CHAPTER 3

Reserved Problems

3.1R A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the *P*-value.

One-way ANOVA

Source	DF	SS	MS	F	P
Factor	3	36.15	?	?	?
Error	?	?	?		
Total	19	196.04			

3.2R An article in *Bioelectromagnetics* ("Electromagnetic Effects on Forearm Disuse Osteopenia: A Randomized, Double-Blind, Sham-Controlled Study," Vol. 32, 2011, pp. 273–282) described a randomized, double-blind, sham-controlled, feasibility and dosing study to determine if a common pulsing electromagnetic field (PEMF) treatment could moderate the substantial osteopenia that occurs after forearm disuse. Subjects were randomized into four groups after a distal radius fracture, or carpal surgery requiring immobilization in a cast. Active or identical sham PEMF transducers were worn on the distal forearm for 1, 2, or 4 h/day for 8 weeks starting after cast removal ("baseline") when bone density continues to decline. Bone mineral density (BMD) and bone geometry were measured in the distal forearm by dual energy X-ray absorptiometry (DXA) and peripheral quantitative computed tomography (pQCT). The data below are the percent losses in BMD measurements on the radius after 16 weeks for patients wearing the active or sham PEMF transducers for 1, 2, or 4 h/day (data were constructed to match the means and standard deviations read from a graph in the paper).

(a) Is there evidence to support a claim that PEMF usage affects BMD loss? If so, analyze the data to determine which specific treatments produce the differences.

(b) Analyze the residuals from this experiment and comment on the underlying assumptions and model adequacy.

Sham	PEMF 1 h/day	PEMF 2 h/day	PEMF 4 h/day
4.51	5.32	4.73	7.03
7.95	6.00	5.81	4.65
4.97	5.12	5.69	6.65
3.00	7.08	3.86	5.49
7.97	5.48	4.06	6.98
2.23	6.52	6.56	4.85
3.95	4.09	8.34	7.26
5.64	6.28	3.01	5.92
9.35	7.77	6.71	5.58
6.52	5.68	6.51	7.91
4.96	8.47	1.70	4.90
6.10	4.58	5.89	4.54
7.19	4.11	6.55	8.18
4.03	5.72	5.34	5.42
2.72	5.91	5.88	6.03
9.19	6.89	7.50	7.04
5.17	6.99	3.28	5.17
5.70	4.98	5.38	7.60
5.85	9.94	7.30	7.90
6.45	6.38	5.46	7.91

3.3R An article in the *ACI Materials Journal* (Vol. 84, 1987, pp. 213–216) describes several experiments investigating the rodding of concrete to remove entrapped air. A 3-inch × 6-inch cylinder was used, and the number of times this rod was used is the design variable. The res ulting compressive strength of the concrete specimen is the response. The data are shown in the following table:

Rodding Level	Compressive Strength		
10	1530	1530	1440
15	1610	1650	1500
20	1560	1730	1530
25	1500	1490	1510

(a) Is there any difference in compressive strength due to the rodding level? Use $\alpha = 0.05$.

(b) Find the P-value for the F-statistic in part (a).

(c) Analyze the residuals from this experiment. What conclusions can you draw about the underlying model assumptions?

(d) Construct a graphical display to compare the treatment means as described in Section 3.5.3.

3.4R Four different designs for a digital computer circuit are being studied to compare the amount of noise present. The following data have been obtained:

Circuit Design	Noise Observed				
1	19	20	19	30	8
2	80	61	73	56	80
3	47	26	25	35	50
4	95	46	83	78	97

(a) Is the same amount of noise present for all four designs? Use $\alpha = 0.05$.

(b) Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?

(c) Which circuit design would you select for use? Low noise is best.

3.5R An experiment was performed to investigate the effectiveness of five insulating materials. Four samples of each material were tested at an elevated voltage level to accelerate the time to failure. The failure times (in minutes) are shown below:

Material	Failure Time (minutes)			
1	110	157	194	178
2	1	2	4	18
3	880	1256	5276	4355
4	495	7040	5307	10,050
5	7	5	29	2

(a) Do all five materials have the same effect on mean failure time?

(b) Plot the residuals versus the predicted response. Construct a normal probability plot of the residuals. What information is conveyed by these plots?

(c) Based on your answer to part (b), conduct another analysis of the failure time data and draw appropriate conclusions.

3.6R A manufacturer suspects that the batches of raw material furnished by his supplier differ significantly in calcium content. There are a large number of batches currently in the warehouse. Five of these are randomly selected for study. A chemist makes five determinations on each batch and obtains the following data:

Batch 1	Batch 2	Batch 3	Batch 4	Batch 5
23.46	23.59	23.51	23.28	23.29
23.48	23.46	23.64	23.40	23.46
23.56	23.42	23.46	23.37	23.37
23.39	23.49	23.52	23.46	23.32
23.40	23.50	23.49	23.39	23.38

(a) Is there significant variation in calcium content from batch to batch? Use $\alpha = 0.05$.

(b) Estimate the components of variance.

(c) Find a 95 percent confidence interval for $\sigma_\tau^2/(\sigma_\tau^2 + \sigma^2)$.

(d) Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?

(e) Use the REML method to analyze this data. Compare the 95 percent confidence interval on the error variance from REML with the exact chi-square confidence interval.

3.7R An article in the *Journal of Quality Technology* (Vol. 13, No. 2, 1981, pp. 111–114) describes an experiment that investigates the effects of four bleaching chemicals on pulp brightness. These four chemicals were selected at random from a large population of potential bleaching agents. The data are as follows:

Oven	Temperature				
1	77.199	74.466	92.746	76.208	82.876
2	80.522	79.306	81.914	80.346	73.385
3	79.417	78.017	91.596	80.802	80.626
4	78.001	78.358	77.544	77.364	77.386

(a) Is there a difference in the chemical types? Use $\alpha = 0.05$.

(b) Estimate the variability due to chemical types.

(c) Estimate the variability due to random error.

(d) Analyze the residuals from this experiment and comment on model adequacy.

3.8R Consider the single-factor random effects model discussed in this chapter. Develop a procedure for finding a $100(1-\alpha)$ percent confidence interval on the ratio $\sigma^2/(\sigma_\tau^2 + \sigma^2)$. Assume that the experiment is balanced.

3.9R A textile mill has a large number of looms. Each loom is supposed to provide the same output of cloth per minute. To investigate this assumption, five looms are chosen at random, and their output is noted at different times. The following data are obtained:

Loom	Output (lb/min)				
1	14.0	14.1	14.2	14.0	14.1
2	13.9	13.8	13.9	14.0	14.0
3	14.1	14.2	14.1	14.0	13.9
4	13.6	13.8	14.0	13.9	13.7
5	13.8	13.6	13.9	13.8	14.0

(a) Explain why this is a random effects experiment. Are the looms equal in output? Use $\alpha = 0.05$.

(b) Estimate the variability between looms.

(c) Estimate the experimental error variance.

(d) Find a 95 percent confidence interval for $\sigma_\tau^2/(\sigma_\tau^2 + \sigma^2)$.

(e) Analyze the residuals from this experiment. Do you think that the analysis of variance assumptions are satisfied?

(f) Use the REML method to analyze this data. Compare the 95 percent confidence interval on the error variance from REML with the exact chi-square confidence interval.

3.10R An experiment with a single factor has been conducted as a completely randomized design and analyzed using computer software. A portion of the output is shown below.

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Source   DF       SS      MS      F
Factor    ?        ?   25.69   3.65
Error    12    84.35       ?
Total    15   161.42
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(a) Fill in the missing information.

(b) How many levels of the factor were used in this experiment?

(c) How many replicates were used in this experiment?

(d) Find bounds on the P-value.

3.11R An experiment with a single factor (A) was run as a completely randomized design (CRD). A portion of the ANOVA for this experiment is shown below.

Response: y

ANOVA for Selected Model

Source	Sum of Squares	DF	Mean Square	F Value
A	______	___	______	______
Pure Error	825.00	25	______	
Cor Total	1516.67	29		

(e) Complete the analysis of variance table.

(f) The number of treatments or levels of factor A is _________.

(g) The number of replicates in this experiment is __________.

(h) Does the test statistic computed in the ANOVA table indicate that there is a statistically significant difference (using $\alpha = 0.05$) among the means of the levels of factor A?

Yes No (Place an open circle around the **correct** answer)

(i) The *P*-value for the ANOVA *F*- statistic lies between _______ and __________.

(j) Suppose that the actual *P*-value is 0.0073. Does this mean that the probability that the null hypothesis of equal treatment means is true is 0.0073?

Yes **No** (Place an open circle around the **correct** answer)

3.12R Reconsider the experiment from reserve problem 1 above. Suppose that two of the treatment means are $\bar{y}_{1.} = 101.83$ and $\bar{y}_{2.} = 109.83$. Test the hypothesis that these two treatment means are equal (assuming a two-sided alternative). Hint: the number of degrees of freedom for this test equals the number of error degrees of freedom in the ANOVA table from problem 1. Report your conclusions at a five percent significance level.

3.13R The square root of the residual mean square in the ANOVA estimates the standard deviation of an observation from any factor level. **True** **False**

3.14R The basic ANOVA treats both quantitative and qualitative factors identically so far as the sums of squares calculations are concerned. **True** **False**

3.15R Fisher's LSD is a multiple comparison procedure for comparing pairs of treatment means. **True** **False**

3.16R The analysis of variance is very sensitive to the assumption of statistical independence of the observations and treatments. **True** **False**

3.17R The assumption of normality in the analysis of variance is not a very important assumption. **True** **False**

3.18R There is no effective way to check the validity of the assumption of normality of the observations in a designed experiment. **True** **False**

3.19R Cochran's theorem establishes the independence of the mean squares on the right hand side of the ANOVA partitioning. **True** **False**

3.20R An experiment was conducted with four levels of a single fixed factor. The data from this experiment follows:

Level 1: 23, 24, 27, 22

Level 2: 26, 25, 30, 29

Level 3: 20, 21, 18, 23

Level 4: 27, 31, 32, 19

(a) Is there a significant difference in means between the levels of this factor?

(b) Use the Fisher LSD method to determine exactly which means differ.

(c) Analyze the residuals from this experiment. Do you see any problems or possible violations of the assumptions?

3.21R An experiment was conducted with three levels of a single factor. Assume that the levels of this factor were chosen at random. The data from this experiment follow:

Level 1: 498, 493, 485

Level 2: 488, 484, 479, 477, 480

Level 3: 499, 502, 506, 510

(a) Is there a statistically significant difference in the factor levels?

(b) Estimate the variance components using the ANOVA (method of moments) method.

(c) Estimate the variance components using the REML method. Compare these estimates with those you found in part b above.

3.22R A single-factor completely randomized design has five levels of the factor and six replicates. The total sum of squares is 1000.00 and the treatment sum of squares is 850.25.

(a) What is the estimate of the error variance?

(b) What proportion of the variability in the response variable is explained by the treatment effect?