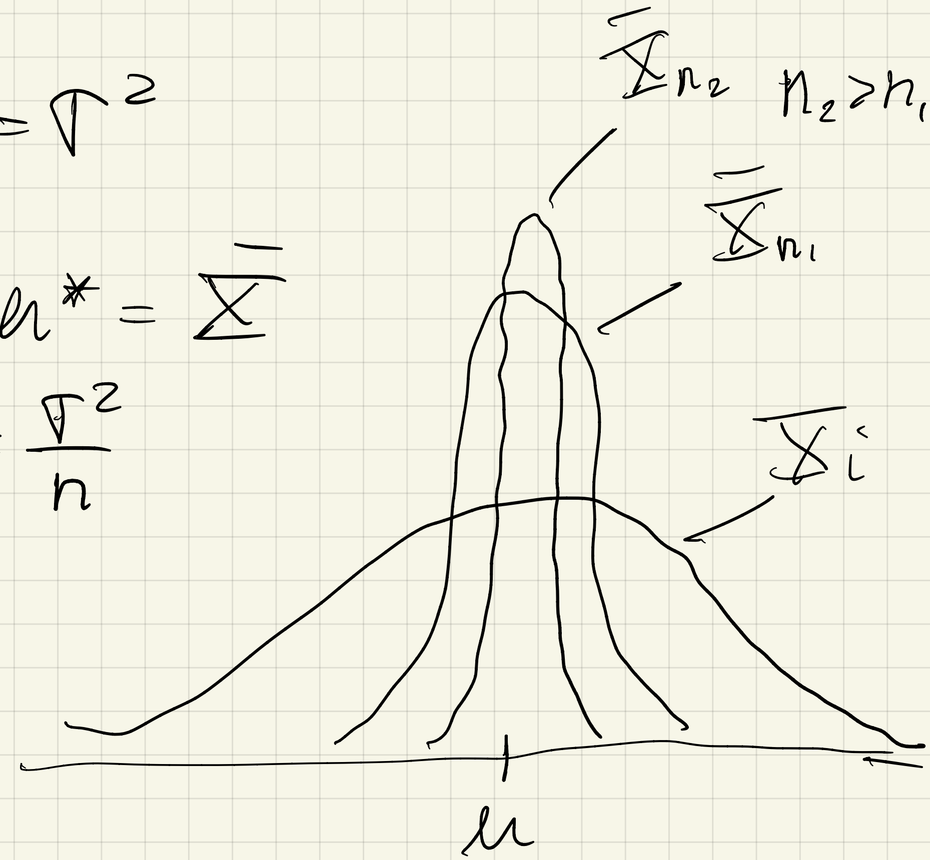
3/11 Stickprov $X_1, \dots, X_n$

$E(X_i) = \mu$, $V(X_i) = \sigma^2$

"Bästa" sättet skatta μ är $\mu^* = \bar{X}$

$E(\bar{X}) = \mu$ och $V(\bar{X}) = \frac{\sigma^2}{n}$

$\mu^* = \bar{X} \underset{\text{CGS}}{\approx} N\left(\mu; \frac{\sigma}{\sqrt{n}}\right)$, $n \geq 30$



X_i kontinuerlig

$\mu^* = \bar{X}$

$\in N\left(\mu; \frac{\sigma}{\sqrt{n}}\right)$, n litet Antag: $X_i \in N(\mu; \sigma)$

$\underset{\text{CGS}}{\approx} N\left(\mu; \frac{\sigma}{\sqrt{n}}\right)$ n stort. Antag $X_i \in \bullet(\mu; \sigma)$

ML - metoden

Ex $X_i \in Po(\lambda) \quad i = 1, \dots, n$

$$P(X_i = x_i) = e^{-\lambda} \frac{\lambda^{x_i}}{x_i!} \qquad x_i = 0, 1, 2, \dots$$

λ okänd Bestäm ML-skattning

- $(P(X_1 = x_1) P(X_2 = x_2) \cdot \dots \cdot P(X_n = x_n))$
- $L(\lambda) = \prod_1^n e^{-\lambda} \frac{\lambda^{x_i}}{x_i!} = e^{-\lambda n} \frac{\lambda^{\sum_1^n x_i}}{x_1! \cdot x_2! \cdot \dots \cdot x_n!}$ (likelihood-funktion)
- logaritmera $L(\lambda)$

$$\ln L(\lambda) = -\lambda n + \sum_1^n x_i \cdot \ln(\lambda) - \ln(x_1! \cdot x_2! \cdot \dots \cdot x_n!)$$

- derivera $D(\ln L(\lambda)) = -n + \frac{1}{\lambda} \sum_1^n x_i = 0 \Longrightarrow$

$\lambda = \frac{1}{n} \sum_1^n x_i = \bar{x}$ | ML-skattningen av λ i $Po(\lambda)$ är $\lambda^*_{ML} = \bar{x}$

$$\lambda^* = \bar{X} \qquad \left(\text{om } X_i \in Po(\lambda) \quad E(X_i) = V(X_i) = \lambda \right)$$

VVR då $E(\lambda^*) = E(\bar{X}) = \lambda$

$$V(\lambda^*) = V(\bar{X}) = \frac{\lambda}{n} \Rightarrow D(\lambda^*) = \sqrt{\frac{\lambda}{n}}$$

Konsistent

$$\lambda^* = \bar{X} \underset{\text{CGS}}{\approx} N\left(\lambda; \sqrt{\frac{\lambda}{n}}\right)$$

Def 11.9 s 271

Enskattning $D(\theta^*)$ kallas medelfel för skattning θ^* och betecknas med $d(\theta^*)$

<u>Ex</u> • X_i = antal ....

(Antag) • $X_i \in Po(\lambda)$, $i = 1, \ldots, 30$

resultat från stickprovet: $\bar{x} = 10$, $n = 30$

Skattar λ med $\lambda^*_{obs} = \bar{x} = 10$

• $\lambda^* = \bar{X} \underset{\substack{\uparrow \\ CGS}}{\approx} N\left(\lambda; \sqrt{\frac{\lambda}{30}}\right)$

Skatta medelfelet för λ

$D(\lambda^*) = \sqrt{\frac{\lambda}{30}}$ $\quad d(\lambda^*) = \sqrt{\frac{10}{30}} = \frac{1}{\sqrt{3}} \approx 0{,}6$

$X_i \in N(\mu;\sigma) \quad i=1,\ldots,n$

μ o σ okända parametrar

$$L(\mu;\sigma^2) = \prod_{1}^{n} \frac{1}{2\pi\sigma^2} \cdot e^{-(x_i-\mu)^2/2\sigma^2}$$

$$= \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\sum \frac{(x_i-\mu)^2}{2\sigma^2}}$$

$\vdots$

$$\mu^*_{ML} = \bar{x} \quad , \quad (\sigma^2)^*_{ML} = \frac{1}{n}\sum_{1}^{n}(x_i-\bar{x})^2$$

Justerad ML-skattning för v.v.R. $(\sigma^2)^* = \frac{1}{n-1}\sum_{1}^{n}(x_i-\bar{x})^2$

Ex 9. från tentan 221028.

$$f(x) = \frac{1}{\theta^2} x \cdot e^{-\frac{x}{\theta}}, \quad x > 0$$

X är en stok. var med $E(X) = 2\theta$

Likelihood funktion

- $$L(\theta) = \prod_{1}^{n} \frac{1}{\theta^2} x_i e^{-\frac{x_i}{\theta}} = \frac{1}{\theta^2} x_1 e^{-\frac{x_1}{\theta}} \cdot \frac{1}{\theta^2} x_2 e^{-\frac{x_2}{\theta}} \cdot \ldots \cdot \frac{1}{\theta^2} x_n e^{-\frac{x_n}{\theta}}$$

$$= \frac{1}{\theta^{2n}} (x_1 \cdot x_2 \cdot \ldots x_n) e^{-\frac{1}{\theta}\sum_{1}^{n} x_i} > 0$$

- logaritmera

$$\ln(L(\theta)) = -2n\ln(\theta) + \ln(x_1 \cdot x_2 \cdots x_n) - \frac{1}{\theta}\sum_{1}^{n} x_i$$

- Derivera m.a.p. θ

$$\frac{L'(\theta)}{L(\theta)} = -\frac{2n}{\theta} + \frac{1}{\theta^2}\sum_{1}^{n} x_i = 0 \Rightarrow \theta = \frac{1}{2n}\sum_{1}^{n} x_i = \frac{\bar{x}}{2}$$

ML-skattning av Θ är $\Theta^*_{ML} = \frac{1}{2n}\sum_1^n x_i$

$$E(\Theta^*) = E\left(\frac{1}{2n}\sum_1^n X_i\right) = \frac{1}{2n}\sum_1^n E(X_i) = ☆$$

$$E(X_i) = \int_0^\infty x \cdot f(x)\,dx = \int_0^\infty x\,\frac{1}{\Theta^2}\,x e^{-\frac{x}{\Theta}}\,dx$$

$$= \int_0^\infty \underset{g(x)}{\frac{1}{\Theta^2}x^2}\,\underset{f(x)}{e^{-\frac{x}{\Theta}}}\,dx = \underbrace{\left[\frac{1}{\Theta^2}x^2\left(-\Theta e^{-\frac{x}{\Theta}}\right)\right]_0^\infty}_{=0} + \int_0^\infty \frac{1}{\Theta}2x e^{-\frac{x}{\Theta}}\,dx$$

$$= \underbrace{\left[\frac{1}{\Theta}2x\left(-\Theta e^{-\frac{x}{\Theta}}\right)\right]_0^\infty}_{=0} + \int_0^\infty 2e^{-\frac{x}{\Theta}}\,dx = \left[-2\Theta e^{-\frac{x}{\Theta}}\right]_0^\infty =$$

$$= 2\Theta$$

$$☆ = \frac{1}{2n}\sum_1^n 2\Theta = \frac{n2\Theta}{2n} = \Theta$$ dvs $E(\Theta^*) = \Theta$ V.V.R.

$\underline{Ex}$ • $X \in Bin(n; p)$ $E(X) = np$, p - okänd

Bestäm Minsta-Kvadrat skattningen av p

• $Q(p) = (x - np)^2$

• $\frac{dQ}{dp} = 2(x - np)(-n) = 0 \Leftrightarrow x - np = 0 \Rightarrow p = \frac{x}{n}$

• $p^*_{MK} = \frac{x}{n}$

$E(p^*_{MK}) = E\left(\frac{X}{n}\right) = \frac{1}{n} np = p$ V.V.R

$V(p^*_{MK}) = V\left(\frac{X}{n}\right) = \frac{1}{n^2} np(1-p) = \frac{p(1-p)}{n}$

• $p^* = \frac{X}{n} \approx N\left(p; \sqrt{\frac{p(1-p)}{n}}\right)$

<u>Ex</u> Två oberoende stickprov

$$X_i \in N(\mu_1; \sigma) \quad i = 1, \ldots, n_1 \qquad \mu_1^* = \bar{x} \quad \sigma^* = s_x$$

$$Y_j \in N(\mu_2; \sigma) \quad j = 1, \ldots, n_2 \qquad \mu_2^* = \bar{y} \quad \sigma^* = s_y$$

Viktad skattning av σ

$$\sigma^* = s_p = \sqrt{\frac{(n_1-1)s_x^2 + (n_2-1)s_y^2}{n_1+n_2-2}}$$, är V.V.R skattning av σ

Pop 1

$\bar{X}$

μ_1

Pop 2

$\bar{Y}$

μ_2

$$\mu_1 = \mu_2 \,?$$

$$\Longleftrightarrow$$

$$\mu_1 - \mu_2 = 0$$

$$(\mu_1 - \mu_2)^* = \bar{X} - \bar{Y} \in N\left(\mu_1 - \mu_2 \,;\, \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$\sigma^* = S_p$$