

CHAPTER 2

Reserved Problems

2.1R Consider the computer output shown below.

One-Sample Z					
Test of mu = 30 vs not = 30					
The assumed standard deviation = 1.2					
N	Mean	SE Mean	95% CI	Z	P
16	31.2000	0.3000	(30.6120, 31.7880)	?	?

- Fill in the missing values in the output. What conclusion would you draw?
- Is this a one-sided or two-sided test?
- Use the output and the normal table to find a 99 percent CI on the mean.
- What is the P -value if the alternative hypothesis is $H_1 : \mu > 30$?

2.2R Consider the computer output shown below.

One-Sample T: Y							
Test of mu = 91 vs. not = 91							
Variable	N	Mean	Std. Dev.	SE Mean	95% CI	T	P
Y	25	92.5805	?	0.4673	(91.6160, ?)	3.38	0.002

- Fill in the missing values in the output. Can the null hypothesis be rejected at the 0.05 level? Why?
- Is this a one-sided or a two-sided test?
- If the hypotheses had been $H_0 : \mu = 90$ versus $H_1 : \mu \neq 90$, would you reject the null hypothesis at the 0.05 level?
- Use the output and the t table to find a 99 percent two-sided CI on the mean.
- What is the P -value if the alternative hypothesis is $H_1 : \mu > 91$?

2.3R The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of

$\sigma = 0.0001$ inch. A random sample of 10 shafts has an average diameter of 0.2545 inches.

- Set up appropriate hypotheses on the mean μ .
- Test these hypotheses using $\alpha = 0.05$. What are your conclusions?
- Find the P -value for this test.
- Construct a 95 percent confidence interval on the mean shaft diameter.

2.4R A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2$, $S_2^2 = 94.73$, $n_2 = 9$.

- Can you conclude that the two variances are equal? Use $\alpha = 0.05$.
- Has the filtering device reduced the percentage of impurity significantly? Use $\alpha = 0.05$.

2.5R Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

5.34	6.65	4.76	5.98	7.25
6.00	7.55	5.54	5.62	6.21
5.97	7.35	5.44	4.39	4.98
5.25	6.35	4.61	6.00	5.32

- Construct a 95 percent confidence interval estimate of σ^2 .
- Test the hypothesis that $\sigma^2 = 1.0$. Use $\alpha = 0.05$. What are your conclusions?
- Discuss the normality assumption and its role in this problem.
- Check normality by constructing a normal probability plot. What are your conclusions?

2.6R Why is the random sampling assumption important in statistical inference? Suppose that you had to select a random sample of 100 items from a production line. How would you

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propose to do this? Should you take into account factors such as the production rate, or whether the line operates continuously or only intermittently?

2.7R Power calculation for hypothesis testing are relatively easy to do with modern statistical software. What do you think “adequate power” should be for an experiment? What issues need to be considered in answering this question?

2.8R Suppose that you are testing the equality of two means and you can safely assume that the two variances are known. Find the P -values for the following observed values of the test statistic. Assume a two-sided alternative.

- (a) $Z_0 = 2.12$ (b) $Z_0 = 1.75$
 (c) $Z_0 = -1.34$ (d) $Z_0 = -2.00$

2.9R Suppose that you are testing the equality of two means and you cannot assume that the two variances are known, but they are likely equal. The sample sizes are $n_1 = n_2 = 15$. Find bounds on the P -values for the following observed values of the test statistic. Assume a two-sided alternative.

- (a) $t_0 = 2.23$ (b) $t_0 = 1.82$
 (c) $t_0 = -1.77$ (d) $t_0 = -2.16$

2.10R You are investigating the breaking strength of a wire that will be used in attaching a semiconductor to an external device. You have tested four samples of this type of wire and the sample results are (in psi): 139, 146, 143, and 137. You want the mean breaking strength to exceed 140 psi. Is there sufficient evidence to conclude that the mean breaking strength does exceed the specification? Construct a 95% confidence interval on the mean. Interpret the results.

2.11R The mean diameter of a shaft should be 0.25 inches. A random sample of 10 of these shafts have a sample mean diameter of 0.2512 inches. The standard deviation of diameter is known to be 0.03 inches. Is there sufficient evidence to conclude that the mean diameter meets the specification? Construct a 95% confidence interval on the mean. Interpret the results.

2.12R You are investigating the fill volume of wine bottles. A random sample of 10 bottles from a recent filling operation yields the following fill volume data (in ml): 750.1, 750.6, 750.2, 750.2, 748.6, 751.2, 749.3, 748.9, 750.4, 750.0. specification require that the mean fill volume of these bottles must meet or exceed 570.0 ml.

- (a) What are the appropriate hypotheses for this decision problem?
 (b) Calculate an appropriate test statistic.
 (c) What is the P -value for the test statistic?

(d) Is there reason to doubt the normality assumption for these data?

(e) Construct a 95% confidence interval on the mean fill volume.

2.13R Two different formulations of a chemical flare are being investigated. Ten samples of flares made from each formulation are tested for burning time in minutes with the following results: Formulation 1, sample mean = 82, sample standard deviation = 4.25; Formulation 2, sample mean = 79.8, sample standard deviation = 3.79.

- (a) Test the hypothesis that the two variances are equal.
 (b) Test the hypothesis that the mean burning times are the same.
 (c) What are the P -values for the two tests above?
 (d) Construct a 95% confidence interval on the difference in the two means.

2.14R Suppose that two levels of a factor are being compared and that the observed sample means are $\bar{y}_1 = 101.83$ and $\bar{y}_2 = 109.83$. Both variances are assumed equal and an estimate of the common variance is $S = 8.5$. Test the hypothesis that these populations two means are equal (assuming a two-sided alternative). Report your conclusions at a five percent significance level. What is the P -value for this test?

2.15R The two-sample Z -test has been used to compare the difference in two means. The alternative hypothesis is one-sided and upper-tailed. The computed value of the test statistic is $Z_0 = 2.34$. The P -value associated with this test is:

- (a) 0.01191 (b) 0.02382 (c) 0.05000
 (d) 0.02500 (e) None of the above

2.16R The two-sample t -test has been used to compare the difference between two means. The alternative hypothesis is two-sided. The sample sizes used in the experiment were $n_1 = n_2 = 15$. The computed value of the test statistic is $t_0 = 2.67$. A lower bound on the P -value is:

- (a) 0.01 (b) 0.025 (c) 0.050
 (d) 0.020 (e) None of the above

2.17R Suppose that we want to construct a 95% two-sided confidence interval on the difference in two means where the standard deviations are known to be $\sigma_1 = 3$ and $\sigma_2 = 9$. The total sample size is restricted to 40. What is the length of the 95% confidence interval if both sample sizes are the same (20)? What is the length of the 95% confidence interval if the optimal sample size allocation had been used?