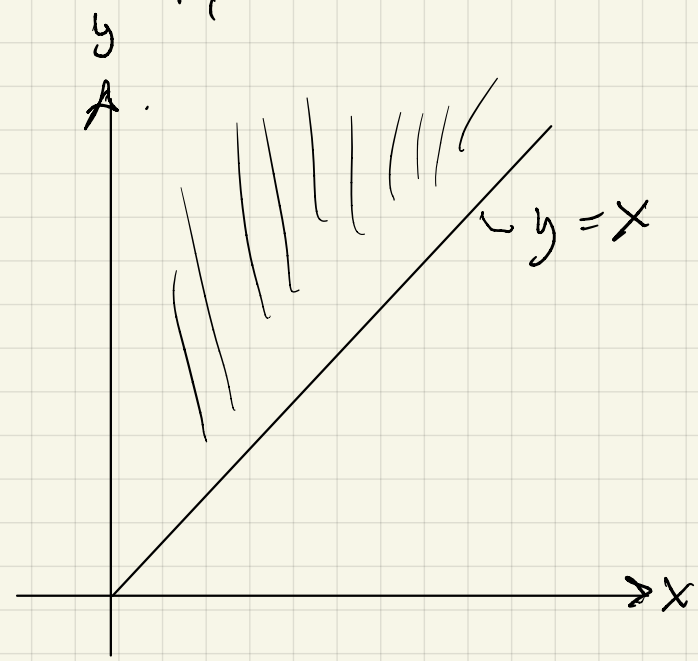
26/9 X = tid till napp för A, $X \in Exp(a)$, $f_X(x) = ae^{-ax}, x \geq 0$

4.12 Y = tid till napp för B, $Y \in Exp(b)$, $f_Y(y) = be^{-by}, y \geq 0$



X o Y är ober. stoch. var. $\Rightarrow$

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

$$P(A \text{ vinner}) = \underline{P(Y > X)} =$$

$$\int_0^\infty \int_x^\infty ae^{-ax} \cdot \underline{b}e^{-by}\, dy\, dx =$$

$$\int_0^\infty ae^{-ax} \left[-e^{-by} \right]_x^\infty dx = \int_0^\infty ae^{-ax} \cdot e^{-bx}\, dx$$

$$\int_0^\infty a\, e^{-(a+b)x}\, dx = \left[-\frac{a}{a+b} e^{-(a+b)x} \right]_0^\infty = \frac{a}{a+b}$$

$a = 2b$

$P(A \text{ vinner}) =$

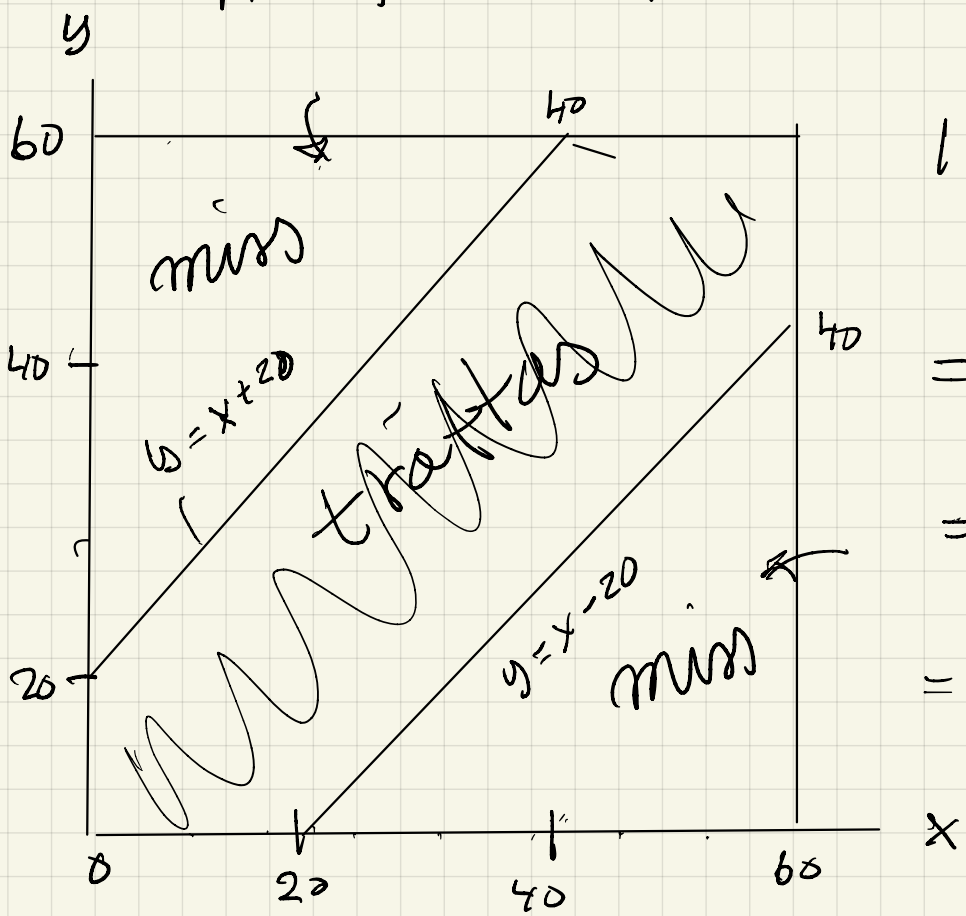
$$\frac{2b}{2b+b} = \frac{2}{3}$$

4.8 X = ankomst tid i min efter kl 12, $X \in U(0,60)$, $f_X(x) = \frac{1}{60}, 0<x<60$

Y = ankomst tid i min. efter kl 12, $Y \in U(0,6)$; $f_Y(y) = \frac{1}{60}, 0<y<60$

X & Y oberoende stok. var. $\Rightarrow f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) = \frac{1}{3600}$

$$P(\text{träffas}) = P(|Y-X|<20) = P(\underline{X-20<Y<X+20}) =$$



$Y > X+20$

$$1 - P(\text{inte träffas}) = 1 - \frac{2 \cdot \frac{40 \cdot 40}{2}}{3600} =$$

$$= 1 - \frac{4}{9} = \frac{5}{9}$$

$$= 1 - 2P(Y < X+20) =$$

$$= 1 - 2\int_0^{40}\int_{x+20}^{60} \frac{1}{3600}\, dy\, dx$$

KAP 4.6 Största o minsta värde

$$Z_+ = \max(X_1; X_2) = (X_1 \le z; X_2 \le z),$$

X_1 o X_2 ober stok. var $z \in \Omega_{Z_+}$

Bestäm fördelningsfunktion för Z_+

$$F_{Z_+}(z) = P(Z_+ \le z) = P(X_1 \le z; X_2 \le z)$$

$$\underset{\text{OBER}}{=} P(X_1 < z)\, P(X_2 \le z) = F_{X_1}(z) \cdot F_{X_2}(z)$$

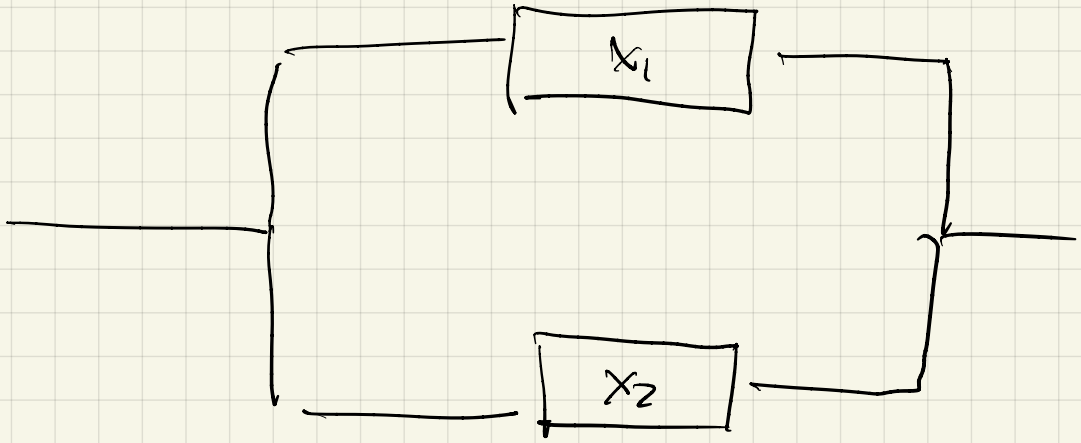
$$Z_- = \min(X_1; X_2) = (X_1 > z, X_2 > z)$$

$$F_{Z_-}(z) = P(Z_- \le z) = 1 - P(Z_- > z) =$$

$$= 1 - P(X_1 > z; X_2 > z) \underset{\text{OBER}}{=} 1 - P(X_1 > z)\, P(X_2 > z)$$

$$= 1 - (1 - F_{X_1}(z))(1 - F_{X_2}(z))$$

Min



MAX

4.16 $X_1, X_2, \ldots, X_n$ oberoende var. $f_{X_i}(x) = \lambda_i e^{-\lambda_i x}, \; x \geq 0$

$X_i \in Exp(\lambda_i), \; i = 1, \ldots, n$ $\qquad F_{X_i}(x) = 1 - e^{-\lambda_i x}, \; x \geq 0$

$P(X_i \leq x)$

Bestäm tätheten för

$Z_- = \min(X_1, \ldots, X_n)$ oberoende

$$F_{Z_-}(z) = P(Z_- \leq z) = 1 - P(Z_- > z) = 1 - P(X_1 > z)P(X_2 > z) \ldots P(X_n > z)$$

$$= 1 - e^{-\lambda_1 z} \cdot e^{-\lambda_2 z} \cdot \ldots \cdot e^{-\lambda_n z} = 1 - e^{-(\sum_1^n \lambda_i) \cdot z}$$

Sätt $\lambda = \sum_1^n \lambda_i$

$$F_{Z_-}(z) = 1 - e^{-\lambda z} \Rightarrow F'_{Z_-}(z) = \lambda e^{-\lambda z}, \; z \geq 0$$

$$Z_- \in Exp\left(\sum_1^n \lambda_i\right)$$

4.23 $Z = X + Y$ X o Y är ober med $f_X(x) = 2e^{-2x}, x \geq 0$

o. $f_Y(y) = 3e^{-3y}; y \geq 0$

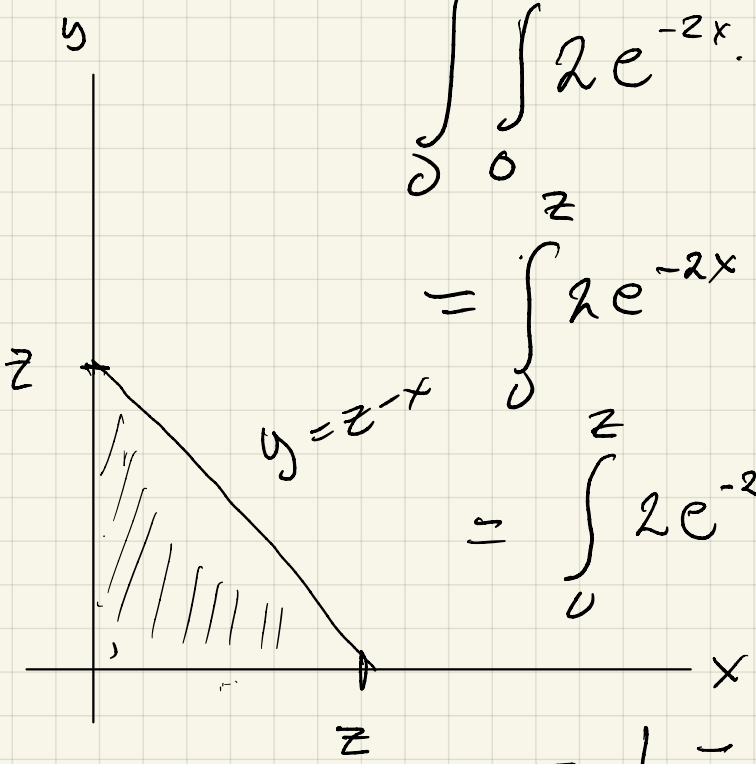
Bestäm $f_Z(z)$, $z \geq 0$

$$F_Z(z) = P(Z \leq z) = P(X + Y \leq z) = P(Y \leq z - X) =$$

$$\int_0^z \int_0^{z-x} 2e^{-2x} \cdot 3e^{-3y}\, dy\, dx = \int_0^z 2e^{-2x} \left[-e^{-3y}\right]_{y=0}^{y=z-x} dx$$

$$= \int_0^z 2e^{-2x} \left[1 - e^{-3(z-x)}\right] dx = \int_0^z 2e^{-2x} \left(1 - e^{-3z} \cdot e^{3x}\right) dx$$

$$= \int_0^z 2e^{-2x} - 2e^{-3z} e^{x}\, dx = \left[-e^{-2x} - 2e^{-3z} e^{x}\right]_{x=0}^{x=z}$$



$$= 1 - e^{-2z} - 2e^{-2z} + 2e^{-3z} = 1 - 3e^{-2z} + 2e^{-3z}$$

$$F'(z) = f_Z(z) = 6e^{-2z} - 6e^{-3z}, \quad z \geq 0$$

$Y = X_1 + X_2 + \ldots + X_n$, p Ber. stok. var

$$\overline{X} = \frac{1}{n} \sum_{1}^{n} X_i$$

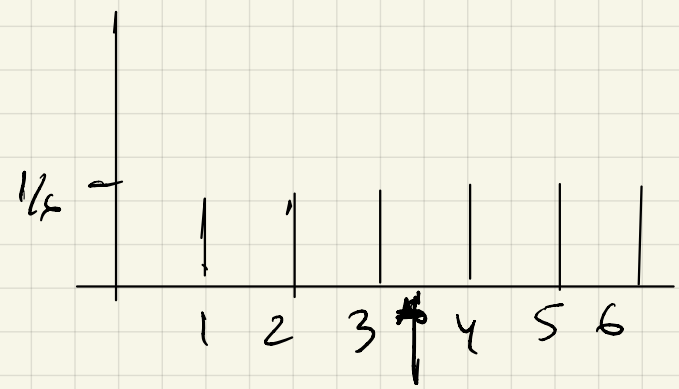
KAP 5 Vänte värde

Ex Slå en tärning en gång

X = antal prickar

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$P(x) = \frac{1}{6}, \quad x \in \Omega$



$\mu = 3{,}5$

Bestäm Väntevärdet $E(X)$

$$\mu = E(X) = \sum_{x=1}^{6} x \cdot \frac{1}{6} = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = \frac{7}{2} = 3{,}5$$

Ex $X \in Exp(\lambda)$, $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$ Bestäm $\mu = E(X)$

$$\mu = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^{\infty} x \cdot \lambda e^{-\lambda x}\,dx = \underbrace{\left[x \cdot (-e^{-\lambda x}\right]_0^{\infty}}_{=0} + \int e^{-\lambda x}\,dx$$

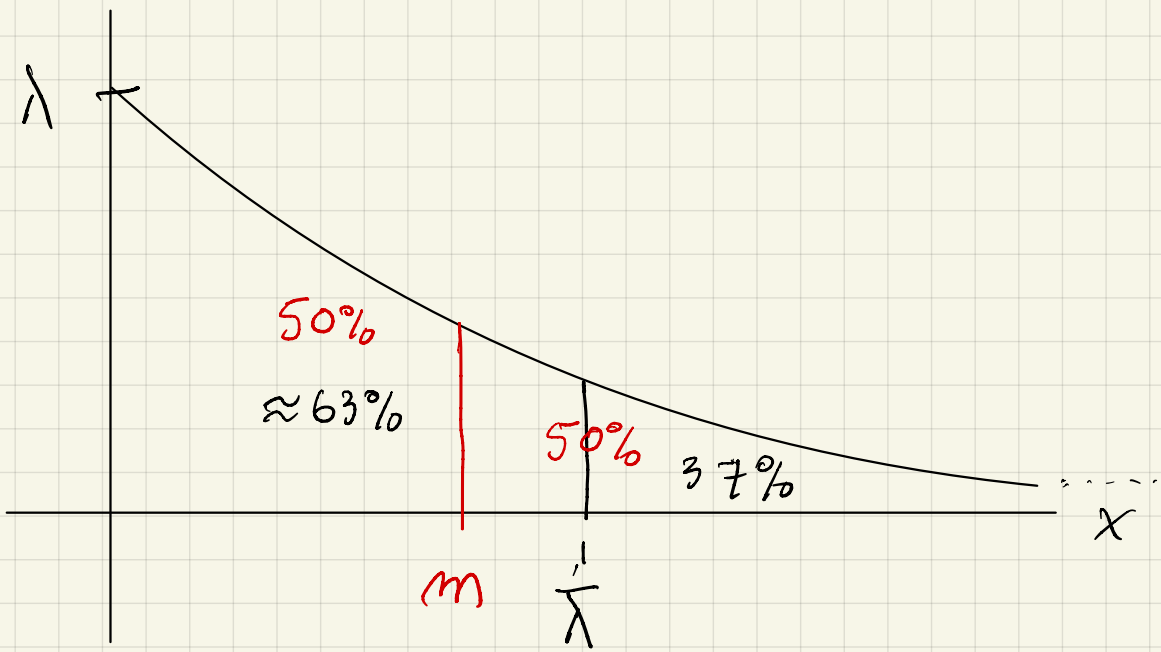
$$= \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^{\infty} = \frac{1}{\lambda}$$

$$\boxed{X \in Exp(\lambda)} \rightarrow E(X) = \frac{1}{\lambda}, \quad f(x) = \lambda e^{-\lambda x}$$

$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0$$

$$P\left(X \leq \frac{1}{\lambda}\right) = \int_0^{\frac{1}{\lambda}} \lambda e^{-\lambda x}\, dx = F\left(\frac{1}{\lambda}\right) = 1 - e^{-\lambda \cdot \frac{1}{\lambda}} = 1 - e^{-1}$$

$$\approx 0{,}63$$



$$V(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\,dx =$$

$$= \int_{-\infty}^{\infty} (x^2 - 2\mu x + \mu^2)\, f(x)\,dx =$$

$$= \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - 2\mu \underbrace{\int_{-\infty}^{\infty} x \cdot f(x)\,dx}_{\mu = E(X)} + \mu^2 \underbrace{\int_{-\infty}^{\infty} f(x)\,dx}_{=1} =$$

$$= \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - \mu^2 = \sigma^2$$

Bevis av sats 6, X kont. stok. var