CHAPTER 8

Reserved Problems

8.1R Suppose that in Problem 6.3R, only a one-half fraction of the 2^4 design could be run. Construct the design and perform the analysis, using the data from replicate I.

8.2R R. D. Snee ("Experimenting with a Large Number of Variables," in *Experiments in Industry: Design, Analysis and Interpretation of Results*, by R. D. Snee, L. B. Hare, and J. B. Trout, Editors, ASQC, 1985) describes an experiment in which a 2^{5-1} design with $I = ABCDE$ was used to investigate the effects of five factors on the color of a chemical product. The factors are A = solvent/reactant, B = catalyst/reactant, C = temperature, D = reactant purity, and E = reactant pH. The responses obtained are as follows:

$e = -0.63$	$d = 6.79$
$a = 2.51$	$ade = 5.47$
$b = -2.68$	$bde = 3.45$
$abe = 1.66$	$abd = 5.68$
$c = 2.06$	$cde = 5.22$
$ace = 1.22$	$acd = 4.38$
$bce = -2.09$	$bcd = 4.30$
$abc = 1.93$	$abcde = 4.05$

(a) Prepare a normal probability plot of the effects. Which effects seem active?

(b) Calculate the residuals. Construct a normal probability plot of the residuals and plot the residuals versus the fitted values. Comment on the plots.

(c) If any factors are negligible, collapse the 2^{5-1} design into a full factorial in the active factors. Comment on the resulting design, and interpret the results.

8.3R An article in *Industrial and Engineering Chemistry* ("More on Planning Experiments to Increase Research Efficiency," 1970, pp. 60–65) uses a 2^{5-2} design to investigate the effect of A = condensation temperature, B = amount of material 1, C = solvent volume, D = condensation time, and E = amount of material 2 on yield. The results obtained are as follows:

$e = 23.2$	$ad = 16.9$	$cd = 23.8$	$bde = 16.8$
$ab = 15.5$	$bc = 16.2$	$ace = 23.4$	$abcde = 18.1$

(a) Verify that the design generators used were $I = ACE$ and $I = BDE$.

(b) Write down the complete defining relation and the aliases for this design.

(c) Estimate the main effects.

(d) Prepare an analysis of variance table. Verify that the AB and AD interactions are available to use as error.

(e) Plot the residuals versus the fitted values. Also construct a normal probability plot of the residuals. Comment on the results.

8.4R Construct a 2^{7-2} design by choosing two four-factor interactions as the independent generators. Write down the complete alias structure for this design. Outline the analysis of variance table. What is the resolution of this design?

8.5R Construct a 2_{III}^{6-3} design. Determine the effects that may be estimated if a full fold over of this design is performed.

8.6R Consider the 2_{III}^{6-3} design in Problem 8.5R. Determine the effects that may be estimated if a single factor fold over of this design is run with the signs for factor A reversed.

8.7R Consider the isatin yield data from the experiment described in Problem 6.8R. The original experiment was a 2^4 full factorial. Suppose that the original experimenters could only afford eight runs. Set up the 2^{4-1} fractional factorial design with $I = ABCD$ and select the responses for the runs from the full factorial data in Problem 6.8R. Analyze the data and draw conclusions. Compare your findings with those from the full factorial in Problem 6.8R.

8.8R An article in *Thin Solid Films* (504, "A Study of Si/SiGe Selective Epitaxial Growth by Experimental Design

Approach," 2006, Vol. 504, pp. 95–100) describes the use of a fractional factorial design to investigate the sensitivity of low-temperature (740–760°C) Si/SiGe selective epitaxial growth to changes in five factors and their two-factor interactions. The five factors are SiH_2Cl_2, GeH_4, HCl, B_2H_6, and temperature. The factor levels studied are as follows:

Factors	Levels (−)	Levels (+)
SiH_2Cl_2 (sccm)	8	12
GeH_4 (sccm)	7.2	10.8
HCl (sccm)	3.2	4.8
B_2H_6 (sccm)	4.4	6.6
Temperature (°C)	740	760

Table P8.1R contains the design matrix and the three measured responses. Bede RADS Mercury software based on the Takagi–Taupin dynamical scattering theory was used to extract the Si cap thickness, SiGe thickness, and Ge concentration of each sample.

(a) What design did the experimenters use? What is the defining relation?

(b) Will the experimenters be able to estimate all main effects and two-factor interactions with this experimental design?

(c) Analyze all three responses and draw conclusions.

(d) Is there any indication of curvature in the responses?

(e) Analyze the residuals and comment on model adequacy.

8.9R An unreplicated 2^{4-1} fractional factorial experiment with four center points has been run. The experimenter has used the following factors:

Factor	Natural Levels	Coded Levels (x's)
***A*—time**	10, 50 (minutes)	−1, 1
***B*—temperature**	200, 300 (deg C)	−1, 1
***C*—concentration**	70, 90 (percent)	−1, 1
***D*—pressure**	260, 300 (psi)	−1, 1

■ **TABLE P8.1R**
The Epitaxial Growth Experiment in Problem 8.8R

Run Order	Factors *A*	*B*	*C*	*D*	*E*	Si Cap Thickness (Å)	SiGe Thickness (Å)	Ge Concentration (at.%)
7	−	−	−	−	+	371.18	475.05	8.53
17	−	−	−	+	−	152.36	325.21	9.74
6	−	−	+	−	−	91.69	258.60	9.78
10	−	−	+	+	+	234.48	392.27	9.14
16	−	+	−	−	−	151.36	440.37	12.13
2	−	+	−	+	+	324.49	623.60	10.68
15	−	+	+	−	+	215.91	518.50	11.42
4	−	+	+	+	−	97.91	356.67	12.96
9	+	−	−	−	−	186.07	320.95	7.87
13	+	−	−	+	+	388.69	487.16	7.14
18	+	−	+	−	+	277.39	422.35	6.40
5	+	−	+	+	−	131.25	241.51	8.54
14	+	+	−	−	+	378.41	630.90	9.17
3	+	+	−	+	−	192.65	437.53	10.35
1	+	+	+	−	−	128.99	346.22	10.95
12	+	+	+	+	+	298.40	526.69	9.73
8	0	0	0	0	0	215.70	416.44	9.78
11	0	0	0	0	0	212.21	419.24	9.80

(a) Suppose that the average of the 16 factorial design points is 100 and the average of the center points is 120, what is the sum of squares for pure quadratic curvature?

(b) Suppose that the prediction equation that results from this experiment is $\hat{y} = 50 + 5x_1 + 2x_2 - 2x_1x_2$. Find the predicted response at $A = 20$, $B = 250$, $C = 80$, and $D = 275$.

8.10R A 16-run fractional factorial experiment in 10 factors on sand-casting of engine manifolds was conducted by engineers at the Essex Aluminum Plant of the Ford Motor Company and described in the article "Evaporative Cast Process 3.0 Liter Intake Manifold Poor Sandfill Study," by D. Becknell (*Fourth Symposium on Taguchi Methods*, American Supplier Institute, Dearborn, MI, 1986, pp. 120–130). The purpose was to determine which of 10 factors has an effect on the proportion of defective castings. The design and the resulting proportion of nondefective castings $\hat{p}$ observed on each run are shown in Table P8.2R. This is a resolution III fraction with generators $E = CD$, $F = BD$, $G = BC$, $H = AC$, $J = AB$, and $K = ABC$. Assume that the number of castings made at each run in the design is 1000.

(a) Find the defining relation and the alias relationships in this design.

(b) Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

(c) Fit an appropriate model using the factors identified in part (b).

(d) Plot the residuals from this model versus the predicted proportion of nondefective castings. Also prepare a normal probability plot of the residuals. Comment on the adequacy of these plots.

(e) In part (d) you should have noticed an indication that the variance of the response is not constant. (Considering that the response is a proportion, you should have expected this.) The previous table also shows a transformation on $\hat{p}$, the arcsin square root, that is a widely used *variance stabilizing transformation* for proportion data (refer to the discussion of variance stabilizing transformations in Chapter 3). Repeat parts (a) through (d) using the transformed response and comment on your results. Specifically, are the residual plots improved?

(f) There is a modification to the arcsin square root transformation, proposed by Freeman and Tukey ("Transformations Related to the Angular and the Square Root," *Annals of Mathematical Statistics*, Vol. 21, 1950, pp. 607–611), that improves its performance in the tails. F&T's modification is

$$\begin{aligned}&[\arcsin \sqrt{n\hat{p}/(n+1)}\\&+ \arcsin \sqrt{(n\hat{p}+1)/(n+1)}]/2\end{aligned}$$

■ **TABLE P8.2R**
The Sand-Casting Experiment in Problem 8.10R

Run	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*	$\hat{p}$	Arcsin $\sqrt{\hat{p}}$	F&T's Modification
1	−	−	−	−	+	+	+	+	+	−	0.958	1.364	1.363
2	+	−	−	−	+	+	+	−	−	+	1.000	1.571	1.555
3	−	+	−	−	+	−	−	+	−	+	0.977	1.419	1.417
4	+	+	−	−	+	−	−	−	+	−	0.775	1.077	1.076
5	−	−	+	−	−	+	−	−	+	+	0.958	1.364	1.363
6	+	−	+	−	−	+	−	+	−	−	0.958	1.364	1.363
7	−	+	+	−	−	−	+	−	−	−	0.813	1.124	1.123
8	+	+	+	−	−	−	+	+	+	+	0.906	1.259	1.259
9	−	−	−	+	−	−	+	+	+	−	0.679	0.969	0.968
10	+	−	−	+	−	−	+	−	−	+	0.781	1.081	1.083
11	−	+	−	+	−	+	−	+	−	+	1.000	1.571	1.556
12	+	+	−	+	−	+	−	−	+	−	0.896	1.241	1.242
13	−	−	+	+	+	−	−	−	+	+	0.958	1.364	1.363
14	+	−	+	+	+	−	−	+	−	−	0.818	1.130	1.130
15	−	+	+	+	+	+	+	−	−	−	0.841	1.161	1.160
16	+	+	+	+	+	+	+	+	+	+	0.955	1.357	1.356

Rework parts (a) through (d) using this transformation and comment on the results. (For an interesting discussion and analysis of this experiment, refer to "Analysis of Factorial Experiments with Defects or Defectives as the Response," by S. Bisgaard and H. T. Fuller, *Quality Engineering*, Vol. 7, 1994–95, pp. 429–443.)

8.11R How could an "optimal design" approach be used to augment a fractional factorial design to de-alias effects of potential interest?

8.12R 2^{k-1} fractional factorial contains full factorials in every subset of k – 1 of the original k factors. This is called ______________________________.

8.13R When there are several variables, the system or process is likely to be driven primarily by some of the main effects and some low-order interactions. This idea is referred to as ______________________________.

8.14R Consider the 2^{5-1} fractional factorial design with generator E = ABC. This design is of resolution ______________________________.

8.15R If an interaction say AB is significant we usually include both main effects A and B in the model. This is called ______________________________.

8.16R An experimenter has conducted a 2^{5-1} fractional factorial experiment. The test matrix and the resulting response observations are shown below.

Run	A	B	C	D	E	Y	Block
1	−1	−1	−1	−1	−1	63	
2	1	−1	−1	−1	1	21	
3	−1	1	−1	−1	1	36	
4	1	1	−1	−1	−1	99	
5	−1	−1	1	−1	1	24	
6	1	−1	1	−1	−1	66	
7	−1	1	1	−1	−1	71	
8	1	1	1	−1	1	54	
9	−1	−1	−1	1	1	23	
10	1	−1	−1	1	−1	74	
11	−1	1	−1	1	−1	80	
12	1	1	−1	1	1	33	
13	−1	−1	1	1	−1	63	
14	1	−1	1	1	1	21	
15	−1	1	1	1	1	44	
16	1	1	1	1	−1	96	

(a) The resolution of the design is __________________.

(b) What are the design generators that have been used to construct this design? What is the complete defining relation?

(c) Write down the aliases, showing only main effects and two-factor interactions.

(d) Suppose that after analyzing the data, the two largest effects were B and E. Dropping the negligible factors results in a ______________________ design in factors B and E. (Fill in the blank with the type of design).

(e) Suppose that after analyzing the data, the three largest effects were A, C, and E. Dropping the negligible factors results in a ______________________ design in factors A, C, and E. (Fill in the blank with the type of design).

(f) Suppose that the original experiment would only allow eight runs to be made on each day. You wish to consider using the two days as two blocks. What factor (and its aliases) would you choose to confound with blocks? Use the column labeled "Blocks" in the design matrix above to identify which runs would be assigned to block 1 and which runs would be assigned to block 2.

(g) Suppose it was possible to conduct only 8 runs for the initial design. What design would you recommend (it is not necessary to write out the test matrix)? What will be the resolution of the design and what will be the alias structure?

8.17R Consider reserve problem 5. Suppose that the experimenters had included $n_C = 4$ center points. The average of the four runs at the center is 1520. If the sample standard deviation of these four center points is 10, conduct a test for second-order curvature in the model. What are the results of this test?

8.18R The resolution of a two-level fractional factorial design is the number of words in the defining relation.

(a) True

(b) False

8.19R With a resolution R design, main effects are confounded with interactions of order R-1, two factor interactions are confounded with interactions of order R-2, and so forth.

(a) True

(b) False

8.20R For a half fraction of a two-level factorial design the maximum resolution possible is equal to the number of factors.

(a) True

(b) False

8.21R is a good practice to keep the number of factor levels low and region of interest small in a screening experiment.

(a) True

(b) False

8.22R In a resolution IV design

(a) all main effects are aliased with three-factor interactions

(b) all two factor interactions are aliased with at least one other two-factor interaction

(c) main effects and two-factor interactions are clear of each other

(d) all of the above statements are true

(e) none of the above statements are true

8.23R In the 2^{7-3} fractional factorial design with $E = ABC$, $F = BCD$, and $G = ACD$, if factors A, B, and C are dropped, the remaining factors form

(a) an unreplicated full factorial design

(b) a fractional factorial design in four factors

(c) a replicated full factorial design in 16 runs

(d) none of the above statements are true

8.24R In a resolution III design

(a) at least one main effect is aliased with some two-factor interactions

(b) some two factor interactions may be aliased with other two-factor interactions

(c) may be a good choice as a screening design if there are lots of factors

(d) all of the above statements are true

(e) none of the above statements are true

8.25R When there are several variables, the system or process is likely to be driven primarily by some of the main effects and low interactions. This idea is referred to as:

(a) The blocking effect

(b) The projection property

(c) The sparsity of effects principle

(d) Sequential experimentation

8.26R An experimenter has conducted a 2^{5-2} fractional factorial experiment. The test matrix and the resulting response observations are shown below. The response variable is camber of the substrate.

Std	**A**	**B**	**C**	**D**	**E**	**Block (1 or 2)**	**Y**
1	−1	−1	−1	−1	1		50
2	1	−1	−1	1	1		21
3	−1	1	−1	1	−1		45
4	1	1	−1	−1	−1		25
5	−1	−1	1	1	−1		43
6	1	−1	1	−1	−1		33
7	−1	1	1	−1	1		45
8	1	1	1	1	1		34

(a) Write the design generators that have been used to construct this design.

(b) What is the complete defining relation?

(c) What is the resolution of this design?

(d) Calculate the effect of A and write the aliases for this main effect.

(e) Calculate the sum of squares for A.

(f) If this experiment had been run in 2 blocks, confounding the effect of AB with block effect, fill in the column indicated in table with the number of the block for each run.

(g) Suppose that the experiment was run in 4 blocks. Set up the design so that the main effects are not confounded with the block effects. What effects are confounded with blocks?

Write the number of the block for each run for the design that you choose.

Std	A	B	C	D	E	Block (1, 2, 3 or 4)
1	−1	−1	−1	−1	1	
2	1	−1	−1	1	1	
3	−1	1	−1	1	−1	
4	1	1	−1	−1	−1	
5	−1	−1	1	1	−1	
6	1	−1	1	−1	−1	
7	−1	1	1	−1	1	
8	1	1	1	1	1	

(h) Suppose that after analyzing the data from the original design with no blocking, the two largest effects were A and C. Dropping the negligible factors results in a ______________________ design in factors A and C. (Fill in the blank with the type of design)

(i) Is there a way to obtain a better design for 5 factors in 8 runs? If so, the generator(s) for that design would be _____________.

(j) Write down a model involving all five main effects that you could use to predict the response over the design space. Write this model in the coded factors.

(k) Use this model to predict the response at the point in the design space where the first four factors are at the high level and factor E is at the middle of its range.

(l) Assume that the factors that should be included in the model are A, D, and AD. Write this model using the coded factors.