3/10 $X_1, X_2, \ldots, X_n$ Stickprov

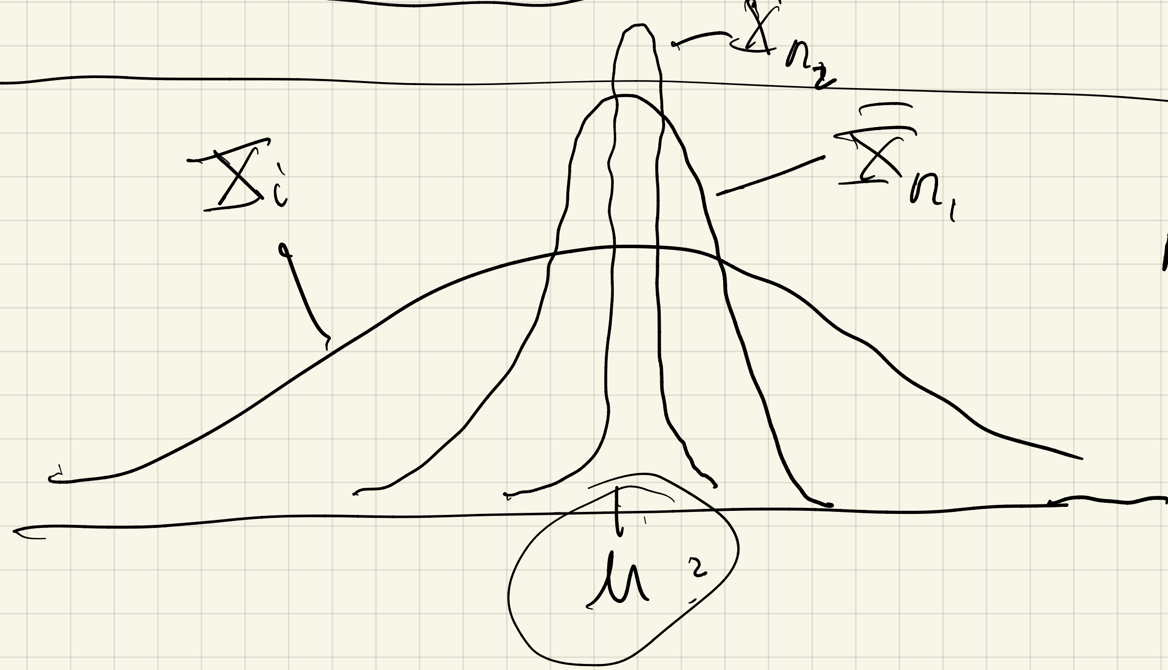
ober. stok. var

$E(X_i) = \mu$ $V(X_i) = \sigma^2$

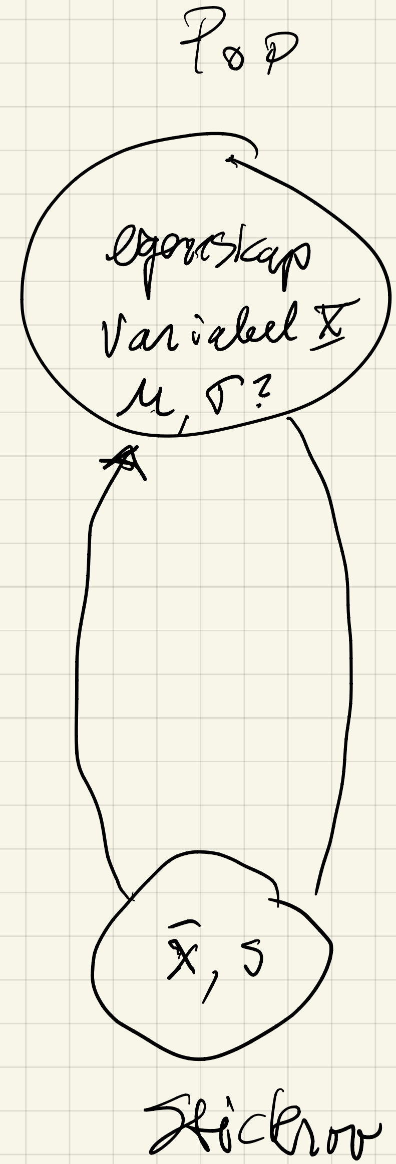
$$\boxed{\bar{X} = \frac{1}{n}\sum_{1}^{n} X_i}$$

$E(\bar{X}) = \mu$

$V(\bar{X}) = \frac{\sigma^2}{n}$



$n_2 > n_1$



5.30 X_i = vikt för barn nr $i = 1, 2, 3$

$E(X_i) = 36 = \mu$ $\quad D(X_i) = 3 = \sigma$

$V(X_i) = 3^2 = \sigma^2$

$$\bar{X} = \frac{1}{3}\sum_{1}^{3} X_i$$

$$E(\bar{X}) = E\left(\frac{1}{3}\sum_{1}^{3} X_i\right) = \frac{1}{3}\sum_{1}^{3} E(X_i) = \frac{1}{3}\,3\cdot 36 = \underline{\underline{36}}$$

$$V(\bar{X}) = V\left(\frac{1}{3}\sum_{1}^{3} X_i\right) \underset{\text{OBER.}}{=} \frac{1}{3^2}\sum_{1}^{3} V(X_i) = \frac{1}{9}\cdot 3\cdot 3^2 = 3 = \sigma_{\bar{x}}^2$$

$$D(\bar{X}) = \sqrt{3}$$

$$V(\bar{X}) = \frac{\sigma^2}{n} \qquad D(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

5.31 X_i = Smältpunkt $i=1,\dots,n$ Ober. mellan mätningarna

$$V(\bar{X}) = 0{,}4^2 \qquad D(\bar{X}) = 0{,}4$$

$$V(\bar{X}) = \frac{\sigma^2}{n} \qquad \underline{\sigma = 2}$$

Bestämma n så att

$$\frac{2^2}{n} \leq 0{,}4^2 \implies n \geq \frac{4}{0{,}16} = 25$$

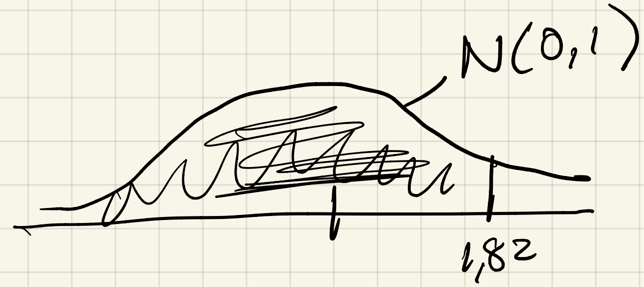
minst 25 mätningar krävs

Öv $\quad X \in N(0,1)$

6.1

a) $P(X \leq 1{,}82) = F(1{,}82) = \Phi(1{,}82)$

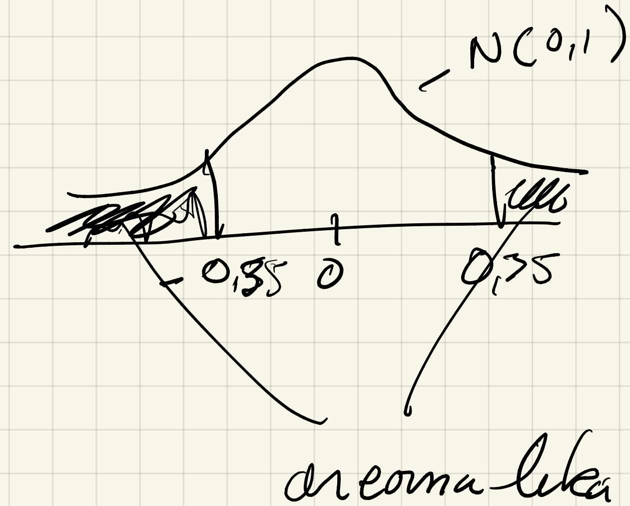
$= 0{,}9656$



b) $P(X \leq -0{,}35) = \Phi(-0{,}35) =$

$= 1 - \Phi(0{,}35)$

$= 1 - 0{,}6368 = 0{,}3632$



c) $P(-1{,}2 < X < 0{,}5) = \Phi(0{,}5) - \Phi(-1{,}2)$

$= \Phi(0{,}5) - (1 - \Phi(1{,}2))$

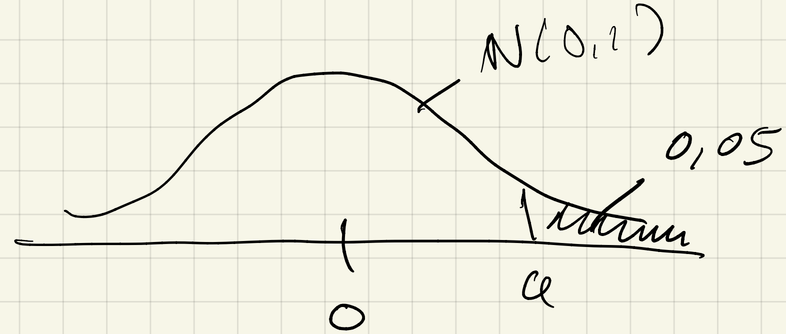
$= 0{,}6915 - (1 - 0{,}8849) = 0{,}5764$

d) Bestäm a så att

$$P(\bar{X} > a) = 0{,}05$$

$\Leftrightarrow$ tabell

$$a = 1{,}6449$$

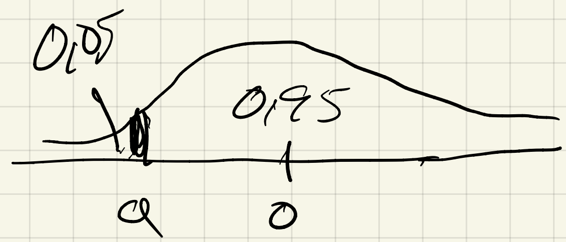


d*) Bestäm a så att

$$P(\bar{X} < a) = 0{,}05$$

$\Leftrightarrow$

$$a = -1{,}6449$$

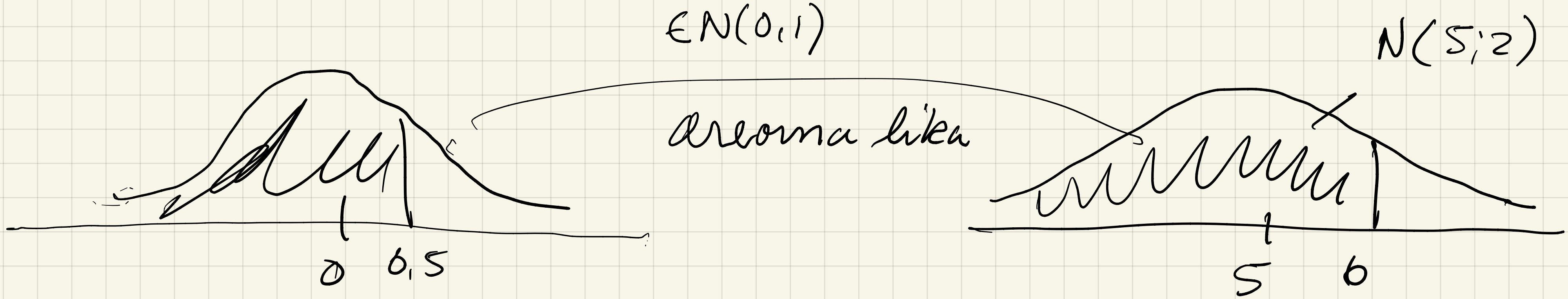


$$\Phi(-x) = 1 - \Phi(x)$$

$$P(\bar{X} > a) = 0{,}95 \Rightarrow a = -1{,}6449$$

6,4 $\underline{X} \in N(5;2)$

$$P(\underline{X} \le 6) = P\Big(\underbrace{\frac{\underline{X}-5}{2}}_{\in N(0,1)} \le \frac{6-5}{2}\Big) = \Phi(0{,}5) = 0{,}6915$$



$$P(1{,}8 < \underline{X} < 7{,}2) = \Phi\Big(\frac{7{,}2-5}{2}\Big) - \Phi\Big(\frac{1{,}8-5}{2}\Big)$$

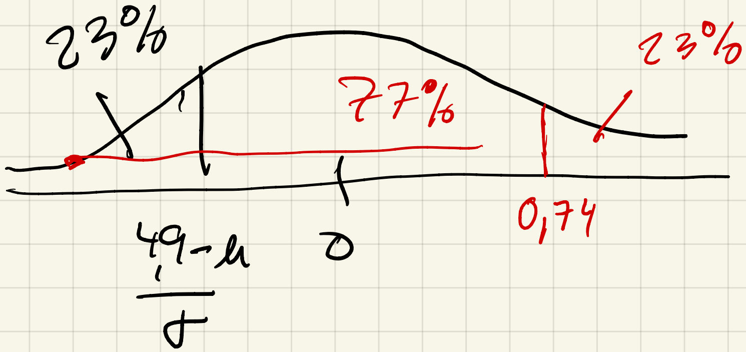
$$= \Phi(1{,}1) - \Phi(-1{,}6)$$

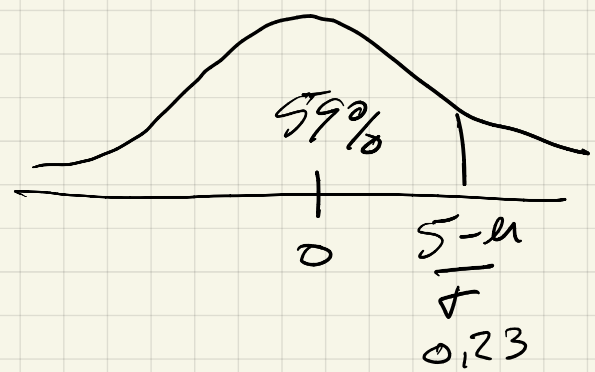
$$= \Phi(1{,}1) - (1 - \Phi(1{,}6)) =$$

6.10 X = diametern av en kula

$X \in N(\mu; \sigma)$

$$\begin{cases} P(X \le 4{,}9) = 0{,}23 \\ P(X \le 5{,}0) = 0{,}59 \end{cases} \iff \begin{cases} \Phi\left(\frac{4{,}9-\mu}{\sigma}\right) = 0{,}23 \\ \Phi\left(\frac{5-\mu}{\sigma}\right) = 0{,}59 \end{cases}$$





$$\begin{cases} \frac{5-\mu}{\sigma} \simeq 0{,}23 \\ \frac{4{,}9-\mu}{\sigma} \simeq -0{,}74 \end{cases}$$

6.12 X_i = vikt av kund nr i

$X_i \in N(70; 10)$, Antag att olika kunders vikter är ober.

$$Y = \sum_1^{10} X_i = \quad \Big| \quad E(Y) = E\left(\sum_1^{10} X_i\right) = \sum_1^{10} E(X_i) = \overset{n \cdot \mu}{10 \cdot 70} = \underline{700}$$

Vikt av 10 kunder

$$V(Y) = V\left(\sum_1^{10} X_i\right) \underset{\text{OBER}}{=} \sum_1^{10} V(X_i) = \overset{n \cdot \sigma^2}{10 \cdot 10^2} = \underline{1000}$$

$$Y \in N(700; \sqrt{1000})$$

$$P(\text{överlast}) = P(Y > 800) = 1 - \Phi\left(\frac{800 - 700}{10\sqrt{10}}\right) =$$

$$1 - \Phi(\sqrt{10}) = 1 - \Phi(3{,}17) \approx$$

$$1 - 0{,}999 = 0{,}001$$

Ex X_i = kroppsvikt $\quad X_i \in N(70;10)$

Vad är sannolikheten att genomsnittsvikten av 10 kunder överstiger 75 kg?

$$P(\bar{X} > 75) = \left[\begin{array}{l} E(\bar{X}) = 70 \\ V(\bar{X}) = \frac{10^2}{10} = \left[\frac{\sigma^2}{n}\right] = 10 = \\ \bar{X} \in N(70; \sqrt{10}) \end{array}\right]$$

$$\bar{X} = \frac{1}{10}\sum_{1}^{10} X_i$$

$$1 - \Phi\left(\frac{75-70}{\sqrt{10}}\right) = 1 - \Phi(1{,}58)$$

$$= 1 - 0{,}9429 = 0{,}0571$$