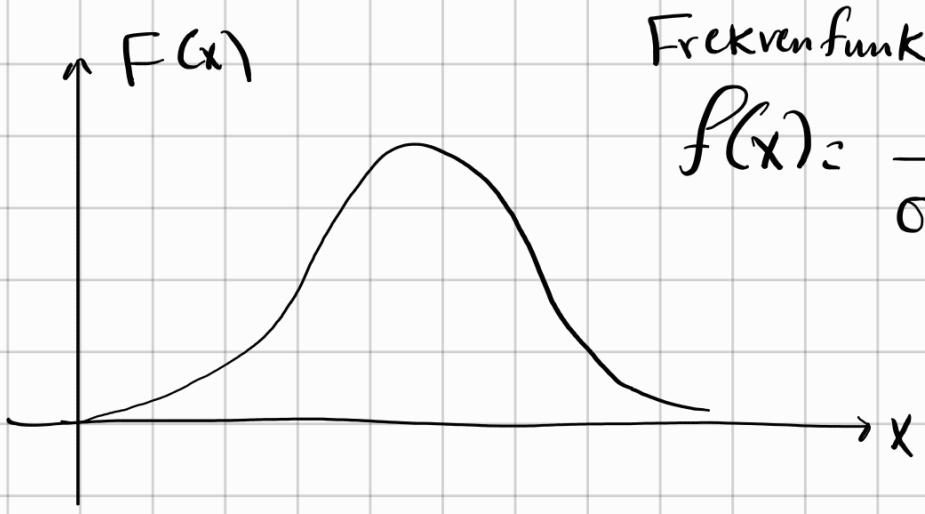
3.10

Normal fördelning

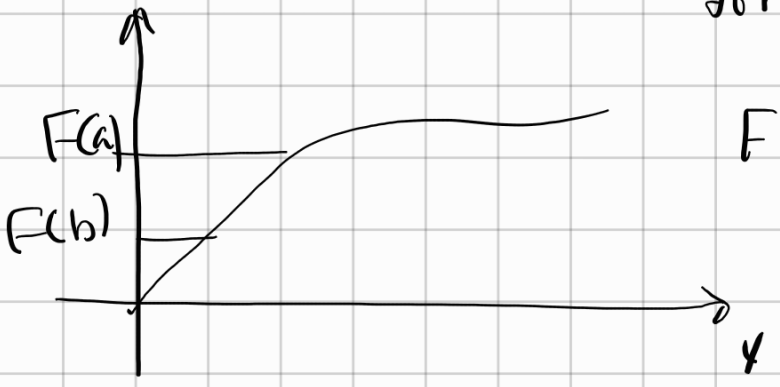
$X \in N(\mu, \sigma)$



Frekvensfunk

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$-\infty < x < \infty$



Fördelningsfunk:

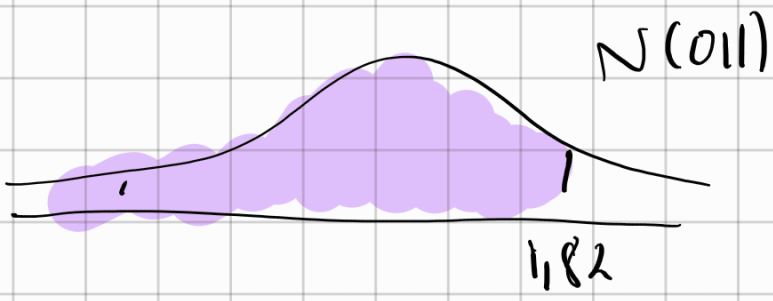
$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

Sats 1 $X \in N(\mu, \sigma) \Rightarrow Y = \frac{X-\mu}{\sigma}$

6.1) $X \in N(0,1)$

Standard normal fördelning.

a) $P(X \leq 1{,}182) = F(1{,}182) = \Phi(1{,}182)$



b) $P(X \leq -0{,}135) = \Phi(-0{,}135)$

$= 1 - \Phi(0{,}135)$

$= 1 - 0{,}6368 = 0{,}3626$

c) $P(-1{,}2 < X < 0{,}5) = \Phi(0{,}5) - \Phi(-1{,}2)$

$= \Phi(0{,}5) - (1 - \Phi(1{,}2))$

$= 0{,}6915 - (1 - 0{,}8849)$

d) bestäm a så att

$P(X < a) = 0{,}05$

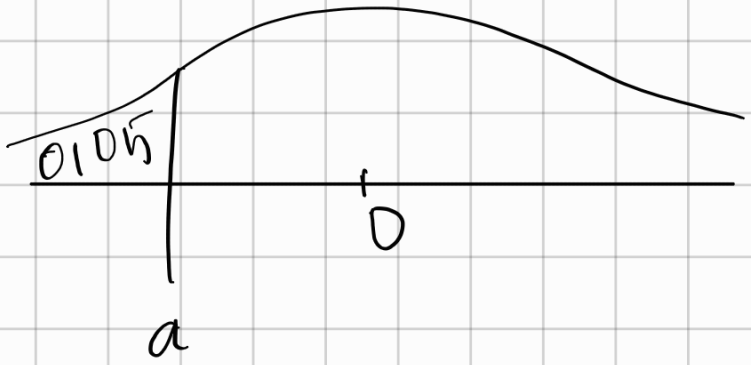
tabell

$\Leftrightarrow a = -1{,}6449$

finns inte i boken

Bestäm a så att

$$P(X < a) = 0{,}05$$



$\Leftrightarrow \quad a = -1{,}6449 \qquad \Phi(-x) = 1 - \Phi(x).$

$$P(X > a) = 0{,}95 \Rightarrow a = -1{,}6449.$$

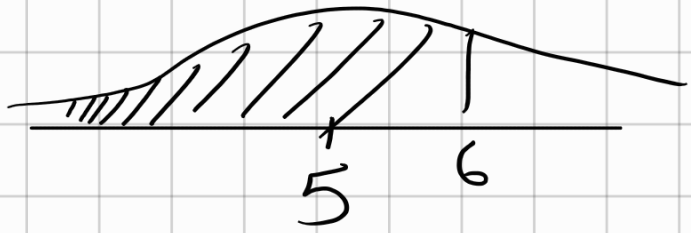
6,14 $\qquad X \in N(5, 2)$

$$P(X \leq 6) = P\left(\frac{X-5}{2} \leq \frac{6-5}{2}\right).$$

$\hookrightarrow N(0,1)$

$$= \Phi(0{,}5) = 0{,}6915.$$

Standardisering.



$$P(1{,}8 < \bar{X} < 7{,}2) = \Phi\left(\frac{7{,}2-5}{2}\right) - \Phi\left(\frac{1{,}8-5}{2}\right)$$

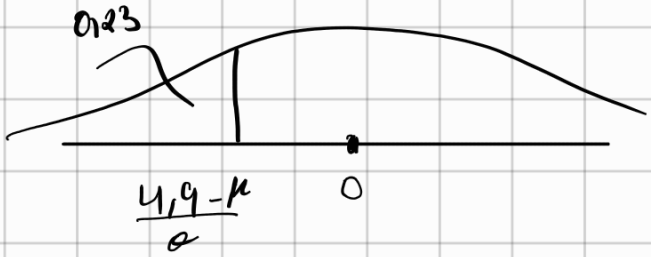
$$= \Phi(1{,}1) - \Phi(-1{,}6) = \Phi(1{,}1) - (1 - \Phi(1{,}6)).$$

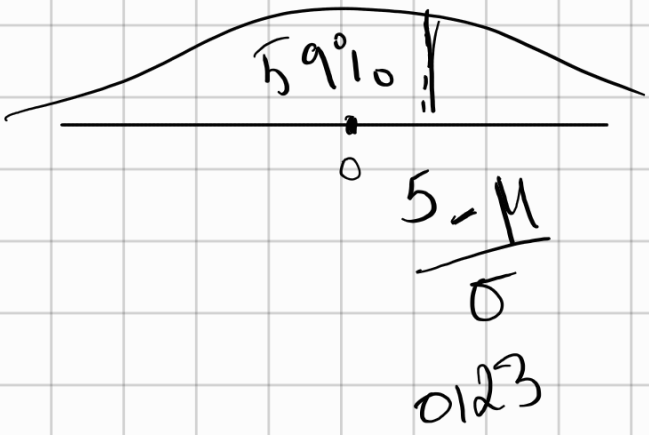
6.10 X = diameten på en kula

$$X \in N(\mu, \sigma)$$

$$\begin{cases} P(X \leq 4{,}5) = 0{,}23 \\ P(X \leq 5{,}0) = 0{,}59 \end{cases}$$

$$\Leftrightarrow \begin{cases} \Phi\left(\frac{4{,}5-\mu}{\sigma}\right) = 0{,}23 \\ \Phi\left(\frac{5-\mu}{\sigma}\right) = 0{,}59 \end{cases}$$





$$\begin{cases} \frac{5-\mu}{\sigma} = 0,123 \\ \frac{4,9-\mu}{\sigma} = -0,174 \end{cases}$$

6.12 X_i = Vikt kund nr i

$X \in N(70, 10)$, Oberoende S.V.

$Y = \sum_{1}^{10} X_i$ $\quad E(Y) =$

$E\left(\sum_{1}^{10} X_i\right) = \sum_{1}^{10} E(X_i)$

$= n \cdot \mu = 10 \cdot 70 = 700$

$$V(Y) =$$

$$= V\left(\sum_{1}^{10} X_i\right) = \sum_{1}^{10} V(X_i) = 10 \cdot 10^2 = 10^3$$

$$Y \in N(700, \sqrt{1000})$$

$$p(\text{överlast}) = P(Y > 800)$$

$$= 1 - \Phi\left(\frac{800-700}{10\sqrt{10}}\right)$$

$$= 1 - \Phi(\sqrt{10}) = 1 - \Phi(3{,}17).$$

$$\approx 1 - 0{,}999 = 0{,}001.$$

X_i = kropp vikt $\quad X_i \in N(70;10)$

Sannolikhet kund över stiger 75 kg ?

$$P(\bar{X} > 75) = \quad E(\bar{X}) = 70$$

$$V(\bar{X}) = \frac{10^2}{10}$$

$$= \left[\frac{\sigma^2}{n}\right]$$

$$= 10$$

$$\bar{X} \in N(70, \sqrt{10})$$

SH

$$1 - \Phi\left(\frac{75-70}{\sqrt{10}}\right) = 1 - \Phi(1{,}58).$$

$$= 1 - 0{,}9429 = 0{,}0571.$$