14/11 Konfidensintervall, ej Normalfördelning

Ex $X_i \in Po(\lambda)$ $E(X_i) = \lambda = V(X_i)$

$$\lambda^* = \bar{X} \approx N\left(\lambda; \sqrt{\frac{\lambda}{n}}\right)$$

CGS $n \geq 30$

$$\lambda^*_{OBS} = \bar{x}$$

$$\lambda \in \left(\bar{x} \pm \lambda_{\alpha/2} \sqrt{\frac{\bar{x}}{n}}\right), \approx (1-\alpha)\cdot 100\%$$

Ex 12.31

X = antal def. i urvalet

$X \in Hyp(N; 600; p) \approx Bin(600; p)$

antag $\frac{n}{N} < 0{,}1$

$$p^* = \frac{X}{n} \approx N\left(p; \sqrt{\frac{p(1-p)}{n}}\right)$$

Resultat: $x = 24 \quad n = 600$

$$p^*_{OBS} = \frac{24}{600} = 0{,}04$$

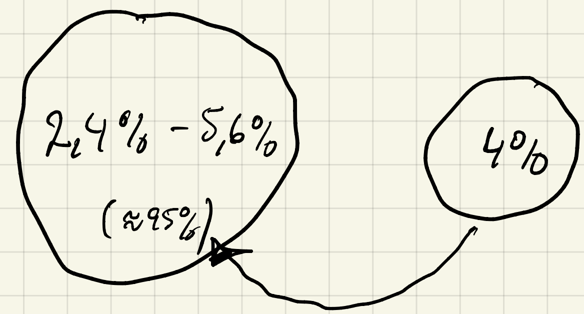
95% K.I för p: $\lambda_{0{,}025} = 1{,}96$

$$p \in \left(p^*_{OBS} \pm 1{,}96\sqrt{\frac{p^*_{OBS}(1-p^*)}{600}}\right)$$

$$p \in (0{,}04 \pm 0{,}01568) \approx (95\%)$$

$$p \in (0{,}024; 0{,}056) \approx (95\%)$$

POP.



12.32 Hur stort n behövs för att

$$P \in (P^* \pm 0{,}005), \ (\approx 95\%)$$

a) P okänd

felmarginal $e = 1{,}96 \sqrt{\dfrac{P^*(1-P^*)}{n}} \leq 0{,}005$

$P(1-P) \leq \frac{1}{4}$, $e = 1{,}96 \sqrt{\dfrac{1}{4n}} \leq 0{,}005 \Rightarrow$

$$n \geq \left(\frac{1{,}96}{0{,}005}\right)^2 \cdot \frac{1}{4} = 38416$$

b) Vi vet $0 < P < 0{,}04$ $\left(P^*_{OBS} = 0{,}04\right)$

$$e = 1{,}96 \sqrt{\frac{0{,}04 \cdot 0{,}96}{n}} \leq 0{,}005 \Rightarrow n \geq \left(\frac{1{,}96}{0{,}005}\right)^2 \cdot 0{,}04 \cdot 0{,}96 = 5900$$

12.33 $\quad X$ = antal borgerliga symp. okt, $X \approx Bin(1704, p_1)$

Y = antal borgerliga symp. Nov, $Y \approx Bin(1689, p_2)$

$$p_1^* = \frac{X}{1704} \approx N\left(p_1 ; \sqrt{\frac{p_1(1-p_1)}{1704}}\right)$$

CGS

$$p_2^* = \frac{Y}{1689} \approx N\left(p_2 ; \sqrt{\frac{p_2(1-p_2)}{1689}}\right)$$

$\underline{p_1 - p_2 = 0?}$; 95 K.I

$$(p_1 - p_2)^* = \frac{X}{1704} - \frac{Y}{1689} \approx N\left(p_1 - p_2 ; \sqrt{\frac{p_1(1-p_1)}{1704} + \frac{p_2(1-p_2)}{1689}}\right)$$

Resultat: $p^*_{1\,obs} = 0{,}465$ & $p^*_{2\,obs} = 0{,}456$

$$p_1 - p_2 \in \left(\underbrace{0{,}465 - 0{,}456}_{0{,}009} \pm \underbrace{1{,}96\sqrt{\frac{0{,}465 \cdot 0{,}535}{1704} + \frac{0{,}456 \cdot 0{,}544}{1689}}}_{e}\right)$$

om $e < 0{,}009$ stat. säker förändring $\alpha \approx 5\%$

om $e \geq 0{,}009$ ingen stat. säker förändring $\alpha \approx 5\%$

$P_1 - P_2 = 0$

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Okänd parameter K. I

$\boxed{\text{Skattning av, } \mu} \pm \boxed{\text{tabell-värde}} \boxed{\text{medel-fel, } \mu^*}$

Normalfördelning?

JA

NEJ

$\boxed{\sigma \text{ KÄND?}}$

JA

$\mu \in \left(\bar{x} \pm \lambda_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$

Nej

$\mu \in \left(\bar{x} \pm t_{\alpha/2}(n-1) \frac{s}{\sqrt{n}}\right)$

$\boxed{n > 30?}$

Nej teckenintervall för median

JA

$\mu \in \left(\bar{x} \pm \lambda_{\alpha/2} \frac{s}{\sqrt{n}}\right)$

Ex teckenintervall för medianen, m

12.18 X_i = plant längd

$X_i \in N(\mu; \sigma)$, σ okänd

$\mu \in (\bar{x} \pm t_{0,025}(15) \frac{s}{\sqrt{16}})$, (95%)

$X_i \in \bullet(\mu, \sigma)$

storleksordnar stickprovet

$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	.....	$x_{(13)}$	$x_{(14)}$	$x_{(15)}$	$x_{(16)}$
3,5	3,9	4,0	4,1		5,3	5,5	5,8	5,9

$P(X_i \leq m) = 0,5$

$m \in (x_{(1)}; x_{(16)})$

$m \in (x_{(2)}; x_{(15)})$

Vilken konfidensgrad har dessa intervall?

$$P(X_{(1)} < m < X_{(16)}) = \left[\begin{array}{l} Y = \text{antal obs t.v. om median} \\ Y \in Bin(16; \tfrac{1}{2}) \end{array} \right]$$

$$= 1 - P(Y=16) - P(Y=0)$$

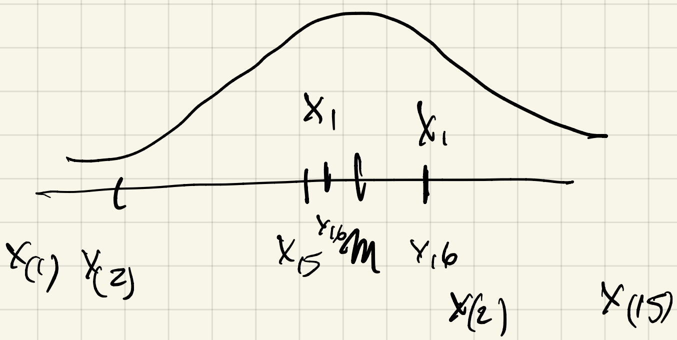
$$= 1 - 2\cdot\left(\tfrac{1}{2}\right)^{16} \approx 0{,}99996$$

$x_{(1)}$ — $x_{(16)}$ $\qquad$ $x_{(1)}$ — $x_{(16)}$

$$P(X_{(2)} < m < X_{(15)}) = 1 - P(X_{(2)} < X_{(15)} < m) - P(m < X_{(2)} < X_{(15)})$$

$$1 - P(Y \geq 15) - P(Y \leq 1) =$$

$$1 - 2P(Y \leq 1) = 1 - \underbrace{2 \cdot 0{,}000026}_{\alpha} = 0{,}99948$$



$$P(X_{(4)} < m < X_{(13)}) = 1 - 2P(Y \leq 3) = 1 - 2\cdot 0{,}0164 \approx 0{,}978$$