Birla Institute of Technology and Science, Pilani
Work Integrated Learning Programmes Division
I Semester 2019-2020

Mathematical Foundations for Data Science
DSE CLZC416

Assignment 1

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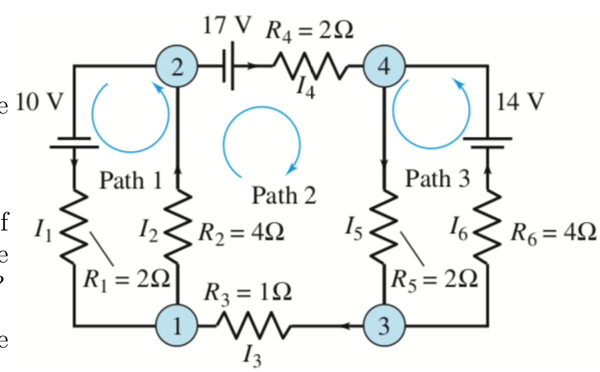
Marks: 5 x 2 = 10

1. Describe the set of vectors b, for $Ax = b$ to be consistent in each of the cases
   A. A is diagonal non-zero matrix
   B. $A = [B \;\; I_n]$ where $B \in R^{n\times n}$
   C. $A = [I_n \;\; 0_n]^\top$ where $0_n \in R^{n\times n}$ is the zero matrix
   D. $A = [0_n \;\; I_n]$ where $0_n \in R^{n\times n}$ is the zero matrix

2. Refer to the figure below and answer the following questions.
   A. How many equations do Kirchoff's laws, both for voltages and currents, yield?
   B. Do the equations in (A) above directly yield a unique solution for the currents I_1, I_2, I_3, I_4, I_5, I_6 ?
   C. While principles in Physics tell us that the values of the currents need to be unique, how do we rewrite the equations to get the values of I_1, I_2, I_3, I_4, I_5 and I_6?
   D. Please use a software package (like Octave) to solve the resulting reduced system.



3. Let V be a subspace of R^m. Suppose that $S = \{v_1, v_2 ... v_m\}$ is a basis for V, then prove that any set of $m + 1$ or more vectors in V is linearly dependent. Construct examples for $m = 2$ and understand the statement first before formally proving the same.

4. Find a basis for the following subspaces of $\mathbb{R}^4$
   A. Vectors for which $x_1 = 2x_4$
   B. Vectors for which $x_1 + x_2 + x_3 = 0$ and $x_3 + x_4 = 0$
   C. Subspace spanned by $[1\ 1\ 1\ 1]^\top$, $[1\ 2\ 3\ 4]^\top$, $[2\ 3\ 4\ 5]^\top$

5. Show that a 5×7 matrix A must have $2 \leq \text{nullity}(A) \leq 7$. Give an example of a 5×7 matrix A with $\text{nullity}(A) = 2$ and an example of a 5×7 matrix A with $\text{nullity}(A) = 7$.