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ÖV 5.1 1000 lotter 0,5 kr/st

Vinst	0	5	20	100
Antal	964	30	5	1
$P(X=k)$	0,964	0,03	0,005	0,001

Bestäm $\mu = E(X)$

$$E(X) = \sum_{0}^{100} x\,P(x) = 5\cdot 0{,}03 + 20\cdot 0{,}005 + 100\cdot 0{,}001.$$
$$= 0{,}35$$

För väntad Brutto Vinsten 35 öre/lott

Y = netto vinst

$$Y = X - 0{,}5 = g(X).$$

y	$-0{,}5$	$4{,}5$	$19{,}5$	$99{,}5$
	0,964	0,003	0,005	0,001

$$E(Y) = \sum_y y\,P(y).$$

$$= \sum_x (x - 0{,}5)\,P(x)$$

$$= \sum_x x\,P(x) - 0{,}5 \underbrace{\sum P(x)}_{=1}$$

$$= E(X) - 0{,}5$$

$$= 0{,}15.$$

5.7 $X \in Exp(2)$ $\quad f(x) = 2e^{-2x}, \; x \geq 0$

$$E(X) = \frac{1}{2}$$

$$Y = e^{X}$$

$$E(Y) = E(g(x)) = \int_0^\infty g(x) \cdot f(x)\,dx$$

$$= \int_0^\infty e^x 2e^{-2x}\,dx = \int_0^\infty 2e^{-x}\,dx$$

$$= \left[-2e^{-x} \right]_0^\infty = 2$$

Ex Slå en tärning en gång.

$X \in U(6)$

$E(X) = 3{,}5 = \mu$

$V(X) = \frac{35}{12} \approx 3 = \sigma^2$

$D(X) = \sqrt{\frac{35}{12}} \approx 1{,}7 = \sigma$

X = antal prickar.

$E[X] = 3{,}5$

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	...	...		$\frac{1}{6}$	$\frac{1}{6}$

Bestäm $V(X) = \sigma^2$

$$V(X) = E((X-\mu)^2) = \sum_{x=1}^{6} (x-\mu)^2 p(x) =$$

$$= E(X^2) - \mu^2$$

$$= \sum_{1}^{6} x^2 p(x) - \left(\frac{7}{2}\right)^2 = \frac{1}{6}(1+2^2+3^2+4^2+5^2+6^2) - \frac{49}{4}$$

$$= \frac{91}{6} - \frac{49}{4} = \frac{35}{12}.$$

Standard avvikelse $\sigma = \sqrt{V(X)} = \sqrt{\frac{35}{12}} \approx 1{,}7$

Ex $X \in Exp(\lambda)$ $\quad f(x) = \lambda e^{-\lambda x}, x \geq 0.$

Bestäm $V(X)$ "varians" och $D(X)$ "std"

$$V(X) = E(X^2) - \mu^2 = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx - \frac{1}{\lambda^2} =$$

$$\int_0^\infty x^2 \lambda e^{-\lambda x}\, dx = \underbrace{\left[\underset{g(x)}{x^2} \underset{F(x)}{(-e^{-\lambda x})} \right]_0^\infty}_{=0} - \int_0^\infty \underset{g'(x)}{-2x} \underset{F(x)}{e^{-\lambda x}}\, dx$$

$$= \int_0^\infty 2x e^{-\lambda x}\, dx = \left[2x \left(-\frac{1}{\lambda} e^{-\lambda x} \right) \right]_0^\infty$$

$$+ \int_0^\infty \frac{2}{\lambda} e^{-\lambda x}\, dx$$

$$= \left[\frac{2}{\lambda^2} e^{-\lambda x} \right]_0^\infty = \frac{2}{\lambda^2} \leftarrow$$

Ex Linjär Transformation:

X kontinuerlig s.v. med täthetsfunktion $f(x)$.

$Y = aX + b = g(x)$

$$\mu_Y = E(Y) = E(aX + b) = \int_{-\infty} (ax+b) \cdot f(x)\, dx.$$

$$= a \underbrace{\int_{-2} x f(x)\, dx}_{=1}$$

$$\sigma_Y^2 = V(Y) = V(aX+b) = E\left[(aX+b - a\mu_X(t)b)^2\right]$$

$$= E\left[(aX - a\mu_X)^2\right] = E\left(a^2(X-\mu_X)^2\right)$$

$$= a^2 E\left[(X-\mu_X)^2\right] = a^2 V(X).$$

$$D(Y) = |a|\, D(X).$$

Beroendetmått

Def

$\boxed{5.16}$ 2 dim s.v. (X, Y) beskrivs av $P_{XY}(x,y)$.

$y \backslash x$	0	1	2	$P_Y(y)$
1	0,1	0,2	0,3	0,6
2	0,4	0	0	0,4
P_X	0,5	0,2	0,3	1

Om Oberoende s.v.

$$P_{XY}(x,y) = P_X(x) \cdot P_Y(y)$$

$$P(1,2) = 0 \neq P_X(1) \cdot P_Y(2) = 0{,}4 \cdot 0{,}2 = 0{,}08.$$

$\therefore$ X o Y är beroende.

$$E(X) = \sum_{x=0}^{2} x\, P_X(x) = 1 \cdot 0{,}2 + 2 \cdot 0{,}3 = 0{,}8.$$

$$E(Y) = \sum_{y=1}^{2} y\, P_Y(y) = 1 \cdot 0{,}6 + 2 \cdot 0{,}4$$

$$E(XY) = \sum_{y=0}^{2} \sum_{x=0}^{2} x \cdot y\, P_{XY}(x,y) = 0{,}4$$

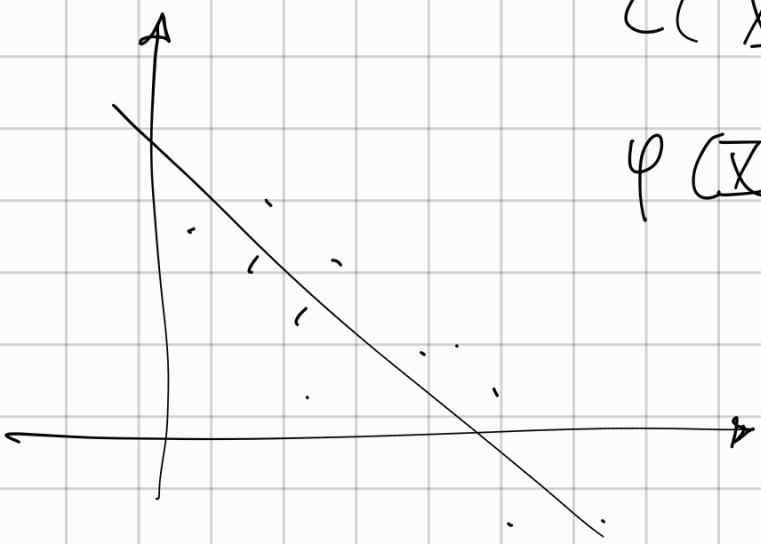
Kurv = Beroende mätt.

$C(X,Y) > 0$

$\varphi(X,Y) > 0$

$C(X,Y) < 0$

$\varphi(X,Y) < 0$



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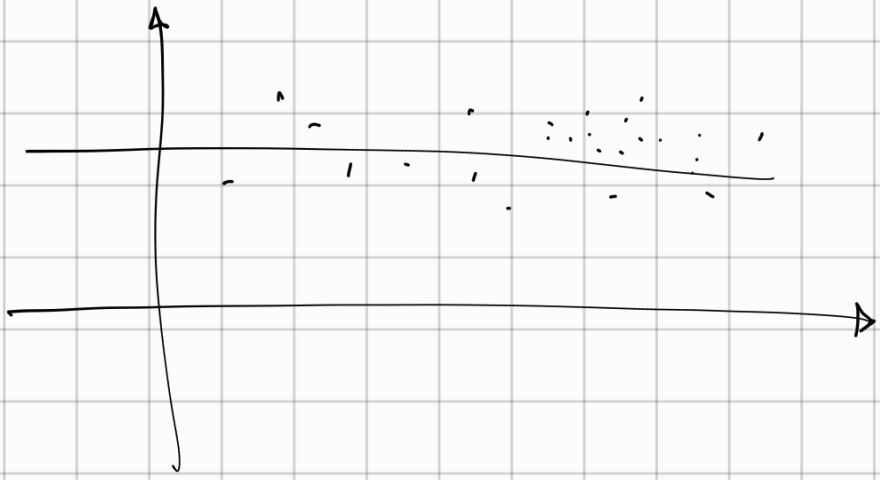

$$C(X,Y) \approx 0$$

$$\varphi(X,Y) \approx 0$$

<u>Ex</u> Stick prover

$X_1 \ldots X_n$ Oberoende s.v.

$$Y = \sum_i^n X_i \quad , \quad \bar{X} = \frac{1}{n}\sum_i^n X_i$$

X_1, X_2 Oberoende s.v. med

$$f_{X_1}(x_1) = f_{X_2}(x_2).$$

$$Y = X_1 + X_2.$$

$$E(Y) = E(X_1 + X_2) = \iint_{\Omega_{X_1}\,\Omega_{X_2}} (x_1 + x_2)\cdot f_{X_1}(x_1) f_{X_2}(x_2)\, dx_2\, dx_1$$

$$= \int$$

$$\int_{\Omega_{X_1}}\int_{\Omega_{X_2}} x_1 f_{X_1}(x_1) f_{X_2}(x_2)\, dx_2\, dx_1 +$$

$$\int_{\Omega_{X_1}}\int_{\Omega_{X_2}} x_2 f_{X_2}(x_2) f_{X_1}(x_1)\, dx_1 dx_2$$

$$= \int_{\Omega_{X_1}} x_1 f_{X_1}(x_1)\, dx_1 + \int_{\Omega_{X_2}} x_2 f_{X_2}(x)\, dx_2$$

$$= E(X_1) + E(X_2).$$

$$V(Y) = V(X_1 + X_2) = E\left(\left(X_1 + X_2 - (\mu_1 + \mu_2)\right)^2\right)$$

$$= E\left[\left((X_1 - \mu_1) + (X_2 - \mu_2)\right)^2\right]$$

$$= E\left[(X_1 - \mu_1)^2 + (X_2 - \mu_2)^2 + 2(X_1 - \mu_1)(X_2 - \mu_2)\right]$$

$$= E\left((X_1 - \mu_1)^2\right) + E\left((X_1 - \mu_2)^2\right)$$

$$+ 2E\left((X_1 - \mu_1)(X_2 - \mu_2)\right)$$

$$= V(X_1) + V(X_2) + 2C(X_1; X_2).$$

X_1 o X_2 är oberoende $\Rightarrow C(X, Y) = 0.$

$$V(X_1 \pm X_2) = V(X_1) + V(X_2).$$

Ex/ slå En fyr sidig tärning 2 ggr

X_1 = 1:a slag

X_2 = 2:a slag

$P_{X_1}(x) = P_{X_2}(x) = \frac{1}{4}$

$Y_1 = X_1 + X_2$

y_1	2	3	4	5	6	7	8
$p(y_1)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$Y_2 = X_1 - X_2$

y_2	-3	-2	-1	0	1	2	3

$V(Y_1) = V(Y_2)$