CHAPTER 11

Reserved Problems

11.1R An industrial engineer has developed a computer simulation model of a two-item inventory system. The decision variables are the order quantity and the reorder point for each item. The response to be minimized is total inventory cost. The simulation model is used to produce the data shown in Table P11.1R. Identify the experimental design. Find the path of steepest descent.

■ **TABLE P11.1R**
The Inventory Experiment, Problem 11.1R

Item 1 Order Quantity (ξ_1)	Item 1 Reorder Point (ξ_2)	Item 2 Order Quantity (ξ_3)	Item 2 Reorder Point (ξ_4)	Total Cost
100	25	250	40	625
140	45	250	40	670
140	25	300	40	663
140	25	250	80	654
100	45	300	40	648
100	45	250	80	634
100	25	300	80	692
140	45	300	80	686
120	35	275	60	680
120	35	275	60	674
120	35	275	60	681

11.2R The region of experimentation for three factors are time ($40 \leq T_1 \leq 80$ min), temperature ($200 \leq T_2 \leq 300$°C), and pressure ($20 \leq P \leq 50$ psig). A first-order model in coded variables has been fit to yield data from a 2^3 design. The model is

$$\hat{y} = 30 + 5x_1 + 2.5x_2 + 3.5x_3$$

Is the point $T_1 = 85$, $T_2 = 325$, $P = 60$ on the path of steepest ascent?

11.3R The hexagon design in Table P11.2R is used in an experiment that has the objective of fitting a second-order model:

■ **TABLE P11.2R**
A Hexagon Design

x_1	x_2	y
1	0	68
0.5	$\sqrt{0.75}$	74
−0.5	$\sqrt{0.75}$	65
−1	0	60
−0.5	$-\sqrt{0.75}$	63
0.5	$-\sqrt{0.75}$	70
0	0	58
0	0	60
0	0	57
0	0	55
0	0	69

(a) Fit the second-order model.

(b) Perform the canonical analysis. What type of surface has been found?

(c) What operating conditions on x_1 and x_2 lead to the stationary point?

(d) Where would you run this process if the objective is to obtain a response that is as close to 65 as possible?

11.4R ***The rotatable central composite design.*** It can be shown that a second-order design is rotatable if $\sum_{u=1}^{n} x_{iu}^{a} x_{ju}^{b} = 0$, if a or b or both are odd, and if $\sum_{u=1}^{n} x_{iu}^{4} = 3\sum_{u=1}^{n} x_{iu}^{2} x_{ju}^{2}$. Show that for the central composite design these conditions lead to $\alpha = (n_F)^{1/4}$ for rotatability, where n_F is the number of points in the factorial portion.

11.5R A chemical engineer wishes to fit a calibration curve for a new procedure used to measure the concentration of a particular ingredient in a product manufactured in his facility. Twelve samples can be prepared, having known concentration. The engineer wants to build a model for the measured concentrations. He suspects that a linear calibration curve will be adequate to model the measured concentration as a function of the known concentrations; that is, $y = \beta_0 + \beta_1 x + \varepsilon$, where x is the actual concentration. Four experimental designs are under consideration. Design 1 consists of six runs at known concentration 1 and six runs at known concentration 10. Design 2 consists of four runs at concentrations 1, 5.5, and 10. Design 3 consists of three runs at concentrations 1, 4, 7, and 10. Finally, design 4 consists of three runs at concentrations 1 and 10 and six runs at concentration 5.5.

(a) Plot the scaled variance of prediction for all four designs on the same graph over the concentration range $1 \leq x \leq 10$. Which design would be preferable?

(b) For each design, calculate the determinant of $(\mathbf{X}'\mathbf{X})^{-1}$. Which design would be preferred according to the D-criterion?

(c) Calculate the D-efficiency of each design relative to the "best" design that you found in part (b).

(d) For each design, calculate the average variance of prediction over the set of points given by $x = 1, 1.5, 2, 2.5, \ldots, 10$. Which design would you prefer according to the V-criterion?

(e) Calculate the V-efficiency of each design relative to the best design that you found in part (d).

(f) What is the G-efficiency of each design?

11.6R Rework problem 11.5R assuming that the model the engineer wishes to fit is a quadratic. Obviously, only designs 2, 3, and 4 can now be considered.

11.7R Consider the first-order model in Problem 11.36. The design used to fit this model was a 2^2 factorial with three center points. The variance of the predicted response at the point $x_1 = 1.5$, $x_2 = 1.0$ is ______.

11.8R Suppose that you have fit the following response surface model in terms of coded design variables:

$$\hat{y} = 200 + 10x_1 + 5x_2 - 2x_3$$

The design used to fit this model was a 2^3 factorial.

(a) The experimenter is using the method of steepest ascent. He/she decides to make the step size exactly one coded unit in the x_2 direction. Find the coordinates of this point from the design center on the path of steepest ascent in terms of coded design variables.

(b) Suppose that the experimenter had chosen x_1 as the variable to define the step size for steepest ascent. Once again, the length of the step is one coded unit. Do you think that this is a better choice for defining step size than was used in part (a)? Explain why or why not.

(c) Suppose that the levels of the natural variables ξ_1, ξ_2, and $\xi_3$used in the 2^3 design were (100, 150), (20, 40), and (60, 80), respectively. What are the coordinates of the point on the path of steepest ascent from part (a) in terms of the natural variables?

(d) Find the distance separating the two points from parts (a) and (b).

11.9R An engineer has conducted an experiment with three factors using a central composite design with 6 center points. The results of this model fitting process are summarized below.

Model Term	Extra Sum of Squares	Degrees of Freedom	Mean Square	F
Mean (Intercept)	150	1		
Linear or First-Order	110			
Interaction	75			
Quadratic	70			
Higher-Order	10			
Pure Error	5			X
Total (uncorrected)	420		XXXXXXX	X

(a) Fill in the blanks in the columns labeled "degrees of freedom", "mean square" and "F".

(b) What type of model seems appropriate here and why?

(c) What terms are included in the grouping "higher-order" referred to in the above table?

(d) What hypothesis is the F-test shown for the linear or first – order terms in the above table actually testing?

(e) What hypothesis is the F-test shown for the quadratic terms in the above table actually testing? How is this "quadratic" sum of squares for this test computed?

11.10R An experimenter has fit a second-order response surface model in four variables. The experimental design was a central composite design with $\alpha = 2$. The canonical analysis reveals that the eigenvalues of the $\widehat{\mathrm{B}}$ matrix are as follows: $\lambda_1 = 8, \lambda_2 = 6, \lambda_3 = -.2, \lambda_4 = .01$.

(a) The estimate of the stationary point is $\mathbf{x}_s' = [1.9, 1.7, -1.5, 1.3]$. What type of response surface have we found (i.e., minimum, maximum . . .)?

(b) How would your answer change (if at all) if the coordinates of the stationary point were $\mathbf{x}_s' = [1.5, 2.5, -0.5, 1.1]$?

(c) How would you interpret a fitted response surface if the eigenvalues of the $\widehat{\mathrm{B}}$ matrix were $\lambda_1 = 8, \lambda_2 = 6, \lambda_3 = 3, \lambda_4 = .2$,and the standard errors of the eigenvalues were 0.5?

11.11R When using steepest ascent, the usual model that is employed is

(a) First order

(b) First order + interaction

(c) Reduced quadratic

(d) Full quadratic

11.12R Response surface methods depend upon having a model that provides a good fit to the response of interest. Statistical measure that can be used to judge the adequacy of model fit are

(a) Adjusted R^2

(b) Ordinary R^2

(c) PRESS

(d) All of the above.

(e) None of the above

11.13R When adding second-order terms to a first order model the value of the PRESS statistic will

(a) Always increase

(b) Always decrease

(c) Change to reflect the fit of the model to the data

(d) Change to reflect the ability of the model to predict new data

(e) None of the above

11.14R A first-order model in four variables has been fit using a $2^{4\text{-}1}$ fractional factorial design ($n = 8$ runs). The variance of the predicted response at any corner of the design is

(a) $5\sigma^2$

(b) $\sigma^2/2$

(c) $5\sigma^2/8$

(d) $\sigma^2/8$

(e) None of the above

11.15R Adding center points to a two-level design

(a) Provides an estimate of error

(b) Allows estimation of all second-order terms

(c) Reduces the variance of all model regression coefficients

(d) All of the above

(e) None of the above

11.16R When conducting optimization experiments, examining contour plots are the only way to interpret the nature of a fitted response surface.

(a) True

(b) False

11.17R A CCD in k variables will always have more runs than a 3^3 factorial design.

(a) True

(b) False