Alt. Method: Plug in for g_{TT}, g_{zz} instead of dt^2, dr^2

Hartman Ch. 11

Extremal Reissner-Nordstrom Black Hole

$$f(r) = \left(1 - \frac{Q}{r}\right)^2$$

$$g_{tt} = -f(r), \quad g_{rr} = \frac{1}{f(r)}$$

$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + r^2 d\Omega_2^2$$

Parametrized x-form to new coordinates:

$$\rightarrow r = Q\left(1 + \frac{\lambda}{z}\right), \quad t = \frac{QT}{\lambda}$$

Method:

- Assume form: $ds^2 = g_{TT}\, dT^2 + g_{zz}\, dz^2$
$$+ r^2 d\Omega_2^2$$

- Use Jacobian, inverse-transformation to find g_{TT}, g_{zz}.

1. $$F(r) = \left(1 - \frac{Q}{r}\right)^2 = \left(1 - \frac{Q}{Q(1+\lambda/z)}\right)^2$$
$$= \left(1 - \frac{1}{1+\frac{\lambda}{z}}\right)^2 = \left(1 - \frac{1}{\frac{z+\lambda}{z}}\right)^2 = \left(1 - \frac{z}{z+\lambda}\right)^2$$
$$= \left(\frac{z+\lambda - z}{z+\lambda}\right)^2 = \left(\frac{\lambda}{z+\lambda}\right)^2$$

2. $$g_{tt} = -F(r)$$
$$g_{tt} = -\left(\frac{\lambda}{z+\lambda}\right)^2$$

3. $g_{rr} = \frac{1}{f(r)} = \left(\frac{z+\lambda}{\lambda}\right)^2$

4. $g_{\rho\sigma} = \frac{\partial x^\mu}{\partial x^\rho} \frac{\partial x^\nu}{\partial x^\sigma} g_{\mu\nu}$

$$g_{TT} = \left(\frac{\partial t}{\partial T}\right)^2 g_{tt} + \cancel{\cdots} \to 0$$

$$t = \frac{Q}{\lambda} T \Rightarrow \left(\frac{\partial t}{\partial T}\right)^2 = \frac{Q^2}{\lambda^2}$$

$$g_{TT} = -\frac{Q^2}{\lambda^2}\left(\frac{\lambda}{z+\lambda}\right)^2 = -Q^2\left(\frac{1}{z+\lambda}\right)^2$$

$$\lim_{\lambda\to 0} g_{TT}\, dT^2 = -\frac{Q^2}{z^2} dT^2$$

5. $g_{zz} = \left(\frac{\partial r}{\partial z}\right)^2 g_{rr}$

$$r = Q\left(1 + \frac{\lambda}{z}\right)$$

$$\Rightarrow \frac{\partial r}{\partial z} = -\frac{Q\lambda}{z^2}$$

$$\Rightarrow \left(\frac{\partial r}{\partial z}\right)^2 = \frac{Q^2 \lambda^2}{z^4}$$

$$g_{zz} = \frac{Q^2\lambda^2}{z^4}\left(\frac{1}{f(r)}\right) = \frac{Q^2\lambda^2}{z^4}\left(\frac{z+\lambda}{\lambda}\right)^2$$

$$g_{zz} = \frac{Q^2}{z^4}(z+\lambda)^2$$

$$\lim_{\lambda \to 0} g_{zz}\, dz^2 = \frac{Q^2}{z^2} dz^2$$

6. $\lim_{\lambda \to 0} r^2 = Q^2$

$$\downarrow$$

$$\boxed{ds^2 = \frac{Q^2}{z^2}\left(-dT^2 + dz^2\right) + Q^2 d\Omega_2^2}$$

Both methods work! But I

don't know if they're equivalent...