**Chapter 2**

## Simple Comparative Experiments

# Solutions

**2.1.** Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | N | Mean | SE Mean | Std. Dev. | Variance | Minimum | Maximum |
| Y | 9 | 19.96 | ? | 3.12 | ? | 15.94 | 27.16 |

SE Mean = 1.04 Variance = 9.73

**2.2S.** Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variable | N | Mean | SE Mean | Std. Dev. | Sum |
| Y | 16 | ? | 0.159 | ? | 399.851 |

Mean = 24.991 Std. Dev. = 0.636

**2.3.** Suppose that we are testing *H0*: *µ* = *µ0* versus *H1*: *µ* ≠ *µ0*. Calculate the *P*-value for the following observed values of the test statistic:

(a) *Z0* = 2.25 *P*-value = 0.02445

(b) *Z0* = 1.55 *P*-value = 0.12114

(c) *Z0* = 2.10 *P*-value = 0.03573

(d) *Z0* = 1.95 *P*-value = 0.05118

(e) *Z0* = -0.10 *P*-value = 0.92034

**2.4S.** Suppose that we are testing *H0*: *µ* = *µ0* versus *H1*: *µ* > *µ0*. Calculate the *P*-value for the following observed values of the test statistic:

(a) *Z0* = 2.45 *P*-value = 0.00714

(b) *Z0* = -1.53 *P*-value = 0.93699

(c) *Z0* = 2.15 *P*-value = 0.01578

(d) *Z0* = 1.95 *P*-value = 0.02559

(e) *Z0* = -0.25 *P*-value = 0.59871

**2.5.** Suppose that we are testing *H0*: *µ1* = *µ2* versus *H1*: *µ1* = *µ2* with a sample size of *n1 = n2* = 12. Both sample variances are unknown but assumed equal. Find bounds on the *P*-value for the following observed values of the test statistic:

(a) *t0* = 2.30 Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.0313

(b) *t0* = 3.41 Table *P*-value = 0.002, 0.005 Computer *P*-value = 0.0025

(c) *t0* = 1.95 Table *P*-value = 0.1, 0.05 Computer *P*-value = 0.0640

(d) *t0* = -2.45 Table *P*-value = 0.05, 0.02 Computer *P*-value = 0.0227

Note that the degrees of freedom is (12 +12) – 2 = 22. This is a two-sided test

**2.6.** Suppose that we are testing *H0*: *µ1* = *µ2* versus *H1*: *µ1* > *µ2* with a sample size of *n1 = n2* = 10. Both sample variances are unknown but assumed equal. Find bounds on the *P*-value for the following observed values of the test statistic:

(a) *t0* = 2.31 Table *P*-value = 0.01, 0.025 Computer *P*-value = 0.01648

(b) *t0* = 3.60 Table *P*-value = 0.001, 0.0005 Computer *P*-value = 0.00102

(c) *t0* = 1.95 Table *P*-value = 0.05, 0.025 Computer *P*-value = 0.03346

(d) *t0* = 2.19 Table *P*-value = 0.01, 0.025 Computer *P*-value = 0.02097

Note that the degrees of freedom is (10 +10) – 2 = 18. This is a one-sided test.

**2.7.** Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

Minitab Output



According to the output, the Anderson-Darling Normality Test has a *P*-Value of 0.435. The data can be considered normal. The 95% confidence interval on the mean is (9.526,12.378). This confidence interval contains 10, therefore there is evidence that the population mean is 10.

**2.8.** A computer program has produced the following output for the hypothesis testing problem:

|  |
| --- |
| Difference in sample means: 2.35  Degrees of freedom: 18  Standard error of the difference in the sample means: ? |
| Test statistic: *t*o = 2.01  *P*-Value = 0.0298 |

(a) What is the missing value for the standard error?



(b) Is this a two-sided or one-sided test? One-sided test for a *t*0 = 2.01 is a *P*-value of 0.0298.

(c) If **=0.05, what are your conclusions? Reject the null hypothesis and conclude that there is a difference in the two samples.

(d) Find a 90% two-sided CI on the difference in the means.



**2.9S.** A computer program has produced the following output for the hypothesis testing problem:

|  |
| --- |
| Difference in sample means: 11.5  Degrees of freedom: 24  Standard error of the difference in the sample means: ? |
| Test statistic: *t*o = -1.88  *P*-Value = 0.0723 |

(a) What is the missing value for the standard error?



(b) Is this a two-sided or one-sided test? Two-sided test for a *t*0 = -1.88 is a *P*-value of 0.0723.

(c) If **=0.05, what are your conclusions? Accept the null hypothesis, there is no difference in the means.

(d) Find a 90% two-sided CI on the difference in the means.



**2.10.** A two-sample *t*-test has been conducted and the sample sizes are *n*1 = *n*2 = 10. The computed value of the test statistic is *t0* = 2.15. If the null hypothesis is two-sided, an upper bound on the *P*-value is

(a) 0.10

**(b) 0.05**

(c) 0.025

(d) 0.01

(e) None of the above.

**2.11.** A two-sample *t*-test has been conducted and the sample sizes are *n*1 = *n*2 = 12. The computed value of the test statistic is *t0* = 2.27. If the null hypothesis is two-sided, an upper bound on the *P*-value is

(a) 0.10

**(b) 0.05**

(c) 0.025

(d) 0.01

(e) None of the above.

**2.12S.** Suppose that we are testing *H0*: *µ* = *µ0* versus *H1*: *µ* > *µ0* with a sample size of *n* = 15. Calculate bounds on the *P*-value for the following observed values of the test statistic:

(a) *t0* = 2.35 Table *P*-value = 0.01, 0.025 Computer *P*-value = 0.01698

(b) *t0* = 3.55 Table *P*-value = 0.001, 0.0025 Computer *P*-value = 0.00160

(c) *t0* = 2.00 Table *P*-value = 0.025, 0.005 Computer *P*-value = 0.03264

(d) *t0* = 1.55 Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.07172

The degrees of freedom are 15 – 1 = 14. This is a one-sided test.

**2.13.** Suppose that we are testing *H0*: *µ* = *µ0* versus *H1*: *µ* ≠ *µ0* with a sample size of *n* = 10. Calculate bounds on the *P*-value for the following observed values of the test statistic:

(a) *t0* = 2.48 Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.03499

(b) *t0* = -3.95 Table *P*-value = 0.002, 0.005 Computer *P*-value = 0.00335

(c) *t0* = 2.69 Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.02480

(d) *t0* = 1.88 Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.09281

(e) *t0* = -1.25 Table *P*-value = 0.20, 0.50 Computer *P*-value = 0.24282

**2.14.** Consider the computer output shown below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| One-Sample T: Y | | | | | | | |
| Test of mu = 25 vs > 25 | | | | | | | |
| Variable | N | Mean | Std. Dev. | SE Mean | 95% Lower Bound | T | P |
| Y | 12 | 25.6818 | ? | 0.3360 | ? | ? | 0.034 |

(a) How many degrees of freedom are there on the *t*-test statistic?

(N-1) = (12 – 1) = 11

(b) Fill in the missing information.

Std. Dev. = 1.1639 95% Lower Bound = 2.0292

**2.15.** Consider the computer output shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Two-Sample T-Test and CI: Y1, Y2 | | | | |
| Two-sample T for Y1 vs Y2 | | | | |
|  | N | Mean | Std. Dev. | SE Mean |
| Y1 | 20 | 50.19 | 1.71 | 0.38 |
| Y2 | 20 | 52.52 | 2.48 | 0.55 |
| Difference = mu (X1) – mu (X2) | | | | |
| Estimate for difference: -2.33341 | | | | |
| 95% CI for difference: (-3.69547, -0.97135) | | | | |
| T-Test of difference = 0 (vs not = ) : T-Value = -3.47 | | | | |
| P-Value = 0.01 DF = 38 | | | | |
| Both use Pooled Std. Dev. = 2.1277 | | | | |

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

Yes, the *P*-Value of 0.001 is much less than 0.05.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) If the hypothesis had been *H0*: *µ1* - *µ2* = 2 versus *H1*: *µ1* - *µ2* ≠ 2 would you reject the null hypothesis at the 0.05 level?

Yes.

(d) If the hypothesis had been H0: µ1 - µ2 = 2 versus H1: µ1 - µ2 < 2 would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

Yes, no additional calculations are required because the test is naturally becoming more significant with the change from -2.33341 to -4.33341.

(e) Use the output and the *t* table to find a 95 percent upper confidence bound on the difference in means?

95% upper confidence bound = -1.21.

(f) What is the *P*-value if the alternative hypotheses are *H0*: *µ1* - *µ2* = 2 versus *H1*: *µ1* - *µ2* ≠ 2?

*P*-value = 1.4E-07.

**2.16.** The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is = 3 psi. A random sample of four specimens is tested. The results are *y*1=145, *y*2=153, *y*3=150 and *y*4=147.

(a) State the hypotheses that you think should be tested in this experiment.

*H*0: = 150 *H*1: > 150

(b) Test these hypotheses using = 0.05. What are your conclusions?

*n* = 4, = 3, = 1/4 (145 + 153 + 150 + 147) = 148.75



Since *z*0.05 = 1.645, do not reject.

(c) Find the *P*-value for the test in part (b).

From the *z-*table: 

(d) Construct a 95 percent confidence interval on the mean breaking strength.

The 95% confidence interval is





**2.17.** The viscosity of a liquid detergent is supposed to average 800 centistokes at 25C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is = 25 centistokes.

(a) State the hypotheses that should be tested.

*H*0: = 800 *H*1:  800

(b) Test these hypotheses using = 0.05. What are your conclusions?

 Since *z*/2 = *z*0.025 = 1.96, do not reject.

(c) What is the *P*-value for the test?

(d) Find a 95 percent confidence interval on the mean.



The 95% confidence interval is





**2.18.** A normally distributed random variable has an unknown mean and a known variance 2 = 9. Find the sample size required to construct a 95 percent confidence interval on the mean that has total length of 1.0.

Since *y* N(,9), a 95% two-sided confidence interval on is

If the total interval is to have width 1.0, then the half-interval is 0.5. Since *z*/2 = *z*0.025 = 1.96,



**2.19S.** The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

|  |  |
| --- | --- |
| Days | |
| 108 | 138 |
| 124 | 163 |
| 124 | 159 |
| 106 | 134 |
| 115 | 139 |

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

*H*0: = 120 *H*1: > 120

(b) Test these hypotheses using = 0.01. What are your conclusions?

 = 131

*S*2 = 3438 / 9 = 382





since *t*0.01,9 = 2.821; do not reject *H*0

Minitab Output

**T-Test of the Mean**

Test of mu = 120.00 vs mu > 120.00

Variable N Mean StDev SE Mean T P

Shelf Life 10 131.00 19.54 6.18 1.78 0.054

**T Confidence Intervals**

Variable N Mean StDev SE Mean 99.0 % CI

Shelf Life 10 131.00 19.54 6.18 ( 110.91, 151.09)

(c) Find the *P*-value for the test in part (b). *P*=0.054

(d) Construct a 99 percent confidence interval on the mean shelf life.

The 99% confidence interval is  with = 0.01.





**2.20.** Consider the shelf life data in Problem 2.19S. Can shelf life be described or modeled adequately by a normal distribution? What effect would violation of this assumption have on the test procedure you used in solving Problem 2.22?

A normal probability plot, obtained from Minitab, is shown. There is no reason to doubt the adequacy of the normality assumption. If shelf life is not normally distributed, then the impact of this on the *t*-test in problem 2.19 is not too serious unless the departure from normality is severe.



**2.21.** The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Hours | | | |
| 159 | 280 | 101 | 212 |
| 224 | 379 | 179 | 264 |
| 222 | 362 | 168 | 250 |
| 149 | 260 | 485 | 170 |

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

*H*0: = 225 *H*1: > 225

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use = 0.05.

= 241.50

*S*2 =146202 / (16 - 1) = 9746.80





since *t*0.05,15 = 1.753; do not reject *H*0

Minitab Output

**T-Test of the Mean**

Test of mu = 225.0 vs mu > 225.0

Variable N Mean StDev SE Mean T P

Hours 16 241.5 98.7 24.7 0.67 0.26

**T Confidence Intervals**

Variable N Mean StDev SE Mean 95.0 % CI

Hours 16 241.5 98.7 24.7 ( 188.9, 294.1)

(c) Find the *P*-value for this test. *P*=0.26

(d) Construct a 95 percent confidence interval on mean repair time.

The 95% confidence interval is 





**2.22.** Reconsider the repair time data in Problem 2.21. Can repair time, in your opinion, be adequately modeled by a normal distribution?

The normal probability plot below does not reveal any serious problem with the normality assumption.



**2.23.** Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviation of 1 = 0.015 and 2 = 0.018. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

|  |  |  |  |
| --- | --- | --- | --- |
| Machine 1 | | Machine 2 | |
| 16.03 | 16.01 | 16.02 | 16.03 |
| 16.04 | 15.96 | 15.97 | 16.04 |
| 16.05 | 15.98 | 15.96 | 16.02 |
| 16.05 | 16.02 | 16.01 | 16.01 |
| 16.02 | 15.99 | 15.99 | 16.00 |

(a) State the hypotheses that should be tested in this experiment.

*H*0: 1 = 2 *H*1: 1 2

(b) Test these hypotheses using =0.05. What are your conclusions?

 



*z*0.025 = 1.96; do not reject

(c) What is the *P*-value for the test? *P* = 0.1770

(d) Find a 95 percent confidence interval on the difference in the mean fill volume for the two machines.

The 95% confidence interval is







**2.24.** Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that 1 = 2 = 1.0 psi. From random samples of *n*1 = 10 and *n*2 = 12 we obtain 1 = 162.5 and 2 = 155.0. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this questions, set up and test appropriate hypotheses using = 0.01. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

*H*0: 1 - 2 =10 *H*1: 1 - 2 >10

 



*z*0.01 = 2.325; do not reject

The 99 percent confidence interval is







**2.25.** The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the means and variance of the burning times.

|  |  |  |  |
| --- | --- | --- | --- |
| Type 1 | | Type 2 | |
| 65 | 82 | 64 | 56 |
| 81 | 67 | 71 | 69 |
| 57 | 59 | 83 | 74 |
| 66 | 75 | 59 | 82 |
| 82 | 70 | 65 | 79 |

(a) Test the hypotheses that the two variances are equal. Use = 0.05.



Do not reject.

(b) Using the results of (a), test the hypotheses that the mean burning times are equal. Use = 0.05. What is the *P*-value for this test?

Do not reject.

From the computer output, *t*=0.05; do not reject. Also from the computer output *P*=0.96

Minitab Output

**Two Sample T-Test and Confidence Interval**

Two sample T for Type 1 vs Type 2

N Mean StDev SE Mean

Type 1 10 70.40 9.26 2.9

Type 2 10 70.20 9.37 3.0

95% CI for mu Type 1 - mu Type 2: ( -8.6, 9.0)

T-Test mu Type 1 = mu Type 2 (vs not =): T = 0.05 P = 0.96 DF = 18

Both use Pooled StDev = 9.32

(c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

The assumption of normality is required in the theoretical development of the *t*-test. However, moderate departure from normality has little impact on the performance of the *t*-test. The normality assumption is more important for the test on the equality of the two variances. An indication of nonnormality would be of concern here. The normal probability plots shown below indicate that burning time for both formulations follow the normal distribution.





**2.26S.** An article in *Solid State Technology*, "Orthogonal Design of Process Optimization and Its Application to Plasma Etching" by G.Z. Yin and D.W. Jillie (May, 1987) describes an experiment to determine the effect of C2F6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| C2F6 | Uniformity Observation | | | | | |
| (SCCM) | 1 | 2 | 3 | 4 | 5 | 6 |
| 125 | 2.7 | 4.6 | 2.6 | 3.0 | 3.2 | 3.8 |
| 200 | 4.6 | 3.4 | 2.9 | 3.5 | 4.1 | 5.1 |

(a) Does the C2F6 flow rate affect average etch uniformity? Use = 0.05.

No, C2F6 flow rate does not affect average etch uniformity.

Minitab Output

**Two Sample T-Test and Confidence Interval**

Two sample T for Uniformity

Flow Rat N Mean StDev SE Mean

125 6 3.317 0.760 0.31

200 6 3.933 0.821 0.34

95% CI for mu (125) - mu (200): ( -1.63, 0.40)

T-Test mu (125) = mu (200) (vs not =): T = -1.35 P = 0.21 DF = 10

Both use Pooled StDev = 0.791

(b) What is the *P*-value for the test in part (a)? From the *Minitab* output, *P*=0.21

(c) Does the C2F6 flow rate affect the wafer-to-wafer variability in etch uniformity? Use = 0.05.



Do not reject; C2F6 flow rate does not affect wafer-to-wafer variability.

(d) Draw box plots to assist in the interpretation of the data from this experiment.

The box plots shown below indicate that there is little difference in uniformity at the two gas flow rates. Any observed difference is not statistically significant. See the *t*-test in part (a).



**2.27.** Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kÅ) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

|  |  |  |
| --- | --- | --- |
| 95 ºC | 100 ºC | |
| 11.176 | | 5.623 |
| 7.089 | | 6.748 |
| 8.097 | | 7.461 |
| 11.739 | | 7.015 |
| 11.291 | | 8.133 |
| 10.759 | | 7.418 |
| 6.467 | | 3.772 |
| 8.315 | | 8.963 |

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use = 0.05.



Since *t*0.05,14 = 1.761, reject *H*0. There appears to be a lower mean thickness at the higher temperature. This is also seen in the computer output.

Minitab Output

**Two-Sample T-Test and CI: Thickness, Temp**

Two-sample T for Thick@95 vs Thick@100

N Mean StDev SE Mean

Thick@95 8 9.37 2.10 0.74

Thick@10 8 6.89 1.60 0.56

Difference = mu Thick@95 - mu Thick@100

Estimate for difference: 2.475

95% lower bound for difference: 0.833

T-Test of difference = 0 (vs >): T-Value = 2.65 P-Value = 0.009 DF = 14

Both use Pooled StDev = 1.86

(b) What is the *P*-value for the test conducted in part (a)? *P* = 0.009

(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

From the computer output the 95% lower confidence bound is . This lower confidence bound is greater than 0; therefore, there is a difference in the two temperatures on the thickness of the photoresist.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality of the photoresist thickness.





There are no significant deviations from the normality assumptions.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kÅ.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 1.86

Sample

Difference Size Power

2.5 8 0.7056

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kÅ with a power of at least 0.9?.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 1.86

Sample Target Actual

Difference Size Power Power

1.5 34 0.9000 0.9060

This result makes intuitive sense. More samples are needed to detect a smaller difference.

**2.28S.** Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 seconds and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are shown below.

|  |  |  |  |
| --- | --- | --- | --- |
| 10 Seconds | | 20 Seconds | |
| 1 | 3 | 7 | 6 |
| 2 | 6 | 8 | 9 |
| 1 | 5 | 5 | 5 |
| 3 | 3 | 9 | 7 |
| 5 | 2 | 5 | 4 |
| 1 | 1 | 8 | 6 |
| 5 | 6 | 6 | 8 |
| 2 | 8 | 4 | 5 |
| 3 | 2 | 6 | 8 |
| 5 | 3 | 7 | 7 |

(a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use = 0.05.

From the analysis shown below, there is evidence that the longer cool-down time results in fewer appearance defects.

Minitab Output

**Two-Sample T-Test and CI: 10 seconds, 20 seconds**

Two-sample T for 10 seconds vs 20 seconds

N Mean StDev SE Mean

10 secon 20 3.35 2.01 0.45

20 secon 20 6.50 1.54 0.34

Difference = mu 10 seconds - mu 20 seconds

Estimate for difference: -3.150

95% upper bound for difference: -2.196

T-Test of difference = 0 (vs <): T-Value = -5.57 P-Value = 0.000 DF = 38

Both use Pooled StDev = 1.79

(b) What is the *P*-value for the test conducted in part (a)? From the *Minitab* output, *P* = 0.000

(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

From the *Minitab* output, . This lower confidence bound is less than 0. The two samples are different. The 20 second cooling time gives a cosmetically better housing.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality for the data from this experiment.





There are no significant departures from normality.

**2.29.** The diameter of a ball bearing was measured by 12 inspectors, each using two different kinds of calipers. The results were:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Inspector | Caliper 1 | Caliper 2 | Difference | Difference^2 |
| 1 | 0.265 | 0.264 | 0.001 | 0.000001 |
| 2 | 0.265 | 0.265 | 0 | 0 |
| 3 | 0.266 | 0.264 | 0.002 | 0.000004 |
| 4 | 0.267 | 0.266 | 0.001 | 0.000001 |
| 5 | 0.267 | 0.267 | 0 | 0 |
| 6 | 0.265 | 0.268 | -0.003 | 0.000009 |
| 7 | 0.267 | 0.264 | 0.003 | 0.000009 |
| 8 | 0.267 | 0.265 | 0.002 | 0.000004 |
| 9 | 0.265 | 0.265 | 0 | 0 |
| 10 | 0.268 | 0.267 | 0.001 | 0.000001 |
| 11 | 0.268 | 0.268 | 0 | 0 |
| 12 | 0.265 | 0.269 | -0.004 | 0.000016 |
|  |  |  |  |  |

1. Is there a significant difference between the means of the population of measurements represented by the two samples? Use *α* = 0.05.

 or equivalently 

Minitab Output

**Paired T-Test and Confidence Interval**

Paired T for Caliper 1 - Caliper 2

N Mean StDev SE Mean

Caliper 12 0.266250 0.001215 0.000351

Caliper 12 0.266000 0.001758 0.000508

Difference 12 0.000250 0.002006 0.000579

95% CI for mean difference: (-0.001024, 0.001524)

T-Test of mean difference = 0 (vs not = 0): T-Value = 0.43 P-Value = 0.674

1. Find the *P*-value for the test in part (a). *P*=0.674
2. Construct a 95 percent confidence interval on the difference in the mean diameter measurements for the two types of calipers.



**2.30.** An article in the journal of *Neurology* (1998, Vol. 50, pp.1246-1252) observed that the monozygotic twins share numerous physical, psychological and pathological traits. The investigators measured an intelligence score of 10 pairs of twins. The data are obtained as follows:

|  |  |  |
| --- | --- | --- |
| Pair | Birth Order: 1 | Birth Order: 2 |
| 1 | 6.08 | 5.73 |
| 2 | 6.22 | 5.80 |
| 3 | 7.99 | 8.42 |
| 4 | 7.44 | 6.84 |
| 5 | 6.48 | 6.43 |
| 6 | 7.99 | 8.76 |
| 7 | 6.32 | 6.32 |
| 8 | 7.60 | 7.62 |
| 9 | 6.03 | 6.59 |
| 10 | 7.52 | 7.67 |

(a) Is the assumption that the difference in score is normally distributed reasonable?

Minitab Output



By plotting the differences, the output shows that the Anderson-Darling Normality Test shows a P-Value of 0.860. The data is assumed to be normal.

(b) Find a 95% confidence interval on the difference in the mean score. Is there any evidence that mean score depends on birth order?

The 95% confidence interval on the difference in mean score is (-0.366415, 0.264415) contains the value of zero. There is no difference in birth order.

(c) Test an appropriate set of hypothesis indicating that the mean score does not depend on birth order.

 or equivalently 

Minitab Output

Paired T for Birth Order: 1 - Birth Order: 2

N Mean StDev SE Mean

Birth Order: 1 10 6.967 0.810 0.256

Birth Order: 2 10 7.018 1.053 0.333

Difference 10 -0.051 0.441 0.139

95% CI for mean difference: (-0.366, 0.264)

T-Test of mean difference = 0 (vs not = 0): T-Value = -0.37 P-Value = 0.723

Do not reject. The *P*-value is 0.723.

**2.31S.** An article in the *Journal of Strain Analysis* (vol.18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Girder | Karlsruhe Method | Lehigh Method | Difference | Difference^2 |
| S1/1 | 1.186 | 1.061 | 0.125 | 0.015625 |
| S2/1 | 1.151 | 0.992 | 0.159 | 0.025281 |
| S3/1 | 1.322 | 1.063 | 0.259 | 0.067081 |
| S4/1 | 1.339 | 1.062 | 0.277 | 0.076729 |
| S5/1 | 1.200 | 1.065 | 0.135 | 0.018225 |
| S2/1 | 1.402 | 1.178 | 0.224 | 0.050176 |
| S2/2 | 1.365 | 1.037 | 0.328 | 0.107584 |
| S2/3 | 1.537 | 1.086 | 0.451 | 0.203401 |
| S2/4 | 1.559 | 1.052 | 0.507 | 0.257049 |
|  |  | Sum = | 2.465 | 0.821151 |
|  |  | Average = | 0.274 |  |

(a) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use *α* = 0.05.

 or equivalently 







, reject the null hypothesis.

Minitab Output

**Paired T-Test and Confidence Interval**

Paired T for Karlsruhe - Lehigh

N Mean StDev SE Mean

Karlsruh 9 1.3401 0.1460 0.0487

Lehigh 9 1.0662 0.0494 0.0165

Difference 9 0.2739 0.1351 0.0450

95% CI for mean difference: (0.1700, 0.3777)

T-Test of mean difference = 0 (vs not = 0): T-Value = 6.08 P-Value = 0.000

(b) What is the *P*-value for the test in part (a)?

*P*=0.0002

(c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load.



(d) Investigate the normality assumption for both samples.

The normal probability plots of the observations for each method follow. There are no serious concerns with the normality assumption, but there is an indication of a possible outlier (1.178) in the Lehigh method data.





(a) Investigate the normality assumption for the difference in ratios for the two methods.



There is no issue with normality in the difference of ratios of the two methods.

(b) Discuss the role of the normality assumption in the paired *t*-test.

As in any *t*-test, the assumption of normality is of only moderate importance. In the paired *t*-test, the assumption of normality applies to the distribution of the differences. That is, the individual sample measurements do not have to be normally distributed, only their difference.

**2.32.** The deflection temperature under load for two different formulations of ABS plastic pipe is being studied. Two samples of 12 observations each are prepared using each formulation, and the deflection temperatures (in °F) are reported below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Formulation 1 |  |  |  | Formulation 2 |  |
| 206 | 193 | 192 |  | 177 | 176 | 198 |
| 188 | 207 | 210 |  | 197 | 185 | 188 |
| 205 | 185 | 194 |  | 206 | 200 | 189 |
| 187 | 189 | 178 |  | 201 | 197 | 203 |

(a) Construct normal probability plots for both samples. Do these plots support assumptions of normality and equal variance for both samples?





(b) Do the data support the claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2? Use *α* = 0.05.

No, formulation 1 does not exceed formulation 2 per the *Minitab* output below.

Minitab Output

**Two Sample T-Test and Confidence Interval**

N Mean StDev SE Mean

Form 1 12 194.5 10.2 2.9

Form 2 12 193.08 9.95 2.9

Difference = mu Form 1 - mu Form 2

Estimate for difference: 1.42

95% lower bound for difference: -5.64

T-Test of difference = 0 (vs >): T-Value = 0.34 P-Value = 0.367 DF = 22

Both use Pooled StDev = 10.1

(c) What is the *P*-value for the test in part (a)?

*P* = 0.367

**2.33.** Refer to the data in problem 2.32. Do the data support a claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2 by at least 3 °F?

No, formulation 1 does not exceed formulation 2 by at least 3 °F.

Minitab Output

**Two-Sample T-Test and CI: Form1, Form2**

Two-sample T for Form 1 vs Form 2

N Mean StDev SE Mean

Form 1 12 194.5 10.2 2.9

Form 2 12 193.08 9.95 2.9

Difference = mu Form 1 - mu Form 2

Estimate for difference: 1.42

95% lower bound for difference: -5.64

T-Test of difference = 3 (vs >): T-Value = -0.39 P-Value = 0.648 DF = 22

Both use Pooled StDev = 10.1

**2.34S.** In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution and the observed etch rates (in mils/min) are shown below:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Solution 1 | | |  |  |  | Solution 2 | | |  |
| 9.9 | |  | 10.6 | |  | 10.2 | |  | 10.6 | |
| 9.4 | |  | 10.3 | |  | 10.0 | |  | 10.2 | |
| 10.0 | |  | 9.3 | |  | 10.7 | |  | 10.4 | |
| 10.3 | |  | 9.8 | |  | 10.5 | |  | 10.3 | |

(a) Do the data indicate that the claim that both solutions have the same mean etch rate is valid? Use = 0.05 and assume equal variances.

No, the solutions do not have the same mean etch rate. See the Minitab output below.

Minitab Output

**Two Sample T-Test and Confidence Interval**

Two-sample T for Solution 1 vs Solution 2

N Mean StDev SE Mean

Solution 8 9.950 0.450 0.16

Solution 8 10.363 0.233 0.082

Difference = mu Solution 1 - mu Solution 2

Estimate for difference: -0.413

95% CI for difference: (-0.797, -0.028)

T-Test of difference = 0 (vs not =): T-Value = -2.30 P-Value = 0.037 DF = 14

Both use Pooled StDev = 0.358

(b) Find a 95% confidence interval on the difference in mean etch rate.

From the Minitab output, -0.797 to –0.028.

(c) Use normal probability plots to investigate the adequacy of the assumptions of normality and equal variances.





Both the normality and equality of variance assumptions are valid.

**2.35.** Two popular pain medications are being compared on the basis of the speed of absorption by the body. Specifically, tablet 1 is claimed to be absorbed twice as fast as tablet 2. Assume that  and  are known. Develop a test statistic for

*H*0: 21 = 2

*H*1: 21 2

, assuming that the data is normally distributed.

The test statistic is: , reject if 

**2.36. Continuation of Problem 2.35.** An article in *Nature* (1972, pp.225-226) reported on the levels of monoamine oxidase in blood platelets for a sample of 43 schizophrenic patients resulting in = 2.69 and *s*1 = 2.30 while for a sample of 45 normal patients the results were = 6.35 and *s*2 = 4.03. The units are nm/mg protein/h. Use the results of the previous problem to test the claim that the mean monoamine oxidase level for normal patients is at least twice the mean level for schizophrenic patients. Assume that the sample sizes are large enough to use the sample standard deviations as the true parameter values.



*z*0 = -1.05; using α =0.05, , do not reject.

**2.37.** Suppose we are testing

*H*0: 1 = 2

*H*1: 1 2

where  >  are known. Our sampling resources are constrained such that *n*1 + *n*2 = *N*. Show that an allocation of the observation *n*1 and *n*2to the two samples leads to the most powerful test in the ratio *n*1 / *n*2 = *σ*1 / *σ*2.

The most powerful test is attained by the *n*1 and *n*2 that maximize *z*o for given .

Thus, we chose *n*1 and *n*2 to , subject to *n*1 + *n*2 = *N*.

This is equivalent to min , subject to *n*1 + *n*2 = *N*.

Now , implies that *n*1 / *n*2 = *σ*1 / *σ*2.

Thus *n*1 and *n*2 are assigned proportionally to the ratio of the standard deviations. This has intuitive appeal, as it allocates more observations to the population with the greatest variability.

**2.38. Continuation of Problem 2.37.** Suppose that we want to construct a 95% two-sided confidence interval on the difference in two means where the two sample standard deviations are known to be *σ*1 = 4 and *σ*2 = 8. The total sample size is restricted to *N* = 30. What is the length of the 95% CI if the sample sizes used by the experimenter are *n*1 = *n*2 = 15? How much shorter would the 95% CI have been if the experiment had used the optimal sample size calculation?

The 95% confidence interval for *n*1 = *n*2 = 15 is



The 95% confidence interval for the proportions is,

Therefore *n*2 = 20 and *n*1 = 10



The confidence interval decreases from a multiple of 2.31 to a multiple of 2.19.

**2.39.** Develop Equation 2.46 for a 100(1 - *α*) percent confidence interval for the variance of a normal distribution.

 . Thus,  . Therefore,

,

so  is the 100(1 - *α*)% confidence interval on *σ* 2.

**2.40.** Develop Equation 2.50 for a 100(1 - *α*) percent confidence interval for the ratio  / , where  and  are the variances of two normal distributions.



 or



**2.41.** Develop an equation for finding a 100(1 - *α*) percent confidence interval on the difference in the means of two normal distributions where  . Apply your equation to the portland cement experiment data, and find a 95% confidence interval.







where 

Using the data from Table 2.1





where 



**2.42.** Construct a data set for which the paired *t*-test statistic is very large, but for which the usual two-sample or pooled *t*-test statistic is small. In general, describe how you created the data. Does this give you any insight regarding how the paired *t*-test works?

|  |  |  |
| --- | --- | --- |
| A | B | delta |
| 7.1662 | 8.2416 | -1.0754 |
| 2.3590 | 2.4555 | -0.0965 |
| 19.9977 | 21.1018 | -1.1041 |
| 0.9077 | 2.3401 | -1.4324 |
| -15.9034 | -15.0013 | -0.9021 |
| -6.0722 | -5.5941 | -0.4781 |
| 9.9501 | 10.6910 | -0.7409 |
| -1.0944 | -0.1358 | -0.9586 |
| -4.6907 | -3.3446 | -1.3461 |
| -6.6929 | -5.9303 | -0.7626 |

Minitab Output

**Paired T-Test and Confidence Interval**

Paired T for A - B

N Mean StDev SE Mean

A 10 0.59 10.06 3.18

B 10 1.48 10.11 3.20

Difference 10 -0.890 0.398 0.126

95% CI for mean difference: (-1.174, -0.605)

T-Test of mean difference = 0 (vs not = 0): T-Value = -7.07 P-Value = 0.000

**Two Sample T-Test and Confidence Interval**

Two-sample T for A vs B

N Mean StDev SE Mean

A 10 0.6 10.1 3.2

B 10 1.5 10.1 3.2

Difference = mu A - mu B

Estimate for difference: -0.89

95% CI for difference: (-10.37, 8.59)

T-Test of difference = 0 (vs not =): T-Value = -0.20 P-Value = 0.846 DF = 18

Both use Pooled StDev = 10.1

These two sets of data were created by making the observation for *A* and *B* moderately different within each pair (or block), but making the observations between pairs very different. The fact that the difference between pairs is large makes the pooled estimate of the standard deviation large and the two-sample *t*-test statistic small. Therefore the fairly small difference between the means of the two treatments that is present when they are applied to the same experimental unit cannot be detected. Generally, if the blocks are very different, then this will occur. Blocking eliminates the variability associated with the nuisance variable that they represent.

**2.43S.** Consider the experiment described in problem 2.25. If the mean burning times of the two flames differ by as much as 2 minutes, find the power of the test. What sample size would be required to detect an actual difference in mean burning time of 1 minute with a power of at least 0.90?

From the *Minitab* output below, the power is 0.0740. This answer was obtained by using the pooled estimate of *σ* from Problem 2-11, *Sp* = 9.32. Because the difference in means is very small relative to the standard deviation, the power is very low.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 9.32

Sample

Difference Size Power

2 10 0.0740

From the *Minitab* output below, the required sample size is 1827. The sample size is huge because the difference in means is very small relative to the standard deviation.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 9.32

Sample Target Actual

Difference Size Power Power

1 1827 0.9000 0.9001

**2.44.** Reconsider the bottle filling experiment described in Problem 2.23. Rework this problem assuming that the two population variances are unknown but equal.

Minitab Output

**Two-Sample T-Test and CI: Machine 1, Machine 2**

Two-sample T for Machine 1 vs Machine 2

N Mean StDev SE Mean

Machine 10 16.0150 0.0303 0.0096

Machine 10 16.0050 0.0255 0.0081

Difference = mu Machine 1 - mu Machine 2

Estimate for difference: 0.0100

95% CI for difference: (-0.0163, 0.0363)

T-Test of difference = 0 (vs not =): T-Value = 0.80 P-Value = 0.435 DF = 18

Both use Pooled StDev = 0.0280

The hypothesis test is the same: *H*0: 1 = 2 *H*1: 1 2

The conclusions are the same as Problem 2.19, do not reject *H*0. There is no difference in the machines. The *P*-value for this analysis is 0.435.

The confidence interval is (-0.0163, 0.0363). This interval contains 0. There is no difference in machines.

**2.45.** Consider the data from problem 2.23. If the mean fill volume of the two machines differ by as much as 0.25 ounces, what is the power of the test used in problem 2.19? What sample size could result in a power of at least 0.9 if the actual difference in mean fill volume is 0.25 ounces?

The power is 1.0000 as shown in the analysis below.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 0.028

Sample

Difference Size Power

0.25 10 1.0000

The required sample size is 2 as shown below.

Minitab Output

**Power and Sample Size**

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)

Calculating power for mean 1 = mean 2 + difference

Alpha = 0.05 Sigma = 0.028

Sample Target Actual

Difference Size Power Power

0.25 2 0.9000 0.9805

**2.46.** An experiment has been performed with a factor that has only two levels. Samples of size *n*1 = *n*2 = 12 have been taken and the resulting sample data is as follows:

Can you conclude that there is no difference in means using = 0.05? What are bounds on the *P*-value for this test? Find a 95% confidence interval on the difference in the two means. Does the confidence interval provide any information that is useful in interpreting the test of the hypothesis on the difference in the two means?

Minitab Output

**Two-Sample T-Test and CI**

Sample N Mean StDev SE Mean

1 12 12.50 1.80 0.52

2 12 13.10 2.10 0.61

Difference = μ (1) - μ (2)

Estimate for difference: -0.600

95% CI for difference: (-2.256, 1.056)

T-Test of difference = 0 (vs ≠): T-Value = -0.75 P-Value = 0.460 DF = 22

Both use Pooled StDev = 1.9558

There is no difference in the means. The *P*-value is 0.460. If using the *t*-table in the book, the bounds are 0.50 < P < 0.80. The 95% confidence interval is (-2.256, 1.056). This interval contains zero, meaning that there is no difference in the means.

**2.47.** Reconsider the situation in Problem 2.46. Suppose that the two sample sizes were *n*1 = *n*2 = 5. What difference in conclusions (if any) would you have obtained from the hypothesis test? From the CI?

Minitab Output

**Two-Sample T-Test and CI**

Sample N Mean StDev SE Mean

1 5 12.50 1.80 0.80

2 5 13.10 2.10 0.94

Difference = μ (1) - μ (2)

Estimate for difference: -0.60

95% CI for difference: (-3.45, 2.25)

T-Test of difference = 0 (vs ≠): T-Value = -0.49 P-Value = 0.641 DF = 8

Both use Pooled StDev = 1.9558

There is not difference in the conclusion, there is no difference in the means. The confidence interval for *n*=5 is much wider than the confidence interval for *n*=12.

**2.48.** Suppose that you are testing the hypothesis *H*0: = 50 against the usual two-sided alternative.

The data are normally distributed with known standard deviation = 1. The sample average obtained in the experiment is 50.5, and it is known that if the true population mean is actually 50.5 then this has no practical significance on the problem that motivated the experiment. Find the *P*-value for the *t*-test for the following sample sizes:

1. *n* = 5
2. *n* = 10
3. *n* = 25
4. *n* = 50
5. *n* = 100
6. *n* = 1000

Discuss your findings. What does this tell you about relying on *P*-values in hypothesis testing situations when sample sizes are large?

Minitab Output

**One-Sample T**

Test of μ = 50 vs ≠ 50

N Mean StDev SE Mean 95% CI T P

5 50.500 1.000 0.447 (49.258, 51.742) 1.12 0.326

N Mean StDev SE Mean 95% CI T P

10 50.500 1.000 0.316 (49.785, 51.215) 1.58 0.148

N Mean StDev SE Mean 95% CI T P

25 50.500 1.000 0.200 (50.087, 50.913) 2.50 0.020

N Mean StDev SE Mean 95% CI T P

50 50.500 1.000 0.141 (50.216, 50.784) 3.54 0.001

N Mean StDev SE Mean 95% CI T P

100 50.500 1.000 0.100 (50.302, 50.698) 5.00 0.000

N Mean StDev SE Mean 95% CI T P

1000 50.5000 1.0000 0.0316 (50.4379, 50.5621) 15.81 0.000

As the sample size increases, the SE Mean decreases and the 95% confidence interval gets smaller, driving the P-value smaller. For this problem, the test becomes statistically significant between sample sizes of 10 and 25. Having more data in this case just means that you are proving that there is a statistical difference, not a practical difference.

**2.49.** Consider the situation in Problem 2.48. Calculate the 95% confidence interval on the mean for each of the sample sizes given. How does the length of the confidence interval change with sample size?

The 95% confidence interval decreases as sample sizes increases.

**2.50.** Is the assumption of sampling from a normal distribution critical in the application of the *t*-test? Justify your Answer.

The normal distribution is an assumption to develop the formal test procedure. Moderate departures from normality do not seriously affect the results of a *t*-test. The *t*-test is a good approximation of a randomized design and can be without much concern for normality. A quick procedure for checking normality is the normal probability plot.

**2.51.** An experiment has been performed with a factor that has only two levels. Sample of size *n*1 = *n*2 = 10 have been taken and the resulting sample data is as follows:

It seems likely that the two population variances are not the same. Can you conclude that there is no difference in means using = 0.05? What are bounds on the P-value for this test? Find a 95% confidence interval on the difference in the two means. Does the confidence interval provide any information that is useful in interpreting the test of the hypothesis on the difference in the two means?

Minitab Output

**Test and CI for Two Variances**

Method

Null hypothesis σ(First) / σ(Second) = 1

Alternative hypothesis σ(First) / σ(Second) ≠ 1

Significance level α = 0.05

F method was used. This method is accurate for normal data only.

Statistics

95% CI for

Sample N StDev Variance StDevs

First 10 1.500 2.250 (1.032, 2.738)

Second 10 4.100 16.810 (2.820, 7.485)

Ratio of standard deviations = 0.366

Ratio of variances = 0.134

95% Confidence Intervals

CI for

CI for StDev Variance

Method Ratio Ratio

F (0.182, 0.734) (0.033, 0.539)

Tests

Test

Method DF1 DF2 Statistic P-Value

F 9 9 0.13 0.006

Two-Sample T-Test and CI

Sample N Mean StDev SE Mean

1 10 10.70 1.50 0.47

2 10 15.10 4.10 1.3

Difference = μ (1) - μ (2)

Estimate for difference: -4.40

95% CI for difference: (-7.44, -1.36)

T-Test of difference = 0 (vs ≠): T-Value = -3.19 P-Value = 0.009 DF = 11

The variances of the two samples is different. The confidence interval on the variance is (0033, 0.539). This interval does not include 1, therefore they are different. The *P*-value is 0.006. Using the two-sample *t*-test, with unequal variances, the means are different as well. The confidence interval of the difference of the means at 95% is (-7.44, -1.36). This interval does not include zero, therefore we can conclude that the means are different.

**2.52.** Do you think that using a significance level of = 0.05 is appropriate for all experiments? In the early stages of research and development work is there a lot of harm in identifying a factor as important when it really isn’t? Would that seem to justify higher levels of significance such as = 0.10 or perhaps even = 0.15 in some situations?

Not necessarily. With the use of P-values, one should not immediately reject the null hypothesis if the P-value were say, 0.055. The variable would probably become statistically significant with only a small increase in sample size. For many research and development projects, we may want to consider an = 0.10 or higher. In the R&D environment, typically the sample sizes are very small and we are interested to understand if there might be an effect of the variable that we are sampling.

**2.53.** In the early stages of research and development experimentation which type of error do you think is most important, type I or type II? Justify your answer.

Type II. If a Type II error is made, an important factor is not identified, and it may never be considered again. A Type I error leads to a factor that is not important but thought to be active. This will eventually be identified.