# CHAPTER 9

# Reserved Problems

**9.1R** A medical researcher is studying the effect of lidocaine on the enzyme level in the heart muscle of beagle dogs. Three different commercial brands of lidocaine ($A$), three dosage levels ($B$), and three dogs ($C$) are used in the experiment, and two replicates of a $3^3$ factorial design are run. The observed enzyme levels follow. Analyze the data from this experiment.

| | | Replicate I | | |
|---|---|---|---|---|
| | | Dog | | |
| Lidocaine Brand | Dosage Strength | 1 | 2 | 3 |
| | 1 | 96 | 84 | 85 |
| 1 | 2 | 94 | 99 | 98 |
| | 3 | 101 | 106 | 98 |
| | 1 | 85 | 84 | 86 |
| 2 | 2 | 95 | 98 | 97 |
| | 3 | 108 | 114 | 109 |
| | 1 | 84 | 83 | 81 |
| 3 | 2 | 95 | 97 | 93 |
| | 3 | 105 | 100 | 106 |

| | | Replicate II | | |
|---|---|---|---|---|
| | | Dog | | |
| | | 1 | 2 | 3 |
| | 1 | 84 | 85 | 86 |
| 1 | 2 | 95 | 97 | 90 |
| | 3 | 105 | 104 | 103 |
| | 1 | 80 | 82 | 84 |
| 2 | 2 | 93 | 99 | 95 |
| | 3 | 110 | 102 | 100 |
| | 1 | 83 | 80 | 79 |
| 3 | 2 | 92 | 96 | 93 |
| | 3 | 102 | 111 | 108 |

**9.2R** Compute the $I$ and $J$ components of the two-factor interactions for Problem 9.1R.

**9.3R** An article by W. D. Baten in the 1956 volume of *Industrial Quality Control* described an experiment to study the effect of three factors on the lengths of steel bars. Each bar was subjected to one of two heat treatment processes and was cut on one of four machines at one of three times during the day (8 A.M., 11 A.M., or 3 P.M.). The coded length data are as follows:

| Time of Day | Process Heat Treatment Process | Machine 1 | | Machine 2 | | Machine 3 | | Machine 4 | |
|---|---|---|---|---|---|---|---|---|---|
| 8 A.M.. | 1 | 6 | 9 | 7 | 9 | 1 | 2 | 6 | 6 |
| | | 1 | 3 | 5 | 5 | 0 | 4 | 7 | 3 |
| | 2 | 4 | 6 | 6 | 5 | −1 | 0 | 4 | 5 |
| | | 0 | 1 | 3 | 4 | 0 | 1 | 5 | 4 |
| 11 A.M. | 1 | 6 | 3 | 8 | 7 | 3 | 2 | 7 | 9 |
| | | 1 | −1 | 4 | 8 | 1 | 0 | 11 | 6 |
| | 2 | 3 | 1 | 6 | 4 | 2 | 0 | 9 | 4 |
| | | 1 | −2 | 1 | 3 | −1 | 1 | 6 | 3 |
| 3 P.M. | 1 | 5 | 4 | 10 | 11 | −1 | 2 | 10 | 5 |
| | | 9 | 6 | 6 | 4 | 6 | 1 | 4 | 8 |
| | 2 | 6 | 0 | 8 | 7 | 0 | −2 | 4 | 3 |
| | | 3 | 7 | 10 | 0 | 4 | −4 | 7 | 0 |

**(a)** Analyze the data from this experiment, assuming that the four observations in each cell are replicates.

**(b)** Analyze the residuals from this experiment. Is there any indication that there is an outlier in one cell? If you find an outlier, remove it and repeat the analysis from part (a). What are your conclusions?

**(c)** Suppose that the observations in the cells are the lengths (coded) of bars processed together in heat treatment and then cut sequentially (that is, in order) on the four machines. Analyze the data to determine the effects of the three factors on mean length.

**(d)** Calculate the log variance of the observations in each cell. Analyze this response. What conclusions can you draw?

**(e)** Suppose the time at which a bar is cut really cannot be controlled during routine production. Analyze the average length and the log variance of the length for each of the 12 bars cut at each machine/heat treatment process combination. What conclusions can you draw?

**9.4R** Reconsider the experiment in Problem 9.3R. Suppose that it was necessary to estimate all main effects and two-factor interactions, but the full factorial with 24 runs (not counting replication) was too expensive. Recommend an alternative design.

**9.5R** Suppose that you need to design an experiment for five factors. Three of these factors are categorical with three levels and the remaining two factors are continuous. You are interested in the main effects and the two-factor interactions of all factors, along with the pure quadratic effects of the two continuous factors.

**(a)** How many runs are required for this experiment?

**(b)** Construct a *D*-optimal design with the minimum number of runs. What are the relative variances of the model parameters?

**(c)** Construct an *I*-optimal design with the minimum number of runs. What are the relative variances of the model parameters? How does this design compare to the *D*-optimal design you constructed in part (b)?

**9.6R** Suppose that you want to design an experiment for nine continuous two-level factors but you can only afford 12 runs. A coworker suggests that you should choose 9 columns of the 11 columns from the 12-run Plackett–Burman design.

**(a)** How many possible designs are there?

**(b)** Are all of the designs from part (a) the same? Or does it matter which subset of the 11 columns of the Plackett–Burman design you choose?

**(c)** Construct a *D*-optimal design for this problem. Should this design be one of the possible choices you identified in part (a)?

**9.7R** The concentration of a substance in a chemical product is thought to be affected by the time and temperature the product is exposed to during a particular stage of processing. A $3^2$ factorial experiment with two replicates is conducted to study this issue. The experimental results follow.

| | **Temperature (°C)** | | |
|---|---|---|---|
| **Time (Min)** | **200** | **225** | **250** |
| 20 | 20, 22 | 21, 23 | 22, 25 |
| 25 | 24, 26 | 27, 28 | 28, 30 |
| 30 | 28, 29 | 30, 28 | 32, 30 |

Analyze the data from this experiment and draw conclusions.

**9.8R** Compute the *I* and *J* components of the two-factor interaction in reserve problem 1 above.

**9.9R** Fit a second-order model to the data from reserve problem 9.7R. Are all terms in the model significant?

**9.10R** Reconsider reserve problem 9.7R. Suppose that this experiment had been run in blocks. Set up the design with the *AB* component of the two-factor interaction confounded with blocks. Compare this with the design in Example 9.2, where the $AB^2$ interaction is confounded with blocks. Assume that only one replicate was run (the first observation in each cell) and assign these observations to the blocks. Analyze the data.

**9.11R** Consider the $3^4$ factorial design. Construct this design in 3 blocks confounding $A^2BCD$ with blocks.

**9.12R** Construct a $3^{4-1}$ design using $I = ABC^2D$. Write out the alias relationships for this design.

**9.13R** It is necessary to design an experiment for three continuous two-level factors and two categorical three-level factors. It is important to be able to estimate all main effects and two-factor interactions. What is the smallest number of runs that you can use? Construct this design using the *D* criterion. What are the relative variances of the model parameters?

**9.14R** Reconsider the design problem in problem 9.13R. Construct the *I* optimal design and find the relative variances of the model parameters. Compare this design to the *D* optimal design from problem 9.13R.

**9.15R** You need to design an experiment for four continuous factors. You want to estimate all of the main effects but you are only interested in the *AB* and *BC* interactions. Construct a *D*-optimal design with the minimum number of runs. What are the relative variances of the model parameters?

**9.16R** You need to design an experiment with eight two-level factors and you can only afford to use 12 runs. A colleague suggests that you use 8 columns from the 12-run Plackett-Burman design.

**(a)** How many possible designs are there?

**(b)** Are all of these designs equivalent in some sense?

**(c)** Construct one of these design. What are the relative variances of the coefficients?

**(d)** Find the alias matrix for this design assuming that all two-factor interactions should be considered as potential aliased terms.

**(e)** Find a *D*-optimal design for the experiment. Compare this to the design you selected in part c above.