Let $\gamma_{s,t}^w$ be a path from the node s to t of length less than w

Let $\mathcal{N}^w(s) := \{v : \forall \gamma_{s,v}\}$

Let $f(\theta, \beta) := \prod_{s \in \mathcal{V}} \prod_{t \in \mathcal{N}^w(s)} p(t|s) = \prod_{s \in \mathcal{V}} \prod_{t \in \mathcal{N}^w(s)} \sum_{\gamma_{s,t}} \prod_{(x_i, x_{i+1}) \in \gamma_{s,t}} \sigma(\beta(x_i)\theta(x_{i+1}))$

and $\log f = \sum_{s \in \mathcal{V}} \sum_{t \in \mathcal{N}^w(s)} \log \left(\sum_{\gamma_{s,t}} p(t|s) \right)$

Let $l_{s,t}(\theta, \beta) = \log \left(\sum_{\gamma_{s,t}} p(t|s) \right) = log \left(\sum_{\gamma_{s,t}} \sigma(\theta(t)\beta(s)) \right)$ Yanlis

Update equations

Let $\zeta := x_1, x_2, \cdots, x_k$ be a path of size k from s to t where $k \leq w$, $x_1 = s$, and $x_k = t$

$$\frac{\delta l(\theta, \beta)}{\theta(t)} = \frac{\sum_{\zeta \in \gamma_{s,t}} \prod_{(x_i, x_{i+1}) \in \zeta} \sigma(\beta(x_{i-1})\theta(x_i))(1 - \sigma(\beta(x_{|\zeta|-1})\theta(t)))\beta(x_{|\zeta|-1})}{\sum_{\zeta \in \gamma_{s,t}} \prod_{(x_i, x_{i+1}) \in \zeta} \sigma(\beta(x_i)\theta(x_{i+1}))}$$

and

$$\frac{\delta l(\theta, \beta)}{\beta(s)} = \frac{\sum_{\zeta \in \gamma_{s,t}} \prod_{(x_i, x_{i+1}) \in \zeta} \sigma(\beta(x_{i-1})\theta(x_i))(1 - \sigma(\beta(s)\theta(x_2)))\theta(x_2)}{\sum_{\zeta \in \gamma_{s,t}} \prod_{(x_i, x_{i+1}) \in \zeta} \sigma(\beta(x_i)\theta(x_{i+1}))} \star$$