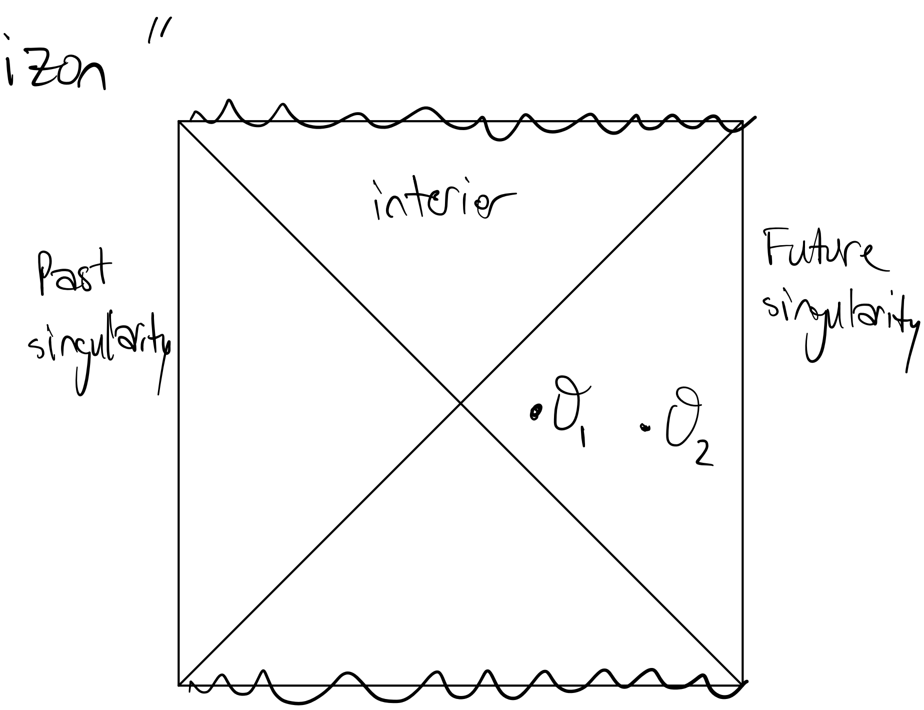
(1): "Correlation fn. of two simple bulk operators separated in time a finite distance above the black hole horizon"



"Above horizon" $\rightarrow$ not in interior

Assume not causally disconnected $\rightarrow$ same cone

Assume <u>only</u> separated in time $\rightarrow$ same vertical/spatial displacement in diagram

What is described in (1) is:

$$A = \langle \psi | \mathcal{O}_1(t_1, \vec{x}) \mathcal{O}_2(t_2, \vec{x}) | \psi \rangle$$

$\downarrow$ Time-origin arbitrary

$$= \langle \psi | \mathcal{O}_1(0, \vec{x}) \mathcal{O}_2(T, \vec{x}) | \psi \rangle; \quad T = t_2 - t_1 \geq 0$$

"Finite entropy" $\leftrightarrow$ "Finite temperature" $\rightarrow$ <u>Thermal</u> correlator

Per Hartman 4.32, thermal correlator given by

$$A = \frac{1}{Z} \mathrm{Tr}\, e^{-\beta H} \mathcal{O}(0,\vec{x})\, \mathcal{O}(T,\vec{x}) = \frac{1}{Z} \mathrm{Tr}\, e^{-\beta H} \mathcal{O}(0)\, \mathcal{O}(T)$$

$$= \frac{1}{Z} \sum_n \langle n | e^{-\beta H} \mathcal{O}(0)\, \mathcal{O}(T) | n \rangle$$

$* \; |\Psi\rangle \sim \int e^{-\frac{\beta}{2} H(\phi)} |\phi\rangle \, d\phi$

For thermal state. Weights given by $e^{-\frac{\beta}{2} H}$ 2.

$\downarrow$ Heisenberg picture: $\mathcal{O}(T) = e^{iHT} \mathcal{O}(0)\, e^{-iHT}$

$$= \frac{1}{Z} \sum_n \langle n | e^{-\beta H} \mathcal{O}(0)\, e^{iHT} \mathcal{O}(0)\, e^{-iHT} | n \rangle$$

$\downarrow$ $\mathcal{O}(0) \equiv \mathcal{O}$

$$= \frac{1}{Z} \sum_n \langle n | e^{-\beta H} \mathcal{O}\, e^{iHT} \mathcal{O}\, e^{-iHT} | n \rangle$$

$$= \frac{1}{Z} \sum_n \langle n | e^{-\frac{\beta}{2} H} e^{-\frac{\beta}{2} H} \mathcal{O}\, e^{iHT} \mathcal{O}\, e^{-iHT} | n \rangle$$

$$= \frac{1}{Z} \sum_n \langle n | e^{-\frac{\beta}{2} E_n} e^{-\frac{\beta}{2} H} \mathcal{O}\, e^{iHT} \mathcal{O}\, e^{-iE_n T} | n \rangle$$

$$= \frac{1}{Z} \sum_n e^{-(\frac{\beta}{2} + iT) E_n} \langle n | e^{-\frac{\beta}{2} H} \mathcal{O}\, e^{iHT} \mathcal{O} | n \rangle$$

Insert identity

$$= \frac{1}{Z} \sum_{n,m} e^{-(\frac{\beta}{2}+iT)E_n} \langle n| e^{-\frac{\beta}{2}H} \mathcal{O} |m\rangle \langle m| e^{iHT} \mathcal{O} |n\rangle$$

$$= \frac{1}{Z} \sum_{n,m} e^{-(\frac{\beta}{2}+iT)E_n} \langle m| e^{-\frac{\beta}{2}H} \mathcal{O} |n\rangle \langle m| e^{iHT} \mathcal{O} |n\rangle$$

$$= \frac{1}{Z} \sum_{n,m} e^{-(\frac{\beta}{2}+iT)E_n} e^{-\frac{\beta}{2}E_m} \langle m|\mathcal{O}|n\rangle e^{iE_mT} \langle m|\mathcal{O}|n\rangle$$

$$= \frac{1}{Z} \sum_{n,m} e^{-(\frac{\beta}{2}+iT)E_n} e^{-(\frac{\beta}{2}-iT)E_m} |\langle n|\mathcal{O}|m\rangle|^2 \checkmark$$

We can see that, for large T, we have oscillation, not exponential decay. This oscillation not visible in perturbative (CFT?) approach, so exact bulk solution might be useful.

Eigenstate thermalization hypothesis: E'states should behave like statistical ensemble for large t, N. (My words). More-or-less.

Define the simpler observable, stripped of the matrix elements and relabeled $\beta \to 2\beta$

$$Z(\beta+iT)\,Z(\beta-iT)=\sum_{n,m} e^{-(\beta+iT)E_n}\, e^{-(\beta-iT)E_m}$$

$$=\sum_{m,n} e^{-\beta(E_m+E_n)+iT(E_m-E_n)}$$

For $\beta=0$, this is called the spectral form factor

$$\left|Z(\beta+iT)\right|^2=\left|\sum_{n,m} e^{-(\beta+iT)E_n}\, e^{-(\beta+iT)E_m}\right|$$

$$=\sum_{m,n}\left|e^{-\beta(E_m+E_n)+iT(E_m+E_n)}\right|$$

$$=\sum_{m,n}\left|e^{-\beta(E_m+E_n)}\, e^{iT(E_m+E_n)}\right|$$

$$=\sum_{m,n}\left|e^{-\beta(E_m+E_n)}\right|\underbrace{\left|e^{iT(E_m+E_n)}\right|}_{=1}$$

$$=\sum_{m,n}\left|e^{-\beta(E_m+E_n)}\right|$$