CHAPTER 4

Reserved Problems

4.1R The ANOVA from a randomized complete block experiment output is shown below.

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

(a) Fill in the blanks. You may give bounds on the *P*-value.

(b) How many blocks were used in this experiment?

(c) What conclusions can you draw?

4.2R An article in the *Fire Safety Journal* ("The Effect of Nozzle Design on the Stability and Performance of Turbulent Water Jets," Vol. 4, August 1981) describes an experiment in which a shape factor was determined for several different nozzle designs at six levels of jet efflux velocity. Interest focused on potential differences between nozzle designs, with velocity considered as a nuisance variable. The data are shown below:

Nozzle Design	Jet Efflux Velocity (m/s)					
	11.73	**14.37**	**16.59**	**20.43**	**23.46**	**28.74**
1	0.78	0.80	0.81	0.75	0.77	0.78
2	0.85	0.85	0.92	0.86	0.81	0.83
3	0.93	0.92	0.95	0.89	0.89	0.83
4	1.14	0.97	0.98	0.88	0.86	0.83
5	0.97	0.86	0.78	0.76	0.76	0.75

(a) Does nozzle design affect the shape factor? Compare the nozzles with a scatter plot and with an analysis of variance, using $\alpha = 0.05$.

(b) Analyze the residuals from this experiment.

(c) Which nozzle designs are different with respect to shape factor? Draw a graph of the average shape factor for each nozzle type and compare this to a scaled *t* distribution. Compare the conclusions that you draw from this plot to those from Duncan's multiple range test.

4.3R ***Two missing values in a randomized block.*** Suppose that in Problem 4.7 the observations for chemical type 2 and bolt 3 and chemical type 4 and bolt 4 are missing.

(a) Analyze the design by iteratively estimating the missing values, as described in Section 4.1.3.

(b) Differentiate SS_E with respect to the two missing values, equate the results to zero, and solve for estimates of the missing values. Analyze the design using these two estimates of the missing values.

(c) Derive general formulas for estimating two missing values when the observations are in *different* blocks.

(d) Derive general formulas for estimating two missing values when the observations are in the *same* block.

4.4R ***Extended incomplete block designs.*** Occasionally, the block size obeys the relationship $a < k < 2a$. An extended incomplete block design consists of a single replicate of each treatment in each block along with an incomplete block design with $k^* = k - a$. In the balanced case, the incomplete block design will have parameters $k^* = k - a$, $r^* = r - b$, and λ^*. Write out the statistical analysis. (*Hint:* In the extended incomplete block design, we have $\lambda = 2r - b + \lambda^*$.)

4.5R Consider the direct mail marketing experiment in Problem 4.12. Suppose that this experiment had been run as a completely randomized design, ignoring potential regional differences, but that exactly the same data were obtained. Reanalyze the experiment under this new assumption. What difference would ignoring blocking have on the results and conclusions?

4.6R Suppose that a single factor experiment with 4 levels of the factor has been conducted. There are 6 replicates and the experiment has been conducted in blocks. The error sum of squares is 500 and the block sum of squares is 250. If the experiment had been conducted as a completely randomized design the estimate of the error variance σ^2 would be

(a) 25.0 **(b)** 25.5 **(c)** 35.0

(d) 37.5 **(e)** None of the above

4.7R A single factor experiment has been conducted with 4 levels of the factor and 5 replicates. The total sum of squares is 1500, the error sum of squares is 300, and the block sum of squares is 400. Construct the appropriate analysis of variance table. What conclusions would you draw about the equality of treatment means?

4.8R Consider Problem 3.16 in the textbook. Suppose that this experiment could not be easily run as a CRD because the material required is only available in small batches. Assume that each column in the data table represents observations taken using material from a specific batch.

(a) This turns the experiment into what type of design?

(b) Show the ANOVA for this experiment. What conclusions can you draw about dosage?

4.9R In a randomized complete block design the number of treatments and blocks must be equal. **True** **False**

4.10R In a randomized complete block design, if the blocks are chosen randomly from a population of blocks, the conclusions extend to the population of blocks. **True** **False**

4.11R Consider the chocolate and cardovascular health experiment in Section 3.8.1 of the textbook. The analysis of this experiment in Chapter 3 assumed that this was a completely randomized design. Suppose that the experiment was actually conducted in blocks, with each subject considered a block. Analyze this experiment as an RCBD and draw conclusions. Compere your results with those given in Section 3.8.1.

4.12R Consider the following RCBD:

	Blocks			
Factor Levels	**1**	**2**	**3**	**4**
1	120	125	110	116
2	107	111	101	103
3	113	109	99	98

Analyze the data from this experiment and draw conclusions.

4.13R Reconsider the experiment in reserve problem 4.12R above.

(a) Assume that blocks are random. Find the ANOVA estimates of the variance components.

(b) Find the REML estimates of the variance components.

4.14R Reconsider the experiment in reserve problem 4.12R. Suppose tha the observation for factor level 3 in block 2 (111) is missing.

(a) Estimate the missing value and analyze the resulting data. Compare your results with the analysis where all observations are known.

(b) Analyze the data using the exact method. Compare your results with the analysis using an estimate of the missing value and the results where all observations are known.

4.15R Suppose that you need to design an experiment for 4 treatment levels. However, the block size can only be 3. There is only enough time and material to run 6 blocks. Set up a D-optimal design for this experiment. Is the resulting design a balanced incomplete block design?