13/9 EV

- X = antal kunder / tidsintervall
- $X \in Po(4)$ $\quad p(k) = e^{-4}\frac{4^k}{k!}$ $\quad k = \boxed{0, 1, 2, 3, 4,} 5$

$X \le x$

$X \le 2$

$$P(\text{flera än 2 men färre än 5}) = P(2 < X < 5) =$$

$$P(3 \le X \le 4) = e^{-4}\left(\frac{4^3}{3!} + \frac{4^4}{4!}\right) = P(3) + P(4)$$

$$\underset{\text{tabell}}{=} \underline{\underline{F(4)}} - \underline{\underline{F(2)}} = 0{,}62884 - 0{,}23810 = 0{,}39074$$

T 82-84

Distr

$\boxed{\text{VARS}}$

μ, x

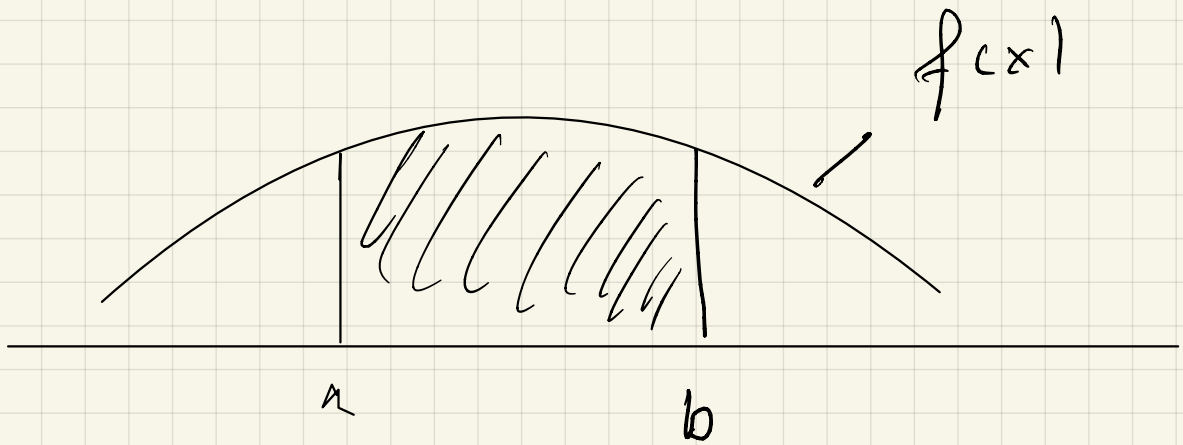
PoissonCDF(4,4) − PoissonCDF(4,2)

PoissonPDF(4,3) + PoissonPDF(4,4)

Kontinuerlig stok. var $\underline{\circ}$ $a, b \in \mathbb{R}$

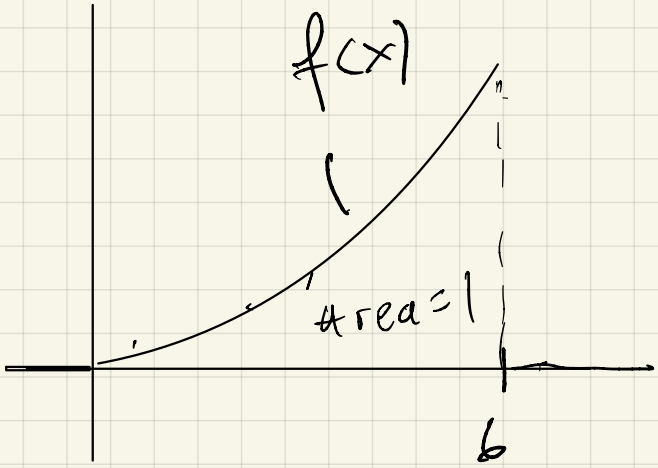
$$P(a < X \leq b) = P(a \leq X \leq b) = P(a \leq X < b) = P(a < X < b) =$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



3.13 Bestäm $C \in \mathbb{R}$ så att

$$f(x) = \begin{cases} Cx^2, & 0 < x < 6 \\ 0 & \text{f.ö.} \end{cases}$$



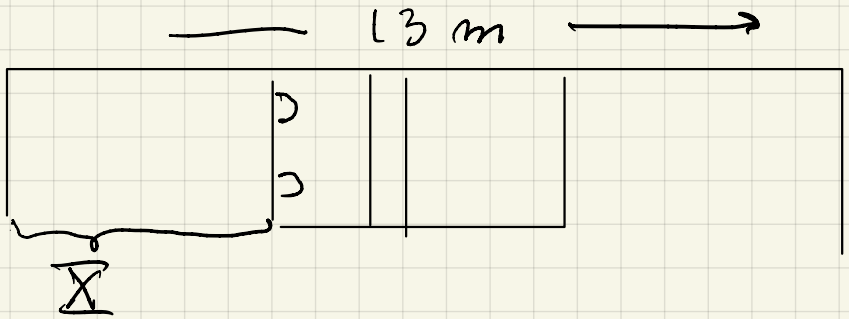
blir en täthetsfunktion för någon stok. var. X.

$$P(\Omega) = 1 \iff \int_0^6 Cx^2\,dx = 1 \iff \left[C\frac{x^3}{3}\right]_0^6 = 1 \Rightarrow C = \frac{1}{72}$$

$$f(x) = \begin{cases} \frac{1}{72}x^2, & 0 < x < 6 \\ 0 \end{cases} \qquad (f(x) \geq 0)$$

3.21 P-ruta 13 m lång

Bil 5 m lång



- X = plats framför bilen
- $\Omega = \{x,\ 0 \le x \le 8\}$
- antag att alla värden i Ω är lika sannolika.

$X \in U(0,8)$ $\quad f(x) = \begin{cases} \frac{1}{8} & 0 \le x \le 8 \\ 0 & \text{för} \end{cases}$

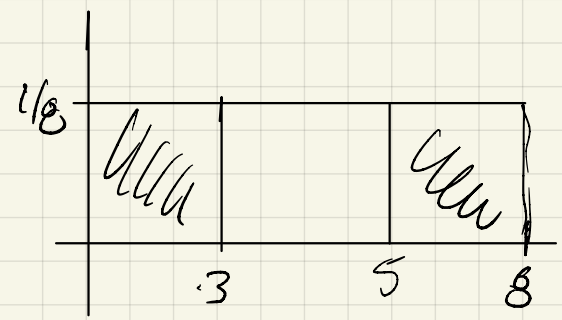
P(en 5m-bil i rutan) =

P(framför) + P(bakom) =

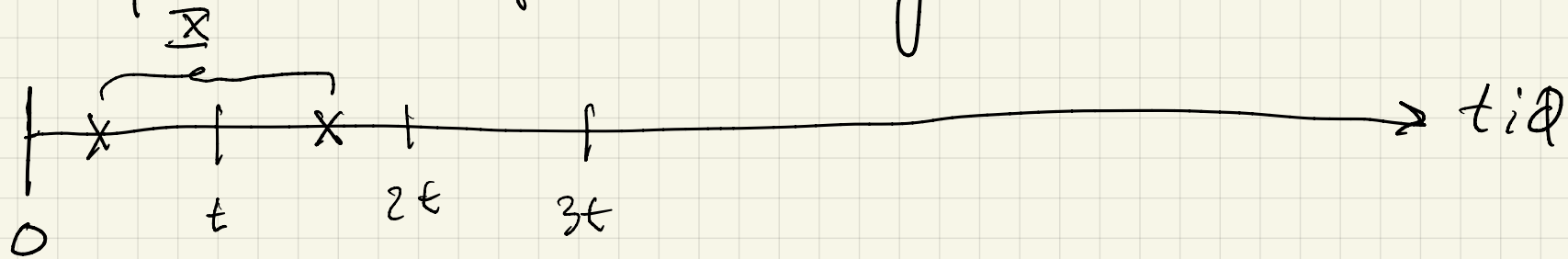
$P(X \ge 5) + P(X \le 3) =$

$$\int_5^8 \frac{1}{8}\,dx + \int_0^3 \frac{1}{8}\,dx =$$

$$1 - \int_3^5 \frac{1}{8}\,dx = \frac{3}{4}$$



Exponential fördelning



X = tidsavståndet mellan 2 händelser

$X \in Exp(\lambda)$ $\qquad f(x) = \lambda e^{-\lambda x}, \; x \geq 0$

Ex 3.20 $\quad X$ = samtalstid i minuter

$$P(X > t) = e^{-\lambda t}, \; \lambda > 0$$

- Fördelningsfunktion för X

$$F(t) = P(X \leq t) = 1 - P(X > t) = 1 - e^{-\lambda t} \quad (Exp(\lambda))$$

- täthets(frekvens)funktionen för X

$$f(t) = F'(t) = \lambda e^{-\lambda t}, \; x \geq 0$$

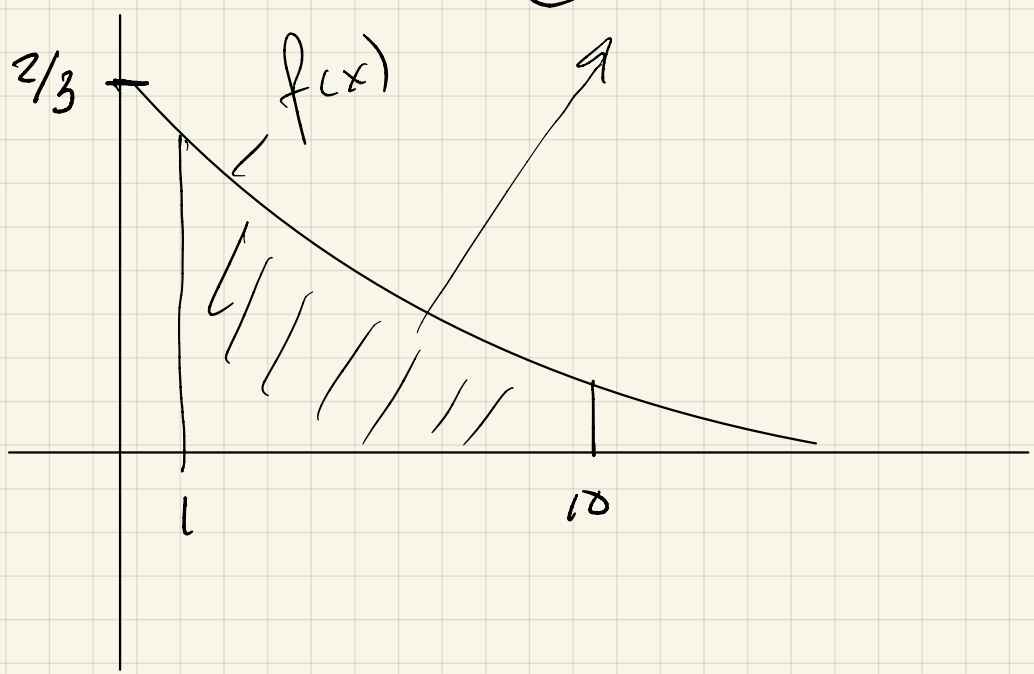
om $\lambda = \frac{2}{3}$ $\quad \underline{X} \in Exp(\frac{2}{3})$ $\quad f(t) = \frac{2}{3} e^{-\frac{2}{3}t}$

$F(t) = 1 - e^{-\frac{2}{3}t}$

$$P(1 < \underline{X} \leq 10) = \int_1^{10} \frac{2}{3} e^{-\frac{2}{3}t} \, dt = F(10) - F(1)$$

$$= 1 - e^{-\frac{20}{3}} - \left(1 - e^{-\frac{2}{3}}\right)$$

$$= e^{-\frac{2}{3}} - e^{-20/3}$$

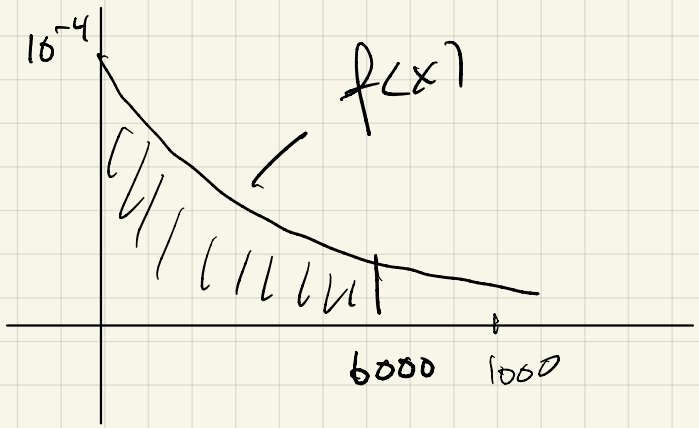


3.22 X = livslängd i timmar

$\lambda = 10^{-4}$ $\quad X \in Exp(10^{-4})$

$f(x) = 10^{-4} e^{-x \cdot 10^{-4}}, \quad x \geq 0$

$F(x) = 1 - e^{-x \cdot 10^{-4}}$



a) $P(X < 6000) = F(6000) = 1 - e^{-0,6} \approx 0,45 = p$

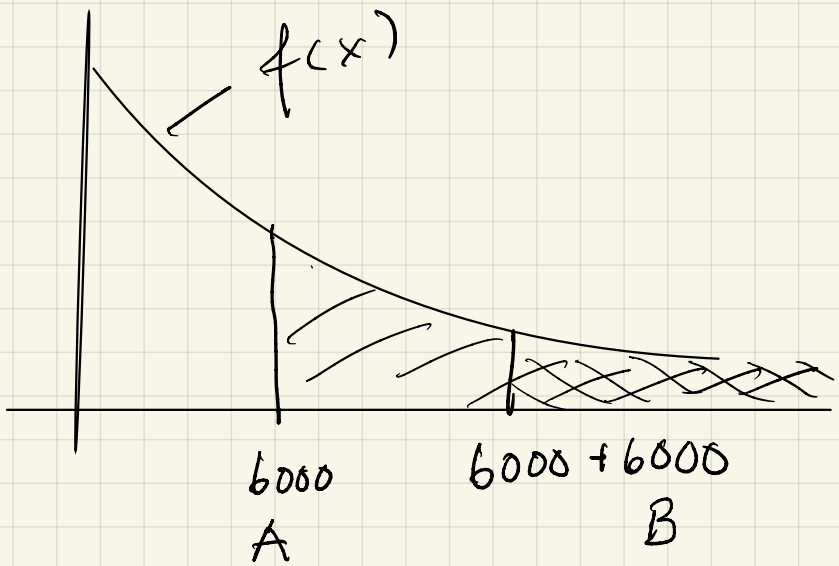
b) Y = antal transistorer med livslängd < 6000h

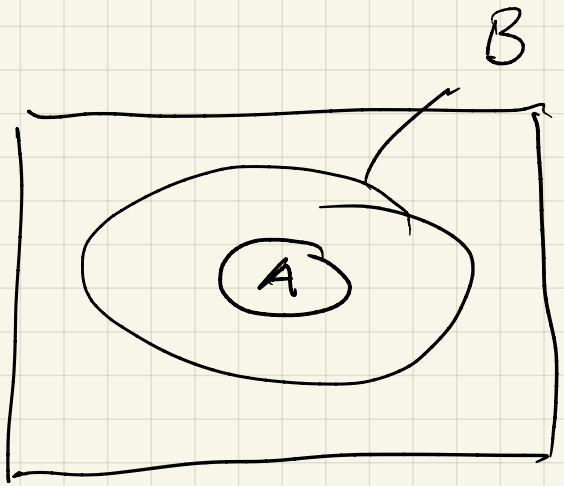
(oberoende upprepningar m. samma p)

$Y \in Bin(5; 0,45)$ $\quad P(y) = \binom{5}{y} 0,45^y 0,55^{5-y}, \quad y = 0, \boxed{1, 2, 3, 4, 5}$

$P(Y \geq 1) = 1 - P(Y = 0) = 1 - 0,55^5 \approx 0,95$

c*) Om vi vet att transistorn funkar efter 6000h. Hur stor är sannolikhet att den funkar efter ytterligare 6000 h?





$$P(X > 6000+6000 \mid X > 6000) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} =$$

$$\frac{e^{-(6000+6000)\cdot 10^{-4}}}{e^{-6000\cdot 10^{-4}}} = \frac{e^{-\frac{3}{5}} \cdot \cancel{e^{-\frac{3}{5}}}}{\cancel{e^{-\frac{3}{5}}}} = e^{-\frac{3}{5}} = P(X > 6000)$$

Anm 3.2 s. 61