**Chapter 5**

**Introduction to Factorial Designs**

**Solutions**

**5.1.** An interaction effect in the model from a factorial experiment involving quantitative factors is a way of incorporating curvature into the response surface model representation of the results.

**True** False

**5.2.** A factorial experiment may be conducted as a RCBD by running each replicate of the experiment in a unique block.

**True** False

**5.3.** If an interaction effect in a factorial experiment is significant the main effects of the factors involved in that interaction are difficult to interpret individually.

**True** False

**5.4S.** A biomedical researcher has conducted a two-factor factorial experiment as part of the research to develop a new product. She performed the statistical analysis using a computer software package. A portion of the output is shown below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **ANOVA for Selected Factorial Model** | | | | | |
| **Analysis of variance table [Partial sum of Squares]** | | | | | |
| Source | *SS* | *DF* | *MS* | *F* | *P* |
| Model | 874.00 | 5 | 174.80 | 3.28 | 0.0904 |
| *A* | 776.00 | ? | 388.00 | 7.27 | 0.0249 |
| *B* | 5.33 | 1 | 5.33 | 0.10 | 0.7625 |
| *AB* | 92.67 | 2 | 46.33 | 0.87 | 0.4663 |
| Error | 320.00 | ? | 53.33 |  |  |
| Total | 1194.00 | 11 |  |  |  |

The complete ANOVA with the missing degrees of freedom is

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **ANOVA for Selected Factorial Model** | | | | | |
| **Analysis of variance table [Partial sum of Squares]** | | | | | |
| Source | *SS* | *DF* | *MS* | *F* | *P* |
| Model | 874.00 | 5 | 174.80 | 3.28 | 0.0904 |
| *A* | 776.00 | 2 | 388.00 | 7.27 | 0.0249 |
| *B* | 5.33 | 1 | 5.33 | 0.10 | 0.7625 |
| *AB* | 92.67 | 2 | 46.33 | 0.87 | 0.4663 |
| Error | 320.00 | 6 | 53.33 |  |  |
| Total | 1194.00 | 11 |  |  |  |

(a) Interpret the *F*-statistic in the “Model” row of the ANOVA. Specifically, what hypotheses are being tested?

H0: The model is not significant; none of the parameters in the model is likely significant

H1: The model is significant; at least one of the parameters in the model is likely significant

With an *F* value of 3.28, and a *P-*value of 0.0904, marginally significant, at least one of the parameters in the model might be significant.

(b) What conclusions should be drawn regarding the individual model effects?

The individual parameter *F* values and corresponding *P-*values show that factor *A* is significant. *B* and the *AB* interaction are not significant.

(c) How many levels of factor *A* were used in this experiment?

With 2 degrees of freedom, there are three levels for factor *A*.

(d) How many replicates were run?

Based on the degrees of freedom for factors A and B, this is a 3x2 factorial experiment for a total of six runs. With 11 total degrees of freedom, a total of 12 runs were made. Therefore, two replicates were made for each experimental run.

**5.5.** A biomedical researcher has conducted a two-factor factorial experiment as part of the research to develop a new product. She performed the statistical analysis using a computer software package. A portion of the output is shown below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | ? | 1 | 50.00 | ? |
| *B* | 80.00 | ? | 40.00 | ? |
| *AB* | 30.00 | 2 | 15.00 | ? |
| Error | ? | 12 | ? |  |
| Total | 172.00 | 17 |  |  |

(a) Complete the ANOVA calculations.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.00 | 50.00 |
| *B* | 80.00 | 2 | 40.00 | 40.00 |
| *AB* | 30.00 | 2 | 15.00 | 15.00 |
| Error | 12.00 | 12 | 1.00 |  |
| Total | 172.00 | 17 |  |  |

(b) Provide an interpretation of this experiment.

Both main effects *A* and *B*, as well as the *AB* interaction are significant.

(c) The pure error estimate of the standard deviation of the sample observations is 1.

**True** False

**5.6S.** The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Two-way ANOVA: y versus A, B | | | | | |
| Source | DF | SS | MS | F | P |
| A | 1 | ? | 0.0002 | ? | ? |
| B | ? | 180.378 | ? | ? | ? |
| Interaction | 3 | 8.479 | ? | ? | 0.932 |
| Error | 8 | 158.797 | ? |  |  |
| Total | 15 | 347.653 |  |  |  |

(a) Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Two-way ANOVA: y versus A, B | | | | | |
| Source | DF | SS | MS | F | P |
| A | 1 | 0.0002 | 0.0002 | 0.00001 | 0.998 |
| B | 3 | 180.378 | 60.1260 | 3.02907 | 0.093 |
| Interaction | 3 | 8.479 | 2.8263 | 0.14239 | 0.932 |
| Error | 8 | 158.797 | 19.8496 |  |  |
| Total | 15 | 347.653 |  |  |  |

(b) How many levels were used for factor *B*?

4 levels.

(c) How many replicates of the experiment were performed?

2 replicates.

(d) What conclusions would you draw about this experiment?

Factor *B* is moderately significant with a *P*-value of 0.93. Factor A and the two-factor interaction are not significant.

**5.7.** The yield of a chemical process is being studied. The two most important variables are thought to be the pressure and the temperature. Three levels of each factor are selected, and a factorial experiment with two replicates is performed. The yield data follow:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Pressure |  |
| Temperature | 200 | 215 | 230 |
| 150 | 90.4 | 90.7 | 90.2 |
|  | 90.2 | 90.6 | 90.4 |
| 160 | 90.1 | 90.5 | 89.9 |
|  | 90.3 | 90.6 | 90.1 |
| 170 | 90.5 | 90.8 | 90.4 |
|  | 90.7 | 90.9 | 90.1 |

(a) Analyze the data and draw conclusions. Use *α* = 0.05.

Both pressure (*A*) and temperature (*B*) are significant, the interaction is not.

# Design Expert Output

**Response:** **Surface Finish**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.14 8 0.14 8.00 0.0026 significant

*A* *0.77* *2* *0.38* *21.59* *0.0004*

*B* *0.30* *2* *0.15* *8.47* *0.0085*

*AB* *0.069* *4* *0.017* *0.97* *0.4700*

Residual 0.16 9 0.018

*Lack of Fit* *0.000* *0*

*Pure Error* *0.16* *9* *0.018*

Cor Total 1.30 17

The Model F-value of 8.00 implies the model is significant. There is only a 0.26% chance that a

"Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

(b) Prepare appropriate residual plots and comment on the model’s adequacy.

The residual plots show no serious deviations from the assumptions.





(c) Under what conditions would you operate this process?



Set pressure at 215 and Temperature at the high level, 170 degrees C, as this gives the highest yield.

The standard analysis of variance treats all design factors as if they were qualitative. In this case, both factors are quantitative, so some further analysis can be performed. In Section 5.5, we show how response curves and surfaces can be fit to the data from a factorial experiment with at least one quantative factor. Since both factors in this problem are quantitative and have three levels, we can fit linear and quadratic effects of both temperature and pressure. The Design-Expert output, including the response surface plots, now follows.

# Design Expert Output

**Response:** **Surface Finish**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.13 5 0.23 16.18 < 0.0001 significant

*A 0.10 1 0.10 7.22 0.0198*

*B 0.067 1 0.067 4.83 0.0483*

*A2 0.67 1 0.67 47.74 < 0.0001*

*B2 0.23 1 0.23 16.72 0.0015*

*AB 0.061 1 0.061 4.38 0.0582*

Residual 0.17 12 0.014

*Lack of Fit 7.639E-003 3 2.546E-003 0.14 0.9314 not significant*

### Pure Error 0.16 9 0.018

Cor Total 1.30 17

The Model F-value of 16.18 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, A2, B2 are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

Std. Dev. 0.12 R-Squared 0.8708

Mean 90.41 Adj R-Squared 0.8170

C.V. 0.13 Pred R-Squared 0.6794

PRESS 0.42 Adeq Precision 11.968

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 90.52 1 0.062 90.39 90.66

A-Pressure -0.092 1 0.034 -0.17 -0.017 1.00

B-Temperature 0.075 1 0.034 6.594E-004 0.15 1.00

A2 -0.41 1 0.059 -0.54 -0.28 1.00

B2 0.24 1 0.059 0.11 0.37 1.00

AB -0.087 1 0.042 -0.18 3.548E-003 1.00

**Final Equation in Terms of Coded Factors:**

Yield =

+90.52

-0.092 \* A

+0.075 \* B

-0.41 \* A2

+0.24 \* B2

-0.087 \* A \* B

**Final Equation in Terms of Actual Factors:**

Yield =

+48.54630

+0.86759 \* Pressure

-0.64042 \* Temperature

-1.81481E-003 \* Pressure2

+2.41667E-003 \* Temperature2

-5.83333E-004 \* Pressure \* Temperature



**5.8.** An engineer suspects that the surface finish of a metal part is influenced by the feed rate and the depth of cut. She selects three feed rates and four depths of cut. She then conducts a factorial experiment and obtains the following data:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Depth of | Cut (in) |  |
| Feed Rate (in/min) | 0.15 | 0.18 | 0.20 | 0.25 |
|  | 74 | 79 | 82 | 99 |
| 0.20 | 64 | 68 | 88 | 104 |
|  | 60 | 73 | 92 | 96 |
|  |  |  |  |  |
|  | 92 | 98 | 99 | 104 |
| 0.25 | 86 | 104 | 108 | 110 |
|  | 88 | 88 | 95 | 99 |
|  |  |  |  |  |
|  | 99 | 104 | 108 | 114 |
| 0.30 | 98 | 99 | 110 | 111 |
|  | 102 | 95 | 99 | 107 |

(a) Analyze the data and draw conclusions. Use *α* = 0.05.

The depth (*A*) and feed rate (*B*) are significant, as is the interaction (*AB*).

# Design Expert Output

**Response:** **Surface Finish**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 5842.67 11 531.15 18.49 < 0.0001 significant

*A-Depth* *2125.11* *3* *708.37* *24.66* *< 0.0001*

*B-Feed* *3160.50* *2* *1580.25* *55.02* *< 0.0001*

*AB* *557.06* *6* *92.84* *3.23* *0.0180*

Residual 689.33 24 28.72

*Lack of Fit* *0.000* *0*

*Pure Error* *689.33* *24* *28.72*

Cor Total 6532.00 35

The Model F-value of 18.49 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

(b) Prepare appropriate residual plots and comment on the model’s adequacy.

The residual plots shown indicate nothing unusual.





(c) Obtain point estimates of the mean surface finish at each feed rate.

|  |  |
| --- | --- |
| Feed Rate | Average |
| 0.20 | 81.58 |
| 0.25 | 97.58 |
| 0.30 | 103.83 |



(d) Find *P*-values for the tests in part (a).

The *P*-values are given in the computer output in part (a).

**5.9.** For the data in Problem 5.8, compute a 95 percent interval estimate of the mean difference in response for feed rates of 0.20 and 0.25 in/min.

We wish to find a confidence interval on, where  is the mean surface finish for 0.20 in/min and  is the mean surface finish for 0.25 in/min.





Therefore, the 95% confidence interval for *μ*1 – *μ*2 is –16.000 ± 9.032.

**5.10S.** An article in *Industrial Quality Control* (1956, pp. 5-8) describes an experiment to investigate the effect of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Glass |  | Phosphor Type |  |
| Type | 1 | 2 | 3 |
|  | 280 | 300 | 290 |
| 1 | 290 | 310 | 285 |
|  | 285 | 295 | 290 |
|  |  |  |  |
|  | 230 | 260 | 220 |
| 2 | 235 | 240 | 225 |
|  | 240 | 235 | 230 |

(a) Is there any indication that either factor influences brightness? Use *α* = 0.05.

Both factors, phosphor type (*A*) and Glass type (*B*) influence brightness.

# Design Expert Output

**Response:** **Current** **in microamps**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 15516.67 5 3103.33 58.80 < 0.0001 significant

*A* *933.33* *2* *466.67* *8.84* *0.0044*

*B* *14450.00* *1* *14450.00* *273.79* *< 0.0001*

*AB* *133.33* *2* *66.67* *1.26* *0.3178*

Residual 633.33 12 52.78

*Lack of Fit* *0.000* *0*

*Pure Error* *633.33* *12* *52.78*

Cor Total 16150.00 17

The Model F-value of 58.80 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

(b) Do the two factors interact? Use *α* = 0.05.

There is no interaction effect.

(c) Analyze the residuals from this experiment.

The residual plot of residuals versus phosphor content indicates a very slight inequality of variance. It is not serious enough to be of concern, however.





**5.11S.** The factors that influence the breaking strength of a synthetic fiber are being studied. Four production machines and three operators are chosen and a factorial experiment is run using fiber from the same production batch. The results are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Operator | Machine | | | |
|  | 1 | 2 | 3 | 4 |
| 1 | 109 | 110 | 108 | 110 |
|  | 110 | 115 | 109 | 108 |
|  |  |  |  |  |
| 2 | 110 | 110 | 111 | 114 |
|  | 112 | 111 | 109 | 112 |
|  |  |  |  |  |
| 3 | 116 | 112 | 114 | 120 |
|  | 114 | 115 | 119 | 117 |

(a) Analyze the data and draw conclusions. Use *α* = 0.05.

Only the Operator (*A*) effect is significant.

Design Expert Output

**Response:** **Stength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 217.46 11 19.77 5.21 0.0041 significant

*A* *160.33* *2* *80.17* *21.14* *0.0001*

*B* *12.46* *3* *4.15* *1.10* *0.3888*

*AB* *44.67* *6* *7.44* *1.96* *0.1507*

Residual 45.50 12 3.79

*Lack of Fit* *0.000* *0*

*Pure Error* *45.50* *12* *3.79*

Cor Total 262.96 23

The Model F-value of 5.21 implies the model is significant.

There is only a 0.41% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A are significant model terms.

(b) Prepare appropriate residual plots and comment on the model’s adequacy.

The residual plot of residuals versus predicted shows that variance increases very slightly with strength. There is no indication of a severe problem.





**5.12.** An experiment is conducted to study the influence of operating temperature and three types of face-plate glass in the light output of an oscilloscope tube. The following data are collected:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Temperature |  |
| Glass Type | 100 | 125 | 150 |
|  | 580 | 1090 | 1392 |
| 1 | 568 | 1087 | 1380 |
|  | 570 | 1085 | 1386 |
|  |  |  |  |
|  | 550 | 1070 | 1328 |
| 2 | 530 | 1035 | 1312 |
|  | 579 | 1000 | 1299 |
|  |  |  |  |
|  | 546 | 1045 | 867 |
| 3 | 575 | 1053 | 904 |
|  | 599 | 1066 | 889 |

(a) Use *α* = 0.05 in the analysis. Is there a significant interaction effect? Does glass type or temperature affect the response? What conclusions can you draw?

# Design Expert Output

**Response:**  **Light Output**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 2.412E+006 8 3.015E+005 824.77 < 0.0001 significant

*A 1.509E+005 2 75432.26 206.37 < 0.0001*

*B 1.780E+006 1 1.780E+006 4869.13 < 0.0001*

*B2 1.906E+005 1 1.906E+005 521.39 < 0.0001*

*AB 2.262E+005 2 1.131E+005 309.39 < 0.0001*

*AB2 64373.93 2 32186.96 88.06 < 0.0001*

*Pure Error 6579.33 18 365.52*

Cor Total 2.418E+006 26

The Model F-value of 824.77 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 19.12 R-Squared 0.9973

Mean 940.19 Adj R-Squared 0.9961

C.V. 2.03 Pred R-Squared 0.9939

PRESS 14803.50 Adeq Precision 75.466

From the analysis of variance, both factors, Glass Type (*A*) and Temperature (*B*) are significant, as well as the interaction (*AB*). *B*2 and *AB*2 interation terms are also significant. The interaction and pure quadratic tems can be clearly seen in the plot shown below. For glass types 1 and 2 the temperature is fairly linear, for glass type 3, there is a quadratic effect.



(b) Fit an appropriate model relating light output to glass type and temperature.

The model, both coded and uncoded are shown in the *Design Expert* output below.

# Design Expert Output

**Final Equation in Terms of Coded Factors:**

Light Output =

+1059.00

+28.33 \* A[1]

-24.00 \* A[2]

+314.44 \* B

-178.22 \* B2

+92.22 \* A[1]B

+65.56 \* A[2]B

+70.22 \* A[1]B2

+76.22 \* A[2]B2

**Final Equation in Terms of Actual Factors:**

Glass Type 1

Light Output =

-3646.00000

+59.46667 \* Temperature

-0.17280 \* Temperature2

Glass Type 2

Light Output =

-3415.00000

+56.00000 \* Temperature

-0.16320 \* Temperature2

Glass Type 3

Light Output =

-7845.33333

+136.13333 \* Temperature

-0.51947 \* Temperature2

(c) Analyze the residuals from this experiment. Comment on the adequacy of the models you have considered.

The only concern from the residuals below is the inequality of variance observed in the residuals versus glass type plot shown below.





**5.13.** Consider the data in Problem 5.7. Fit an appropriate model to the response data. Use this model to provide guidance concerning operating conditions for the process.

See the alternative analysis shown in Problem 5.7 part (c).

**5.14.** Use Tukey’s test to determine which levels of the pressure factor are significantly different for the data in Problem 5.7.

Because the *AB* interaction is not significant, the sum of squares for the interaction is included as lack of fit in the residual error sum of squares for a *SS*E of 0.23 and degrees of freedom of 13. The sample size is also assumed to be increased from 2 to 6.

The three pressure averages, arranged in ascending order are

|  |  |  |
| --- | --- | --- |
|  |  |  |

and



Comparing the differences with *T*0.05, we have







Therefore, the difference in yield between a pressure of 215 and 230 psig is statistically significant as is the difference in yield between 215 and 200 psig. However, the difference in yield between 200 and 230 psig is not statistically significant.

**5.15S.** An experiment was conducted to determine if either firing temperature or furnace position affects the baked density of a carbon anode. The data are shown below.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Temperature (°C) |  |
| Position | 800 | 825 | 850 |
|  | 570 | 1063 | 565 |
| 1 | 565 | 1080 | 510 |
|  | 583 | 1043 | 590 |
|  |  |  |  |
|  | 528 | 988 | 526 |
| 2 | 547 | 1026 | 538 |
|  | 521 | 1004 | 532 |

Suppose we assume that no interaction exists. Write down the statistical model. Conduct the analysis of variance and test hypotheses on the main effects. What conclusions can be drawn? Comment on the model’s adequacy.

The model for the two-factor, no interaction model is . Both factors, furnace position (*A*) and temperature (*B*) are significant. Other than the residual representing standard order 14 being marginally low, the residual plots show nothing unusual.

Design Expert Output

**Response:** **Density**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 9.525E+005 3 3.175E+005 718.24 < 0.0001 significant

*A* *7160.06* *1* *7160.06* *16.20* *0.0013*

*B* *9.453E+005* *2* *4.727E+005* *1069.26* *< 0.0001*

Residual 6188.78 14 442.06

*Lack of Fit* *818.11* *2* *409.06* *0.91* *0.4271* *not significant*

*Pure Error* *5370.67* *12* *447.56*

Cor Total 9.587E+005 17

The Model F-value of 718.24 implies the model is significant.

There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.



**5.16.** Derive the expected mean squares for a two-factor analysis of variance with one observation per cell, assuming that both factors are fixed.

|  |  |
| --- | --- |
|  | Degrees of Freedom |
|  | *a*-1 |
|  | *b*-1 |
|  |  |

**5.17.** Consider the following data from a two-factor factorial experiment. Analyze the data and draw conclusions. Perform a test for nonadditivity. Use *α* = 0.05.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Column | Factor |  |
| Row Factor | 1 | 2 | 3 | 4 |
| 1 | 36 | 39 | 36 | 32 |
| 2 | 18 | 20 | 22 | 20 |
| 3 | 30 | 37 | 33 | 34 |

Design Expert Output

**Response:** **data**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 609.42 5 121.88 25.36 0.0006 significant

*A* *580.50* *2* *290.25* *60.40* *0.0001*

*B* *28.92* *3* *9.64* *2.01* *0.2147*

Residual 28.83 6 4.81

Cor Total 638.25 11

The Model F-value of 25.36 implies the model is significant. There is only

a 0.06% chance that a "Model F-Value" this large could occur due to noise.

The row factor (*A*) is significant.

The test for nonadditivity is as follows:



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| Row | 580.50 | 2 | 290.25 | 57.3780 |
| Column | 28.91667 | 3 | 9.63889 | 1.9054 |
| Nonadditivity | 3.54051 | 1 | 3.54051 | 0.6999 |
| Error | 25.29279 | 5 | 5.058558 |  |
| Total | 638.25 | 11 |  |  |

**5.18.** The shear strength of an adhesive is thought to be affected by the application pressure and temperature. A factorial experiment is performed in which both factors are assumed to be fixed. Analyze the data and draw conclusions. Perform a test for nonadditivity.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Temperature (°F) |  |
| Pressure (lb/in2) | 250 | 260 | 270 |
| 120 | 9.60 | 11.28 | 9.00 |
| 130 | 9.69 | 10.10 | 9.57 |
| 140 | 8.43 | 11.01 | 9.03 |
| 150 | 9.98 | 10.44 | 9.80 |

Design Expert Output

**Response:** **Strength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 5.24 5 1.05 2.92 0.1124 not significant

*A* *0.58* *3* *0.19* *0.54* *0.6727*

*B* *4.66* *2* *2.33* *6.49* *0.0316*

Residual 2.15 6 0.36

Cor Total 7.39 11

The "Model F-value" of 2.92 implies the model is not significant relative to the noise.

There is a 11.24 % chance that a "Model F-value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case B are significant model terms.

Temperature (*B*) is a significant factor.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| Row | 0.5806917 | 3 | 0.1935639 | 0.5815 |
| Column | 4.65765 | 2 | 2.328825 | 6.9960 |
| Nonadditivity | 0.48948 | 1 | 0.48948 | 1.4704 |
| Error | 1.6644 | 5 | 0.33288 |  |
| Total | 7.392225 | 11 |  |  |

**5.19S.** Consider the three-factor model

 

Notice that there is only one replicate. Assuming the factors are fixed, write down the analysis of variance table, including the expected mean squares. What would you use as the “experimental error” in order to test hypotheses?

|  |  |  |
| --- | --- | --- |
| Source | Degrees of Freedom | Expected Mean Square |
| *A* | *a*-1 |  |
| *B* | *b*-1 |  |
| *C* | *c*-1 |  |
| *AB* | (*a*-1)(*b*-1) |  |
| *BC* | (*b*-1)(*c*-1) |  |
| Error (*AC + ABC*) | *b*(*a*-1)(*c*-1) |  |
| Total | *abc*-1 |  |

**5.20.** The percentage of hardwood concentration in raw pulp, the vat pressure, and the cooking time of the pulp are being investigated for their effects on the strength of paper. Three levels of hardwood concentration, three levels of pressure, and two cooking times are selected. A factorial experiment with two replicates is conducted, and the following data are obtained:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Percentage | Cooking Time 3.0 Hours | | |  | Cooking Time 4.0 Hours | | |
| of Hardwood | Pressure | | |  | Pressure | | |
| Concentration | 400 | 500 | 650 |  | 400 | 500 | 650 |
| 2 | 196.6 | 197.7 | 199.8 |  | 198.4 | 199.6 | 200.6 |
|  | 196.0 | 196.0 | 199.4 |  | 198.6 | 200.4 | 200.9 |
|  |  |  |  |  |  |  |  |
| 4 | 198.5 | 196.0 | 198.4 |  | 197.5 | 198.7 | 199.6 |
|  | 197.2 | 196.9 | 197.6 |  | 198.1 | 198.0 | 199.0 |
|  |  |  |  |  |  |  |  |
| 8 | 197.5 | 195.6 | 197.4 |  | 197.6 | 197.0 | 198.5 |
|  | 196.6 | 196.2 | 198.1 |  | 198.4 | 197.8 | 199.8 |

(a) Analyze the data and draw conclusions. Use *α* = 0.05.

Design Expert Output

**Response:** **strength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 59.73 17 3.51 9.61 < 0.0001 significant

*A* *7.76* *2* *3.88* *10.62* *0.0009*

*B* *20.25* *1* *20.25* *55.40* *< 0.0001*

*C* *19.37* *2* *9.69* *26.50* *< 0.0001*

*AB* *2.08* *2* *1.04* *2.85* *0.0843*

*AC* *6.09* *4* *1.52* *4.17* *0.0146*

*BC* *2.19* *2* *1.10* *3.00* *0.0750*

*ABC* *1.97* *4* *0.49* *1.35* *0.2903*

Residual 6.58 18 0.37

*Lack of Fit* *0.000* *0*

*Pure Error* *6.58* *18* *0.37*

Cor Total 66.31 35

The Model F-value of 9.61 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C, AC are significant model terms.

All three main effects, concentration (*A*), pressure (*C*) and time (*B*), as well as the concentration x pressure interaction (*AC*) are significant at the 5% level. The concentration x time (*AB*) and pressure x time interactions (*BC*) are significant at the 10% level.

(b) Prepare appropriate residual plots and comment on the model’s adequacy.





There is nothing unusual about the residual plots.

(c) Under what set of conditions would you run the process? Why?





For the highest strength, run the process with the percentage of hardwood at 2, the pressure at 650, and the time at 4 hours.

The standard analysis of variance treats all design factors as if they were qualitative. In this case, all three factors are quantitative, so some further analysis can be performed. In Section 5.5, we show how response curves and surfaces can be fit to the data from a factorial experiment with at least one quantative factor. Since the factors in this problem are quantitative and two of them have three levels, we can fit a linear term for the two-level factor and linear and quadratic components for the three-level factors. The Minitab output, with the *ABC* interaction removed due to insignificance, now follows. Also included is the Design Expert output; however, if the student choses to use Design Expert, sequential sum of squares must be selected to assure that the sum of squares for the model equals the total of the sum of squares for each factor included in the model.

Minitab Output

**General Linear Model: Strength versus**

Factor Type Levels Values

Analysis of Variance for Strength, using Adjusted SS for Tests

Source DF Seq SS Adj SS Adj MS F P

Hardwood 1 6.9067 4.9992 4.9992 13.23 0.001

Time 1 20.2500 1.3198 1.3198 3.49 0.074

Pressure 1 15.5605 1.5014 1.5014 3.97 0.058

Hardwood\*Hardwood 1 0.8571 2.7951 2.7951 7.40 0.012

Pressure\*Pressure 1 3.8134 1.8232 1.8232 4.83 0.038

Hardwood\*Time 1 0.7779 1.5779 1.5779 4.18 0.053

Hardwood\*Pressure 1 2.1179 3.4564 3.4564 9.15 0.006

Time\*Pressure 1 0.0190 2.1932 2.1932 5.81 0.024

Hardwood\*Hardwood\*Time 1 1.3038 1.3038 1.3038 3.45 0.076

Hardwood\*Hardwood\*

Pressure 1 2.1885 2.1885 2.1885 5.79 0.025

Hardwood\*Pressure\*

Pressure 1 1.6489 1.6489 1.6489 4.36 0.048

Time\*Pressure\*Pressure 1 2.1760 2.1760 2.1760 5.76 0.025

Error 23 8.6891 8.6891 0.3778

Total 35 66.3089

Term Coef SE Coef T P

Constant 236.92 29.38 8.06 0.000

Hardwood 10.728 2.949 3.64 0.001

Time -14.961 8.004 -1.87 0.074

Pressure -0.2257 0.1132 -1.99 0.058

Hardwood\*Hardwood -0.6529 0.2400 -2.72 0.012

Pressure\*Pressure 0.000234 0.000107 2.20 0.038

Hardwood\*Time -1.1750 0.5749 -2.04 0.053

Hardwood\*Pressure -0.020533 0.006788 -3.02 0.006

Time\*Pressure 0.07450 0.03092 2.41 0.024

Hardwood\*Hardwood\*Time 0.10278 0.05532 1.86 0.076

Hardwood\*Hardwood\*Pressure 0.000648 0.000269 2.41 0.025

Hardwood\*Pressure\*Pressure 0.000012 0.000006 2.09 0.048

Time\*Pressure\*Pressure -0.000070 0.000029 -2.40 0.025

Unusual Observations for Strength

Obs Strength Fit SE Fit Residual St Resid

6 198.500 197.461 0.364 1.039 2.10R

R denotes an observation with a large standardized residual.

# Design Expert Output

**Response:**  **Strength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 57.62 12 4.80 12.71 < 0.0001 significant

*A 6.91 1 6.91 18.28 0.0003*

*B 20.25 1 20.25 53.60 < 0.0001*

*C 15.56 1 15.56 41.19 < 0.0001*

*A2 0.86 1 0.86 2.27 0.1456*

*C2 3.81 1 3.81 10.09 0.0042*

*AB 0.78 1 0.78 2.06 0.1648*

*AC 2.12 1 2.12 5.61 0.0267*

*BC 0.019 1 0.019 0.050 0.8245*

*A2B 1.30 1 1.30 3.45 0.0761*

*A2C 2.19 1 2.19 5.79 0.0245*

*AC2 1.65 1 1.65 4.36 0.0479*

*BC2 2.18 1 2.18 5.76 0.0249*

Residual 8.69 23 0.38

*Lack of Fit 2.11 5 0.42 1.15 0.3691 not significant*

*Pure Error 6.58 18 0.37*

Cor Total 66.31 35

The Model F-value of 12.71 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C2, AC, A2C, AC2, BC2 are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

Std. Dev. 0.61 R-Squared 0.8690

Mean 198.06 Adj R-Squared 0.8006

C.V. 0.31 Pred R-Squared 0.6794

PRESS 21.26 Adeq Precision 15.040

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 197.21 1 0.26 196.67 197.74

A-Hardwood -0.98 1 0.23 -1.45 -0.52 3.36

B-Cooking Time 0.78 1 0.26 0.24 1.31 6.35

C-Pressure 0.19 1 0.25 -0.33 0.71 4.04

A2 0.42 1 0.25 -0.093 0.94 1.04

C2 0.79 1 0.23 0.31 1.26 1.03

AB -0.22 1 0.13 -0.48 0.039 1.06

AC -0.46 1 0.15 -0.78 -0.14 1.08

BC 0.062 1 0.13 -0.20 0.32 1.02

A2B 0.46 1 0.25 -0.053 0.98 3.96

A2C 0.73 1 0.30 0.10 1.36 3.97

AC2 0.57 1 0.27 5.625E-003 1.14 3.32

BC2 -0.55 1 0.23 -1.02 -0.075 3.30

**Final Equation in Terms of Coded Factors:**

Strength =

+197.21

-0.98 \* A

+0.78 \* B

+0.19 \* C

+0.42 \* A2

+0.79 \* C2

-0.22 \* A \* B

-0.46 \* A \* C

+0.062 \* B \* C

+0.46 \* A2 \* B

+0.73 \* A2 \* C

+0.57 \* A \* C2

-0.55 \* B \* C2

**Final Equation in Terms of Actual Factors:**

Strength =

+236.91762

+10.72773 \* Hardwood

-14.96111 \* Cooking Time

-0.22569 \* Pressure

-0.65287 \* Hardwood2

+2.34333E-004 \* Pressure2

-1.17500 \* Hardwood \* Cooking Time

-0.020533 \* Hardwood \* Pressure

+0.074500 \* Cooking Time \* Pressure

+0.10278 \* Hardwood2 \* Cooking Time

+6.48026E-004 \* Hardwood2 \* Pressure

+1.22143E-005 \* Hardwood \* Pressure2

-7.00000E-005 \* Cooking Time \* Pressure2



Cooking Time: B = 4.00

**5.21.** The quality control department of a fabric finishing plant is studying the effect of several factors on the dyeing of cotton-synthetic cloth used to manufacture men’s shirts. Three operators, three cycle times, and two temperatures were selected, and three small specimens of cloth were dyed under each set of conditions. The finished cloth was compared to a standard, and a numerical score was assigned. The results follow. Analyze the data and draw conclusions. Comment on the model’s adequacy.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Temperature |  |  |  |
|  |  | 300°C |  |  |  | 350°C |  |
|  |  | Operator |  |  |  | Operator |  |
| Cycle Time | 1 | 2 | 3 |  | 1 | 2 | 3 |
|  | 23 | 27 | 31 |  | 24 | 38 | 34 |
| 40 | 24 | 28 | 32 |  | 23 | 36 | 36 |
|  | 25 | 26 | 29 |  | 28 | 35 | 39 |
|  |  |  |  |  |  |  |  |
|  | 36 | 34 | 33 |  | 37 | 34 | 34 |
| 50 | 35 | 38 | 34 |  | 39 | 38 | 36 |
|  | 36 | 39 | 35 |  | 35 | 36 | 31 |
|  |  |  |  |  |  |  |  |
|  | 28 | 35 | 26 |  | 26 | 36 | 28 |
| 60 | 24 | 35 | 27 |  | 29 | 37 | 26 |
|  | 27 | 34 | 25 |  | 25 | 34 | 24 |

All three main effects, and the *AB, AC*, and *ABC* interactions are significant. There is nothing unusual about the residual plots.

Design Expert Output

**Response:** **Score**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1239.33 17 72.90 22.24 < 0.0001 significant

*A* *436.00* *2* *218.00* *66.51* *< 0.0001*

*B* *261.33* *2* *130.67* *39.86* *< 0.0001*

*C* *50.07* *1* *50.07* *15.28* *0.0004*

*AB* *355.67* *4* *88.92* *27.13* *< 0.0001*

*AC* *78.81* *2* *39.41* *12.02* *0.0001*

*BC* *11.26* *2* *5.63* *1.72* *0.1939*

*ABC* *46.19* *4* *11.55* *3.52* *0.0159*

Residual 118.00 36 3.28

*Lack of Fit* *0.000* *0*

*Pure Error* *118.00* *36* *3.28*

Cor Total 1357.33 53

The Model F-value of 22.24 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C, AB, AC, ABC are significant model terms.







**5.22.** In Problem 5.7, suppose that we wish to reject the null hypothesis with a high probability if the difference in the true mean yield at any two pressures is as great as 0.5. If a reasonable prior estimate of the standard deviation of yield is 0.1, how many replicates should be run?

2 replications will be enough to detect the given difference.

**5.23.** Consider the data in Problem 5.12. Analyze the data, assuming that replicates are blocks.

Design Expert Output

**Response:** **Warping**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 11.28 1 11.28

Model 968.22 15 64.55 9.96 < 0.0001 significant

*A* *698.34* *3* *232.78* *35.92* *< 0.0001*

*B* *156.09* *3* *52.03* *8.03* *0.0020*

*AB* *113.78* *9* *12.64* *1.95* *0.1214*

Residual 97.22 15 6.48

Cor Total 1076.72 31

The Model F-value of 9.96 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Both temperature and copper content are significant. This agrees with the analysis in Problem 5.12.

**5.24S.** Consider the data in Problem 5.11S. Analyze the data, assuming that replicates are blocks.

Design-Expert Output

**Response:** **Stength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 2.04 1 2.04

Model 217.46 11 19.77 5.00 0.0064 significant

*A 160.33 2 80.17 20.29 0.0002*

*B 12.46 3 4.15 1.05 0.4087*

*AB 44.67 6 7.44 1.88 0.1716*

Residual 43.46 11 3.95

Cor Total 262.96 23

The Model F-value of 5.00 implies the model is significant. There is only

a 0.64% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A are significant model terms.

Only the operator factor (*A*) is significant. This agrees with the analysis in Problem 5.11S.

**5.25S.** An article in the *Journal of Testing and Evaluation* (Vol. 16, no.2, pp. 508-515) investigated the effects of cyclic loading and environmental conditions on fatigue crack growth at a constant 22 MPa stress for a particular material. The data from this experiment are shown below (the response is crack growth rate).

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Environment |  |
| Frequency | Air | H2O | Salt H2O |
|  | 2.29 | 2.06 | 1.90 |
| 10 | 2.47 | 2.05 | 1.93 |
|  | 2.48 | 2.23 | 1.75 |
|  | 2.12 | 2.03 | 2.06 |
|  |  |  |  |
|  | 2.65 | 3.20 | 3.10 |
| 1 | 2.68 | 3.18 | 3.24 |
|  | 2.06 | 3.96 | 3.98 |
|  | 2.38 | 3.64 | 3.24 |
|  |  |  |  |
|  | 2.24 | 11.00 | 9.96 |
| 0.1 | 2.71 | 11.00 | 10.01 |
|  | 2.81 | 9.06 | 9.36 |
|  | 2.08 | 11.30 | 10.40 |

(a) Analyze the data from this experiment (use *α* = 0.05).

Design Expert Output

**Response:** **Crack Growth**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 376.11 8 47.01 234.02 < 0.0001 significant

*A* *209.89* *2* *104.95* *522.40* *< 0.0001*

*B* *64.25* *2* *32.13* *159.92* *< 0.0001*

*AB* *101.97* *4* *25.49* *126.89* *< 0.0001*

Residual 5.42 27 0.20

*Lack of Fit* *0.000* *0*

*Pure Error* *5.42* *27* *0.20*

Cor Total 381.53 35

The Model F-value of 234.02 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

Both frequency and environment, as well as their interaction are significant.

(b) Analyze the residuals.

The residual plots indicate that there may be some problem with inequality of variance. This is particularly noticable on the plot of residuals versus predicted response and the plot of residuals versus frequency.





(c) Repeat the analyses from parts (a) and (b) using ln(*y*) as the response. Comment on the results.

Design Expert Output

**Response:** **Crack Growth** **Transform:** **Natural log** **Constant:** **0.000**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 13.46 8 1.68 179.57 < 0.0001 significant

*A* *7.57* *2* *3.79* *404.09* *< 0.0001*

*B* *2.36* *2* *1.18* *125.85* *< 0.0001*

*AB* *3.53* *4* *0.88* *94.17* *< 0.0001*

Residual 0.25 27 9.367E-003

*Lack of Fit* *0.000* *0*

*Pure Error* *0.25* *27* *9.367E-003*

Cor Total 13.71 35

The Model F-value of 179.57 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

Both frequency and environment, as well as their interaction are significant. The residual plots based on the transformed data look better.





**5.26.** An article in the *IEEE Transactions on Electron Devices* (Nov. 1986, pp. 1754) describes a study on polysilicon doping. The experiment shown below is a variation of their study. The response variable is base current.

|  |  |  |  |
| --- | --- | --- | --- |
| Polysilicon | Anneal Temperature (°C) | | |
| Doping (ions) | 900 | 950 | 1000 |
| 1 x 10 20 | 4.60 | 10.15 | 11.01 |
| 4.40 | 10.20 | 10.58 |
|  | | | |
| 2 x 10 20 | 3.20 | 9.38 | 10.81 |
| 3.50 | 10.02 | 10.60 |

(a) Is there evidence (with *α* = 0.05) indicating that either polysilicon doping level or anneal temperature affect base current?

Design Expert Output

**Response:** **Base Current**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 112.74 5 22.55 350.91 < 0.0001 significant

*A* *0.98* *1* *0.98* *15.26* *0.0079*

*B* *111.19* *2* *55.59* *865.16* *< 0.0001*

*AB* *0.58* *2* *0.29* *4.48* *0.0645*

Residual 0.39 6 0.064

*Lack of Fit* *0.000* *0*

*Pure Error* *0.39* *6* *0.064*

Cor Total 113.13 11

The Model F-value of 350.91 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Both factors, doping and anneal are significant. Their interaction is significant at the 10% level.

(b) Prepare graphical displays to assist in interpretation of this experiment.



(c) Analyze the residuals and comment on model adequacy.





There is a funnel shape in the plot of residuals versus predicted, indicating some inequality of variance.

(d) Is the model  supported by this experiment (*x*1 = doping level, *x*2 = temperature)? Estimate the parameters in this model and plot the response surface.

Design Expert Output

**Response:** **Base Current**

**ANOVA for Response Surface Reduced Quadratic Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 112.73 4 28.18 493.73 < 0.0001 significant

*A* *0.98* *1* *0.98* *17.18* *0.0043*

*B* *93.16* *1* *93.16* *1632.09* *< 0.0001*

*B2* *18.03* *1* *18.03* *315.81* *< 0.0001*

*AB* *0.56* *1* *0.56* *9.84* *0.0164*

Residual 0.40 7 0.057

*Lack of Fit* *0.014* *1* *0.014* *0.22* *0.6569* *not significant*

*Pure Error* *0.39* *6* *0.064*

Cor Total 113.13 11

The Model F-value of 493.73 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, B2, AB are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 9.94 1 0.12 9.66 10.22

A-Doping -0.29 1 0.069 -0.45 -0.12 1.00

B-Anneal 3.41 1 0.084 3.21 3.61 1.00

B2 -2.60 1 0.15 -2.95 -2.25 1.00

AB 0.27 1 0.084 0.065 0.46 1.00

All of the coefficients in the assumed model are significant. The quadratic effect is easily observable in the response surface plot.



**5.27.** An experiment was conducted to study the life (in hours) of two different brands of batteries in three different devices (radio, camera, and portable DVD player). A completely randomized two-factor experiment was conducted, and the following data resulted.

|  |  |  |  |
| --- | --- | --- | --- |
| Brand | Device | | |
| of Battery | Radio | Camera | DVD Player |
| A | 8.6 | 7.9 | 5.4 |
| 8.2 | 8.4 | 5.7 |
|  | | | |
| B | 9.4 | 8.5 | 5.8 |
| 8.8 | 8.9 | 5.9 |

(a) Analyze the data and draw conclusions, using *α* = 0.05.

Both brand of battery (*A*) and type of device (*B*) are significant, the interaction is not.

# Design Expert Output

**Response:** **Life**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Terms added sequentially (first to last)]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 23.33 5 4.67 54.36 < 0.0001 significant

*A 0.80 1 0.80 9.33 0.0224*

*B 22.45 2 11.22 130.75 < 0.0001*

*AB 0.082 2 0.041 0.48 0.6430*

*Pure Error 0.52 6 0.086*

Cor Total 23.84 11

The Model F-value of 54.36 implies the model is significant. There is only a 0.01% chance that a

"Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

(b) Investigate model adequacy by plotting the residuals.

The residual plots show no serious deviations from the assumptions.





(c) Which brand of batteries would you recommend?

Battery brand B is recommended.



**5.28.** A manufacturer of laundry products is investigating the performance of a newly formulated stain remover. The new formulation is compared to the original formulation with respect to its ability to remove a standard tomato-like stain in a test article of cotton cloth using a factorial experiment. The other factors in the experiment are the number of times the test article is washed (1 or 2), and whether or not a detergent booster is used. The response variable is the stain shade after washing (12 is the darkest, 0 is the lightest). The data are shown in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Formulation | Number of Washings | | Number of Washings | |
| 1 | 2 | 1 | 2 |
| Booster | | Booster | |
| Yes | No | Yes | No |
| New | 6 | 6 | 3 | 4 |
| 5 | 5 | 2 | 1 |
|  | | | | |
| Original | 10 | 11 | 10 | 9 |
| 9 | 11 | 9 | 10 |

(a) Conduct an anlysis of variance. Using *α* = 0.05, what conclusions can you draw?

The formulation, number of washings, and the interaction between these to factors appear to be significant. Continued analysis is required as a result of the residual plots in part (b). Conclusions are presented in part (b).

# Design Expert Output

**Response:** **Stain Shade**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Terms added sequentially (first to last)]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 159.44 7 22.78 24.30 < 0.0001 significant

*A 138.06 1 138.06 147.27 < 0.0001*

*B 14.06 1 14.06 15.00 0.0047*

*C 0.56 1 0.56 0.60 0.4609*

*AB 5.06 1 5.06 5.40 0.0486*

*AC 0.56 1 0.56 0.60 0.4609*

*BC 0.56 1 0.56 0.60 0.4609*

*ABC 0.56 1 0.56 0.60 0.4609*

*Pure Error 7.50 8 0.94*

Cor Total 166.94 15

The Model F-value of 24.30 implies the model is significant. There is only a 0.01% chance that a

"Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

(b) Investigate model adequacy by plotting the residuals.

The residual plots shown below identify a violation from our assumptions; nonconstant variance. A power transformation was chosen to correct the violation. λ can be found through trial and error; or the use of a Box-Cox plot that is described in a later chapter. A Box-Cox plot is shown below that identifies a power transformation λ of 1.66.







The analysis of variance was performed with the transformed data and is shown below. This time, only the formulation and number of washings appear to be significant; the interaction between these two factors is no longer significant after the data transformation. The residual plots show no deviations from the assumptions. The plot of the effects below identfies the new formulation along with two washings produces the best results. The booster is not significant.

# Design Expert Output

**Response:** **Stain Shade**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Terms added sequentially (first to last)]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 5071.22 7 724.46 38.18 < 0.0001 significant

*A 4587.21 1 4587.21 241.74 < 0.0001*

*B 312.80 1 312.80 16.48 0.0036*

*C 37.94 1 37.94 2.00 0.1951*

*AB 38.24 1 38.24 2.01 0.1935*

*AC 28.55 1 28.55 1.50 0.2548*

*BC 28.55 1 28.55 1.50 0.2548*

*ABC 37.94 1 37.94 2.00 0.1951*

*Pure Error 151.81 8 18.98*

Cor Total 5223.03 15

The Model F-value of 38.18 implies the model is significant. There is only a 0.01% chance that a

"Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.







**5.29.** Bone anchors are used by orthopedic surgeons in repairing torn rotator cuffs (a common shoulder tendon injury among baseball players). The bone anchor is a threaded insert that is screwed into a hole that has been drilled into the shoulder bone near the site of the torn tendon. The torn tendon is then sutured to the anchor. In a successful operation, the tendon is stabilized and reattaches itself to the bone. However, bone anchors can pull out if they are subjected to high loads. An experiment was performed to study the force required to pull out the anchor for three anchor types and two different foam densities (the foam simulates the natural variability found in real bone). Two replicates of the experiment were performed. The experimental design and the pullout force response data are as follows.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Anchor Type | Foam Density | | | |
| Low | | High | | |
| A | 190 | 200 | 241 | 255 | |
| B | 185 | 190 | 230 | 237 | |
| C | 210 | 205 | 256 | 260 | |

(a) Analyze the data from this experiment.

# Design Expert Output

**Response:** **Force**

**ANOVA for selected factorial model**

**Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 8465.42 5 1693.08 49.43 < 0.0001 significant

*A-Anchor Type* *990.17* *2* *495.08* *14.45* *0.0051*  
  *B-Foam Density* *7450.08* *1* *7450.08* *217.52* *< 0.0001*  
  *AB* *25.17* *2* *12.58* *0.37* *0.7071*  
 Pure Error 205.50 6 34.25  
 Cor Total 8670.92 11

The Model F-value of 49.43 implies the model is significant. There is only  
 a 0.01% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B are significant model terms.   
 Values greater than 0.1000 indicate the model terms are not significant.   
 If there are many insignificant model terms (not counting those required to support hierarchy),   
 model reduction may improve your model.

(b) Investigate model adequacy by constructing appropriate residual plots.

The residuals versus Anchor Type appear to show some inequality of variance; however, not enough to be of concern.





(c) What conclusions can you draw?

Both factors are significant; the interaction is not significant.



**5.30.** An experiment was performed to investigate the keyboard feel on a computer (crisp or mushy) and the size of the keys (small, medium, or large). The response variable is typing speed. Three replicates of the experiment were performed. The experimental design and the data are as follow.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Key Size | Keyboard Feel | | | | | |
| Mushy | | | Crisp | | | |
| Small | 31 | 33 | 35 | 36 | 40 | 41 |
| Medium | 36 | 35 | 33 | 40 | 41 | 42 |
| Large | 37 | 34 | 33 | 38 | 36 | 39 |

(a) Analyze the data from this experiment.

# Design Expert Output

**Response:** **Speed**

**ANOVA for selected factorial model**

**Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 140.00 5 28.00 8.00 0.0016 significant

A-Key Size 12.33 2 6.17 1.76 0.2134

B-Keyboard Feel 117.56 1 117.56 33.59 < 0.0001

AB 10.11 2 5.06 1.44 0.2741

Pure Error 42.00 12 3.50

Cor Total 182.00 17

The Model F-value of 8.00 implies the model is significant. There is only

a 0.16% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case B are significant model terms.

Values greater than 0.1000 indicate the model terms are not significant.

If there are many insignificant model terms (not counting those required to support hierarchy),

model reduction may improve your model.

(b) Investigate model adequacy by constructing appropriate residual plots.

The residual plots show no deviations from the assumptions.

 

 

(c) What conclusions can you draw?

Only factor *B*, Keyboard Feel, appears to be significant. From the plot below, the faster typing speed is observed with the crisp keyboard feel.



**5.31.** An article in *Quality Progress* (May 2011, pp.42-48) describes the use of factorial experiments to improve a silver powder productin process. This product is used in conductive pastes to manufacture a wide variety of products ranging from silicon wafers to elastic membrane switches. Powder density (g/cm2) and surface area (cm2/g) are the two critical characteristics of this product. The experiments involved three factors – reaction temperature, ammonium percent, and stirring rate. Each of these factors had two levels and the design was replicated twice. The design is shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Ammonium (%) | Stir Rate (RPM) | Temperature (°C) | Density | Surface Area |
| 2 | 100 | 8 | 14.68 | 0.40 |
| 2 | 100 | 8 | 15.18 | 0.43 |
| 30 | 100 | 8 | 15.12 | 0.42 |
| 30 | 100 | 8 | 17.48 | 0.41 |
| 2 | 150 | 8 | 7.54 | 0.69 |
| 2 | 150 | 8 | 6.66 | 0.67 |
| 30 | 150 | 8 | 12.46 | 0.52 |
| 30 | 150 | 8 | 12.62 | 0.36 |
| 2 | 100 | 40 | 10.95 | 0.58 |
| 2 | 100 | 40 | 17.68 | 0.43 |
| 30 | 100 | 40 | 12.65 | 0.57 |
| 30 | 100 | 40 | 15.96 | 0.54 |
| 2 | 150 | 40 | 8.03 | 0.68 |
| 2 | 150 | 40 | 8.84 | 0.75 |
| 30 | 150 | 40 | 14.96 | 0.41 |
| 30 | 150 | 40 | 14.96 | 0.41 |

(a) Analyze the density response. Are any interactions significant? Draw appropriate conclusions about the effects of the significant factors on the response.

The *AB* interaction is significant, along with factors *A* and *B*. See ANOVA table below.

Design Expert Output

**Response** **1** **Density**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Partial sum of squares - Type III]**  
 **Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 155.19 7 22.17 5.58 0.0136 significant

*A-Ammonium* *44.39* *1* *44.39* *11.18* *0.0102*  
  *B-Stir Rate* *70.69* *1* *70.69* *17.80* *0.0029*  
  *C-Temperature* *0.33* *1* *0.33* *0.083* *0.7812*  
  *AB* *28.12* *1* *28.12* *7.08* *0.0288*  
  *AC* *0.022* *1* *0.022* *5.480E-003* *0.9428*  
  *BC* *10.13* *1* *10.13* *2.55* *0.1489*  
  *ABC* *1.52* *1* *1.52* *0.38* *0.5534*  
 Pure Error 31.76 8 3.97  
 Cor Total 186.95 15  
  
 The Model F-value of 5.58 implies the model is significant. There is only  
 a 1.36% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B, AB are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 12.86 1 0.50 11.71 14.01

A-Ammonium 1.67 1 0.50 0.52 2.81 1.00  
 B-Stir Rate -2.10 1 0.50 -3.25 -0.95 1.00  
 C-Temperature 0.14 1 0.50 -1.01 1.29 1.00  
 AB 1.33 1 0.50 0.18 2.47 1.00  
 AC -0.037 1 0.50 -1.19 1.11 1.00  
 BC 0.80 1 0.50 -0.35 1.94 1.00  
 ABC 0.31 1 0.50 -0.84 1.46 1.00

(b) Prepare appropriate residual plots and comment on model adequacy.

The residual plots below show concerns with the assumptions. A transformation may be appropriate.











The inverse transformation was applied to the density data and the analysis and residual plots are shown below.

Design Expert Output

**Response** **1** **Density**  
 **Transform:** **Inverse**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 0.012 7 1.678E-003 13.42 0.0008 significant

*A-Ammonium* *3.723E-003* *1* *3.723E-003* *29.78* *0.0006*  
  *B-Stir Rate* *4.445E-003* *1* *4.445E-003* *35.55* *0.0003*  
  *C-Temperature* *9.349E-005* *1* *9.349E-005* *0.75* *0.4124*  
  *AB* *2.767E-003* *1* *2.767E-003* *22.13* *0.0015*  
  *AC* *3.537E-005* *1* *3.537E-005* *0.28* *0.6093*  
  *BC* *6.654E-004* *1* *6.654E-004* *5.32* *0.0499*  
  *ABC* *1.377E-005* *1* *1.377E-005* *0.11* *0.7485*  
 Pure Error 1.000E-003 8 1.250E-004  
 Cor Total 0.013 15

The Model F-value of 13.42 implies the model is significant. There is only  
 a 0.08% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B, AB, BC are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 0.085 1 2.795E-003 0.079 0.091

A-Ammonium -0.015 1 2.795E-003 -0.022 -8.808E-003 1.00  
 B-Stir Rate 0.017 1 2.795E-003 0.010 0.023 1.00  
 C-Temperature -2.417E-003 1 2.795E-003 -8.863E-003 4.029E-003 1.00  
 AB -0.013 1 2.795E-003 -0.020 -6.705E-003 1.00  
 AC 1.487E-003 1 2.795E-003 -4.959E-003 7.933E-003 1.00  
 BC -6.449E-003 1 2.795E-003 -0.013 -2.601E-006 1.00  
 ABC 9.276E-004 1 2.795E-003 -5.518E-003 7.374E-003 1.00











(c) Construct contour plots to aid in practical interpretation of the density response.



(d) Analyze the surface area response. Are any interactions significant? Draw appropriate conclusions about the effects of the significant factors on the response.

The *A* and *B* factors are significant along with the *AB* interaction.

Design Expert Output

**Response** **2**  **Surface Area**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Partial sum of squares - Type III]**  
 **Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 0.21 7 0.030 8.73 0.0033 significant

*A-Ammonium* *0.061* *1* *0.061* *17.72* *0.0030*  
  *B-Stir Rate* *0.032* *1* *0.032* *9.12* *0.0166*  
  *C-Temperature* *0.014* *1* *0.014* *3.99* *0.0807*  
  *AB* *0.089* *1* *0.089* *25.61* *0.0010*  
  *AC* *5.625E-005* *1* *5.625E-005* *0.016* *0.9016*  
  *BC* *0.013* *1* *0.013* *3.66* *0.0920*  
  *ABC* *3.306E-003* *1* *3.306E-003* *0.96* *0.3567*  
 Pure Error 0.028 8 3.456E-003  
 Cor Total 0.24 15  
  
 The Model F-value of 8.73 implies the model is significant. There is only  
 a 0.33% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B, AB are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 0.52 1 0.015 0.48 0.55

A-Ammonium -0.062 1 0.015 -0.096 -0.028 1.00  
 B-Stir Rate 0.044 1 0.015 0.010 0.078 1.00  
 C-Temperature 0.029 1 0.015 -4.517E-003 0.063 1.00  
 AB -0.074 1 0.015 -0.11 -0.040 1.00  
 AC -1.875E-003 1 0.015 -0.036 0.032 1.00  
 BC -0.028 1 0.015 -0.062 5.767E-003 1.00  
 ABC -0.014 1 0.015 -0.048 0.020 1.00

(e) Prepare appropriate residual plots and comment on model adequacy.

The residual plots below do not identify any concerns with model adequacy.











(f) Construct contour plots to aid in practical interpretation of the surface area response.



**5.32.** Continuation of Problem 5.31. Suppose that the specifications require that surface area must be between 0.3 and 0.6 cm2/g and that density must be less than 14 g/cm2. Find a set of operating conditions that will result in a product that meets these requirements.



**5.33.** An article in *Biotechnology Progress* (2001, Vol. 17, pp. 366-368) described an experiment to investigate nisin extraction in aqueous two-phase solutions. A two-factor factorial experiment was conducted using factors *A* = concentration of PEG and *B* = concentration of Na2SO4. Data similar to that reported in the paper are shown below.

|  |  |  |
| --- | --- | --- |
| *A* | *B* | Extraction (%) |
| 13 | 11 | 62.9 |
| 13 | 11 | 65.4 |
| 15 | 11 | 76.1 |
| 15 | 11 | 72.3 |
| 13 | 13 | 87.5 |
| 13 | 13 | 84.2 |
| 15 | 13 | 102.3 |
| 15 | 13 | 105.6 |

(a) Analyze the extraction response. Draw appropriate conclusions about the effects of the significant factors on the response.

Factors *A* and *B* are significant. The *AB* interaction is moderately significant.

**Response** **1** **Extraction**  
  **ANOVA for selected factorial model**  
 **Analysis of variance table [Partial sum of squares - Type III]**  
 **Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 1752.16 3 584.05 110.02 0.0003 significant

*A-PEG* *396.21* *1* *396.21* *74.63* *0.0010*  
  *B-Na2SO4* *1323.55* *1* *1323.55* *249.32* *< 0.0001*  
  *AB* *32.40* *1* *32.40* *6.10* *0.0689*  
 Pure Error 21.24 4 5.31  
 Cor Total 1773.40 7  
  
 The Model F-value of 110.02 implies the model is significant. There is only  
 a 0.03% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 82.04 1 0.81 79.78 84.30

A-PEG 7.04 1 0.81 4.78 9.30 1.00  
 B-Na2SO4 12.86 1 0.81 10.60 15.12 1.00  
 AB 2.01 1 0.81 -0.25 4.27 1.00

(b) Prepare appropriate residual plots and comment on model adequacy.









(c) Construct contour plots to aid in practical interpretation of the density response.



**5.34.** Reconsider the experiment in Problem 5.8. Suppose that this experiment had been conducted in three blocks, with each replicate a block. Assume that the observations in the data table are given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does it appear that blocking was useful in this experiment?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Feed Rate | Block |  | Depth of | Cut (in) |  |
| (in/min) |  | 0.15 | 0.18 | 0.20 | 0.25 |
|  | 1 | 74 | 79 | 82 | 99 |
| 0.20 | 2 | 64 | 68 | 88 | 104 |
|  | 3 | 60 | 73 | 92 | 96 |
|  |  |  |  |  |  |
|  | 1 | 92 | 98 | 99 | 104 |
| 0.25 | 2 | 86 | 104 | 108 | 110 |
|  | 3 | 88 | 88 | 95 | 99 |
|  |  |  |  |  |  |
|  | 1 | 99 | 104 | 108 | 114 |
| 0.30 | 2 | 98 | 99 | 110 | 111 |
|  | 3 | 102 | 95 | 99 | 107 |

The *MS*E was reduced from 28.72 to 23.12. This had very little effect on the results. The variance component estimate for the blocks is:



# Design Expert Output

**Response:** **Surface Finish**

**ANOVA for selected factorial model**  
 **Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 180.67 2 90.33

Model 5842.67 11 531.15 22.97 < 0.0001 significant  
*A-Feed Rate* *3160.50* *2* *1580.25* *68.35* *< 0.0001*  
*B-Depth of Cut* *2125.11* *3* *708.37* *30.64* *< 0.0001*  
*AB* *557.06* *6* *92.84* *4.02* *0.0073*

Residual 508.67 22 23.12  
Cor Total 6532.00 35

**5.35.** Reconsider the experiment in Problem 5.10. Suppose that this experiment had been conducted in three blocks, with each replicate a block. Assume that the observations in the data table are given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does it appear that blocking was useful in this experiment?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Glass | Block |  | Phosphor Type |  |
| Type |  | 1 | 2 | 3 |
|  | 1 | 280 | 300 | 290 |
| 1 | 2 | 290 | 310 | 285 |
|  | 3 | 285 | 295 | 290 |
|  |  |  |  |  |
|  | 1 | 230 | 260 | 220 |
| 2 | 2 | 235 | 240 | 225 |
|  | 3 | 240 | 235 | 230 |

The ANOVA below identifies a very small impact by including the blocks in the analysis. In fact, the *MSE* actually increases from 52.78 in Problem 5.10 to 62.50 with the inclusion of the blocks due to the reduction of the residual degrees of freedom from 12 to 10. Because the *MSE* is greater than the *MS*Blocks, the variance component estimate for blocks is zero.

# Design Expert Output

**Response:** **Current**

**ANOVA for selected factorial model**  
 **Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 8.33 2 4.17

Model 15516.67 5 3103.33 49.65 < 0.0001 significant  
*A-Phosphor Type* *933.33* *2* *466.67* *7.47* *0.0104*  
*B-Glass Type* *14450.00* *1* *14450.00* *231.20* *< 0.0001*  
*AB* *133.33* *2* *66.67* *1.07* *0.3803*

Residual 625.00 10 62.50  
 Cor Total 16150.00 17

**5.46.** Reconsider the experiment in Problem 5.11. Suppose that this experiment had been conducted in two blocks, with each replicate a block. Assume that the observations in the data table are given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does it appear that blocking was useful in this experiment?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Operator | Block | Machine | | | |
|  |  | 1 | 2 | 3 | 4 |
| 1 | 1 | 109 | 110 | 108 | 110 |
|  | 2 | 110 | 115 | 109 | 108 |
|  |  |  |  |  |  |
| 2 | 1 | 110 | 110 | 111 | 114 |
|  | 2 | 112 | 111 | 109 | 112 |
|  |  |  |  |  |  |
| 3 | 1 | 116 | 112 | 114 | 120 |
|  | 2 | 114 | 115 | 119 | 117 |

The ANOVA below identifies a very small impact by including the blocks in the analysis. In fact, the *MSE* actually increases from 3.79 in Problem 5.11 to 3.95 with the inclusion of the blocks due to the reduction of the residual degrees of freedom from 12 to 11. Because the *MSE* is greater than the *MS*Blocks, the variance component estimate for blocks is zero.

Design Expert Output

**Response:** **Stength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Clssical sum of squares – Type II]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 2.04 1 2.04

Model 217.46 11 19.77 5.00 0.0064 significant  
  *A-Operator* *160.33* *2* *80.17* *20.29* *0.0002*  
  *B-Machine* *12.46* *3* *4.15* *1.05* *0.4087*  
  *AB* *44.67* *6* *7.44* *1.88* *0.1716*  
 Residual 43.46 11 3.95  
 Cor Total 262.96 23

**5.37.** Reconsider the three-factor factorial experiment in Problem 5.20. Suppose that this experiment had been conducted in two blocks, with each replicate a block. Assume that the observations in the data table given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does is appear that blocking was useful in this experiment?

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Percentage | Block | Cooking Time 3.0 Hours | | |  | Cooking Time 4.0 Hours | | |
| of Hardwood |  | Pressure | | |  | Pressure | | |
| Concentration |  | 400 | 500 | 650 |  | 400 | 500 | 650 |
| 2 | 1 | 196.6 | 197.7 | 199.8 |  | 198.4 | 199.6 | 200.6 |
|  | 2 | 196.0 | 196.0 | 199.4 |  | 198.6 | 200.4 | 200.9 |
|  |  |  |  |  |  |  |  |  |
| 4 | 1 | 198.5 | 196.0 | 198.4 |  | 197.5 | 198.7 | 199.6 |
|  | 2 | 197.2 | 196.9 | 197.6 |  | 198.1 | 198.0 | 199.0 |
|  |  |  |  |  |  |  |  |  |
| 8 | 1 | 197.5 | 195.6 | 197.4 |  | 197.6 | 197.0 | 198.5 |
|  | 2 | 196.6 | 196.2 | 198.1 |  | 198.4 | 197.8 | 199.8 |

The ANOVA below identifies a very small impact by including the blocks in the analysis; the *SS*Blocks estimate is zero. In fact, the *MSE* actually increases from 0.37 in Problem 5.20 to 0.39 with the inclusion of the blocks due to the reduction of the residual degrees of freedom from 18 to 17. Because the *MSE* is greater than the *MS*Blocks, the variance component estimate for blocks is zero.

**Response:** **Strength**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Clssical sum of squares – Type II]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 0.000 1 0.000

Model 59.73 17 3.51 9.08 < 0.0001 significant  
  *A-Concentration* *7.76* *2* *3.88* *10.03* *0.0013*  
  *B-Time* *20.25* *1* *20.25* *52.32* *< 0.0001*  
  *C-Pressure* *19.37* *2* *9.69* *25.03* *< 0.0001*  
  *AB* *2.08* *2* *1.04* *2.69* *0.0967*  
  *AC* *6.09* *4* *1.52* *3.93* *0.0194*  
  *BC* *2.19* *2* *1.10* *2.84* *0.0866*  
  *ABC* *1.97* *4* *0.49* *1.27* *0.3186*  
 Residual 6.58 17 0.39  
 Cor Total 66.31 35

**5.38S.** Reconsider the three-factor factorial experiment in Problem 5.21. Suppose that this experiment had been conducted in three blocks, with each replicate a block. Assume that the observations in the data table given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does is appear that blocking was useful in this experiment?

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Temperature |  |  |  |
|  |  |  | 300°C |  |  |  | 350°C |  |
|  |  |  | Operator |  |  |  | Operator |  |
| Cycle Time | Block | 1 | 2 | 3 |  | 1 | 2 | 3 |
|  | 1 | 23 | 27 | 31 |  | 24 | 38 | 34 |
| 40 | 2 | 24 | 28 | 32 |  | 23 | 36 | 36 |
|  | 3 | 25 | 26 | 29 |  | 28 | 35 | 39 |
|  |  |  |  |  |  |  |  |  |
|  | 1 | 36 | 34 | 33 |  | 37 | 34 | 34 |
| 50 | 2 | 35 | 38 | 34 |  | 39 | 38 | 36 |
|  | 3 | 36 | 39 | 35 |  | 35 | 36 | 31 |
|  |  |  |  |  |  |  |  |  |
|  | 1 | 28 | 35 | 26 |  | 26 | 36 | 28 |
| 60 | 2 | 24 | 35 | 27 |  | 29 | 37 | 26 |
|  | 3 | 27 | 34 | 25 |  | 25 | 34 | 24 |

The ANOVA below identifies a very small impact by including the blocks in the analysis. The *MSE* improved from 3.28 in Problem 5.21 to 3.27 with the inclusion of the blocks. The variance component estimate for the blocks is:



Design Expert Output

**Response:** **Score**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Clssical sum of squares – Type II]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 6.78 2 3.39

Model 1239.33 17 72.90 22.29 < 0.0001 significant  
  *A-Cycle Time* *436.00* *2* *218.00* *66.64* *< 0.0001*  
  *B-Operator* *261.33* *2* *130.67* *39.94* *< 0.0001*  
  *C-Temperature* *50.07* *1* *50.07* *15.31* *0.0004*  
  *AB* *355.67* *4* *88.92* *27.18* *< 0.0001*  
  *AC* *78.81* *2* *39.41* *12.05* *0.0001*  
  *BC* *11.26* *2* *5.63* *1.72* *0.1941*  
  *ABC* *46.19* *4* *11.55* *3.53* *0.0163*  
 Residual 111.22 34 3.27  
 Cor Total 1357.33 53

**5.39.** Reconsider the bone anchor experiment in Problem 5.29. Suppose that this experiment had been conducted in two blocks, with each replicate a block. Assume that the observations in the data table are given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does it appear that blocking was useful in this experiment?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Anchor Type | Foam Density | | | |
| Low | | High | | |
| A | 190 | 200 | 241 | 255 | |
| B | 185 | 190 | 230 | 237 | |
| C | 210 | 205 | 256 | 260 | |

The *MS*E was reduced from 28.72 to 23.12. This had very little effect on the results. The *MS*E was reduced from 34.25 in Problem 5.29 to 20.68 with the inclusion of the blocks. This had a small effect on the results. The variance component estimate for the blocks is:



# Design Expert Output

**Response:** **Force**

**ANOVA for selected factorial model**

**Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 102.08 1 102.08

Model 8465.42 5 1693.08 81.86 < 0.0001 significant  
  *A-Anchor Type* *990.17* *2* *495.08* *23.94* *0.0028*  
  *B-Foam Density* *7450.08* *1* *7450.08* *360.20* *< 0.0001*  
  *AB* *25.17* *2* *12.58* *0.61* *0.5801*  
 Residual 103.42 5 20.68  
 Cor Total 8670.92 11

**5.40.** Reconsider the keyboard experiment in Problem 5.30. Suppose that this experiment had been conducted in three blocks, with each replicate a block. Assume that the observations in the data table are given in order, that is, the first observation in each cell comes from the first replicate, and so on. Reanalyze the data as a factorial experiment in blocks and estimate the variance component for blocks. Does it appear that blocking was useful in this experiment?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Key Size | Keyboard Feel | | | | | |
| Mushy | | | Crisp | | | |
| Small | 31 | 33 | 35 | 36 | 40 | 41 |
| Medium | 36 | 35 | 33 | 40 | 41 | 42 |
| Large | 37 | 34 | 33 | 38 | 36 | 39 |

The ANOVA below identifies a very small impact by including the blocks in the analysis. In fact, the *MSE* actually increases from 3.50 in Problem 5.30 to 3.97 with the inclusion of the blocks due to the reduction of the residual degrees of freedom from 12 to 10. Because the *MSE* is greater than the *MS*Blocks, the variance component estimate for blocks is zero.

# Design Expert Output

**Response:** **Speed**

**ANOVA for selected factorial model**

**Analysis of variance table [Classical sum of squares - Type II]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 2.33 2 1.17

Model 140.00 5 28.00 7.06 0.0045 significant  
  *A-Key Size* *12.33* *2* *6.17* *1.55* *0.2583*  
  *B-Keyboard Feel* *117.56* *1* *117.56* *29.64* *0.0003*  
  *AB* *10.11* *2* *5.06* *1.27* *0.3213*  
 Residual 39.67 10 3.97  
 Cor Total 182.00 17

**5.41.** The C. F. Eye Care company manufactures lenses for transplantation into the eye following cataract surgery. An engineering group has conducted an experiment involving two factors to determine their effect on the lens polishing process. The results of this experiment are summarized in the following ANOVA display:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *DF* | *SS* | *MS* | *F* | *P-*value |
| Factor *A* | ? | ? | 0.0833 | 0.05 | 0.952 |
| Factor *B* | ? | 96.333 | 96.333 | 57.80 | <0.001 |
| Interaction | 2 | 12.167 | 6.0833 | 3.65 | ? |
| Error | 6 | 10.000 | ? |  |  |
| Total | 11 | 118.667 |  |  |  |

(a) The sum of squares for factor *A* is 0.1666.

(b) The number of degrees of freedom for factor *A* in the experiment is 2.

(c) The number of degrees of freedom for factor *B* is 1.

(d) The mean square for error is 1.666.

(e) An upper bound for the *P*-value for the interaction test statistic is 0.1.

(f) The engineers used 3 levels of factor *A* in this experiment.

(g) The engineers used 2 levels of factor *B* in this experiment.

(h) There are 2 replicates of this experiment.

(i) Would you conclude that the effect of factor *B* depends on the level of factor *A* (Yes or No)? No, the *P*-value is 0.092.

(j) An estimate of the standard deviation of the response variable is 1.29.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *DF* | *SS* | *MS* | *F* | *P-*value |
| Factor *A* | 2 | 0.1666 | 0.0833 | 0.05 | 0.952 |
| Factor *B* | 1 | 96.333 | 96.333 | 57.80 | <0.001 |
| Interaction | 2 | 12.167 | 6.0833 | 3.65 | 0.092 |
| Error | 6 | 10.000 | 1.6666 |  |  |
| Total | 11 | 118.667 |  |  |  |

**5.42.** Reconsider the lens polishing experiment in Problem 5.41. Suppose that this experiment has been conducted as a randomized complete box design. The sum of squares for blocks is 4.00. Reconstruct the ANOVA given this new information. What impact does the blocking have on the conclusions from the original experiment?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *DF* | *SS* | *MS* | *F* | *P-*value |
| Factor *A* | 2 | 0.1666 | 0.0833 | 0.05 | 0.952 |
| Factor *B* | 1 | 96.333 | 96.333 | 57.80 | <0.001 |
| Interaction | 2 | 12.167 | 6.0833 | 3.65 | 0.092 |
| Block | 1 | 4.000 | 4.000 | 0.67 | 0.452 |
| Error | 6 | 10.000 | 1.6666 |  |  |
| Total | 11 | 118.667 |  |  |  |

Blocking has no impact.

**5.43.** In Problem 4.54 you met physics PhD Laura Van Ertia who had conducted a single-factor experiment in her pursuit of the unified theory. She is at it again, and this time she has moved on to a two-factor factorial conducted as a completely randomized design. From her experiment, Laura has constructed the following incomplete ANOVA display:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 350.00 | 2 | ? | ? |
| *B* | 300.00 | ? | 150 | ? |
| *AB* | 200.00 | ? | 50 | ? |
| Error | 150.00 | 18 |  |  |
| Total | 1000.00 |  |  |  |

(a) How many levels of factor B did she use in the experiment? 3

(b) How many degrees of freedom are associated with the interaction? 4

(c) The error mean square is 8.333.

(d) The mean square for factor A is 175.

(e) How many replicates of the experiment were conducted? 3.

(f) What are your conclusions about interaction and the two main effects? All are significant

(g) An estimate of the standard deviation of the response variable is 2.886.

(h) If this experiment had been run in blocks ther would have been 2 degrees of freedom for blocks.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* | *P* |
| *A* | 350.00 | 2 | 175 | 21 | 0.00001968 |
| *B* | 300.00 | 2 | 150 | 18 | 0.00005081 |
| *AB* | 200.00 | 4 | 50 | 6 | 0.002996 |
| Error | 150.00 | 18 | 8.333 |  |  |
| Total | 1000.00 |  |  |  |  |

**5.44.** Continuation of Problem 5.43. Suppose that Laura did actually conduct the experiment in Problem 5.48 as a randomized complete block design. Assume that the block sum of squares is 60.00. Reconstruct the ANOVA display under the new set of assumptions.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* | *P* |
| *A* | 350.00 | 2 | 175 | 31.1 | 3.0712E-06 |
| *B* | 300.00 | 2 | 150 | 26.7 | 7.9815E-06 |
| *AB* | 200.00 | 4 | 50 | 8.9 | 0.0005573 |
| Block | 60.00 | 2 | 30 | 5.3 | 0.01714 |
| Error | 90.00 | 16 | 5.625 |  |  |
| Total | 1000.00 |  |  |  |  |

In addition to both main effects and the interaction, the block is significant.

**5.46.** Consider the following incomplete ANOVA table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.00 | ? |
| *B* | 80.00 | 2 | 40.00 | ? |
| *AB* | 30.00 | 2 | 15.00 | ? |
| Error | ? | 12 | ? |  |
| Total | 172.00 | 17 |  |  |

In addition to the ANOVA table, you know that the experiment has been replicated three times, and that the totals of the three replicates are 10, 12, and 14, repectively. The original experiment was run as a completely randomized design. Answer the following questions:

(a) The pure error estimate of the standard deviation of the sample observations is 1 (Yes or No)? Yes

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.00 | 50.00 |
| *B* | 80.00 | 2 | 40.00 | 40.00 |
| *AB* | 30.00 | 2 | 15.00 | 15.00 |
| Error | 12.00 | 12 | 1.00 |  |
| Total | 172.00 | 17 |  |  |

(b) Suppose that the experiment has been run in blocks, so that it is a randomized complete block design. The number of degrees of freedom for blocks would be 2 .

(c) The block sum of squares is \_\_\_\_?



(d) The error sum of squares in the randomized complete block design is now 10.66?

(e) For the randomized complete block design, what is the extimate of the standard deviation of the sample observations? 1.03.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.000 | 46.86 |
| *B* | 80.00 | 2 | 40.000 | 37.49 |
| *AB* | 30.00 | 2 | 15.000 | 14.06 |
| Block | 1.333 | 2 | 0.667 | 0.63 |
| Error | 10.66 | 10 | 1.067 |  |
| Total | 172.00 | 17 |  |  |

**5.52.** Consider the following incomplete ANOVA table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.00 | ? |
| *B* | 80.00 | 2 | 40.00 | ? |
| *AB* | 30.00 | 2 | 15.00 | ? |
| Block | 10.00 | 1 | ? | ? |
| Error | ? | ? | ? |  |
| Total | 185.00 | 11 |  |  |

(a) The pure error estimate of the standard deviation of the sample observations is 1.73.

**True** False

(b) Suppose that the experiment had not been run in blocks; that is, it is now a CRD. The number of degrees of freedom for error would now be 6 .

(c) The error mean square in the CRD would be 3.5 .

(d) The *F*-test statistic for interaction in the CRD is significant at = 0.05.

**True** False

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F* |
| *A* | 50.00 | 1 | 50.00 | 16.67 |
| *B* | 80.00 | 2 | 40.00 | 13.33 |
| *AB* | 30.00 | 2 | 15.00 | 5.00 |
| Block | 10.00 | 1 | 10.00 | 3.33 |
| Error | 15.00 | 5 | 3.00 |  |
| Total | 185.00 | 11 |  |  |