CHAPTER 5

Reserved Problems

5.1R The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

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Two-way ANOVA: y versus, A, B

Source          DF       SS      MS       F       P
A                1    0.322       ?       ?       ?
B                ?   80.554  40.2771    4.59       ?
Interaction      ?        ?       ?       ?       ?
Error           12  105.327  8.7773
Total           17  231.551
```

(a) Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.

(b) How many levels were used for factor *B*?

(c) How many replicates of the experiment were performed?

(d) What conclusions would you draw about this experiment?

5.2R Johnson and Leone (*Statistics and Experimental Design in Engineering and the Physical Sciences*, Wiley, 1977) describe an experiment to investigate warping of copper plates. The two factors studied were the temperature and the copper content of the plates. The response variable was a measure of the amount of warping. The data are as follows:

	Copper Content (%)			
Temperature (°C)	**40**	**60**	**80**	**100**
50	17, 20	16, 21	24, 22	28, 27
75	12, 9	18, 13	17, 12	27, 31
100	16, 12	18, 21	25, 23	30, 23
125	21, 17	23, 21	23, 22	29, 31

(a) Is there any indication that either factor affects the amount of warping? Is there any interaction between the factors? Use $\alpha = 0.05$.

(b) Analyze the residuals from this experiment.

(c) Plot the average warping at each level of copper content and compare them to an appropriately scaled t distribution. Describe the differences in the effects of the different levels of copper content on warping. If low warping is desirable, what level of copper content would you specify?

(d) Suppose that temperature cannot be easily controlled in the environment in which the copper plates are to be used. Does this change your answer for part (c)?

5.3R A mechanical engineer is studying the thrust force developed by a drill press. He suspects that the drilling speed and the feed rate of the material are the most important factors. He selects four feed rates and uses a high and low drill speed chosen to represent the extreme operating conditions. He obtains the following results. Analyze the data and draw conclusions. Use $\alpha = 0.05$.

	Feed Rate			
Drill Speed	**0.015**	**0.030**	**0.045**	**0.060**
125	2.70	2.45	2.60	2.75
	2.78	2.49	2.72	2.86
200	2.83	2.85	2.86	2.94
	2.86	2.80	2.87	2.88

5.4R The yield of a chemical process is being studied. The two factors of interest are temperature and pressure. Three levels of each factor are selected; however, only nine runs can be made in one day. The experimenter runs a complete replicate of the design on each day. The data are shown in the following table. Analyze the data, assuming that the days are blocks.

	Day 1 Pressure			Day 2 Pressure		
Temperature	**250**	**260**	**270**	**250**	**260**	**270**
Low	86.3	84.0	85.8	86.1	85.2	87.3
Medium	88.5	87.3	89.0	89.4	89.9	90.3
High	89.1	90.2	91.3	91.7	93.2	93.7

5.5R I have recently purchased new golf clubs, which I believe will significantly improve my game. Below are the scores of three rounds of golf played at three different golf courses with the old and the new clubs.

	Course		
Clubs	**Ahwatukee**	**Karsten**	**Foothills**
Old	90	91	88
	87	93	86
	86	90	90
New	88	90	86
	87	91	85
	85	88	88

(a) Conduct an analysis of variance. Using $\alpha = 0.05$, what conclusions can you draw?

(b) Investigate model adequacy by plotting the residuals.

5.6R Consider the following ANOVA for a two-factor factorial experiment:

Source	DF	SS	MS	F	P
A	2	8.0000	4.00000	2.00	0.216
B	1	8.3333	8.33333	4.17	0.087
Interaction	2	10.6667	5.33333	2.67	0.148
Error	6	12.0000	2.00000		
Total	11	39.0000			

In addition to the ANOVA, you are given the following data totals. Row totals (factor A) $= 18, 10, 14$; column totals (factor B) $= 16, 26$; cell totals $= 10, 8, 2, 8, 4, 10$, and replicate totals $= 19, 23$. The grand total is 42. The original experiment was a completely randomized design. Now suppose that the experiment had been run in two complete blocks. Answer the following questions about the ANOVA for the blocked experiment.

(a) The block sum of squares is ___.

(b) There are ___ degrees of freedom for blocks.

(c) The error sum of squares is now ___.

(d) The interaction effect is now significant at 1 percent.

Yes

No

5.7R Consider the factorial experiment in problem 5.30 in the textbook. Answer the following questions.

(a) Suppose that the ANOVA sources of variation are Model, Error, and Total. Assuming the full factorial model, how many degrees of freedom are there for the Model source (your answer must be an integer)?

(b) Suppose that the pure error sum of squares is 30, what is the value of the error mean square?

(c) Assuming the full factorial model, the residuals in the first cell (Doping $= 1 \times 10^6$ and temperature $= 900$) are 0.10 and -0.10 (answer yes or no).

(d) If the F-statistic for the model is 9.25, you would conclude that none of the factors in the model are significant (answer yes or no)?

5.8R In a two-factor factorial experiment where both factors have three levels and there are 3 replicates, the number of degrees of freedom for error will be

(a) 4 **(b)** 18

(c) 27 **(d)** 12

5.9R In a factorial experiment the total sum of squares is 1000.00, the model sum of squares is 800.00 and the value of the PRESS statistic is 300.00. The value of the R^2 for prediction is 0.80.

True **False**

5.10R The C. F. Eye Care company manufactures lenses for transplantation into the eye following cataract surgery. An engineering group has conducted an experiment involving two factors to determine their effect on the lens polishing process. The results of this experiment are summarized in the following ANOVA:

Source	**DF**	**SS**	**MS**	**F**	**P-value**
Factor A	_____	_____	0.0833	0.05	0.952
Factor B	C	96.333	96.3333	57.80	0.000
Interaction	2	12.167	6.0833	3.65	E
Error	6	10.000	D		
Total	11	118.667			

Answer the following questions about this experiment.

(a) The sum of squares for factor A is_____.

(b) The number of degrees of freedom for Factor A in the experiment is _____.

(c) The number of degrees of freedom for Factor B is _____.

(d) The mean square for error is _____ .

(e) An upper bound for the P-value for the interaction test statistic is_____.

(f) The engineers used _____ levels of the factor A in this experiment.

(g) The engineers used _____ levels of the factor B in this experiment.

(h) There are _____ replicates of this experiment.

(i) Would you conclude that the effect of factor B depends on the level of factor A?

(j) An estimate of the standard deviation of the response variable is _____ .

5.11R Consider the following incomplete ANOVA table:

Source	SS	DF	MS	F
A	350.00	2		
B	300.00		150	
AB	200.00		50	
Error	150.00	18		
Total	1000.00			

(a) How many levels of factor B were used in the experiment? _____

(b) How many degrees of freedom are associated with interaction? _____

(c) The error mean square is _____.

(d) The mean square for factor A is _____.

(e) How many replicates of the experiment were conducted? _____.

(f) Suppose that the error mean square had been 10.00. The computed value of the F statistic for factor A would be _____ .

(g) Suppose that the computed value of the F statistic had been 20. An upper bound on the P-value for this statistic would be _____.

(h) An estimate of the standard deviation of the response variable is _____ .

(i) Assume that the error mean square is 10.00. Is there a significant interaction effect at the $\alpha = 0.05$ level of significance? _____

(j) If this experiment had been run in blocks there would have been _____ degrees of freedom.

5.12R Consider the following two-factor factorial experiment:

	Factor B			
Factor A	**1**	**2**	**3**	**4**
1	120, 122	125, 130	110, 104	116, 112
2	107, 109	111, 117	101, 95	103, 99
3	113, 120	109, 103	99, 92	98, 90
4	125, 130	110, 118	100, 93	111, 119

(a) Analyze the data from this experiment and draw conclusions, assuming a fixed effects model.

(b) What is the estimate of the error variance?

5.13R A two-factor factorial experiment has 2 levels of factor A and 3 levels of factor B. The experiment is replicated three times. The ANOVA indicates that both main effects are significant but the interaction is not significant. The interaction term is dropped from the model. The number of pure error degrees of freedom for the reduced model are

(a) 12 **(b)** 14 **(c)** 6

(d) 10 **(e)** 8 **(f)** None of the above

5.14R A two-factor factorial experiment has 2 levels of factor A and 5 levels of factor B. If the experiment is replicated twice the number of error or residual degrees of freedom are

(a) 10 **(b)** 12 **(c)** 15

(d) 20 **(e)** 30 **(f)** None of the above

5.15R Consider the following ANOVA:

Source_____DF_____SS_____MS_____F
A_____2__20.0000__10.0000__3.75
B_____2___8.0000___4.0000__1.50
Interaction___4__14.0000___3.5000__1.31
Error_____9__24.0000___2.6667
Total_____17__66.0000

If the experimenter drops the interaction term from the model, the value of R^2 is

(a) 0.3125 **(b)** 0.6563 **(c)** 0.4025

(d) 0.3750 **(e)** None of the above

5.16R A two-factor factorial experiment has been conducted with 3 levels of factor A and 2 levels of factor B. There are two replicates. The total sum of squares is 100, and the sums of squares for the other factors are as follows: $SS_A = 40$, $SS_B = 25$, and $SS_{AB} = 16$. The estimate of the error variance is

(a) 2.5 **(b)** 3.0 **(c)** 3.5

(d) 4.0 **(e)** None of the above