$j = \gcd(a, b)$

$i = \mathrm{lcm}(a, b)$

$ak \equiv bl \rightarrow$ kpk

$O\log(N)$ $\quad$ $O\log(N)$

$\cancel{\gcd(a, b) \times \mathrm{lcm}(a, b)} =$ $\qquad$ $\dfrac{\cancel{\gcd(a, b)} \times \mathrm{lcm}}{ab}$

$O(\log N)$ $\quad$ $O(\log N)$

$ij = \dfrac{ij}{ab}$

$O(\log N) \longrightarrow O(1)$

$ab - 1$

$$\frac{a \times b}{\gcd(a,b)} = \operatorname{lcm}(a,b)$$

$$\operatorname{lcm}(a,b) \times \gcd(a,b) = ab$$

$$ij = ab \rightarrow ij - \frac{ij}{ab} = ab - \frac{ab}{ab}$$

$$= ab - 1$$

Ketinggian = N

Jml balok M

$$M = \frac{N(N+1)}{2}$$

$$\frac{N(N+1)}{2} \le M$$

$N \quad A_N$

$\vdots$

4

3

$2 \quad A_2$

$1 \quad A_1$

Jumlah balok

$$1+2+3+\dots+N = \frac{N(N+1)}{2}$$

$$i < j$$

$$A_i > A_j$$

Supaya tinggi maksimal

A_i minimal $\rightarrow$ $A_i = A_j + 1$

$$N(N+1) \le 2M$$

$N^2 + N \le 2M \rightarrow$ ① lower bound $O(\log N)$

② For loop $O(N)$

$M = 1000 \rightarrow \quad N = \lfloor \sqrt{2M} \rfloor \quad \underline{\underline{N = 44}}$

$N = \lfloor \sqrt{2000} \rfloor$

$N = 10 \rightarrow \quad M = \frac{N \times (N+1)}{2} = \frac{110}{2} = 55$

$10 = \lfloor \sqrt{2.55} \rfloor$

$10 = 10$

$\sqrt{7} = 2$

$\lfloor \sqrt{x} \rfloor =$

$O(1) \rightarrow \lfloor \sqrt{2M} \rfloor$

ganjil $\rightarrow \frac{i+1}{2}$, genap $\rightarrow \frac{N+1-i}{2}$

$N = 7$

$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$
1	7	2	6	3	5	4

$L \rightarrow R$ Total Panjang

$1 \rightarrow 2 = 1+7 = 8$

$1 \rightarrow 3 = 1+7+2 = 10$

$5 \rightarrow 7 = 3+5+4 = 12$

$f(L,R)$ = hasil jumlah panjang lakuan dari $L \to R$

* Naive = brute force → setiap query : $O(N)$
comp : $O(QN)$

$$f(L,R) = \begin{cases} F_{odd}(L,R) & \text{Jumlah semua panjang lakuan pada index i ganjil} \\ F_{even}(L,R) & \text{index i genap} \end{cases}$$

$N = 7 \to$ 1 7 2 6 3 5 4 → F_{ganjil}

$$F_{ganjil} = \frac{N \times (N+1)}{2}, \quad F_{genap} = \sum_{\substack{i=2 \\ i \text{ even}}}^{N} \frac{N-i}{2}$$

$N = 7$ $\qquad$ $\cancel{ceil(\frac{7}{2})+1}$ $\qquad$ $N = 8$ $\qquad$ $\cancel{ceil(\frac{8}{2})+1}$ $\qquad$ $N = 9$ $\qquad$ $ceil(\frac{9}{2})+1$

genap : $7+6+5 \rightarrow A_{N-1}$ $\qquad$ $8+7+6+5$ A_N $\qquad$ $9+8+7+6$

$$F_{genap} = 1+2+3+\dots+N - (1+2+3+\dots+N/2)$$

$$= \frac{N \times (N+1)}{2} - \left|\frac{(N-1)(N)}{2}\right|$$

$F(L, R)$

$F(\overset{O}{1}, \overset{E}{8}) \rightarrow$ ganjil : $\overset{L}{1} \rightarrow \overset{R-1}{7}$ R

genap : $\overset{L+1}{2} \rightarrow 8$

$F(\overset{E}{2}, \overset{O}{7}) \rightarrow$ ganjil : $\overset{L+1}{3} \rightarrow \overset{R}{7}$

genap : $\overset{L}{2} \rightarrow \overset{R-1}{6}$

$F(\overset{E}{2}, \overset{E}{8}) \rightarrow$ ganjil : $\overset{L+1}{3} \rightarrow \overset{R-1}{7}$

genap : $\overset{2}{2} \rightarrow \overset{R}{8}$

$F(\overset{O}{3}, \overset{O}{7}) \rightarrow$ ganjil : $\overset{2}{3} \rightarrow \overset{R}{7}$

genap : $\overset{L+1}{4} \rightarrow \overset{R-1}{6}$

$F(L, R)$

L	R	
ganjil	genap	$\rightarrow F_{odd}(L, R-1) + F_{even}(L+1, R)$
genap	ganjil	$\rightarrow F_{odd}(L+1, R) + F_{even}(L, R-1)$
ganjil	ganjil	$\rightarrow F_{odd}(L, R) + F_{even}(L+1, R-1$
genap	genap	$\rightarrow F_{odd}(L+1, R-1) + F_{even}(L, R)$

$F_{ganjil} = \sum$ range ganjil $L \rightarrow R$

$$\sum 1 \rightarrow R = \frac{R \times (R+1)}{2}$$

$$L \rightarrow R = \qquad \sum 1 - (R) - \sum 1 - (L-1)$$

$$\sum L \to R = \frac{R \times (R+1)}{2} - \frac{(L-1)(L)}{2}$$

$$F_{ganjil} = \frac{R \times (R+1)}{2} - \frac{(L-1)(L)}{2}$$

$F_{genap} = \sum$ Range genap $L \to R$

$$\sum_{genap} 1 \to N = N + N-1 + N-2 + \dots + \left\lceil \frac{N}{2} \right\rceil + 1$$

$$= \sum 1 \to N - \sum 1 \to A$$

A: $N-1 \to N$ ganjil, $N \to N$ genap

$$= \frac{N \times (N+1)}{2} -$$

$$\sum_{genap} L \to R = \sum 1 \to R - \sum 1 (L^2 - 1)$$

$$\sum_{\text{genap}} L \to R = \underbrace{\frac{R \times (R+1)}{2} - \frac{\left(\left\lceil \frac{L-1}{2} \right\rceil\right) \times \left(\left\lceil \frac{L-1}{2} \right\rceil + 1\right)}{2}}_{F_{\text{genap}}}$$

$$\left\lceil \frac{N}{2} \right\rceil + 1$$

$$8 + 7 + 6 + 5 \rightarrow R = N$$

R

2 4 6 8

$R = N - 1 \quad R = N - 2$

$2 \to 6 = 8 + 7 + 6$

$1 \to 4 = 8 + 7$

$2 \to 4 = \sum 1 \to 8 - \sum 1 \to 6$

$2 \to 6 = \sum 1 \to 8 - \sum 1 \to 5$

$$2 \to 8 = \Sigma 1 \to 8 - \Sigma 1 \to 4$$

$$N + 1 - \frac{R}{2}$$

$6 \to$, $5 \to 6 - 1$

$4 \to 8/2$

1 2 3 4 5

$pref[i] = \sum 1 \rightarrow i$

$= \sum arr[0] \rightarrow (i-1)$

$1 \rightarrow 3 = pref(3)$

$1 \rightarrow 5 = pref[5]$

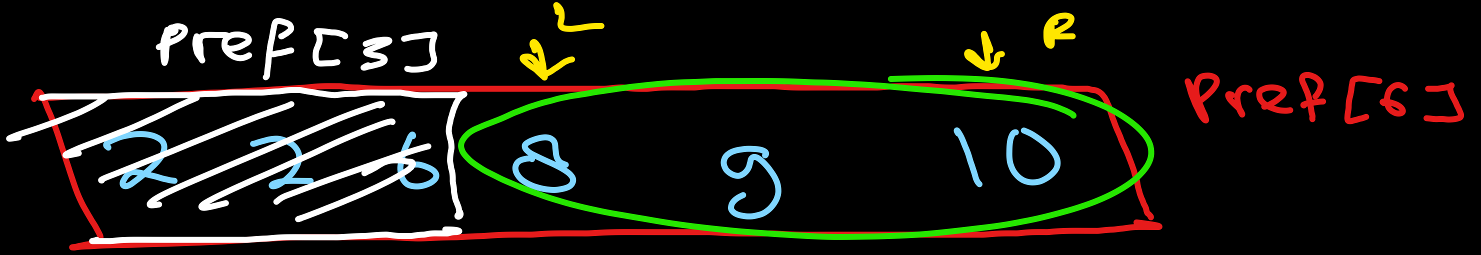
$sum(L, R)$

$\cancel{1}$ $\boxed{\boxed{2 \quad 3 \quad 4}}$ 5

$sum(2, 4) = 2+3+4$

$= 9$

$sum(L, R) = pref(R) - pref(L-1)$



$$\mathrm{sum}(4, 6) = \mathrm{pref}(6) - \mathrm{pref}(3)$$

$$\mathrm{pref}[i] = \mathrm{pref}[i-1] + \mathrm{arr}[i]$$

$$\mathrm{pref}(4) = \underbrace{2+2+6}_{\mathrm{pref}(3)} + \underbrace{8}_{\mathrm{arr}[4]}$$

$$\mathrm{pref}(3) = \underbrace{2+2}_{\mathrm{pref}(2)} + \underbrace{6}_{\mathrm{arr}[3]}$$

Pointer

int x = 9 →
y = x →

	x	y
addres	0x88F	0x99F
value	9	

duplicate (x) = 9

Variable assignment

Pass by value

Pass by refference

	x	y
addr	0x88F	0x99F
val		

y as pointer

f (int x)

x = 5

f (x)

y = 4

f (y)

kaidua(x)

addr		
val		

int a = 10

kaidua (a) → x = a

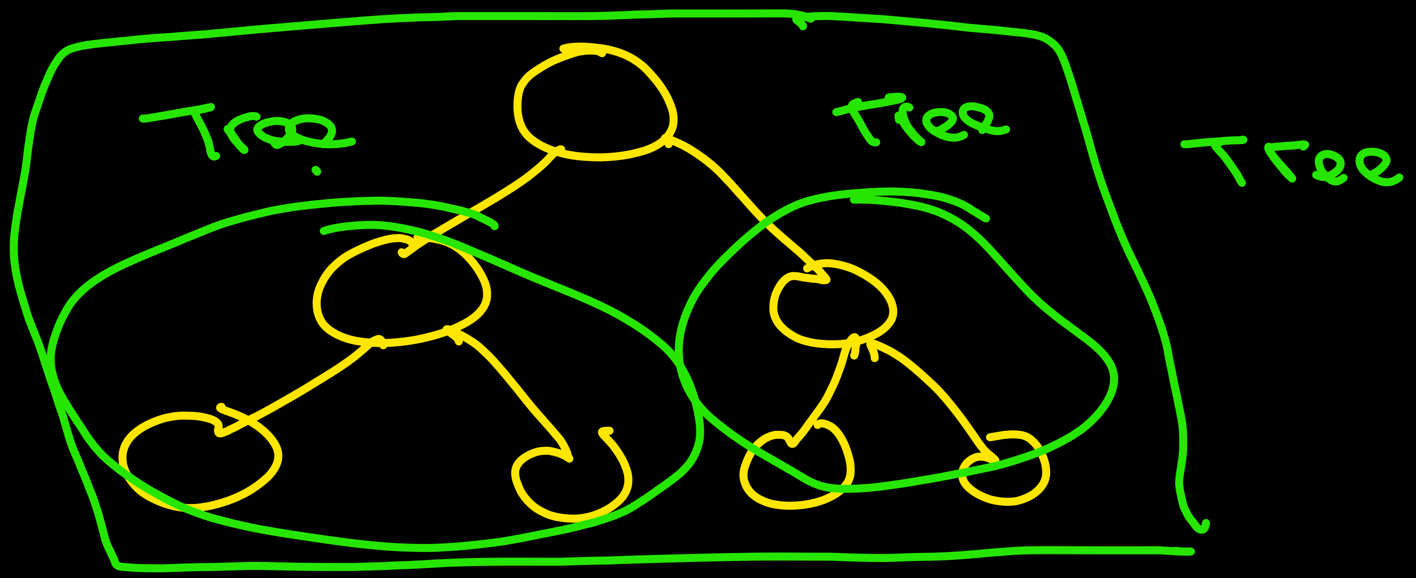
	a	x
addr	0x88F	0x996
val	10	10 x2 = 20

x

x ⇄ y

struct +pointer

object



Tree → Tree → left

Tree → left → right

Recursive $\rightarrow$ Orde

Orde

$$f(n) = f(n-1) + f(n-2) + \dots + f(n-i)$$

Orde : i_{max}

$$f(n) = f(n-1) + f(n-2)$$

2

Orde : 2

$$f(n) = f(n-1) + f(n-3) + f(n-5) \xrightarrow{} \text{orde}$$

orde : 5

Jumlah base case minimal sebanyak orde. orde = k

$f(0), f(1), \ldots, f(k)$

* Base case
* Function Return'f

Bottom-up

Top-down

Bottom up: pre-compute

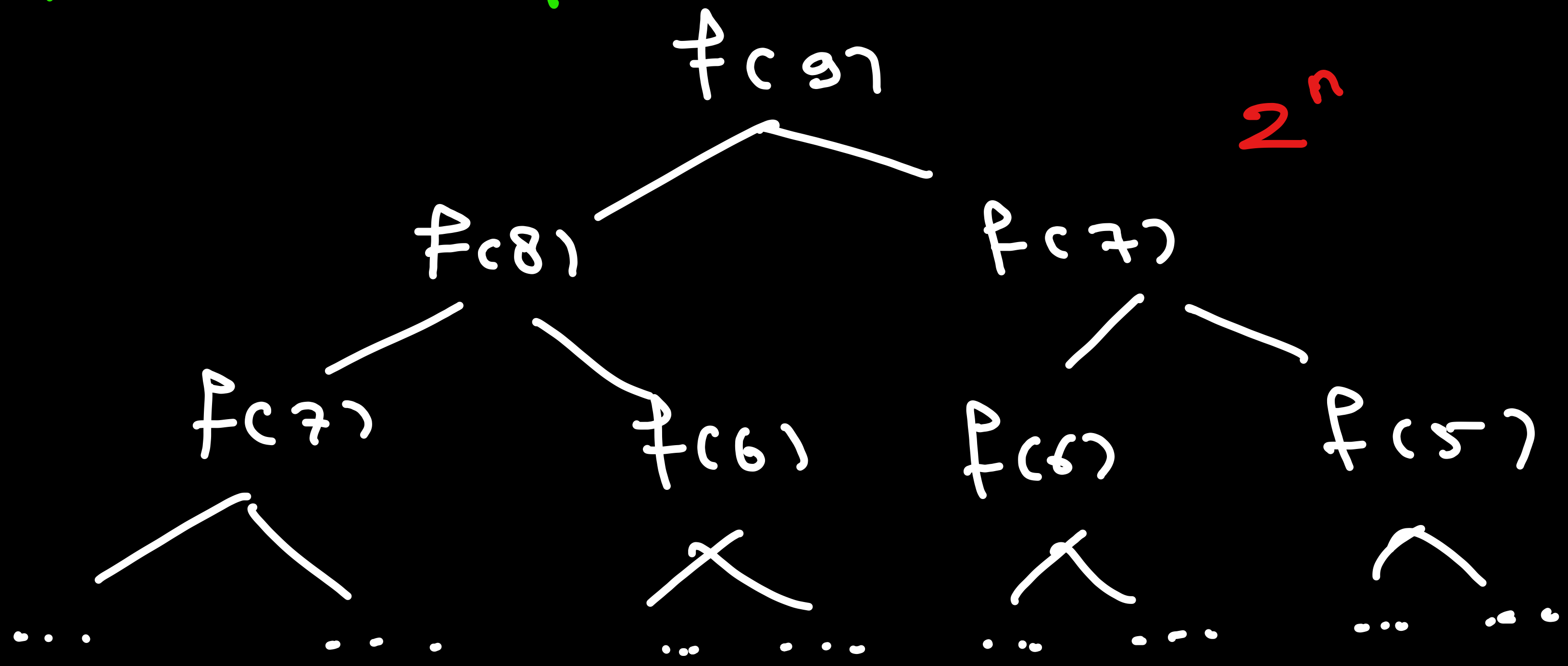
$$f(n) = n \times f(n-1)$$

$$f(5) \Rightarrow f(4) \Rightarrow f(3) \Rightarrow f(2) \Rightarrow \ldots \Rightarrow f(0)$$

Paradigma
- DP : Memoization + Pruning
- DnC : Rec

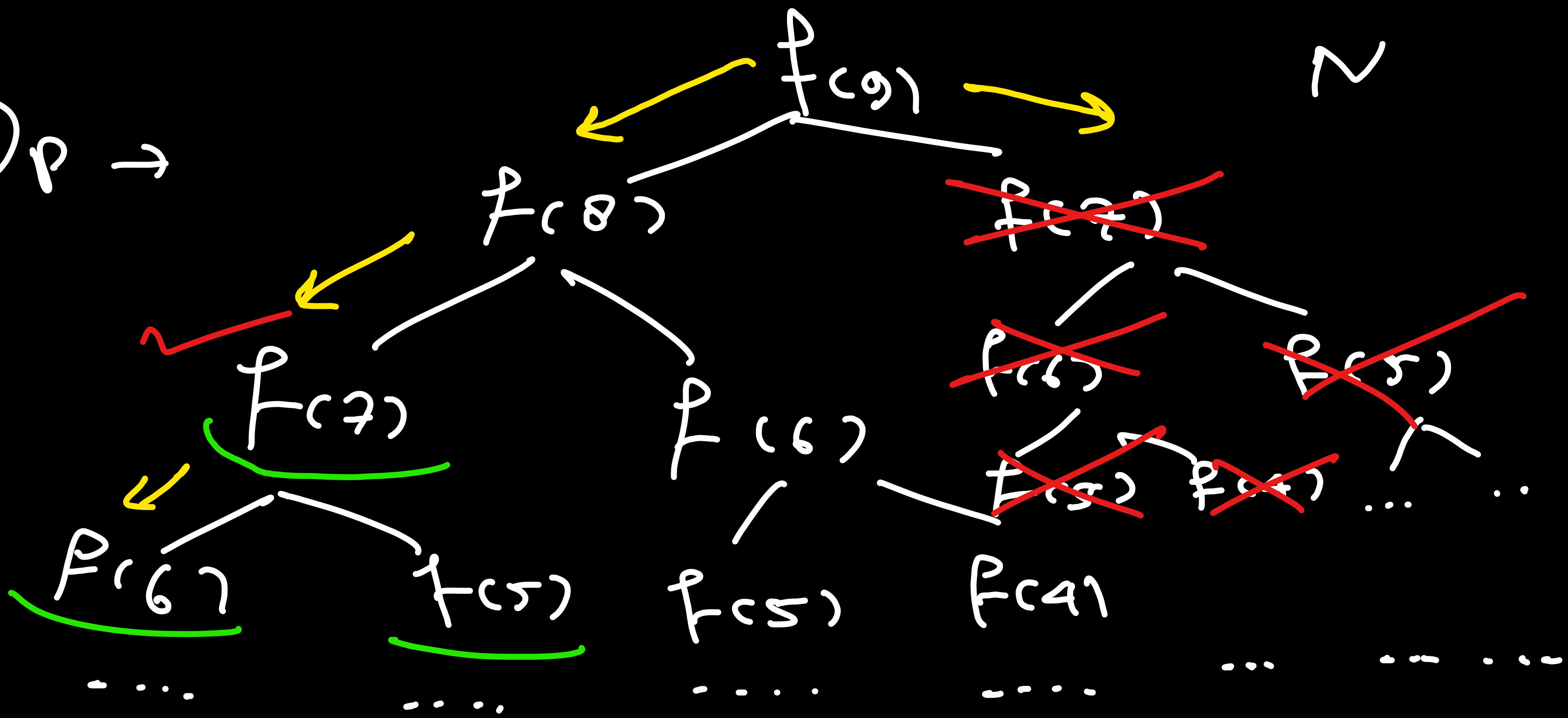
$$f(n) = f(n-1) + f(n-2)$$

DnC →



2^n

Dp →



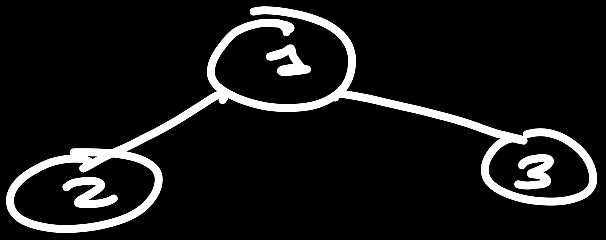
$$f(n) = f(n-1) + f(n-2)$$

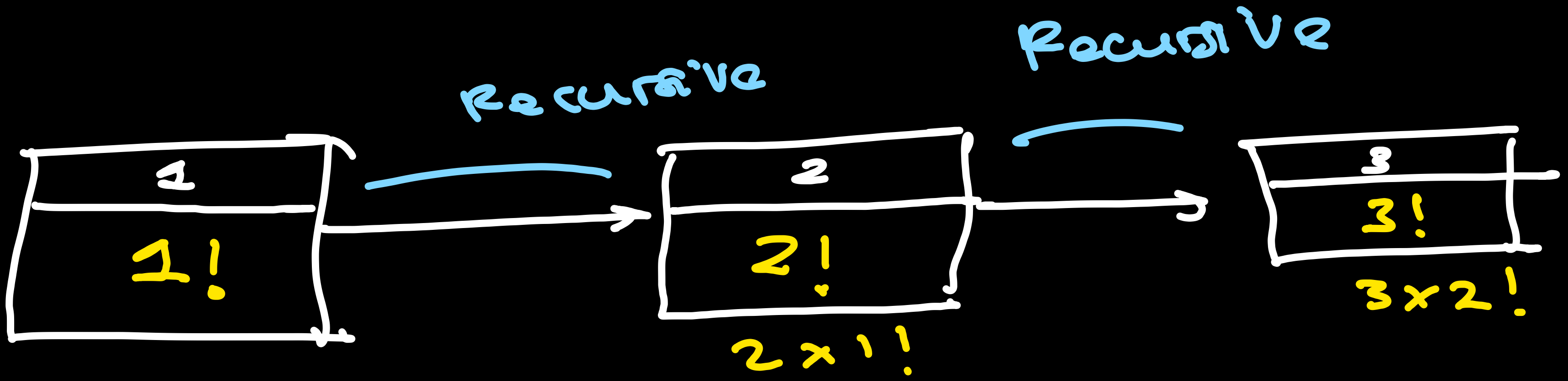
Zero base : $f(0) = 1, \quad f(1) = 1$

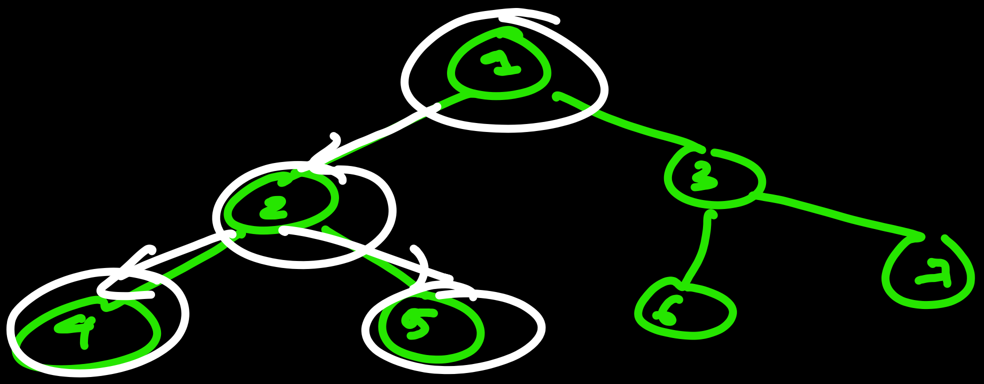
$f(2) = 1 + 1 \quad (f(0) + f(1))$

$= 2$

One base : $f(1) = 2, \quad f(2) = 1$


1
2
3


Recursive
Recursive
1
1!
2
2!
2×1!
3
3!
3×2!

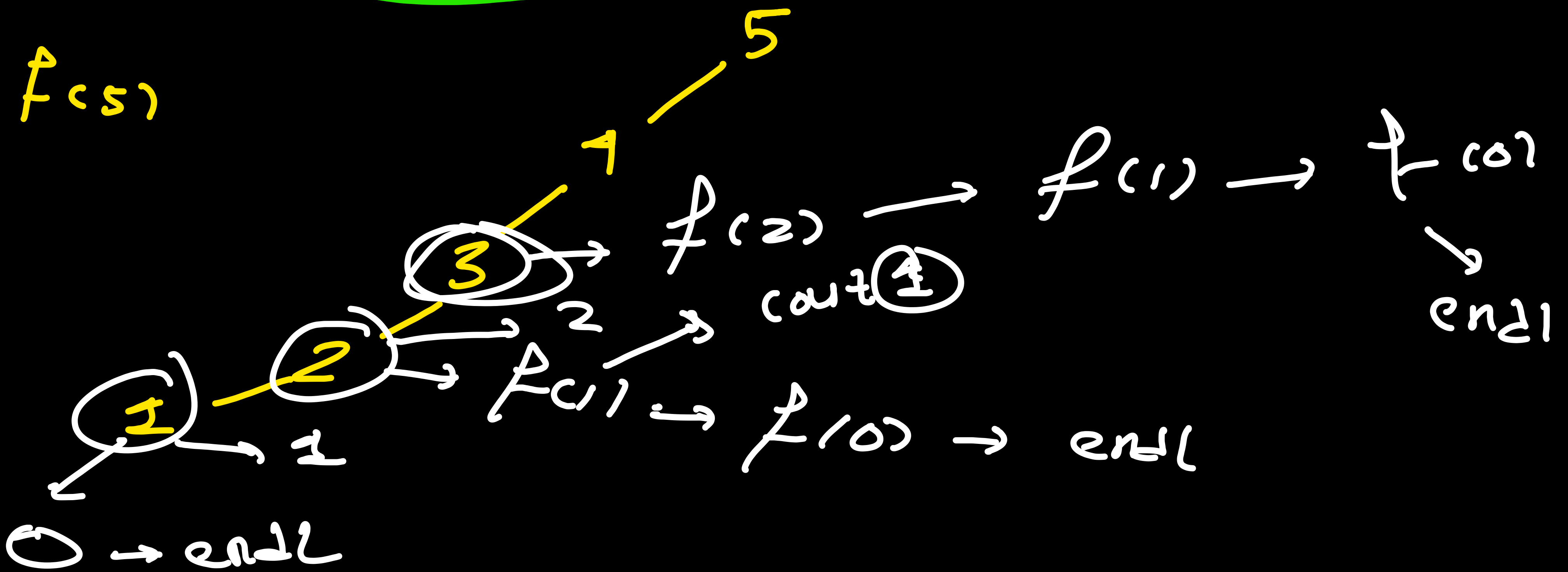

1
2
3
4
5
6
7

$f(n) =$

$f(n-1)$

cout << n

$f(n-1)$



2

$2 \to f(1) \to \cancel{f(0)}$

endl

cout 2 cout 1

$\to f(0) \to$ endl

$\to f(2-1) = 1 \to$ cout 1 endl

1 21

T

$N_1 \longrightarrow N_1\,!\ ?$

$N_2 \longrightarrow N_2\,!\ ?$

N_3 ...

$\vdots$

N_T

5

$1 \to f(1) \to f(0) = 1$

$2 \to f(1) \to\ = 2$

$3 \to f(2) \to f(1) = 6$

$4 \to f(3) \to f(2)$ ...

5

Setiap N akan N operasi

ada $T \to O(NT)$

$\cancel{10^{10}}$

5

$1 \rightarrow 1$

$2 \rightarrow F(1) = 2 \times 1 = 2$

$3 \rightarrow 3 \times F(2) = 3 \times 2 = 6$

$4 \rightarrow 4 \times F(3) = 4 \times 6 = 24$

5

$\longrightarrow O(N)$

Memo [1005]

Memset (memo, -1)

Faktorial (5) = 120

memo [5] = 120 bukan -1 → dp

0

dp = f(n) = 0

~~f(n) ≠ 0~~

gak mungkin faktorial

array [1005]

array [0] = 1
array [1] = 2

1	2				....

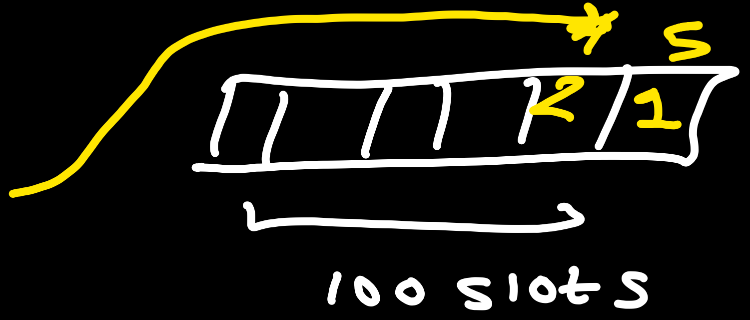
1005

Map <int , int> map 2

Map [0] = 1
Map [0] = 2
Memo [5] → key

key	val
0	~~1~~
5	....

array [100] →



array[5] = 1

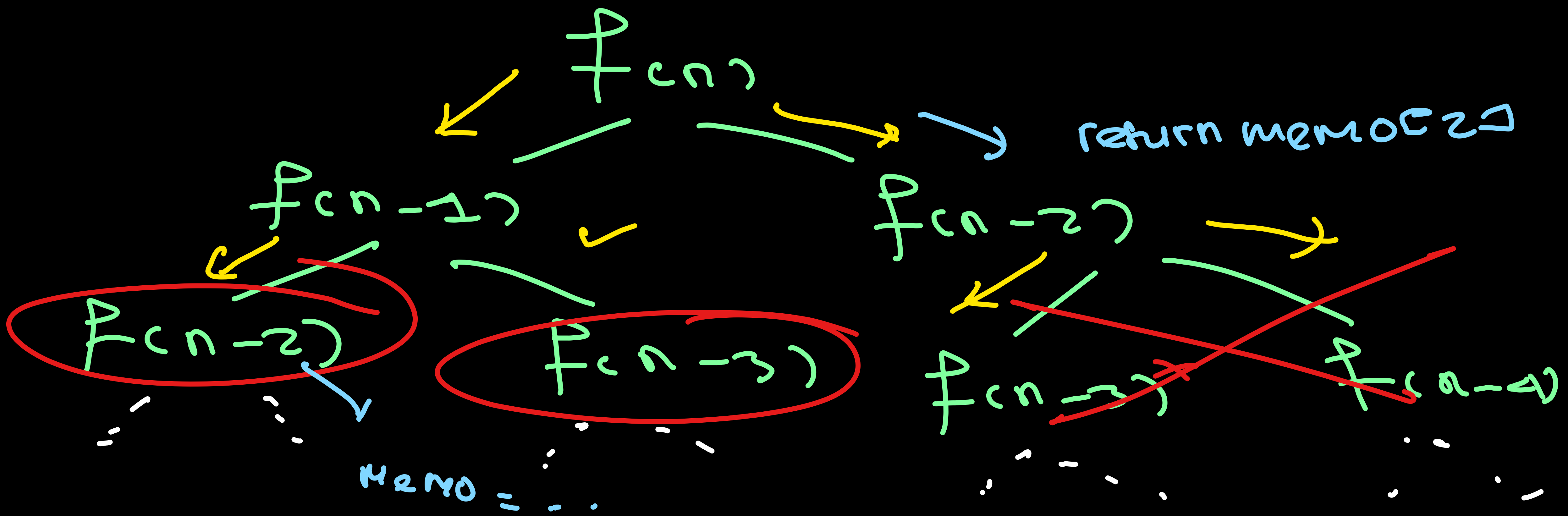
array[4] = 2

Map →

key	val
5	1
4	2

Map[5]=1

Map[4]=2


f(n)
f(n-1)
f(n-2)
return memo[2]
f(n-2)
f(n-3)
f(n-3)
f(n-4)
memo = ...

$$nC_1 = n$$

$$nC_n = 1$$

$$nC_k = nC_{n-k}$$

if (k > n-k): $\Big\}$ state reduction
    k = n-k

$$15C_{13} = f(15,13) \rightarrow f(14,12) \rightarrow \cdots$$

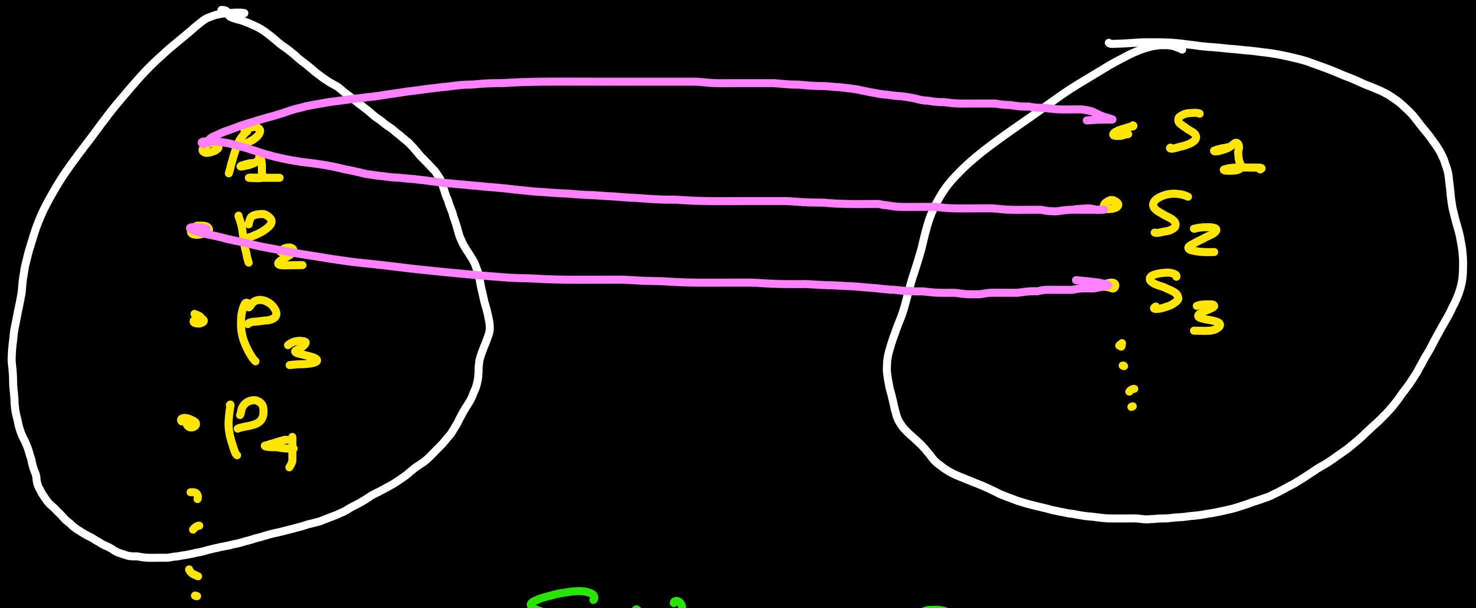
$$15C_2 = f(15,2) \rightarrow f(15,1)$$

=

Berapa banyak pemetaan tak kosong n pem → n peserta

N

Pemandu × peserta



$A \to B$

$n(A)^{n(B)}$

Setiap Pemandu minimal memandu 1 orang

invalid case : Pemandu tdk memandu samsek → $f(x) = 0$

$n(A)^{n(B)}$ — Pemandu tdk memandu

$n(A)^{n(B)}$ — $n(A)$

Binary exp $n^x - n$

1 → kasus khusus

1 ... → 1 kelompok

x^n → 1 ~~$9 = 9^1 - 1$~~

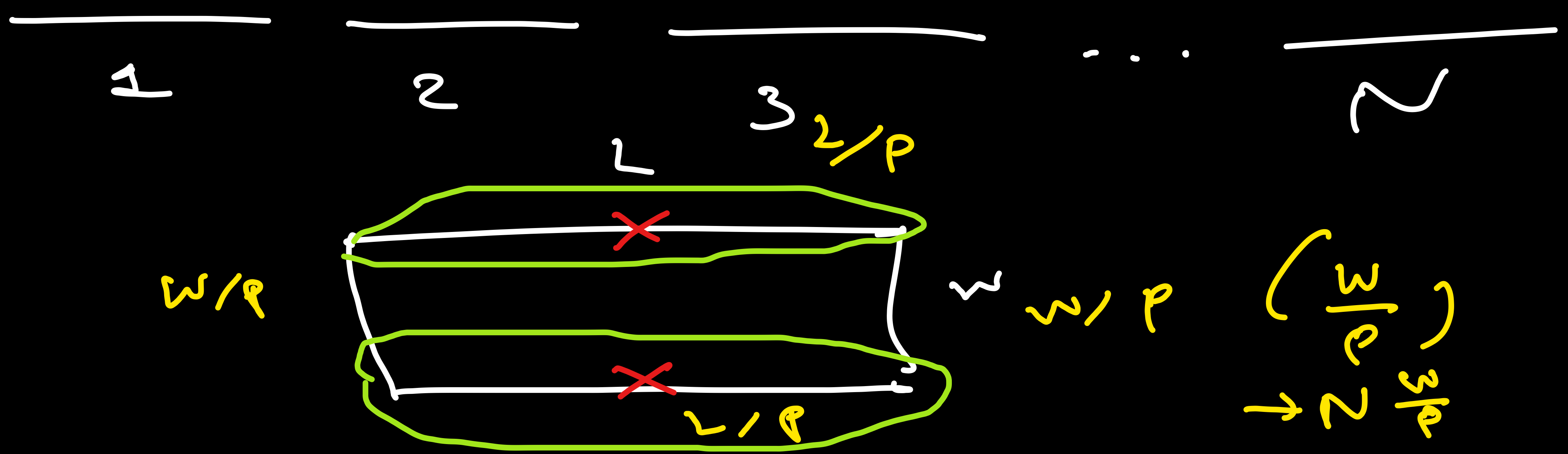
$= 8$

$2, 3 \to$

- $A = \{1,2\}, \{1,3\}, \{2,3\}$

  $B = \{3\}, \{2\}, \{1\}$

6



$$\left(\frac{W}{P}\right) \to N^{\frac{W}{P}}$$

$$\left(\frac{L}{P}\right)$$

$$\frac{N \times N \times N \times N}{N^{\frac{L}{P}} \times N^{\frac{L}{P}}} \qquad \frac{L}{P}$$

$$= \left(N^{\frac{L}{P}}\right)^2 \times N\left(\frac{W}{P}^2\right)$$

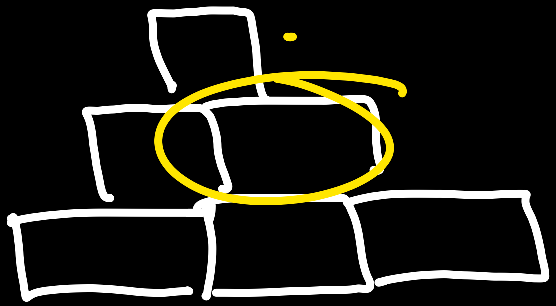
$$= \left(N^{\frac{L+W}{P}} \right)^2$$

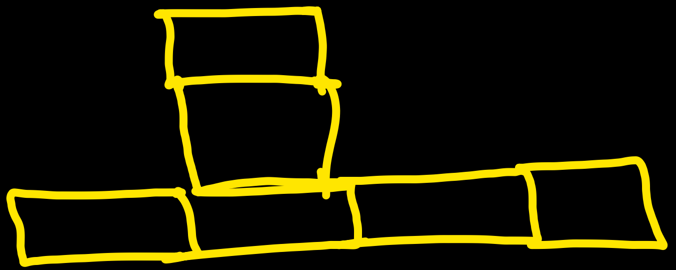
$$= 2^{\frac{2+1}{1}} \rightarrow (2^3)^2 = \underline{64}$$

L_3 :

L_2 :

L_1 :





Banyak balok pada lantai ke-i → x_i

$$x_1 + x_2 + x_3 + \dots + x_i = M$$

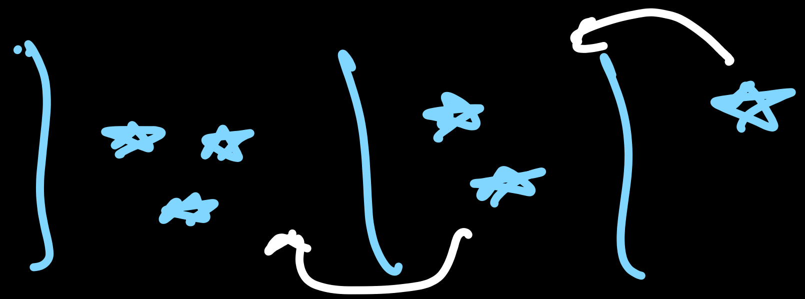
i = ketinggian <u>Max</u> $\qquad i = \sqrt{2M}$

$$x_i \geq 1$$

ada berapa konfigurasi x_i

$$x_1 + x_2 + x_3 + \dots + x_i = M$$

$$\binom{M+i-1}{M}$$



$$x_i \geq 1 \to x_i - 1 \geq 0$$

Stars & bars $\to$ $a_1 + a_2 + \dots + a_i = r$

$$a_i \geq 0$$

$$a_i = x_i - 1$$

$$x_i = a_i + 1$$

$$x_1 + x_2 + x_3 + \dots + x_i = M$$

$$a_1 + 1 + a_2 + 1 + a_3 + 1 + \dots + a_i + 1 = M$$

$$a_1 + a_2 + a_3 + \dots + a_i = 36$$

$$a_i \geq 0$$

$$\sqrt{90} = \ldots 9 \binom{M-i+i-1}{M-i}$$

$$= \binom{M-1}{M-i} = \binom{M-1}{i-1}$$

$$\binom{36+9-1}{36} = \binom{44}{36}$$

$$\binom{44}{36} =$$