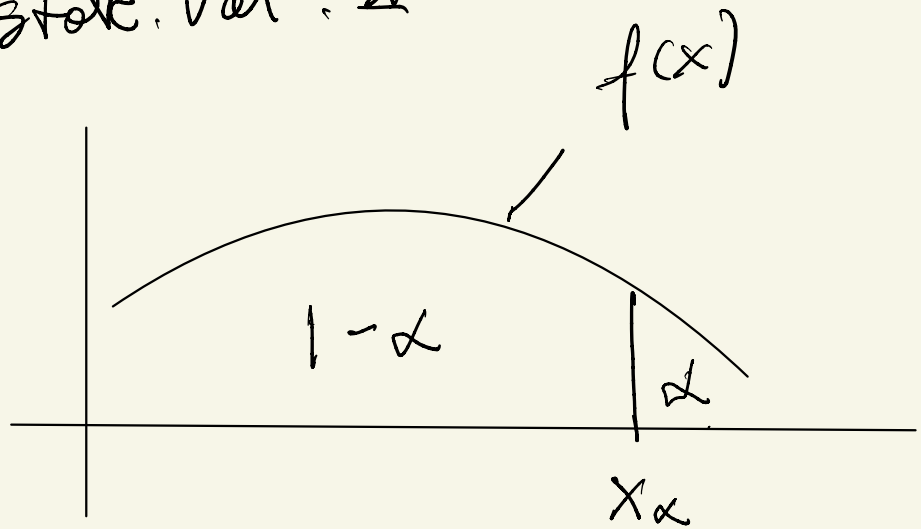
19/9 Fördelningsfunktionen $F(x) = P(X \le x)$

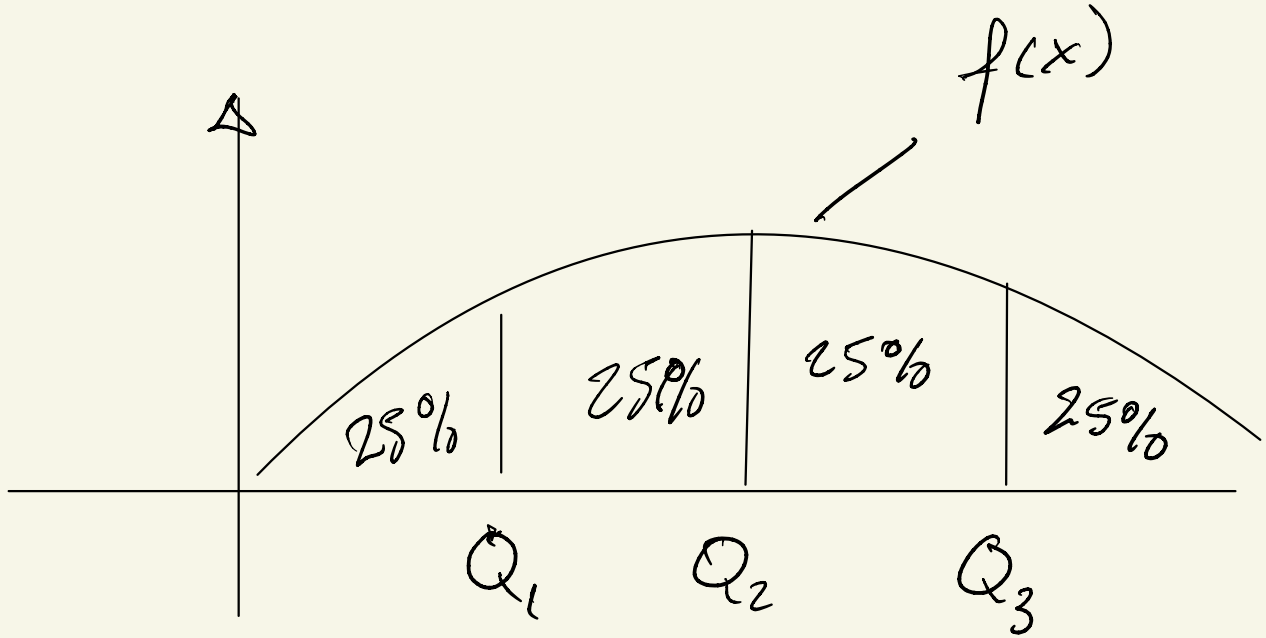
$$F(x) = \begin{cases} \sum_{k=0}^{x} p(x), & X \text{ diskret} \\ \int_{-\infty}^{x} f(t)\,dt, & X \text{ kont.} \end{cases}$$

$F(x) \to 0,\ x \to -\infty$

$F(x) \to 1 \quad x \to \infty$

Def s. 68-69 lösning $x = x_\alpha$ till ekvationen $F(x_\alpha) = 1 - \alpha$ kallas α-kvantilen för stok. var. X





Q - kvartil $\quad Q_2$ = median

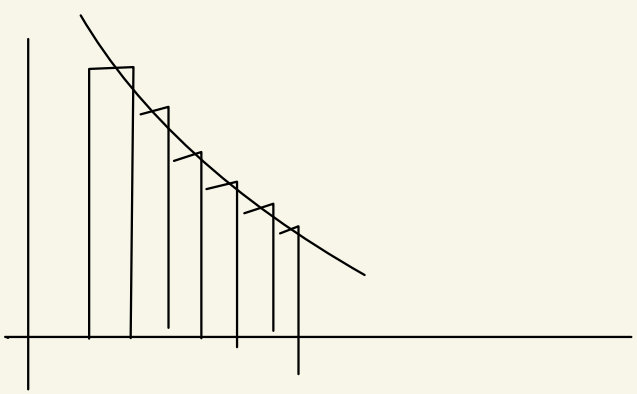
Ex 3.15 En stok. var X beskrivs av

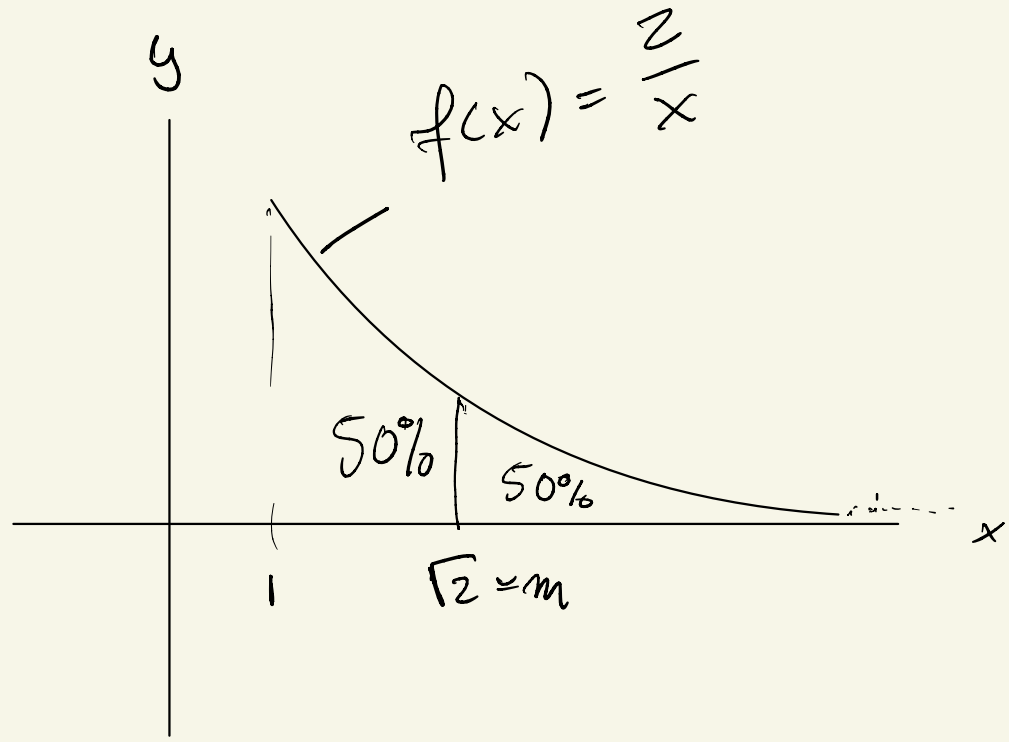
$$F(x) = \begin{cases} 0, & x < 1 \\ 1 - \frac{1}{x^2}, & x \geq 1 \end{cases}$$

Beräkna fördelningsmedian, $m = Q_2$

$$F(m) = P(X \leq m) = \frac{1}{2} \Rightarrow 1 - \frac{1}{m^2} = \frac{1}{2} \Rightarrow m = (\pm)\sqrt{2}$$

$$F'(x) = f(x) = \begin{cases} 0, & x < 1 \\ \frac{2}{x}, & x \geq 1 \end{cases}$$

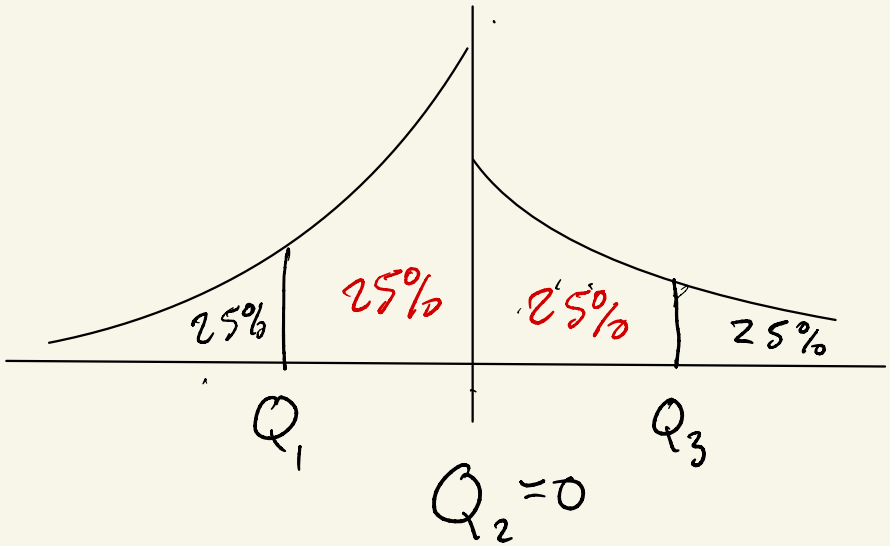




3.16 Beräkna de 3 kvartilerna för den stok. var X med

$$f(x) = \begin{cases} \frac{3}{2} e^{3x} & x < 0 \\ \frac{1}{2} e^{-x} & x \geq 0 \end{cases}$$

$$\int_0^\infty \frac{1}{2} e^{-x} dx = \left[-\frac{1}{2} e^{-x} \right]_0^\infty = \frac{1}{2}$$



Funktioner av stokastiska variabler

$g_1(X) = aX + b \quad , a, b \in \mathbb{R}$ $\qquad g(X) = -\ln(X)$

$g_2(X) = (X - \mu)^2 \quad , \mu \in \mathbb{R}$

$g_3(X) = \frac{1}{n}\sum_{1}^{n} X_i = \bar{X} \quad , \quad Y = \frac{\bar{X}}{n}$

Ex X = prickar på tärning $\qquad Y = 2X + 3$

$\Omega_X = \{1, \dots, 6\} \qquad \Omega_Y = \{5, 7, 9, 11, 13, 15\}$

$P_X(x) = \frac{1}{6}$

Bestäm $P_Y(y)$

x	1	2	3	4	5	6
y	5	7	9	11	13	15
$P_X(x) = P_Y(y)$	$\frac{1}{6}$	.	.	.	.	$\frac{1}{6}$

3.27 X är likformigt fördelad på $\{0, 1, \ldots, 9\}$

$Y = |X - 3|$ Bestämma $p_Y(y)$.

x	0	1	2	3	4	5	6	7	8	9
y	3	2	1	0	1	2	3	4	5	6
$p(x)$	$\frac{1}{10}$	.	.	.	.	.	.	.	.	$\frac{1}{10}$

$p(x) = P(X = x)$

y	0	1	2	3	4	5	6
$p_Y(y)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$P(X = 3) = p(3)$

Ex Låt $X \in U(0,1)$ och sätt $Y = -\ln(X)$

$f_X(x) = 1, \; 0 < x < 1$

Bestäm $f_Y(y)$!

- $\Omega_X = \{x; 0 < x < 1\} \Rightarrow \Omega_Y = \{y > 0\}$

Välj ett godtyckligt $y > 0$

lös ut X

- $F_Y(y) = P(Y \le y) = P(-\ln(X) \le y) = P(X \ge e^{-y})$

$= \int_{e^{-y}}^{1} 1 \, dx = 1 - e^{-y}$, $\quad F_Y(y) = 1 - e^{-y}, \; y > 0$

$F_Y'(y) = f_Y(y) = e^{-y}, \; y > 0$

$Y \in Exp(1)$

$X \in U(0,1) \quad Y = -\lambda \ln(X) \Rightarrow Y \in Exp(\lambda)$

X_i - mätresultat

$\mathbb{X} = X_1, X_2, \ldots, X_n$ - stickprov

$g_1(\mathbb{X}) = X_1 + X_2 + \ldots + X_n$

$g_2(\mathbb{X}) = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

KAP 4 2-dim stok. var

4.2 X o Y är oberoende stok. var med

x	1	3	5	7	9
$P_X(x)$	0,1	0,2	0,4	0,2	0,1

y	2	4	6	8
$P_Y(y)$	0,1	0,2	0,3	0,4

Bestäm

$$P(\underset{A}{X=3};\ \underset{B}{Y=6}) \overset{\text{OBER}}{=} P(X=3)P(Y=6) = 0{,}2 \cdot 0{,}3 = 0{,}06$$

$$P(X \le 3;\ Y \le 6) = F_{X,Y}(3,6) \overset{\text{OBER}}{=} F_X(3)F_Y(6) = (0{,}1+0{,}2)(0{,}1+0{,}2+0{,}3)$$

$$= 0{,}18$$

$X \backslash Y$	2	4	6	8	$P_X(x)$
1	0,01	0,02	0,03		0,1
3	0,02	0,04	$\boxed{0{,}06}$		0,2
5					0,4
7					0,2
9					0,1
$P_Y(y)$	0,1	0,2	0,3	0,4	1

$P_X(x)$ – Marginella sannolikhetsfunktion för X

Marginal Y

Ex En stokastisk variabel (X, Y) har täthet

$$f(x,y) = \frac{1}{\pi^2} \frac{1}{1+x^2} \cdot \frac{1}{1+y^2} \qquad (x,y) \in \mathbb{R}^2$$

Bestäm $F(x,y)$

$$F(x,y) = P(X \le x, Y \le y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{1}{\pi^2} \frac{1}{1+t^2} \frac{1}{1+s^2} \, ds \, dt =$$

$$= \int_{-\infty}^{x} \frac{1}{\pi^2} \frac{1}{1+t^2} \Big[\arctan(s)\Big]_{-\infty}^{y} = \int_{-\infty}^{x} \frac{1}{\pi^2} \frac{1}{1+t^2} \left(\arctan(y) + \frac{\pi}{2}\right) dt$$

$$= \frac{1}{\pi^2} \left(\arctan(x) + \frac{\pi}{2}\right)\left(\arctan(y) + \frac{\pi}{2}\right)$$

$$F(x,y) = \frac{1}{\pi^2}\left(\arctan(x) + \frac{\pi}{2}\right)\left(\arctan(y) + \frac{\pi}{2}\right)$$

bestäm marginella $F_X(x)$ o $F_Y(y)$

$$F_X(x) = F(x;\infty) = \int_{-\infty}^{\infty} f(x,y)\,dy = \frac{1}{\pi^2}\left(\arctan(x) + \frac{\pi}{2}\right)\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= \frac{1}{\pi}\left(\arctan(x) + \frac{\pi}{2}\right)$$

$$F_Y(y) = F(\infty;y) = \frac{1}{\pi}\left(\arctan(y) + \frac{\pi}{2}\right)$$

$$F_X(x) \cdot F_Y(y) = F(x,y)$$

$$f_X(x) = F_X'(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

$$f_Y(y) = F_Y'(y) = \frac{1}{\pi}\frac{1}{1+y^2}$$

$$f_X(x) \cdot f_Y(y) = f_{X,Y}(x,y) = \frac{1}{\pi^2}\frac{1}{1+x^2}\cdot\frac{1}{1+y^2}$$

$\Rightarrow$ X o Y är oberoende stok. var.