10/11 Konfidens intervall för μ i en $N(\mu;\sigma)$ med okänt σ.

$$\mu^* = \bar{X} \in N\left(\mu; \frac{\sigma}{\sqrt{n}}\right)$$

$$\sigma^* = S = \sqrt{\frac{1}{n-1}\sum(X_i - \bar{X})^2}$$

$$\frac{\bar{X} - \mu}{\sigma^*/\sqrt{n}} \in t(n-1)$$

$$P\left(-t_{\alpha/2}(n-1) < \frac{\bar{X} - \mu}{\sigma^*/\sqrt{n}} < t_{\alpha/2}(n-1)\right) = 1 - \alpha$$

$\Longleftrightarrow$ lös ut μ

$$P\left(\bar{X} - t_{\alpha/2}(n-1)\frac{\sigma^*}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2}(n-1)\frac{\sigma^*}{\sqrt{n}}\right) = 1 - \alpha$$

$$\mu \in \left(\bar{X} \pm t_{\alpha/2}(n-1)\frac{S}{\sqrt{n}}\right), (1-\alpha)100\%$$

Ex X_i = syrekonc. i vatten $\quad X_i \in N(\mu; \sigma)$, σ okänd

$$\mu^* = \bar{X} \in N\left(\mu; \frac{\sigma}{\sqrt{n}}\right)$$

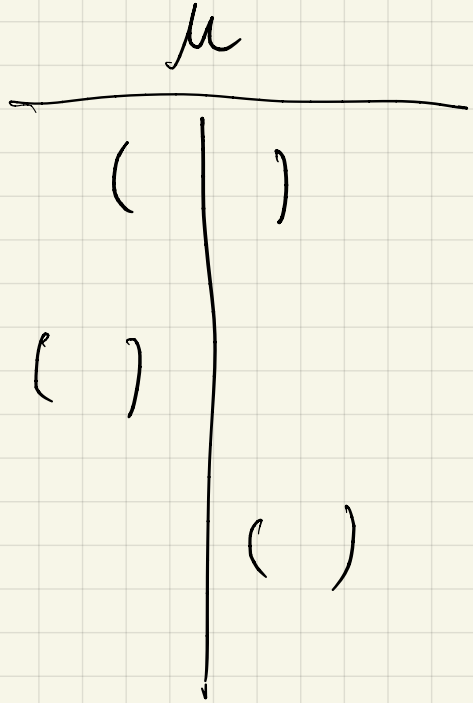
resultat: $n = 30$, $\bar{x} = 2{,}52$ & $\sigma^*_{obs} = s = 2{,}0$

99% konfidensgrad (1% felrisk)
$\quad \overset{\shortparallel}{\alpha}$

$$t^{(29)}_{0{,}005} = 2{,}76$$

$$\mu \in \left(2{,}52 \pm 2{,}76 \frac{2}{\sqrt{30}}\right), (99\%)$$

$$\mu \in (1{,}52 ; 3{,}52), (99\%)$$



Normalfördelning

JA: σ känd $\quad \mu \in \left(\bar{x} \pm \lambda_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$

NEJ: σ okänd $\quad \mu \in \left(\bar{x} \pm t^{(n-1)}_{\alpha/2} \frac{s}{\sqrt{n}}\right)$

Ex X_i = syre konc i vatten $X_i \in N(\mu; 2)$ σ känd.

Hur många mätningar krävs för att felmarginalen ska bli hälften så stor?

$\sigma = 2$, $\lambda_{0,005} = 2,5758$ $n = 30 \rightarrow e = 0,94$

$$\frac{e}{2} = 2,5758 \frac{2}{\sqrt{n}} \leq \frac{0,94}{2} \Rightarrow n \geq \left(\frac{2,5758 \cdot 2 \cdot 2}{0,94}\right)^2 = 120 = 4 \cdot 30$$

$n = 120$

$\mu \in (\bar{x} \pm 0,47)$, (99%)

Ex ensidigt uppåt eller nedåt begränsade K. I.

X_i = syretkonc. i vatten $X_i \in N(\mu; \sigma)$ $\bar{x} = 2{,}52$, $n = 30$

$\sigma = 2$

$\mu \in (\bar{x} \pm \lambda_{\alpha/2} \frac{\sigma}{\sqrt{n}})$ 2-sidigt konfidensintervall

$$\left.\begin{array}{l} \mu \in (\bar{x} - \lambda_\alpha \frac{\sigma}{\sqrt{n}}, \infty), (1-\alpha)100\% \\ \mu \in (-\infty; \bar{x} + \lambda_\alpha \frac{\sigma}{\sqrt{n}}], (1-\alpha)100\% \end{array}\right\} \text{1-sidigt K. I.}$$

$\lambda_{0{,}01} = 2{,}3263$ felmarginalen $e = 2{,}3263 \frac{2}{\sqrt{30}} \approx 0{,}85$

nedåt begr.

$\mu \in (1{,}67; \infty), (99\%)$

$\mu > 1{,}67$, $\alpha = 1\%$

uppåt begr.

$\mu \in (-\infty; 3{,}37), (99\%)$

$\mu < 3{,}37$, $\alpha = 1\%$

$\mu \in (1{,}67;\ 3{,}37)$, (98%)

Ex Beräkna ett K.I för σ^2 o σ med konfidensgrad 95%.

X_i = syrekonc. i vatten $X_i \in N(\mu; \sigma)$

$\mu^* = \bar{x} = 2{,}52$, $\sigma^* = s = 2$, $n = 30$

$$\frac{1}{\sigma^2}\sum_1^n (X_i - \bar{X})^2 \in \chi^2(n-1), \quad (\sigma^2)^* = s^2 = \frac{1}{n-1}\sum_1^n (X_i - \bar{X})^2$$

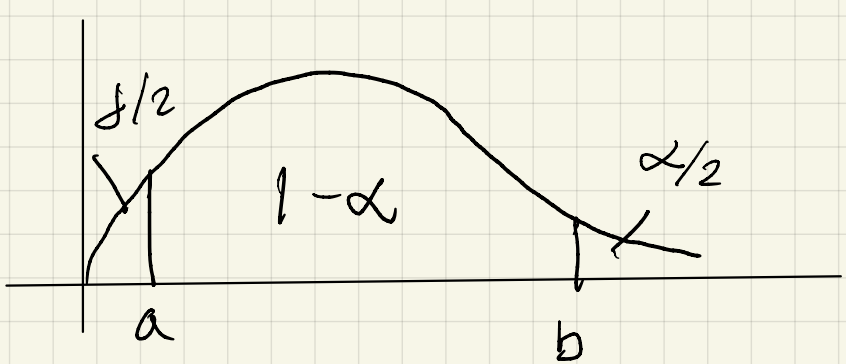
$$\Longleftrightarrow$$

$$\frac{(n-1)s^2}{\sigma^2} \in \chi^2(n-1)$$

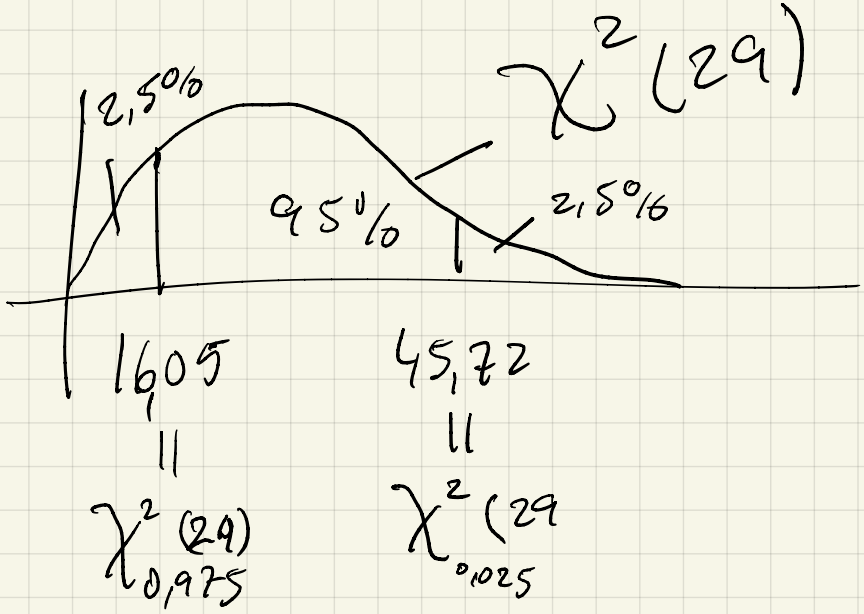
$$P\left(a < \frac{(n-1)s^2}{\sigma^2} < b\right) = 1-\alpha$$

$\Longleftrightarrow$ lös ut σ^2

$$P\left(\frac{(n-1)s^2}{b} < \sigma^2 < \frac{(n-1)s^2}{a}\right) = 1-\alpha$$



$\alpha = 5\%$ $\quad n = 30$



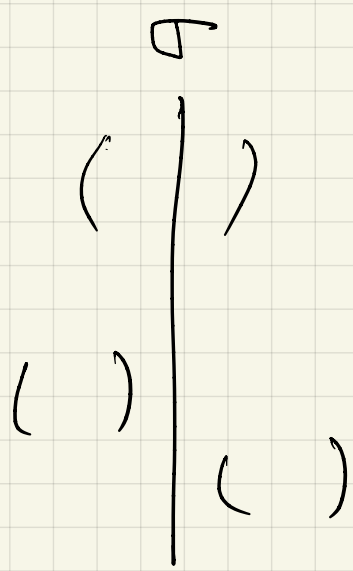
$$\sigma^2 \in \left(\frac{29 \cdot 2^2}{45,72} ; \frac{29 \cdot 2^2}{16,05} \right), (95\%)$$

$$\sigma^2 \in (2,53 ; 7,25), (95\%)$$

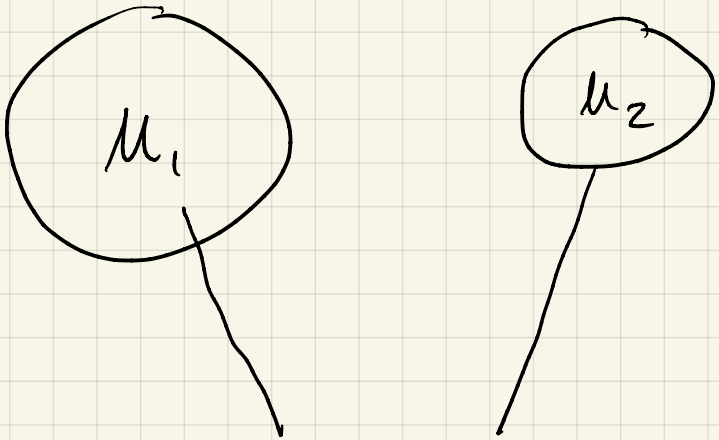
$$\sigma \in \left(\sqrt{2,53} ; \sqrt{7,25} \right), (95\%)$$

$$\sigma \in (1,59 ; 2,70), (95\%)$$

"med 95% säkerhet (5% felrisk) är det sanna värde på σ i intervallet"



PoP 1 PoP 2



$$\mu_1 = \mu_2$$

$$\Longleftrightarrow$$

$$\mu_1 - \mu_2 = 0 \ ?$$

$$X_i \in N(\mu_1; \sigma), \ i = 1, \dots, n_1$$

$$Y_j \in N(\mu_2; \sigma), \ j = 1, \dots, n_2$$

σ känd

$$\mu_1^* = \bar{X} \in N\left(\mu_1; \frac{\sigma}{\sqrt{n_1}}\right)$$

$$\mu_2 = \bar{Y} \in N\left(\mu_2; \frac{\sigma}{\sqrt{n_2}}\right)$$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \in N(0,1)$$

$$(\mu_1 - \mu_2)^* = \bar{X} - \bar{Y} \in N\left(\mu_1 - \mu_2; \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

Konfidensintervall för $\mu_1 - \mu_2$

$$\mu_1 - \mu_2 \in \left(\bar{x} - \bar{y} \pm \lambda_{\alpha/2}\, \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right), \ (1-\alpha)100\%$$

$X_i \in N(\mu_1; \sigma), \quad i = 1, \dots, n_1$

$Y_j \in N(\mu_2; \sigma): j = 1, \dots, n_2$

$\underline{\sigma \text{ okänd}}$

Stickprovsgen

$\bar{x}, s_1, n_1$

$\bar{y}, s_2, n_2$

$\left.\begin{array}{l} \sigma^* = s_1 \\ \sigma^* = s_2 \end{array}\right\}$ vilketsad skattning av σ

$$\sigma_p^* = s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

Konfidensintervall för

$\mu_1 - \mu_2$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \in t(n_1 + n_2 - 2)$$

$$\mu_1 - \mu_2 \in \left(\bar{x} - \bar{y} \pm t_{\alpha/2}^{(n_1+n_2-2)} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right), \ (1-\alpha)\,100\%$$

12.21 X_i = överhöjning fabrik A, $X_i \in N(\mu_A; \sigma)$

Y_j = överhöjning fabrik B, $Y_j \in N(\mu_B; \sigma)$

Stickprovsresultat

$\mu_A^* = \bar{x} = 18{,}1$, $\sigma^* = s_A = 5$ $n_A = 9$

$\mu_B^* = \bar{y} = 14{,}6$, $\sigma^* = s_B = 7{,}1$ $n_B = 16$

} 99% K.I för $\mu_A - \mu_B$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{9} + \frac{1}{16}}} \in t(23)$$

$$(\mu_A - \mu_B)^* = \bar{x} - \bar{y} = 3{,}5$$

$$\sigma^* = s_p = \sqrt{\frac{(9-1)\,5^2 + (16-1)\,7{,}1^2}{16+9-2}} \approx 6{,}45$$

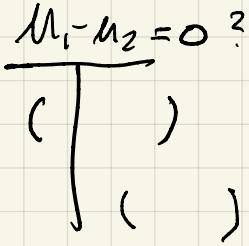
$$t_{0{,}005}^{(23)} = 2{,}81$$

felmarginalen

$$e = t_{0{,}005}^{(23)}\, s_p \sqrt{\frac{1}{9} + \frac{1}{16}} \approx 7{,}45$$

$\mu_1 - \mu_2 \in (3{,}5 \pm 7{,}45)$, (99%)

$\mu_1 - \mu_2 \in (-4{,}05; 11{,}05)$, (99%)

ingen stat. säker skillnad mellan A o B

$\mu_1 - \mu_2 = 0$?