

Chapter 7: the central limit theorem

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| Exercise 1. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *a. What is the distribution for the weights of one 25-pound lifting weight? What is the mean and standard deviation?*  *b. What is the distribution for the mean weight of 100 25-pound lifting weights?*  *c. Find the probability that the mean actual weight for the 100 weights is less than 24.9.* |
| Solution | a. *U*(24, 26), 25, 0.5774  b. *N*(25, 0.0577)  c. 0.0416 |
| Exercise 2. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Draw the graph from* ***Exercise 7.1****.* |
| Solution |  |
| Exercise 3. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Find the probability that the mean actual weight for the 100 weights is greater than 25.2.* |
| Solution | 0.0003 |
| Exercise 4. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Draw the graph from* ***Exercise 7.3****.* |
| Solution |  |
| Exercise 5. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Find the 90th percentile for the mean weight for the 100 weights.* |
| Solution | 25.07 |
| Exercise 6. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Draw the graph from* ***Exercise 7.5****.* |
| Solution |  |
| Exercise 7. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *a. What is the distribution for the sum of the weights of 100 25-pound lifting weights?*  *b. Find P(Σx < 2,450).* |
| Solution | a. *N*(2,500, 5.7735)  b. 0 |
| Exercise 8. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Draw the graph from* ***Exercise 7.7****.* |
| Solution |  |
| Exercise 9. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Find the 90th percentile for the total weight of the 100 weights.* |
| Solution | 2,507.40 |
| Exercise 10. | *A manufacturer produces 25-pound lifting weights. The lowest actual weight is 24 pounds, and the highest is 26 pounds. Each weight is equally likely so the distribution of weights is uniform. A sample of 100 weights is taken.*  *Draw the graph from* ***Exercise 7.9****.* |
| Solution |  |
| Exercise 11. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *a. What is the standard deviation?*  *b. What is the parameter m?* |
| Solution | a. 10  b. |
| Exercise 12. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *What is the distribution for the length of time one battery lasts?* |
| Solution |  |
| Exercise 13. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *What is the distribution for the mean length of time 64 batteries last?* |
| Solution |  |
| Exercise 14. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *What is the distribution for the total length of time 64 batteries last?* |
| Solution | *N*(640, 80) |
| Exercise 15. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *Find the probability that the sample mean is between seven and 11.* |
| Solution | 0.7799 |
| Exercise 16. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *Find the 80th percentile for the total length of time 64 batteries last.* |
| Solution | 707.3 |
| Exercise 17. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *Find the IQR for the mean amount of time 64 batteries last.* |
| Solution | 1.69 |
| Exercise 18. | *The length of time a particular smartphone's battery lasts follows an exponential distribution with a mean of ten months. A sample of 64 of these smartphones is taken.*  *Find the middle 80% for the total amount of time 64 batteries last.* |
| Solution | 205.05 |
| Exercise 19. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find P(Σx > 420).* |
| Solution | 0.0072 |
| Exercise 20. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find the 90th percentile for the sums.* |
| Solution | 410.46 |
| Exercise 21. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find the 15th percentile for the sums.* |
| Solution | 391.54 |
| Exercise 22. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find the first quartile for the sums.* |
| Solution | 394.49 |
| Exercise 23. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find the third quartile for the sums.* |
| Solution | 405.51 |
| Exercise 24. | *A uniform distribution has a minimum of six and a maximum of ten. A sample of 50 is taken.*  *Find the 80th percentile for the sums.* |
| Solution | 406.87  **Exercises 25-32 to come. Answers are in the textbook.** |
| Exercise 33 | *A question is asked of a class of 200 freshmen, and 23% of the students know the correct answer. If a sample of 50 students is taken repeatedly, what is the expected value of the mean of the sampling distribution of sample proportions?* |
| Solution | Based on the CLT the sampling distribution has a mean 23% |
| Exercise 34 | *A question is asked of a class of 200 freshmen, and 23% of the students know the correct answer. If a sample of 50 students is taken repeatedly, what is the standard deviation of the mean of the sampling distribution of sample proportions?* |
| Solution | Based on CLT the sampling distribution has a standard deviation = 0.0595 or 0.06 |
| Exercise 35 | *A game is played repeatedly. A player wins one-fifth of the time. If samples of 40 times the game is played are taken repeatedly, what is the expected value of the mean of the sampling distribution of sample proportions?* |
| Solution | 1/5 |
| Exercise 36 | *A game is played repeatedly. A player wins one-fifth of the time. If samples of 40 times the game is played are taken repeatedly, what is the standard deviation of the mean of the sampling distribution of sample proportions?* |
| Solution |  |
| Exercise 37 | *A virus attacks one in three of the people exposed to it. An entire large city is exposed. If samples of 70 people are taken, what is the expected value of the mean of the sampling distribution of sample proportions?* |
| Solution | 1/3 |
| Exercise 38 | *A virus attacks one in three of the people exposed to it. An entire large city is exposed. If samples of 70 people are taken, what is the standard deviation of the mean of the sampling distribution of sample proportions?* |
| Solution |  |
| Exercise 39 | *A company inspects products coming through its production process, and rejects detected products. One-tenth of the items are rejected. If samples of 50 items are taken, what is the expected value of the mean of the sampling distribution of sample proportions?* |
| Solution | 1/10 |
| Exercise 40 | *A company inspects products coming through its production process, and rejects detected products. One-tenth of the items are rejected. If samples of 50 items are taken, what is the standard deviation of the mean of the sampling distribution of sample proportions?* |
| Solution |  |
| Exercise 41 | *A fishing boat has 1,000 fish on board, with an average weight of 120 pounds and a standard deviation of 6.0 pounds. If sample sizes of 50 fish are checked, what is the probability the fish in a sample will have mean weight within 2.8 pounds the true mean of the population?* |
| Solution | The sampling distribution is normal with mean 120 and standard deviation =0.8274  P(sample mean is less than 122.8)-P(sample mean is more than 117.2)=0.99964-0.00035737=0.999 |
| Exercise 42 | *An experimental garden has 500 sunflowers plants. The plants are being treated so they grow to unusual heights. The average height is 9.3 feet with a standard deviation of 0.5 foot. If sample sizes of 60 plants are taken, what is the probability the plants in a given sample will have an average height within 0.1 foot of the true mean of the population?* |
| Solution | The sampling distribution is normal with mean 9.3 and standard deviation =0.0606  P(sample average is less than 9.4)-P(sample average is more than 9.2)=0.9505074-0.0494926=0.901 |
| Exercise 43 | *A company has 800 employees. The average number of workdays between absence for illness is 123 with a standard deviation of 14 days. Samples of 50 employees are examined. What is the probability a sample has a mean of workdays with no absence for illness of at least 124 days?* |
| Solution | The sampling distribution is normal with mean 123 and standard deviation =1.918  P(sample mean is more than 124) =1-NORM.DIST(124,123,1.918,TRUE)=0.301 |
| Exercise 44 | *Cars pass an automatic speed check device that monitors 2,000 cars on a given day. This population of cars has an average speed of 67 miles per hour with a standard deviation of 2 miles per hour. If samples of 30 cars are taken, what is the probability a given sample will have an average speed within 0.50 mile per hour of the population mean?* |
| Solution | The sampling distribution is normal with mean 67 and standard deviation =0.36249  P(sample mean is less than 67.5)-P(sample mean is more than 66.5)=0.91610629-0.083893=0.832 |
| Exercise 45 | *A town keeps weather records. From these records it has been determined that it rains on an average of 37% of the days each year. If 30 days are selected at random from one year, what is the probability that at least 5 and at most 11 days had rain?* |
| Solution | The same methodology for proportion of days between 11/30=0.3667 and 5/11=0.1667 |
| Exercise 46 | *A maker of yardsticks has an ink problem that causes the markings to smear on 4% of the yardsticks. The daily production run is 2,000 yardsticks. What is the probability if a sample of 100 yardsticks is checked, there will be ink smeared on at most 4 yardsticks?* |
| Solution | 4/100=4%. Since this is the mean of the sampling distribution the probability is 50% |
| Exercise 47 | *A school has 300 students. Usually, there are an average of 21 students who are absent. If a sample of 30 students is taken on a certain day, what is the probability that at most 2 students in the sample will be absent?* |
| Solution | We apply the proportion formula for 2/30=0.0667 |
| Exercise 48 | *A college gives a placement test to 5,000 incoming students each year. On the average 1,213 place in one or more developmental courses. If a sample of 50 is taken from the 5,000, what is the probability at most 12 of those sampled will have to take at least one developmental course?* |
| Solution | We apply the proportion formula for 12/50=0.24  The average is 1213/5000=0.2426 |
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| Exercise 49. | *Previously, De Anza statistics students estimated that the amount of change daytime statistics students carry is exponentially distributed with a mean of $0.88. Suppose that we randomly pick 25 daytime statistics students.*  *a. In words, Χ = \_\_\_\_\_\_\_\_\_\_\_\_*  *b. Χ ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *c. In words, = \_\_\_\_\_\_\_\_\_\_\_\_*  *d. ~ \_\_\_\_\_\_ (\_\_\_\_\_\_, \_\_\_\_\_\_)*  *e. Find the probability that an individual had between $0.80 and $1.00. Graph the situation, and shade in the area to be determined.*  *f. Find the probability that the average of the 25 students was between $0.80 and $1.00. Graph the situation, and shade in the area to be determined.*  *g. Explain why there is a difference in part e and part f.* |
| Solution | a. *Χ* = amount of change students carry  b. *Χ* ~ *E*(0.88, 0.88)  c. = average amount of change carried by a sample of 25 students.  d. ~ *N*(0.88, 0.176)  e. 0.0819  f. 0.1882  g. The distributions are different. Part a is exponential and part b is normal. |
| Exercise 50. | *Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet. We randomly sample 49 fly balls.*  *a. If = average distance in feet for 49 fly balls, then ~ \_\_\_\_\_\_\_(\_\_\_\_\_\_\_,\_\_\_\_\_\_\_)*  *b. What is the probability that the 49 balls traveled an average of less than 240 feet? Sketch the graph. Scale the horizontal axis for . Shade the region corresponding to the probability. Find the probability.*  *c. Find the 80th percentile of the distribution of the average of 49 fly balls.* |
| Solution | a.  b. 0.0808; for graph, check student’s solution.  c. 256.01 feet |
| Exercise 51. | *According to the Internal Revenue Service, the average length of time for an individual to complete (keep records for, learn, prepare, copy, assemble, and send) IRS Form 1040 is 10.53 hours (without any attached schedules). The distribution is unknown. Let us assume that the standard deviation is two hours. Suppose we randomly sample 36 taxpayers.*  *a. In words, Χ = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *b. In words, = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *c. ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *d. Would you be surprised if the 36 taxpayers finished their Form 1040s in an average of more than 12 hours? Explain why or why not in complete sentences.*  *e. Would you be surprised if one taxpayer finished his or her Form 1040 in more than 12 hours? In a complete sentence, explain why.* |
| Solution | a. length of time for an individual to complete IRS form 1040, in hours.  b. mean length of time for a sample of 36 taxpayers to complete IRS form 1040, in hours.  c.  d. Yes. I would be surprised, because the probability is almost 0.  e. No. I would not be totally surprised because the probability is 0.2312 |
| Exercise 52. | *Suppose that a category of world-class runners are known to run a marathon (26 miles) in an average of 145 minutes with a standard deviation of 14 minutes. Consider 49 of the races. Let the average of the 49 races.*  *a. ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *b. Find the probability that the runner will average between 142 and 146 minutes in these 49 marathons.*  *c. Find the 80th percentile for the average of these 49 marathons.*  *d. Find the median of the average running times.* |
| Solution | a.  b. 0.6247  c. 146.68  d. 145 minutes |
| Exercise 53. | *The length of songs in a collector’s iTunes album collection is uniformly distributed from two to 3.5 minutes. Suppose we randomly pick five albums from the collection. There are a total of 43 songs on the five albums.*  *a. In words, Χ = \_\_\_\_\_\_\_\_\_*  *b. Χ ~ \_\_\_\_\_\_\_\_\_\_\_\_\_*  *c. In words, = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *d. ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *e. Find the first quartile for the average song length, .*  *f. The IQR(interquartile range) for the average song length, , is \_\_\_\_\_\_\_\_\_.* |
| Solution | a. the length of a song, in minutes, in the collection  b. *U*(2, 3.5)  c. the average length, in minutes, of the songs from a sample of five albums from collection  d. *N*(2.75, 0.0660)  e. 2.71 minutes  f. 0.09 minutes |
| Exercise 54. | *In 1940 the average size of a U.S. farm was 174 acres. Let’s say that the standard deviation was 55 acres. Suppose we randomly survey 38 farmers from 1940.*  *a. In words, Χ = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *b. In words, = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *c. ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *d. The IQR for is from \_\_\_\_\_\_\_ acres to \_\_\_\_\_\_\_ acres.* |
| Solution | a. the size of a U.S. farm in 1940  b. the average size of a U.S. farm as estimated from a sample of 38, in acres  c.  d. 168.0, 180.0 |
| Exercise 55. | *Determine which of the following are true and which are false. Then, in complete sentences, justify your answers.*  *a. When the sample size is large, the mean of is approximately equal to the mean of Χ.*  *b. When the sample size is large, is approximately normally distributed.*  *c. When the sample size is large, the standard deviation of is approximately the same as the standard deviation of Χ.* |
| Solution | a. True. The mean of a sampling distribution of the means is approximately the mean of the data distribution.  b. True. According to the Central Limit Theorem, the larger the sample, the closer the sampling distribution of the means becomes normal.  c. True. The standard deviation of the sampling distribution of the means will decrease making it approximately the same as the standard deviation of *X* as the sample size increases. |
| Exercise 56. | *The percent of fat calories that a person in America consumes each day is normally distributed with a mean of about 36 and a standard deviation of about ten. Suppose that 16 individuals are randomly chosen. Let = average percent of fat calories.*  *a. ~ \_\_\_\_\_\_(\_\_\_\_\_\_, \_\_\_\_\_\_)*  *b. For the group of 16, find the probability that the average percent of fat calories consumed is more than five. Graph the situation and shade in the area to be determined.*  *c. Find the first quartile for the average percent of fat calories.* |
| Solution | a.  b. 1  c. 34.31 |
| Exercise 57. | *The distribution of income in some Third World countries is considered wedge shaped (many very poor people, very few middle income people, and even fewer wealthy people). Suppose we pick a country with a wedge shaped distribution. Let the average salary be $2,000 per year with a standard deviation of $8,000. We randomly survey 1,000 residents of that country.*  *a. In words, Χ = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *b. In words, = \_\_\_\_\_\_\_\_\_\_\_\_\_*  *c. ~ \_\_\_\_\_(\_\_\_\_\_,\_\_\_\_\_)*  *d. How is it possible for the standard deviation to be greater than the average?*  *e. Why is it more likely that the average of the 1,000 residents will be from $2,000 to $2,100 than from $2,100 to $2,200?* |
| Solution | a. *X* = the yearly income of someone in a third world country  b. the average salary from samples of 1,000 residents of a third world country  c.  d. Very wide differences in data values can have averages smaller than standard deviations.  e. The distribution of the sample mean will have higher probabilities closer to the population mean.  *P*(2000 < < 2100) = 0.1537  *P*(2100 < < 2200) = 0.1317 |
| Exercise 58. | *Which of the following is NOT TRUE about the distribution for averages?*  *a. The mean, median, and mode are equal.*  *b. The area under the curve is one.*  *c. The curve never touches the x-axis.*  *d. The curve is skewed to the right.* |
| Solution | d |
| Exercise 59. | *The cost of unleaded gasoline in the Bay Area once followed an unknown distribution with a mean of $4.59 and a standard deviation of $0.10. Sixteen gas stations from the Bay Area are randomly chosen. We are interested in the average cost of gasoline for the 16 gas stations. The distribution to use for the average cost of gasoline for the 16 gas stations is:*  *a.*  *b.*  *c.*  *d.* |
| Solution | b |

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