Chapter 7

**Blocking and Confounding in the 2*k* Factorial Design**

# Solutions

**7.1S.** Consider the experiment described in Problem 6.5. Analyze this experiment assuming that each replicate represents a block of a single production shift.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| Cutting Speed (*A*) | 0.67 | 1 | 0.67 | <1 |
| Tool Geometry (*B*) | 770.67 | 1 | 770.67 | 22.38\* |
| Cutting Angle (*C*) | 280.17 | 1 | 280.17 | 8.14\* |
| *AB* | 16.67 | 1 | 16.67 | <1 |
| *AC* | 468.17 | 1 | 468.17 | 13.60\* |
| *BC* | 48.17 | 1 | 48.17 | 1.40 |
| *ABC* | 28.17 | 1 | 28.17 | <1 |
| Blocks | 0.58 | 2 | 0.29 |  |
| Error | 482.08 | 14 | 34.43 |  |
| Total | 2095.33 | 23 |  |  |

Design Expert Output

**Response:** **Life** **in hours**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 0.58 2 0.29

Model 1519.67 4 379.92 11.23 0.0001 significant

*A* *0.67* *1* *0.67* *0.020* *0.8900*

*B* *770.67* *1* *770.67* *22.78* *0.0002*

*C* *280.17* *1* *280.17* *8.28* *0.0104*

*AC* *468.17* *1* *468.17* *13.84* *0.0017*

Residual 575.08 17 33.83

Cor Total 2095.33 23

The Model F-value of 11.23 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case B, C, AC are significant model terms.

These results agree with the results from Problem 6.5. Tool geometry, cutting angle and the interaction between cutting speed and cutting angle are significant at the 5% level. The Design Expert program also includes factor *A*, cutting speed, in the model to preserve hierarchy.

**7.2.** Consider the experiment described in Problem 6.9. Analyze this experiment assuming that each one of the four replicates represents a block.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| Bit Size (*A*) | 1107.23 | 1 | 1107.23 | 364.22\* |
| Cutting Speed (*B*) | 227.26 | 1 | 227.26 | 74.76\* |
| *AB* | 303.63 | 1 | 303.63 | 99.88\* |
| Blocks | 44.36 | 3 | 14.79 |  |
| Error | 27.36 | 9 | 3.04 |  |
| Total | 1709.83 | 15 |  |  |

These results agree with those from Problem 6.9. Bit size, cutting speed and their interaction are significant at the 1% level.

Design Expert Output

**Response:** **Vibration**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 44.36 3 14.79

Model 1638.11 3 546.04 179.61 < 0.0001 significant

*A* *1107.23* *1* *1107.23* *364.21* *< 0.0001*

*B* *227.26* *1* *227.26* *74.75* *< 0.0001*

*AB* *303.63* *1* *303.63* *99.88* *< 0.0001*

Residual 27.36 9 3.04

Cor Total 1709.83 15

The Model F-value of 179.61 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

**7.3S.** Consider the data from the first replicate of Problem 6.5. Suppose that these observations could not all be run using the same bar stock. Set up a design to run these observations in two blocks of four observations each with *ABC* confounded. Analyze the data.

|  |  |
| --- | --- |
| Block 1 | Block 2 |
| (1) | *a* |
| *ab* | *b* |
| *ac* | *c* |
| *bc* | *abc* |

From the normal probability plot of effects, *B*, *C*, and the *AC* interaction are significant. Factor *A* was included in the analysis of variance to preserve hierarchy.



Design Expert Output

**Response:** **Life** **in hours**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 91.13 1 91.13

Model 896.50 4 224.13 7.32 0.1238 not significant

*A* *3.13* *1* *3.13* *0.10* *0.7797*

*B* *325.12* *1* *325.12* *10.62* *0.0827*

*C* *190.12* *1* *190.12* *6.21* *0.1303*

*AC* *378.13* *1* *378.13* *12.35* *0.0723*

Residual 61.25 2 30.62

Cor Total 1048.88 7

The "Model F-value" of 7.32 implies the model is not significant relative to the noise. There is a

12.38 % chance that a "Model F-value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case there are no significant model terms.

This design identifies the same significant factors as Problem 6.5.

**7.4.** Consider the data from the first replicate of Problem 6.11. Construct a design with two blocks of eight observations each with *ABCD* confounded. Analyze the data.

|  |  |
| --- | --- |
| Block 1 | Block 2 |
| (1) | *a* |
| *ab* | *b* |
| *ac* | *c* |
| *bc* | *d* |
| *ad* | *abc* |
| *bd* | *abd* |
| *cd* | *acd* |
| *abcd* | *bcd* |

The significant effects are identified in the normal probability plot of effects below:



*AC*, *BC*, and *BD* were included in the model to preserve hierarchy.

Design Expert Output

**Response:** **yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 42.25 1 42.25

Model 892.25 11 81.11 9.64 0.0438 significant

*A* *400.00* *1* *400.00* *47.52* *0.0063*

*B* *2.25* *1* *2.25* *0.27* *0.6408*

*C* *2.25* *1* *2.25* *0.27* *0.6408*

*D* *100.00* *1* *100.00* *11.88* *0.0410*

*AB* *81.00* *1* *81.00* *9.62* *0.0532*

*AC* *1.00* *1* *1.00* *0.12* *0.7531*

*AD* *56.25* *1* *56.25* *6.68* *0.0814*

*BC* *6.25* *1* *6.25* *0.74* *0.4522*

*BD* *9.00* *1* *9.00* *1.07* *0.3772*

*ABC* *144.00* *1* *144.00* *17.11* *0.0256*

*ABD* *90.25* *1* *90.25* *10.72* *0.0466*

Residual 25.25 3 8.42

Cor Total 959.75 15

The Model F-value of 9.64 implies the model is significant. There is only

a 4.38% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, D, ABC, ABD are significant model terms.

**7.5.** Repeat Problem 7.4 assuming that four blocks are required. Confound *ABD* and *ABC* (and consequently *CD*) with blocks.

The block assignments are shown in the table below. The normal probability plot of effects identifies factors *A* and *D*, and the interactions *AB*, *AD*, and the *ABCD* as strong candidates for the model. For hierarchal purposes, factor *B* was included in the model; however, hierarchy is not preserved for the *ABCD* interaction allowing an estimate for error.

|  |  |  |  |
| --- | --- | --- | --- |
| Block 1 | Block 2 | Block 3 | Block 4 |
| (1) | *ac* | *c* | *a* |
| *ab* | *bc* | *abc* | *b* |
| *acd* | *d* | *ad* | *cd* |
| *bcd* | *abd* | *bd* | *abcd* |



Design Expert Output

**Response:** **yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 243.25 3 81.08

Model 681.75 6 113.63 19.62 0.0011 significant

*A 400.00 1 400.00 69.06 0.0002*

*B 2.25 1 2.25 0.39 0.5560*

*D 100.00 1 100.00 17.27 0.0060*

*AB 81.00 1 81.00 13.99 0.0096*

*AD 56.25 1 56.25 9.71 0.0207*

*ABCD 42.25 1 42.25 7.29 0.0355*

Residual 34.75 6 5.79

Cor Total 959.75 15

The Model F-value of 19.62 implies the model is significant. There is only

a 0.11% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, D, AB, AD, ABCD are significant model terms.

**7.6S.** Using the data from the 25 design in Problem 6.24, construct and analyze a design in two blocks with *ABCDE* confounded with blocks.

|  |  |  |  |
| --- | --- | --- | --- |
| Block 1 | Block 1 | Block 2 | Block 2 |
| (1) | *ae* | *a* | *e* |
| *ab* | *be* | *b* | *abe* |
| *ac* | *ce* | *c* | *ace* |
| *bc* | *abce* | *abc* | *bce* |
| *ad* | *de* | *d* | *ade* |
| *bd* | *abde* | *abd* | *bde* |
| *cd* | *acde* | *acd* | *cde* |
| *abcd* | *bcde* | *bcd* | *abcde* |

The normal probability plot of effects identifies factors *A*, *B*, *C*, and the *AB* interaction as being significant. This is confirmed with the analysis of variance.



Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 0.28 1 0.28

Model 11585.13 4 2896.28 958.51 < 0.0001 significant

*A* *1116.28* *1* *1116.28* *369.43* *< 0.0001*

*B* *9214.03* *1* *9214.03* *3049.35* *< 0.0001*

*C* *750.78* *1* *750.78* *248.47* *< 0.0001*

*AB* *504.03* *1* *504.03* *166.81* *< 0.0001*

Residual 78.56 26 3.02

Cor Total 11663.97 31

The Model F-value of 958.51 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C, AB are significant model terms.

**7.7S.** Repeat Problem 7.6S assuming that four blocks are necessary. Suggest a reasonable confounding scheme.

Use *ABC* and *CDE*, and consequently *ABDE*. The four blocks follow.

|  |  |  |  |
| --- | --- | --- | --- |
| Block 1 | Block 2 | Block 3 | Block 4 |
| (1) | *a* | *ac* | *c* |
| *ab* | *b* | *bc* | *abc* |
| *acd* | *cd* | *d* | *ad* |
| *bcd* | *abcd* | *abd* | *bd* |
| *ace* | *ce* | *e* | *ae* |
| *bce* | *abce* | *abe* | *be* |
| *de* | *ade* | *acde* | *cde* |
| *abde* | *bde* | *bcde* | *abcde* |

The normal probability plot of effects identifies the same significant effects as in Problem 7.7.



Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 13.84 3 4.61

Model 11585.13 4 2896.28 1069.40 < 0.0001 significant

*A* *1116.28* *1* *1116.28* *412.17* *< 0.0001*

*B* *9214.03* *1* *9214.03* *3402.10* *< 0.0001*

*C* *750.78* *1* *750.78* *277.21* *< 0.0001*

*AB* *504.03* *1* *504.03* *186.10* *< 0.0001*

Residual 65.00 24 2.71

Cor Total 11663.97 31

The Model F-value of 1069.40 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C, AB are significant model terms.

**7.8S.** Consider the data from the 25 design in Problem 6.24. Suppose that it was necessary to run this design in four blocks with *ACDE* and *BCD* (and consequently *ABE*) confounded. Analyze the data from this design.

|  |  |  |  |
| --- | --- | --- | --- |
| Block 1 | Block 2 | Block 3 | Block 4 |
| *(1)* | *a* | *b* | *c* |
| *ae* | *e* | *abe* | *ace* |
| *cd* | *acd* | *bcd* | *d* |
| *abc* | *bc* | *ac* | *ab* |
| *acde* | *cde* | *abcde* | *ade* |
| *bce* | *abce* | *ce* | *be* |
| *abd* | *bd* | *ad* | *abcd* |
| *bde* | *abde* | *de* | *bcde* |

Even with four blocks, the same effects are identified as significant per the normal probability plot and analysis of variance below:



Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 2.59 3 0.86

Model 11585.13 4 2896.28 911.62 < 0.0001 significant

*A* *1116.28* *1* *1116.28* *351.35* *< 0.0001*

*B* *9214.03* *1* *9214.03* *2900.15* *< 0.0001*

*C* *750.78* *1* *750.78* *236.31* *< 0.0001*

*AB* *504.03* *1* *504.03* *158.65* *< 0.0001*

Residual 76.25 24 3.18

Cor Total 11663.97 31

The Model F-value of 911.62 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C, AB are significant model terms.

**7.9S.** Consider the fill height deviation experiment in Problem 6.20. Suppose that each replicate was run on a separate day. Analyze the data assuming that the days are blocks.

Design Expert Output

**Response:** **Fill Deviation**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 1.00 1 1.00

Model 70.75 4 17.69 28.30 < 0.0001 significant

*A 36.00 1 36.00 57.60 < 0.0001*

*B 20.25 1 20.25 32.40 0.0002*

*C 12.25 1 12.25 19.60 0.0013*

*AB 2.25 1 2.25 3.60 0.0870*

Residual 6.25 10 0.62

Cor Total 78.00 15

The Model F-value of 28.30 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C are significant model terms.

The analysis is very similar to the original analysis in chapter 6. The same effects are significant.

**7.10.** Consider the fill height deviation experiment in Problem 6.20. Suppose that only four runs could be made on each shift. Set up a design with *ABC* confounded in replicate 1 and *AC* confounded in replicate 2. Analyze the data and comment on your findings.

Design Expert Output

**Response:** **Fill Deviation**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 1.50 3 0.50

Model 70.75 4 17.69 24.61 0.0001 significant

*A 36.00 1 36.00 50.09 0.0001*

*B 20.25 1 20.25 28.17 0.0007*

*C 12.25 1 12.25 17.04 0.0033*

*AB 2.25 1 2.25 3.13 0.1148*

Residual 5.75 8 0.72

Cor Total 78.00 15

The Model F-value of 24.61 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, C are significant model terms.

The analysis is very similar to the original analysis of Problem 6.20 and that of problem 7.9S. The *AB* interaction is less significant in this scenario.

**7.11S.** Consider the putting experiment in Problem 6.21. Analyze the data considering each replicate as a block.

The analysis is similar to that of Problem 6.21. Blocking has not changed the significant factors, however, the residual plots show that the normality assumption has been violated. The transformed data also has similar analysis to the transformed data of Problem 6.21. The ANOVA shown is for the transformed data.

Design Expert Output

**Response:** **Distance from cup Transform: Square root Constant: 0**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 13.50 6 2.25

Model 37.26 2 18.63 7.83 0.0007 significant

A 21.61 1 21.61 9.08 0.0033

B 15.64 1 15.64 6.57 0.0118

Residual 245.13 103 2.38

Cor Total 295.89 111

The Model F-value of 7.83 implies the model is significant. There is only

a 0.07% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

**7.12.** The experiment in Problem 6.35 is a 25 factorial. Suppose that this design had been run in four blocks of eight runs each.

(a) Recommend a blocking scheme and set up the design.

Interactions *ABC* and *BDE* are confounded with the blocks such that:

|  |  |  |
| --- | --- | --- |
| Block | *ABC* | *BDE* |
| 1 | – | + |
| 2 | + | – |
| 3 | – | – |
| 4 | + | + |

Note, the *ACDE* interaction is also confounded with the blocks. The experimental runs with the blocks are shown below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Block | A | B | C | D | E | y |
| Block 1 | -1 | -1 | -1 | -1 | -1 | 8.11 |
| Block 2 | 1 | -1 | -1 | -1 | -1 | 5.56 |
| Block 4 | -1 | 1 | -1 | -1 | -1 | 5.77 |
| Block 3 | 1 | 1 | -1 | -1 | -1 | 5.82 |
| Block 2 | -1 | -1 | 1 | -1 | -1 | 9.17 |
| Block 1 | 1 | -1 | 1 | -1 | -1 | 7.8 |
| Block 3 | -1 | 1 | 1 | -1 | -1 | 3.23 |
| Block 4 | 1 | 1 | 1 | -1 | -1 | 5.69 |
| Block 3 | -1 | -1 | -1 | 1 | -1 | 8.82 |
| Block 4 | 1 | -1 | -1 | 1 | -1 | 14.23 |
| Block 2 | -1 | 1 | -1 | 1 | -1 | 9.2 |
| Block 1 | 1 | 1 | -1 | 1 | -1 | 8.94 |
| Block 4 | -1 | -1 | 1 | 1 | -1 | 8.68 |
| Block 3 | 1 | -1 | 1 | 1 | -1 | 11.49 |
| Block 1 | -1 | 1 | 1 | 1 | -1 | 6.25 |
| Block 2 | 1 | 1 | 1 | 1 | -1 | 9.12 |
| Block 3 | -1 | -1 | -1 | -1 | 1 | 7.93 |
| Block 4 | 1 | -1 | -1 | -1 | 1 | 5 |
| Block 2 | -1 | 1 | -1 | -1 | 1 | 7.47 |
| Block 1 | 1 | 1 | -1 | -1 | 1 | 12 |
| Block 4 | -1 | -1 | 1 | -1 | 1 | 9.86 |
| Block 3 | 1 | -1 | 1 | -1 | 1 | 3.65 |
| Block 1 | -1 | 1 | 1 | -1 | 1 | 6.4 |
| Block 2 | 1 | 1 | 1 | -1 | 1 | 11.61 |
| Block 1 | -1 | -1 | -1 | 1 | 1 | 12.43 |
| Block 2 | 1 | -1 | -1 | 1 | 1 | 17.55 |
| Block 4 | -1 | 1 | -1 | 1 | 1 | 8.87 |
| Block 3 | 1 | 1 | -1 | 1 | 1 | 25.38 |
| Block 2 | -1 | -1 | 1 | 1 | 1 | 13.06 |
| Block 1 | 1 | -1 | 1 | 1 | 1 | 18.85 |
| Block 3 | -1 | 1 | 1 | 1 | 1 | 11.78 |
| Block 4 | 1 | 1 | 1 | 1 | 1 | 26.05 |

(b) Analyze the data from this blocked design. Is blocking important?

Blocking does not appear to be important; however, if the *ADE* or *ABE* interaction had been chosen to define the blocks, then blocking would have appeared as important. The *ADE* and *ABE* are significant effects in the analysis below.



Design Expert Output

**Response 1 y  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 2.58 3 0.86

Model 879.62 11 79.97 45.38 < 0.0001 significant  
  *A-A* *83.56* *1* *83.56* *47.41* *< 0.0001*  
  *B-B* *0.060* *1* *0.060* *0.034* *0.8553*  
  *D-D* *285.78* *1* *285.78* *162.16* *< 0.0001*  
  *E-E* *153.17* *1* *153.17* *86.91* *< 0.0001*  
  *AB* *48.93* *1* *48.93* *27.76* *< 0.0001*  
  *AD* *88.88* *1* *88.88* *50.43* *< 0.0001*  
  *AE* *33.76* *1* *33.76* *19.16* *0.0004*  
  *BE* *52.71* *1* *52.71* *29.91* *< 0.0001*  
  *DE* *61.80* *1* *61.80* *35.07* *< 0.0001*  
  *ABE* *44.96* *1* *44.96* *25.51* *< 0.0001*  
  *ADE* *26.01* *1* *26.01* *14.76* *0.0013*  
 Residual 29.96 17 1.76  
 Cor Total 912.16 31  
  
 The Model F-value of 45.38 implies the model is significant. There is only  
 a 0.01% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, D, E, AB, AD, AE, BE, DE, ABE, ADE are significant model terms.

**7.13.** Repeat Problem 7.12 using a design in two blocks.

(a) Recommend a blocking scheme and set up the design.

Interaction *ABCDE* is confounded with the blocks. The design is shown below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Block | A | B | C | D | E | y |
| Block 1 | -1 | -1 | -1 | -1 | -1 | 8.11 |
| Block 2 | 1 | -1 | -1 | -1 | -1 | 5.56 |
| Block 2 | -1 | 1 | -1 | -1 | -1 | 5.77 |
| Block 1 | 1 | 1 | -1 | -1 | -1 | 5.82 |
| Block 2 | -1 | -1 | 1 | -1 | -1 | 9.17 |
| Block 1 | 1 | -1 | 1 | -1 | -1 | 7.8 |
| Block 1 | -1 | 1 | 1 | -1 | -1 | 3.23 |
| Block 2 | 1 | 1 | 1 | -1 | -1 | 5.69 |
| Block 2 | -1 | -1 | -1 | 1 | -1 | 8.82 |
| Block 1 | 1 | -1 | -1 | 1 | -1 | 14.23 |
| Block 1 | -1 | 1 | -1 | 1 | -1 | 9.2 |
| Block 2 | 1 | 1 | -1 | 1 | -1 | 8.94 |
| Block 1 | -1 | -1 | 1 | 1 | -1 | 8.68 |
| Block 2 | 1 | -1 | 1 | 1 | -1 | 11.49 |
| Block 2 | -1 | 1 | 1 | 1 | -1 | 6.25 |
| Block 1 | 1 | 1 | 1 | 1 | -1 | 9.12 |
| Block 2 | -1 | -1 | -1 | -1 | 1 | 7.93 |
| Block 1 | 1 | -1 | -1 | -1 | 1 | 5 |
| Block 1 | -1 | 1 | -1 | -1 | 1 | 7.47 |
| Block 2 | 1 | 1 | -1 | -1 | 1 | 12 |
| Block 1 | -1 | -1 | 1 | -1 | 1 | 9.86 |
| Block 2 | 1 | -1 | 1 | -1 | 1 | 3.65 |
| Block 2 | -1 | 1 | 1 | -1 | 1 | 6.4 |
| Block 1 | 1 | 1 | 1 | -1 | 1 | 11.61 |
| Block 1 | -1 | -1 | -1 | 1 | 1 | 12.43 |
| Block 2 | 1 | -1 | -1 | 1 | 1 | 17.55 |
| Block 2 | -1 | 1 | -1 | 1 | 1 | 8.87 |
| Block 1 | 1 | 1 | -1 | 1 | 1 | 25.38 |
| Block 2 | -1 | -1 | 1 | 1 | 1 | 13.06 |
| Block 1 | 1 | -1 | 1 | 1 | 1 | 18.85 |
| Block 1 | -1 | 1 | 1 | 1 | 1 | 11.78 |
| Block 2 | 1 | 1 | 1 | 1 | 1 | 26.05 |

(b) Analyze the data from this blocked design. Is blocking important?

The analysis below shows that the blocking does not appear to be very important.



Design Expert Output

**Response 1 y  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 4.04 1 4.04

Model 879.62 11 79.97 53.31 < 0.0001 significant  
  *A-A* *83.56* *1* *83.56* *55.71* *< 0.0001*  
  *B-B* *0.060* *1* *0.060* *0.040* *0.8431*  
  *D-D* *285.78* *1* *285.78* *190.54* *< 0.0001*  
  *E-E* *153.17* *1* *153.17* *102.12* *< 0.0001*  
  *AB* *48.93* *1* *48.93* *32.62* *< 0.0001*  
  *AD* *88.88* *1* *88.88* *59.26* *< 0.0001*  
  *AE* *33.76* *1* *33.76* *22.51* *0.0001*  
  *BE* *52.71* *1* *52.71* *35.14* *< 0.0001*  
  *DE* *61.80* *1* *61.80* *41.20* *< 0.0001*  
  *ABE* *44.96* *1* *44.96* *29.98* *< 0.0001*  
  *ADE* *26.01* *1* *26.01* *17.34* *0.0005*  
 Residual 28.50 19 1.50  
 Cor Total 912.16 31  
  
 The Model F-value of 53.31 implies the model is significant. There is only  
 a 0.01% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, D, E, AB, AD, AE, BE, DE, ABE, ADE are significant model terms.

**7.14.** The design in Problem 6.38 is a 23 factorial replicated twice. Suppose that each replicate was a block. Analyze all of the responses from this blocked design. Are the results comparable to those from Problem 6.46? Is the block effect large?

The block effect is not large and does not appear to be important for the analysis on any of the four the responses as shown below. The results are comparable to those from Problem 6.46.

Design Expert Output

**Response 1 Fishbone Pb  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 0.000 1 0.000

Model 12.19 7 1.74 2300.73 < 0.0001 significant  
  *A-Apatite* *10.99* *1* *10.99* *14514.07* *< 0.0001*  
  *B-pH* *0.35* *1* *0.35* *459.75* *< 0.0001*  
  *C-Pb* *0.27* *1* *0.27* *350.30* *< 0.0001*  
  *AB* *0.36* *1* *0.36* *475.47* *< 0.0001*  
  *AC* *0.19* *1* *0.19* *249.92* *< 0.0001*  
  *BC* *0.022* *1* *0.022* *29.72* *0.0010*  
  *ABC* *0.020* *1* *0.020* *25.89* *0.0014*  
 Residual 5.300E-003 7 7.571E-004  
 Cor Total 12.20 15

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 0.85 1 6.879E-003 0.84 0.87

Block 1 0.000 1  
 Block 2 0.000

A-Apatite -0.83 1 6.879E-003 -0.85 -0.81 1.00  
 B-pH -0.15 1 6.879E-003 -0.16 -0.13 1.00  
 C-Pb 0.13 1 6.879E-003 0.11 0.15 1.00  
 AB 0.15 1 6.879E-003 0.13 0.17 1.00  
 AC -0.11 1 6.879E-003 -0.13 -0.092 1.00  
 BC 0.037 1 6.879E-003 0.021 0.054 1.00  
 ABC -0.035 1 6.879E-003 -0.051 -0.019 1.00

Design Expert Output

**Response 1 Fishbone pH  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 2.256E-003 1 2.256E-003

Model 21.09 7 3.01 102.87 < 0.0001 significant  
  *A-Apatite* *9.84* *1* *9.84* *336.14* *< 0.0001*  
  *B-pH* *8.14* *1* *8.14* *277.85* *< 0.0001*  
  *C-Pb* *1.12* *1* *1.12* *38.19* *0.0005*  
  *AB* *0.61* *1* *0.61* *20.91* *0.0026*  
  *AC* *1.17* *1* *1.17* *40.01* *0.0004*  
  *BC* *0.098* *1* *0.098* *3.33* *0.1106*  
  *ABC* *0.11* *1* *0.11* *3.66* *0.0972*  
 Residual 0.20 7 0.029  
 Cor Total 21.30 15  
  
 The Model F-value of 102.87 implies the model is significant. There is only  
 a 0.01% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B, C, AB, AC are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 5.05 1 0.043 4.95 5.15

Block 1 -0.012 1  
 Block 2 0.012

A-Apatite 0.78 1 0.043 0.68 0.89 1.00  
 B-pH -0.71 1 0.043 -0.81 -0.61 1.00  
 C-Pb -0.26 1 0.043 -0.37 -0.16 1.00  
 AB 0.20 1 0.043 0.094 0.30 1.00  
 AC -0.27 1 0.043 -0.37 -0.17 1.00  
 BC -0.078 1 0.043 -0.18 0.023 1.00  
 ABC -0.082 1 0.043 -0.18 0.019 1.00

Design Expert Output

**Response 1 Hydroxyapatite Pb  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 2.250E-004 1 2.250E-004

Model 4.01 7 0.57 937.82 < 0.0001 significant  
  *A-Apatite* *2.45* *1* *2.45* *4010.43* *< 0.0001*  
  *B-pH* *0.27* *1* *0.27* *434.29* *< 0.0001*  
  *C-Pb* *0.54* *1* *0.54* *884.58* *< 0.0001*  
  *AB* *0.17* *1* *0.17* *275.25* *< 0.0001*  
  *AC* *0.50* *1* *0.50* *825.43* *< 0.0001*  
  *BC* *0.036* *1* *0.036* *59.11* *0.0001*  
  *ABC* *0.046* *1* *0.046* *75.69* *< 0.0001*  
 Residual 4.275E-003 7 6.107E-004  
 Cor Total 4.01 15  
  
 The Model F-value of 937.82 implies the model is significant. There is only  
 a 0.01% chance that a "Model F-Value" this large could occur due to noise.  
  
 Values of "Prob > F" less than 0.0500 indicate model terms are significant.   
 In this case A, B, C, AB, AC, BC, ABC are significant model terms.

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 0.42 1 6.178E-003 0.40 0.43

Block 1 3.750E-003 1  
 Block 2 -3.750E-003

A-Apatite -0.39 1 6.178E-003 -0.41 -0.38 1.00  
 B-pH 0.13 1 6.178E-003 0.11 0.14 1.00  
 C-Pb 0.18 1 6.178E-003 0.17 0.20 1.00  
 AB -0.10 1 6.178E-003 -0.12 -0.088 1.00  
 AC -0.18 1 6.178E-003 -0.19 -0.16 1.00  
 BC -0.048 1 6.178E-003 -0.062 -0.033 1.00  
 ABC 0.054 1 6.178E-003 0.039 0.068 1.00

Design Expert Output

**Response 1 Hydroxyapatite pH  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 2.025E-003 1 2.025E-003

Model 20.44 7 2.92 1494.46 < 0.0001 significant  
  *A-Apatite* *8.15* *1* *8.15* *4172.37* *< 0.0001*  
  *B-pH* *8.82* *1* *8.82* *4515.27* *< 0.0001*  
  *C-Pb* *0.084* *1* *0.084* *43.05* *0.0003*  
  *AB* *3.24* *1* *3.24* *1658.50* *< 0.0001*  
  *AC* *0.014* *1* *0.014* *7.37* *0.0300*  
  *BC* *0.13* *1* *0.13* *64.51* *< 0.0001*  
  *ABC* *2.250E-004* *1* *2.250E-004* *0.12* *0.7443*  
 Residual 0.014 7 1.954E-003  
 Cor Total 20.45 15

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 3.77 1 0.011 3.74 3.79

Block 1 0.011 1  
 Block 2 -0.011

A-Apatite 0.71 1 0.011 0.69 0.74 1.00  
 B-pH -0.74 1 0.011 -0.77 -0.72 1.00  
 C-Pb -0.073 1 0.011 -0.099 -0.046 1.00  
 AB -0.45 1 0.011 -0.48 -0.42 1.00  
 AC -0.030 1 0.011 -0.056 -3.871E-003 1.00  
 BC 0.089 1 0.011 0.063 0.11 1.00  
 ABC 3.750E-003 1 0.011 -0.022 0.030 1.00

**7.15.** Consider the 26 design in eight blocks of eight runs each with *ABCD*, *ACE*, and *ABEF* as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confound with blocks.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Block 1 | Block 2 | Block 3 | Block 4 | Block 5 | Block 6 | Block 7 | Block 8 |
| *b* | *abc* | *a* | *c* | *ac* | (1) | *bc* | *ab* |
| *acd* | *d* | *bcd* | *abd* | *bd* | *abcd* | *ad* | *cd* |
| *ce* | *ae* | *abce* | *be* | *abe* | *bce* | *e* | *ace* |
| *abde* | *bcde* | *de* | *acde* | *cde* | *ade* | *abcde* | *bde* |
| *abcf* | *bf* | *cf* | *af* | *f* | *acf* | *abf* | *bcf* |
| *df* | *acdf* | *abdf* | *bcdf* | *abcdf* | *bdf* | *cdf* | *adf* |
| *aef* | *cef* | *bef* | *abcef* | *bcef* | *abef* | *acef* | *ef* |
| *bcdef* | *abdef* | *acdef* | *def* | *adef* | *cdef* | *bdef* | *abcdef* |

The factors that are confounded with blocks are *ABCD, ABEF, ACE, BDE, CDEF, BCF*, and *ADF*.

**7.16S.** Consider the 22 design in two blocks with *AB* confounded. Prove algebraically that *SSAB* = *SS*Blocks.

If *AB* is confounded, the two blocks are:

|  |  |
| --- | --- |
| Block 1 | Block 2 |
| (1) | *a* |
| *ab* | *b* |
| (1) + *ab* | *a + b* |







**7.17.** Consider the data in Example 7.2. Suppose that all the observations in block 2 are increased by 20. Analyze the data that would result. Estimate the block effect. Can you explain its magnitude? Do blocks now appear to be an important factor? Are any other effect estimates impacted by the change you made in the data?



This is the block effect estimated in Example 7.2 plus the additional 20 units that were added to each observation in block 2. All other effects are the same.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| *A* | 1870.56 | 1 | 1870.56 | 89.93 |
| *C* | 390.06 | 1 | 390.06 | 18.75 |
| *D* | 855.56 | 1 | 855.56 | 41.13 |
| *AC* | 1314.06 | 1 | 1314.06 | 63.18 |
| *AD* | 1105.56 | 1 | 1105.56 | 53.15 |
| Blocks | 5967.56 | 1 | 5967.56 |  |
| Error | 187.56 | 9 | 20.8 |  |
| Total | 11690.93 | 15 |  |  |

Design Expert Output

**Response:** **Filtration** **in gal/hr**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 5967.56 1 5967.56

Model 5535.81 5 1107.16 53.13 < 0.0001 significant

*A* *1870.56* *1* *1870.56* *89.76* *< 0.0001*

*C* *390.06* *1* *390.06* *18.72* *0.0019*

*D* *855.56* *1* *855.56* *41.05* *0.0001*

*AC* *1314.06* *1* *1314.06* *63.05* *< 0.0001*

*AD* *1105.56* *1* *1105.56* *53.05* *< 0.0001*

Residual 187.56 9 20.84

Cor Total 11690.94 15

The Model F-value of 53.13 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, C, D, AC, AD are significant model terms.

**7.18.** Suppose that the data in Problem 6.5 we had confounded *ABC* in replicate I, *AB* in replicate II, and *BC* in replicate III. Construct the analysis of variance table.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Replicate I | |  | | Replicate II | |  | | Replicate III | |  | |
|  | (*ABC* Confounded) | |  | | (*AB* Confounded) | |  | | (*BC* Confounded) | |  | |
| Block-> | 1 | 2 | | 1 | | 2 | | 1 | | 2 | |
|  | (1) | *a* | | (1) | | *a* | | (1) | | *b* | |
|  | *ab* | *b* | | *ab* | | *b* | | *bc* | | *c* | |
|  | *ac* | *c* | | *abc* | | *ac* | | *abc* | | *ab* | |
|  | *bc* | *abc* | | *c* | | *bc* | | *a* | | *ac* | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| *A* | 0.67 | 1 | 0.67 | <1 |
| *B* | 770.67 | 1 | 770.67 | 20.77 |
| *C* | 280.17 | 1 | 280.17 | 7.55 |
| *AB* (reps 1 and III) | 25.00 | 1 | 25.00 | <1 |
| *AC* | 468.17 | 1 | 468.17 | 12.62 |
| *BC* (reps I and II) | 22.56 | 1 | 22.56 | <1 |
| *ABC* (reps II and III) | 0.06 | 1 | 0.06 | <1 |
| Blocks within replicates | 119.25 | 3 | 39.75 |  |
| Replicates | 0.58 | 2 |  |  |
| Error | 408.21 | 11 | 37.11 |  |
| Total | 2095.33 | 23 |  |  |

**7.19.** Repeat the analysis of Problem 6.5 assuming that *ABC* was confounded with blocks in each replicate.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | Replicate I, II, and III | |  | |
|  | | (*ABC* Confounded) | |  | |
| Block-> | | 1 | | 2 | |
|  | | (1) | | *a* | |
|  | | *ab* | | *b* | |
|  | | *ac* | | *c* | |
|  | | *bc* | | *abc* | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| *A* | 0.67 | 1 | 0.67 | <1 |
| *B* | 770.67 | 1 | 770.67 | 22.38 |
| *C* | 280.17 | 1 | 280.17 | 8.14 |
| *AB* | 16.67 | 1 | 16.67 | <1 |
| *AC* | 468.17 | 1 | 468.17 | 13.59 |
| *BC* | 48.17 | 1 | 48.17 | 1.40 |
| Blocks (or *ABC*) | 28.17 | 1 | 28.17 |  |
| Replicates/Lack of Fit | 0.58 | 2 |  |  |
| Error | 482.09 | 14 | 34.44 |  |
| Total | 2095.33 | 23 |  |  |

**7.20.** Suppose that in Problem 6.11 *ABCD* was confounded in replicate I and *ABC* was confounded in replicate II. Perform the statistical analysis of variance.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of | Sum of | Degrees of | Mean |  |
| Variation | Squares | Freedom | Square | F0 |
| *A* | 657.03 | 1 | 657.03 | 84.89 |
| *B* | 13.78 | 1 | 13.78 | 1.78 |
| *C* | 57.78 | 1 | 57.78 | 7.46 |
| *D* | 124.03 | 1 | 124.03 | 16.02 |
| *AB* | 132.03 | 1 | 132.03 | 17.06 |
| *AC* | 3.78 | 1 | 3.78 | <1 |
| *AD* | 38.28 | 1 | 38.28 | 4.95 |
| *BC* | 2.53 | 1 | 2.53 | <1 |
| *BD* | 0.28 | 1 | 0.28 | <1 |
| *CD* | 22.78 | 1 | 22.78 | 2.94 |
| *ABC* | 144.00 | 1 | 144.00 | 18.64 |
| *ABD* | 175.78 | 1 | 175.78 | 22.71 |
| *ACD* | 7.03 | 1 | 7.03 | <1 |
| *BCD* | 7.03 | 1 | 7.03 | <1 |
| *ABCD* | 10.56 | 1 | 10.56 | 1.36 |
| Replicates | 11.28 | 1 | 11.28 |  |
| Blocks | 118.81 | 2 | 59.41 |  |
| Error | 100.65 | 13 | 7.74 |  |
| Total | 1627.47 | 31 |  |  |

**7.21.** Construct a 23 design with *ABC* confounded in the first two replicates and *BC* confounded in the third. Outline the analysis of variance and comment on the information obtained.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Replicate I | |  | | Replicate II | |  | | Replicate III | |  | |
|  | (*ABC* Confounded) | |  | | (*ABC* Confounded) | | | | (*BC* Confounded) | |  | |
| Block-> | 1 | 2 | | 1 | | 2 | | 1 | | 2 | |
|  | (1) | *a* | | (1) | | *a* | | (1) | | *b* | |
|  | *ab* | *b* | | *ab* | | *b* | | *bc* | | *c* | |
|  | *ac* | *c* | | *ac* | | *c* | | *abc* | | *ab* | |
|  | *bc* | *abc* | | *bc* | | *abc* | | *a* | | *ac* | |

|  |  |
| --- | --- |
| Source of | Degrees of |
| Variation | Freedom |
| *A* | 1 |
| *B* | 1 |
| *C* | 1 |
| *AB* | 1 |
| *AC* | 1 |
| *BC* | 1 |
| *ABC* | 1 |
| Replicates | 2 |
| Blocks | 3 |
| Error | 11 |
| Total | 23 |

This design provides “two-thirds” information on *BC* and “one-third” information on *ABC*.

**7.22.** The block effect in a two-level design with two blocks can be calculated directly as the difference in the average response between the two blocks.

**True** False

**7.23.** When constructing the design confounded in 8 blocks, three independent effects are chosen to generate the blocks, and there are a total of 8 interaction confounded with blocks.

True **False**

**7.24.** Consider the factor design in two blocks. If ABCDE is confounded with blocks, then which of the following runs is in the same block as run *acde*?

(a) *a*

(b) *acd*

(c) *bcd*

(d) *be*

(e) *abe*

(f) **None of the above**

**7.25.** The information on the interaction confounded with the block can always be separated from the block effect.

True **False**

**7.26.** Consider the full 25 factorial design problem in 6.42. Suppose that this experiment had been run in two blocks with *ABCDE* confounded with the blocks. Set up the blocked design and perform the analysis. Compare your results with the results obtained for the completely randomized design in Problem 6.42.

Minitab Output



Since no four factor interactions appear to be important, remove them from the model to get the ANOVA table. Also, Factor *C* and its interaction are not important and they are removed from the model.

Minitab Output

**Factorial Regression: Y versus Blocks, A, B, D, E**

Analysis of Variance

Source DF Adj SS Adj MS F-Value P-Value

Model 15 887.537 59.169 38.44 0.000

Blocks 1 4.040 4.040 2.62 0.125

Linear 4 522.574 130.643 84.88 0.000

A 1 83.560 83.560 54.29 0.000

B 1 0.060 0.060 0.04 0.845

D 1 285.784 285.784 185.68 0.000

E 1 153.169 153.169 99.52 0.000

2-Way Interactions 6 286.088 47.681 30.98 0.000

A\*B 1 48.931 48.931 31.79 0.000

A\*D 1 88.878 88.878 57.75 0.000

A\*E 1 33.764 33.764 21.94 0.000

B\*D 1 0.006 0.006 0.00 0.952

B\*E 1 52.711 52.711 34.25 0.000

D\*E 1 61.799 61.799 40.15 0.000

3-Way Interactions 4 74.835 18.709 12.16 0.000

A\*B\*D 1 3.816 3.816 2.48 0.135

A\*B\*E 1 44.959 44.959 29.21 0.000

A\*D\*E 1 26.010 26.010 16.90 0.001

B\*D\*E 1 0.050 0.050 0.03 0.859

Error 16 24.626 1.539

Total 31 912.162

Model Summary

S R-sq R-sq(adj) R-sq(pred)

1.24061 97.30% 94.77% 89.20%



The Block effect is not important and results are the same as Problem 6.42. The residuals are acceptable for this model.

**7.27.** Suppose that you are designing and experiment for four factors and that due to material properties it is necessary to conduct the experiment in blocks. Material availability restricts you to the use of two blocks; however, each batch of material is only sufficient for six runs. So, the standard factorial in two blocks of eight runs each with *ABCD* confounded will not work. Recommend a design. Suggestion: this is a reasonable application for a *D*-optimal design. What type of design do you find in each block?

This is a reasonable application of the *D*-optimal Design. In the design below, the blocks are basically a fractional factorial with a couple of corner points missing.

JMP Table

