12/10 Poissonfördelning

X = antal händelser / min $\quad X \in Po(1)$

Y = antal händelser / tim

$$Y = \sum_{1}^{60} X_i \in Po(60)$$

7.21 $X \in Po(7{,}5) \qquad P(x) = e^{-7{,}5} \dfrac{7{,}5^x}{x!}, \quad x = 0, 1, \ldots$

$P(X \leq 4) = 0{,}13206$

$P(6 \leq X \leq 11) = F(11) - F(5) = 0{,}92076 - 0{,}24144$

$P(X \leq 11) - P(X \leq 5) = 0{,}67932$

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0{,}7761 = 0{,}22359$

$P(X = 8) = F(8) - F(7) = e^{-7{,}5} \dfrac{7{,}5^8}{8!}$

a-c) DISTR $\boxed{\text{VARS}}$ Poisson CDF(7,5, x) $\sim F(x) = P(X \leq x)$

d) Poisson PDF(7,5, x) $P(x) = P(X = x)$

Ex over.

$X_1 \in Bin(10^6, 10^{-6}) \approx Po(1)$

$X_2 \in Bin(10^6; 2 \cdot 10^{-6}) \approx Po(2)$

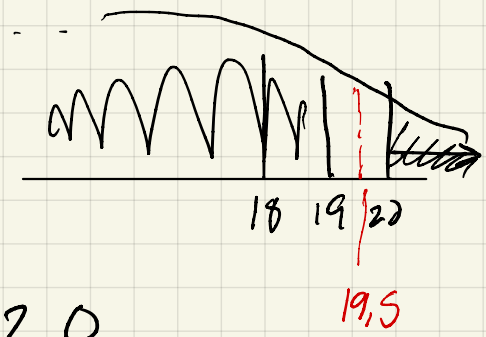
$Y = X_1 + X_2 \approx Po(1+2) = Po(3)$

$P(Y \geq 10) = 1 - P(Y \leq 9) \approx 1 - 0{,}99890 \approx 0{,}0011$

Uppg. 5 Dugga 220430

X_k = antal inkommande samtal / tim

$X_k \in Po(\lambda)$, $P(x) = e^{-\lambda} \dfrac{\lambda^x}{x!}$, $x = 0, 1, \ldots$



$P(X = 0) = 0{,}135$

$\Longleftrightarrow$

$e^{-\lambda} \dfrac{\lambda^0}{0!} = 0{,}135 \Rightarrow \lambda = -\ln(0{,}135) \approx 2{,}0$

$X_k \in Po(2)$

a) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0{,}6767 = 0{,}3233$

b) $Y = X_1 + \ldots X_8$ = antal samtal / 8 tim (ober. stok. var)

$Y \in Po(2 \cdot 8) = Po(16) \overset{CGS}{\approx} N(16; \sqrt{16})$

$P(X \geq 20) = 1 - P(X \leq 19) \approx 1 - \Phi\left(\dfrac{19.5 - 16}{\sqrt{16}}\right) = 1 - \Phi(0{,}88)$

$= 1 - 0{,}8106 \simeq 0{,}19$ $\quad$ 1 − Poisson(16, 19) = 0,1878

UPP 1. X = tid till nästa olycka i månader

$$\boxed{X \in Exp(\lambda)} \quad E(X) = 6 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{6}$$

$$f(x) = \frac{1}{6} e^{-\frac{x}{6}}, \; x \geq 0, \quad F(x) = 1 - e^{-\frac{x}{6}}$$

$$P(\text{nästa olycka inom 12 mån}) = P(X \leq 12) =$$

$$= \int_0^{12} \frac{1}{6} e^{-\frac{x}{6}} dx = \left[-e^{-\frac{x}{6}} \right]_0^{12} = 1 - e^{-2} \approx 0{,}865$$

MPP2 Dugga 220930

R = Regn > 50 mm H = Högt lufttryck, M = Måttligt, L = Lågt

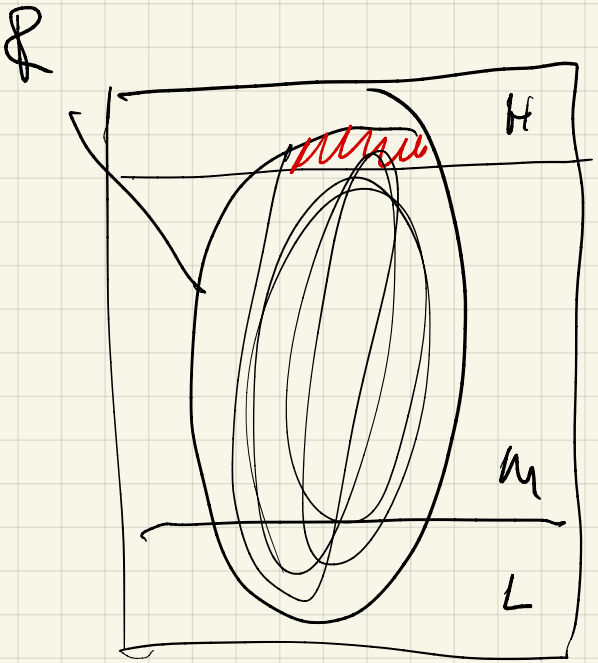
$P(L) = 0{,}2$ $P(M) = 0{,}72$ $P(H) = 0{,}08$

$P(R|L) = 0{,}9$ $P(R|M) = 0{,}47$ $P(R|H) = 0{,}06$

$$P(H|R) = \frac{P(H \cap R)}{P(R)} =$$

$$P(R) = P(L \cap R) + P(M \cap R) + P(H \cap R)$$
$$= P(L) \cdot P(R|L) + P(M) \cdot P(R|M) + P(H) \cdot P(R|H) =$$
$$\star = 0{,}2 \cdot 0{,}9 + 0{,}72 \cdot 0{,}47 + 0{,}08 \cdot 0{,}06 =$$

$$= \frac{0{,}08 \cdot 0{,}06}{\star} \approx 0{,}00917$$



3) X_1 = tid till leveransadress $\quad E(X_1) = 12 \quad D(X_1) = 8$

X_2 = tid för leverans $\quad X \in Exp(\lambda) \quad E(X_2) = 3 \quad D(X_2) = 3$

$Y = X_1 + X_2$ = tid för hela leveransen

$E(Y) = E(X_1 + X_2) = E(X_1) + E(X_2) = 12 + 3 = 15$

$V(Y) = V(X_1 + X_2) \underset{\text{OBER}}{=} V(X_1) + V(X_2) = 8^2 + 3^2 = 73$

$D(Y) = \sqrt{73}$

$Z = \sum_1^{25} Y_i$ = tid för 25 leveranser

$Z \underset{\text{CGS}}{\approx} N(\underbrace{25 \cdot 15}_{n \cdot \mu}; \underbrace{5 \cdot \sqrt{73}}_{\sqrt{n} \cdot \sigma})$

375

6 timmar — 360 min

P(minst 25 leveranser)

$P(Z \leq 360) \approx \Phi\left(\frac{360 - 375}{5\sqrt{73}}\right)$

$= \Phi(-0{,}35) = 1 - \Phi(0{,}35) =$

$= 0{,}3632$

uppg 4) Dugga 220930

X = antal rätta svar av 10, Y = antal fel svar av 10

$X \in Bin(10; 0{,}55)$ $\qquad$ $Y \in Bin(10; 0{,}45)$

$P(\text{klara testet}) = P(X \geq 6) = 1 - P(X \leq 5) = P(Y \leq 4) = 0{,}5044 = p$

$\sum_{x=0}^{4} \binom{10}{x} 0{,}45^x 0{,}55^{10-x}$

Z = antal studenter som klarar testet på 1:a försöket av 250

$Z \in Bin(250; 0{,}5044)$

$E(Z) = 250 \cdot 0{,}5044 = 126{,}1$

$V(Z) = 250 \cdot 0{,}5044(1 - 0{,}5044) = 62{,}4952$

$Z \underset{\text{cgs}}{\approx} N(126{,}1; \sqrt{62{,}4952})$

$P(\text{minst } 150) = P(Z \geq 150) = 1 - P(Z \leq 149) \approx$

$1 - \Phi\left(\frac{149{,}5 - 126{,}1}{\sqrt{62{,}4952}}\right) = 1 - \Phi(2{,}96) =$

$= 1 - 0{,}9985 = 0{,}0015$

W = antal test tills man klarar sig

$W \in fg(0{,}5044)$

$$E(W) = \frac{1}{0{,}5044} \approx 2$$