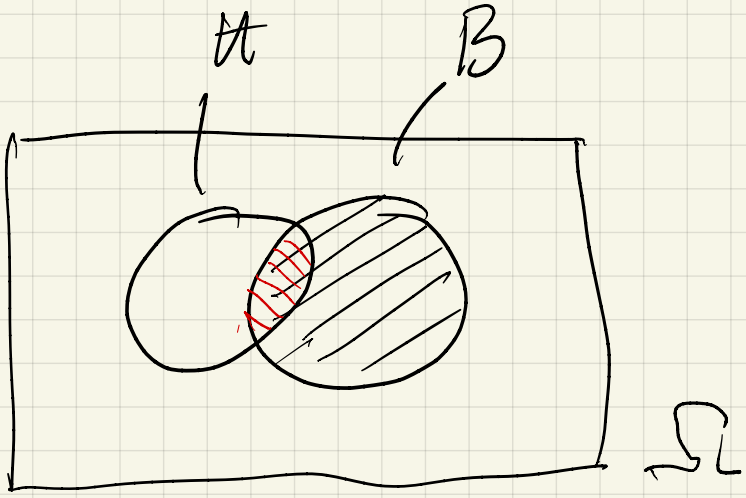
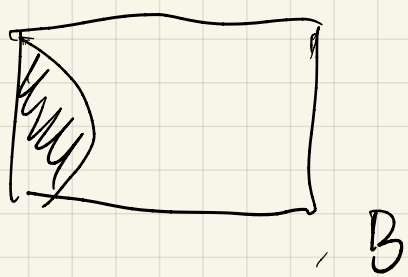
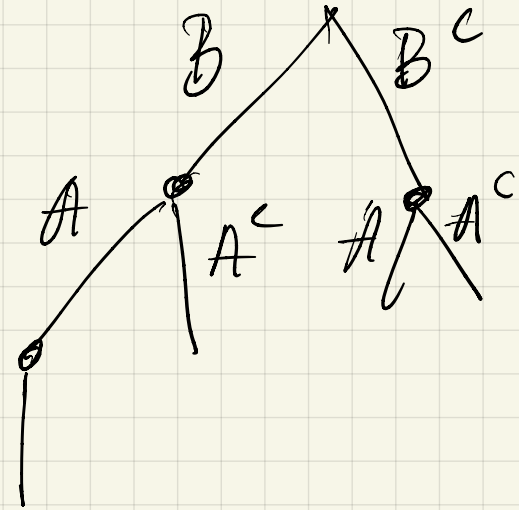
5/9 Betingad sannolikhet

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



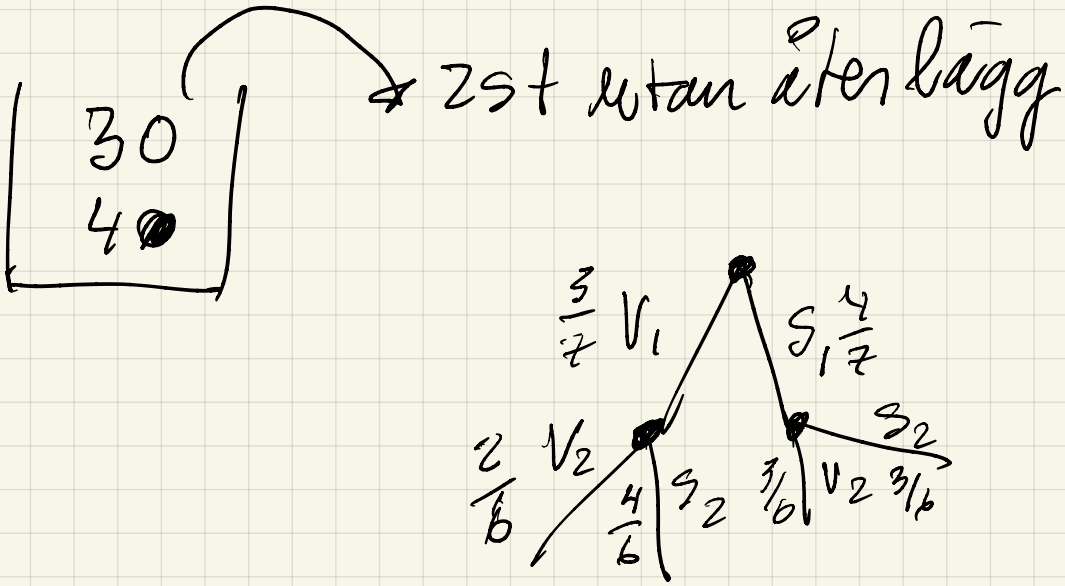




$$P(A \cap B) = P(B)\,P(A \mid B)$$

$$P(A^c \cap B^c) = P(B^c)\,P(A^c \mid B^c)$$

Ex



$$P(\text{2st vita}) = \underbrace{\frac{3}{7} \cdot \frac{2}{6}} = \frac{\binom{3}{2}\binom{4}{0}}{\binom{7}{2}}$$

$$P(V_2 \mid V_1) = \frac{2}{6}$$

$$P(V_2 \mid S_1) = \frac{3}{6}$$

$$P(V_2) = P(V_2 \cap V_1) + P(V_2 \cap S_1)$$

2.29

$P(A) = 0{,}5$

$P(B) = 0{,}3$

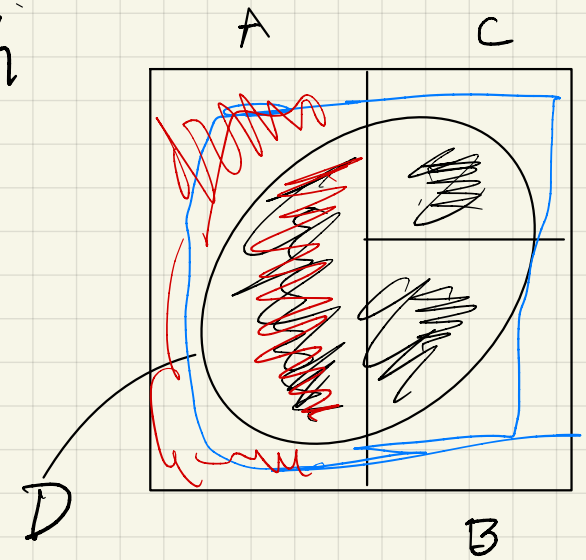
$P(C) = 0{,}2$

$D = D$ luftstid $\geq 10\,h$

$P(D|A) = 0{,}95$

$P(D|B) = 0{,}97$

$P(D|C) = 0{,}98$

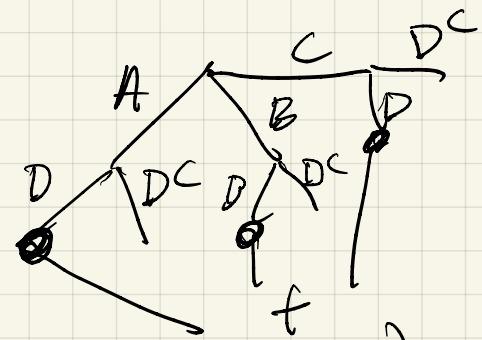


a) $P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$

$= P(A)P(D|A) + P(B)P(D|B) + P(D|C)$

$= 0{,}5 \cdot 0{,}95 + 0{,}3 \cdot 0{,}97 + 0{,}2 \cdot 0{,}98 = 0{,}963$

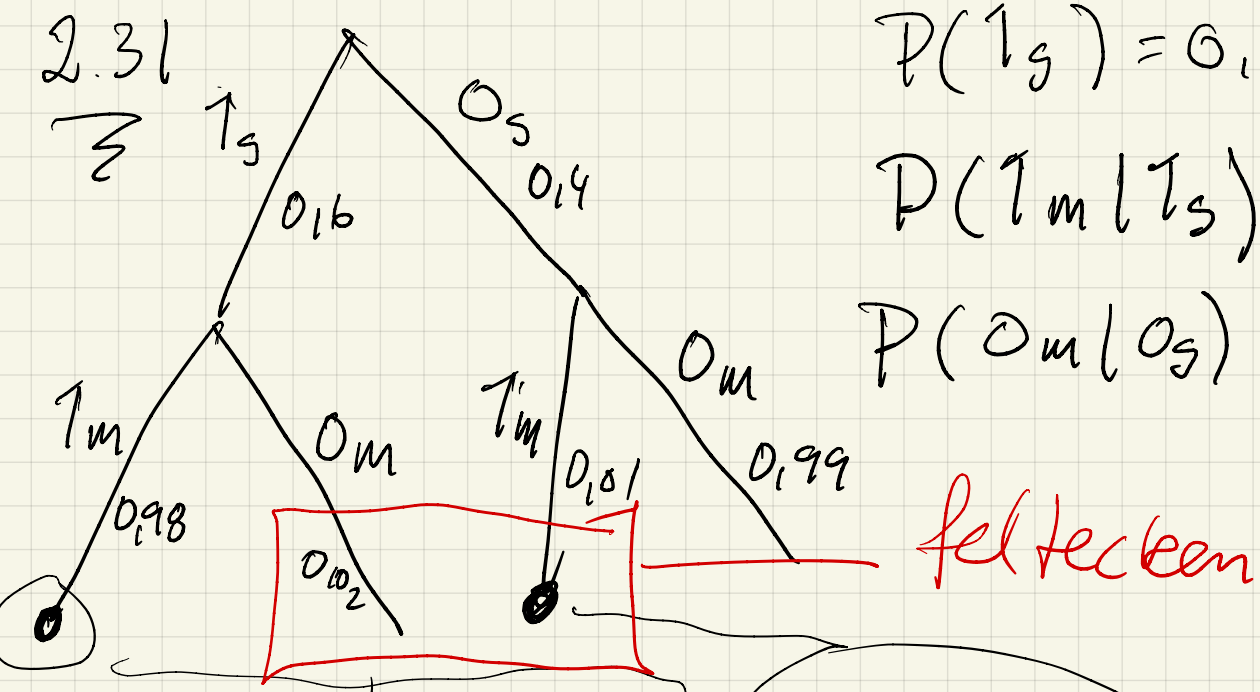
b) $P(A|D) = \dfrac{P(A \cap D)}{P(D)} = \dfrac{0{,}95 \cdot 0{,}5}{0{,}963} = 0{,}493$



c) $P(D^c) = 1 - P(D) = 0{,}037$

$P(A|D^c) = \dfrac{P(A \cap D^c)}{P(D^c)} = \dfrac{P(A)P(D^c|A)}{P(D^c)} = \dfrac{0{,}5 \cdot 0{,}05}{0{,}037}$

2.31



$P(1_s) = 0{,}6 \quad P(0_s) = 0{,}4$

$P(1_m | 1_s) = 0{,}98 \quad P(0_m | 1_s) = 0{,}02$

$P(0_m | 0_s) = 0{,}99 \quad P(1_m | 0_s) = 0{,}01$

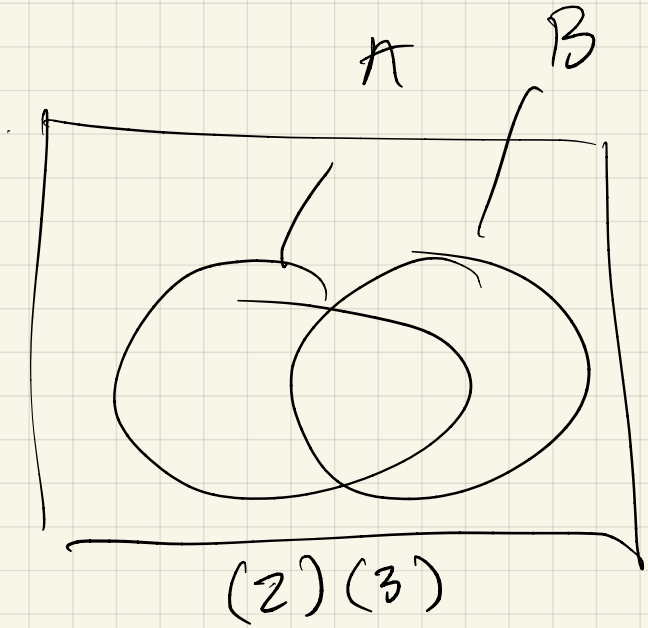
$$P(1_m) = 0{,}6 \cdot 0{,}98 + 0{,}4 \cdot 0{,}01$$

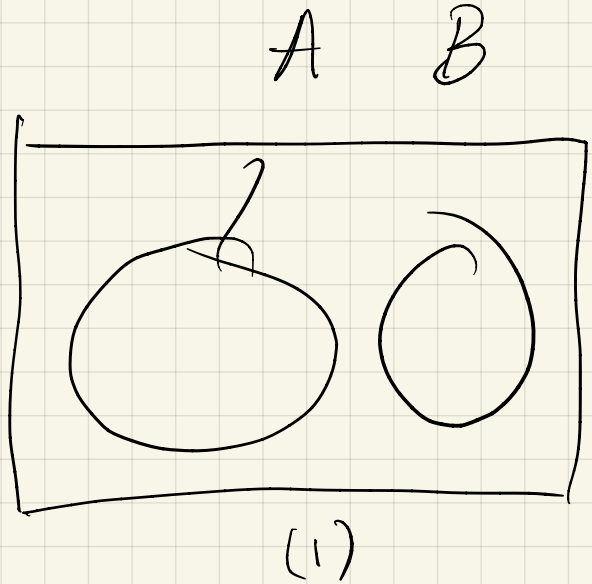
a)

$$P(1_s | 1_m) = \frac{P(1_s \cap 1_m)}{P(1_m} = \frac{0{,}6 \cdot 0{,}98}{0{,}6 \cdot 0{,}98 + 0{,}4 \cdot 0{,}01} = 0{,}997\,?$$

b) P(feltecken hos mottagaren) $= P(1_s \cap 0_m) + P(0_s \cap 1_m)$

$$= 0{,}6 \cdot 0{,}02 + 0{,}4 \cdot 0{,}01 = 0{,}016$$

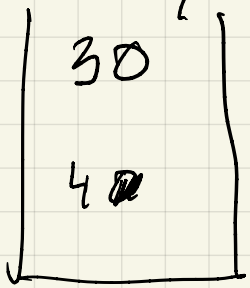




$$P(A \cap B) = \begin{cases} 0, & A \cap B = \emptyset \quad (1) \\ P(A)P(B|A) = P(B)P(A|B) & (2) \\ P(A)P(B), & A \text{ o } B \text{ oberoende} \quad (3) \end{cases}$$

beroende händelser

Ex



$N = 7$

$n = 3$ gr med återlägg (utan hänsyn till ordningen)

A_k = k st vita i urvalet

$P(A_k)$, $k = 0, 1, 2, 3$

$P(V) = \frac{3}{7}$ o $P(S) = \frac{4}{7}$ i alla val

dragn	1	2	3	
$\binom{3}{3} = 1$	V	V	V	$k = 3$
$\binom{3}{2} = 3$	V	V	S	$k = 2$
	V	S	V	
	S	V	V	
$\binom{3}{1} = 3$	V	S	S	$k = 1$
	S	V	S	
	S	S	V	
$\binom{3}{0} = 1$	S	S	S	$k = 0$

$$P(A_k) = \binom{3}{k}\left(\frac{3}{7}\right)^k\left(\frac{4}{7}\right)^{3-k} \qquad k = 0, 1, 2, 3$$

$$P(A_2) = \binom{3}{2}\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^1 = \frac{3 \cdot 9 \cdot 4}{7^3}$$

Binomial fördelning

Definitionen av ober. händelser på ett utvidga

A, B och C ober. händelser om $P(A \cap B \cap C) = P(A)P(B)P(C)$

Om A och B ober. händelser

A och B^c, A^c och B och A^c och B^c också oberoende

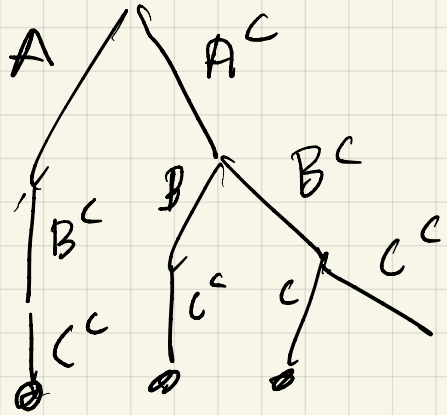
2.36 Tolka fel, A, B och C, uppkommer ober. av varandra på en produkt med

$P(A) = 0{,}2 \quad P(B) = 0{,}05$ och $P(C) = 0{,}10$

$P(\text{minst 1 av felen på en produkt}) = P(A \cup B \cup C) \overset{\text{OBER.}}{=}$

$1 - P(\text{inget fel på en produkt}) = 1 - P(A^c \cap B^c \cap C^c) =$

$1 - P(A^c)P(B^c)P(C^c) = 1 - 0{,}8 \cdot 0{,}95 \cdot 0{,}9 = \underline{\underline{0{,}316}}$



$$P(\text{exakt 1 fel}) = P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C^c)$$

Sats 2.11 om händelserna $A_1, \ldots, A_n$ oberoende.

$0 < P(A_k) = P_k$ så är sannolikheten att minst 1 av händelserna inträffar

$$P(\text{minst 1 av } A_1, \ldots, A_n) = 1 - P(\text{ingen inträffar}) =$$

$$1 - (1-P_1)(1-P_2)(1-P_3) \cdots (1-P_n)$$

Ex A_k = 7 rätt på lottodragning nr k, $k = 1, 2, 3 \ldots\ldots$

$$P(\text{7 rätt}) = \frac{1}{6724323} = p$$

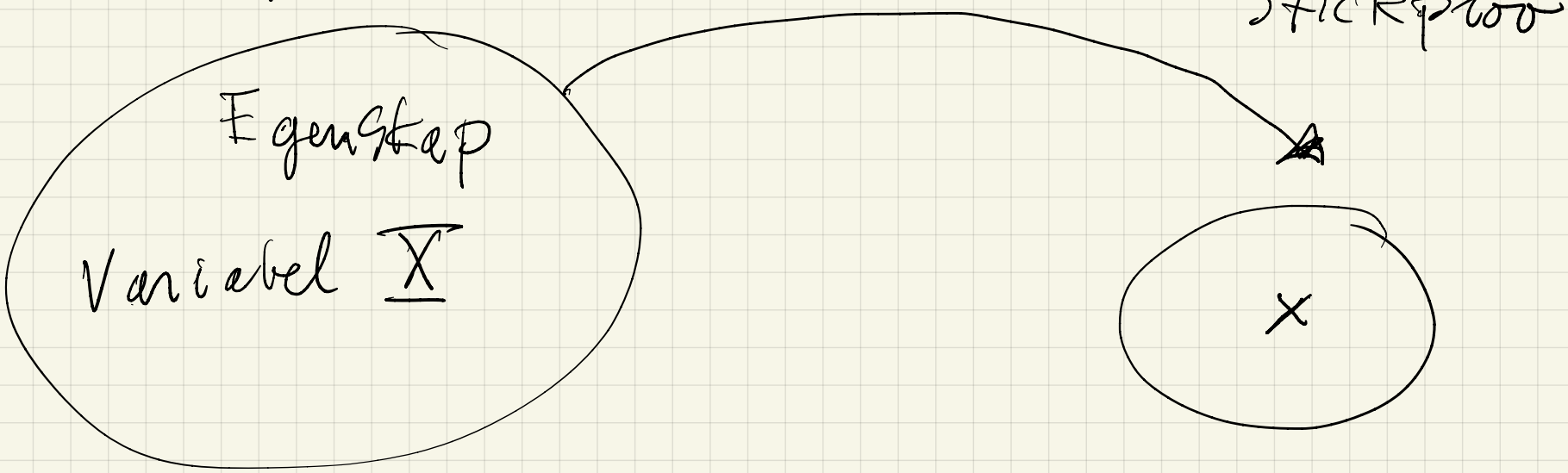
$$P(\text{7 rätt minst 1 gång på } n \text{ dragningar}) = P(A_1 \cup A_2 \cup \ldots \cup A_n)$$

$$= 1 - P(\text{ingen 7:a}) = 1 - P(A_1^c \cap A_2^c \cap \ldots \cap A_n^c) =$$

$$1 - (1-p)^n \longrightarrow 1, \quad n \to \infty$$

KAP 3 Stokastiska (slump) variabler

PoP.



$P(X > x)$ $\quad P(X = x)$ $\quad P(X < x)$

$P(X \geq x)$ $\quad P(X \leq x)$