10/10 Binomial

$$X_i = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

$$Bin(1; p)$$

$$E(X_i) = p$$

$$V(X_i) = p - p^2 = p(1-p)$$

$$X = \sum_{1}^{n} X_i \in Bin(n; p).$$

$$E(X) = \sum_{1}^{n} E(X_i) = np$$

$$Y = \frac{X}{n}$$

relativa frekvensen hur många gånger lyckades jag.

$$Y \in Bin(n; p)$$

$$E(Y) = E\left(\frac{X}{n}\right) = \frac{1}{n} E(X) = \frac{1}{n} np = p$$

$$V(Y) = V\left(\frac{X}{n}\right) = \frac{1}{n^2} V(X) = \frac{1}{n^2} np(1-p) = \frac{1}{n} p(1-p)$$

$$Y \approx N\left(p; \sqrt{\frac{p(1-p)}{n}}\right)$$

Ex X: antal lyckades försök $X \in \text{Bin}(160; 0{,}7)$

$P(X \leq 10)$.

Y: antal misslyckades försök

$Y \in \text{Bin}(160; 0{,}3)$.

$X \leq 10$

$x = 0, 1, \ldots, 9, 10, 11, \ldots, 15$

$y = \underbrace{15, 14, 13, \ldots 6, 5}_{Y \geq 5}, \underbrace{4, \ldots, 0}_{Y \leq 4}$

Ex

X = antal lyck—

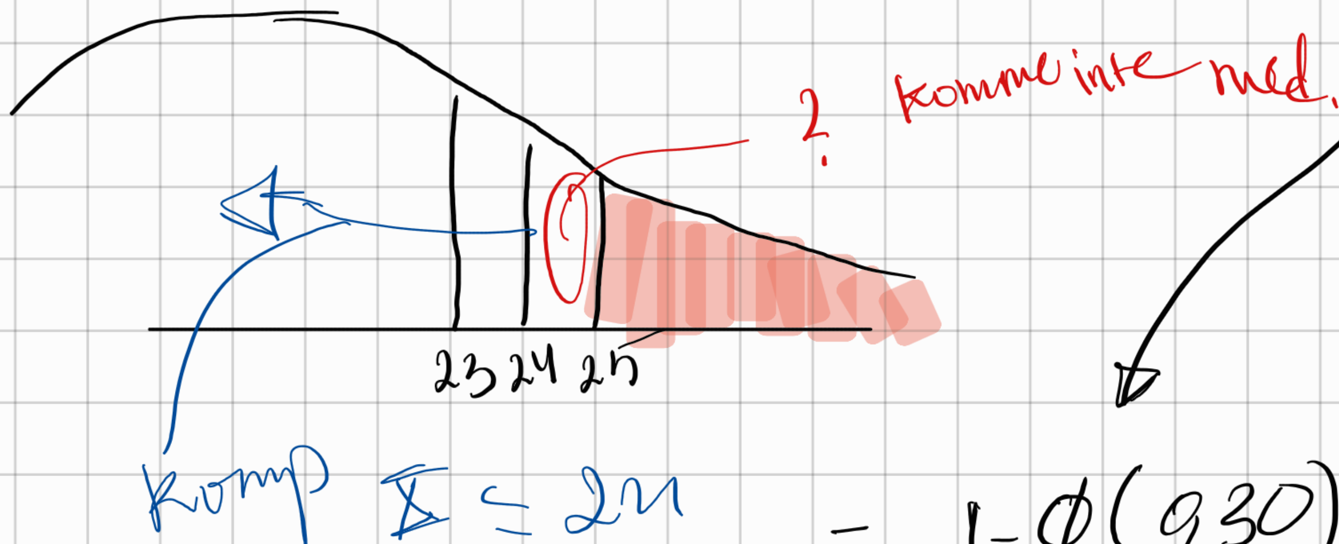
X = Bin(50; 0,4)

$E(X) = 50 \cdot 0{,}4 = 20$

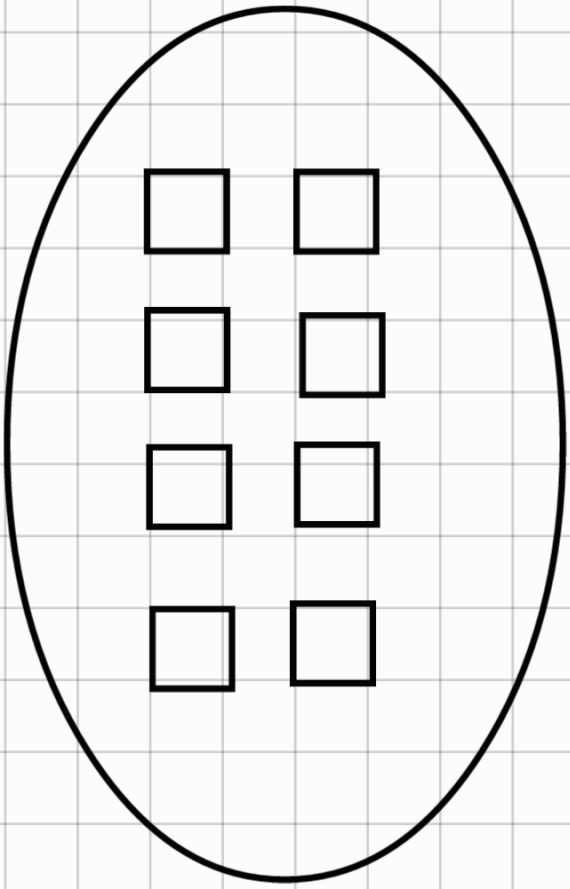
$V(X) = \underset{n}{50} \cdot \underset{p}{0{,}4}(1-0{,}4) = 12 > 10$

$X \approx N(20; \sqrt{12})$

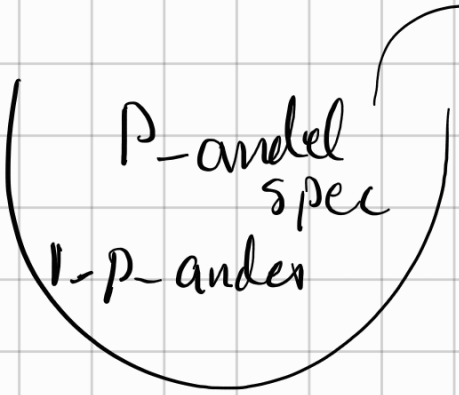
$P(X \geq 25) \approx 1 - \Phi\left(\frac{24-20}{\sqrt{12}}\right)$



$= 1 - \Phi(1{,}30)$
$= 0{,}0968.$



Ex Hyper



X antal spec i intervall

$X \in Hyp(N; n; p)$

$Y = \frac{X}{n}$ $\quad E(Y) = \frac{1}{n} np = p = \mu$

$V(Y) = \frac{1}{n^2} npq \frac{N-n}{N-1} = \frac{p(1-p)}{n} \cdot \frac{N-n}{N-1} = \sigma^2$

okänd

$T \approx N(\mu, \sigma)$.

COS

7.16 $X \in Hyp(100; 5; 0,06)$

$$P(X) = \frac{\binom{6}{x}\binom{94}{5-x}}{\binom{100}{5}}$$

$$E(X) = n \cdot p$$

$$V(X) = np(p-1)\frac{N-n}{N-1}$$

c) $P(Acc) = P(X \leq 1)$

$$= \frac{\binom{6}{0}\binom{94}{5} + \binom{6}{1}\binom{94}{4}}{\binom{100}{5}}$$

$\frac{n}{N} = \frac{5}{100} = 0,05 < 0,1$