10/10 X_1 = tid för relä 1 $X_1 \in N(1;0{,}1)$

1.15 X_2 = tid för relä 2 $X_2 \in N(1{,}5;0{,}2)$

X_1 o X_2 oberoende.

$$P(\text{relä 2 före relä 1}) = P(X_2 < X_1) =$$

$$P(X_2 - X_1 < 0) = \sqrt{X_2 - X_1 \in N(1{,}5-1, \sqrt{0{,}1^2+0{,}2^2})} =$$

$$X_2 - X_1 \in N(0{,}5; \sqrt{0{,}05})$$

$$= \Phi\left(\frac{0-0{,}5}{\sqrt{0{,}05}}\right) = \Phi(-2{,}24) = 1 - \Phi(2{,}24)$$

$$= 1 - 0{,}98745 = 0{,}01255$$

6.20 X_i = vikt i ton i en vagn

$$X_i \in \bullet(\overset{\mu}{10}; \overset{\sigma}{0{,}5})$$

okänd fördelning

Antag

$X_i \in N(10; 0{,}5)$

$$Y = \sum_{1}^{25} X_i = \text{vikt 25 vagnar}$$

$$Y \underset{\text{C.G.S.}}{\approx} N(\overset{n \cdot \mu}{25 \cdot 10}; \overset{\sqrt{n} \cdot \sigma}{\sqrt{25} \cdot 0{,}5})$$

$$P(Y > 255) = 1 - P(Y \leq 255) \approx 1 - \Phi\left(\frac{255-250}{2{,}5}\right)$$

$$= 1 - \Phi(2) = 1 - 0{,}97725 \approx 0{,}023$$

6.21 X_i = antal barn i en familj

x	0	1	2	3
$p(x)$	0,4	0,2	0,3	0,1

$$Y = \sum_{1}^{1000} X_i = \text{antal barn i 1000 familjer}$$

$$P(\text{får dagisplats}) = P(Y \leq a) = 0{,}90$$

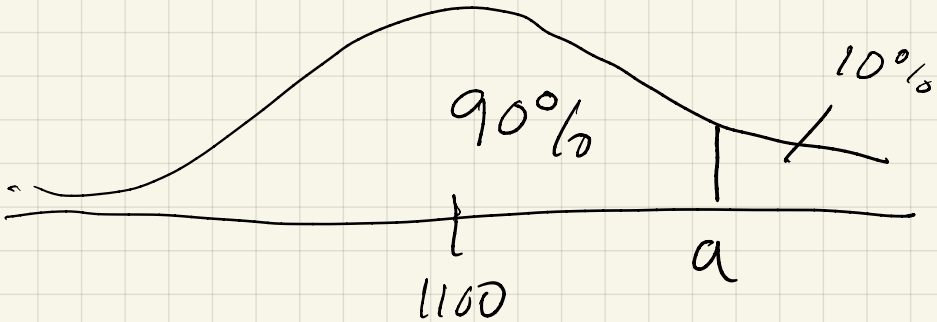
$$E(Y) = \sum_{1}^{1000} E(X_i) = 1000\mu$$

$$V(Y) = \sum_{1}^{1000} V(X_i) = 1000\,\sigma^2$$

$$\mu = E(X_i) = \sum_{0}^{3} x\,p(x) = 1\cdot 0{,}2 + 2\cdot 0{,}3 + 3\cdot 0{,}1 = 1{,}1$$

$$\sigma^2 = V(X_i) = \sum_{1}^{3} x^2 p(x) - \mu^2 = 1^2\cdot 0{,}2 + 2^2\cdot 0{,}3 + 3^2\cdot 0{,}1 - 1{,}1^2 = 1{,}09$$

$$Y \approx N(\overbrace{1100}^{n\cdot\mu};\ \overbrace{\sqrt{1000}}^{\sqrt{n}}\ \overbrace{\sqrt{1{,}09}}^{\sigma})$$



$$P(Y \le a) \approx \Phi\left(\frac{a-1100}{\sqrt{1090}}\right) = 0{,}9 \iff \frac{a-1100}{\sqrt{1090}} = 1{,}2816 \Rightarrow$$

$$a = 1100 + 1{,}2816\sqrt{1090} \approx 1143$$

1143 platser för att "alla" barnslen får plats på dagis med 90% sannolikhet.

6.23 X_i = livslängd i tim. för ett rör.

$X_i \in Exp(\lambda)$ $\quad \mu = \frac{1}{\lambda} = 200 = \sigma$

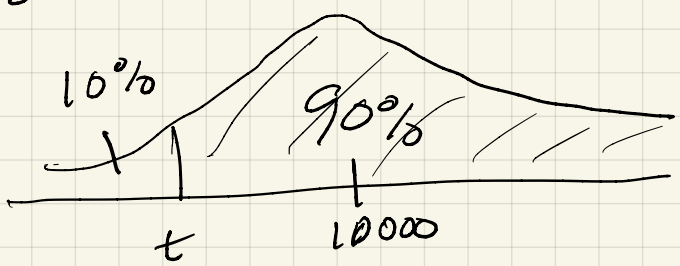
$Y = \sum_1^{50} X_i$ = livslängd för 50 rör

$P(Y > t) = 0{,}90 \Leftrightarrow P(Y \leq t) = 0{,}10 \overset{C.G.S.}{\Longleftrightarrow}$

$E(Y) = \overset{n}{50} \cdot \overset{\mu}{200} = 10000$

$V(Y) = 50 \cdot 200^2 \Rightarrow D(Y) = \sqrt{50} \cdot 200$

$Y \approx N(10000; \sqrt{50} \cdot 200)$



$$\Phi\left(\frac{t - 10000}{\sqrt{50} \cdot 200}\right) = 0{,}1 \Rightarrow$$

$$\frac{t - 10000}{\sqrt{50} \cdot 200} = -1{,}2816 \Rightarrow t \approx 8190 \text{ tim}$$

KAP 7 Binomialfördelningen

$$X_i = \begin{cases} 1 & , p \\ 0 & 1-p \end{cases} \qquad X_i \in Bin(1; p)$$

$$E(X_i) = p$$

$$V(X_i) = p - p^2 = p(1-p)$$

$$X = \sum_1^n X_i \in Bin(n; p)$$

$$E(X) = \sum_1^n E(X_i) = np$$

$$\underset{\text{OBER.}}{V(X)} = \sum_1^n V(X_i) = np(1-p) \quad , \quad \underset{\text{CGS}}{X \approx} N(np; \sqrt{np(1-p)})$$

$X \in Bin(n; p)$ $\quad X$ = antal lyckade försök

$$Y = \frac{X}{n} \in Bin(n; p)$$

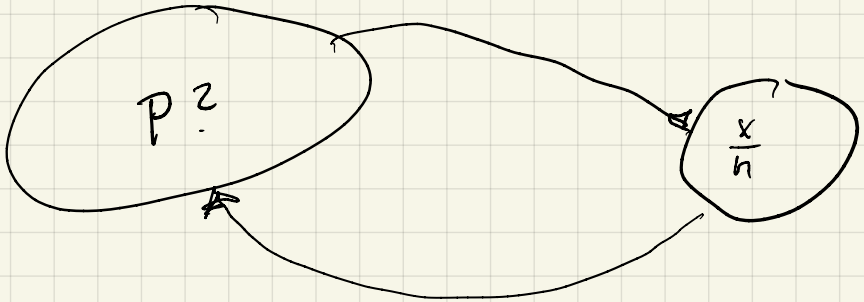
x	0,	1	$\ldots$		n
y	0	$\frac{1}{n}$	$\ldots$		1

$$E(Y) = E\left(\frac{X}{n}\right) = \frac{1}{n} E(X) = \frac{1}{n} np = p$$

$$V(Y) = E\left(\frac{X}{n}\right) = \frac{1}{n^2} V(X) = \frac{1}{n^2} np(1-p) = \frac{p(1-p)}{n}$$

$p \circ p$

$$Y \approx N\left(p; \sqrt{\frac{p(1-p)}{n}}\right)$$



Ex X = antal lyckade försök $X \in Bin(15; 0{,}7)$

$P(X \leq 10) = P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - 0{,}51549 = 0{,}485$

Y = antal misslyckade försök $Y \in Bin(15; 0{,}3)$

$$x = \overbrace{0\ 1\ 2\ \ldots,\ 9,\ 10}^{X \leq 10},\ 11,\ \ldots\ 15$$

$$y = \underbrace{15,\ 14,\ 13\ \ \ \

Ex X = antal lyckade

$X \sim Bin(50; 0{,}4)$

$$E(X) = \underset{n}{50} \cdot \underset{p}{0{,}4} = 20$$

$$V(X) = \underset{n}{50} \cdot \underset{p}{0{,}4} \cdot \underset{(1-p)}{0{,}6} = 12 > 10$$

$$X \approx N(20; \sqrt{12})$$

$$P(X \geq 25) \approx 1 - \Phi\left(\frac{24{,}5 - 20}{\sqrt{12}}\right)$$

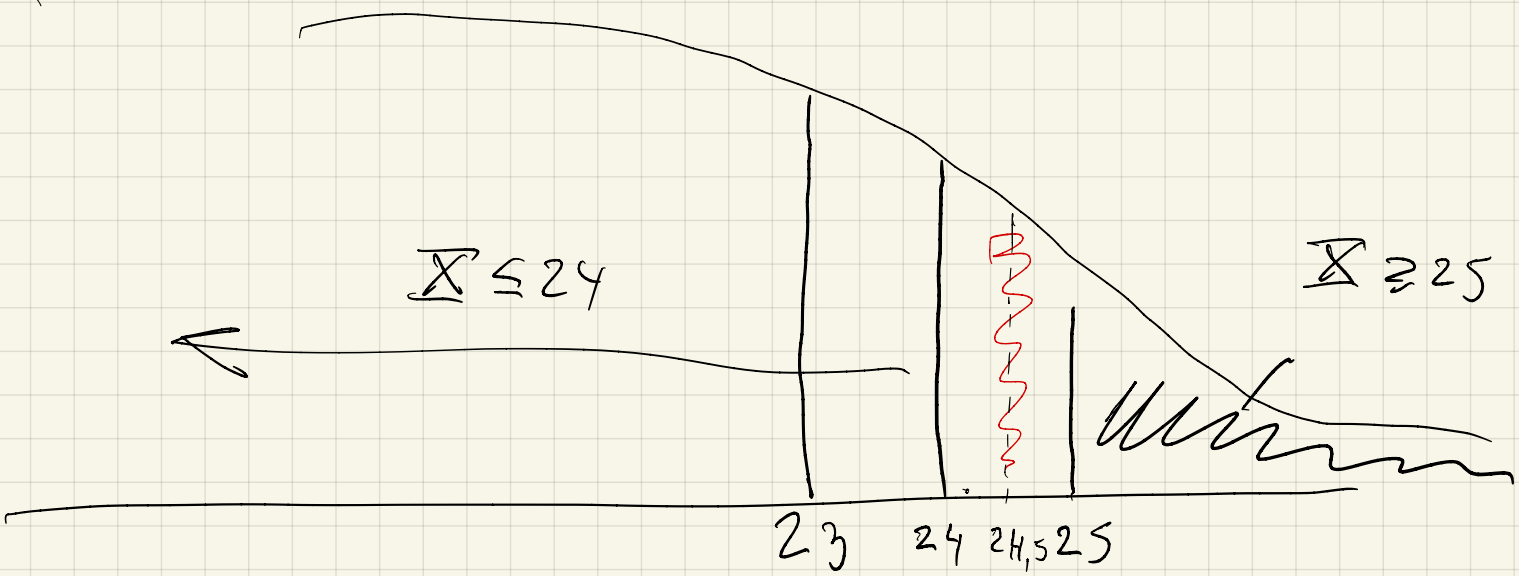
$$= 1 - \Phi(1{,}30) = 0{,}0968$$

$$1 - \text{BinomCDF}(50; 0{,}4; 24)$$

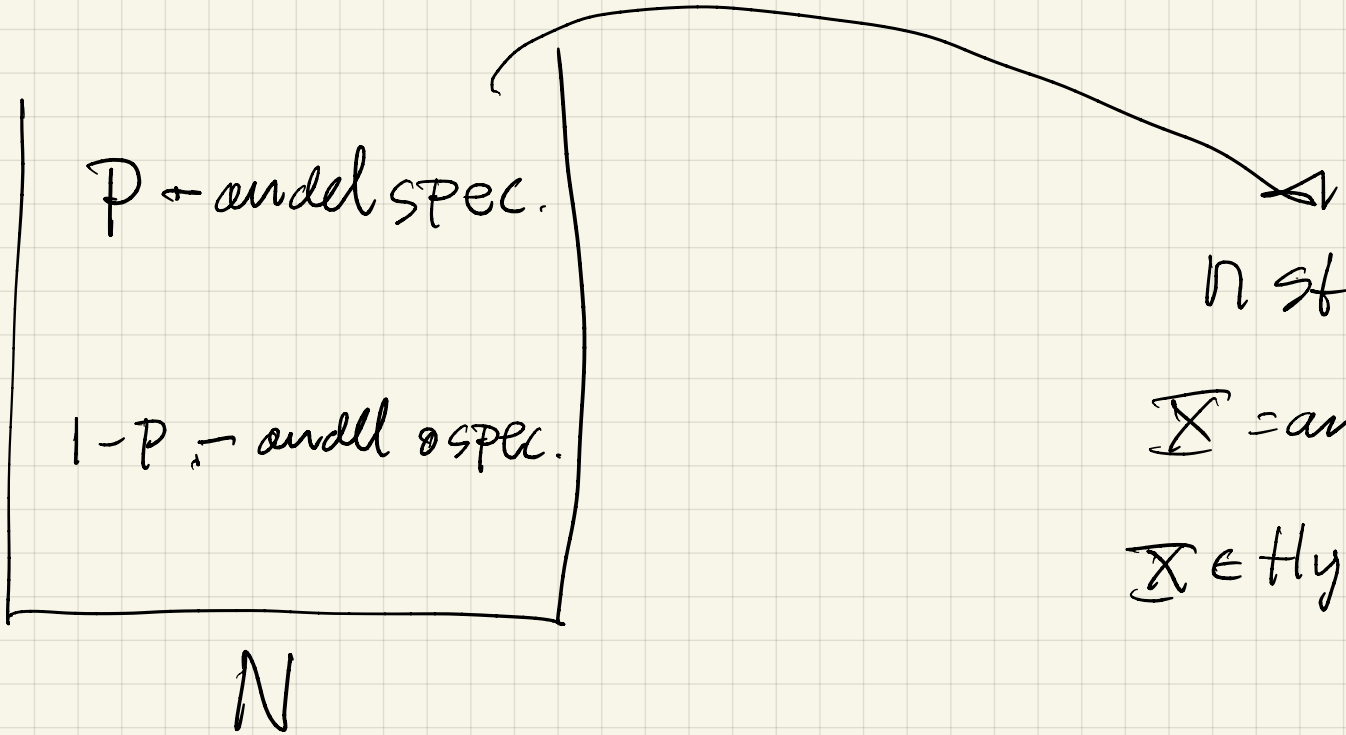
$$= 0{,}098$$

DISTR

$\boxed{\text{VAB}}$



Ex Hypergeometrisk fördelning



n st utan återläggning

X = antal spec. i urvalet

$X \in Hyp(N; n; P)$

okänd

$$Y = \frac{X}{n}$$

$$E(Y) = \frac{1}{n} nP = P = \mu$$

$$V(Y) = \frac{1}{n^2} nP(1-P)\frac{N-n}{N-1} = \frac{P(1-P)}{n}\left(\frac{N-n}{N-1}\right) = \sigma^2$$

$$Y \underset{\text{CGS}}{\approx} N(\mu; \sigma)$$

7.16 (2.21)

$n = 5$ st utan återlägg

94 ok
6 def

$N = 100$

X = antal def i urvalet

a) $X \in Hyp(100; 5; 0{,}06)$

$$P(x) = \frac{\binom{6}{x}\binom{94}{5-x}}{\binom{100}{5}} \qquad x = 0, 1, 2, 3, 4, 5$$

b) $E(X) = n \cdot p = 5 \cdot 0{,}06 = 0{,}3$

$$V(X) = n\,p(1-p)\,\frac{N-n}{N-1} = 5 \cdot 0{,}06 \cdot 0{,}94 \cdot \frac{95}{99}$$

c) $$P(ACC) = P(X \le 1) = \frac{\binom{6}{0}\binom{94}{5} + \binom{6}{1}\binom{94}{4}}{\binom{100}{5}} \approx 0{,}0968$$

$\frac{n}{N} = \frac{5}{100} = 0{,}05 < 0{,}1 \quad X \approx Bin(5; 0{,}06)$

$$P(X \le 1) = 0{,}94^5 + 5 \cdot 0{,}06 \cdot 0{,}94^4$$