynomial is an algebraic expression which includes constants, variables and exponents. <u>It has only positive integer in power.</u>

<u>Ex:</u> (i) $2x^2 + 2x + 4$ ✓

(ii) $4x^3 + x^{-1} - 4$ ✗✗

(iii) $3x^{3/2} + 2x - 3$ ✗✗

In polynomial $\boxed{2x^2} + \boxed{2x} + \boxed{4}$, $2x^2$, $2x$ & 4 are called terms of polynomial.

$3y^2 + 5y$ has two terms.

$\downarrow$

coefficient.

<u>Degree of Polynomial</u> :- The highest power in a polynomial is called degree of polynomial.

Types of Polynomial

1. Constant Polynomial :- Degree of polynomial is 0.

Ex :- 2, 3, 5, - - - -

$2 = 2x^0 \rightarrow 0$ degree.

It is also called zero polynomial.

2. Linear Polynomial -

Degree is 1.

Ex : $x+2$, $y-5$

3. Quadratic Polynomial -

Degree is 2

Ex! - $2x^2 + x + 5$, $x^2 + \frac{5}{2}x$

4. Cubic Polynomial :-

Degree is 3.

Ex! - $8x^3$, $2x^3 + x^2 + x + 1$

Polynomials in One Variable

If there is only one variable in the polynomial then it is called polynomial in one variable.

Ex :- (i) $x^3 + x - 4$. It is one variable polynomial and is denoted by $p(x)$

(ii) $r^2 + 2$, $p(r)$

(iii) $x^2 + y + 4$ is polynomial in 2 variable.

Types of Polynomial based on terms

1. **Zero Polynomial** :- It has zero terms or only a constant. Ex:- 2, 1, −3, ...

2. **Monomial** :- It has only one term.

Ex:- (i) x^2

(ii) y

(iii) $-x$

3. <u>Binomial</u>: It has two terms.

Ex:- (i) $x^2 + 1$

(ii) $4x^2 + 3x$

(iii) $x^{50} - 90$

4. <u>Trinomial</u>: It has three terms.

Ex: (i) $x^{99} - 3x + 8$

(ii) $x^{199} + x^{198} + 4x$

(iii) $3x^3 + 1 + 5x$

Types

- Degree
  - Constant (0)
  - Linear (1)
  - Quadratic (2)
  - Cubic (3)
- Terms
  - Zero (No term)
  - Monomial (1 term)
  - Binomial (2 term)
  - Trinomial (3 term)

EX 2.1 HW

$$\frac{1}{(\sqrt{7}+2)} \times \frac{(\sqrt{7}-2)}{(\sqrt{7}-2)}$$

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

$a = \sqrt{7}$, ✓

$b = 2$ ✓

$$\frac{1 \times (\sqrt{7}-2)}{(\sqrt{7})^2 - (2)^2}$$

$$= \frac{\sqrt{7}-2}{7-4}$$

$$= \frac{\sqrt{7} - 2}{3} \quad \text{Ans.}$$

1

$$\frac{1}{\boxed{\sqrt{3} + \sqrt{2}}} \qquad \frac{\sqrt{2}}{\boxed{\sqrt{5} - \sqrt{6}}} \times (\sqrt{5} + 2)$$

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

2 $\dfrac{1}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{7}}{7}$ Ans.

$$\frac{1}{\sqrt{7}}$$

$\boxed{2}$ 5 $\sqrt{5}$

$$\frac{5}{2\ \ 5\times\underline{\underline{\sqrt{5}}}} = \frac{\sqrt{5}}{2\times 5}$$

$$= \frac{\sqrt{5}}{10} \text{ Ans.}$$

$$2\times 2 = 2^2$$

Q.

$$\frac{\sqrt{3}\ \ \ \sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})\times(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$(a-b)^2 = a^2-2ab+b^2$$

$$\frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = \frac{(\sqrt{3})^2-2\cdot\sqrt{3}\cdot\sqrt{2}+(\sqrt{2})^2}{1}$$

$$\boxed{\begin{aligned} \sqrt{p} \times \sqrt{p} &= p \\ (\sqrt{p})^2 &= p \end{aligned}}$$

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$= 3 - 2\sqrt{3} \cdot \sqrt{2} + 2$$

$$= 5 - 2\sqrt{6}$$

Simplify

Q. $(\sqrt{3} + 2)(\sqrt{2} - 2)$ ✓

$$= \sqrt{3}(\sqrt{2} - 2) + 2(\sqrt{2} - 2)$$

$$= \sqrt{3} \cdot \sqrt{2} - \sqrt{3} \cdot 2 + 2\sqrt{2} - 4$$

$$= \sqrt{6} - 2\sqrt{3} + 2\sqrt{2} - 4$$

Ans.

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$$

$$a \cancel{\times a} = a^2$$

Q. $(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2})$

$$= \sqrt{3}(\sqrt{3} - \sqrt{2}) - \sqrt{2}(\sqrt{3} - \sqrt{2})$$

$$= \sqrt{3} \cdot \sqrt{3} - \sqrt{3} \cdot \sqrt{2} - \sqrt{2} \cdot \sqrt{3} + \sqrt{2} \cdot \sqrt{2}$$

$$= 3 - \sqrt{6} - \sqrt{6} + 2$$

$x + x = 2x$

$$= 5 - (\sqrt{6} + \sqrt{6})$$

$$\sqrt{6} + \sqrt{6}$$

$$= 5 - 2\sqrt{6}$$ Ans.

$x + 4x = 5x$ $\quad 1x + 2x = 3x$

$1\sqrt{6} + 1\sqrt{6} = 2\sqrt{6}$

$x + 2y = x + 2y$

$x \quad y$

$1\sqrt{6} + 2\sqrt{6} = 3\sqrt{6}$

$\sqrt{6} + 4\sqrt{3} = \times\times$

Q. $\dfrac{\sqrt{2}}{5 - \sqrt{3}}$ $\quad \dfrac{\sqrt{2}(5+\sqrt{3})}{\quad}$

$(a+b)(a-b) = a^2 - b^2$

$$\frac{\sqrt{2} \times (5+\sqrt{3})}{22}$$

$$\frac{\sqrt{2}\times 5 + \sqrt{2}\times\sqrt{3}}{22}$$

$5\sqrt{2}$

Q. I $\quad \dfrac{N}{(\sqrt{a}+b)} \,/\, \dfrac{N}{(\sqrt{a}-b)}$

$\downarrow \qquad\qquad \downarrow$

$(a+b)(a-b) = a^2 - b^2$

Q. I.

$$\frac{N}{\sqrt{a}}$$

multiply & divide by same $\sqrt{a}$ to numerator & denom.

Q. Which polynomial is of one variable.

coefficients.

(i) $4s^2 + r + 6 \to s, r$

$\to 3$

(ii) $4s^2 - s + 12 \to s$

3

(iii) $4x^2 - y + 2 \to$ two; x & y

2

(iv) $x^{10} + y^3 + t^{50}$

3

$x, y, t = 3$ variable

Write the coefficients of x^2

Q. (i) $2 + x^2 + x \rightarrow +1 = 1$

(ii) $2 - 1x^2 + x^3 \rightarrow -1$

(iii) $\frac{\pi}{2} x^2 + x \rightarrow \frac{\pi}{2}$

(iv) $\sqrt{2}x - 1 \rightarrow 0$

Degree of Polynomial

$x^{49} + x^{40} - 3x^3 = \underline{49}$

Types of polynomial based on degree of polynomial.

Identify whether below is linear, quadratic or cubic.

(i) $x^2 + x \rightarrow$ Quadratic

(ii) $x - x^3 \rightarrow$ cubic.

(iii) $r^2 \rightarrow$ Quad

(iv) $7x^3$ $\Rightarrow$ cubic

(v) $y + y^2 + y$ $\rightarrow$ Qu

(vi) $3t$ $\rightarrow$ Linear.

Polynomials based on terms

→ zero Polynomial → 0 terms.

→ Monomial → Only 1 term

→ Binomial → 2 terms.

→ Trinomial → 3 terms.

Q. Give one example of binomial of degree 35.

1

$x^{35} + 7$ ✓

Monomial of degree 140.

y^{140}

Zeroes of Polynomial

$p(y) = y^{100}$

$\Rightarrow p(x) = 5x^3 - 2x^2 + 3x - 2$

Replace x by 2,

$p(2) = 5 \cdot (2)^3 - 2 \cdot (2)^2 + 3 \cdot 2 - 2$

$p(2) = 5 \cdot 8 - 2 \cdot 4 + 6 - 2$

$= 40 - 8 + 6 - 2$

$p(2) = 36$

Q. Find the value of

(i) $p(x) = 5x^2 - 3x + 7$

at $x = 1$, $\underline{p(1) = 9}$

$p(1) = 5 \cdot 1^2 - 3 \cdot 1 + 7$

(ii) $q(y) = 3y^3 - 4y + \sqrt{11}$

at $y = 2$

$\underline{q(2) = 16 + \sqrt{11}}$

Zeroes of Polynomial.

$p(x)$

for any value of x,

$p(x) = 0$ then

that value of x is

called zeroes of polynomi-al-

Q: Find the value of

$p(x) = x + 2$ at

$x = 2$ & $x = -2$

Sol. $P(2) = 4$

$P(-2) = 0$

At $x = -2$, $P(-2) = 0$

then $x = -2$ is zeroes of polynomial.

Q: Check if 2 and 0 are zeroes of polynomial $x^2 - 2x$.

Sol. $P(x) = x^2 - 2x$

$P(2) = 0$

$$P(0) = 0$$

Hence, 2 and 0 are zeroes of polynomial $x^2 - 2x$.

Q. Find zero of polynomial

$$P(x) = 2x + 1.$$

$\cancel{x + 1 = 0}$

$\cancel{x = -1}$

Sol.

$$P(x) = 0$$

$$\Rightarrow 2x + 1 = 0$$

$$\Rightarrow 2x = 0 - 1$$

$\Rightarrow \quad 2x = -1$

$\Rightarrow \quad x = \dfrac{-1}{2}$ Ans.

Q. Find zero of polynomial

$p(x) = 2x + 5$

$0 + 1 = 1$

PIZZA ← Process

$0 - 1 = -1$

Sol. Eat

$p(x) = 0$

$0 \times 5 = 0$

$x \quad \boxed{\Rightarrow 2x + 5 = 0}$

$\Rightarrow \quad 2x = 0 - 5$

$\Rightarrow \quad 2x = -5$

$$\Rightarrow \quad x = -\frac{5}{2}$$

$x = -\frac{5}{2}$ Ans.

Addition → Subtraction

Subtraction → Addition

Multiplication → Division

Division → Multiplication

H/W Find value of x

(i) $x + 5 = 0$

(ii) $3x - 2 = 0$

(iii) $3x = 0$

(iv) $x - 5 = 0$

(v) $cx + d = 0$

Ch - 1 TEST

NCERT - Exercise

Examples.

Polynomial

Q: Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x+1$

Sol.

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x+1 & \cancel{x^3} + 1 \\
 & \underset{-}{\cancel{x^3}} \underset{-}{+} x^2 \\
\hline
 & -\cancel{x^2} + 1 \\
 & \underset{+}{-}\cancel{x^2} \underset{+}{-} x \\
\hline
 & \cancel{1} + \cancel{x}
\end{array}
$$

$x \times ? = -x^2$

$x \times ? = x$

Degree equal or greater

$1 \pm x$

$0 \to$ email
er

Q. Find the remainder when $x^4+x^3-2x^2+x+1$ is divided by $x-1$.

Sol.

$x \times ? = -x$

$$
\begin{array}{r}
x^3 + 2x^2 + 1 \\
x-1 \overline{\big)\, x^4 + x^3 - 2x^2 + x + 1} \\
\cancel{x^4} - x^3 \\
- \quad + \\
\hline
2\cancel{x^3} - \cancel{2x^2} + x + 1 \\
\cancel{2x^3} - \cancel{2x^2} \\
- \quad + \\
\hline
\end{array}
$$

$$\cancel{x}+1$$
$$\cancel{x}-1$$
$$- \qquad +$$
$$2$$

$$x-1=0$$
$$\Rightarrow \boxed{x=1}$$

$$P(x)=x^4+x^3-2x^2+x+1$$
$$P(1)=1^4+1^3-\underline{2\cdot1^2}+1+1$$
$$=\underline{1+1}-2+\underline{1+1}$$
$$=2-2+2$$

$= 0 + 2$

$= 2 \rightarrow$ Remainder.

Q. Divide $4t^3 + 4t^2 - t - 1$ by $2t + 1$.

$4t^2 - 2t^2$

Sol.

$$\begin{array}{r} 2t^2 + t - 1 \\ 2t+1 \overline{\smash{)}\ \cancel{4t^3} + 4t^2 - t - 1} \\ \cancel{4t^3} + 2t^2 \\ - \quad - \\ \hline \cancel{2t^2} - t - 1 \\ \cancel{2t^2} \pm t \\ - \quad \underline{\ } \end{array}$$

$$-t-t = -2t$$

$$\begin{array}{rr} \cancel{-2t} & \cancel{-1} \\ \cancel{-2t} & \cancel{-1} \\ + & + \\ \hline & 0 \\ \hline \end{array}$$

Q.

Find remainder when x^3+3x^2+3x+1 is divided by $x+1$ by long division method.

Q. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$

Sol.

$$x - a = 0$$

$$\Rightarrow \boxed{x = a}$$

Let $p(x) = x^3 - ax^2 + 6x - a$

$$p(a) = a^3 - a^1 \cdot a^2 + 6 \cdot a - a$$

$$= \cancel{a^3} - \cancel{a^3} + 6a - a$$

$= 0 + 7a$

$= 7a.$

This method is called Remainder theorem.

Q. Use remainder theorem to find remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $5 + 2x$.

Ex 2.3

3

check whether $7+3x$ is factor of $3x^3+7x$.

$$x = -\frac{7}{3}$$

$$\begin{array}{r} 49 \\ \times 7 \\ \hline 343 \end{array}$$

$$3 \times \left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$$

$(-)^3$

$= - \times - \times -$

$= + \times -$

$= -$

$$\cancel{3} \times \frac{-343}{\cancel{27}_{9}} - \frac{49}{3}$$

$$3 \times \left(\frac{-343}{27}\right) \quad 9$$

$$\frac{-343-147}{9}$$

$$= \frac{-\cancel{390}\;\overset{-590}{}\;130}{\cancel{9}\;3}$$

$$= \frac{-130}{3}.$$

Q. $0.\overline{232}$ in form of $\frac{p}{q}$.

Sol. Let $x = 0.\overline{232}$

$x = 0.232232232\ldots$ — (i)

multiplying both sides by 1000

$1000x = 232.232232\ldots$ — (ii)

Eq. (ii) − Eq. (i)

$$1000x - x = 232.0$$

$$\Rightarrow 999x = 232$$

$$x = \frac{232}{999}$$ Ans

Q. Express $32.12\overline{35}$ in the form of $\frac{p}{q}$.

Sol. $x = 32.\underline{12}\overline{35}$

Factorisation of Polynomial

Q. Examine whether $x+2$ is factor of $x^3 + 3x^2 + 5x + 6$ and of $2x+4$

Sol.
$$x + 2 = 0$$
$$x = -2$$

Let $p(x) = x^3 + 3x^2 + 5x + 6$

$$p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$$

$$= -8 + 3 \times 4 - 10 + 6$$

$$= -8 + 12 - 10 + 6$$

$$= 4 - 4$$

$$p(-2) = 0$$

$\therefore$ $x+2$ is factor of $p(x)$.

let $g(x) = 2x + 4$

$$g(-2) = 2(-2) + 4$$

$$= -4 + 4$$

$$= 0$$

$\therefore$ +2 is also factor of $g(x)$.

Q. Fin the value of K, if -1 is factor of $4x^3\ 13x^2 - 4x + K$.

Factorisation of Polynomial

1. Splitting the middle terms. It is ony applicable for quadratic polynomial.

Q. $6x^2 + 17x + 5$

Sol.

$$17x = a , b$$

$$a + b = 17x$$

$$a \times b = 5 \times 6x^2$$

$$= \underline{30x^2}$$

we split the middle term in such a way that middle term = sum of two number i.e. $a + b$

and

$a \times b$ = first term $\times$ last term.

$$6x^2 + 17x + 5$$

= find two number a & b

$$a + b = 17x$$

$$a \times b = 5 \times 6x^2$$

$$= 30x^2.$$

$$30 = 1 \times 30$$

$= 2 \times 15$

$= 3 \times 10$

$2x + 15x$

$= 5 \times 6$

$= 17x$

$2x \times 15x = 30x^2 \rightarrow 6 \times 5$

$\underline{2 \,\&\, 15}$

$6x^2 + 17x + 5$

$\underline{2x \,\&\, 15x}$

$= 6x^2 + 2x + 15x + 5$

$= 2x(3x+1) + 5(3x+1)$

$= (3x+1)(2x+5)$ Ans.

Q. Factorise y^2-5y+6 using splitting the middle term method.

Q. $12x^2-7x+1$

$12x^2-3x-4x+1$

$3x(4x-1)-1(4x-1)$

$(3x-1)(4x-1)$ ✓

II. Factor theorem

Q. $y^2 - 5y + 6$.

Sol. First term $\times$ Last term

$= y^2 \times 6 \quad = 6y^2$

Factors of 6 $= 1, 2, 3$

$P(1) = 1^2 - 5 \times 1 + 6$

$P(1) = 1 - 5 + 6$

$= 2 \neq 0$

$p(x) = y^2 - 5y + 6$

$$p(2) = 2^2 - 5\times 2 + 6$$

$$= 4 - 10 + 6$$

$$= -6 + 6$$

$$= 0$$

$\therefore (y-2)$ is factor of

$$y^2 - 5y + 6$$

$$p(y) = y^2 - 5y + 6$$

$$p(3) = 3^2 - 5\times 3 + 6$$

$$p(3) = 9 - 15 + 6$$

$$p(3) = -6 + 6$$

$$p(3) = 0$$

$(y-3)$ is factor of $y^2 - 5y + 6$.

$\therefore \underline{y^2 - 5y + 6} = (y-2)(y-3)$

Q. $3x^2 - x - 4$ using factor theorem.

$\cancel{+12x^2}$

$3x^2 \times (-4) = -12x^2$

$12 \rightarrow \pm 1, \pm 2, \pm 3, \pm 6$

$$P(1) = 3\times1^2 - 1 - 4$$
$$= 3-1-4 = -2$$

$$P(-1) = 3\times(-1)^2 - (-1) - 4$$
$$= 3+1-4$$
$$= 4-4 = 0$$

$(x-(-1))$ is factor

$(x+1)$ is factor of $\underline{3x^2-x-4}$.

Algebraic Identities.

1. $(x+y)^2 = x^2 + 2xy + y^2$

2. $(x-y)^2 = x^2 - 2xy + y^2$

3. $(x^2-y^2) = (x+y)(x-y)$

4. $(x+a)(y+b) =$

$$x^2 + (a+b)x + ab.$$

5. $(x+y+z)^3 = x^2+y^2+z^2+$

$$2xy+2yz+2zx$$

6. $(x+y)^3 = x^3+y^3+3xy(x+y)$.

7. $(x-y)^3 = x^3-y^3-3xy(x-y)$

8. $x^3+y^3+z^3-3xyz =$

$(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Q. (1) $(x+3)(x+3)$

Sol. we know,

$(x+y)^2 = x^2+2xy+y^2$

$$(x+3)(x+3) = (x+3)^2$$

$$= x^2 + 2 \cdot x \cdot 3 + 3^2$$

$$= x^2 + 6x + 9 \quad \underline{\text{Ans.}}$$

(ii) $(x-3)(x+5)$

<u>Sol.</u> $(x+a)(x+b) = x^2 + (a+b)x + ab.$

$$(x-3)(x+5) = x^2 + (-3+5)x + (-3) \cdot 5$$

$$= x^2 + 2x + (-15)$$

$$= x^2 + 2x - 15 \quad \underline{\underline{\text{Ans.}}}$$

Q. $(x+4)(x+10)$

Q. $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Sol. $(y^2)^2 - \left(\frac{3}{2}\right)^2$

$y^4 - \frac{9}{4}$ Ans.

Q. Evaluate 105×106 without multiplying.

Sol $(100+5)\times(100+6)$

$(x+a)(x+b)$

$x = 100$, $a = 5$, $b = 6$

$10000 + 11 \times 100 + 30$

~~$10000 + 11 \times 130$~~

$10000 + 1100 + 30$

$10000 + 1130$

11130 Ans

Q. 104×96

$(100+4)(100-4)$

$(x+y)(x-y) = x^2 - y^2$

Q. $(3a+4b+5c)^2$

Q. $(104)^3 = (100+4)^3$

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$$(3a+4b+5c)^2 = (3a)^2 + (4b)^2 + (5c)^2 + 2(3a)\cdot(4b) + 2(4b)\cdot(5c) + 2(5c)\cdot(3a)$$

$$= 9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac.$$

Q. Find :-

(1) $(-12)^3 + (7)^3 + (5)^3$

$$x^3 + y^2 + z^3$$

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

$$x^3+y^3+z^3=\underline{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}+3xyz$$

$$(-12)^3+(7)^3+(5)^3=\underbrace{(-12+7+5)}_{}{}^{=0}\left((-12)^2+(7)^2+(5)^2-(-12)(7)-(7)(5)-(5)(-12)\right)+$$

$3(-12)(7)(5)$

$$= 0 \times \left[(-12)^2 + (7)^2 + (5)^2 - (-12)(7) - (7)(5) - (5)(-12)\right] +$$

$$3(-12)(7)(5)$$

Ep 2.5

~~Friday~~

$dx_1 + dx_2$

$$0 + 3(-12)(7)(5)$$

$$= 3(-12)(35)$$

$$= -36 \times 35$$

$$= -1260 \text{ Ans.}$$