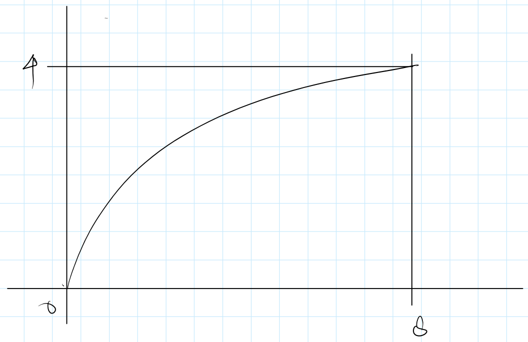
$y = x^{\frac{2}{3}}$



$f(x) = x^{\frac{2}{3}}$

$f'(x) = \frac{2}{3x^{\frac{1}{3}}}$

$$\int_0^8 \sqrt{1+[f'(x)]^2}\,dx$$

$$\lim_{a\to 0^+}\int_a^8 \sqrt{1+\frac{4}{9x^{\frac{2}{3}}}}\,dx$$

$$\lim_{a\to 0^+}\int_a^8 \frac{1}{3x^{\frac{1}{3}}}\sqrt{4+9x^{\frac{2}{3}}}\,dx$$

misal $4+9x^{\frac{2}{3}} = u$

$$\frac{du}{dx} = \frac{9\cdot 2}{3\cdot x^{\frac{1}{3}}}$$

$$\lim_{a\to 0^+}\int_a^8 u^{\frac{1}{2}}\,\frac{du}{18}$$

$$\frac{1}{18}\cdot\lim_{a\to 0^+}\left(\frac{2}{3}x^{\frac{3}{2}}\Big|_a^8\right)$$

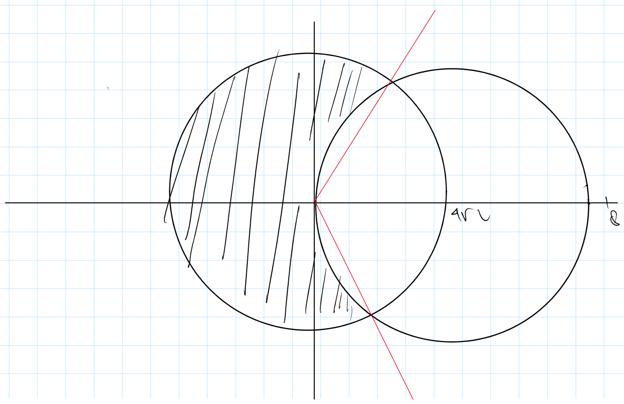
$$\frac{1}{18}\lim_{a\to 0^+}\left(\frac{2}{3}\left(4+9x^{\frac{2}{3}}\right)^{\frac{3}{2}}\Big|_a^8\right)$$

$$\frac{1}{27}\cdot\lim_{a\to 0^+}\left(40-4\right)$$

$$\frac{4}{3}$$

2. 2. Seekor kambing berada pada kandang yang dibatasi oleh kurva $r = 4\sqrt{2}$ tetapi di luar kurva $r = 8\cos\theta$. Tentukan luas kandang kambing tersebut.

Jawab :



3

1, Tentukan luas yg di batasi oleh kurva $y = x^2$ dan $y = x + 2$ serta sketlah grafiknya

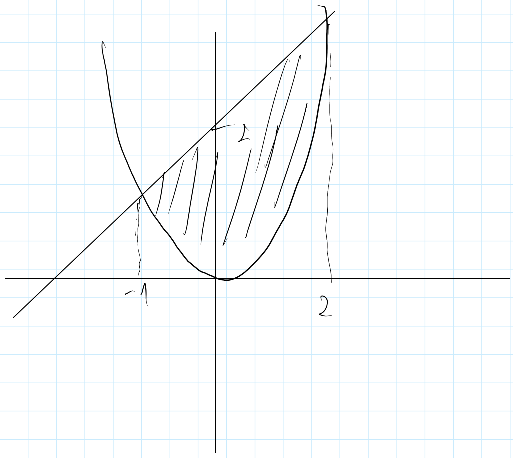
$y = x^2$; $y = x + 2$

titik potong kurva & garis

$x^2 = x + 2$

$(x-2)(x+1) = 0$

$x = 2 \lor x = -1$



$L = \int_{-1}^{2} (f_1 - f_2)\,dx$

$L = \int_{-1}^{2} \left[(x+2) - (x^2) \right] dx$

$= \int_{-1}^{2} \left[-x^2 + x + 2 \right] dx$

$-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_{-1}^{2}$

$\left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$

$\left(-3 + 4 - \frac{1}{2}\right)$

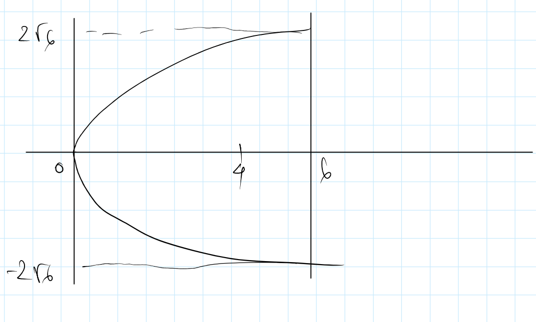
$= \frac{1}{2}$ //

4

2. Dapat titik berat yg terjadi jika bidang datary g dibatsi oleh $y^2 = 4x$ dan $x =$ 6 di putar pada sbb x

$y^2 = 4x$; $x = 6$ diputar terhadap sumbu X

$\bar{x} = \frac{\int_0^6 x y^2\,dx}{\int_0^6 y^2\,dx}$



$\bar{x} = \frac{\int_0^6 x(4x)\,dx}{\quad}$

6

4. Tentukan luas daerah di dalam lingkaran $r = 5\sin\theta$ dan di luar limacon $r = 1 + \sin\theta$, serta sketlah grafiknya

$r = 5\sin\theta \quad ; \quad r = 1 + \sin\theta$

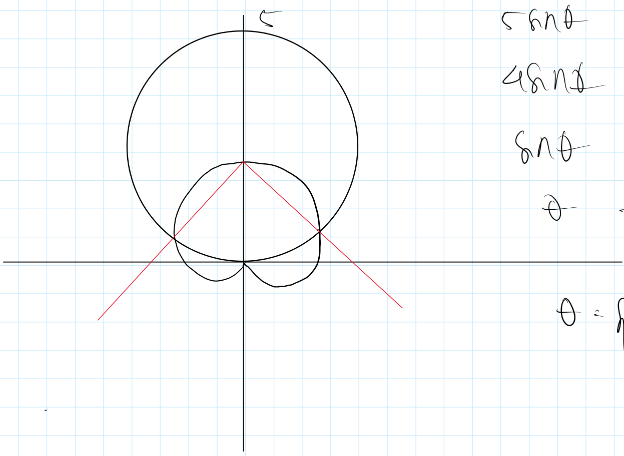
titik potong :

$5\sin\theta = 1 + \sin\theta$

$4\sin\theta = 1$

$\sin\theta = \frac{1}{4}$

$\theta = \arcsin\left(\frac{1}{4}\right)$



$\theta = \left\{ \arcsin\left(\frac{1}{4}\right) ; 180 - \arcsin\left(\frac{1}{4}\right) \right\}$

$$L = \int_{\arcsin(\frac{1}{4})}^{\pi/2} \frac{1}{2}(5\sin\theta)^2 \, d\theta - \int_{\arcsin(\frac{1}{4})}^{\pi/2} \frac{1}{2}(1+\sin\theta)^2 \, d\theta$$

$$\int_{\arcsin(\frac{1}{4})}^{\pi/2} 25\sin^2\theta \, d\theta - \int_{\arcsin(\frac{1}{4})}^{\pi/2} (1 + \sin^2\theta + 2\sin\theta) \, d\theta$$

$$25 \int_{\arcsin(\frac{1}{4})}^{\pi/2} \sin^2\theta \, d\theta - \int_{\arcsin(\frac{1}{4})}^{\pi/2} 1 \, d\theta - \int_{\arcsin(\frac{1}{4})}^{\pi/2} \sin^2\theta \, d\theta - 2\int_{\arcsin(\frac{1}{4})}^{\pi/2} \sin\theta \, d\theta$$

$$= 24 \int_{\arcsin(\frac{1}{4})}^{\pi/2} \sin^2\theta - \int_{\arcsin(\frac{1}{4})}^{\pi/4} 1 \, d\theta - 2\int_{\arcsin(\frac{1}{4})}^{\pi/4} \sin\theta \, d\theta$$

$$= 24\left(\frac{1}{2}\theta - \frac{\sin(2\theta)}{4}\right)\Bigg|_{\arcsin(\frac{1}{4})}^{\pi/2} - \theta\Bigg|_{\arcsin(\frac{1}{4})}^{\pi/2} + 2\cos\theta\Bigg|_{\arcsin(\frac{1}{4})}^{\pi/2}$$

$$= 12\left(\frac{\pi}{2} - \frac{\sin(0)}{2} - \frac{1}{2}\left(\arcsin\left(\frac{1}{4}\right)\right) + \sin\left(\arcsin\left(\frac{1}{4}\right)\right) \cdot \cos\left(\arcsin\left(\frac{1}{4}\right)\right)\right)$$

$$- \left(\frac{\pi}{2} - \arcsin\left(\frac{1}{4}\right)\right)$$

$$+ 2\cos\left(\frac{\pi}{2}\right) - 2\cos\left(\arcsin\left(\frac{1}{4}\right)\right)$$

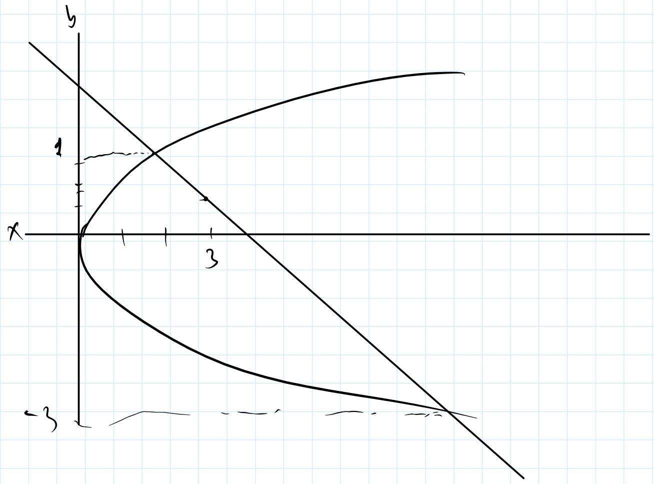
$$12\left(\frac{\pi}{2} - \frac{1}{2}\arcsin\left(\frac{1}{4}\right) + \frac{1}{4}\left(\sqrt{1 - \left(\frac{1}{4}\right)^2}\right)\right)$$

$$- \frac{\pi}{2} + \arcsin\left(\frac{1}{4}\right)$$

$$2(0) - 2\left(\sqrt{1 - \left(\frac{1}{4}\right)^2}\right)$$

$$\Rightarrow 6\pi - 6\arcsin\left(\frac{1}{4}\right) + \frac{3}{4}\sqrt{15} - \frac{\pi}{2} + \arcsin\left(\frac{1}{4}\right) - \frac{2}{4}\sqrt{15}$$

7\. 1. Dapatkan luas daerah yang dibatasi oleh $x = y^2$ dan $2y + x = 3$.



tiltik potong

$y^2 = 3 - 2y$

$y^2 + 2y - 3 = 0$

$(x+3)(x-1)$

$x = -3 \lor x = 1$

$L = \int_{-3}^{1} 3 - 2y - y^2 \, dy$

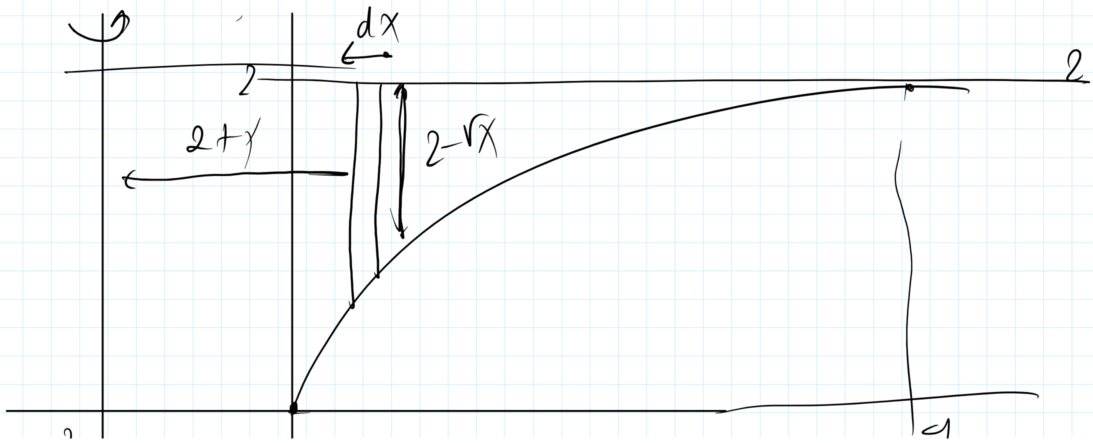
$= 3y - y^2 - \frac{y^3}{3} \Big|_{-3}^{1}$

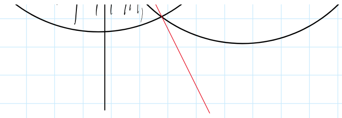
$= \left(3 - 1 - \frac{1}{3}\right) - \left(-9 \; \cancel{-9 + 9}\right)$

$= 11 - \frac{1}{3}$

$= \frac{32}{3}$

8\. 2. Gambarkan daerah yang dibatasi oleh kurva-kurva $y = \sqrt{x}$, $y = 2$ dan $x = 0$, kemudian dapatkan volume benda putar jika daerah tersebut diputar pada garis $x = -2$.





titik potong :

$4\sqrt{2} = 8\cos\theta$

$\cos\theta = \frac{1}{2}\sqrt{2}$

$\theta = \{\pi/4 , -\pi/4\}$

$L = L_1 - L_2$

$= \int_{\pi/4}^{\pi} \frac{1}{2}(4\sqrt{2})^2\, d\theta - \int_{\pi/4}^{\pi/2} \frac{1}{2}(8\cos\theta)^2\, d\theta$

$= 16\theta \Big|_{\pi/4}^{\pi} - 32\int_{\pi/4}^{\pi/2} \frac{\cos 2\theta + 1}{2}\, d\theta$

$= 16\theta \Big|_{\pi/4}^{\pi} - 16\left(\frac{\sin(2\theta)}{2} + \theta \Big|_{\pi/4}^{\pi/2}\right)$

$= 12\pi - 16\left(\frac{\sin(0)}{2} + \frac{\pi}{2} - \left(\frac{\sin(90)}{2} + \frac{\pi}{4}\right)\right)$

$= 12\pi - 16\left(\frac{\pi}{2} - \frac{1}{2} - \frac{\pi}{4}\right)$

$= 12\pi - 16\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= 8\pi + 8$

Karena simetri :

$16\pi + 16$ //

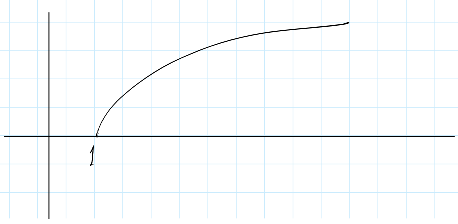
9

3. Diberikan persamaan parametrik $x = t^2 + 1$, $y = t$, $0 \le t \le 5$.

(a) Buatlah sketsa kurva tersebut dengan mengeliminasi parameter t

(b) Dapatkan persamaan garis singgung dari persamaan parametrik tersebut saat $t = \frac{1}{2}$.

a. $x = y^2 + 1$



b. $\frac{dx}{dt} = 2t$; $\frac{dy}{dt} = 1$

$m = \frac{dy}{dx} = \frac{1}{2t}$

$t = 1 \Rightarrow m = 1$

$$\bar{x} = \frac{\int_0^6 x(4x)\,dx}{\int_0^6 4x\,dx}$$

$$= \frac{\frac{4}{3}x^3 \Big|_0^6}{2x^2 \Big|_0^6} \qquad \frac{4\cdot\cancel{6}\cdot\cancel{6}\cdot\cancel{6}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{6}\cdot\cancel{6}} = 4$$

5. **3.** ***Dapatkan persamaan garis singgung pada kurva x = 2t+ 4, y =*** $t^2 - 2t + 4$ ***pada t = 1***

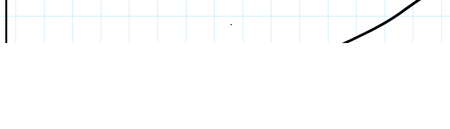
$$\frac{dx}{dt} = 2 \quad ; \quad \frac{dy}{dt} = 2t-2$$

$$m = \frac{dy}{dx} = \frac{2t-2}{2} = t-1$$

$$m\Big|_{t=1} = (1)-1$$
$$= 0 \,//$$

---

12 1. Dapatkan luas daerah yang dibatasi oleh $y = x^2 - 4x + 3$ dan $y = x + 3$.



titik potong

$$x+\cancel{3} = x^2 - 4x + \cancel{3}$$

$= \frac{11}{2}\pi - 5 \arcsin\left(\frac{1}{a}\right) - \frac{1}{a}\sqrt{15}$

6

5.dapatkan deret Taylor dari $f(x) = \frac{1}{x+2}$ disekitar $x = 3$

$f(x) = \frac{1}{x+2}$ disekitar $x = 3$

$$\boxed{\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k}$$

$f^{(0)} = \frac{1}{x+2} \longrightarrow f^{(0)}(3) = \frac{1}{5}$

$f^{(1)} = -\frac{1}{(x+2)^2} \longrightarrow f^{(1)}(3) = -\frac{1}{25}$

$f^{(2)} = 2\frac{1}{(x+2)^3} \longrightarrow f^{(2)}(3) = \frac{2}{125}$

$f^{(3)} = -6\frac{1}{(x+2)^4} \longrightarrow f^{(3)}(3) = \frac{-6}{625}$

$f^{(4)} = 24\frac{1}{(x+2)^5} \longrightarrow f^{(4)}(3) = \frac{24}{3125}$

k=0 k=1 k=2 k=3 k=4

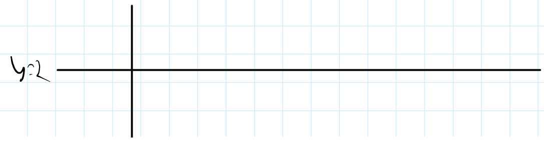
$\frac{1}{5} - \frac{1}{25} + \frac{2}{125} - \frac{6}{625} + \frac{24}{3125} \cdots$

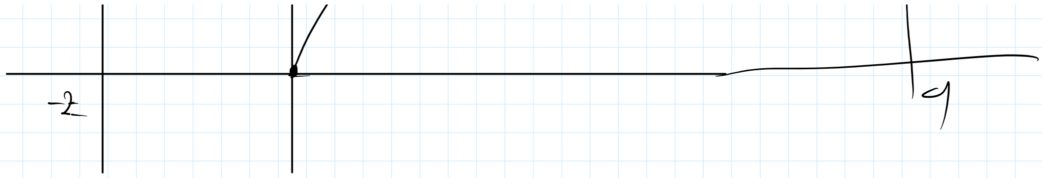
$$\sum_{k=0}^{\infty} \frac{(-1)^k k!}{(5)^{k+1}} \cdot \frac{(x-3)^k}{k!}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-3)^k}{(5)^{k+1}} =$$

14.

1. Dapatkan luas daerah yang dibatasi oleh ...
2. Gambarkan daerah yang dibatasi oleh kurva-kurva $y = 2x - x^2$ dan $y = x^2 - 2x$. Menggunakan Dalil Guldin I, dapatkan volume benda padat jika daerah tersebut diputar terhadap garis $y = 2$.





$$V = \int_0^9 2\pi (x+2)(2-\sqrt{x})\,dx$$

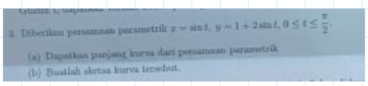
$$= 2\pi \int_0^9 \left(2x - x^{\frac{3}{2}} + 4 - 2x^{\frac{1}{2}}\right) dx$$

$$= 2\pi \left(x^2 - \frac{2}{5}x^{\frac{5}{2}} + 4x - \frac{2}{3}x^{\frac{3}{2}} \Big|_0^4 \right)$$

$$= 2\pi \left(16 - \frac{2}{5}\,16\sqrt{4} + 4(4) - \frac{2}{3}\,4\sqrt{4} \right)$$

$$= 2\pi \left(16 - \frac{2}{5}\cdot 16 \cdot 2 + 16 - \frac{2}{3}\,4 \cdot 2 \right)$$

$$= 2\pi \left(32 - \frac{64}{5} - \frac{16}{3} \right) //$$

15. 
3. Diberikan persamaan parametrik $x = \sin t$, $y = 1 + 2\sin t$, $0 \le t \le \frac{\pi}{2}$.

(a) Dapatkan panjang kurva dari persamaan parametrik

(b) Buatlah sketsa kurva tersebut.

$x = \sin t$; $y = 1 + 2\sin t$; $0 \le t \le \pi/2$

$$s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \sqrt{5} \left(\sin t \Big|_0^{\pi/2} \right)$$

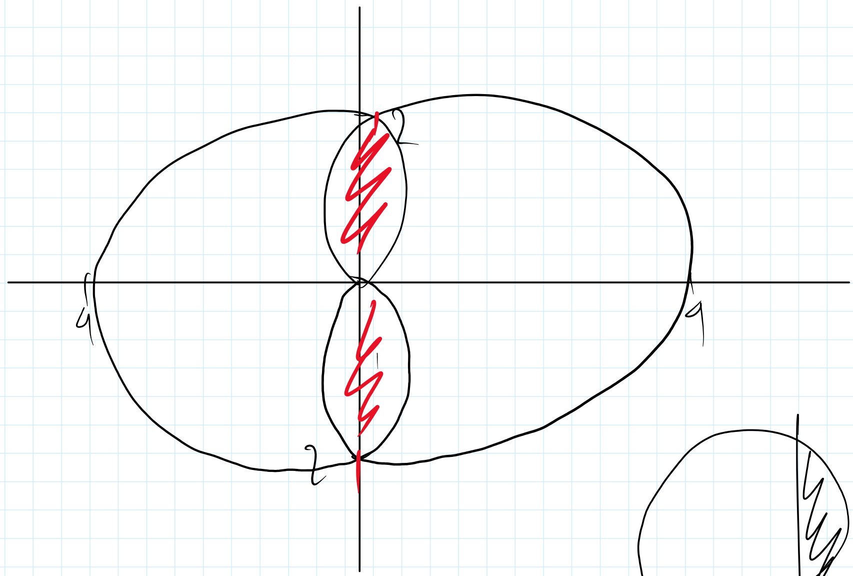
$$= \sqrt{5} //$$

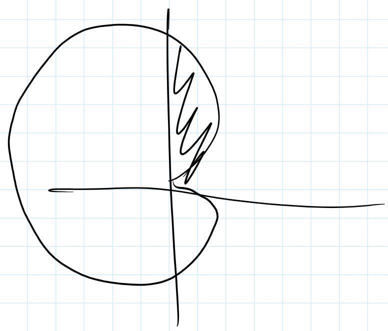
$\frac{dx}{dt} = \cos t$; $\frac{dy}{dt} = 2\cos t$

$t=1 \Rightarrow m = 1$

10. 4. Dapatkan luas daerah dari irisan kardioida $r = 2 - 2\cos\theta$ dan kardioida $r = 2 + 2\cos\theta$.

$r = 2 - 2.\cos\theta$; $r = 2 + 2\cos\theta$





$L = L_1$

$= \int_0^{\pi/2} \frac{1}{2}(2 - 2\cos\theta)^2 \, d\theta$

$= \int_0^{\pi/2} \frac{1}{2}(4 + 4\cos^2\theta - 8\cos\theta) \, d\theta$

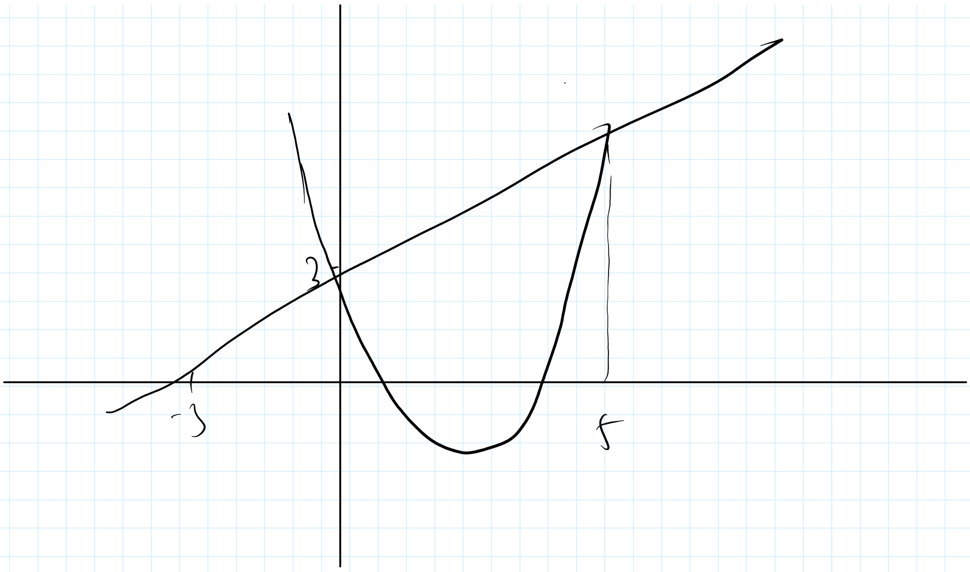
$= \int_0^{\pi/2} 2 + 2\cos^2\theta - 4\cos\theta \, d\theta$

$= 2\theta + \theta + \frac{\sin(2\theta)}{2} - 4\sin\theta \Big|_0^{\pi/2}$

$= \pi + \pi/2 + 0 - 4(1)$

$= \frac{3}{2}\pi - 4$

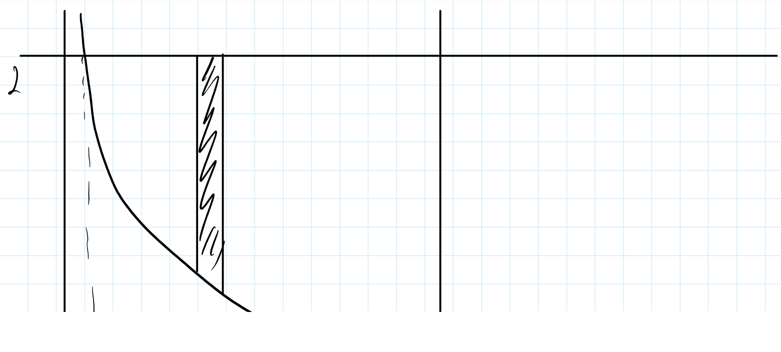
dengan menggunakan kesimetrisan bentuk dapat diperoleh :

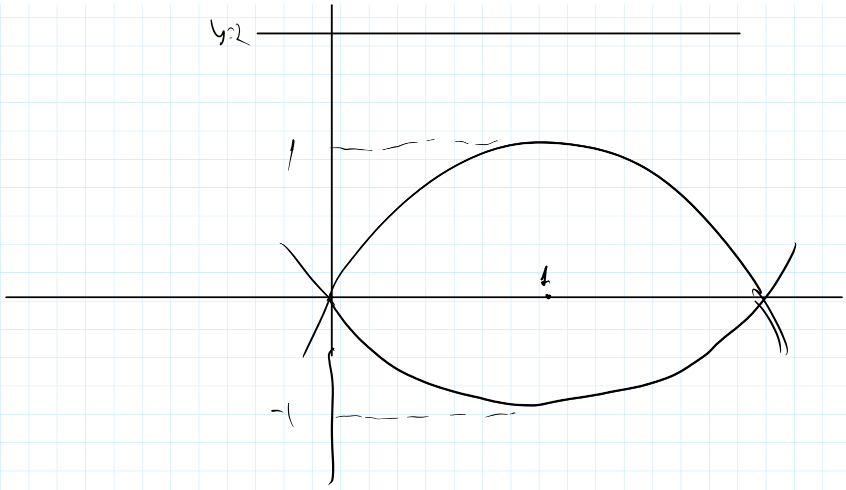


$$\cancel{x+3} = x^2 - 4x + \cancel{3}$$
$$x^2 - 5x = 0$$
$$x = 0 \vee x = 5$$

$$L = \int_0^5 (x+3 - x^2 + 4x - 3)\,dx$$
$$= \int_0^5 (-x^2 + 5x)\,dx$$
$$= \left(-\frac{1}{3}x^3 + \frac{5}{2}x^2 \Big|_0^5\right)$$
$$= \frac{-125}{3} + \frac{5 \cdot 5 \cdot 5}{2}$$
$$= \frac{125}{6}$$

13. 2. Dapatkan volume benda putar jika daerah yang dibatasi oleh kurva-kurva $y = \dfrac{1}{x}$, $x = 2$ dan $y = 2$ diputar terhadap sumbu-x. Buatlah sketsa daerah tersebut.





$$\bar{x} = \frac{\int_0^2 x(y_1-y_2)\,dx}{\int_0^2 (y_1-y_2)\,dx}$$

$$= \frac{\int_0^2 x(2x-x^2-x^2+2x)\,dx}{\int_0^2 2x-x^2-x^2+2x\,dx}$$

$$= \frac{\int_0^2 -2x^3+4x^2\,dx}{\int_0^2 -2x^2+4x\,dx}$$

$$= \frac{-\frac{1}{2}x^4 + \frac{4}{3}x^3 \Big|_0^2}{-\frac{2}{3}x^3 + 2x^2 \Big|_0^2} = \frac{-8 + \frac{32}{3}}{-\frac{16}{3} + 8}$$

$$= \frac{-24+32}{-16+24} = \frac{8}{8} = 1$$

$$\bar{y} = \frac{\int_0^2 y_1^2 - y_2^2\,dx}{\int_0^2 y_1-y_2\,dx}$$

$$= 0$$

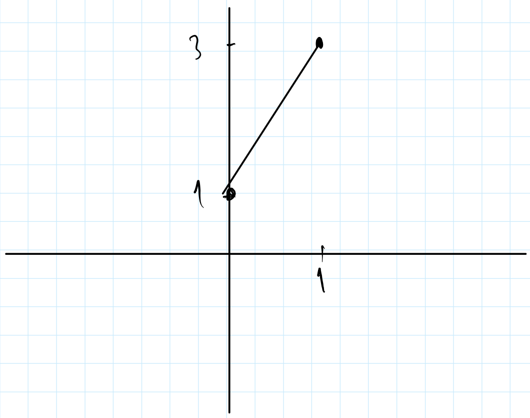
$= \sqrt{5}$ //

$\frac{dx}{dt} = \cos t \quad ; \quad \frac{dy}{dt} = 2\cos t$

$$\int_0^{\pi/2} \sqrt{\cos^2 t + 4\cos^2 t} \; dt$$

$$\int_0^{\pi/2} \cos t \sqrt{5} \; dt$$

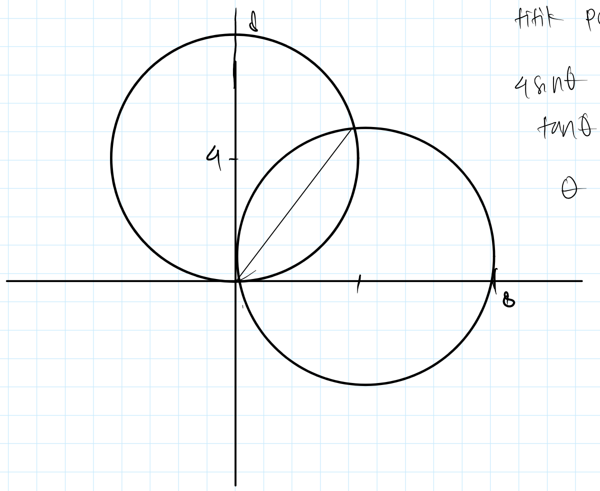
$$\sqrt{5} \int_0^{\pi/2} \cos t \; dt$$



16. (b) Buatlah sketsa kurva ...

4. Dapatkan luas daerah yang berada di dalam lingkaran $r = 4\sin\theta$ dan di luar lingkaran $r = 4\cos\theta$.

$r = 4 \sin\theta \quad ; \quad r = 4 \cos\theta$



titik potong

$4 \sin\theta = 4 \cos\theta$

$\tan\theta = 1$

$\theta = \pi/4$

$$L = \int_{\pi/4}^{\pi} \tfrac{1}{2}(4 \sin\theta)^2 d\theta - \int_{\pi/4}^{\pi/2} \tfrac{1}{2}(4 \cos^2\theta) \, dt$$

$$= 8 \int_{\pi/4}^{\pi} \sin^2\theta \, d\theta - 8 \int_{\pi/4}^{\pi/2} \cos^2\theta \, d\theta$$

$$= 8\left(\int_{\pi/4}^{\pi} \tfrac{1}{2}(1 - \cos 2\theta) \, d\theta \right) - 8 \int_{\pi/4}^{\pi/2} \tfrac{1}{2}(1 + \cos 2\theta) \, d\theta$$

$$= 4\left(\theta - \frac{\sin(2\theta)}{2} \Big|_{\pi/4}^{\pi} \right) - 4\left(\theta + \frac{\sin(2\theta)}{2} \Big|_{\pi/4}^{\pi/2} \right)$$

dengan menggunakan kesimetrisan bentuk dapat diperoleh :

$$L_{Total} = 4L$$
$$= 4\left(\frac{3}{2}\pi - 4\right)$$
$$= 6\pi - 16$$

11\. 5. Dapatkan lima suku pertama polinomial Maclaurin untuk fungsi $f(x) = e^{-x^2}$.

$$f^{(0)} = e^{-x^2}$$

$$f^{(1)} = e^{-x^2}(-2x)$$

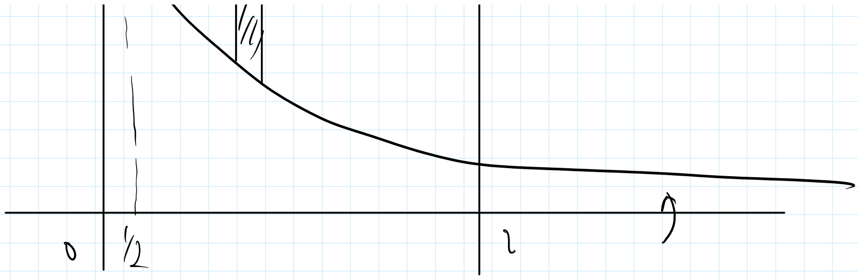
$$f^{(2)} = 4x^2 e^{-x^2} - 2e^{-x^2}$$

$$f^{(3)} = 8x e^{-x^2} - 8x^3 e^{-x^2} + 4x e^{-x^2} = 12x e^{-x^2} - 8x^3 e^{-x^2}$$

$$f^{(4)} = 8e^{-x^2} - 16x^2 e^{-x^2} - 24x^2 e^{-x^2} + 16x^4 e^{-x^2} + 4e^{-x^2} - 8x^2 e^{-x^2}$$
$$= 12e^{-x^2} - 48x^2 e^{-x^2} + 16x^4 e^{-x^2}$$

$$f^{(5)} = -16x e^{-x^2} - 32x e^{-x^2} + 32x^3 e^{-x^2} - 24x e^{-x^2} + 48x^3 e^{-x^2} +$$
$$64x^3 e^{-x^2} - 32x^5 e^{-x^2} - 8x e^{-x^2} - 16x e^{-x^2} + 16x^3 e^{-x^2}$$

$$= -96x e^{-x^2} + 160x^3 e^{-x^2} - 32x^5 e^{-x^2}$$



$$\pi \int_{1/2}^{2} 4 - \frac{1}{x^2} \, dx$$

$$\pi \left(4x \Big|_{1/2}^{2} - \left(-\frac{1}{x} \Big|_{1/2}^{2} \right) \right)$$

$$\pi \left(6 - \left(-\frac{1}{2} + 2 \right) \right)$$

$$\pi \left(6 - \frac{3}{2} \right)$$

$$\frac{9}{2} \pi$$

$= 0$ //

titik pusat = $\{1, 0\}$

$\bar{d} = 2$

$$L = \int_0^2 (2x - x^2) - (x^2 - 2x)\,dx$$

$$= \int_0^2 4x - 2x^2\,dx$$

$$= 2x^2 - \frac{2}{3}x^3 \Big|_0^2$$

$$= 2(4) - \frac{2}{3}8$$

$$= \frac{8}{3}$$

$$V = 2\pi d L$$

$$= 2\pi(2)\frac{8}{3}$$

$$= \frac{32\pi}{3}$$ //

$$= 4\left(\theta - \frac{\sin(2\theta)}{2}\Big|_{\pi/4}^{\pi}\right) - 4\left(\theta + \frac{\sin(2\theta)}{2}\Big|_{\pi/4}^{\pi/2}\right)$$

$$= 4\left(\left(\pi - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right) - 4\left(\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} + \frac{1}{2}\right)\right)$$

$$4\left(\frac{3}{4}\pi + \frac{1}{2}\right) - 4\left(\frac{1}{4}\pi + \frac{1}{2}\right)$$

$$3\pi + 2 - \pi + 2$$

$$2\pi + 4$$