

Chapter 5: Continuous Random Variables

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| Exercise 1. | *Which type of distribution does the graph illustrate?*    *Figure 5.23* | |
| Solution | Uniform Distribution | |
| Exercise 2. | *Which type of distribution does the graph illustrate?*    *Figure 5.24* | |
| Solution | Exponential Distribution | |
| Exercise 3. | *Which type of distribution does the graph illustrate?*    *Figure 5.25* | |
| Solution | Normal Distribution | |
| Exercise 4. | *What does the shaded area represent? P(\_\_\_< x < \_\_\_)*    *Figure 5.26* | |
| Solution | *P*(2 < *x* < 5) | |
| Exercise 5. | *What does the shaded area represent? P(\_\_\_< x < \_\_\_)*  *Figure 5.27* | |
| Solution | *P*(6 < *x* <7) | |
| Exercise 6. | *For a continuous probability distribution, 0 ≤ x ≤ 15. What is P(x > 15)?* | |
| Solution | *f*(*x*) = 0 | |
| Exercise 7. | *What is the area under f(x) if the function is a continuous probability density function?* | |
| Solution | one | |
| Exercise 8. | *For a continuous probability distribution, 0 ≤ x ≤ 10. What is P(x = 7)?* | |
| Solution | zero | |
| Exercise 9. | *A continuous probability function is restricted to the portion between x = 0 and 7. What is P(x = 10)?* | |
| Solution | zero | |
| Exercise 10. | *f(x) for a continuous probability function is  , and the function is restricted to 0 ≤ x ≤ 5. What is P(x < 0)?* | |
| Solution | zero | |
| Exercise 11. | *f(x), a continuous probability function, is equal to  , and the function is restricted to 0 ≤ x ≤ 12. What is P(0 < x < 12)?* | |
| Solution | one | |
| Exercise 12. | *Find the probability that x falls in the shaded area.*    *Figure 5.28* | |
| Solution | 0.2222 | |
| Exercise 13. | *Find the probability that x falls in the shaded area.*    *Figure 5.29* | |
| Solution | 0.625 | |
| Exercise 14. | *Find the probability that x falls in the shaded area.*    *Figure 5.30* | |
| Solution | 0.3 | |
| Exercise 15. | *f(x), a continuous probability function, is equal to and the function is restricted to 1 ≤ x ≤ 4.Describe ?* | |
| Solution | The probability is equal to the area from *x* = to *x* = 4 above the *x*-axis and up to *f*(*x*) = . | |
| Exercise 16. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What type of distribution is this?* | |
| Solution | a uniform distribution | |
| Exercise 17. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *In this distribution, outcomes are equally likely. What does this mean?* | |
| Solution | It means that the value of *x* is equally likely to be any number between 1.5 and 4.5. | |
| Exercise 18. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What is the height of f(x) for the continuous probability distribution?* | |
| Solution |  | |
| Exercise 19. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What are the constraints for the values of x?* | |
| Solution | 1.5 ≤ *x* ≤ 4.5 | |
| Exercise 20. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *Graph P(2 < x < 3).* | |
| Solution |  | |
| Exercise 21. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What is P(2 < x < 3)?* | |
| Solution | 0.3333 | |
| Exercise 22. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What is P(x < 3.5| x < 4)?* | |
| Solution | 0.8 | |
| Exercise 23. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *What is P(x = 1.5)?* | |
| Solution | Zero | |
| Solution | 4.2 thousand feet squared (4,200 square feet) | |
| Exercise 24. | *The data that follow are the square footage (in 1000 feet squared) of 28 homes.*   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 1.5 | 2.4 | 3.6 | 2.6 | 1.6 | 2.4 | 2.0 | | 3.5 | 2.5 | 1.8 | 2.4 | 2.5 | 3.5 | 4.0 | | 2.6 | 1.6 | 2.2 | 1.8 | 3.8 | 2.5 | 1.5 | | 2.8 | 1.8 | 4.5 | 1.9 | 1.9 | 3.1 | 1.6 |   *Table 5.2*  *The sample mean = 2.50 and the sample standard deviation = 0.8302.*  *The distribution can be written as X ~ U(1.5, 4.5).*  *Find the probability that a randomly selected home has more than 3,000 square feet given that you already know the house has more than 2,000 square feet.* | |
| Solution | 0.6 | |
| Exercise 25. | *A distribution is given as X ~ U(0, 12).*  *What is a? What does it represent?* | |
| Solution | *a* is zero, and it represents the lowest value of *x*. | |
| Exercise 26. | *A distribution is given as X ~ U(0, 12).*  *What is b? What does it represent?* | |
| Solution | *b* is 12, and it represents the highest value of *x*. | |
| Exercise 27. | *A distribution is given as X ~ U(0, 12).*  *What is the probability density function?* | |
| Solution | *f(x)* =  for 0 ≤ *x* ≤ 12 | |
| Exercise 28. | *A distribution is given as X ~ U(0, 12).*  *What is the theoretical mean?* | |
| Solution | Six | |
| Exercise 29. | *A distribution is given as X ~ U(0, 12).*  *What is the theoretical standard deviation?* | |
| Solution | 3.46 | |
| Exercise 30. | *A distribution is given as X ~ U(0, 12).*  *Draw the graph of the distribution for P(x > 9).* | |
| Solution | Figure 5.52 | |
| Exercise 31. | *A distribution is given as X ~ U(0, 12).*  *Find P(x > 9).* | |
| Solution | 0.25 | |
| ~~Exercise 33.~~ | *~~A distribution is given as X ~ U(0, 12).~~*  *~~Find the 40~~~~th~~ ~~percentile.~~* | |
| ~~Solution~~ | ~~4.8~~ | |
| Exercise 32. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *What is being measured here?* | |
| Solution | The age of cars in the staff parking lot | |
| Exercise 33. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *In words, define the random variable X.* | |
| Solution | *X* = The age (in years) of cars in the staff parking lot | |
| Exercise 34. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Are the data discrete or continuous?* | |
| Solution | Continuous | |
| Exercise 35. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *The interval of values for x is* \_\_\_\_\_\_. | |
| Solution | 0.5 to 9.5 | |
| Exercise 36. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *The distribution for X is* \_\_\_\_\_\_. | |
| Solution | *X* ~ *U*(0.5,9.5) | |
| Exercise 37. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Write the probability density function?* | |
| Solution | *f*(*x*) = where x is between 0.5 and 9.5, inclusive. | |
| Exercise 38. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Graph the probability distribution.*  *a. Sketch the graph of the probability distribution.*    *Figure 5.31*  *b. Identify the following values:*  *i. Lowest value for* *:* \_\_\_\_\_\_.  *ii. Highest value for* *:* \_\_\_\_\_\_.  *iii. Height of the rectangle:* \_\_\_\_\_\_.  *iv. Label the x-axis (words):* \_\_\_\_\_\_.  *v. Label for y-axis (words):* \_\_\_\_\_\_. | |
| Solution | a. Check student’s solution.  b.  i. 0.5  ii. 9.5  iii.  iv. Age of Cars  v. f(x) | |
| Exercise 39. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Find the average age of the cars in the lot.* | |
| Solution | *μ* = 5 | |
| Exercise 40. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Find the probability that a randomly chosen car in the lot was less than four years old.*  *a. Sketch the graph. Shade the area of interest.*    *Figure 5.46*  *b.* *Find the probability. P*(*x < 4*) *=\_\_\_\_* | |
| Solution | *a. Check student’s solution.*  b. *P*(*x* < 4) = 0.39 | |
| Exercise 41. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Considering only the cars less than 7.5 years old, find the probability that a randomly chosen car in the lot was less than four years old.*   1. *Sketch the graph, shade the area of interest.*     *Figure 5.47*   1. *Find the probability. P(x < 4|x < 7.5) =\_\_\_\_* | |
| Solution | a. Check student’s solution.  b. | |
| Exercise 42. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *What has changed in the previous two problems that made the solutions different?* | |
| Solution | The population that the car is randomly chosen from is different in each case. In the first case, the car is chosen from all of the cars in the lot. In the second case, the car is being chosen from only those cars in the lot that are less than 7.5 years old. | |
| Exercise 43. | *The age of cars in the staff parking lot of a suburban college is uniformly distributed from six months (0.5 years) to 9.5 years.*  *Find the third quartile of ages of cars in the lot. This means you will have to find the value such that , or 75%, of the cars are at most (less than or equal to) that age.*  *a. Sketch the graph. Shade the area of interest.*    *Figure 5.34*  *b. Find the value k such that P(x < k) = 0.75*  *c. The third quartile is*\_\_\_\_\_\_\_ | |
| Solution | a. Check student’s solution.  b. *k* = 7.25  c. 7.25 | |
| Exercise 44. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution:*  *X ~ Exp(0.2)*  *What type of distribution is this?* | |
| Solution | an exponential distribution | |
| Exercise 45. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution:*  *Are outcomes equally likely in this distribution? Why or why not?* | |
| Solution | No, outcomes are not equally likely. In this distribution, more people require a little bit of time, and fewer people require a lot of time, so it is more likely that someone will require less time. | |
| Exercise 46. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution:*  *What is m? What does it represent?* | |
| Solution | The value of *m* is 0.2, and it represents the decay parameter. | |
| Exercise 47. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution:*  *What is the mean?* | |
| Solution | Five | |
| Exercise 48. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *What is the standard deviation?* | |
| Solution | Five | |
| Exercise 49. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *State the probability density function.* | |
| Solution | *f(x)* = 0.2e-0.2*x* | |
| Exercise 50. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *Graph the distribution.* | |
| Solution |  | |
| Exercise 51. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *Find P(2 < x < 10).* | |
| Solution | 0.5350 | |
| Exercise 52. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *Find P(x > 6).* | |
| Solution | 0.3012 | |
| Exercise 53. | *A customer service representative must spend different amounts of time with each customer to resolve various concerns. The amount of time spent with each customer can be modeled by the following distribution: X ~ Exp (0.2).*  *Find the 70th percentile.* | |
| Solution | 6.02 | |
| Exercise 54. | *A distribution is given as X ~ Exp(0.75).*  *What is m?* | |
| Solution | The value of *m* is 0.75. | |
| Exercise 55. | *A distribution is given as X ~ Exp(0.75).*  *What is the probability density function?* | |
| Solution | *f(x)* = 0.75e-0.75*x* | |
| Exercise 56. | *A distribution is given as X ~ Exp(0.75).*  *What is the cumulative distribution function?* | |
| Solution | *P*(*x* < *X*) = 1 – e-0.75*x* | |
| Exercise 57. | *A distribution is given as X ~ Exp(0.75).*  *Draw the distribution.* | |
| Solution | Figure 5.53 | |
| Exercise 58. | *A distribution is given as X ~ Exp(0.75).*  *Find P(x < 4).* | |
| Solution | 0.9502 | |
| Exercise 59. | *A distribution is given as X ~ Exp(0.75).*  *Find the 30th percentile.* | |
| Solution | 0.4756 | |
| Exercise 60. | *A distribution is given as X ~ Exp(0.75).*  *Find the median.* | |
| Solution | 0.9242 | |
| Exercise 61. | *A distribution is given as X ~ Exp(0.75).*  *Which is larger, the mean or the median?* | |
| Solution | The mean is larger. The mean is  , which is greater than 0.9242. | |
| Exercise 62. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *What is being measured here?* | |
| Solution | The time (in years) that it takes carbon-14 to decay. | |
| Exercise 63. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *Are the data discrete or continuous?* | |
| Solution | continuous | |
| Exercise 64. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *In words, define the random variable X.* | |
| Solution | *X* = Time (years) for carbon-14 to decay. | |
| Exercise 65. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *What is the decay rate (m)?* | |
| Solution | *m* = 0.000121 | |
| Exercise 66. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *The distribution for X is \_\_\_\_\_\_\_.* | |
| Solution | *X* ~ *Exp*(0.000121) | |
| Exercise 67. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *Find the amount (percent of one gram) of carbon-14 lasting less than 5,730 years. This means, find P(x < 5,730).*   1. *Sketch the graph. and shade the area of interest.*     *Figure 5.35*   1. *Find the probability. P(X < 5,730) = ­­­­\_\_\_\_* | |
| Solution | a. Check student’s solution.  b. *P*(*x* < 5,730) = 0.5001 | |
| Exercise 68. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *Find the percentage of carbon-14 lasting longer than 10,000 years.*  *a. Sketch the graph. and shade the area of interest.*    *Figure 5.36*  *b. Find the probability P(x > 10,000).* | |
| Solution | a. Check student’s solution.  b. *P*(*x* > 10,000) = 0.2982 | |
| Exercise 69. | *Carbon-14 is a radioactive element with a half-life of about 5,730 years. Carbon-14 is said to decay exponentially. The decay rate is 0.000121. We start with one gram of carbon-14. We are interested in the time (years) it takes to decay carbon-14 .*  *Thirty percent (30%) of carbon-14 will decay within how many years?*  *a. Sketch the graph. and shade the area of interest.*    *Figure 537*  *b. Find the value k such that P(x < k) = 0.30* | |
| Solution | a. Check student’s solution.  b. *k* = 2947.73 | |
|  | *For each probability and percentile problem, Draw the Picture* | |
| Exercise 70. | *Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer “yes” or “no.” You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.*  *a. What part of the experiment will yield discrete data?*  *b. What part of the experiment will yield continuous data?* | |
| Solution | a. Number of yes and no answers are discrete data.  b. Computed percentage is continuous data. | |
| Exercise 71. | *When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?* | |
| Solution | Age is a measurement, regardless of the accuracy used. | |
| Exercise 72. | *Births are approximately uniformly distributed throughout the year. They can be said to follow a uniform distribution from 0 to 52 (spread of 52 weeks).*  *a. Graph the probability distribution.*  *b. f(x) =\_\_\_\_\_\_\_\_*  *c. μ = \_\_\_\_\_\_\_*  *d. σ = \_\_\_\_\_\_\_*  *e. Find the probability that a person is born at the exact moment week 19 ends. That is, find P(x = 19) =\_\_\_\_\_\_\_\_*  *f. P(2 < x < 31) = \_\_\_\_\_\_\_\_*  *g. Find the probability that a person is born after week 40.*  *h. P(12 < x|x < 28) =\_\_\_\_\_\_\_\_* | |
| Solution | a. Check student’s solution.  b.  c. 26  d. 15.01  e. zero  f.  g.  h. | |
| Exercise 73. | *A random number generator picks a number from one to nine in a uniform manner.*  *a. X ~\_\_\_\_\_\_\_\_*  *b. Graph the probability distribution.*  *c. f(x) =\_\_\_\_\_\_\_\_\_*  *d. μ =\_\_\_\_\_\_\_*  *e. σ = \_\_\_\_\_\_*  *f. P(3.5 < x < 7.25) =\_\_\_\_\_\_\_*  *g. P(x > 5.67) = \_\_\_\_\_\_\_\_*  *h. P(x > 5|x > 3) =\_\_\_\_\_\_\_\_\_*  *i. Find the 90th percentile.* | |
| Solution | a. *X ~ U(*1, 9)  b. Check student’s solution.  c.  d. five  e. 2.3  f.  g.  h.  i. 8.2 | |
| Exercise 74. | *According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between six and 15 pounds a month until they approach trim body weight. Let’s suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month.*  *a. Define the random variable. X =\_\_\_\_\_\_*  *b. X ~\_\_\_\_\_\_*  *c. Graph the probability distribution.*  *d. f(x) = \_\_\_\_\_\_*  *e. μ =\_\_\_\_\_\_*  *f. σ =\_\_\_\_\_\_*  *g. Find the probability that the individual lost more than ten pounds in a month.*  *h. Suppose it is known that the individual lost more than ten pounds in a month. Find the probability that he lost less than 12 pounds in the month.*  *i. P(7 < x < 13|x > 9) =\_\_\_\_\_\_. State this in a probability question, similar to parts g and h, draw the picture, and find the probability.* | |
| Solution | a. *X* represents the weight loss of a randomly selected individual from Dr. John McDougall’s live-in weight loss program.  b. *X ~* *U*(6, 15)  c. Check student’s solution.  d.  e. 10.5  f. 2.6  g.  h.  i.  . Answers may vary when formulating a probability question. | |
| Exercise 75. | *A subway train on the Red Line arrives every eight minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.*  *a. Define the random variable. X =\_\_\_\_\_\_*  *b. X ~\_\_\_\_\_\_*  *c. Graph the probability distribution.*  *d. f(x) = \_\_\_\_\_\_*  *e. μ =\_\_\_\_\_\_*  *f. σ =\_\_\_\_\_\_\_*  *g. Find the probability that the commuter waits less than one minute.*  *h. Find the probability that the commuter waits between three and four minutes.*  *i. Sixty percent of commuters wait more than how long for the train? State this in a probability question, similarly to parts g and h, draw the picture, and find the probability.* | |
| Solution | a. *X* represents the length of time a commuter must wait for a train to arrive on the Red Line.  b. *X* ~ *U*(0,8)  c. Check student’s solution.  d. *f*(*x*) = where 0 ≤ *x* ≤ 8  e. four  f. 2.31  g.  h.  i. 3.2; answers may vary when formulating a probability question. | |
| Exercise 76. | *The age of a first grader on September 1 at Garden Elementary School is uniformly distributed from 5.8 to 6.8 years. We randomly select one first grader from the class.*  *a. Define the random variable. X =\_\_\_\_\_\_*  *b. X ~\_\_\_\_\_\_\_*  *c. Graph the probability distribution.*  *d. f(x) = \_\_\_\_\_\_\_*  *e. μ =\_\_\_\_\_\_\_*  *f. σ =\_\_\_\_\_\_\_*  *g. Find the probability that she is over 6.5 years.*  *h. Find the probability that she is between four and six years.*  *i. Find the 70th percentile for the age of first graders on September 1 at Garden Elementary School.* | |
| Solution | a. *X* represents the age of a first grader on September 1 at Garden Elementary School  b. *X* ~ *U*(5.8, 6.8)  c. Check student’s solution.  d. *f*(*x*) = 1 where  e. 6.3  f. 0.29  g. 0.3  h. 0.2  i. 6.5 years | |
| Exercise 77. | *The Sky Train from the terminal to the rental–car and long–term parking center is supposed to arrive every eight minutes. The waiting times for the train are known to follow a uniform distribution. What is the average waiting time (in minutes)?*  *a. zero*  *b. two*  *c. three*  *d. four* | |
| Solution | d | |
| ~~Exercise 78.~~ | *~~The Sky Train from the terminal to the rental–car and long–term parking center is supposed to arrive every eight minutes. The waiting times for the train are known to follow a uniform distribution .Find the 30~~~~th~~ ~~percentile for the waiting times (in minutes).~~*  *~~a. two~~*  *~~b. 2.4~~*  *~~c. 2.75~~*  *~~d. three~~* | |
| Solution | b | |
| Exercise 78. | *The Sky Train from the terminal to the rental–car and long–term parking center is supposed to arrive every eight minutes. The waiting times for the train are known to follow a uniform distribution. The probability of waiting more than seven minutes given that a person has waited more than four minutes is?*  *a. 0.125*  *b. 0.250*  *c. 0.5*  *d. 0.75* | |
| Solution | b | |
| Exercise 79. | *The time (in minutes) until the next bus departs from a major bus depot follows a distribution with f(x) = where x has a range of from 25 to 45 minutes.*  *a. Define the random variable. X =\_\_\_\_\_\_*  *b. X ~\_\_\_\_\_\_*  *c. Graph the probability distribution.*  *d. The distribution is \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (name of distribution). It is \_\_\_\_\_\_\_\_\_\_\_\_\_ (discrete or continuous).*  *e. μ =\_\_\_\_\_\_*  *f. σ =\_\_\_\_\_\_*  *g. Find the probability that the time is at most 30 minutes. Sketch and label a graph of the distribution. Shade the area of interest. Write the answer in a probability statement.*  *h. Find the probability that the time is between 30 and 40 minutes. Sketch and label a graph of the distribution. Shade the area of interest. Write the answer in a probability statement.*  *i. P(25 < x < 55) = \_\_\_\_\_\_\_\_\_. State this in a probability statement, similarly to parts g and h, draw the picture, and find the probability.*  *j. Find the 90th percentile. This means that 90% of the time, the time is less than \_\_\_\_\_ minutes.*  *k. Find the 75th percentile. In a complete sentence, state what this means. (See part j.)*  *l. Find the probability that the time is more than 40 minutes given (or knowing that) it is at least 30 minutes.* | |
| Solution | a. *X* represents the time (in minutes) until the next bus departs a major bus depot.  b. X ~ U (25,45)  c. Check student’s solution.  d. uniform; continuous  e. 35 minutes  f. 5.8 minutes  g. 0.25. Check student’s graph. The probability of a waiting time of 30 minutes or less is 0.30, given waiting times ~U(25, 45)  h. 0.5  i. one. Check student’s graph. The probability of a waiting time between 25 and 55 minutes is 1.0, given waiting times ~U(25, 45).  j. 43 minutes. Ninety percent of waiting times will be 40 minutes or less, given waiting times ~U(25, 45).  k. 40 minutes. Three-quarters of waiting times will be 40 minutes or less, given waiting times ~U(25, 45).  l. 0.3333 | |
| Exercise 80. | *Suppose that the value of a stock varies each day from $16 to $25 with a uniform distribution.*  *a. Find the probability that the value of the stock is more than $19.*  *b. Find the probability that the value of the stock is between $19 and $22.*  *c. Find the upper quartile - 25% of all days the stock is above what value? Draw the graph.*  *d. Given that the stock is greater than $18, find the probability that the stock is more than $21.* | |
| Solution | a. The probability density function of *X* is .  *P*(*X* > 19) = (25 – 19) (1/9) = 6/9 = 2/3.  G:\Clients\Connexions\CONNEX120012_Statistics\07_Art\Ch05\Phase_1_Rendered_from_ASI\JPEG\CNX_Stats_C05_RWP_001.jpg  Figure 5.54  b. *P*(19 < *X* < 22) = (22– 19) (1/9) = 3/9 = 1/3.  G:\Clients\Connexions\CONNEX120012_Statistics\07_Art\Ch05\Phase_1_Rendered_from_ASI\JPEG\CNX_Stats_C05_RWP_002.jpg  Figure 5.55  c. The area must be 0.25, and 0.25 = (width)(1/9), so width = (0.25)(9) = 2.25. Thus, the value is 25 – 2.25 = 22.75.  d. This is a conditional probability question. *P*(*x* > 21| *x* > 18). You can do this two ways:   * Draw the graph where a is now 18 and b is still 25. The height is 1/(25 – 18) = 1/7   So, *P*(*x* > 21| *x* > 18) = (25 – 21)(1/7) = 4/7.   * Use the formula: *P*(*x* > 21|*x* > 18) = *P*(*x* > 21 AND *x* > 18)/*P*(*x* > 18)   = *P*(*x* > 21)/*P*(*x* >18) = (25 – 21)/(25 – 18) = 4/7. | |
| Exercise 81 | *A fireworks show is designed so that the time between fireworks is between one and five seconds, and follows a uniform*  *distribution.*  *a. Find the average time between fireworks.*  *b. Find probability that the time between fireworks is greater than four seconds.* | |
| Solution | a. The average time is (5 + 1)/2 = 3 seconds.  b. *P*(*X* > 4) = (5 – 4)(1/(5 – 1)) = 1/4. | |
| Exercise 82. | *The number of miles driven by a truck driver falls between 300 and 700, and follows a uniform distribution.*  *a. Find the probability that the truck driver goes more than 650 miles in a day.*  *b. Find the probability that the truck drivers goes between 400 and 650 miles in a day.*  *~~c. At least how many miles does the truck driver travel on the furthest 10% of days~~?* | |
| Solution | a. P(X > 650) = .  b. P(400 < X < 650) =  c. , so width = 400(0.10) = 40. Since 700 – 40 = 660, the drivers travel at least 660 miles on the furthest 10% of days. | |
| Exercise 83. | *Suppose that the length of long distance phone calls, measured in minutes, is known to have an exponential distribution with the average length of a call equal to eight minutes.*  *a. Define the random variable. X =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*  *b. Is X continuous or discrete?*  *c. X ~\_\_\_\_\_\_\_*  *d. μ =\_\_\_\_\_\_\_*  *e. σ =\_\_\_\_\_\_\_*  *f. Draw a graph of the probability distribution. Label the axes.*  *g. Find the probability that a phone call lasts less than nine minutes.*  *h. Find the probability that a phone call lasts more than nine minutes.*  *i. Find the probability that a phone call lasts between seven and nine minutes.*  *j. If 25 phone calls were made one after another, on average, what would you expect the total to be? Why?* | |
| Solution | a. *X* = the length of long distance phone calls, measured in minutes.  b. *X* is continuous.  c. *X* ~  d.  e.  f. Check student’s solution.  g. 0.6753  h. 0.33247  i. 0.0922  j. The expected total would be 25(8) = 200 minutes, since the average length of a call is eight minutes. | |
| Exercise 84. | *Suppose that the useful life of a particular car battery, measured in months, decays with parameter 0.025. We are interested in the life of the battery.*  *a. Define the random variable. X =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*  *b. Is X continuous or discrete?*  *c. X ~ \_\_\_\_\_\_*  *d. On average, how long would you expect one car battery to last?*  *e. On average, how long would you expect nine car batteries to last, if they are used one after another?*  *f. Find the probability that a car battery lasts more than 36 months.*  *g. Seventy percent of the batteries last at least how long?* | |
| Solution | a. *X* =the useful life of a particular car battery, measured in months.  b. *X* is continuous.  c. *X* ~ *Exp*(0.025)  d. 40 months  e. 360 months  f. 0.4066  g. 14.27 | |
| Exercise 85. | *The percent of persons (ages five and older) in each state who speak a language at home other than English is approximately exponentially distributed with a mean of 9.848. Suppose we randomly pick a state. (Source: Bureau of the Census, U.S. Dept. of Commerce)*  *a. Define the random variable. X =\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*  *b. Is X continuous or discrete?*  *c. X ~\_\_\_\_\_\_*  *d. μ = \_\_\_\_\_\_*  *e. σ = \_\_\_\_\_\_*  *f. Draw a graph of the probability distribution. Label the axes.*  *g. Find the probability that the percent is less than 12.*  *h. Find the probability that the percent is between eight and 14.*  *i. The percent of all individuals living in the United States who speak a language at home other than English is 13.8.*  *i. Why is this number different from 9.848%?*  *ii. What would make this number higher than 9.848%?* | |
| Solution | a. *X* = the percent of persons (ages five and older) in each state who speak a language at home other than English.  b. *X* is continuous.  c. *X* ~  d.  e.  f. Check student’s solution.  g. 0.7043  h. 0.2025  i.  i. This number differs from 9.848, because each state has a different population size.  ii. If states that have larger populations tend to have a higher percentage of individuals who speak a language at home other than English, then that would account for a higher percentage when all the states’ residents are pooled together. | |
| Exercise 86. | *The time (in years)* ***after*** *reaching age 60 that it takes an individual to retire is approximately exponentially distributed with a mean of about five years. Suppose we randomly pick one retired individual. We are interested in the time after age 60 to retirement.*  *a. Define the random variable. X =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*  *b. Is X continuous or discrete?*  *c. X ~ \_\_\_\_\_\_\_*  *d. μ = \_\_\_\_\_\_*  *e. σ = \_\_\_\_\_\_\_*  *f. Draw a graph of the probability distribution. Label the axes.*  *g. Find the probability that the person retired after age 70.*  *h. Do more people retire before age 65 or after age 65?*  *i. In a room of 1,000 people over age 80, how many do you expect will NOT have retired yet?* | |
| Solution | a. *X* = the time (in years) after reaching age 60 that it takes an individual to retire  b. *X* is continuous.  c. *X* ~ *Exp*  d. five  e. five  f. Check student’s solution.  g. 0.1353  h. before  i. 18.3 | |
| Exercise 87. | *The cost of all maintenance for a car during its first year is approximately exponentially distributed with a mean of $150.*  *a. Define the random variable. X =\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*  *b. X ~\_\_\_\_\_\_\_*  *c. μ =\_\_\_\_\_\_*  *d. σ =\_\_\_\_\_\_*  *e. Draw a graph of the probability distribution. Label the axes.*  *f. Find the probability that a car required over $300 for maintenance during its first year.* | |
| Solution | a. *X* = the cost of all maintenance for a car during its first year.  b. *X* ~  c.  d.  e. Check student’s solution.  f. 0.1353 | |
| Exercise 88. | *The average lifetime of a certain new cell phone is three years. The manufacturer will replace any cell phone failing within two years of the date of purchase. The lifetime of these cell phones is known to follow an exponential distribution.*  *The decay rate is:*  *a. 0.3333*  *b. 0.5000*  *c. 2*  *d. 3* | |
| Solution | A | |
| Exercise 89. | *The average lifetime of a certain new cell phone is three years. The manufacturer will replace any cell phone failing within two years of the date of purchase. The lifetime of these cell phones is known to follow an exponential distribution.*  *What is the probability that a phone will fail within two years of the date of purchase?*  *a. 0.8647*  *b. 0.4866*  *c. 0.2212*  *d. 0.9997* | |
| Solution | B | |
| Exercise 90. | *The average lifetime of a certain new cell phone is three years. The manufacturer will replace any cell phone failing within two years of the date of purchase. The lifetime of these cell phones is known to follow an exponential distribution.*  *What is the median lifetime of these phones (in years)?*  *a. 0.1941*  *b. 1.3863*  *c. 2.0794*  *d. 5.5452* | |
| Solution | C | |
| Exercise 91. | *At a 911 call center, calls come in at an average rate of one call every two minutes. Assume that the time that elapses*  *from one call to the next has the exponential distribution.*  *a. On average, how much time occurs between five consecutive calls?*  *b. Find the probability that after a call is received, it takes more than three minutes for the next call to occur.*  *c. Ninety-percent of all calls occur within how many minutes of the previous call?*  *d. Suppose that two minutes have elapsed since the last call. Find the probability that the next call will occur within*  *the next minute.*  *e. Find the probability that less than 20 calls occur within an hour.* | |
| Solution | a. Since one call occurs every two minutes, we expect that five calls occur every ten minutes on average.  Use for b – d:  Let *T* = time (min) elapsed between calls. We know that *μ* = 2 minutes (1 call every 2 minutes tells us, that for the exponential, *µ* = 2 minutes between calls, on average), so the decay parameter is *m* = 1/2.  The cumulative distribution function is .  b. Therefore, .  c. We want to find the 90th percentile, so we need to solve for t.  Substituting in the cumulative distribution function, .  Rearranging terms gives and converting this to logarithmic form gives . Solving for *t*, .  Hence, 90% of calls occur within 1.151 minutes (or 69.1 seconds) of the previous call.  d. We want to find the conditional probability *P*(*T* > 3 |*T* > 2).  By the memoryless property*P*(*X* > *r* + *t*|*X* > *r*) = *P*(*X* > *t*)), *P*(*T* > 3 |*T* > 2) = *P*(*T* > 1).  Then .  e. Let *X* = the number of calls that occur within an hour (60 minutes). We use the fact that *X* has the Poisson distribution with mean of *λ* = 30 calls per hour.  We need to find *P*(*X* < 20), which is equal to *P*(*X* ≤ 19) ≈ 0.0219. | |
| Exercise 92. | *In major league baseball, a no-hitter is a game in which a pitcher, or pitchers, doesn't give up any hits throughout*  *the game. No-hitters occur at a rate of about three per season. Assume that the duration of time between no-hitters is*  *exponential.*  *a. What is the probability that an entire season elapses with a single no-hitter?*  *b. If an entire season elapses without any no-hitters, what is the probability that there are no no-hitters in the*  *following season?*  *c. What is the probability that there are more than 3 no-hitters in a single season?* | |
| Solution | a. Let *X* = the number of no-hitters throughout a season. Since the duration of time between no-hitters is exponential, the number of no-hitters per season is Poisson with mean *λ* = 3.  Therefore, .  Note: you could let *T* = duration of time between no-hitters. Since the time is exponential and there are 3 no-hitters per season, then the time between no-hitters is 1/3 season. For the exponential, *µ* = 1/3.  Therefore, *m* = 1/*µ* = 3 and *T* ∼ *Exp*(3).  The desired probability is *P*(*T* > 1) = 1 – *P*(*T* < 1) =  b. Let *T* = duration of time between no-hitters. We find *P*(*T* > 2 | *T* > 1), and by the **memoryless property** this is simply *P*(*T* > 1), which we found to be 0.0498 in part a.  c. Let *X* = the number of no-hitters is a season. Assume that *X* is Poisson with mean *λ* = 3. Then *P*(*X* > 3) = 1 − *P*(*X* ≤ 3) = 0.3528. | |
| Exercise 93. | *During the years 1998–2012, a total of 29 earthquakes of magnitude greater than 6.5 have occurred in Papua New*  *Guinea. Assume that the time spent waiting between earthquakes is exponential.*  *a. What is the probability that the next earthquake occurs within the next three months?*  *b. Given that six months has passed without an earthquake in Papua New Guinea, what is the probability that the*  *next three months will be free of earthquakes?*  *c. What is the probability of zero earthquakes occurring in 2014?*  *d. What is the probability that at least two earthquakes will occur in 2014?* | |
| Solution | Let T = time between earthquakes. During the 15 year period from 1998 to 2012, there have been 29 earthquakes, so assume an average of 29/15 = 1.933 earthquakes per year.  For the exponential, there are 15/29 years between earthquakes so *µ* = 15/29 and *m* = 1/ *µ* = 29/15.  Then *T* ∼ *Exp*(29/15), and the cumulative distribution function is  a. We can then find 3833.  b. *P*(*T* > 0.75 |*T* > 0.5) = *P*(*T* > 0.25) = 1 − *P*(*T* < 0.25) = .  (6 months + 3 months = 9 months. 9/12 = 0.75 years)  c. *P*(*T* > 1) = 1 – *P*(*T* < 1) = .  (or: Let *X* = number of earthquakes during a year. Assume that *X* ∼ Poisson(29/15)  where*λ* = 29/15. Therefore, *P*(*X* = 0) = .  d. Let *X* = number of earthquakes that occur during a year. Then *X* ∼ Poisson(29/15).  Thus, *P*(*X* ≥ 2) = *P*(*X* > 1) = 1 − *P*(*X* ≤ 1) ≈ 0.5756. |
| Exercise 94. | *According to the American Red Cross, about one out of nine people in the U.S. have Type B blood. Suppose the blood*  *types of people arriving at a blood drive are independent. In this case, the number of Type B blood types that arrive roughly*  *follows the Poisson distribution.*  *a. If 100 people arrive, how many on average would be expected to have Type B blood?*  *b. What is the probability that over 10 people out of these 100 have type B blood?*  *c. What is the probability that more than 20 people arrive before a person with type B blood is found?* | |
| Solution | a. 100/9 = 11.11  b. *P*(*X* > 10) = 1 – *P*(*X* ≤ 10) = 1 – Poissoncdf(11.11, 10) ≈ 0.5532.  c. The number of people with Type B blood encountered roughly follows the Poisson distribution, so the number of people *X* who arrive between successive Type B arrivals is roughly exponential with mean *μ* = 9 and *m* = 1/9. The cumulative distribution function of *X* is . Thus, .  Note: we could also deduce that each person arriving has a 8/9 chance of not having Type B blood. So the probability that none of the first 20 people arrive have Type B blood is . (The geometric distribution is more appropriate than the exponential because the number of people between Type B people is discrete instead of continuous.) |
| Exercise 95. | *A web site experiences traffic during normal working hours at a rate of 12 visits per hour. Assume that the duration*  *between visits has the exponential distribution.*  *a. Find the probability that the duration between two successive visits to the web site is more than ten minutes.*  *b. The top 25% of durations between visits are at least how long?*  *c. Suppose that 20 minutes have passed since the last visit to the web site. What is the probability that the next visit*  *will occur within the next 5 minutes?*  *d. Find the probability that less than 7 visits occur within a one-hour period.* | |
| Solution | Let *T* = duration (in minutes) between successive visits.  a. Then *μ* = 5 minutes, and *m* = 1/5 = 0.20, so *T* ∼ Exp(0.20), and the cdf is *P*(*T* < *t*) = . a)Therefore, *P*(*T* > 10) = 1 − *P*(*T* < 10) = .  b. *P*(*T* > 10) = 0.25. We need to solve 0.25 = *P*(*T* > *t*) for *t*, so .  Solving for *t* gives minutes. Or use *ln*(area\_to\_the\_right)/(–*m*).  c. *P*(*T* < 25 |*T* > 20) = 1 – *P*(*T* > 25 |*T* > 20) = 1 – i(*T* > 5) = .  d. Let *X* = # of visitors during a one-hour period. Use the Poisson distribution, *X* ∼ Poisson(12).  Find *P*(*X* < 7) = *P*(*X* ≤ 6) ≈ 0.0895. | |
| Exercise 96. | *At an urgent care facility, patients arrive at an average rate of one patient every seven minutes. Assume that the*  *duration between arrivals is exponentially distributed.*  *a. Find the probability that the time between two successive visits to the urgent care facility is less than 2 minutes.*  *b. Find the probability that the time between two successive visits to the urgent care facility is more than 15 minutes.*  *c. If 10 minutes have passed since the last arrival, what is the probability that the next person will arrive within the*  *next five minutes?*  *d. Find the probability that more than eight patients arrive during a half-hour period.* | |
| Solution | Let *T* = duration (in minutes) between successive visits. Since patients arrive at a rate of one patient every seven minutes, *μ* = 7 and the decay constant is *m* = 1/7. The cdf is *P*(*T* < *t*) = .  a. *P*(*T* < 2) = 0.2485.  b. *P*(*T* > 15) = .  c. *P*(*T* > 15 |*T* > 10) = *P*(*T* > 5) = .  d. Let *X* = # of patients arriving during a half-hour period. Then *X* has the Poisson distribution with a mean of 30/7, *X* ∼ Poisson(30/7). Find *P*(*X* > 8) = 1 – *P*(*X* ≤ 8) ≈ 0.0311. |

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