

Calculus For Dummies

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Introduction

Much of calculus is really just very advanced algebra, geometry, and trig. It builds upon and is a logical extension of those subjects. If you can do **algebra**, **geometry**, and **trig**, you can do *calculus*.

Part I

An Overview of Calculus

- A brief and straightforward explanation of just what calculus is. Hint: it's got a lot to do with **curves** and with things that are **constantly changing**.
- Examples of where you might see calculus at work in the real world: curving cables, curving domes, and the curving path of a spacecraft.
- The first of the two big ideas in calculus: **differentiation**, which means finding a **derivative**. A derivative is basically just the fancy calculus version of a **slope**; and it's a simple rate — a this per that.
- The second big calculus idea: **integration**. It's the fancy calculus version of adding up small parts of something to get the total.
- An honest-to-goodness explanation of why calculus works: In short, it's because when you **zoom in on curves** (infinitely far), they become **straight**.

1 What Is Calculus?

Calculus is basically just advanced algebra and geometry. It can be seen as an extension of these two fields of study.

For instance, in figure 1, the man in the left image is pushing a box up a straight incline. On the right however, the man is pushing up the same box up a curving incline. The problem in both cases is to determine the amount of energy required to push the crate up the slope.

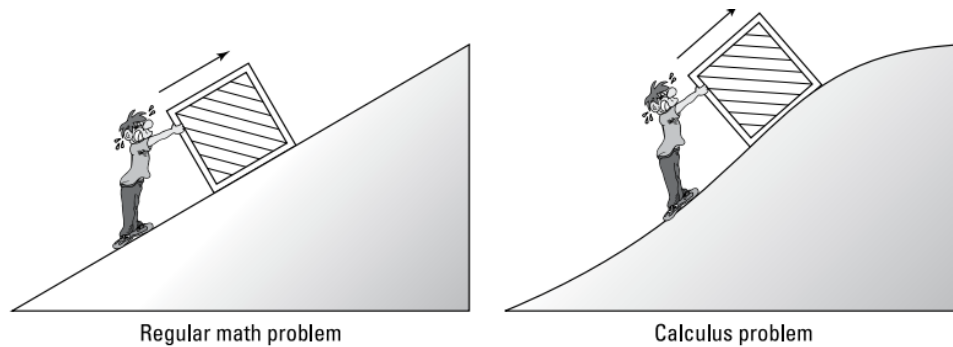


Figure 1: The difference between regular math and calculus: In a word, it's the curve

Work Done

$$W = F * d$$

“Work done” is concerned with the energy transferred from one object to another as a result of a force applied over a distance. So let's say we apply a force of 20N to a box over a distance of 0.5m, the amount of work done would be equal to 10 Joules. This is all nice and dandy, but what if we want to calculate the amount of work done when the force applied is **not constant** - this is where calculus comes into play.

Straight Incline

For the straight line, the man is pushing the box with an *unchanging force*, and the crate goes up the incline at an *unchanging speed*. With some simple physics formulas and regular math (including algebra and trigonometry), you can compute how many calories of energy are required to push the crate up the incline slope.

***The amount of energy expended each second remains the same**

Curving Incline

For the curving incline, things are *constantly* changing. The steepness of the incline is *changing* - and not just in increments (one steepness for the first 3 feet then a different steepness for the next 3 feet). It's *constantly changing*. The man is also pushing the box in *constantly changing force* - the steeper the incline the harder the push. As a result, the amount of energy expended is also changing, not every second or every thousandth of a second, but constantly changing from one moment to the next. That's what makes it a calculus problem. By this time, it should come as no surprise to you that calculus is described as "*the mathematics of change*."

For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that — in contrast to the straight incline problem, which you can sort of do in a single shot — you've got to break up the curving incline problem into small chunks and do each chunk separately. Figure 1-2 shows a small portion of the curving incline blown up to several times its size.

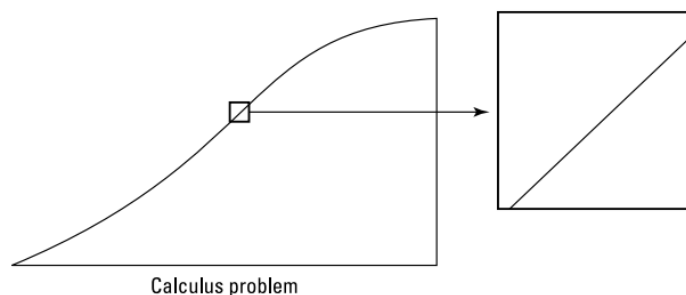


Figure 2: Zooming in on the curve — voilà, it's straight (almost).

What makes the invention of calculus such a fantastic achievement is that it does what seems impossible: it zooms in infinitely. As a matter of fact, everything in calculus involves infinity in one way or another, because if something is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

Real-World Examples of Calculus

With regular math we can determine the length of a straight line that runs diagonally from one point to another using Pythagorean theorem. With calculus we can determine the length of a curved line. See Figure 3.

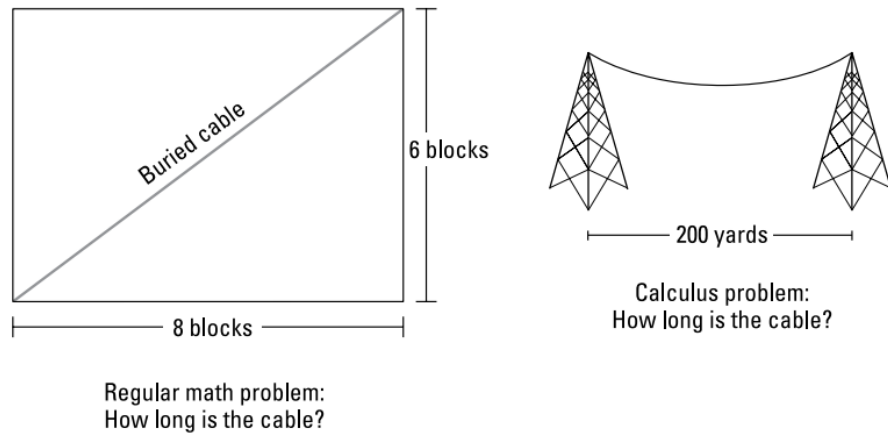


Figure 3: Without and with calculus.

We can also calculate the area of domes and other circular structures. See Figure 4.

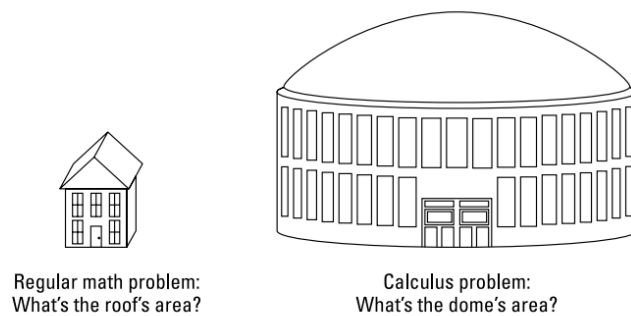


Figure 4: Without and with calculus.

2 The Two Big Ideas of Calculus: Differentiation and Integration — plus Infinite Series