Solvable Models of Quantum Black Holes: A Review on Jackiw-Teitelboim Gravity

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https://arxiv.org/pdf/2210.10846.pdf

References of interest

- 5
- 6
- 10
- 11, maybe

1 Introduction

Starting point: "the problem of non-renormalizability[1] of pure Eintein-Hilbert (EH) gravity in 3+1d". This has "led us through higher dimensions, string theory, compactification, branes....", but remains elusive.

To simplify, work in lower dimensions[2] and use holography.

In two spacetime-dimensions, simplest candidate model is EH gravity

$$S = \frac{1}{16\pi G_N} \int d^2x \sqrt{-g} R + ... \tag{1}$$

[3] "However, this model is topological[4] since its Euclidean action is just the Euler characteristic[5], and the Einstein tensor vanishes identially... Additionally coupling this model to a matter action S_m, the gravitational equations lead to $T^m_{\mu\nu} = 0$ and no energy flows exist. Hence when using it as a classical toy model for black hole formation and evaporation, this model has little value"

To get something more interesting, couple the Ricci scalar to the scalar "dilaton"[6] field Φ

$$S = \frac{1}{16\pi G_N} \int d^2x \sqrt{-g} \Phi R + ... \tag{2}$$

We can interpret the dilaton field as a "spacetime-varying gravitational coupling".

Jackiw-Teitelboim (JT) gravity is of the form (2). "Interestingly, JT also captures the dynamics close to the horizon of near-extremal black holes in higher dimensions, see the review [5]"

[1] Would be nice to see a rigorous statement of EH being nonrenormalizable.

[2] Why must we work in lower dimensions in order for Euclidean path integral formulation to make sense? It doesn't *really* make sense to me in any number of dimensions.

[3] *By "...", they mean the boundary terms, right?

[4] *What does it mean for a model to be topological? Trivial? Unsolveable?

[5] *What is the less math-y definition of the Euler characteristic?

[6] We will later find that this is a bit of a misnomer, since Φ is not really related to rescalings of the metric

“In section 2 we introduce the model, and fully solve its classical equations of motion, crucially incorporating boundary conditions at the holographic boundary that allow us to map the dynamics to its boundary Schwarzian[7] description. Section 3 proceeds with the exact quantization of the model, by computing the Euclidean gravitational path integral in the Schwarzian language. In section 4 we furthermore include non-trivial topological corrections (or wormholes[8]) to the quantum amplitudes, that modify the heavy quantum regime even further. Finally, section 5 contains several applications of the exact solvability of JT gravity. We do not treat these in technical detail, but refer to the literature for more information. In particular, in the past few years significant progress has been made on Hawking’s information loss paradox...”

2 Classical Jackiw-Teitelboim Gravity

“We begin by introducing and motivating the JT model and its coupling to matter. In this section, we study the classical solution of this model including gravitational backreaction. Our endeavors will ultimately lead us to a description in terms of a boundary Schwarzian model that will be the starting point for a quantum mechanical solution in the next section 3”

2.1 Dilaton gravity models

Working in **Euclidean signature**, most general theory of dilaton gravity in two dimensions with a two-derivative action[9][10] can be written as

$$I = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} d^2x\sqrt{g}\left(U_1(\tilde{\Phi})R + U_2(\tilde{\Phi})g^{\mu\nu}\partial_\mu\tilde{\Phi}\partial_\nu\tilde{\Phi} + U_3(\tilde{\Phi})\right) \tag{3}$$

Notice the first term is a curvature term, the second term is a kinetic term, and the third term is a potential.

“The 2d Newton’s constant G_N is dimensionless[11], and hence unlike in higher dimensions does not set the scale of physics. Later on, around equation (2.57), we will see an effective scale emerge nonetheless.”

We will simplify (3) as such:

1. Field redefinition: $\Phi = U_1(\tilde{\Phi})$
   - Requires assumption that there is no value of $\tilde{\Phi}$ such that $U_1'(\tilde{\Phi}) = 0$, otherwise field redefinition non-invertible and kinetic term ill-defined.
2. Weyl transformation[12][13]

$$g'_{\mu\nu} = e^{2\omega}g_{\mu\nu} \Longrightarrow g'^{1/2}R' = g^{1/2}(R - 2\nabla^2\omega) \tag{4}$$

   where

$$\omega(x) = \frac{1}{2}\int^{\Phi(x)} U_2(\Phi')d\Phi' \tag{5}$$

   - This cancels the term in (3) proportional to U_2 i.e. the kinetic term

These simplifications give us

$$I[g,\Phi] = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} d^2x\sqrt{g}(\Phi R + U(\Phi)) \tag{6}$$

Where $U(\Phi)$ is called the **dilaton potential**. “Notice that the above field transformations from (3) to (6) are done at the classical level. In the full quantum theory, care has to be taken when performing these steps, in particular with regard to the thermodynamics of the resulting models.”

[7] *Where to learn about this ‘Schwarzian’ stuff? See **The Schwarzian Theory - Origins**, top of chapter 1

[8] *Very interesting. ‘Wormholes’ here are interpreted in the rigorous topological way: a hole is a non-trivial deformation of the topology. In the same way that a wormhole non-trivially affects the connectivity of the spacetime, a hole non-trivially affects the topological set i.e. the connectivity of the topology.

[9] Implications and relaxations of this discussed in reference [17]. Including higher-derivative terms gives a model that is “power-counting non-renormalizable”

[10] *What does it mean to be *power-counting* non-renormalizable?

[11] *Why is G_N dimensionless in 2d?

[12] i.e. a local rescaling

[13] recall: a global transformation acts identically on each point

JT gravity corresponds to a specific *linear* choice of dilaton potential: $U(\Phi) = -\Lambda\Phi$, where Λ is indeed the cosmological constant.

$$I_{JT}^{\Lambda}[g,\Phi] = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} \sqrt{g}\Phi(R-\Lambda) \tag{7}$$

Note: $R-\Lambda$ is the local curvature minus the universal curvature.

We will focus on AdS, i.e. $\Lambda < 0$. Choosing $\Lambda = -2/L^2$ and using units such that $L = 1$:

$$I_{JT}[g,\Phi] = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} \sqrt{g}\Phi(R+2) - \frac{1}{8\pi G_N}\oint_{\partial\mathcal{M}} \sqrt{h}\Phi(K-1) \tag{8}$$

Where we've included the Gibbons-Hawking-York (GHY) boundary term (K) and a holographic counterterm (-1) a posteriori (see your own notes: **Deriving the Gibbons-Hawking-York Boundary Term**).

Later we will find it necessary to add the *topological* EH term[14], $S_0\chi$, where χ is the "Euler characteristic of the manifold"

$$\chi = \frac{1}{4\pi}\int_{\mathcal{M}} \sqrt{g}R + \frac{1}{2\pi}\oint_{\partial\mathcal{M}} \sqrt{h}K \tag{9}$$

The final action we will study throughout this review is hence

$$I[g,\Phi] = -S_0\chi + I_{JT}[g,\Phi] \tag{10}$$

We will also briefly consider the flat and dS cases in chapter 5.

2.1.1 First-order formulation

[14] *What's the topological EH term?