Chapter 8

Two-Level Fractional Factorial Designs

**Solutions**

**8.1.** Suppose that in the chemical process development experiment in Problem 6.11, it was only possible to run a one-half fraction of the 24 design. Construct the design and perform the statistical analysis, using the data from replicate 1.

The required design is a 24-1 with *I=ABCD*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=ABC* |  |  |
| – | – | – | – | (1) | 90 |
| + | – | – | + | *ad* | 72 |
| – | + | – | + | *bd* | 87 |
| + | + | – | – | *ab* | 83 |
| – | – | + | + | *cd* | 99 |
| + | – | + | – | *ac* | 81 |
| – | + | + | – | *bc* | 88 |
| + | + | + | + | *abcd* | 80 |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A -12 288 64.2857

Model B -1 2 0.446429

Model C 4 32 7.14286

Model D -1 2 0.446429

Model AB 6 72 16.0714

Model AC -1 2 0.446429

Model AD -5 50 11.1607

Error BC Aliased

Error BD Aliased

Error CD Aliased

Error ABC Aliased

Error ABD Aliased

Error ACD Aliased

Error BCD Aliased

Error ABCD Aliased

Lenth's ME 22.5856

Lenth's SME 54.0516



The largest effect is *A*. The next largest effects are the *AB* and *AD* interactions. A plausible tentative model would be *A*, *AB* and *AD*, along with *B* and *D* to preserve hierarchy.

Design Expert Output

**Response:** **yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 414.00 5 82.80 4.87 0.1791 not significant

*A* *288.00* *1* *288.00* *16.94* *0.0543*

*B* *2.00* *1* *2.00* *0.12* *0.7643*

*D* *2.00* *1* *2.00* *0.12* *0.7643*

*AB* *72.00* *1* *72.00* *4.24* *0.1758*

*AD* *50.00* *1* *50.00* *2.94* *0.2285*

Residual 34.00 2 17.00

Cor Total 448.00 7

The "Model F-value" of 4.87 implies the model is not significant relative to the noise. There is a

17.91 % chance that a "Model F-value" this large could occur due to noise.

Std. Dev. 4.12 R-Squared 0.9241

Mean 85.00 Adj R-Squared 0.7344

C.V. 4.85 Pred R-Squared -0.2143

PRESS 544.00 Adeq Precision 6.441

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 85.00 1 1.46 78.73 91.27

A-A -6.00 1 1.46 -12.27 0.27 1.00

B-B -0.50 1 1.46 -6.77 5.77 1.00

D-D -0.50 1 1.46 -6.77 5.77 1.00

AB 3.00 1 1.46 -3.27 9.27 1.00

AD -2.50 1 1.46 -8.77 3.77 1.00

**Final Equation in Terms of Coded Factors:**

yield =

+85.00

-6.00 \* A

-0.50 \* B

-0.50 \* D

+3.00 \* A \* B

-2.50 \* A \* D

**Final Equation in Terms of Actual Factors:**

yield =

+85.00000

-6.00000 \* A

-0.50000 \* B

-0.50000 \* D

+3.00000 \* A \* B

-2.50000 \* A \* D

The Design-Expert output indicates that we really only need the main effect of factor *A*. The updated analysis is shown below:

Design Expert Output

**Response:** **yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 288.00 1 288.00 10.80 0.0167 significant

*A* *288.00* *1* *288.00* 10.80 0.0167

Residual 160.00 6 26.67

Cor Total 448.00 7

The Model F-value of 10.80 implies the model is significant. There is only

a 1.67% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 5.16 R-Squared 0.6429

Mean 85.00 Adj R-Squared 0.5833

C.V. 6.08 Pred R-Squared 0.3651

PRESS 284.44 Adeq Precision 4.648

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 85.00 1 1.83 80.53 89.47

A-A -6.00 1 1.83 -10.47 -1.53 1.00

**Final Equation in Terms of Coded Factors:**

yield =

+85.00

-6.00 \* A

**Final Equation in Terms of Actual Factors:**

yield =

+85.00000

-6.00000 \* A

**8.2S.** Consider the plasma etch experiment described in Example 6.1. Suppose that only a one-half fraction of the design could be run. Set up the design and analyze the data.

Because Example 6.1 is a replicated 23 factorial experiment, a half fraction of this design is a 23-1 with four runs. The experiment is replicated to assure an adequate estimate of the *MSE*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | Etch |  |  |  |  |
|  |  |  | Rate |  |  | Factor | Levels |
| *A* | *B* | *C=AB* | (A/min) |  |  | Low (-) | High (+) |
| – | – | + | 1037 |  | *A* (Gap, cm) | 0.80 | 1.20 |
| – | – | + | 1052 |  | *B* (C2F6 flow, SCCM) | 125 | 200 |
| + | – | – | 669 |  | *C* (Power, W) | 275 | 325 |
| + | – | – | 650 |  |  |  |  |
| – | + | – | 633 |  |  |  |  |
| – | + | – | 601 |  |  |  |  |
| + | + | + | 729 |  |  |  |  |
| + | + | + | 860 |  |  |  |  |

The analysis shown below identifies all three main effects as significant. Because this is a resolution III design, the main effects are aliased with two factor interactions. The original analysis from Example 6.1 identifies factors *A*, *C*, and the *AC* interaction as significant. In our replicated half fraction experiment, factor *B* is aliased with the *AC* interaction. This problem points out the concerns of running small resolution III designs.

Design Expert Output

**Response:** **Etch Rate**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 2.225E+005 3 74169.79 31.61 0.0030 significant

*A 21528.13 1 21528.13 9.18 0.0388*

*B 42778.13 1 42778.13 18.23 0.0130*

*C 1.582E+005 1 1.582E+005 67.42 0.0012*

Pure Error 9385.50 4 2346.37

Cor Total 2.319E+005 7

The Model F-value of 31.61 implies the model is significant. There is only

a 0.30% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 48.44 R-Squared 0.9595

Mean 778.88 Adj R-Squared 0.9292

C.V. 6.22 Pred R-Squared 0.8381

PRESS 37542.00 Adeq Precision 12.481

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 778.88 1 17.13 731.33 826.42

A-Gap -51.88 1 17.13 -99.42 -4.33 1.00

B-C2F6 Flow -73.13 1 17.13 -120.67 -25.58 1.00

C-Power 140.63 1 17.13 93.08 188.17 1.00

**Final Equation in Terms of Coded Factors:**

Etch Rate =

+778.88

-51.88 \* A

-73.13 \* B

+140.63 \* C

**Final Equation in Terms of Actual Factors:**

Etch Rate =

-332.37500

-259.37500 \* Gap

-1.95000 \* C2F6 Flow

+5.62500 \* Power

**8.3S.** Problem 6.24 describes a process improvement study in the manufacturing process of an integrated circuit. Suppose that only eight runs could be made in this process. Set up an appropriate 25-2 design and find the alias structure. Use the appropriate observations from Problem 6.24 as the observations in this design and estimate the factor effects. What conclusions can you draw?

*I = ABD = ACE = BCDE*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *(ABD)* | *=BD* |  | *A* | *(ACE)* | *=CE* |  | *A* | *(BCDE)* | *=ABCDE* | *A=BD=CE=ABCDE* |
| *B* | *(ABD)* | *=AD* |  | *B* | *(ACE)* | *=ABCE* |  | *B* | *(BCDE)* | *=CDE* | *B=AD=ABCE=CDE* |
| *C* | *(ABD)* | *=ABCD* |  | *C* | *(ACE)* | *=AE* |  | *C* | *(BCDE)* | *=BDE* | *C=ABCD=AE=BDE* |
| *D* | *(ABD)* | *=AB* |  | *D* | *(ACE)* | *=ACDE* |  | *D* | *(BCDE)* | *=BCE* | *D=AB=ACDE=BCE* |
| *E* | *(ABD)* | *=ABDE* |  | *E* | *(ACE)* | *=AC* |  | *E* | *(BCDE)* | *=BCD* | *E=ABDE=AC=BCD* |
| *BC* | *(ABD)* | *=ACD* |  | *BC* | *(ACE)* | *=ABE* |  | *BC* | *(BCDE)* | *=DE* | *BC=ACD=ABE=DE* |
| *BE* | *(ABD)* | *=ADE* |  | *BE* | *(ACE)* | *=ABC* |  | *BE* | *(BCDE)* | *=CD* | *BE=ADE=ABC=CD* |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=AC* |  |  |
| – | – | – | + | + | *de* | 6 |
| + | – | – | – | – | *a* | 9 |
| – | + | – | – | + | *be* | 35 |
| + | + | – | + | – | *abd* | 50 |
| – | – | + | + | – | *cd* | 18 |
| + | – | + | – | + | *ace* | 22 |
| – | + | + | – | – | *bc* | 40 |
| + | + | + | + | + | *abcde* | 63 |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 11.25 253.125 8.91953

Model B 33.25 2211.13 77.9148

Model C 10.75 231.125 8.1443

Model D 7.75 120.125 4.23292

Error E 2.25 10.125 0.356781

Error BC -1.75 6.125 0.215831

Error BE 1.75 6.125 0.215831

Lenth's ME 28.232

Lenth's SME 67.5646



The main *A*, *B*, *C*, and *D* are large. However, recall that we are really estimating *A+BD+CE*, *B+AD*, *C+DE* and *D+AD*. There are other possible interpretations of the experiment because of the aliasing.

Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 2815.50 4 703.88 94.37 0.0017 significant

*A* *253.13* *1* *253.13* *33.94* *0.0101*

*B* *2211.12* *1* *2211.12* *296.46* *0.0004*

*C* *231.13* *1* *231.13* *30.99* *0.0114*

*D* *120.13* *1* *120.13* *16.11* *0.0278*

Residual 22.38 3 7.46

Cor Total 2837.88 7

The Model F-value of 94.37 implies the model is significant. There is only

a 0.17% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 2.73 R-Squared 0.9921

Mean 30.38 Adj R-Squared 0.9816

C.V. 8.99 Pred R-Squared 0.9439

PRESS 159.11 Adeq Precision 25.590

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 30.38 1 0.97 27.30 33.45

A-Aperture 5.63 1 0.97 2.55 8.70 1.00

B-Exposure Time 16.63 1 0.97 13.55 19.70 1.00

C-Develop Time 5.37 1 0.97 2.30 8.45 1.00

D-Mask Dimension 3.87 1 0.97 0.80 6.95 1.00

**Final Equation in Terms of Coded Factors:**

Yield =

+30.38

+5.63 \* A

+16.63 \* B

+5.37 \* C

+3.87 \* D

**Final Equation in Terms of Actual Factors:**

Aperture small

Mask Dimension Small

Yield =

-6.00000

+0.83125 \* Exposure Time

+0.71667 \* Develop Time

Aperture large

Mask Dimension Small

Yield =

+5.25000

+0.83125 \* Exposure Time

+0.71667 \* Develop Time

Aperture small

Mask Dimension Large

Yield =

+1.75000

+0.83125 \* Exposure Time

+0.71667 \* Develop Time

Aperture large

Mask Dimension Large

Yield =

+13.00000

+0.83125 \* Exposure Time

+0.71667 \* Develop Time

**8.4S. Continuation of Problem 8.3S.** Suppose you have made the eight runs in the 25-2 design in Problem 8.3S. What additional runs would be required to identify the factor effects that are of interest? What are the alias relationships in the combined design?

We could fold over the original design by changing the signs on the generators *D = AB* and *E = AC* to produce the following new experiment.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=-AB* | *E=-AC* |  |  |
| – | – | – | – | – | (1) | 7 |
| + | – | – | + | + | *ade* | 10 |
| – | + | – | + | – | *bd* | 32 |
| + | + | – | – | + | *abe* | 52 |
| – | – | + | – | + | *ce* | 15 |
| + | – | + | + | – | *acd* | 21 |
| – | + | + | + | + | *bcde* | 41 |
| + | + | + | – | – | *abc* | 60 |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | (–ABD) | =–BD |  | A | (–ACE) | =–CE |  | A | (BCDE) | =ABCDE | A=–BD=–CE=ABCDE |
| B | (–ABD) | =–AD |  | B | (–ACE) | =–ABCE |  | B | (BCDE) | =CDE | B=–AD=–ABCE=CDE |
| C | (–ABD) | =–ABCD |  | C | (–ACE) | =–AE |  | C | (BCDE) | =BDE | C=–ABCD=–AE=BDE |
| D | (–ABD) | =–AB |  | D | (–ACE) | =–ACDE |  | D | (BCDE) | =BCE | D=–AB=–ACDE=BCE |
| E | (–ABD) | =–ABDE |  | E | (–ACE) | =–AC |  | E | (BCDE) | =BCD | E=–ABDE=–AC=BCD |
| BC | (–ABD) | =–ACD |  | BC | (–ACE) | =–ABE |  | BC | (BCDE) | =DE | BC=–ACD=–ABE=DE |
| BE | (–ABD) | =–ADE |  | BE | (–ACE) | =–ABC |  | BE | (BCDE) | =CD | BE=–ADE=–ABC=CD |

Assuming all three factor and higher interactions to be negligible, all main effects can be separated from their two-factor interaction aliases in the combined design.

**8.5.** In the Example 6.6, a 24 factorial design was used to improve the response rate to a credit card marketing offer. Suppose that the researchers had used the 24-1 fraction factorial design with *I=ABCD* instead. Set up the design and select the responses for the runs from the full factorial data in Example 6.6. Analyze the data and draw conclusions. Compare your findings with those from the full factorial in Example 6.6.

Based on the Pseudo *p*-Value, the effects appear to be much less significant than those found in Example 6.6. The estimates for the Long-term Interest Rate, Account Opening Fee, and Annual Fee \* Long-term Interest Rate are similar to the estimates found in Example 6.6; however, the other estimates are not as similar.

JMP Output

**Response Response rate**

**Summary of Fit**

|  | |  | |
| --- | --- | --- | --- |
| RSquare | 1 | |
| RSquare Adj | . | |
| Root Mean Square Error | . | |
| Mean of Response | 2.3375 | |
| Observations (or Sum Wgts) | 8 | |

**Sorted Parameter Estimates**

| **Term** |  | **Estimate** | **Relative Std Error** | **Pseudo t-Ratio** | **Pseudo t-Ratio** | **Pseudo p-Value** |
| --- | --- | --- | --- | --- | --- | --- |
| Long-term interest rate[Low] |  | 0.275 | 0.353553 | 1.22 |  | 0.3307 |
| Account-opening fee[No] |  | 0.255 | 0.353553 | 1.13 |  | 0.3600 |
| Initial interest[Current] |  | -0.17 | 0.353553 | -0.76 |  | 0.5188 |
| Annual fee[Current] |  | -0.15 | 0.353553 | -0.67 |  | 0.5649 |
| Annual fee[Current]\*Long-term interest rate[Low] |  | -0.0775 | 0.353553 | -0.34 |  | 0.7591 |
| Annual fee[Current]\*Account-opening fee[No] |  | -0.0725 | 0.353553 | -0.32 |  | 0.7739 |
| Annual fee[Current]\*Initial interest[Current] |  | 0.0525 | 0.353553 | 0.23 |  | 0.8344 |

No error degrees of freedom, so ordinary tests uncomputable. Relative Std Error corresponds to residual standard error of 1. Pseudo t-Ratio and p-Value calculated using Lenth PSE = 0.225 and DFE=2.3333

**Prediction Profiler**



**8.6.** **Continuation of Problem 8.5.** In Example 6.6, we found that all four main effects and the two-factor *AB* interaction were significant. Show that if the alternate fraction (*I=-ABCD*) is added to the 24-1 design in Problem 8.5 that the analysis from the combined design produced results identical to those found in Example 6.6

The estimates for the effects from the alternate fraction are shown below. By combining the estimates from Problem 8.5 with the estimates below, the same estimates as those shown in Example 6.6 are found. For example:







The analysis can also be performed as full factorial in two blocks with the block effect confounded with the *ABCD* interaction. This JMP analysis for the full factorial in two blocks is also shown below. The effect estimates are the same as those shown in Example 6.6.

JMP Output

**Response Response rate**

**Summary of Fit**

|  | |  | |
| --- | --- | --- | --- |
| RSquare | 1 | |
| RSquare Adj | . | |
| Root Mean Square Error | . | |
| Mean of Response | 2.39 | |
| Observations (or Sum Wgts) | 8 | |

**Sorted Parameter Estimates**

| **Term** |  | **Estimate** | **Relative Std Error** | **Pseudo t-Ratio** | **Pseudo t-Ratio** | **Pseudo p-Value** |
| --- | --- | --- | --- | --- | --- | --- |
| Account-opening fee[No] |  | 0.2625 | 0.353553 | 0.79 |  | 0.5035 |
| Annual fee[Current] |  | -0.2575 | 0.353553 | -0.77 |  | 0.5109 |
| Annual fee[Current]\*Account-opening fee[No] |  | -0.23 | 0.353553 | -0.69 |  | 0.5529 |
| Long-term interest rate[Low] |  | 0.2225 | 0.353553 | 0.67 |  | 0.5649 |
| Initial interest[Current] |  | -0.0825 | 0.353553 | -0.25 |  | 0.8248 |
| Annual fee[Current]\*Initial interest[Current] |  | -0.05 | 0.353553 | -0.15 |  | 0.8929 |
| Annual fee[Current]\*Long-term interest rate[Low] |  | -0.03 | 0.353553 | -0.09 |  | 0.9355 |

No error degrees of freedom, so ordinary tests uncomputable. Relative Std Error corresponds to residual standard error of 1. Pseudo t-Ratio and p-Value calculated using Lenth PSE = 0.33375 and DFE=2.3333

**Prediction Profiler**



JMP Output

**Response Response rate**

**Summary of Fit**

|  | |  | |
| --- | --- | --- | --- |
| RSquare | 1 | |
| RSquare Adj | . | |
| Root Mean Square Error | . | |
| Mean of Response | 2.36375 | |
| Observations (or Sum Wgts) | 16 | |

**Sorted Parameter Estimates**

| **Term** |  | **Estimate** | **Relative Std Error** | **Pseudo t-Ratio** | **Pseudo t-Ratio** | **Pseudo p-Value** |
| --- | --- | --- | --- | --- | --- | --- |
| Account-opening fee[No] |  | 0.25875 | 0.25 | 3.63 |  | 0.0150\* |
| Long-term interest rate[Low] |  | 0.24875 | 0.25 | 3.49 |  | 0.0174\* |
| Annual fee[Current] |  | -0.20375 | 0.25 | -2.86 |  | 0.0354\* |
| Annual fee[Current]\*Account-opening fee[No] |  | -0.15125 | 0.25 | -2.12 |  | 0.0872 |
| Initial interest rate[Current] |  | -0.12625 | 0.25 | -1.77 |  | 0.1366 |
| Initial interest rate[Current]\*Long-term interest rate[Low] |  | 0.07875 | 0.25 | 1.11 |  | 0.3194 |
| Annual fee[Current]\*Long-term interest rate[Low] |  | -0.05375 | 0.25 | -0.75 |  | 0.4846 |
| Account-opening fee[No]\*Initial interest rate[Current]\*Long-term interest rate[Low] |  | 0.05375 | 0.25 | 0.75 |  | 0.4846 |
| Account-opening fee[No]\*Long-term interest rate[Low] |  | 0.05125 | 0.25 | 0.72 |  | 0.5042 |
| Annual fee[Current]\*Account-opening fee[No]\*Long-term interest rate[Low] |  | -0.04375 | 0.25 | -0.61 |  | 0.5661 |
| Block[1] |  | -0.02625 | 0.25 | -0.37 |  | 0.7276 |
| Annual fee[Current]\*Account-opening fee[No]\*Initial interest rate[Current] |  | 0.02625 | 0.25 | 0.37 |  | 0.7276 |
| Account-opening fee[No]\*Initial interest rate[Current] |  | -0.02375 | 0.25 | -0.33 |  | 0.7524 |
| Annual fee[Current]\*Initial interest rate[Current]\*Long-term interest rate[Low] |  | -0.00375 | 0.25 | -0.05 |  | 0.9601 |
| Annual fee[Current]\*Initial interest rate[Current] |  | 0.00125 | 0.25 | 0.02 |  | 0.9867 |

No error degrees of freedom, so ordinary tests uncomputable. Relative Std Error corresponds to residual standard error of 1. Pseudo t-Ratio and p-Value calculated using Lenth PSE = 0.07125 and DFE=5

**Prediction Profiler**



**8.7.** **Continuation of Problem 8.5.** Reconsider the 24-1 fractional factorial design with *I=ABCD* from Problem 8.6. Set a partial fold-over of this fraction to isolate the *AB* interaction. Select the appropriate set of responses from the full factorial data in Example 6.6 and analyze the resulting data.

The fold-over could be based on either factor *A* or *B* to isolate the *AB* interaction. The partial fold-over shown below in block 2 was based on factor *B*, with the + level chosen (Yes).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Annual Fee | Account opening fee | Initial interest Rate | Long-term interest rate | Block | Response rate |
| Current | No | Current | Low | 1 | 2.45 |
| Lower | Yes | Current | Low | 1 | 2.29 |
| Lower | No | Lower | Low | 1 | 3.39 |
| Current | Yes | Lower | Low | 1 | 2.32 |
| Lower | No | Current | High | 1 | 2.24 |
| Current | Yes | Current | High | 1 | 1.69 |
| Current | No | Lower | High | 1 | 2.29 |
| Lower | Yes | Lower | High | 1 | 2.03 |
| Current | Yes | Current | Low | 2 | 2.16 |
| Lower | Yes | Lower | Low | 2 | 2.44 |
| Lower | Yes | Current | High | 2 | 1.87 |
| Current | Yes | Lower | High | 2 | 2.04 |

The analysis is shown in the JMP Output below.

JMP Output

**Response Response rate**

**Summary of Fit**

|  |  |
| --- | --- |
| RSquare | 0.956417 |
| RSquare Adj | 0.760293 |
| Root Mean Square Error | 0.205548 |
| Mean of Response | 2.2675 |
| Observations (or Sum Wgts) | 12 |

**Sorted Parameter Estimates**

| **Term** |  | **Estimate** | **Std Error** | **t Ratio** | **t Ratio** | **Prob>|t|** |
| --- | --- | --- | --- | --- | --- | --- |
| Long-term interest rate[Low] |  | 0.23625 | 0.062936 | 3.75 |  | 0.0642 |
| Account-opening fee[No] |  | 0.255 | 0.072672 | 3.51 |  | 0.0725 |
| Initial interest rate[Current] |  | -0.13625 | 0.062936 | -2.16 |  | 0.1628 |
| Annual fee[Current] |  | -0.15 | 0.072672 | -2.06 |  | 0.1751 |
| Annual fee[Current]\*Account-opening fee[No] |  | -0.0975 | 0.072672 | -1.34 |  | 0.3118 |
| Annual fee[Current]\*Long-term interest rate[Low] |  | -0.04375 | 0.062936 | -0.70 |  | 0.5589 |
| Initial interest rate[Current]\*Long-term interest rate[Low] |  | 0.025 | 0.072672 | 0.34 |  | 0.7636 |
| Block[1] |  | -0.0225 | 0.072672 | -0.31 |  | 0.7861 |
| Annual fee[Current]\*Initial interest rate[Current] |  | 0.01375 | 0.062936 | 0.22 |  | 0.8473 |

**Prediction Profiler**



**8.8.** An article by J.J. Pignatiello, Jr. And J.S. Ramberg in the *Journal of Quality Technology*, (Vol. 17, 1985, pp. 198-206) describes the use of a replicated fractional factorial to investigate the effects of five factors on the free height of leaf springs used in an automotive application. The factors are *A* = furnace temperature, *B* = heating time, *C* = transfer time, *D* = hold down time, and *E* = quench oil temperature. The data are shown in Table P81.

**Table P8.1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* |  | Free Height |  |
| – | – | – | – | – | 7.78 | 7.78 | 7.81 |
| + | – | – | + | – | 8.15 | 8.18 | 7.88 |
| – | + | – | + | – | 7.50 | 7.56 | 7.50 |
| + | + | – | – | – | 7.59 | 7.56 | 7.75 |
| – | – | + | + | – | 7.54 | 8.00 | 7.88 |
| + | – | + | – | – | 7.69 | 8.09 | 8.06 |
| – | + | + | – | – | 7.56 | 7.52 | 7.44 |
| + | + | + | + | – | 7.56 | 7.81 | 7.69 |
| – | – | – | – | + | 7.50 | 7.25 | 7.12 |
| + | – | – | + | + | 7.88 | 7.88 | 7.44 |
| – | + | – | + | + | 7.50 | 7.56 | 7.50 |
| + | + | – | – | + | 7.63 | 7.75 | 7.56 |
| – | – | + | + | + | 7.32 | 7.44 | 7.44 |
| + | – | + | – | + | 7.56 | 7.69 | 7.62 |
| – | + | + | – | + | 7.18 | 7.18 | 7.25 |
| + | + | + | + | + | 7.81 | 7.50 | 7.59 |

(a) Write out the alias structure for this design. What is the resolution of this design?

*I*=*ABCD*, Resolution IV

|  |  |  |
| --- | --- | --- |
| *A* | *(ABCD)=* | *BCD* |
| *B* | *(ABCD)=* | *ACD* |
| *C* | *(ABCD)=* | *ABD* |
| *D* | *(ABCD)=* | *ABC* |
| *E* | *(ABCD)=* | *ABCDE* |
| *AB* | *(ABCD)=* | *CD* |
| *AC* | *(ABCD)=* | *BD* |
| *AD* | *(ABCD)=* | *BC* |
| *AE* | *(ABCD)=* | *BCDE* |
| *BE* | *(ABCD)=* | *ACDE* |
| *CE* | *(ABCD)=* | *ABDE* |
| *DE* | *(ABCD)=* | *ABCE* |

(b) Analyze the data. What factors influence the mean free height?

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 0.242083 0.703252 24.3274

Model B -0.16375 0.321769 11.1309

Model C -0.0495833 0.0295021 1.02056

Model D 0.09125 0.0999188 3.45646

Model E -0.23875 0.684019 23.6621

Model AB -0.0295833 0.0105021 0.363296

Model AC 0.00125 1.875E-005 0.000648614

Model AD -0.0229167 0.00630208 0.218006

Model AE 0.06375 0.0487687 1.68704

Error BC Aliased

Error BD Aliased

Model BE 0.152917 0.280602 9.70679

Error CD Aliased

Model CE -0.0329167 0.0130021 0.449777

Model DE 0.0395833 0.0188021 0.650415

Error Pure Error 0.627067 21.6919

Lenth's ME 0.088057

Lenth's SME 0.135984



Design Expert Output

**Response:** **Free Height**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.99 4 0.50 23.74 < 0.0001 significant

*A* *0.70* *1* *0.70* *33.56* *< 0.0001*

*B* *0.32* *1* *0.32* *15.35* *0.0003*

*E* *0.68* *1* *0.68* *32.64* *< 0.0001*

*BE* *0.28* *1* *0.28* *13.39* *0.0007*

Residual 0.90 43 0.021

*Lack of Fit* *0.27* *11* *0.025* *1.27* *0.2844* *not significant*

*Pure Error* *0.63* *32* *0.020*

Cor Total 2.89 47

The Model F-value of 23.74 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 0.14 R-Squared 0.6883

Mean 7.63 Adj R-Squared 0.6593

C.V. 1.90 Pred R-Squared 0.6116

PRESS 1.12 Adeq Precision 13.796

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 7.63 1 0.021 7.58 7.67

A-Furnace Temp 0.12 1 0.021 0.079 0.16 1.00

B-Heating Time -0.082 1 0.021 -0.12 -0.040 1.00

E-Quench Temp -0.12 1 0.021 -0.16 -0.077 1.00

BE 0.076 1 0.021 0.034 0.12 1.00

**Final Equation in Terms of Coded Factors:**

Free Height =

+7.63

+0.12 \* A

-0.082 \* B

-0.12 \* E

+0.076 \* B \* E

**Final Equation in Terms of Actual Factors:**

Free Height =

+7.62562

+0.12104 \* Furnace Temp

-0.081875 \* Heating Time

-0.11937 \* Quench Temp

+0.076458 \* Heating Time \* Quench Temp

(c) Calculate the range and standard deviation of the free height for each run. Is there any indication that any of these factors affects variability in the free height?

Design Expert Output (Range)

Term Effect SumSqr % Contribtn

Model Intercept

Model A 0.11375 0.0517563 16.2198

Model B -0.12625 0.0637563 19.9804

Model C 0.02625 0.00275625 0.863774

Error D 0.06125 0.0150063 4.70277

Model E -0.01375 0.00075625 0.236999

Error AB 0.04375 0.00765625 2.39937

Error AC -0.03375 0.00455625 1.42787

Error AD 0.03625 0.00525625 1.64724

Error AE -0.00375 5.625E-005 0.017628

Model BC Aliased

Error BD Aliased

Model BE 0.01625 0.00105625 0.331016

Error CD Aliased

Model CE -0.13625 0.0742562 23.271

Error DE -0.02125 0.00180625 0.566056

Error ABC Aliased

Error ABD Aliased

Error ABE 0.03125 0.00390625 1.22417

Error ACD Aliased

Error ACE 0.04875 0.00950625 2.97914

Error ADE 0.13875 0.0770062 24.1328

Error BCD Aliased

Model BCE Aliased

Error BDE Aliased

Error CDE Aliased

Lenth's ME 0.130136

Lenth's SME 0.264194

Interaction *ADE* is aliased with *BCE*. Although the plot below identifies *ADE*, *BCE* was included in the analysis.



Design Expert Output (Range)

**Response:** **Range**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 0.28 8 0.035 5.70 0.0167 significant

A 0.052 1 0.052 8.53 0.0223

B 0.064 1 0.064 10.50 0.0142

C 2.756E-003 1 2.756E-003 0.45 0.5220

E 7.562E-004 1 7.562E-004 0.12 0.7345

BC 5.256E-003 1 5.256E-003 0.87 0.3831

BE 1.056E-003 1 1.056E-003 0.17 0.6891

CE 0.074 1 0.074 12.23 0.0100

BCE 0.077 1 0.077 12.69 0.0092

Residual 0.042 7 6.071E-003

Cor Total 0.32 15

The Model F-value of 5.70 implies the model is significant. There is only

a 1.67% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 0.078 R-Squared 0.8668

Mean 0.22 Adj R-Squared 0.7146

C.V. 35.52 Pred R-Squared 0.3043

PRESS 0.22 Adeq Precision 7.166

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 0.22 1 0.019 0.17 0.27

A-Furn Temp 0.057 1 0.019 0.011 0.10 1.00

B-Heat Time -0.063 1 0.019 -0.11 -0.017 1.00

C-Transfer Time 0.013 1 0.019 -0.033 0.059 1.00

E-Qnch Temp -6.875E-003 1 0.019 -0.053 0.039 1.00

BC 0.018 1 0.019 -0.028 0.064 1.00

BE 8.125E-003 1 0.019 -0.038 0.054 1.00

CE -0.068 1 0.019 -0.11 -0.022 1.00

BCE 0.069 1 0.019 0.023 0.12 1.00

**Final Equation in Terms of Coded Factors:**

Range =

+0.22

+0.057 \* A

-0.063 \* B

+0.013 \* C

-6.875E-003 \* E

+0.018 \* B \* C

+8.125E-003 \* B \* E

-0.068 \* C \* E

+0.069 \* B \* C \* E

**Final Equation in Terms of Actual Factors:**

Range =

+0.21937

+0.056875 \* Furnace Temp

-0.063125 \* Heating Time

+0.013125 \* Transfer Time

-6.87500E-003 \* Quench Temp

+0.018125 \* Heating Time \* Transfer Time

+8.12500E-003 \* Heating Time \* Quench Temp

-0.068125 \* Transfer Time \* Quench Temp

+0.069375 \* Heating Time \* Transfer Time \* Quench Temp

Design Expert Output (StDev)

Term Effect SumSqr % Contribtn

Model Intercept

Model A 0.0625896 0.0156698 16.873

Model B -0.0714887 0.0204425 22.0121

Model C 0.010567 0.000446646 0.48094

Error D 0.0353616 0.00500176 5.3858

Model E -0.00684034 0.000187161 0.201532

Error AB 0.0153974 0.000948317 1.02113

Error AC -0.0218505 0.00190978 2.05641

Error AD 0.0190608 0.00145326 1.56484

Error AE -0.00329035 4.33057E-005 0.0466308

Model BC Aliased

Error BD Aliased

Model BE 0.0087666 0.000307413 0.331017

Error CD Aliased

Model CE -0.0714816 0.0204385 22.0078

Error DE -0.00467792 8.75317E-005 0.0942525

Error ABC Aliased

Error ABD Aliased

Error ABE 0.0155599 0.000968437 1.0428

Error ACD Aliased

Error ACE 0.0199742 0.00159587 1.7184

Error ADE Aliased

Error BCD Aliased

Model BCE 0.0764346 0.023369 25.1633

Error BDE Aliased

Error CDE Aliased

Lenth's ME 0.0596836

Lenth's SME 0.121166

Interaction *ADE* is aliased with *BCE*. Although the plot below identifies *ADE*, *BCE* was included in the analysis.



Design Expert Output (StDev)

**Response:** **StDev**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 0.082 8 0.010 6.82 0.0101 significant

*A* *0.016* *1* *0.016* *10.39* *0.0146*

*B* *0.020* *1* *0.020* *13.56* *0.0078*

*C* *4.466E-004* *1* *4.466E-004* *0.30* *0.6032*

*E* *1.872E-004* *1* *1.872E-004* *0.12* *0.7350*

*BC* *1.453E-003* *1* *1.453E-003* *0.96* *0.3589*

*BE* *3.074E-004* *1* *3.074E-004* *0.20* *0.6653*

*CE* *0.020* *1* *0.020* *13.55* *0.0078*

*BCE* *0.023* *1* *0.023* *15.50* *0.0056*

Residual 0.011 7 1.508E-003

Cor Total 0.093 15

The Model F-value of 6.82 implies the model is significant. There is only

a 1.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 0.039 R-Squared 0.8863

Mean 0.12 Adj R-Squared 0.7565

C.V. 33.07 Pred R-Squared 0.4062

PRESS 0.055 Adeq Precision 7.826

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 0.12 1 9.708E-003 0.094 0.14

A-Furnace Temp 0.031 1 9.708E-003 8.340E-003 0.054 1.00

B-Heating Time -0.036 1 9.708E-003 -0.059 -0.013 1.00

C-Transfer Time 5.283E-003 1 9.708E-003 -0.018 0.028 1.00

E-Quench Temp -3.420E-003 1 9.708E-003 -0.026 0.020 1.00

BC 9.530E-003 1 9.708E-003 -0.013 0.032 1.00

BE 4.383E-003 1 9.708E-003 -0.019 0.027 1.00

CE -0.036 1 9.708E-003 -0.059 -0.013 1.00

BCE 0.038 1 9.708E-003 0.015 0.061 1.00

**Final Equation in Terms of Coded Factors:**

StDev =

+0.12

+0.031 \* A

-0.036 \* B

+5.283E-003 \* C

-3.420E-003 \* E

+9.530E-003 \* B \* C

+4.383E-003 \* B \* E

-0.036 \* C \* E

+0.038 \* B \* C \* E

**Final Equation in Terms of Actual Factors:**

StDev =

+0.11744

+0.031295 \* Furnace Temp

-0.035744 \* Heating Time

+5.28350E-003 \* Transfer Time

-3.42017E-003 \* Quench Temp

+9.53040E-003 \* Heating Time \* Transfer Time

+4.38330E-003 \* Heating Time \* Quench Temp

-0.035741 \* Transfer Time \* Quench Temp

+0.038217 \* Heating Time \* Transfer Time \* Quench Temp

(d) Analyze the residuals from this experiment, and comment on your findings.

The residual plot follows. All plots are satisfactory.









(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for five factors in 16 runs with a higher resolution than this one?

This was not the best design. A resolution V design is possible by setting the generator equal to the highest order interaction, *ABCDE*.

**8.9.** Consider the leaf spring experiment in Problem 8.8. Suppose that factor *E* (quench oil temperature) is very difficult to control during manufacturing. Where would you set factors *A, B, C* and *D* to reduce variability in the free height as much as possible regardless of the quench oil temperature used?



Run the process with *A* at the low level, *B* at the high level, *C* at the low level and *D* at either level (the low level of *D* may give a faster process).

**8.10.** Consider the 25 design in Problem 6.24. Suppose that only a one-half fraction could be run. Furthermore, two days were required to take the 16 observations, and it was necessary to confound the 25-1 design in two blocks. Construct the design and analyze the data.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E=ABCD* |  | Data | Blocks = *AB* | Block |
| – | – | – | – | + | *e* | 8 | + | 1 |
| + | – | – | – | – | *a* | 9 | – | 2 |
| – | + | – | – | – | *b* | 34 | – | 2 |
| + | + | – | – | + | *abe* | 52 | + | 1 |
| – | – | + | – | – | *c* | 16 | + | 1 |
| + | – | + | – | + | *ace* | 22 | – | 2 |
| – | + | + | – | + | *bce* | 45 | – | 2 |
| + | + | + | – | – | *abc* | 60 | + | 1 |
| – | – | – | + | – | *d* | 8 | + | 1 |
| + | – | – | + | + | *ade* | 10 | – | 2 |
| – | + | – | + | + | *bde* | 30 | – | 2 |
| + | + | – | + | – | *abd* | 50 | + | 1 |
| – | – | + | + | + | *cde* | 15 | + | 1 |
| + | – | + | + | – | *acd* | 21 | – | 2 |
| – | + | + | + | – | *bcd* | 44 | – | 2 |
| + | + | + | + | + | *abcde* | 63 | + | 1 |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 10.875 473.063 8.6343

Model B 33.625 4522.56 82.5455

Model C 10.625 451.562 8.24188

Error D -0.625 1.5625 0.0285186

Error E 0.375 0.5625 0.0102667

Error AB Aliased

Error AC 0.625 1.5625 0.0285186

Error AD 0.875 3.0625 0.0558965

Error AE 1.375 7.5625 0.13803

Error BC 0.875 3.0625 0.0558965

Error BD -0.375 0.5625 0.0102667

Error BE 0.125 0.0625 0.00114075

Error CD 0.625 1.5625 0.0285186

Error CE 0.625 1.5625 0.0285186

Error DE -1.625 10.5625 0.192786

Lenth's ME 2.46263

Lenth's SME 5.0517

The *AB* interaction in the above table is aliased with the three-factor interaction *BCD*, and is also confounded with blocks.



Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 203.06 1 203.06

Model 5447.19 3 1815.73 630.31 < 0.0001 significant

*A* *473.06* *1* *473.06* *164.22* *< 0.0001*

*B* *4522.56* *1* *4522.56* *1569.96* *< 0.0001*

*C* *451.56* *1* *451.56* *156.76* *< 0.0001*

Residual 31.69 11 2.88

Cor Total 5681.94 15

The Model F-value of 630.31 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 1.70 R-Squared 0.9942

Mean 30.44 Adj R-Squared 0.9926

C.V. 5.58 Pred R-Squared 0.9878

PRESS 67.04 Adeq Precision 58.100

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 30.44 1 0.42 29.50 31.37

Block 1 3.56 1

Block 2 -3.56

A-Aperture 5.44 1 0.42 4.50 6.37 1.00

B-Exposure Time 16.81 1 0.42 15.88 17.75 1.00

C-Develop Time 5.31 1 0.42 4.38 6.25 1.00

**Final Equation in Terms of Coded Factors:**

Yield =

+30.44

+5.44 \* A

+16.81 \* B

+5.31 \* C

**Final Equation in Terms of Actual Factors:**

Aperture small

Yield =

-1.56250

+0.84063 \* Exposure Time

+0.70833 \* Develop Time

Aperture large

Yield =

+9.31250

+0.84063 \* Exposure Time

+0.70833 \* Develop Time

**8.11.** Analyze the data in Problem 6.26 as if it came from a  design with *I=ABCD*. Project the design into a full factorial in the subset of the original four factors that appear to be significant.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run |  |  |  |  |  | Yield |  | Factor | Levels |
| Number | *A* | *B* | *C* | *D=ABC* |  | (lbs) |  | Low (-) | High (+) |
| 1 | – | – | – | – | (1) | 12 | *A* (h) | 2.5 | 3.0 |
| 2 | + | – | – | + | *ad* | 25 | *B* (%) | 14 | 18 |
| 3 | – | + | – | + | *bd* | 13 | *C* (psi) | 60 | 80 |
| 4 | + | + | – | – | *ab* | 16 | *D* (C) | 225 | 250 |
| 5 | – | – | + | + | *cd* | 19 |  |  |  |
| 6 | + | – | + | – | *ac* | 15 |  |  |  |
| 7 | – | + | + | – | *bc* | 20 |  |  |  |
| 8 | + | + | + | + | *abcd* | 23 |  |  |  |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 3.75 28.125 18.3974

Error B 0.25 0.125 0.0817661

Model C 2.75 15.125 9.8937

Model D 4.25 36.125 23.6304

Error AB -0.75 1.125 0.735895

Model AC -4.25 36.125 23.6304

Model AD 4.25 36.125 23.6304

Lenth's ME 21.174

Lenth's SME 50.6734



Design Expert Output

**Response:** **Yield** **in lbs**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 151.63 5 30.32 48.52 0.0203 significant

*A* *28.13* *1* *28.13* *45.00* *0.0215*

*C* *15.13* *1* *15.13* *24.20* *0.0389*

*D* *36.12* *1* *36.12* *57.80* *0.0169*

*AC* *36.12* *1* *36.12* *57.80* *0.0169*

*AD* *36.13* *1* *36.13* *57.80* *0.0169*

Residual 1.25 2 0.62

Cor Total 152.88 7

The Model F-value of 48.52 implies the model is significant. There is only

a 2.03% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 0.79 R-Squared 0.9918

Mean 17.88 Adj R-Squared 0.9714

C.V. 4.42 Pred R-Squared 0.8692

PRESS 20.00 Adeq Precision 17.892

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 17.88 1 0.28 16.67 19.08

A-Time 1.87 1 0.28 0.67 3.08 1.00

C-Pressure 1.37 1 0.28 0.17 2.58 1.00

D-Temperature 2.13 1 0.28 0.92 3.33 1.00

AC -2.13 1 0.28 -3.33 -0.92 1.00

AD 2.13 1 0.28 0.92 3.33 1.00

**Final Equation in Terms of Coded Factors:**

Yield =

+17.88

+1.87 \* A

+1.37 \* C

+2.13 \* D

-2.13 \* A \* C

+2.13 \* A \* D

**Final Equation in Terms of Actual Factors:**

Yield =

+227.75000

-94.50000 \* Time

+2.47500 \* Pressure

-1.70000 \* Temperature

-0.85000 \* Time \* Pressure

+0.68000 \* Time \* Temperature

**8.12.** Repeat Problem 8.11 using *I*=*–ABCD*. Does use of the alternate fraction change your interpretation of the data?

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run |  |  |  |  |  | Yield |  | Factor | Levels |
| Number | *A* | *B* | *C* | *D=ABC* |  | (lbs) |  | Low (-) | High (+) |
| 1 | – | – | – | + | *d* | 10 | *A* (h) | 2.5 | 3.0 |
| 2 | + | – | – | – | *a* | 18 | *B* (%) | 14 | 18 |
| 3 | – | + | – | – | *b* | 13 | *C* (psi) | 60 | 80 |
| 4 | + | + | – | + | *abd* | 24 | *D* (C) | 225 | 250 |
| 5 | – | – | + | – | *c* | 17 |  |  |  |
| 6 | + | – | + | + | *acd* | 21 |  |  |  |
| 7 | – | + | + | + | *bcd* | 17 |  |  |  |
| 8 | + | + | + | – | *abc* | 15 |  |  |  |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 5.25 55.125 40.8712

Error B 0.75 1.125 0.834106

Model C 1.25 3.125 2.31696

Model D 2.25 10.125 7.50695

Error AB -0.75 1.125 0.834106

Model AC -4.25 36.125 26.7841

Model AD 3.75 28.125 20.8526

Lenth's ME 12.7044

Lenth's SME 30.404



Design Expert Output

**Response:** **Yield** **in lbs**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 132.63 5 26.52 23.58 0.0412 significant

*A* *55.13* *1* *55.13* *49.00* *0.0198*

*C* *3.13* *1* *3.13* *2.78* *0.2375*

*D* *10.13* *1* *10.13* *9.00* *0.0955*

*AC* *36.13* *1* *36.13* *32.11* *0.0298*

*AD* *28.13* *1* *28.13* *25.00* *0.0377*

Residual 2.25 2 1.12

Cor Total 134.88 7

The Model F-value of 23.58 implies the model is significant. There is only

a 4.12% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 1.06 R-Squared 0.9833

Mean 16.88 Adj R-Squared 0.9416

C.V. 6.29 Pred R-Squared 0.7331

PRESS 36.00 Adeq Precision 14.425

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 16.88 1 0.37 15.26 18.49

A-Time 2.63 1 0.37 1.01 4.24 1.00

C-Pressure 0.63 1 0.37 -0.99 2.24 1.00

D-Temperature 1.13 1 0.37 -0.49 2.74 1.00

AC -2.13 1 0.37 -3.74 -0.51 1.00

AD 1.88 1 0.37 0.26 3.49 1.00

**Final Equation in Terms of Coded Factors:**

Yield =

+16.88

+2.63 \* A

+0.63 \* C

+1.13 \* D

-2.13 \* A \* C

+1.88 \* A \* D

**Final Equation in Terms of Actual Factors:**

Yield =

+190.50000

-72.50000 \* Time

+2.40000 \* Pressure

-1.56000 \* Temperature

-0.85000 \* Time \* Pressure

+0.60000 \* Time \* Temperature

**8.13.** Project the  design in Example 8.1 into two replicates of a 22 design in the factors *A* and *B*. Analyze the data and draw conclusions.

The *Design Expert* output below does not identify a significant effect.

Design Expert Output

**Response:** **Filtration Rate**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 728.50 3 242.83 0.41 0.7523 not significant

*A* *722.00* *1* *722.00* *1.23* *0.3291*

*B* *4.50* *1* *4.50* *7.682E-003* *0.9344*

*AB* *2.00* *1* *2.00* *3.414E-003* *0.9562*

Residual 2343.00 4 585.75

*Lack of Fit* *0.000* *0*

*Pure Error* *2343.00* *4* *585.75*

Cor Total 3071.50 7

The "Model F-value" of 0.41 implies the model is not significant relative to the noise. There is a

75.23 % chance that a "Model F-value" this large could occur due to noise.

Std. Dev. 24.20 R-Squared 0.2372

Mean 70.75 Adj R-Squared -0.3349

C.V. 34.21 Pred R-Squared -2.0513

PRESS 9372.00 Adeq Precision 1.198

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 70.75 1 8.56 46.99 94.51

A-Temperature 9.50 1 8.56 -14.26 33.26 1.00

B-Pressure 0.75 1 8.56 -23.01 24.51 1.00

AB -0.50 1 8.56 -24.26 23.26 1.00

**Final Equation in Terms of Coded Factors:**

Filtration Rate =

+70.75

+9.50 \* A

+0.75 \* B

-0.50 \* A \* B

**Final Equation in Terms of Actual Factors:**

Filtration Rate =

+70.75000

+9.50000 \* Temperature

+0.75000 \* Pressure

-0.50000 \* Temperature \* Pressure

**8.14.** Construct a design. Determine the effects that may be estimated if a full fold over of this design is performed.

The design is shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=AC* |  |
| – | – | – | + | + | *de* |
| + | – | – | – | – | *a* |
| – | + | – | – | + | *be* |
| + | + | – | + | – | *abd* |
| – | – | + | + | – | *cd* |
| + | – | + | – | + | *ace* |
| – | + | + | – | – | *bc* |
| + | + | + | + | + | *abcde* |

The design with the fold over included is as follows.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Block | *A* | *B* | *C* | *D=AB* | *E=AC* |  |
| 1 | – | – | – | + | + | *de* |
| 1 | + | – | – | – | – | *a* |
| 1 | – | + | – | – | + | *be* |
| 1 | + | + | – | + | – | *abd* |
| 1 | – | – | + | + | – | *cd* |
| 1 | + | – | + | – | + | *ace* |
| 1 | – | + | + | – | – | *bc* |
| 1 | + | + | + | + | + | *abcde* |
| 2 | + | + | + | – | – | *abc* |
| 2 | – | + | + | + | + | *bcde* |
| 2 | + | – | + | + | – | *acd* |
| 2 | – | – | + | – | + | *ce* |
| 2 | + | + | – | – | + | *abe* |
| 2 | – | + | – | + | – | *bd* |
| 2 | + | – | – | + | + | *ade* |
| 2 | – | – | – | – | – | (1) |

The effects are shown in the table below.

|  |  |  |
| --- | --- | --- |
| Principal Fraction | Second Fraction | |
| *lA=A+BD+CE* | | *l\*A=A-BD-CE* | |
| *lB= B + AD* | | *l\*B= B - AD* | |
| *lC= C + AE* | | *l\*C= C - AE* | |
| *lD= D + AB* | | *l\*D= D - AB* | |
| *lE= E + AC* | | *l\*E= E - AC* | |
| *lBC= BC + DE* | | *l\*BC= BC + DE* | |
| *lBE= BE + CD* | | *l\*BE= BE + CD* | |

By combining the two fractions we can estimate the following:

|  |  |
| --- | --- |
| ( l*i* +*l\*i*)/2 | ( *li* -*l\*i*)/2 |
| *A* | *BD+CE* |
| *B* | *AD* |
| *C* | *AE* |
| *D* | *AB* |
| *E* | *AC* |
| *BC+DE* |  |
| *BE+CD* |  |

These estimates are confirmed with the Design Expert output shown below.

Design Expert Output

**Design Matrix Evaluation for Factorial Reduced 3FI Model**

**Factorial Effects Aliases**

**[Est. Terms] Aliased Terms**

[Intercept] = Intercept

[Block 1] = Block 1 + ABD + ACE

[Block 2] = Block 2 - ABD - ACE

[A] = A

[B] = B + CDE

[C] = C + BDE

[D] = D + BCE

[E] = E + BCD

[AB] = AB

[AC] = AC

[AD] = AD

[AE] = AE

[BC] = BC + DE

[BD] = BD + CE

[BE] = BE + CD

[ABC] = ABC + ADE

[ABE] = ABE + ACD

**8.15.** Fold over the  design in Table 8.19 to produce an eight-factor design. Verify that the resulting design is a  design. Is this a minimal design?

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *H* | *A* | *B* | *C* | *D=AB* | *E=AC* | *F=BC* | *G=ABC* |
|  | + | – | – | – | + | + | + | – |
| Original | + | + | – | – | – | – | + | + |
| Design | + | – | + | – | – | + | – | + |
|  | + | + | + | – | + | – | – | – |
|  | + | – | – | + | + | – | – | + |
|  | + | + | – | + | – | + | – | – |
|  | + | – | + | + | – | – | + | – |
|  | + | + | + | + | + | + | + | + |
|  | – | + | + | + | – | – | – | + |
| Second | – | – | + | + | + | + | – | – |
| Set of | – | + | – | + | + | – | + | – |
| Runs w/ | – | – | – | + | – | + | + | + |
| all Signs | – | + | + | – | – | + | + | – |
| Switched | – | – | + | – | + | – | + | + |
|  | – | + | – | – | + | + | – | + |
|  | – | – | – | – | – | – | – | – |

After folding the original design over, we add a new factor *H*, and we have a design with generators *D=ABH, E=ACH, F=BCH*, and *G=ABC*. This is a  design. It is a minimal design, since it contains 2*k*=2(8)=16 runs.

**8.16.** Fold over a  design to produce a six-factor design. Verify that the resulting design is a  design. Compare this  design to the in Table 8.10.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *F* | *A* | *B* | *C* | *D=AB* | *E=BC* |
|  | + | – | – | – | + | + |
| Original | + | + | – | – | – | + |
| Design | + | – | + | – | – | – |
|  | + | + | + | – | + | – |
|  | + | – | – | + | + | – |
|  | + | + | – | + | – | – |
|  | + | – | + | + | – | + |
|  | + | + | + | + | + | + |
|  | – | + | + | + | – | – |
| Second | – | – | + | + | + | – |
| Set of | – | + | – | + | + | + |
| Runs w/ | – | – | – | + | – | + |
| all Signs | – | + | + | – | – | + |
| Switched | – | – | + | – | + | + |
|  | – | + | – | – | + | – |
|  | – | – | – | – | – | – |

If we relabel the factors from left to right as *A, B, C, D, E, F*, then this design becomes with generators *I=ABDF* and *I=BCEF*. It is not a minimal design, since 2*k* = 2(6) = 12 runs, and the design contains 16 runs.

**8.17.** An industrial engineer is conducting an experiment using a Monte Carlo simulation model of an inventory system. The independent variables in her model are the order quantity (*A*), the reorder point (B), the setup cost (*C*), the backorder cost (*D*), and the carrying cost rate (*E*). The response variable is average annual cost. To conserve computer time, she decides to investigate these factors using a  design with *I*=*ABD* and *I*=*BCE*. The results she obtains are *de* = 95, *ae* = 134, *b* = 158, *abd* = 190, *cd* = 92, *ac* = 187, *bce* = 155, and *abcde* = 185.

(a) Verify that the treatment combinations given are correct. Estimate the effects, assuming three-factor and higher interactions are negligible.

The treatment combinations given are correct. The effects are shown in the *Design Expert* output and the normal and half normal probability plot of effects identify factors *A* and *B* as important.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=BC* |  |
| – | – | – | + | + | *de* |
| + | – | – | – | *+* | *ae* |
| – | + | – | – | – | *b* |
| + | + | – | + | – | *abd* |
| – | – | + | + | – | *cd* |
| + | – | + | – | – | *ac* |
| – | + | + | – | *+* | *bce* |
| + | + | + | + | *+* | *abcde* |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 49 4802 43.9502

Model B 45 4050 37.0675

Error C 10.5 220.5 2.01812

Error D -18 648 5.93081

Error E -14.5 420.5 3.84862

Error AC 13.5 364.5 3.33608

Error AE -14.5 420.5 3.84862

Lenth's ME 81.8727

Lenth's SME 195.937



(b) Suppose that a second fraction is added to the first, for example *ade* = 136, *e* = 93, *ab* = 187, *bd* = 153, *acd* = 139, *c* = 99, *abce* = 191, and *bcde* = 150. How was this second fraction obtained? Add this data to the original fraction, and estimate the effects.

This second fraction is formed by reversing the signs of factor *A*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=BC* |  |
| + | – | – | + | + | *ade* |
| – | – | – | – | *+* | *e* |
| + | + | – | – | – | *ab* |
| – | + | – | + | – | *bd* |
| + | – | + | + | – | *acd* |
| – | – | + | – | – | *c* |
| + | + | + | – | *+* | *abce* |
| – | + | + | + | *+* | *bcde* |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 44.25 7832.25 39.5289

Model B 49.25 9702.25 48.9666

Error C 6.5 169 0.852932

Error D -8 256 1.29202

Error E -8.25 272.25 1.37403

Error AB -10 400 2.01877

Error AC 7.25 210.25 1.06112

Error AD -4.25 72.25 0.364641

Error AE -6 144 0.726759

Error BD 4.75 90.25 0.455486

Error CD -8.5 289 1.45856

Error DE 6.25 156.25 0.788584

Error ACD -6.25 156.25 0.788584

Error ADE 4 64 0.323004

Lenth's ME 25.1188

Lenth's SME 51.5273



(c) Suppose that the fraction *abc* = 189, *ce* = 96, *bcd* = 154, *acde* = 135, *abe* = 193, *bde* = 152, *ad* = 137, and (1) = 98 was run. How was this fraction obtained? Add this data to the original fraction and estimate the effects.

This second fraction is formed by reversing the signs of all factors.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=BC* |  |
| + | + | + | - | - | *abc* |
| - | + | + | + | *-* | *bcd* |
| + | - | + | + | *+* | *acde* |
| - | - | + | - | *+* | *ce* |
| + | + | - | - | *+* | *abe* |
| - | + | - | + | *+* | *bde* |
| + | - | - | + | *-* | *ad* |
| - | - | - | - | *-* | (1) |

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 43.75 7656.25 38.1563

Model B 50.25 10100.3 50.3364

Error C 4.5 81 0.403678

Error D -8.75 306.25 1.52625

Error E -7.5 225 1.12133

Error AB -9.25 342.25 1.70566

Error AC 6 144 0.71765

Error AD -5.25 110.25 0.549451

Error AE -6.5 169 0.842242

Error BC -7 196 0.976801

Error BD 5.25 110.25 0.549451

Error BE 6 144 0.71765

Error ABC -8 256 1.27582

Error ABE 7.5 225 1.12133

Lenth's ME 26.5964

Lenth's SME 54.5583



**8.18.** Construct a 25-1 design. Show how the design may be run in two blocks of eight observations each. Are any main effects or two-factor interactions confounded with blocks?

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E=ABCD* |  | Blocks = *AB* | Block |
| – | – | – | – | + | *e* | + | 1 |
| + | – | – | – | – | *a* | – | 2 |
| – | + | – | – | – | *b* | – | 2 |
| + | + | – | – | + | *abe* | + | 1 |
| – | – | + | – | – | *c* | + | 1 |
| + | – | + | – | + | *ace* | – | 2 |
| – | + | + | – | + | *bce* | – | 2 |
| + | + | + | – | – | *abc* | + | 1 |
| – | – | – | + | – | *d* | + | 1 |
| + | – | – | + | + | *ade* | – | 2 |
| – | + | – | + | + | *bde* | – | 2 |
| + | + | – | + | – | *abd* | + | 1 |
| – | – | + | + | + | *cde* | + | 1 |
| + | – | + | + | – | *acd* | – | 2 |
| – | + | + | + | – | *bcd* | – | 2 |
| + | + | + | + | + | *abcde* | + | 1 |

Blocks are confounded with *AB* and *CDE.*

**8.19.** Construct a 27-2 design. Show how the design may be run in four blocks of eight observations each. Are any main effects or two-factor interactions confounded with blocks?

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *A* | *B* | *C* | *D* | *E* | *F=CDE* | *G=ABC* |  | Block=ACE | Block=BFG | Block assignment |
| 1 | – | – | – | – | – | – | – | (1) | – | – | 1 |
| 2 | + | – | – | – | – | – | + | *ag* | + | + | 4 |
| 3 | – | + | – | – | – | – | + | *bg* | – | – | 1 |
| 4 | + | + | – | – | – | – | – | *ab* | + | + | 4 |
| 5 | – | – | + | – | – | + | + | *cfg* | + | – | 3 |
| 6 | + | – | + | – | – | + | – | *acf* | – | + | 2 |
| 7 | – | + | + | – | – | + | – | *bcf* | + | – | 3 |
| 8 | + | + | + | – | – | + | + | *abcfg* | – | + | 2 |
| 9 | – | – | – | + | – | + | – | *df* | – | + | 2 |
| 10 | + | – | – | + | – | + | + | *adfg* | + | – | 3 |
| 11 | – | + | – | + | – | + | + | *bdfg* | – | + | 2 |
| 12 | + | + | – | + | – | + | – | *abdf* | + | – | 3 |
| 13 | – | – | + | + | – | – | + | *cdg* | + | + | 4 |
| 14 | + | – | + | + | – | – | – | *acd* | – | – | 1 |
| 15 | – | + | + | + | – | – | – | *bcd* | + | + | 4 |
| 16 | + | + | + | + | – | – | + | *abcdg* | – | – | 1 |
| 17 | – | – | – | – | + | + | – | *ef* | + | + | 4 |
| 18 | + | – | – | – | + | + | + | *aefg* | – | – | 1 |
| 19 | – | + | – | – | + | + | + | *befg* | + | + | 4 |
| 20 | + | + | – | – | + | + | – | *abef* | – | – | 1 |
| 21 | – | – | + | – | + | – | + | *ceg* | – | + | 2 |
| 22 | + | – | + | – | + | – | – | *ace* | + | – | 3 |
| 23 | – | + | + | – | + | – | – | *bce* | – | + | 2 |
| 24 | + | + | + | – | + | – | + | *abceg* | + | – | 3 |
| 25 | – | – | – | + | + | – | – | *de* | + | – | 3 |
| 26 | + | – | – | + | + | – | + | *adeg* | – | + | 2 |
| 27 | – | + | – | + | + | – | + | *bdeg* | + | – | 3 |
| 28 | + | + | – | + | + | – | – | *abde* | – | + | 2 |
| 29 | – | – | + | + | + | + | + | *cdefg* | – | – | 1 |
| 30 | + | – | + | + | + | + | – | *acdef* | + | + | 4 |
| 31 | – | + | + | + | + | + | – | *bcdef* | – | – | 1 |
| 32 | + | + | + | + | + | + | + | *abcdefg* | + | + | 4 |

Blocks are confounded with *ACE, BFG,* and *ABCEFG*.

**8.20. *Nonregular fractions of the 2k* [John (1971)].** Consider a 24 design. We must estimate the four main effects and the six two-factor interactions, but the full 24 factorial cannot be run. The largest possible block contains 12 runs. These 12 runs can be obtained from the four one-quarter fractions defined by *I = ± AB = ± ACD = ± BCD* by omitting the principal fraction. Show how the remaining three 24-2 fractions can be combined to estimate the required effects, assuming that three-factor and higher interactions are negligible. This design could be thought of as a three-quarter fraction.

The four 24-2 fractions are as follows:

(1) I = +*AB = +ACD = +BCD*

Runs: *c,d,ab,abcd*

(2) I = +*AB =* –*ACD =*–*BCD*

Runs: (1), *cd, abc, abd*

(3) I = –*AB = +ACD =* –*BCD*

Runs: *a, bc, bd, acd*

(4) I = –*AB =* –*ACD = +BCD*

Runs: *b, ac, ad, bcd*

If we do not run the principal fraction (1), then we can combine the remaining 3 fractions to 3 one-half fractions of the 24 as follows:

Fraction 1: (2) + (3) implies I = –*BCD*. This fraction estimates: *A, AB, AC*, and *AD*

Fraction 2: (2) + (4) implies I = –*ACD*. This fraction estimates: *B, BC, BD*, and *AB*

Fraction 3: (3) + (4) implies I = –*AB*. This fraction estimates: *C, D*, and *CD*

In estimating these effects we assume that all three-factor and higher interactions are negligible. Note that *AB* is estimated in two of the one-half fractions: 1 and 2. We would average these quantities and obtain a single estimate of *AB*. John (1971, pp. 161-163) discusses this design and shows that the estimates obtained above are also the least squares estimates. John also derives the variances and covariances of these estimators.

**8.21S.** Carbon anodes used in a smelting process are baked in a ring furnace. An experiment is run in the furnace to determine which factors influence the weight of packing material that is stuck to the anodes after baking. Six variables are of interest, each at two levels: *A* = pitch/fines ratio (0.45, 0.55); *B* = packing material type (1, 2); *C* = packing material temperature (ambient, 325 C); *D* = flue location (inside, outside); *E* = pit temperature (ambient, 195 C); and *F* = delay time before packing (zero, 24 hours). A 26-3 design is run, and three replicates are obtained at each of the design points. The weight of packing material stuck to the anodes is measured in grams. The data in run order are as follows: *abd* = (984, 826, 936); *abcdef* = (1275, 976, 1457); *be* = (1217, 1201, 890); *af* = (1474, 1164, 1541); *def* = (1320, 1156, 913); *cd* = (765, 705, 821); *ace* = (1338, 1254, 1294); and *bcf* = (1325, 1299, 1253). We wish to minimize the amount stuck packing material.

(a) Verify that the eight runs correspond to a design. What is the alias structure?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D=AB* | *E=AC* | *F=BC* |  |
| – | – | – | + | + | + | *def* |
| + | – | – | – | – | + | *af* |
| – | + | – | – | + | – | *be* |
| + | + | – | + | – | – | *abd* |
| – | – | + | + | – | – | *cd* |
| + | – | + | – | + | – | *ace* |
| – | + | + | – | – | + | *bcf* |
| + | + | + | + | + | + | *abcdef* |

*I=ABD=ACE=BCF=BCDE=ACDF=ABEF=DEF,* Resolution III

*A=BD=CE=CDF=BEF*

*B=AD=CF=CDE=AEF*

*C=AE=BF=BDE=ADF*

*D=AB=EF=BCE=ACF*

*E=AC=DF=BCD=ABF*

*F=BC=DE=ACD=ABE*

*CD=BE=AF=ABC=ADE=BDF=CEF*

(b) Use the average weight as a response. What factors appear to be influential?

Design Expert Output

Term Effect SumSqr % Contribtn

Model A 137.833 37996.1 12.0947

Error B -8.83333 156.056 0.049675

Error C 11.6667 272.222 0.0866527

Model D -259.667 134854 42.926

Model E 99.8333 19933.4 6.34511

Model F 243.5 118585 37.7473

Error AF -34.3333 2357.56 0.750447

Lenth's ME 563.698

Lenth's SME 1349.04



Factors *A, D, E* and *F* (and their aliases) are apparently important.

(c) Use the range of the weights as a response. What factors appear to be influential?

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Error A 44.5 3960.5 2.13311

Error B 13.5 364.5 0.196319

Model C -129 33282 17.9256

Error D 75.5 11400.5 6.14028

Model E 144 41472 22.3367

Model F 163 53138 28.62

Model AF 145 42050 22.648

Lenth's ME 728.384

Lenth's SME 1743.17

Factors *C, E,* *F* and the *AF* interaction (and their aliases) appear to be large.



(d) What recommendations would you make to the process engineers?

It is not known exactly what to do here, since *A, D, E* and *F* are large effects, and because the design is resolution III, the main effects are aliased with two-factor interactions. Note, for example, that *D* is aliased with *EF* and the main effect could really be an *EF* interaction. If the main effects are really important, then setting all factors at the low level would minimize the amount of material stuck to the anodes. It would be necessary to run additional experiments to confirm these findings.

**8.22S.** A 16-run experiment was performed in a semiconductor manufacturing plant to study the effects of six factors on the curvature or camber of the substrate devices produced. The six variables and their levels are shown in Table P8.2.

**Table P8.2**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Lamination | Lamination | Lamination | Firing | Firing | Firing |
|  | Temperature | Time | Pressure | Temperature | Cycle Time | Dew Point |
| Run | (c) | (s) | (tn) | (c) | (h) | (c) |
| 1 | 55 | 10 | 5 | 1580 | 17.5 | 20 |
| 2 | 75 | 10 | 5 | 1580 | 29 | 26 |
| 3 | 55 | 25 | 5 | 1580 | 29 | 20 |
| 4 | 75 | 25 | 5 | 1580 | 17.5 | 26 |
| 5 | 55 | 10 | 10 | 1580 | 29 | 26 |
| 6 | 75 | 10 | 10 | 1580 | 17.5 | 20 |
| 7 | 55 | 25 | 10 | 1580 | 17.5 | 26 |
| 8 | 75 | 25 | 10 | 1580 | 29 | 20 |
| 9 | 55 | 10 | 5 | 1620 | 17.5 | 26 |
| 10 | 75 | 10 | 5 | 1620 | 29 | 20 |
| 11 | 55 | 25 | 5 | 1620 | 29 | 26 |
| 12 | 75 | 25 | 5 | 1620 | 17.5 | 20 |
| 13 | 55 | 10 | 10 | 1620 | 29 | 20 |
| 14 | 75 | 10 | 10 | 1620 | 17.5 | 26 |
| 15 | 55 | 25 | 10 | 1620 | 17.5 | 20 |
| 16 | 75 | 25 | 10 | 1620 | 29 | 26 |

Each run was replicated four times, and a camber measurement was taken on the substrate. The data are shown in Table P8.3.

**Table P8.3**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Camber | for | Replicate | (in/in) | Total | Mean | Standard |
| Run | 1 | 2 | 3 | 4 | (10-4 in/in) | (10-4 in/in) | Deviation |
| 1 | 0.0167 | 0.0128 | 0.0149 | 0.0185 | 629 | 157.25 | 24.418 |
| 2 | 0.0062 | 0.0066 | 0.0044 | 0.0020 | 192 | 48.00 | 20.976 |
| 3 | 0.0041 | 0.0043 | 0.0042 | 0.0050 | 176 | 44.00 | 4.083 |
| 4 | 0.0073 | 0.0081 | 0.0039 | 0.0030 | 223 | 55.75 | 25.025 |
| 5 | 0.0047 | 0.0047 | 0.0040 | 0.0089 | 223 | 55.75 | 22.410 |
| 6 | 0.0219 | 0.0258 | 0.0147 | 0.0296 | 920 | 230.00 | 63.639 |
| 7 | 0.0121 | 0.0090 | 0.0092 | 0.0086 | 389 | 97.25 | 16.029 |
| 8 | 0.0255 | 0.0250 | 0.0226 | 0.0169 | 900 | 225.00 | 39.420 |
| 9 | 0.0032 | 0.0023 | 0.0077 | 0.0069 | 201 | 50.25 | 26.725 |
| 10 | 0.0078 | 0.0158 | 0.0060 | 0.0045 | 341 | 85.25 | 50.341 |
| 11 | 0.0043 | 0.0027 | 0.0028 | 0.0028 | 126 | 31.50 | 7.681 |
| 12 | 0.0186 | 0.0137 | 0.0158 | 0.0159 | 640 | 160.00 | 20.083 |
| 13 | 0.0110 | 0.0086 | 0.0101 | 0.0158 | 455 | 113.75 | 31.120 |
| 14 | 0.0065 | 0.0109 | 0.0126 | 0.0071 | 371 | 92.75 | 29.510 |
| 15 | 0.0155 | 0.0158 | 0.0145 | 0.0145 | 603 | 150.75 | 6.750 |
| 16 | 0.0093 | 0.0124 | 0.0110 | 0.0133 | 460 | 115.00 | 17.450 |

(a) What type of design did the experimenters use?

The , a 16-run design.

(b) What are the alias relationships in this design?

The defining relation is *I*=*ABCE=ACDF=BDEF.* The aliases are shown below.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *(ABCE)=* | *BCE* | *A* | *(ACDF)=* | *CDF* | *A* | *(BDEF)=* | *ABDEF* | *A=BCE=CDF=ABDEF* |
| *B* | *(ABCE)=* | *ACE* | *B* | *(ACDF)=* | *ABCDF* | *B* | *(BDEF)=* | *DEF* | *B=ACE=ABCDF=DEF* |
| *C* | *(ABCE)=* | *ABE* | *C* | *(ACDF)=* | *ADF* | *C* | *(BDEF)=* | *BCDEF* | *C=ABE=ADF=BCDEF* |
| *D* | *(ABCE)=* | *ABCDE* | *D* | *(ACDF)=* | *ACF* | *D* | *(BDEF)=* | *BEF* | *D=ABCDE=ACF=BEF* |
| *E* | *(ABCE)=* | *ABC* | *E* | *(ACDF)=* | *ACDEF* | *E* | *(BDEF)=* | *BDF* | *E=ABC=ACDEF=BDF* |
| *F* | *(ABCE)=* | *ABCEF* | *F* | *(ACDF)=* | *ACD* | *F* | *(BDEF)=* | *BDE* | *F=ABCEF=ACD=BDE* |
| *AB* | *(ABCE)=* | *CE* | *AB* | *(ACDF)=* | *BCDF* | *AB* | *(BDEF)=* | *ADEF* | *AB=CE=BCDF=ADEF* |
| *AC* | *(ABCE)=* | *BE* | *AC* | *(ACDF)=* | *DF* | *AC* | *(BDEF)=* | *ABCDEF* | *AC=BE=DF=ABCDEF* |
| *AD* | *(ABCE)=* | *BCDE* | *AD* | *(ACDF)=* | *CF* | *AD* | *(BDEF)=* | *ABEF* | *AD=BCDE=CF=ABEF* |
| *AE* | *(ABCE)=* | *BC* | *AE* | *(ACDF)=* | *CDEF* | *AE* | *(BDEF)=* | *ABDF* | *AE=BC=CDEF=ABDF* |
| *AF* | *(ABCE)=* | *BCEF* | *AF* | *(ACDF)=* | *CD* | *AF* | *(BDEF)=* | *ABDE* | *AF=BCEF=CD=ABDE* |
| *BD* | *(ABCE)=* | *ACDE* | *BD* | *(ACDF)=* | *ABCF* | *BD* | *(BDEF)=* | *EF* | *BD=ACDE=ABCF=EF* |
| *BF* | *(ABCE)=* | *ACEF* | *BF* | *(ACDF)=* | *ABCD* | *BF* | *(BDEF)=* | *DE* | *BF=ACEF=ABCD=DE* |

(c) Do any of the process variables affect average camber?

Yes, per the analysis below, variables *A*, *C*, *E*, and *F* affect average camber.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 38.9063 6054.79 10.2962

Error B 5.78125 133.691 0.227344

Model C 56.0313 12558 21.355

Error D -14.2188 808.691 1.37519

Model E -34.4687 4752.38 8.08148

Model F -77.4688 24005.6 40.8219

Error AB 19.1563 1467.85 2.49609

Error AC 22.4063 2008.16 3.4149

Error AD -12.2188 597.191 1.01553

Error AE 18.1563 1318.6 2.24229

Error AF -19.7187 1555.32 2.64483

Error BC Aliased

Error BD 23.0313 2121.75 3.60807

Error BE Aliased

Error BF 7.40625 219.41 0.37311

Error CD Aliased

Error CE Aliased

Error CF Aliased

Error DE Aliased

Error DF Aliased

Error EF Aliased

Error ABC Aliased

Error ABD 0.53125 1.12891 0.00191972

Error ABE Aliased

Error ABF -17.3438 1203.22 2.04609

Lenth's ME 71.9361

Lenth's SME 146.041



Design Expert Output

**Response:** **Camber Avg** **in in/in**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 47370.80 4 11842.70 11.39 0.0007 significant

*A* *6054.79* *1* *6054.79* *5.82* *0.0344*

*C* *12558.00* *1* *12558.00* *12.08* *0.0052*

*E* *4752.38* *1* *4752.38* *4.57* *0.0558*

*F* *24005.63* *1* *24005.63* *23.09* *0.0005*

Residual 11435.01 11 1039.55

Cor Total 58805.81 15

The Model F-value of 11.39 implies the model is significant. There is only

a 0.07% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 32.24 R-Squared 0.8055

Mean 107.02 Adj R-Squared 0.7348

C.V. 30.13 Pred R-Squared 0.5886

PRESS 24193.08 Adeq Precision 11.478

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 107.02 1 8.06 89.27 124.76

A-Lam Temp 19.45 1 8.06 1.71 37.19 1.00

C-Lam Pres 28.02 1 8.06 10.27 45.76 1.00

E-Fire Time -17.23 1 8.06 -34.98 0.51 1.00

F-Fire DP -38.73 1 8.06 -56.48 -20.99 1.00

**Final Equation in Terms of Coded Factors:**

Camber Avg =

+107.02

+19.45 \* A

+28.02 \* C

-17.23 \* E

-38.73 \* F

**Final Equation in Terms of Actual Factors:**

Camber Avg =

+263.17380

+1.94531 \* Lam Temp

+11.20625 \* Lam Pres

-2.99728 \* Fire Time

-12.91146 \* Fire DP

(d) Do any of the process variables affect the variability in camber measurements?

Yes, *A*, *B*, *F*, and *AF* interaction affect the variability in camber measurements.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 15.9035 1011.69 27.6623

Model B -16.5773 1099.22 30.0558

Error C 5.8745 138.039 3.77437

Error D -3.2925 43.3622 1.18564

Error E -2.33725 21.851 0.597466

Model F -9.256 342.694 9.37021

Error AB 0.95525 3.65001 0.0998014

Error AC 2.524 25.4823 0.696757

Error AD -4.6265 85.618 2.34103

Error AE -0.18025 0.12996 0.00355347

Model AF -10.8745 473.019 12.9337

Error BC Aliased

Error BD -4.85575 94.3132 2.57879

Error BE Aliased

Error BF 8.21825 270.159 7.38689

Error CD Aliased

Error CE Aliased

Error CF Aliased

Error DE Aliased

Error DF Aliased

Error EF Aliased

Error ABC Aliased

Error ABD -0.68125 1.85641 0.0507593

Error ABE Aliased

Error ABF 3.39825 46.1924 1.26303

Lenth's ME 17.8392

Lenth's SME 36.2162



**Response:** **Camber StDev**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 2926.62 4 731.65 11.02 0.0008 significant

*A* *1011.69* *1* *1011.69* *15.23* *0.0025*

*B* *1099.22* *1* *1099.22* *16.55* *0.0019*

*F* *342.69* *1* *342.69* *5.16* *0.0442*

*AF* *473.02* *1* *473.02* *7.12* *0.0218*

Residual 730.65 11 66.42

Cor Total 3657.27 15

The Model F-value of 11.02 implies the model is significant. There is only

a 0.08% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 8.15 R-Squared 0.8002

Mean 25.35 Adj R-Squared 0.7276

C.V. 32.15 Pred R-Squared 0.5773

PRESS 1545.84 Adeq Precision 9.516

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 25.35 1 2.04 20.87 29.84

A-Lam Temp 7.95 1 2.04 3.47 12.44 1.00

B-Lam Time -8.29 1 2.04 -12.77 -3.80 1.00

F-Fire DP -4.63 1 2.04 -9.11 -0.14 1.00

AF -5.44 1 2.04 -9.92 -0.95 1.00

**Final Equation in Terms of Coded Factors:**

Camber StDev =

+25.35

+7.95 \* A

-8.29 \* B

-4.63 \* F

-5.44 \* A \* F

**Final Equation in Terms of Actual Factors:**

Camber StDev =

-242.46746

+4.96373 \* Lam Temp

-1.10515 \* Lam Time

+10.23804 \* Fire DP

-0.18124 \* Lam Temp \* Fire DP

(e) If it is important to reduce camber as much as possible, what recommendations would you make?







Run *A* and *C* at the low level and *E* and *F* at the high level. *B* at the low level enables a lower variation without affecting the average camber.

**8.23.** A spin coater is used to apply photoresist to a bare silicon wafer. This operation usually occurs early in the semiconductor manufacturing process, and the average coating thickness and the variability in the coating thickness has an important impact on downstream manufacturing steps. Six variables are used in the experiment. The variables and their high and low levels are as follows:

|  |  |  |
| --- | --- | --- |
| Factor | Low Level | High Level |
| Final Spin Speed | 7350 rpm | 6650 rpm |
| Acceleration Rate | 5 | 20 |
| Volume of Resist Applied | 3 cc | 5 cc |
| Time of Spin | 14 s | 6 s |
| Resist Batch Variation | Batch 1 | Batch 2 |
| Exhaust Pressure | Cover Off | Cover On |

The experimenter decides to use a 26-1 design and to make three readings on resist thickness on each test wafer. The data are shown in table P8.4.

**Table P8.4**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *A* | *B* | *C* | *D* | *E* | *F* | Resist Thickness | | | | |
| Run | Volume | Batch | Time | Speed | Acc. | Cover | Left | Center | Right | Avg. | Range |
| 1 | 5 | 2 | 14 | 7350 | 5 | Off | 4531 | 4531 | 4515 | 4525.7 | 16 |
| 2 | 5 | 1 | 6 | 7350 | 5 | Off | 4446 | 4464 | 4428 | 4446 | 36 |
| 3 | 3 | 1 | 6 | 6650 | 5 | Off | 4452 | 4490 | 4452 | 4464.7 | 38 |
| 4 | 3 | 2 | 14 | 7350 | 20 | Off | 4316 | 4328 | 4308 | 4317.3 | 20 |
| 5 | 3 | 1 | 14 | 7350 | 5 | Off | 4307 | 4295 | 4289 | 4297 | 18 |
| 6 | 5 | 1 | 6 | 6650 | 20 | Off | 4470 | 4492 | 4495 | 4485.7 | 25 |
| 7 | 3 | 1 | 6 | 7350 | 5 | On | 4496 | 4502 | 4482 | 4493.3 | 20 |
| 8 | 5 | 2 | 14 | 6650 | 20 | Off | 4542 | 4547 | 4538 | 4542.3 | 9 |
| 9 | 5 | 1 | 14 | 6650 | 5 | Off | 4621 | 4643 | 4613 | 4625.7 | 30 |
| 10 | 3 | 1 | 14 | 6650 | 5 | On | 4653 | 4670 | 4645 | 4656 | 25 |
| 11 | 3 | 2 | 14 | 6650 | 20 | On | 4480 | 4486 | 4470 | 4478.7 | 16 |
| 12 | 3 | 1 | 6 | 7350 | 20 | Off | 4221 | 4233 | 4217 | 4223.7 | 16 |
| 13 | 5 | 1 | 6 | 6650 | 5 | On | 4620 | 4641 | 4619 | 4626.7 | 22 |
| 14 | 3 | 1 | 6 | 6650 | 20 | On | 4455 | 4480 | 4466 | 4467 | 25 |
| 15 | 5 | 2 | 14 | 7350 | 20 | On | 4255 | 4288 | 4243 | 4262 | 45 |
| 16 | 5 | 2 | 6 | 7350 | 5 | On | 4490 | 4534 | 4523 | 4515.7 | 44 |
| 17 | 3 | 2 | 14 | 7350 | 5 | On | 4514 | 4551 | 4540 | 4535 | 37 |
| 18 | 3 | 1 | 14 | 6650 | 20 | Off | 4494 | 4503 | 4496 | 4497.7 | 9 |
| 19 | 5 | 2 | 6 | 7350 | 20 | Off | 4293 | 4306 | 4302 | 4300.3 | 13 |
| 20 | 3 | 2 | 6 | 7350 | 5 | Off | 4534 | 4545 | 4512 | 4530.3 | 33 |
| 21 | 5 | 1 | 14 | 6650 | 20 | On | 4460 | 4457 | 4436 | 4451 | 24 |
| 22 | 3 | 2 | 6 | 6650 | 5 | On | 4650 | 4688 | 4656 | 4664.7 | 38 |
| 23 | 5 | 1 | 14 | 7350 | 20 | Off | 4231 | 4244 | 4230 | 4235 | 14 |
| 24 | 3 | 2 | 6 | 7350 | 20 | On | 4225 | 4228 | 4208 | 4220.3 | 20 |
| 25 | 5 | 1 | 14 | 7350 | 5 | On | 4381 | 4391 | 4376 | 4382.7 | 15 |
| 26 | 3 | 2 | 6 | 6650 | 20 | Off | 4533 | 4521 | 4511 | 4521.7 | 22 |
| 27 | 3 | 1 | 14 | 7350 | 20 | On | 4194 | 4230 | 4172 | 4198.7 | 58 |
| 28 | 5 | 2 | 6 | 6650 | 5 | Off | 4666 | 4695 | 4672 | 4677.7 | 29 |
| 29 | 5 | 1 | 6 | 7350 | 20 | On | 4180 | 4213 | 4197 | 4196.7 | 33 |
| 30 | 5 | 2 | 6 | 6650 | 20 | On | 4465 | 4496 | 4463 | 4474.7 | 33 |
| 31 | 5 | 2 | 14 | 6650 | 5 | On | 4653 | 4685 | 4665 | 4667.7 | 32 |
| 32 | 3 | 2 | 14 | 6650 | 5 | Off | 4683 | 4712 | 4677 | 4690.7 | 35 |

(a) Verify that this is a 26-1 design. Discuss the alias relationships in this design.

*I*=*ABCDEF.*  This is a resolution VI design where main effects are aliased with five-factor interactions and two-factor interactions are aliased with four-factor interactions.

(b) What factors appear to affect average resist thickness?

Factors *B*, *D*, and *E* appear to affect the average resist thickness.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Error A 9.925 788.045 0.107795

Model B 73.575 43306.2 5.92378

Error C 3.375 91.125 0.0124648

Model D -207.062 342999 46.9182

Model E -182.925 267692 36.6172

Error F -5.6625 256.511 0.0350877

Error AB -9 648 0.0886387

Error AC -7.3 426.32 0.0583155

Error AD -3.8625 119.351 0.0163258

Error AE -7.1 403.28 0.0551639

Error AF -26.9875 5826.6 0.79701

Error BC 10.875 946.125 0.129419

Error BD 18.1125 2624.5 0.359001

Error BE -28.35 6429.78 0.879518

Error BF -30.2375 7314.45 1.00053

Error CD -24.9875 4995 0.683257

Error CE 8.2 537.92 0.0735811

Error CF -6.7875 368.561 0.0504148

Error DE -38.5375 11881.1 1.6252

Error DF -3.2 81.92 0.0112057

Error EF -41.1625 13554.8 1.85414

Error ABC 0.375 1.125 0.000153887

Error ABD Aliased

Error ABE 16.5 2178 0.297925

Error ABF 31.4125 7893.96 1.0798

Error ACD 15.5875 1943.76 0.265883

Error ACE Aliased

Error ACF Aliased

Error ADE 9.5375 727.711 0.0995423

Error ADF Aliased

Error AEF Aliased

Error BCD 29.0875 6768.66 0.925873

Error BCE -1.625 21.125 0.00288965

Error BCF Aliased

Error BDE -1.8875 28.5013 0.00389863

Error BDF 3.95 124.82 0.0170739

Error BEF Aliased

Error CDE Aliased

Error CDF Aliased

Error CEF 3.1375 78.7512 0.0107722

Error DEF Aliased

Lenth's ME 28.6178

Lenth's SME 54.4118



Design Expert Output

**Response:** **Thick Avg**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 6.540E+005 3 2.180E+005 79.21 < 0.0001 significant

*B* *43306.24* *1* *43306.24* *15.74* *0.0005*

*D* *3.430E+005* *1* *3.430E+005* *124.63* *< 0.0001*

*E* *2.677E+005* *1* *2.677E+005* *97.27* *< 0.0001*

Residual 77059.83 28 2752.14

Cor Total 7.311E+005 31

The Model F-value of 79.21 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 52.46 R-Squared 0.8946

Mean 4458.51 Adj R-Squared 0.8833

C.V. 1.18 Pred R-Squared 0.8623

PRESS 1.006E+005 Adeq Precision 24.993

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 4458.51 1 9.27 4439.52 4477.51

B-Batch 36.79 1 9.27 17.79 55.78 1.00

D-Speed -103.53 1 9.27 -122.53 -84.53 1.00

E-Acc -91.46 1 9.27 -110.46 -72.47 1.00

**Final Equation in Terms of Coded Factors:**

Thick Avg =

+4458.51

+36.79 \* B

-103.53 \* D

-91.46 \* E

**Final Equation in Terms of Actual Factors:**

Batch Batch 1

Thick Avg =

+6644.78750

-0.29580 \* Speed

-12.19500 \* Acc

Batch Batch 2

Thick Avg =

+6718.36250

-0.29580 \* Speed

-12.19500 \* Acc

(c) Since the volume of resist applied has little effect on average thickness, does this have any important practical implications for the process engineers?

Yes, less material could be used.

(d) Project this design into a smaller design involving only the significant factors. Graphically display the results. Does this aid in interpretation?



The cube plot usually assists the experimenter in drawing conclusions.

(e) Use the range of resist thickness as a response variable. Is there any indication that any of these factors affect the variability in resist thickness?

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A -0.625 3.125 0.0777387

Model B 2.125 36.125 0.89866

Error C -2.75 60.5 1.50502

Error D 1.625 21.125 0.525514

Model E -5.375 231.125 5.74956

Model F 7.75 480.5 11.9531

Model AB 0.625 3.125 0.0777387

Error AC -3.5 98 2.43789

Error AD -0.125 0.125 0.00310955

Error AE 1.875 28.125 0.699649

Model AF 1.75 24.5 0.609472

Error BC 0 0 0

Error BD 0.125 0.125 0.00310955

Error BE -5.375 231.125 5.74956

Model BF 3.25 84.5 2.10206

Error CD 3.75 112.5 2.79859

Error CE 3.75 112.5 2.79859

Error CF 4.875 190.125 4.72962

Error DE 5.375 231.125 5.74956

Error DF 5.5 242 6.02009

Model EF 8 512 12.7367

Error ABC Aliased

Error ABD Aliased

Error ABE 3.625 105.125 2.61513

Model ABF 9 648 16.1199

Error ACD -6.5 338 8.40822

Error ACE Aliased

Error ACF Aliased

Error ADE -3.375 91.125 2.26686

Error ADF -0.5 2 0.0497528

Error AEF 1 8 0.199011

Error BCD Aliased

Error BCE Aliased

Error BCF Aliased

Error BDE -2.625 55.125 1.37131

Error BDF -0.5 2 0.0497528

Error BEF Aliased

Error CDE Aliased

Error CDF Aliased

Error CEF 2.125 36.125 0.89866

Error DEF 2 32 0.796045

Lenth's ME 9.15104

Lenth's SME 17.3991



Design Expert Output

**Response:** **Thick Range**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 2023.00 9 224.78 2.48 0.0400 significant

*A* *3.13* *1* *3.13* *0.034* *0.8545*

*B* *36.13* *1* *36.13* *0.40* *0.5346*

*E* *231.12* *1* *231.12* *2.55* *0.1248*

*F* *480.50* *1* *480.50* *5.29* *0.0313*

*AB* *3.12* *1* *3.12* *0.034* *0.8545*

*AF* *24.50* *1* *24.50* *0.27* *0.6086*

*BF* *84.50* *1* *84.50* *0.93* *0.3451*

*EF* *512.00* *1* *512.00* *5.64* *0.0267*

*ABF* *648.00* *1* *648.00* *7.14* *0.0139*

Residual 1996.88 22 90.77

Cor Total 4019.88 31

The Model F-value of 2.48 implies the model is significant. There is only

a 4.00% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 9.53 R-Squared 0.5032

Mean 26.56 Adj R-Squared 0.3000

C.V. 35.87 Pred R-Squared -0.0510

PRESS 4224.79 Adeq Precision 5.586

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 26.56 1 1.68 23.07 30.06

A-Volume -0.31 1 1.68 -3.81 3.18 1.00

B-Batch 1.06 1 1.68 -2.43 4.56 1.00

E-Acc -2.69 1 1.68 -6.18 0.81 1.00

F-Cover 3.88 1 1.68 0.38 7.37 1.00

AB 0.31 1 1.68 -3.18 3.81 1.00

AF 0.88 1 1.68 -2.62 4.37 1.00

BF 1.63 1 1.68 -1.87 5.12 1.00

EF 4.00 1 1.68 0.51 7.49 1.00

ABF 4.50 1 1.68 1.01 7.99 1.00

**Final Equation in Terms of Coded Factors:**

Thick Range =

+26.56

-0.31 \* A

+1.06 \* B

-2.69 \* E

+3.88 \* F

+0.31 \* A \* B

+0.88 \* A \* F

+1.63 \* B \* F

+4.00 \* E \* F

+4.50 \* A \* B \* F

**Final Equation in Terms of Actual Factors:**

Batch Batch 1

Cover Off

Thick Range =

+22.39583

+3.00000 \* Volume

-0.89167 \* Acc

Batch Batch 2

Cover Off

Thick Range =

+54.77083

-5.37500 \* Volume

-0.89167 \* Acc

Batch Batch 1

Cover On

Thick Range =

+42.56250

-4.25000 \* Volume

+0.17500 \* Acc

Batch Batch 2

Cover On

Thick Range =

+9.43750

+5.37500 \* Volume

+0.17500 \* Acc

The model for thickness range is not very strong. Notice the small value of *R*2, and in particular, the adjusted *R*2. Often we find that obtaining a good model for a response that expresses variability is not as easy as finding a satisfactory model for a response that measures the mean.

(f) Where would you recommend that the process engineers run the process?

Considering only the average thickness results, the engineers could use factors *B, D* and *E* to put the process mean at target. Then the engineer could consider the other factors on the range model to try to set the factors to reduce the variation in thickness at that mean.

**8.24S.** Harry Peterson-Nedry (a friend of the author) owns a vineyard and winery in Newberg, Oregon. They grow several varieties of grapes and manufacture wine. Harry and Judy have used factorial designs for process and product development in the winemaking segment of their business. This problem describes the experiment conducted for their 1985 Pinot Noir. Eight variables, shown in Table P8.5, were originally studied in this experiment:

**Table P8.5**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Variable | Low Level | High Level |
| *A* | Pinot Noir Clone | Pommard | Wadenswil |
| *B* | Oak Type | Allier | Troncais |
| *C* | Age of Barrel | Old | New |
| *D* | Yeast/Skin Contact | Champagne | Montrachet |
| *E* | Stems | None | All |
| *F* | Barrel Toast | Light | Medium |
| *G* | Whole Cluster | None | 10% |
| *H* | Fermentation Temperature | Low (75 F Max) | High (92 F Max) |

Harry decided to use a design with 16 runs. The wine was taste-tested by a panel of experts on 8 March 1986. Each expert ranked the 16 samples of wine tasted, with rank 1 being the best. The design and taste-test panel results are shown in Table P8.6.

**Table P8.6**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | HPN | JPN | CAL | DCM | RGB | *y*bar | *s* |
| 1 | - | - | - | - | - | - | - | - | 12 | 6 | 13 | 10 | 7 | 9.6 | 3.05 |
| 2 | + | - | - | - | - | + | + | + | 10 | 7 | 14 | 14 | 9 | 10.8 | 3.11 |
| 3 | - | + | - | - | + | - | + | + | 14 | 13 | 10 | 11 | 15 | 12.6 | 2.07 |
| 4 | + | + | - | - | + | + | - | - | 9 | 9 | 7 | 9 | 12 | 9.2 | 1.79 |
| 5 | - | - | + | - | + | + | + | - | 8 | 8 | 11 | 8 | 10 | 9.0 | 1.41 |
| 6 | + | - | + | - | + | - | - | + | 16 | 12 | 15 | 16 | 16 | 15.0 | 1.73 |
| 7 | - | + | + | - | - | + | - | + | 6 | 5 | 6 | 5 | 3 | 5.0 | 1.22 |
| 8 | + | + | + | - | - | - | + | - | 15 | 16 | 16 | 15 | 14 | 15.2 | 0.84 |
| 9 | - | - | - | + | + | + | - | + | 1 | 2 | 3 | 3 | 2 | 2.2 | 0.84 |
| 10 | + | - | - | + | + | - | + | - | 7 | 11 | 4 | 7 | 6 | 7.0 | 2.55 |
| 11 | - | + | - | + | - | + | + | - | 13 | 3 | 8 | 12 | 8 | 8.8 | 3.96 |
| 12 | + | + | - | + | - | - | - | + | 3 | 1 | 5 | 1 | 4 | 2.8 | 1.79 |
| 13 | - | - | + | + | - | - | + | + | 2 | 10 | 2 | 4 | 5 | 4.6 | 3.29 |
| 14 | + | - | + | + | - | + | - | - | 4 | 4 | 1 | 2 | 1 | 2.4 | 1.52 |
| 15 | - | + | + | + | + | - | - | - | 5 | 15 | 9 | 6 | 11 | 9.2 | 4.02 |
| 16 | + | + | + | + | + | + | + | + | 11 | 14 | 12 | 13 | 13 | 12.6 | 1.14 |

(a) What are the alias relationships in the design selected by Harry?

*E = BCD, F = ACD, G = ABC, H = ABD*

Defining Contrast :  *I = BCDE = ACDF = ABEF = ABCG = ADEG = BDFG = CEFG = ABDH*

*= ACEH = BCFH = DEFH = CDGH = BEGH = AFGH = ABCDEFGH*

Aliases:

*A = BCG = BDH = BEF = CDF = CEH = DEG = FGH*

*B = ACG = ADH = AEF = CDE = CFH = DFG = EGH*

*C = ABG = ADF = AEH = BDE = BFH = DGH = EFG*

*D = ABH = ACF = AEG = BCE = BFG = CGH = EFH*

*E = ABF = ACH = ADG = BCD = BGH = CFG = DFH*

*F = ABE = ACD = AGH = BCH = BDG = CEG = DEH*

*G = ABC = ADE = AFH = BDF = BEH = CDH = CEF*

*H = ABD = ACE = AFG = BCF = BEG = CDG = DEF*

*AB = CG = DH = EF*

*AC = BG = DF = EH*

*AD = BH = CF = EG*

*AE = BF = CH = DG*

*AF = BE = CD = GH*

*AG = BC = DE = FH*

*AH = BD = CE = FG*

(b) Use the average ranks () as a response variable. Analyze the data and draw conclusions. You will find it helpful to examine a normal probability plot of effect estimates.

The effects list and normal probability plot of effects are shown below. Factors *D*, *E*, *F*, and *G* appear to be significant. Also note that the *DF* and *FG* interactions were chosen instead of *AC* and *AH* based on the alias structure shown above.

Design Expert Output

Term Effect SumSqr % Contribtn

Require Intercept

Error A 1.75 12.25 4.57636

Error B 1.85 13.69 5.11432

Error C 1.25 6.25 2.33488

Model D -4.6 84.64 31.6198

Model E 2.2 19.36 7.23252

Model F -2 16 5.97729

Model G 3.15 39.69 14.8274

Error H -0.6 1.44 0.537956

Error AB -0.7 1.96 0.732218

Ignore AC Aliased

Error AD -1.75 12.25 4.57636

Error AE 0.95 3.61 1.34863

Error AF 0.75 2.25 0.840556

Error AG 0.9 3.24 1.2104

Ignore AH Aliased

Model DF 2.6 27.04 10.1016

Model FG 2.45 24.01 8.96967

Lenth's ME 6.74778

Lenth's SME 13.699



Design Expert Output

**Response:** **Taste Avg**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 210.74 6 35.12 5.55 0.0115 significant

*D 84.64 1 84.64 13.38 0.0053*

*E 19.36 1 19.36 3.06 0.1142*

*F 16.00 1 16.00 2.53 0.1462*

*G 39.69 1 39.69 6.27 0.0336*

*DF 27.04 1 27.04 4.27 0.0687*

*FG 24.01 1 24.01 3.80 0.0832*

Residual 56.94 9 6.33

Cor Total 267.68 15

The Model F-value of 5.55 implies the model is significant. There is only

a 1.15% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 2.52 R-Squared 0.7873

Mean 8.50 Adj R-Squared 0.6455

C.V. 29.59 Pred R-Squared 0.3277

PRESS 179.96 Adeq Precision 7.183

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 8.50 1 0.63 7.08 9.92

D-D -2.30 1 0.63 -3.72 -0.88 1.00

E-E 1.10 1 0.63 -0.32 2.52 1.00

F-F -1.00 1 0.63 -2.42 0.42 1.00

G-G 1.57 1 0.63 0.15 3.00 1.00

DF 1.30 1 0.63 -0.12 2.72 1.00

FG 1.23 1 0.63 -0.20 2.65 1.00

**Final Equation in Terms of Coded Factors:**

Taste Avg =

+8.50

-2.30 \* D

+1.10 \* E

-1.00 \* F

+1.57 \* G

+1.30 \* D \* F

+1.23 \* F \* G

Factors *D* and *G* and are important. Factor *E* and the *DF* and *FG* interactions are moderately important and were included in the model because the PRESS statistic showed improvement with their inclusion. Factor *F* is added to the model to preserve hierarchy. As stated earlier, the interactions are aliased with other two-factor interactions that could also be important. So the interpretation of the two-factor interaction is somewhat uncertain. Normally, we would add runs to the design to isolate the significant interactions, but that will not work very well here because each experiment requires a full growing season. In other words, it would require a very long time to add runs to de-alias the alias chains of interest.





(c) Use the standard deviation of the ranks (or some appropriate transformation such as log *s*) as a response variable. What conclusions can you draw about the effects of the eight variables on variability in wine quality?



There do not appear to be any significant factors.

(d) After looking at the results, Harry decides that one of the panel members (DCM) knows more about beer than he does about wine, so they decide to delete his ranking. What affect would this have on the results and on conclusions from parts (b) and (c)?

Design Expert Output

Term Effect SumSqr % Contribtn

Require Intercept

Error A 1.625 10.5625 4.02957

Error B 2.0625 17.0156 6.49142

Error C 1.5 9 3.43348

Model D -4.5 81 30.9013

Model E 2.4375 23.7656 9.06652

Model F -2.375 22.5625 8.60753

Model G 2.9375 34.5156 13.1676

Error H -0.6875 1.89063 0.721268

Error AB -0.5625 1.26563 0.482833

Ignore AC Aliased

Error AD -1.5 9 3.43348

Error AE 0.6875 1.89063 0.721268

Error AF 0.875 3.0625 1.16834

Error AG 0.8125 2.64062 1.00739

Ignore AH Aliased

Model DF 2.375 22.5625 8.60753

Model FG 2.3125 21.3906 8.16047

Lenth's ME 6.26579

Lenth's SME 12.7205



Design Expert Output

**Response:** **Taste Avg**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 205.80 6 34.30 5.48 0.0120 significant

*D 81.00 1 81.00 12.94 0.0058*

*E 23.77 1 23.77 3.80 0.0831*

*F 22.56 1 22.56 3.60 0.0901*

*G 34.52 1 34.52 5.51 0.0434*

*DF 22.56 1 22.56 3.60 0.0901*

*FG 21.39 1 21.39 3.42 0.0975*

Residual 56.33 9 6.26

Cor Total 262.13 15

The Model F-value of 5.48 implies the model is significant. There is only

a 1.20% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 2.50 R-Squared 0.7851

Mean 8.50 Adj R-Squared 0.6418

C.V. 29.43 Pred R-Squared 0.3208

PRESS 178.02 Adeq Precision 7.403

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 8.50 1 0.63 7.09 9.91

D-D -2.25 1 0.63 -3.66 -0.84 1.00

E-E 1.22 1 0.63 -0.20 2.63 1.00

F-F -1.19 1 0.63 -2.60 0.23 1.00

G-G 1.47 1 0.63 0.054 2.88 1.00

DF 1.19 1 0.63 -0.23 2.60 1.00

FG 1.16 1 0.63 -0.26 2.57 1.00

**Final Equation in Terms of Coded Factors:**

Taste Avg =

+8.50

-2.25 \* D

+1.22 \* E

-1.19 \* F

+1.47 \* G

+1.19 \* D \* F

+1.16 \* F \* G

The results are very similar for average taste without DCM as they were with DCM.



The standard deviation response is much the same with or without DCM’s responses. Again, there are no significant factors.

(e) Suppose that just before the start of the experiment, Harry discovered that the eight new barrels they ordered from France for use in the experiment would not arrive in time, and all 16 runs would have to be made with old barrels. If Harry just drops column C from their design, what does this do to the alias relationships? Do they need to start over and construct a new design?

The resulting design is a , with defining relations: *I* = *ABEF = ADEG = BDFG = ABDH = DEFH =* *BEGH = AFGH*.

(f) Harry knows from experience that some treatment combinations are unlikely to produce good results. For example, the run with all eight variables at the high level generally results in a poorly rated wine. This was confirmed in the 8 March 1986 taste test. He wants to set up a new design for their 1986 Pinot Noir using these same eight variables, but they do not want to make the run with all eight factors at the high level. What design would you suggest?

By changing the sign of any of the design generators, a design that does not include the principal fraction will be generated. This will give a design without an experimental run combination with all of the variables at the high level.

**8.25.** Consider the 25 factorial in Problem 6.35. Suppose that the experimenters could only afford 16 runs. Set up the 25-1 fractional factorial design with *I=ABCDE* and select the responses from the full factorial data in Problem 6.35.

The runs for the 25-1 fractional factorial design with *I=ABCDE* are shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | C | D | E | y |
| -1 | -1 | -1 | -1 | 1 | 7.93 |
| 1 | -1 | -1 | -1 | -1 | 5.56 |
| -1 | 1 | -1 | -1 | -1 | 5.77 |
| 1 | 1 | -1 | -1 | 1 | 12 |
| -1 | -1 | 1 | -1 | -1 | 9.17 |
| 1 | -1 | 1 | -1 | 1 | 3.65 |
| -1 | 1 | 1 | -1 | 1 | 6.4 |
| 1 | 1 | 1 | -1 | -1 | 5.69 |
| -1 | -1 | -1 | 1 | -1 | 8.82 |
| 1 | -1 | -1 | 1 | 1 | 17.55 |
| -1 | 1 | -1 | 1 | 1 | 8.87 |
| 1 | 1 | -1 | 1 | -1 | 8.94 |
| -1 | -1 | 1 | 1 | 1 | 13.06 |
| 1 | -1 | 1 | 1 | -1 | 11.49 |
| -1 | 1 | 1 | 1 | -1 | 6.25 |
| 1 | 1 | 1 | 1 | 1 | 26.05 |

(a) Analyze the data and draw conclusions.

The analysis which identifies the significant effects is shown below.



Design Expert Output

**Response 1 y  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 460.75 12 38.40 92.45 0.0016 significant

*A-A* *38.01* *1* *38.01* *91.51* *0.0024*  
  *B-B* *0.47* *1* *0.47* *1.13* *0.3658*  
  *C-C* *2.50* *1* *2.50* *6.01* *0.0915*  
  *D-D* *125.78* *1* *125.78* *302.84* *0.0004*  
  *E-E* *71.49* *1* *71.49* *172.13* *0.0010*  
  *AB* *42.64* *1* *42.64* *102.67* *0.0020*  
  *AD* *54.02* *1* *54.02* *130.08* *0.0014*  
  *AE* *28.41* *1* *28.41* *68.40* *0.0037*  
  *BC* *7.98* *1* *7.98* *19.22* *0.0220*  
  *BE* *23.81* *1* *23.81* *57.34* *0.0048*  
  *CD* *22.61* *1* *22.61* *54.44* *0.0051*  
  *DE* *43.03* *1* *43.03* *103.62* *0.0020*  
 Residual 1.25 3 0.42  
 Cor Total 461.99 15

(b) Compare your findings with those from the full factorial in Problem 6.35.

The results are similar to those found in Problem 6.35 with the exception that the *BC* and *CD* interactions are now significant, and factor *C* remains in the model because of these interactions.

(c) Are there any potential interactions that need further study? What additional runs do you recommend? Select these runs from the full factorial design in Problem 6.35 and analyze the new design. Discuss your conclusions.

The *BC* interaction is aliased with the *ADE* interaction, and the *CD* interaction is aliased with the *ABE* interaction. The *ADE* and *ABE* interactions in Problem 6.43 were both found to be significant. Including these effects would also allow us to remove the insignificant factor *C* from the model.

The additional runs are shown below as block 2. This is a partial fold-over where factor A was chosen for the fold-over, and factor *C* at the -1 level was chosen for the runs. This allowed both the *ABE* and *ADE* interactions to be de-aliased.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Block | *A* | *B* | *C* | *D* | *E* | *y* |
| 1 | -1 | -1 | -1 | -1 | 1 | 7.93 |
| 1 | 1 | -1 | -1 | -1 | -1 | 5.56 |
| 1 | -1 | 1 | -1 | -1 | -1 | 5.77 |
| 1 | 1 | 1 | -1 | -1 | 1 | 12 |
| 1 | -1 | -1 | 1 | -1 | -1 | 9.17 |
| 1 | 1 | -1 | 1 | -1 | 1 | 3.65 |
| 1 | -1 | 1 | 1 | -1 | 1 | 6.4 |
| 1 | 1 | 1 | 1 | -1 | -1 | 5.69 |
| 1 | -1 | -1 | -1 | 1 | -1 | 8.82 |
| 1 | 1 | -1 | -1 | 1 | 1 | 17.55 |
| 1 | -1 | 1 | -1 | 1 | 1 | 8.87 |
| 1 | 1 | 1 | -1 | 1 | -1 | 8.94 |
| 1 | -1 | -1 | 1 | 1 | 1 | 13.06 |
| 1 | 1 | -1 | 1 | 1 | -1 | 11.49 |
| 1 | -1 | 1 | 1 | 1 | -1 | 6.25 |
| 1 | 1 | 1 | 1 | 1 | 1 | 26.05 |
| 2 | 1 | -1 | -1 | -1 | 1 | 5 |
| 2 | -1 | -1 | -1 | -1 | -1 | 8.11 |
| 2 | 1 | 1 | -1 | -1 | -1 | 5.82 |
| 2 | -1 | 1 | -1 | -1 | 1 | 7.47 |
| 2 | 1 | -1 | -1 | 1 | -1 | 14.23 |
| 2 | -1 | -1 | -1 | 1 | 1 | 12.43 |
| 2 | 1 | 1 | -1 | 1 | 1 | 25.38 |
| 2 | -1 | 1 | -1 | 1 | -1 | 9.2 |



Design Expert Output

**Response 1 y  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Block 6.81 1 6.81

Model 764.14 15 50.94 89.46 < 0.0001 significant  
  *A-A* *51.83* *1* *51.83* *91.03* *< 0.0001*  
  *B-B* *0.47* *1* *0.47* *0.82* *0.3942*  
  *C-C* *2.50* *1* *2.50* *4.38* *0.0746*  
  *D-D* *178.72* *1* *178.72* *313.85* *< 0.0001*  
  *E-E* *91.03* *1* *91.03* *159.85* *< 0.0001*  
  *AB* *42.64* *1* *42.64* *74.88* *< 0.0001*  
  *AC* *0.074* *1* *0.074* *0.13* *0.7298*  
  *AD* *67.88* *1* *67.88* *119.20* *< 0.0001*  
  *AE* *35.19* *1* *35.19* *61.79* *0.0001*  
  *BC* *0.073* *1* *0.073* *0.13* *0.7310*  
  *BE* *29.02* *1* *29.02* *50.96* *0.0002*  
  *DE* *15.80* *1* *15.80* *27.75* *0.0012*  
  *ABC* *6.68* *1* *6.68* *11.73* *0.0111*  
  *ABE* *34.92* *1* *34.92* *61.32* *0.0001*  
  *ADE* *9.58* *1* *9.58* *16.82* *0.0046*  
 Residual 3.99 7 0.57  
 Cor Total 774.93 23

**8.26.** Consider the 24 factorial experiment for surfactin production in Problem 6.36. Suppose that the experimenters could only afford eight runs. Set up the 24-1 fractional factorial with I *= ABCD* and select the responses for the runs from the full factorial data in Problem 6.36. Analyze and draw conclusions. Compare your findings with those from the full factorial in Problem 6.36.

The runs for the 24-1 fractional factorial with I *= ABCD* are shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run | Glucose (g dm-3) | NH4NO3 (g dm-3) | FeSO4 (g dm-3 x 10-4) | MnSO4 (g dm-3 x 10-2) | y (CMC)-1 |
| 1 | 20.00 | 2.00 | 6.00 | 4.00 | 23 |
| 2 | 60.00 | 2.00 | 6.00 | 20.00 | 16 |
| 3 | 20.00 | 6.00 | 6.00 | 20.00 | 18 |
| 4 | 60.00 | 6.00 | 6.00 | 4.00 | 18 |
| 5 | 20.00 | 2.00 | 30.00 | 20.00 | 36 |
| 6 | 60.00 | 2.00 | 30.00 | 4.00 | 16 |
| 7 | 20.00 | 6.00 | 30.00 | 4.00 | 17 |
| 8 | 60.00 | 6.00 | 30.00 | 20.00 | 34 |

(a) Analyze the data and draw conclusions.

The analysis below identifies the factors *C* and *D*, and the *AB* interaction as being significant.



Design Expert Output

**Response 1 y  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 467.00 5 93.40 74.72 0.0133 significant

*A-Glucose* *12.50* *1* *12.50* *10.00* *0.0871*  
  *B-NH4NO3* *2.00* *1* *2.00* *1.60* *0.3333*  
  *C-FeSO4* *98.00* *1* *98.00* *78.40* *0.0125*  
  *D-MnSO4* *112.50* *1* *112.50* *90.00* *0.0109*  
  *AB* *242.00* *1* *242.00* *193.60* *0.0051*  
 Residual 2.50 2 1.25  
 Cor Total 469.50 7

(b) Compare your findings with those from the full factorial in Problem 6.36.

In this 24-1 fractional factorial with *I = ABCD*, the *AB* interaction is aliased with the *CD* interaction. The full factorial experiment in Problem 6.44 found the *CD* interaction to be significant. Additional runs, such as a fold-over, should be run to de-alias the *CD* from the *AB* interaction.

**8.27.** Consider the 24 factorial experiment in Problem 6.38. Suppose that the experimenters could only afford eight runs. Set up the 24-1 fractional factorial design with *I = ABCD* and select the responses from the full factorial data in Problem 6.38.

Problem 6.38 was originally analyzed as a replicated 23 full factorial experiment. The purpose for this experiment was to compare the fishbone apatite with the hydroxyapatite. However, as shown in the solutions for this problem, the experimenters might have observed better results had they analyzed the experiment as a replicated 24 full factorial experiment. This analysis is shown in the solutions for part (i) of Problem 6.38.

The solution presented here is for the replicated 24-1 2 fractional factorial experiment with *I = ABCD* as shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Apatite | pH | Pb | Type | Pb,mM | pH |
| + | + | + | Hydroxyapatite | 0.11 | 3.49 |
| + | + | + | Hydroxyapatite | 0.12 | 3.46 |
| + | + | – | Fishbone | 0.01 | 6.84 |
| + | + | – | Fishbone | 0 | 6.61 |
| + | – | + | Fishbone | 1.11 | 3.35 |
| + | – | + | Fishbone | 1.04 | 3.34 |
| + | – | – | Hydroxyapatite | 0.03 | 3.36 |
| + | – | – | Hydroxyapatite | 0.05 | 3.24 |
| – | + | + | Fishbone | 2.11 | 5.29 |
| – | + | + | Fishbone | 2.18 | 5.06 |
| – | + | – | Hydroxyapatite | 0 | 5.53 |
| – | + | – | Hydroxyapatite | 0 | 5.43 |
| – | – | + | Hydroxyapatite | 1.34 | 2.82 |
| – | – | + | Hydroxyapatite | 1.26 | 2.79 |
| – | – | – | Fishbone | 0.05 | 4.5 |
| – | – | – | Fishbone | 0.05 | 4.74 |

1. Analyze the data for all of the responses and draw conclusions.

Due to non-constant variance observed in the residuals plots, the same transformations that were used in the solution of Problem 6.38 part (i) were also used here.

The analysis for the Pb Response identifies all for main effects and the *AB*, *AC*, and *AD* interactions as being significant. However, the *AB* interaction is aliased with *CD*, *AC* is aliased with *BD*, and *AD* is aliased with *BC.*

Design Expert Output

**Response 1 Pb Response**

**Transform: Power Lambda: 0.7 Constant: 0  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 6.17 7 0.88 1465.63 < 0.0001 significant

*A-Apatite* *0.67* *1* *0.67* *1109.89* *< 0.0001*  
  *B-pH* *0.071* *1* *0.071* *118.74* *< 0.0001*  
  *C-Pb* *3.87* *1* *3.87* *6425.51* *< 0.0001*  
  *D-Type* *0.47* *1* *0.47* *785.36* *< 0.0001*  
  *AB* *0.42* *1* *0.42* *700.14* *< 0.0001*  
  *AC* *0.67* *1* *0.67* *1113.81* *< 0.0001*  
  *AD* *3.600E-003* *1* *3.600E-003* *5.98* *0.0402*  
 Pure Error 4.813E-003 8 6.016E-004  
 Cor Total 6.18 15

There are no concerns with the residual plots shown below.













The analysis for the pH Response identifies all for main effects and the *AD* interaction as being significant. However, the *AD* interaction is aliased with *BC*.

Design Expert Output

**Response 2 pH Response**

**Transform: Inverse Sqrt Constant: 0  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 0.077 7 0.011 421.02 < 0.0001 significant

*A-Apatite* *2.030E-003* *1* *2.030E-003* *77.93* *< 0.0001*  
  *B-pH* *0.034* *1* *0.034* *1319.61* *< 0.0001*  
  *C-Pb* *0.021* *1* *0.021* *813.91* *< 0.0001*  
  *D-Type* *0.019* *1* *0.019* *719.62* *< 0.0001*  
  *AB* *1.038E-004* *1* *1.038E-004* *3.98* *0.0810*  
  *AC* *1.961E-006* *1* *1.961E-006* *0.075* *0.7907*  
  *AD* *3.119E-004* *1* *3.119E-004* *11.97* *0.0086*  
 Pure Error 2.084E-004 8 2.605E-005  
 Cor Total 0.077 15

There are no concerns with the residual plots shown below.










1. Compare your findings with those from the full factorial in Problem 6.38.

In Problem 6.38, part (i), the main effects, and several two and three factor interactions were found to be very significant. In the 24-1 fractional factorial experiment in this problem, the main effects are aliased with the three factor interactions, and the two factor interactions are aliased with each other. Based on the results of Problem 6.38, part (i), these aliases are a significant concern.

**8.28.** An article in the *Journal of Chromatography A* (“Simultaneous Supercritical Fluid Derivatization and Extraction of Formaldehyde by the Hantzsch Reaction,” 2000, Vol. 896, pp. 51-59) describes an experiment where the Hantzsch reaction is used to produce the chemical derivatization of formaldehyde in a supercritical medium. Pressure, temperature, and other parameters such as static and dynamic extraction time must be optimized to increase the yield of this kinetically controlled reaction. A 25-1 fractional factorial design with one center run was used to study the significant parameters affecting the supercritical process in terms of resolution and sensitivity. Ultraviolet-visible spectrophotometry was used as the detection technique. The experimental design and the responses are shown in the table below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | *P* | *T* | *s* | *d* | *c* |  |  |
| Experiment | (MPa) | (°C) | (min) | (min) | (μl) | Resolution | Sensitivity |
| 1 | 13.8 | 50 | 2 | 2 | 100 | 0.00025 | 0.057 |
| 2 | 55.1 | 50 | 2 | 2 | 10 | 0.33333 | 0.094 |
| 3 | 13.8 | 120 | 2 | 2 | 10 | 0.02857 | 0.017 |
| 4 | 55.1 | 120 | 2 | 2 | 100 | 0.20362 | 1.561 |
| 5 | 13.8 | 50 | 15 | 2 | 10 | 0.00027 | 0.010 |
| 6 | 55.1 | 50 | 15 | 2 | 100 | 0.52632 | 0.673 |
| 7 | 13.8 | 120 | 15 | 2 | 100 | 0.00026 | 0.028 |
| 8 | 55.1 | 120 | 15 | 2 | 10 | 0.52632 | 1.144 |
| 9 | 13.8 | 50 | 2 | 15 | 10 | 0.42568 | 0.142 |
| 10 | 55.1 | 50 | 2 | 15 | 100 | 0.60150 | 0.399 |
| 11 | 13.8 | 120 | 2 | 15 | 100 | 0.06098 | 0.767 |
| 12 | 55.1 | 120 | 2 | 15 | 10 | 0.74165 | 1.086 |
| 13 | 13.8 | 50 | 15 | 15 | 100 | 0.08780 | 0.252 |
| 14 | 55.1 | 50 | 15 | 15 | 10 | 0.40000 | 0.379 |
| 15 | 13.8 | 120 | 15 | 15 | 10 | 0.00026 | 0.028 |
| 16 | 55.1 | 120 | 15 | 15 | 100 | 0.28091 | 3.105 |
| Central | 34.5 | 85 | 8.5 | 8.5 | 55 | 0.75000 | 1.836 |

(a) Analyze the data from this experiment and draw conclusions.

The first analysis shown below is for the Resolution response. Factor *A* and the *CD* interaction are significant with factor *D* being moderately significant. Due to hierarchy, factor *C* is also included in the model. Curvature is also significant.



Design Expert Output

**Response 1 Resolution  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 0.80 4 0.20 15.37 0.0002 significant  
  *A-P* *0.57* *1* *0.57* *43.70* *< 0.0001*  
  *C-s* *0.021* *1* *0.021* *1.59* *0.2339*  
  *D-d* *0.060* *1* *0.060* *4.63* *0.0544*  
  *CD* *0.15* *1* *0.15* *11.56* *0.0059*  
 Curvature 0.22 1 0.22 17.19 0.0016 significant  
 Residual 0.14 11 0.013  
 Cor Total 1.16 16

The following analysis is for the Sensitivity response. Factors *A*, *B*, and the *AB* interaction are significant, with factor *E* as being moderately significant. Curvature is also significant.



Design Expert Output

**Response 1 Sensitivity  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 7.75 4 1.94 9.19 0.0016 significant  
  *A-P* *3.19* *1* *3.19* *15.10* *0.0025*  
  *B-T* *2.05* *1* *2.05* *9.72* *0.0098*  
  *E-c* *0.97* *1* *0.97* *4.60* *0.0551*  
  *AB* *1.55* *1* *1.55* *7.32* *0.0204*  
 Curvature 1.42 1 1.42 6.72 0.0251 significant  
 Residual 2.32 11 0.21  
 Cor Total 11.49 16

(b) Are there any concerns about model adequacy or violations of the assumptions?

The first set of plots shown below are the residuals for the Resolution response. There does not appear to be any concerns with these residual plots.















Residual plots for the Sensitivity response are shown below. There appears to be an outlier with the 3.105 Sensitivity value.















(c) Does the single center point cause any concerns about curvature or the possible need for second-order terms?

Yes, for both Resolution and Sensitivity, there appears to be curvature. Additional runs, such as the axial points for a central composite design, along with additional center points for blocking, could be run to identify the second-order terms.

(d) Do you think that running one center point was a good choice in this design?

It was a good idea to run a center point; however, additional center points would have been an improvement, especially if there was potential need to perform additional runs in a second block to identify the second-order terms.

**8.29.** An article in *Soldering & Surface Mount Technology* (“Characterization of Solder Paste Printing Process and Its Optimization,” 1999, Vol.11, No. 3, pp. 23-26) describes the use of a 28-3 fractional factorial experiment to study the effect of eight factors on two responses: percentage volume matching (PVM) – the ration of the actual printed solder paste volume to the designed volume; and non-conformities per unit (NPU) – the number of solder paste printing defects determined by visual inspection (20’ magnification) after printing according to an industry workmanship standard. The factor levels are shown below and the test matrix and response data are shown in Table 8.9.

|  |  |  |
| --- | --- | --- |
|  | Levels | |
| Parameters | (–) | (+) |
| A. Squeegee pressure, *MPa* | 0.1 | 0.3 |
| B. Printing speed, *mm/s* | 24 | 32 |
| C. Squeegee angle, *deg* | 45 | 65 |
| D. Temperature, °*C* | 20 | 28 |
| E. Viscosity, *kCps* | 1,100-1,150 | 1,250-1,300 |
| F. Cleaning interval, *stroke* | 8 | 15 |
| G. Separation speed, *mm/s* | 0.4 | 0.8 |
| H. Relative humidity, *%* | 30 | 70 |

**Table P8.9 –** The Solder Paste Experiment

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | Parameters | | | | | | | |  |  |  |
| Order | A | B | C | D | E | F | G | H |  | PVM | NPU (%) |
| 4 | – | – | – | – | – | – | – | + |  | 1.00 | 5 |
| 13 | + | – | – | – | – | + | + | + |  | 1.04 | 13 |
| 6 | – | + | – | – | – | + | + | – |  | 1.02 | 16 |
| 3 | + | + | – | – | – | – | – | – |  | 0.99 | 12 |
| 19 | – | – | + | – | – | + | – | – |  | 1.02 | 15 |
| 25 | + | – | + | – | – | – | + | – |  | 1.01 | 9 |
| 21 | – | + | + | – | – | – | + | + |  | 1.01 | 12 |
| 14 | + | + | + | – | – | + | – | + |  | 1.03 | 17 |
| 10 | – | – | – | + | – | – | + | – |  | 1.04 | 21 |
| 22 | + | – | – | + | – | + | – | – |  | 1.14 | 20 |
| 1 | – | + | – | + | – | + | – | + |  | 1.20 | 25 |
| 2 | + | + | – | + | – | – | + | + |  | 1.13 | 21 |
| 30 | – | – | + | + | – | + | + | + |  | 1.14 | 25 |
| 8 | + | – | + | + | – | – | – | + |  | 1.07 | 13 |
| 9 | – | + | + | + | – | – | – | – |  | 1.06 | 20 |
| 20 | + | + | + | + | – | + | + | – |  | 1.13 | 26 |
| 17 | – | – | – | – | + | – | – | – |  | 1.02 | 10 |
| 18 | + | – | – | – | + | + | + | – |  | 1.10 | 13 |
| 5 | – | + | – | – | + | + | + | + |  | 1.09 | 17 |
| 26 | + | + | – | – | + | – | – | + |  | 0.96 | 13 |
| 31 | – | – | + | – | + | + | – | + |  | 1.02 | 14 |
| 11 | + | – | + | – | + | – | + | + |  | 1.07 | 11 |
| 29 | – | + | + | – | + | – | + | – |  | 0.98 | 10 |
| 23 | + | + | + | – | + | + | – | – |  | 0.95 | 14 |
| 32 | – | – | – | + | + | – | + | + |  | 1.10 | 28 |
| 7 | + | – | – | + | + | + | – | + |  | 1.12 | 24 |
| 15 | – | + | – | + | + | + | – | – |  | 1.19 | 22 |
| 27 | + | + | – | + | + | – | + | – |  | 1.13 | 15 |
| 12 | – | – | + | + | + | + | + | – |  | 1.20 | 21 |
| 28 | + | – | + | + | + | – | – | – |  | 1.07 | 19 |
| 24 | – | + | + | + | + | – | – | + |  | 1.12 | 21 |
| 16 | + | + | + | + | + | + | + | + |  | 1.21 | 27 |

(a) Verify that the generators are *I = ABCF*, *I = ABDG*, and *I = BCDEH* for this design.

This can be done by recognizing that Column *F* = *ABC*, Column *G* = *ABD*, and Column *H* = *BCDE*.

(b) What are the aliases for the main effects and two-factor interactions? You can ignore all interactions order three and higher.

The full defining relationship is: I + ABCF + ABDG + CDFG + ADEFH + ACEGH + BCDEH; and the alias structure for the main effects and two-factor interactions are:

A + BCF + BDG + CEGH + DEFH

B + ACF + ADG + CDEH + EFGH

C + ABF + DFG + AEGH + BDEH

D + ABG + CFG + AEFH + BCEH

E + ACGH + ADFH + BCDH + BFGH

F + ABC + CDG + ADEH + BEGH

G + ABD + CDF + ACEH + BEFH

H + ACEG + ADEF + BCDE + BEFG

AB + CF + DG

AC + BF + EGH + ADFG + BCDG

AD + BG + EFH + ACFG + BCDF

AE + CGH + DFH + BCEF + BDEG

AF + BC + DEH + ACDG + BDFG

AG + BD + CEH + ACDF + BCFG

AH + CEG + DEF + BCFH + BDGH

BE + CDH + FGH + ACEF + ADEG

BH + CDE + EFG + ACFH + ADGH

CD + FG + BEH + ABCG + ABDF

CE + AGH + BDH + ABEF + DEFG

CG + DF + AEH + ABCD + ABFG

CH + AEG + BDE + ABFH + DFGH

DE + AFH + BCH + ABEG + CEFG

DH + AEF + BCE + ABGH + CFGH

EF + ADH + BGH + ABCE + CDEG

EG + ACH + BFH + ABDE + CDEF

EH + ACG + ADF + BCD + BFG

FH + ADE + BEG + ABCH + CDGH

GH + ACE + BEF + ABDH + CDFH

(c) Analyze both the PVM and NPU responses.

For both the PVM and NPU responses, only factors D and F appear to be significant.

The PVM analysis is shown below.



Design Expert Output

**Response 1 PVM  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 0.12 2 0.058 37.52 < 0.0001 significant

*D-Temperature* *0.095* *1* *0.095* *60.85* *< 0.0001*  
  *F-Cleaning interval* *0.022* *1* *0.022* *14.18* *0.0008*  
 Residual 0.045 29 1.555E-003  
 Cor Total 0.16 31

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 1.07 1 6.970E-003 1.06 1.09

D-Temperature 0.054 1 6.970E-003 0.040 0.069 1.00  
 F-Cleaning interval 0.026 1 6.970E-003 0.012 0.041 1.00

The analysis for the NPU response is shown below.



Design Expert Output

**Response 2 NPU  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 824.06 2 412.03 48.15 < 0.0001 significant

*D-Temperature* *675.28* *1* *675.28* *78.91* *< 0.0001*  
  *F-Cleaning interval* *148.78* *1* *148.78* *17.39* *0.0003*  
 Residual 248.16 29 8.56  
 Cor Total 1072.22 31

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 17.16 1 0.52 16.10 18.21

D-Temperature 4.59 1 0.52 3.54 5.65 1.00  
 F-Cleaning interval 2.16 1 0.52 1.10 3.21 1.00

(d) Analyze the residuals for both responses. Are there any problems with model adequacy?

The residual plots for the PVM response shown below do not identify any concerns.























The residual plots for the NPU response shown below identify an outlier with run number 32, NPU value of 28. The experimenters should investigate this run.























(e) The ideal value of PVM is unity and the NPU response should be as small as possible. Recommend suitable operating conditions for the process based on the experimental results.

The contour plots of both the PVM and NPU responses show that running the Temperature at 20C and the Cleaning Interval at 8 is the best operating conditions. This is confirmed with the desirability plot also shown below. The other factors should be set based on process controls, costs, robustness, or other process performance requirements, as they are not significant to either the PVM or NPU response.







**8.30.** An article in the *International Journal of Research in Marketing* (“Experimental design on the front lines of marketing: Testing new ideas to increase direct mail sales,” 2006. Vol. 23, pp. 309-319) describes the use of a 20-run Plackett-Burman design to improve the effects of 19 factors to improve the response rate to a direct mail campaign to attract new customers to a credit card. The 19 factors are as follows:

|  |  |  |
| --- | --- | --- |
| Factor | (–) Control | (+) New Idea |
| *A*: Envelope teaser | General offer | Product-specific offer |
| *B*: Return address | Blind | Add company name |
| *C*: "Official" ink-stamp on envelope | Yes | No |
| *D*: Postage | Pre-printed | Stamp |
| *E*: Additional graphic on envelop | Yes | No |
| *F*: Price graphic on letter | Small | Large |
| *G*: Sticker | Yes | No |
| *H*: Personalize letter copy | No | Yes |
| *I*: Copy Message | Targeted | Generic |
| *J*: Letter headline | Headline 1 | Headline 2 |
| *K*: List of Benefits | Standard layout | Creative layout |
| *L*: Postscript on letter | Control version | New P.S. |
| *M*: Signature | Manager | Senior Executive |
| *N*: Product selection | Many | Few |
| *O*: Value of free gift | High | Low |
| *P*: Reply envelope | Control | New Style |
| *Q*: Information on buckslip | Product info | Free gift info |
| *R*: 2nd buckslip | No | Yes |
| *S*: Interest rate | Low | High |

The 20-run Plackett-Burman design is shown in the following table. Each test combination in the table was mailed to 5,000 potential customers, and the response rate is the percentage of customers who responded positively to the offer.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Test Cell | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* | *M* | *N* | *O* | *P* | *Q* | *R* | *S* | Orders | Resp Rate |
| 1 | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | 52 | 1.04% |
| 2 | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | 38 | 0.76% |
| 3 | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | 42 | 0.84% |
| 4 | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | 134 | 2.68% |
| 5 | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | 104 | 2.08% |
| 6 | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | 60 | 1.20% |
| 7 | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | 61 | 1.22% |
| 8 | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | 68 | 1.36% |
| 9 | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | 57 | 1.14% |
| 10 | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | 30 | 0.60% |
| 11 | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | 108 | 2.16% |
| 12 | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | 39 | 0.78% |
| 13 | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | 40 | 0.80% |
| 14 | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | 49 | 0.98% |
| 15 | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | 37 | 0.74% |
| 16 | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | 99 | 1.98% |
| 17 | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | 86 | 1.72% |
| 18 | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | 43 | 0.86% |
| 19 | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | 47 | 0.94% |
| 20 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 104 | 2.08% |

(a) Verify that in this design each main effect is aliased to all two-factor interactions except those that involve that main effect. That is, in the 19 factor design, the main effect for each factor is aliased with all two-factor interactions involving the other 18 factors, or 153 two-factor interactions (18!/2!16!).

The Design Expert output shown below identifies the aliases for the main effects and the two factor interactions. Notice that the main effects for each factor is aliased with all two-factor interactions involving the 18 other factors, but do not include the interactions that involve the same main effect.

Design Expert Output

[A] = A - 0.2 \* BC + 0.2 \* BD + 0.2 \* BE + 0.2 \* BF - 0.2 \* BG + 0.2 \* BH

- 0.6 \* BI - 0.2 \* BJ - 0.2 \* BK + 0.2 \* BL + 0.2 \* BM - 0.2 \* BN + 0.2 \* BO

- 0.2 \* BP - 0.2 \* BQ - 0.2 \* BR - 0.2 \* BS - 0.2 \* CD - 0.2 \* CE - 0.6 \* CF

- 0.2 \* CG + 0.2 \* CH - 0.2 \* CI - 0.2 \* CJ - 0.2 \* CK + 0.2 \* CL + 0.2 \* CM

+ 0.2 \* CN - 0.2 \* CO + 0.2 \* CP + 0.2 \* CQ - 0.2 \* CR + 0.2 \* CS - 0.2 \* DE

+ 0.2 \* DF - 0.2 \* DG + 0.2 \* DH + 0.2 \* DI - 0.2 \* DJ + 0.2 \* DK - 0.2 \* DL

- 0.2 \* DM + 0.2 \* DN - 0.2 \* DO - 0.2 \* DP - 0.2 \* DQ - 0.6 \* DR + 0.2 \* DS

- 0.2 \* EF + 0.2 \* EG - 0.2 \* EH - 0.2 \* EI + 0.2 \* EJ + 0.2 \* EK - 0.2 \* EL

+ 0.2 \* EM - 0.6 \* EN - 0.2 \* EO - 0.2 \* EP + 0.2 \* EQ - 0.2 \* ER + 0.2 \* ES

+ 0.2 \* FG - 0.2 \* FH - 0.2 \* FI - 0.2 \* FJ - 0.2 \* FK - 0.2 \* FL - 0.2 \* FM

+ 0.2 \* FN - 0.2 \* FO + 0.2 \* FP + 0.2 \* FQ + 0.2 \* FR - 0.2 \* FS - 0.2 \* GH

+ 0.2 \* GI + 0.2 \* GJ - 0.6 \* GK + 0.2 \* GL - 0.2 \* GM - 0.2 \* GN - 0.2 \* GO

+ 0.2 \* GP - 0.2 \* GQ - 0.2 \* GR + 0.2 \* GS + 0.2 \* HI + 0.2 \* HJ - 0.2 \* HK

+ 0.2 \* HL - 0.2 \* HM - 0.2 \* HN - 0.2 \* HO - 0.2 \* HP + 0.2 \* HQ - 0.2 \* HR

- 0.6 \* HS + 0.2 \* IJ + 0.2 \* IK - 0.2 \* IL - 0.2 \* IM - 0.2 \* IN + 0.2 \* IO

+ 0.2 \* IP - 0.2 \* IQ - 0.2 \* IR - 0.2 \* IS - 0.2 \* JK - 0.2 \* JL + 0.2 \* JM

+ 0.2 \* JN - 0.2 \* JO - 0.6 \* JP - 0.2 \* JQ + 0.2 \* JR - 0.2 \* JS - 0.2 \* KL

- 0.2 \* KM - 0.2 \* KN + 0.2 \* KO - 0.2 \* KP + 0.2 \* KQ + 0.2 \* KR + 0.2 \* KS

- 0.6 \* LM - 0.2 \* LN + 0.2 \* LO - 0.2 \* LP - 0.2 \* LQ + 0.2 \* LR + 0.2 \* LS

+ 0.2 \* MN + 0.2 \* MO + 0.2 \* MP - 0.2 \* MQ - 0.2 \* MR - 0.2 \* MS - 0.2 \* NO

- 0.2 \* NP - 0.2 \* NQ + 0.2 \* NR + 0.2 \* NS + 0.2 \* OP - 0.6 \* OQ + 0.2 \* OR

- 0.2 \* OS + 0.2 \* PQ - 0.2 \* PR - 0.2 \* PS + 0.2 \* QR - 0.2 \* QS - 0.2 \* RS

[B] = B - 0.2 \* AC + 0.2 \* AD + 0.2 \* AE + 0.2 \* AF - 0.2 \* AG + 0.2 \* AH

- 0.6 \* AI - 0.2 \* AJ - 0.2 \* AK + 0.2 \* AL + 0.2 \* AM - 0.2 \* AN + 0.2 \* AO

- 0.2 \* AP - 0.2 \* AQ - 0.2 \* AR - 0.2 \* AS - 0.2 \* CD + 0.2 \* CE + 0.2 \* CF

+ 0.2 \* CG - 0.2 \* CH + 0.2 \* CI - 0.6 \* CJ - 0.2 \* CK - 0.2 \* CL + 0.2 \* CM

+ 0.2 \* CN - 0.2 \* CO + 0.2 \* CP - 0.2 \* CQ - 0.2 \* CR - 0.2 \* CS - 0.2 \* DE

- 0.2 \* DF - 0.6 \* DG - 0.2 \* DH + 0.2 \* DI - 0.2 \* DJ - 0.2 \* DK - 0.2 \* DL

+ 0.2 \* DM + 0.2 \* DN + 0.2 \* DO - 0.2 \* DP + 0.2 \* DQ + 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF + 0.2 \* EG - 0.2 \* EH + 0.2 \* EI + 0.2 \* EJ - 0.2 \* EK + 0.2 \* EL

- 0.2 \* EM - 0.2 \* EN + 0.2 \* EO - 0.2 \* EP - 0.2 \* EQ - 0.2 \* ER - 0.6 \* ES

- 0.2 \* FG + 0.2 \* FH - 0.2 \* FI - 0.2 \* FJ + 0.2 \* FK + 0.2 \* FL - 0.2 \* FM

+ 0.2 \* FN - 0.6 \* FO - 0.2 \* FP - 0.2 \* FQ + 0.2 \* FR - 0.2 \* FS + 0.2 \* GH

- 0.2 \* GI - 0.2 \* GJ - 0.2 \* GK - 0.2 \* GL - 0.2 \* GM - 0.2 \* GN + 0.2 \* GO

- 0.2 \* GP + 0.2 \* GQ + 0.2 \* GR + 0.2 \* GS - 0.2 \* HI + 0.2 \* HJ + 0.2 \* HK

- 0.6 \* HL + 0.2 \* HM - 0.2 \* HN - 0.2 \* HO - 0.2 \* HP + 0.2 \* HQ - 0.2 \* HR

- 0.2 \* HS + 0.2 \* IJ + 0.2 \* IK - 0.2 \* IL + 0.2 \* IM - 0.2 \* IN - 0.2 \* IO

- 0.2 \* IP - 0.2 \* IQ + 0.2 \* IR - 0.2 \* IS + 0.2 \* JK + 0.2 \* JL - 0.2 \* JM

- 0.2 \* JN - 0.2 \* JO + 0.2 \* JP + 0.2 \* JQ - 0.2 \* JR - 0.2 \* JS - 0.2 \* KL

- 0.2 \* KM + 0.2 \* KN + 0.2 \* KO - 0.2 \* KP - 0.6 \* KQ - 0.2 \* KR + 0.2 \* KS

- 0.2 \* LM - 0.2 \* LN - 0.2 \* LO + 0.2 \* LP - 0.2 \* LQ + 0.2 \* LR + 0.2 \* LS

- 0.6 \* MN - 0.2 \* MO + 0.2 \* MP - 0.2 \* MQ - 0.2 \* MR + 0.2 \* MS + 0.2 \* NO

+ 0.2 \* NP + 0.2 \* NQ - 0.2 \* NR - 0.2 \* NS - 0.2 \* OP - 0.2 \* OQ - 0.2 \* OR

+ 0.2 \* OS + 0.2 \* PQ - 0.6 \* PR + 0.2 \* PS + 0.2 \* QR - 0.2 \* QS + 0.2 \* RS

[C] = C - 0.2 \* AB - 0.2 \* AD - 0.2 \* AE - 0.6 \* AF - 0.2 \* AG + 0.2 \* AH

- 0.2 \* AI - 0.2 \* AJ - 0.2 \* AK + 0.2 \* AL + 0.2 \* AM + 0.2 \* AN - 0.2 \* AO

+ 0.2 \* AP + 0.2 \* AQ - 0.2 \* AR + 0.2 \* AS - 0.2 \* BD + 0.2 \* BE + 0.2 \* BF

+ 0.2 \* BG - 0.2 \* BH + 0.2 \* BI - 0.6 \* BJ - 0.2 \* BK - 0.2 \* BL + 0.2 \* BM

+ 0.2 \* BN - 0.2 \* BO + 0.2 \* BP - 0.2 \* BQ - 0.2 \* BR - 0.2 \* BS - 0.2 \* DE

+ 0.2 \* DF + 0.2 \* DG + 0.2 \* DH - 0.2 \* DI + 0.2 \* DJ - 0.6 \* DK - 0.2 \* DL

- 0.2 \* DM + 0.2 \* DN + 0.2 \* DO - 0.2 \* DP + 0.2 \* DQ - 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF - 0.2 \* EG - 0.6 \* EH - 0.2 \* EI + 0.2 \* EJ - 0.2 \* EK - 0.2 \* EL

- 0.2 \* EM + 0.2 \* EN + 0.2 \* EO + 0.2 \* EP - 0.2 \* EQ + 0.2 \* ER + 0.2 \* ES

- 0.2 \* FG + 0.2 \* FH - 0.2 \* FI + 0.2 \* FJ + 0.2 \* FK - 0.2 \* FL + 0.2 \* FM

- 0.2 \* FN - 0.2 \* FO + 0.2 \* FP - 0.2 \* FQ - 0.2 \* FR - 0.2 \* FS - 0.2 \* GH

+ 0.2 \* GI - 0.2 \* GJ - 0.2 \* GK + 0.2 \* GL + 0.2 \* GM - 0.2 \* GN + 0.2 \* GO

- 0.6 \* GP - 0.2 \* GQ - 0.2 \* GR + 0.2 \* GS + 0.2 \* HI - 0.2 \* HJ - 0.2 \* HK

- 0.2 \* HL - 0.2 \* HM - 0.2 \* HN - 0.2 \* HO + 0.2 \* HP - 0.2 \* HQ + 0.2 \* HR

+ 0.2 \* HS - 0.2 \* IJ + 0.2 \* IK + 0.2 \* IL - 0.6 \* IM + 0.2 \* IN - 0.2 \* IO

- 0.2 \* IP - 0.2 \* IQ + 0.2 \* IR - 0.2 \* IS + 0.2 \* JK + 0.2 \* JL - 0.2 \* JM

+ 0.2 \* JN - 0.2 \* JO - 0.2 \* JP - 0.2 \* JQ - 0.2 \* JR + 0.2 \* JS + 0.2 \* KL

+ 0.2 \* KM - 0.2 \* KN - 0.2 \* KO - 0.2 \* KP + 0.2 \* KQ + 0.2 \* KR - 0.2 \* KS

- 0.2 \* LM - 0.2 \* LN + 0.2 \* LO + 0.2 \* LP - 0.2 \* LQ - 0.6 \* LR - 0.2 \* LS

- 0.2 \* MN - 0.2 \* MO - 0.2 \* MP + 0.2 \* MQ - 0.2 \* MR + 0.2 \* MS - 0.6 \* NO

- 0.2 \* NP + 0.2 \* NQ - 0.2 \* NR - 0.2 \* NS + 0.2 \* OP + 0.2 \* OQ + 0.2 \* OR

- 0.2 \* OS - 0.2 \* PQ - 0.2 \* PR - 0.2 \* PS + 0.2 \* QR - 0.6 \* QS + 0.2 \* RS

[D] = D + 0.2 \* AB - 0.2 \* AC - 0.2 \* AE + 0.2 \* AF - 0.2 \* AG + 0.2 \* AH

+ 0.2 \* AI - 0.2 \* AJ + 0.2 \* AK - 0.2 \* AL - 0.2 \* AM + 0.2 \* AN - 0.2 \* AO

- 0.2 \* AP - 0.2 \* AQ - 0.6 \* AR + 0.2 \* AS - 0.2 \* BC - 0.2 \* BE - 0.2 \* BF

- 0.6 \* BG - 0.2 \* BH + 0.2 \* BI - 0.2 \* BJ - 0.2 \* BK - 0.2 \* BL + 0.2 \* BM

+ 0.2 \* BN + 0.2 \* BO - 0.2 \* BP + 0.2 \* BQ + 0.2 \* BR - 0.2 \* BS - 0.2 \* CE

+ 0.2 \* CF + 0.2 \* CG + 0.2 \* CH - 0.2 \* CI + 0.2 \* CJ - 0.6 \* CK - 0.2 \* CL

- 0.2 \* CM + 0.2 \* CN + 0.2 \* CO - 0.2 \* CP + 0.2 \* CQ - 0.2 \* CR - 0.2 \* CS

- 0.2 \* EF + 0.2 \* EG + 0.2 \* EH + 0.2 \* EI - 0.2 \* EJ + 0.2 \* EK - 0.6 \* EL

- 0.2 \* EM - 0.2 \* EN + 0.2 \* EO + 0.2 \* EP - 0.2 \* EQ + 0.2 \* ER - 0.2 \* ES

- 0.2 \* FG - 0.2 \* FH - 0.6 \* FI - 0.2 \* FJ + 0.2 \* FK - 0.2 \* FL - 0.2 \* FM

- 0.2 \* FN + 0.2 \* FO + 0.2 \* FP + 0.2 \* FQ - 0.2 \* FR + 0.2 \* FS - 0.2 \* GH

+ 0.2 \* GI - 0.2 \* GJ + 0.2 \* GK + 0.2 \* GL - 0.2 \* GM + 0.2 \* GN - 0.2 \* GO

- 0.2 \* GP + 0.2 \* GQ - 0.2 \* GR - 0.2 \* GS - 0.2 \* HI + 0.2 \* HJ - 0.2 \* HK

- 0.2 \* HL + 0.2 \* HM + 0.2 \* HN - 0.2 \* HO + 0.2 \* HP - 0.6 \* HQ - 0.2 \* HR

- 0.2 \* HS + 0.2 \* IJ - 0.2 \* IK - 0.2 \* IL - 0.2 \* IM - 0.2 \* IN - 0.2 \* IO

- 0.2 \* IP + 0.2 \* IQ - 0.2 \* IR + 0.2 \* IS - 0.2 \* JK + 0.2 \* JL + 0.2 \* JM

- 0.6 \* JN + 0.2 \* JO - 0.2 \* JP - 0.2 \* JQ - 0.2 \* JR + 0.2 \* JS + 0.2 \* KL

+ 0.2 \* KM - 0.2 \* KN + 0.2 \* KO - 0.2 \* KP - 0.2 \* KQ - 0.2 \* KR - 0.2 \* KS

+ 0.2 \* LM + 0.2 \* LN - 0.2 \* LO - 0.2 \* LP - 0.2 \* LQ + 0.2 \* LR + 0.2 \* LS

- 0.2 \* MN - 0.2 \* MO + 0.2 \* MP + 0.2 \* MQ - 0.2 \* MR - 0.6 \* MS - 0.2 \* NO

- 0.2 \* NP - 0.2 \* NQ + 0.2 \* NR - 0.2 \* NS - 0.6 \* OP - 0.2 \* OQ + 0.2 \* OR

- 0.2 \* OS + 0.2 \* PQ + 0.2 \* PR + 0.2 \* PS - 0.2 \* QR - 0.2 \* QS + 0.2 \* RS

[E] = E + 0.2 \* AB - 0.2 \* AC - 0.2 \* AD - 0.2 \* AF + 0.2 \* AG - 0.2 \* AH

- 0.2 \* AI + 0.2 \* AJ + 0.2 \* AK - 0.2 \* AL + 0.2 \* AM - 0.6 \* AN - 0.2 \* AO

- 0.2 \* AP + 0.2 \* AQ - 0.2 \* AR + 0.2 \* AS + 0.2 \* BC - 0.2 \* BD - 0.2 \* BF

+ 0.2 \* BG - 0.2 \* BH + 0.2 \* BI + 0.2 \* BJ - 0.2 \* BK + 0.2 \* BL - 0.2 \* BM

- 0.2 \* BN + 0.2 \* BO - 0.2 \* BP - 0.2 \* BQ - 0.2 \* BR - 0.6 \* BS - 0.2 \* CD

- 0.2 \* CF - 0.2 \* CG - 0.6 \* CH - 0.2 \* CI + 0.2 \* CJ - 0.2 \* CK - 0.2 \* CL

- 0.2 \* CM + 0.2 \* CN + 0.2 \* CO + 0.2 \* CP - 0.2 \* CQ + 0.2 \* CR + 0.2 \* CS

- 0.2 \* DF + 0.2 \* DG + 0.2 \* DH + 0.2 \* DI - 0.2 \* DJ + 0.2 \* DK - 0.6 \* DL

- 0.2 \* DM - 0.2 \* DN + 0.2 \* DO + 0.2 \* DP - 0.2 \* DQ + 0.2 \* DR - 0.2 \* DS

- 0.2 \* FG + 0.2 \* FH + 0.2 \* FI + 0.2 \* FJ - 0.2 \* FK + 0.2 \* FL - 0.6 \* FM

- 0.2 \* FN - 0.2 \* FO + 0.2 \* FP + 0.2 \* FQ - 0.2 \* FR + 0.2 \* FS - 0.2 \* GH

- 0.2 \* GI - 0.6 \* GJ - 0.2 \* GK + 0.2 \* GL - 0.2 \* GM - 0.2 \* GN - 0.2 \* GO

+ 0.2 \* GP + 0.2 \* GQ + 0.2 \* GR - 0.2 \* GS - 0.2 \* HI + 0.2 \* HJ - 0.2 \* HK

+ 0.2 \* HL + 0.2 \* HM - 0.2 \* HN + 0.2 \* HO - 0.2 \* HP - 0.2 \* HQ + 0.2 \* HR

- 0.2 \* HS - 0.2 \* IJ + 0.2 \* IK - 0.2 \* IL - 0.2 \* IM + 0.2 \* IN + 0.2 \* IO

- 0.2 \* IP + 0.2 \* IQ - 0.6 \* IR - 0.2 \* IS + 0.2 \* JK - 0.2 \* JL - 0.2 \* JM

- 0.2 \* JN - 0.2 \* JO - 0.2 \* JP - 0.2 \* JQ + 0.2 \* JR - 0.2 \* JS - 0.2 \* KL

+ 0.2 \* KM + 0.2 \* KN - 0.6 \* KO + 0.2 \* KP - 0.2 \* KQ - 0.2 \* KR - 0.2 \* KS

+ 0.2 \* LM + 0.2 \* LN - 0.2 \* LO + 0.2 \* LP - 0.2 \* LQ - 0.2 \* LR - 0.2 \* LS

+ 0.2 \* MN + 0.2 \* MO - 0.2 \* MP - 0.2 \* MQ - 0.2 \* MR + 0.2 \* MS - 0.2 \* NO

- 0.2 \* NP + 0.2 \* NQ + 0.2 \* NR - 0.2 \* NS - 0.2 \* OP - 0.2 \* OQ - 0.2 \* OR

+ 0.2 \* OS - 0.6 \* PQ - 0.2 \* PR + 0.2 \* PS + 0.2 \* QR + 0.2 \* QS - 0.2 \* RS

[F] = F + 0.2 \* AB - 0.6 \* AC + 0.2 \* AD - 0.2 \* AE + 0.2 \* AG - 0.2 \* AH

- 0.2 \* AI - 0.2 \* AJ - 0.2 \* AK - 0.2 \* AL - 0.2 \* AM + 0.2 \* AN - 0.2 \* AO

+ 0.2 \* AP + 0.2 \* AQ + 0.2 \* AR - 0.2 \* AS + 0.2 \* BC - 0.2 \* BD - 0.2 \* BE

- 0.2 \* BG + 0.2 \* BH - 0.2 \* BI - 0.2 \* BJ + 0.2 \* BK + 0.2 \* BL - 0.2 \* BM

+ 0.2 \* BN - 0.6 \* BO - 0.2 \* BP - 0.2 \* BQ + 0.2 \* BR - 0.2 \* BS + 0.2 \* CD

- 0.2 \* CE - 0.2 \* CG + 0.2 \* CH - 0.2 \* CI + 0.2 \* CJ + 0.2 \* CK - 0.2 \* CL

+ 0.2 \* CM - 0.2 \* CN - 0.2 \* CO + 0.2 \* CP - 0.2 \* CQ - 0.2 \* CR - 0.2 \* CS

- 0.2 \* DE - 0.2 \* DG - 0.2 \* DH - 0.6 \* DI - 0.2 \* DJ + 0.2 \* DK - 0.2 \* DL

- 0.2 \* DM - 0.2 \* DN + 0.2 \* DO + 0.2 \* DP + 0.2 \* DQ - 0.2 \* DR + 0.2 \* DS

- 0.2 \* EG + 0.2 \* EH + 0.2 \* EI + 0.2 \* EJ - 0.2 \* EK + 0.2 \* EL - 0.6 \* EM

- 0.2 \* EN - 0.2 \* EO + 0.2 \* EP + 0.2 \* EQ - 0.2 \* ER + 0.2 \* ES - 0.2 \* GH

+ 0.2 \* GI + 0.2 \* GJ + 0.2 \* GK - 0.2 \* GL + 0.2 \* GM - 0.6 \* GN - 0.2 \* GO

- 0.2 \* GP + 0.2 \* GQ + 0.2 \* GR - 0.2 \* GS - 0.2 \* HI - 0.2 \* HJ - 0.6 \* HK

- 0.2 \* HL + 0.2 \* HM - 0.2 \* HN - 0.2 \* HO - 0.2 \* HP + 0.2 \* HQ + 0.2 \* HR

+ 0.2 \* HS - 0.2 \* IJ + 0.2 \* IK - 0.2 \* IL + 0.2 \* IM + 0.2 \* IN - 0.2 \* IO

+ 0.2 \* IP - 0.2 \* IQ - 0.2 \* IR + 0.2 \* IS - 0.2 \* JK + 0.2 \* JL - 0.2 \* JM

- 0.2 \* JN + 0.2 \* JO + 0.2 \* JP - 0.2 \* JQ + 0.2 \* JR - 0.6 \* JS + 0.2 \* KL

- 0.2 \* KM - 0.2 \* KN - 0.2 \* KO - 0.2 \* KP - 0.2 \* KQ - 0.2 \* KR + 0.2 \* KS

- 0.2 \* LM + 0.2 \* LN + 0.2 \* LO - 0.6 \* LP + 0.2 \* LQ - 0.2 \* LR - 0.2 \* LS

+ 0.2 \* MN + 0.2 \* MO - 0.2 \* MP + 0.2 \* MQ - 0.2 \* MR - 0.2 \* MS + 0.2 \* NO

+ 0.2 \* NP - 0.2 \* NQ - 0.2 \* NR - 0.2 \* NS - 0.2 \* OP - 0.2 \* OQ + 0.2 \* OR

+ 0.2 \* OS - 0.2 \* PQ - 0.2 \* PR - 0.2 \* PS - 0.6 \* QR - 0.2 \* QS + 0.2 \* RS

[G] = G - 0.2 \* AB - 0.2 \* AC - 0.2 \* AD + 0.2 \* AE + 0.2 \* AF - 0.2 \* AH

+ 0.2 \* AI + 0.2 \* AJ - 0.6 \* AK + 0.2 \* AL - 0.2 \* AM - 0.2 \* AN - 0.2 \* AO

+ 0.2 \* AP - 0.2 \* AQ - 0.2 \* AR + 0.2 \* AS + 0.2 \* BC - 0.6 \* BD + 0.2 \* BE

- 0.2 \* BF + 0.2 \* BH - 0.2 \* BI - 0.2 \* BJ - 0.2 \* BK - 0.2 \* BL - 0.2 \* BM

- 0.2 \* BN + 0.2 \* BO - 0.2 \* BP + 0.2 \* BQ + 0.2 \* BR + 0.2 \* BS + 0.2 \* CD

- 0.2 \* CE - 0.2 \* CF - 0.2 \* CH + 0.2 \* CI - 0.2 \* CJ - 0.2 \* CK + 0.2 \* CL

+ 0.2 \* CM - 0.2 \* CN + 0.2 \* CO - 0.6 \* CP - 0.2 \* CQ - 0.2 \* CR + 0.2 \* CS

+ 0.2 \* DE - 0.2 \* DF - 0.2 \* DH + 0.2 \* DI - 0.2 \* DJ + 0.2 \* DK + 0.2 \* DL

- 0.2 \* DM + 0.2 \* DN - 0.2 \* DO - 0.2 \* DP + 0.2 \* DQ - 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF - 0.2 \* EH - 0.2 \* EI - 0.6 \* EJ - 0.2 \* EK + 0.2 \* EL - 0.2 \* EM

- 0.2 \* EN - 0.2 \* EO + 0.2 \* EP + 0.2 \* EQ + 0.2 \* ER - 0.2 \* ES - 0.2 \* FH

+ 0.2 \* FI + 0.2 \* FJ + 0.2 \* FK - 0.2 \* FL + 0.2 \* FM - 0.6 \* FN - 0.2 \* FO

- 0.2 \* FP + 0.2 \* FQ + 0.2 \* FR - 0.2 \* FS - 0.2 \* HI + 0.2 \* HJ + 0.2 \* HK

+ 0.2 \* HL - 0.2 \* HM + 0.2 \* HN - 0.6 \* HO - 0.2 \* HP - 0.2 \* HQ + 0.2 \* HR

+ 0.2 \* HS - 0.2 \* IJ - 0.2 \* IK - 0.6 \* IL - 0.2 \* IM + 0.2 \* IN - 0.2 \* IO

- 0.2 \* IP - 0.2 \* IQ + 0.2 \* IR + 0.2 \* IS - 0.2 \* JK + 0.2 \* JL - 0.2 \* JM

+ 0.2 \* JN + 0.2 \* JO - 0.2 \* JP + 0.2 \* JQ - 0.2 \* JR - 0.2 \* JS - 0.2 \* KL

+ 0.2 \* KM - 0.2 \* KN - 0.2 \* KO + 0.2 \* KP + 0.2 \* KQ - 0.2 \* KR + 0.2 \* KS

+ 0.2 \* LM - 0.2 \* LN - 0.2 \* LO - 0.2 \* LP - 0.2 \* LQ - 0.2 \* LR - 0.2 \* LS

- 0.2 \* MN + 0.2 \* MO + 0.2 \* MP - 0.6 \* MQ + 0.2 \* MR - 0.2 \* MS + 0.2 \* NO

+ 0.2 \* NP - 0.2 \* NQ + 0.2 \* NR - 0.2 \* NS + 0.2 \* OP + 0.2 \* OQ - 0.2 \* OR

- 0.2 \* OS - 0.2 \* PQ - 0.2 \* PR + 0.2 \* PS - 0.2 \* QR - 0.2 \* QS - 0.6 \* RS

[H] = H + 0.2 \* AB + 0.2 \* AC + 0.2 \* AD - 0.2 \* AE - 0.2 \* AF - 0.2 \* AG

+ 0.2 \* AI + 0.2 \* AJ - 0.2 \* AK + 0.2 \* AL - 0.2 \* AM - 0.2 \* AN - 0.2 \* AO

- 0.2 \* AP + 0.2 \* AQ - 0.2 \* AR - 0.6 \* AS - 0.2 \* BC - 0.2 \* BD - 0.2 \* BE

+ 0.2 \* BF + 0.2 \* BG - 0.2 \* BI + 0.2 \* BJ + 0.2 \* BK - 0.6 \* BL + 0.2 \* BM

- 0.2 \* BN - 0.2 \* BO - 0.2 \* BP + 0.2 \* BQ - 0.2 \* BR - 0.2 \* BS + 0.2 \* CD

- 0.6 \* CE + 0.2 \* CF - 0.2 \* CG + 0.2 \* CI - 0.2 \* CJ - 0.2 \* CK - 0.2 \* CL

- 0.2 \* CM - 0.2 \* CN - 0.2 \* CO + 0.2 \* CP - 0.2 \* CQ + 0.2 \* CR + 0.2 \* CS

+ 0.2 \* DE - 0.2 \* DF - 0.2 \* DG - 0.2 \* DI + 0.2 \* DJ - 0.2 \* DK - 0.2 \* DL

+ 0.2 \* DM + 0.2 \* DN - 0.2 \* DO + 0.2 \* DP - 0.6 \* DQ - 0.2 \* DR - 0.2 \* DS

+ 0.2 \* EF - 0.2 \* EG - 0.2 \* EI + 0.2 \* EJ - 0.2 \* EK + 0.2 \* EL + 0.2 \* EM

- 0.2 \* EN + 0.2 \* EO - 0.2 \* EP - 0.2 \* EQ + 0.2 \* ER - 0.2 \* ES - 0.2 \* FG

- 0.2 \* FI - 0.2 \* FJ - 0.6 \* FK - 0.2 \* FL + 0.2 \* FM - 0.2 \* FN - 0.2 \* FO

- 0.2 \* FP + 0.2 \* FQ + 0.2 \* FR + 0.2 \* FS - 0.2 \* GI + 0.2 \* GJ + 0.2 \* GK

+ 0.2 \* GL - 0.2 \* GM + 0.2 \* GN - 0.6 \* GO - 0.2 \* GP - 0.2 \* GQ + 0.2 \* GR

+ 0.2 \* GS - 0.2 \* IJ + 0.2 \* IK + 0.2 \* IL + 0.2 \* IM - 0.2 \* IN + 0.2 \* IO

- 0.6 \* IP - 0.2 \* IQ - 0.2 \* IR + 0.2 \* IS - 0.2 \* JK - 0.2 \* JL - 0.6 \* JM

- 0.2 \* JN + 0.2 \* JO - 0.2 \* JP - 0.2 \* JQ - 0.2 \* JR + 0.2 \* JS - 0.2 \* KL

+ 0.2 \* KM - 0.2 \* KN + 0.2 \* KO + 0.2 \* KP - 0.2 \* KQ + 0.2 \* KR - 0.2 \* KS

- 0.2 \* LM + 0.2 \* LN - 0.2 \* LO - 0.2 \* LP + 0.2 \* LQ + 0.2 \* LR - 0.2 \* LS

+ 0.2 \* MN - 0.2 \* MO - 0.2 \* MP - 0.2 \* MQ - 0.2 \* MR - 0.2 \* MS - 0.2 \* NO

+ 0.2 \* NP + 0.2 \* NQ - 0.6 \* NR + 0.2 \* NS + 0.2 \* OP + 0.2 \* OQ - 0.2 \* OR

+ 0.2 \* OS + 0.2 \* PQ + 0.2 \* PR - 0.2 \* PS - 0.2 \* QR - 0.2 \* QS - 0.2 \* RS

[I] = I - 0.6 \* AB - 0.2 \* AC + 0.2 \* AD - 0.2 \* AE - 0.2 \* AF + 0.2 \* AG

+ 0.2 \* AH + 0.2 \* AJ + 0.2 \* AK - 0.2 \* AL - 0.2 \* AM - 0.2 \* AN + 0.2 \* AO

+ 0.2 \* AP - 0.2 \* AQ - 0.2 \* AR - 0.2 \* AS + 0.2 \* BC + 0.2 \* BD + 0.2 \* BE

- 0.2 \* BF - 0.2 \* BG - 0.2 \* BH + 0.2 \* BJ + 0.2 \* BK - 0.2 \* BL + 0.2 \* BM

- 0.2 \* BN - 0.2 \* BO - 0.2 \* BP - 0.2 \* BQ + 0.2 \* BR - 0.2 \* BS - 0.2 \* CD

- 0.2 \* CE - 0.2 \* CF + 0.2 \* CG + 0.2 \* CH - 0.2 \* CJ + 0.2 \* CK + 0.2 \* CL

- 0.6 \* CM + 0.2 \* CN - 0.2 \* CO - 0.2 \* CP - 0.2 \* CQ + 0.2 \* CR - 0.2 \* CS

+ 0.2 \* DE - 0.6 \* DF + 0.2 \* DG - 0.2 \* DH + 0.2 \* DJ - 0.2 \* DK - 0.2 \* DL

- 0.2 \* DM - 0.2 \* DN - 0.2 \* DO - 0.2 \* DP + 0.2 \* DQ - 0.2 \* DR + 0.2 \* DS

+ 0.2 \* EF - 0.2 \* EG - 0.2 \* EH - 0.2 \* EJ + 0.2 \* EK - 0.2 \* EL - 0.2 \* EM

+ 0.2 \* EN + 0.2 \* EO - 0.2 \* EP + 0.2 \* EQ - 0.6 \* ER - 0.2 \* ES + 0.2 \* FG

- 0.2 \* FH - 0.2 \* FJ + 0.2 \* FK - 0.2 \* FL + 0.2 \* FM + 0.2 \* FN - 0.2 \* FO

+ 0.2 \* FP - 0.2 \* FQ - 0.2 \* FR + 0.2 \* FS - 0.2 \* GH - 0.2 \* GJ - 0.2 \* GK

- 0.6 \* GL - 0.2 \* GM + 0.2 \* GN - 0.2 \* GO - 0.2 \* GP - 0.2 \* GQ + 0.2 \* GR

+ 0.2 \* GS - 0.2 \* HJ + 0.2 \* HK + 0.2 \* HL + 0.2 \* HM - 0.2 \* HN + 0.2 \* HO

- 0.6 \* HP - 0.2 \* HQ - 0.2 \* HR + 0.2 \* HS - 0.2 \* JK + 0.2 \* JL + 0.2 \* JM

+ 0.2 \* JN - 0.2 \* JO + 0.2 \* JP - 0.6 \* JQ - 0.2 \* JR - 0.2 \* JS - 0.2 \* KL

- 0.2 \* KM - 0.6 \* KN - 0.2 \* KO + 0.2 \* KP - 0.2 \* KQ - 0.2 \* KR - 0.2 \* KS

- 0.2 \* LM + 0.2 \* LN - 0.2 \* LO + 0.2 \* LP + 0.2 \* LQ - 0.2 \* LR + 0.2 \* LS

- 0.2 \* MN + 0.2 \* MO - 0.2 \* MP - 0.2 \* MQ + 0.2 \* MR + 0.2 \* MS + 0.2 \* NO

- 0.2 \* NP - 0.2 \* NQ - 0.2 \* NR - 0.2 \* NS - 0.2 \* OP + 0.2 \* OQ + 0.2 \* OR

- 0.6 \* OS + 0.2 \* PQ + 0.2 \* PR - 0.2 \* PS + 0.2 \* QR + 0.2 \* QS - 0.2 \* RS

[J] = J - 0.2 \* AB - 0.2 \* AC - 0.2 \* AD + 0.2 \* AE - 0.2 \* AF + 0.2 \* AG

+ 0.2 \* AH + 0.2 \* AI - 0.2 \* AK - 0.2 \* AL + 0.2 \* AM + 0.2 \* AN - 0.2 \* AO

- 0.6 \* AP - 0.2 \* AQ + 0.2 \* AR - 0.2 \* AS - 0.6 \* BC - 0.2 \* BD + 0.2 \* BE

- 0.2 \* BF - 0.2 \* BG + 0.2 \* BH + 0.2 \* BI + 0.2 \* BK + 0.2 \* BL - 0.2 \* BM

- 0.2 \* BN - 0.2 \* BO + 0.2 \* BP + 0.2 \* BQ - 0.2 \* BR - 0.2 \* BS + 0.2 \* CD

+ 0.2 \* CE + 0.2 \* CF - 0.2 \* CG - 0.2 \* CH - 0.2 \* CI + 0.2 \* CK + 0.2 \* CL

- 0.2 \* CM + 0.2 \* CN - 0.2 \* CO - 0.2 \* CP - 0.2 \* CQ - 0.2 \* CR + 0.2 \* CS

- 0.2 \* DE - 0.2 \* DF - 0.2 \* DG + 0.2 \* DH + 0.2 \* DI - 0.2 \* DK + 0.2 \* DL

+ 0.2 \* DM - 0.6 \* DN + 0.2 \* DO - 0.2 \* DP - 0.2 \* DQ - 0.2 \* DR + 0.2 \* DS

+ 0.2 \* EF - 0.6 \* EG + 0.2 \* EH - 0.2 \* EI + 0.2 \* EK - 0.2 \* EL - 0.2 \* EM

- 0.2 \* EN - 0.2 \* EO - 0.2 \* EP - 0.2 \* EQ + 0.2 \* ER - 0.2 \* ES + 0.2 \* FG

- 0.2 \* FH - 0.2 \* FI - 0.2 \* FK + 0.2 \* FL - 0.2 \* FM - 0.2 \* FN + 0.2 \* FO

+ 0.2 \* FP - 0.2 \* FQ + 0.2 \* FR - 0.6 \* FS + 0.2 \* GH - 0.2 \* GI - 0.2 \* GK

+ 0.2 \* GL - 0.2 \* GM + 0.2 \* GN + 0.2 \* GO - 0.2 \* GP + 0.2 \* GQ - 0.2 \* GR

- 0.2 \* GS - 0.2 \* HI - 0.2 \* HK - 0.2 \* HL - 0.6 \* HM - 0.2 \* HN + 0.2 \* HO

- 0.2 \* HP - 0.2 \* HQ - 0.2 \* HR + 0.2 \* HS - 0.2 \* IK + 0.2 \* IL + 0.2 \* IM

+ 0.2 \* IN - 0.2 \* IO + 0.2 \* IP - 0.6 \* IQ - 0.2 \* IR - 0.2 \* IS - 0.2 \* KL

+ 0.2 \* KM + 0.2 \* KN + 0.2 \* KO - 0.2 \* KP + 0.2 \* KQ - 0.6 \* KR - 0.2 \* KS

- 0.2 \* LM - 0.2 \* LN - 0.6 \* LO - 0.2 \* LP + 0.2 \* LQ - 0.2 \* LR - 0.2 \* LS

- 0.2 \* MN + 0.2 \* MO - 0.2 \* MP + 0.2 \* MQ + 0.2 \* MR - 0.2 \* MS - 0.2 \* NO

+ 0.2 \* NP - 0.2 \* NQ - 0.2 \* NR + 0.2 \* NS + 0.2 \* OP - 0.2 \* OQ - 0.2 \* OR

- 0.2 \* OS - 0.2 \* PQ + 0.2 \* PR + 0.2 \* PS + 0.2 \* QR + 0.2 \* QS + 0.2 \* RS

[K] = K - 0.2 \* AB - 0.2 \* AC + 0.2 \* AD + 0.2 \* AE - 0.2 \* AF - 0.6 \* AG

- 0.2 \* AH + 0.2 \* AI - 0.2 \* AJ - 0.2 \* AL - 0.2 \* AM - 0.2 \* AN + 0.2 \* AO

- 0.2 \* AP + 0.2 \* AQ + 0.2 \* AR + 0.2 \* AS - 0.2 \* BC - 0.2 \* BD - 0.2 \* BE

+ 0.2 \* BF - 0.2 \* BG + 0.2 \* BH + 0.2 \* BI + 0.2 \* BJ - 0.2 \* BL - 0.2 \* BM

+ 0.2 \* BN + 0.2 \* BO - 0.2 \* BP - 0.6 \* BQ - 0.2 \* BR + 0.2 \* BS - 0.6 \* CD

- 0.2 \* CE + 0.2 \* CF - 0.2 \* CG - 0.2 \* CH + 0.2 \* CI + 0.2 \* CJ + 0.2 \* CL

+ 0.2 \* CM - 0.2 \* CN - 0.2 \* CO - 0.2 \* CP + 0.2 \* CQ + 0.2 \* CR - 0.2 \* CS

+ 0.2 \* DE + 0.2 \* DF + 0.2 \* DG - 0.2 \* DH - 0.2 \* DI - 0.2 \* DJ + 0.2 \* DL

+ 0.2 \* DM - 0.2 \* DN + 0.2 \* DO - 0.2 \* DP - 0.2 \* DQ - 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF - 0.2 \* EG - 0.2 \* EH + 0.2 \* EI + 0.2 \* EJ - 0.2 \* EL + 0.2 \* EM

+ 0.2 \* EN - 0.6 \* EO + 0.2 \* EP - 0.2 \* EQ - 0.2 \* ER - 0.2 \* ES + 0.2 \* FG

- 0.6 \* FH + 0.2 \* FI - 0.2 \* FJ + 0.2 \* FL - 0.2 \* FM - 0.2 \* FN - 0.2 \* FO

- 0.2 \* FP - 0.2 \* FQ - 0.2 \* FR + 0.2 \* FS + 0.2 \* GH - 0.2 \* GI - 0.2 \* GJ

- 0.2 \* GL + 0.2 \* GM - 0.2 \* GN - 0.2 \* GO + 0.2 \* GP + 0.2 \* GQ - 0.2 \* GR

+ 0.2 \* GS + 0.2 \* HI - 0.2 \* HJ - 0.2 \* HL + 0.2 \* HM - 0.2 \* HN + 0.2 \* HO

+ 0.2 \* HP - 0.2 \* HQ + 0.2 \* HR - 0.2 \* HS - 0.2 \* IJ - 0.2 \* IL - 0.2 \* IM

- 0.6 \* IN - 0.2 \* IO + 0.2 \* IP - 0.2 \* IQ - 0.2 \* IR - 0.2 \* IS - 0.2 \* JL

+ 0.2 \* JM + 0.2 \* JN + 0.2 \* JO - 0.2 \* JP + 0.2 \* JQ - 0.6 \* JR - 0.2 \* JS

- 0.2 \* LM + 0.2 \* LN + 0.2 \* LO + 0.2 \* LP - 0.2 \* LQ + 0.2 \* LR - 0.6 \* LS

- 0.2 \* MN - 0.2 \* MO - 0.6 \* MP - 0.2 \* MQ + 0.2 \* MR - 0.2 \* MS - 0.2 \* NO

+ 0.2 \* NP - 0.2 \* NQ + 0.2 \* NR + 0.2 \* NS - 0.2 \* OP + 0.2 \* OQ - 0.2 \* OR

- 0.2 \* OS + 0.2 \* PQ - 0.2 \* PR - 0.2 \* PS - 0.2 \* QR + 0.2 \* QS + 0.2 \* RS

[L] = L + 0.2 \* AB + 0.2 \* AC - 0.2 \* AD - 0.2 \* AE - 0.2 \* AF + 0.2 \* AG

+ 0.2 \* AH - 0.2 \* AI - 0.2 \* AJ - 0.2 \* AK - 0.6 \* AM - 0.2 \* AN + 0.2 \* AO

- 0.2 \* AP - 0.2 \* AQ + 0.2 \* AR + 0.2 \* AS - 0.2 \* BC - 0.2 \* BD + 0.2 \* BE

+ 0.2 \* BF - 0.2 \* BG - 0.6 \* BH - 0.2 \* BI + 0.2 \* BJ - 0.2 \* BK - 0.2 \* BM

- 0.2 \* BN - 0.2 \* BO + 0.2 \* BP - 0.2 \* BQ + 0.2 \* BR + 0.2 \* BS - 0.2 \* CD

- 0.2 \* CE - 0.2 \* CF + 0.2 \* CG - 0.2 \* CH + 0.2 \* CI + 0.2 \* CJ + 0.2 \* CK

- 0.2 \* CM - 0.2 \* CN + 0.2 \* CO + 0.2 \* CP - 0.2 \* CQ - 0.6 \* CR - 0.2 \* CS

- 0.6 \* DE - 0.2 \* DF + 0.2 \* DG - 0.2 \* DH - 0.2 \* DI + 0.2 \* DJ + 0.2 \* DK

+ 0.2 \* DM + 0.2 \* DN - 0.2 \* DO - 0.2 \* DP - 0.2 \* DQ + 0.2 \* DR + 0.2 \* DS

+ 0.2 \* EF + 0.2 \* EG + 0.2 \* EH - 0.2 \* EI - 0.2 \* EJ - 0.2 \* EK + 0.2 \* EM

+ 0.2 \* EN - 0.2 \* EO + 0.2 \* EP - 0.2 \* EQ - 0.2 \* ER - 0.2 \* ES - 0.2 \* FG

- 0.2 \* FH - 0.2 \* FI + 0.2 \* FJ + 0.2 \* FK - 0.2 \* FM + 0.2 \* FN + 0.2 \* FO

- 0.6 \* FP + 0.2 \* FQ - 0.2 \* FR - 0.2 \* FS + 0.2 \* GH - 0.6 \* GI + 0.2 \* GJ

- 0.2 \* GK + 0.2 \* GM - 0.2 \* GN - 0.2 \* GO - 0.2 \* GP - 0.2 \* GQ - 0.2 \* GR

- 0.2 \* GS + 0.2 \* HI - 0.2 \* HJ - 0.2 \* HK - 0.2 \* HM + 0.2 \* HN - 0.2 \* HO

- 0.2 \* HP + 0.2 \* HQ + 0.2 \* HR - 0.2 \* HS + 0.2 \* IJ - 0.2 \* IK - 0.2 \* IM

+ 0.2 \* IN - 0.2 \* IO + 0.2 \* IP + 0.2 \* IQ - 0.2 \* IR + 0.2 \* IS - 0.2 \* JK

- 0.2 \* JM - 0.2 \* JN - 0.6 \* JO - 0.2 \* JP + 0.2 \* JQ - 0.2 \* JR - 0.2 \* JS

- 0.2 \* KM + 0.2 \* KN + 0.2 \* KO + 0.2 \* KP - 0.2 \* KQ + 0.2 \* KR - 0.6 \* KS

- 0.2 \* MN + 0.2 \* MO + 0.2 \* MP + 0.2 \* MQ - 0.2 \* MR + 0.2 \* MS - 0.2 \* NO

- 0.2 \* NP - 0.6 \* NQ - 0.2 \* NR + 0.2 \* NS - 0.2 \* OP + 0.2 \* OQ - 0.2 \* OR

+ 0.2 \* OS - 0.2 \* PQ + 0.2 \* PR - 0.2 \* PS + 0.2 \* QR - 0.2 \* QS - 0.2 \* RS

[M] = M + 0.2 \* AB + 0.2 \* AC - 0.2 \* AD + 0.2 \* AE - 0.2 \* AF - 0.2 \* AG

- 0.2 \* AH - 0.2 \* AI + 0.2 \* AJ - 0.2 \* AK - 0.6 \* AL + 0.2 \* AN + 0.2 \* AO

+ 0.2 \* AP - 0.2 \* AQ - 0.2 \* AR - 0.2 \* AS + 0.2 \* BC + 0.2 \* BD - 0.2 \* BE

- 0.2 \* BF - 0.2 \* BG + 0.2 \* BH + 0.2 \* BI - 0.2 \* BJ - 0.2 \* BK - 0.2 \* BL

- 0.6 \* BN - 0.2 \* BO + 0.2 \* BP - 0.2 \* BQ - 0.2 \* BR + 0.2 \* BS - 0.2 \* CD

- 0.2 \* CE + 0.2 \* CF + 0.2 \* CG - 0.2 \* CH - 0.6 \* CI - 0.2 \* CJ + 0.2 \* CK

- 0.2 \* CL - 0.2 \* CN - 0.2 \* CO - 0.2 \* CP + 0.2 \* CQ - 0.2 \* CR + 0.2 \* CS

- 0.2 \* DE - 0.2 \* DF - 0.2 \* DG + 0.2 \* DH - 0.2 \* DI + 0.2 \* DJ + 0.2 \* DK

+ 0.2 \* DL - 0.2 \* DN - 0.2 \* DO + 0.2 \* DP + 0.2 \* DQ - 0.2 \* DR - 0.6 \* DS

- 0.6 \* EF - 0.2 \* EG + 0.2 \* EH - 0.2 \* EI - 0.2 \* EJ + 0.2 \* EK + 0.2 \* EL

+ 0.2 \* EN + 0.2 \* EO - 0.2 \* EP - 0.2 \* EQ - 0.2 \* ER + 0.2 \* ES + 0.2 \* FG

+ 0.2 \* FH + 0.2 \* FI - 0.2 \* FJ - 0.2 \* FK - 0.2 \* FL + 0.2 \* FN + 0.2 \* FO

- 0.2 \* FP + 0.2 \* FQ - 0.2 \* FR - 0.2 \* FS - 0.2 \* GH - 0.2 \* GI - 0.2 \* GJ

+ 0.2 \* GK + 0.2 \* GL - 0.2 \* GN + 0.2 \* GO + 0.2 \* GP - 0.6 \* GQ + 0.2 \* GR

- 0.2 \* GS + 0.2 \* HI - 0.6 \* HJ + 0.2 \* HK - 0.2 \* HL + 0.2 \* HN - 0.2 \* HO

- 0.2 \* HP - 0.2 \* HQ - 0.2 \* HR - 0.2 \* HS + 0.2 \* IJ - 0.2 \* IK - 0.2 \* IL

- 0.2 \* IN + 0.2 \* IO - 0.2 \* IP - 0.2 \* IQ + 0.2 \* IR + 0.2 \* IS + 0.2 \* JK

- 0.2 \* JL - 0.2 \* JN + 0.2 \* JO - 0.2 \* JP + 0.2 \* JQ + 0.2 \* JR - 0.2 \* JS

- 0.2 \* KL - 0.2 \* KN - 0.2 \* KO - 0.6 \* KP - 0.2 \* KQ + 0.2 \* KR - 0.2 \* KS

- 0.2 \* LN + 0.2 \* LO + 0.2 \* LP + 0.2 \* LQ - 0.2 \* LR + 0.2 \* LS - 0.2 \* NO

+ 0.2 \* NP + 0.2 \* NQ + 0.2 \* NR - 0.2 \* NS - 0.2 \* OP - 0.2 \* OQ - 0.6 \* OR

- 0.2 \* OS - 0.2 \* PQ + 0.2 \* PR - 0.2 \* PS - 0.2 \* QR + 0.2 \* QS + 0.2 \* RS

[N] = N - 0.2 \* AB + 0.2 \* AC + 0.2 \* AD - 0.6 \* AE + 0.2 \* AF - 0.2 \* AG

- 0.2 \* AH - 0.2 \* AI + 0.2 \* AJ - 0.2 \* AK - 0.2 \* AL + 0.2 \* AM - 0.2 \* AO

- 0.2 \* AP - 0.2 \* AQ + 0.2 \* AR + 0.2 \* AS + 0.2 \* BC + 0.2 \* BD - 0.2 \* BE

+ 0.2 \* BF - 0.2 \* BG - 0.2 \* BH - 0.2 \* BI - 0.2 \* BJ + 0.2 \* BK - 0.2 \* BL

- 0.6 \* BM + 0.2 \* BO + 0.2 \* BP + 0.2 \* BQ - 0.2 \* BR - 0.2 \* BS + 0.2 \* CD

+ 0.2 \* CE - 0.2 \* CF - 0.2 \* CG - 0.2 \* CH + 0.2 \* CI + 0.2 \* CJ - 0.2 \* CK

- 0.2 \* CL - 0.2 \* CM - 0.6 \* CO - 0.2 \* CP + 0.2 \* CQ - 0.2 \* CR - 0.2 \* CS

- 0.2 \* DE - 0.2 \* DF + 0.2 \* DG + 0.2 \* DH - 0.2 \* DI - 0.6 \* DJ - 0.2 \* DK

+ 0.2 \* DL - 0.2 \* DM - 0.2 \* DO - 0.2 \* DP - 0.2 \* DQ + 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF - 0.2 \* EG - 0.2 \* EH + 0.2 \* EI - 0.2 \* EJ + 0.2 \* EK + 0.2 \* EL

+ 0.2 \* EM - 0.2 \* EO - 0.2 \* EP + 0.2 \* EQ + 0.2 \* ER - 0.2 \* ES - 0.6 \* FG

- 0.2 \* FH + 0.2 \* FI - 0.2 \* FJ - 0.2 \* FK + 0.2 \* FL + 0.2 \* FM + 0.2 \* FO

+ 0.2 \* FP - 0.2 \* FQ - 0.2 \* FR - 0.2 \* FS + 0.2 \* GH + 0.2 \* GI + 0.2 \* GJ

- 0.2 \* GK - 0.2 \* GL - 0.2 \* GM + 0.2 \* GO + 0.2 \* GP - 0.2 \* GQ + 0.2 \* GR

- 0.2 \* GS - 0.2 \* HI - 0.2 \* HJ - 0.2 \* HK + 0.2 \* HL + 0.2 \* HM - 0.2 \* HO

+ 0.2 \* HP + 0.2 \* HQ - 0.6 \* HR + 0.2 \* HS + 0.2 \* IJ - 0.6 \* IK + 0.2 \* IL

- 0.2 \* IM + 0.2 \* IO - 0.2 \* IP - 0.2 \* IQ - 0.2 \* IR - 0.2 \* IS + 0.2 \* JK

- 0.2 \* JL - 0.2 \* JM - 0.2 \* JO + 0.2 \* JP - 0.2 \* JQ - 0.2 \* JR + 0.2 \* JS

+ 0.2 \* KL - 0.2 \* KM - 0.2 \* KO + 0.2 \* KP - 0.2 \* KQ + 0.2 \* KR + 0.2 \* KS

- 0.2 \* LM - 0.2 \* LO - 0.2 \* LP - 0.6 \* LQ - 0.2 \* LR + 0.2 \* LS - 0.2 \* MO

+ 0.2 \* MP + 0.2 \* MQ + 0.2 \* MR - 0.2 \* MS - 0.2 \* OP + 0.2 \* OQ + 0.2 \* OR

+ 0.2 \* OS - 0.2 \* PQ - 0.2 \* PR - 0.6 \* PS - 0.2 \* QR + 0.2 \* QS - 0.2 \* RS

[O] = O + 0.2 \* AB - 0.2 \* AC - 0.2 \* AD - 0.2 \* AE - 0.2 \* AF - 0.2 \* AG

- 0.2 \* AH + 0.2 \* AI - 0.2 \* AJ + 0.2 \* AK + 0.2 \* AL + 0.2 \* AM - 0.2 \* AN

+ 0.2 \* AP - 0.6 \* AQ + 0.2 \* AR - 0.2 \* AS - 0.2 \* BC + 0.2 \* BD + 0.2 \* BE

- 0.6 \* BF + 0.2 \* BG - 0.2 \* BH - 0.2 \* BI - 0.2 \* BJ + 0.2 \* BK - 0.2 \* BL

- 0.2 \* BM + 0.2 \* BN - 0.2 \* BP - 0.2 \* BQ - 0.2 \* BR + 0.2 \* BS + 0.2 \* CD

+ 0.2 \* CE - 0.2 \* CF + 0.2 \* CG - 0.2 \* CH - 0.2 \* CI - 0.2 \* CJ - 0.2 \* CK

+ 0.2 \* CL - 0.2 \* CM - 0.6 \* CN + 0.2 \* CP + 0.2 \* CQ + 0.2 \* CR - 0.2 \* CS

+ 0.2 \* DE + 0.2 \* DF - 0.2 \* DG - 0.2 \* DH - 0.2 \* DI + 0.2 \* DJ + 0.2 \* DK

- 0.2 \* DL - 0.2 \* DM - 0.2 \* DN - 0.6 \* DP - 0.2 \* DQ + 0.2 \* DR - 0.2 \* DS

- 0.2 \* EF - 0.2 \* EG + 0.2 \* EH + 0.2 \* EI - 0.2 \* EJ - 0.6 \* EK - 0.2 \* EL

+ 0.2 \* EM - 0.2 \* EN - 0.2 \* EP - 0.2 \* EQ - 0.2 \* ER + 0.2 \* ES - 0.2 \* FG

- 0.2 \* FH - 0.2 \* FI + 0.2 \* FJ - 0.2 \* FK + 0.2 \* FL + 0.2 \* FM + 0.2 \* FN

- 0.2 \* FP - 0.2 \* FQ + 0.2 \* FR + 0.2 \* FS - 0.6 \* GH - 0.2 \* GI + 0.2 \* GJ

- 0.2 \* GK - 0.2 \* GL + 0.2 \* GM + 0.2 \* GN + 0.2 \* GP + 0.2 \* GQ - 0.2 \* GR

- 0.2 \* GS + 0.2 \* HI + 0.2 \* HJ + 0.2 \* HK - 0.2 \* HL - 0.2 \* HM - 0.2 \* HN

+ 0.2 \* HP + 0.2 \* HQ - 0.2 \* HR + 0.2 \* HS - 0.2 \* IJ - 0.2 \* IK - 0.2 \* IL

+ 0.2 \* IM + 0.2 \* IN - 0.2 \* IP + 0.2 \* IQ + 0.2 \* IR - 0.6 \* IS + 0.2 \* JK

- 0.6 \* JL + 0.2 \* JM - 0.2 \* JN + 0.2 \* JP - 0.2 \* JQ - 0.2 \* JR - 0.2 \* JS

+ 0.2 \* KL - 0.2 \* KM - 0.2 \* KN - 0.2 \* KP + 0.2 \* KQ - 0.2 \* KR - 0.2 \* KS

+ 0.2 \* LM - 0.2 \* LN - 0.2 \* LP + 0.2 \* LQ - 0.2 \* LR + 0.2 \* LS - 0.2 \* MN

- 0.2 \* MP - 0.2 \* MQ - 0.6 \* MR - 0.2 \* MS - 0.2 \* NP + 0.2 \* NQ + 0.2 \* NR

+ 0.2 \* NS - 0.2 \* PQ + 0.2 \* PR + 0.2 \* PS - 0.2 \* QR - 0.2 \* QS - 0.2 \* RS

[P] = P - 0.2 \* AB + 0.2 \* AC - 0.2 \* AD - 0.2 \* AE + 0.2 \* AF + 0.2 \* AG

- 0.2 \* AH + 0.2 \* AI - 0.6 \* AJ - 0.2 \* AK - 0.2 \* AL + 0.2 \* AM - 0.2 \* AN

+ 0.2 \* AO + 0.2 \* AQ - 0.2 \* AR - 0.2 \* AS + 0.2 \* BC - 0.2 \* BD - 0.2 \* BE

- 0.2 \* BF - 0.2 \* BG - 0.2 \* BH - 0.2 \* BI + 0.2 \* BJ - 0.2 \* BK + 0.2 \* BL

+ 0.2 \* BM + 0.2 \* BN - 0.2 \* BO + 0.2 \* BQ - 0.6 \* BR + 0.2 \* BS - 0.2 \* CD

+ 0.2 \* CE + 0.2 \* CF - 0.6 \* CG + 0.2 \* CH - 0.2 \* CI - 0.2 \* CJ - 0.2 \* CK

+ 0.2 \* CL - 0.2 \* CM - 0.2 \* CN + 0.2 \* CO - 0.2 \* CQ - 0.2 \* CR - 0.2 \* CS

+ 0.2 \* DE + 0.2 \* DF - 0.2 \* DG + 0.2 \* DH - 0.2 \* DI - 0.2 \* DJ - 0.2 \* DK

- 0.2 \* DL + 0.2 \* DM - 0.2 \* DN - 0.6 \* DO + 0.2 \* DQ + 0.2 \* DR + 0.2 \* DS

+ 0.2 \* EF + 0.2 \* EG - 0.2 \* EH - 0.2 \* EI - 0.2 \* EJ + 0.2 \* EK + 0.2 \* EL

- 0.2 \* EM - 0.2 \* EN - 0.2 \* EO - 0.6 \* EQ - 0.2 \* ER + 0.2 \* ES - 0.2 \* FG

- 0.2 \* FH + 0.2 \* FI + 0.2 \* FJ - 0.2 \* FK - 0.6 \* FL - 0.2 \* FM + 0.2 \* FN

- 0.2 \* FO - 0.2 \* FQ - 0.2 \* FR - 0.2 \* FS - 0.2 \* GH - 0.2 \* GI - 0.2 \* GJ

+ 0.2 \* GK - 0.2 \* GL + 0.2 \* GM + 0.2 \* GN + 0.2 \* GO - 0.2 \* GQ - 0.2 \* GR

+ 0.2 \* GS - 0.6 \* HI - 0.2 \* HJ + 0.2 \* HK - 0.2 \* HL - 0.2 \* HM + 0.2 \* HN

+ 0.2 \* HO + 0.2 \* HQ + 0.2 \* HR - 0.2 \* HS + 0.2 \* IJ + 0.2 \* IK + 0.2 \* IL

- 0.2 \* IM - 0.2 \* IN - 0.2 \* IO + 0.2 \* IQ + 0.2 \* IR - 0.2 \* IS - 0.2 \* JK

- 0.2 \* JL - 0.2 \* JM + 0.2 \* JN + 0.2 \* JO - 0.2 \* JQ + 0.2 \* JR + 0.2 \* JS

+ 0.2 \* KL - 0.6 \* KM + 0.2 \* KN - 0.2 \* KO + 0.2 \* KQ - 0.2 \* KR - 0.2 \* KS

+ 0.2 \* LM - 0.2 \* LN - 0.2 \* LO - 0.2 \* LQ + 0.2 \* LR - 0.2 \* LS + 0.2 \* MN

- 0.2 \* MO - 0.2 \* MQ + 0.2 \* MR - 0.2 \* MS - 0.2 \* NO - 0.2 \* NQ - 0.2 \* NR

- 0.6 \* NS - 0.2 \* OQ + 0.2 \* OR + 0.2 \* OS - 0.2 \* QR + 0.2 \* QS - 0.2 \* RS

[Q] = Q - 0.2 \* AB + 0.2 \* AC - 0.2 \* AD + 0.2 \* AE + 0.2 \* AF - 0.2 \* AG

+ 0.2 \* AH - 0.2 \* AI - 0.2 \* AJ + 0.2 \* AK - 0.2 \* AL - 0.2 \* AM - 0.2 \* AN

- 0.6 \* AO + 0.2 \* AP + 0.2 \* AR - 0.2 \* AS - 0.2 \* BC + 0.2 \* BD - 0.2 \* BE

- 0.2 \* BF + 0.2 \* BG + 0.2 \* BH - 0.2 \* BI + 0.2 \* BJ - 0.6 \* BK - 0.2 \* BL

- 0.2 \* BM + 0.2 \* BN - 0.2 \* BO + 0.2 \* BP + 0.2 \* BR - 0.2 \* BS + 0.2 \* CD

- 0.2 \* CE - 0.2 \* CF - 0.2 \* CG - 0.2 \* CH - 0.2 \* CI - 0.2 \* CJ + 0.2 \* CK

- 0.2 \* CL + 0.2 \* CM + 0.2 \* CN + 0.2 \* CO - 0.2 \* CP + 0.2 \* CR - 0.6 \* CS

- 0.2 \* DE + 0.2 \* DF + 0.2 \* DG - 0.6 \* DH + 0.2 \* DI - 0.2 \* DJ - 0.2 \* DK

- 0.2 \* DL + 0.2 \* DM - 0.2 \* DN - 0.2 \* DO + 0.2 \* DP - 0.2 \* DR - 0.2 \* DS

+ 0.2 \* EF + 0.2 \* EG - 0.2 \* EH + 0.2 \* EI - 0.2 \* EJ - 0.2 \* EK - 0.2 \* EL

- 0.2 \* EM + 0.2 \* EN - 0.2 \* EO - 0.6 \* EP + 0.2 \* ER + 0.2 \* ES + 0.2 \* FG

+ 0.2 \* FH - 0.2 \* FI - 0.2 \* FJ - 0.2 \* FK + 0.2 \* FL + 0.2 \* FM - 0.2 \* FN

- 0.2 \* FO - 0.2 \* FP - 0.6 \* FR - 0.2 \* FS - 0.2 \* GH - 0.2 \* GI + 0.2 \* GJ

+ 0.2 \* GK - 0.2 \* GL - 0.6 \* GM - 0.2 \* GN + 0.2 \* GO - 0.2 \* GP - 0.2 \* GR

- 0.2 \* GS - 0.2 \* HI - 0.2 \* HJ - 0.2 \* HK + 0.2 \* HL - 0.2 \* HM + 0.2 \* HN

+ 0.2 \* HO + 0.2 \* HP - 0.2 \* HR - 0.2 \* HS - 0.6 \* IJ - 0.2 \* IK + 0.2 \* IL

- 0.2 \* IM - 0.2 \* IN + 0.2 \* IO + 0.2 \* IP + 0.2 \* IR + 0.2 \* IS + 0.2 \* JK

+ 0.2 \* JL + 0.2 \* JM - 0.2 \* JN - 0.2 \* JO - 0.2 \* JP + 0.2 \* JR + 0.2 \* JS

- 0.2 \* KL - 0.2 \* KM - 0.2 \* KN + 0.2 \* KO + 0.2 \* KP - 0.2 \* KR + 0.2 \* KS

+ 0.2 \* LM - 0.6 \* LN + 0.2 \* LO - 0.2 \* LP + 0.2 \* LR - 0.2 \* LS + 0.2 \* MN

- 0.2 \* MO - 0.2 \* MP - 0.2 \* MR + 0.2 \* MS + 0.2 \* NO - 0.2 \* NP - 0.2 \* NR

+ 0.2 \* NS - 0.2 \* OP - 0.2 \* OR - 0.2 \* OS - 0.2 \* PR + 0.2 \* PS - 0.2 \* RS

[R] = R - 0.2 \* AB - 0.2 \* AC - 0.6 \* AD - 0.2 \* AE + 0.2 \* AF - 0.2 \* AG

- 0.2 \* AH - 0.2 \* AI + 0.2 \* AJ + 0.2 \* AK + 0.2 \* AL - 0.2 \* AM + 0.2 \* AN

+ 0.2 \* AO - 0.2 \* AP + 0.2 \* AQ - 0.2 \* AS - 0.2 \* BC + 0.2 \* BD - 0.2 \* BE

+ 0.2 \* BF + 0.2 \* BG - 0.2 \* BH + 0.2 \* BI - 0.2 \* BJ - 0.2 \* BK + 0.2 \* BL

- 0.2 \* BM - 0.2 \* BN - 0.2 \* BO - 0.6 \* BP + 0.2 \* BQ + 0.2 \* BS - 0.2 \* CD

+ 0.2 \* CE - 0.2 \* CF - 0.2 \* CG + 0.2 \* CH + 0.2 \* CI - 0.2 \* CJ + 0.2 \* CK

- 0.6 \* CL - 0.2 \* CM - 0.2 \* CN + 0.2 \* CO - 0.2 \* CP + 0.2 \* CQ + 0.2 \* CS

+ 0.2 \* DE - 0.2 \* DF - 0.2 \* DG - 0.2 \* DH - 0.2 \* DI - 0.2 \* DJ - 0.2 \* DK

+ 0.2 \* DL - 0.2 \* DM + 0.2 \* DN + 0.2 \* DO + 0.2 \* DP - 0.2 \* DQ + 0.2 \* DS

- 0.2 \* EF + 0.2 \* EG + 0.2 \* EH - 0.6 \* EI + 0.2 \* EJ - 0.2 \* EK - 0.2 \* EL

- 0.2 \* EM + 0.2 \* EN - 0.2 \* EO - 0.2 \* EP + 0.2 \* EQ - 0.2 \* ES + 0.2 \* FG

+ 0.2 \* FH - 0.2 \* FI + 0.2 \* FJ - 0.2 \* FK - 0.2 \* FL - 0.2 \* FM - 0.2 \* FN

+ 0.2 \* FO - 0.2 \* FP - 0.6 \* FQ + 0.2 \* FS + 0.2 \* GH + 0.2 \* GI - 0.2 \* GJ

- 0.2 \* GK - 0.2 \* GL + 0.2 \* GM + 0.2 \* GN - 0.2 \* GO - 0.2 \* GP - 0.2 \* GQ

- 0.6 \* GS - 0.2 \* HI - 0.2 \* HJ + 0.2 \* HK + 0.2 \* HL - 0.2 \* HM - 0.6 \* HN

- 0.2 \* HO + 0.2 \* HP - 0.2 \* HQ - 0.2 \* HS - 0.2 \* IJ - 0.2 \* IK - 0.2 \* IL

+ 0.2 \* IM - 0.2 \* IN + 0.2 \* IO + 0.2 \* IP + 0.2 \* IQ - 0.2 \* IS - 0.6 \* JK

- 0.2 \* JL + 0.2 \* JM - 0.2 \* JN - 0.2 \* JO + 0.2 \* JP + 0.2 \* JQ + 0.2 \* JS

+ 0.2 \* KL + 0.2 \* KM + 0.2 \* KN - 0.2 \* KO - 0.2 \* KP - 0.2 \* KQ + 0.2 \* KS

- 0.2 \* LM - 0.2 \* LN - 0.2 \* LO + 0.2 \* LP + 0.2 \* LQ - 0.2 \* LS + 0.2 \* MN

- 0.6 \* MO + 0.2 \* MP - 0.2 \* MQ + 0.2 \* MS + 0.2 \* NO - 0.2 \* NP - 0.2 \* NQ

- 0.2 \* NS + 0.2 \* OP - 0.2 \* OQ - 0.2 \* OS - 0.2 \* PQ - 0.2 \* PS - 0.2 \* QS

[S] = S - 0.2 \* AB + 0.2 \* AC + 0.2 \* AD + 0.2 \* AE - 0.2 \* AF + 0.2 \* AG

- 0.6 \* AH - 0.2 \* AI - 0.2 \* AJ + 0.2 \* AK + 0.2 \* AL - 0.2 \* AM + 0.2 \* AN

- 0.2 \* AO - 0.2 \* AP - 0.2 \* AQ - 0.2 \* AR - 0.2 \* BC - 0.2 \* BD - 0.6 \* BE

- 0.2 \* BF + 0.2 \* BG - 0.2 \* BH - 0.2 \* BI - 0.2 \* BJ + 0.2 \* BK + 0.2 \* BL

+ 0.2 \* BM - 0.2 \* BN + 0.2 \* BO + 0.2 \* BP - 0.2 \* BQ + 0.2 \* BR - 0.2 \* CD

+ 0.2 \* CE - 0.2 \* CF + 0.2 \* CG + 0.2 \* CH - 0.2 \* CI + 0.2 \* CJ - 0.2 \* CK

- 0.2 \* CL + 0.2 \* CM - 0.2 \* CN - 0.2 \* CO - 0.2 \* CP - 0.6 \* CQ + 0.2 \* CR

- 0.2 \* DE + 0.2 \* DF - 0.2 \* DG - 0.2 \* DH + 0.2 \* DI + 0.2 \* DJ - 0.2 \* DK

+ 0.2 \* DL - 0.6 \* DM - 0.2 \* DN - 0.2 \* DO + 0.2 \* DP - 0.2 \* DQ + 0.2 \* DR

+ 0.2 \* EF - 0.2 \* EG - 0.2 \* EH - 0.2 \* EI - 0.2 \* EJ - 0.2 \* EK - 0.2 \* EL

+ 0.2 \* EM - 0.2 \* EN + 0.2 \* EO + 0.2 \* EP + 0.2 \* EQ - 0.2 \* ER - 0.2 \* FG

+ 0.2 \* FH + 0.2 \* FI - 0.6 \* FJ + 0.2 \* FK - 0.2 \* FL - 0.2 \* FM - 0.2 \* FN

+ 0.2 \* FO - 0.2 \* FP - 0.2 \* FQ + 0.2 \* FR + 0.2 \* GH + 0.2 \* GI - 0.2 \* GJ

+ 0.2 \* GK - 0.2 \* GL - 0.2 \* GM - 0.2 \* GN - 0.2 \* GO + 0.2 \* GP - 0.2 \* GQ

- 0.6 \* GR + 0.2 \* HI + 0.2 \* HJ - 0.2 \* HK - 0.2 \* HL - 0.2 \* HM + 0.2 \* HN

+ 0.2 \* HO - 0.2 \* HP - 0.2 \* HQ - 0.2 \* HR - 0.2 \* IJ - 0.2 \* IK + 0.2 \* IL

+ 0.2 \* IM - 0.2 \* IN - 0.6 \* IO - 0.2 \* IP + 0.2 \* IQ - 0.2 \* IR - 0.2 \* JK

- 0.2 \* JL - 0.2 \* JM + 0.2 \* JN - 0.2 \* JO + 0.2 \* JP + 0.2 \* JQ + 0.2 \* JR

- 0.6 \* KL - 0.2 \* KM + 0.2 \* KN - 0.2 \* KO - 0.2 \* KP + 0.2 \* KQ + 0.2 \* KR

+ 0.2 \* LM + 0.2 \* LN + 0.2 \* LO - 0.2 \* LP - 0.2 \* LQ - 0.2 \* LR - 0.2 \* MN

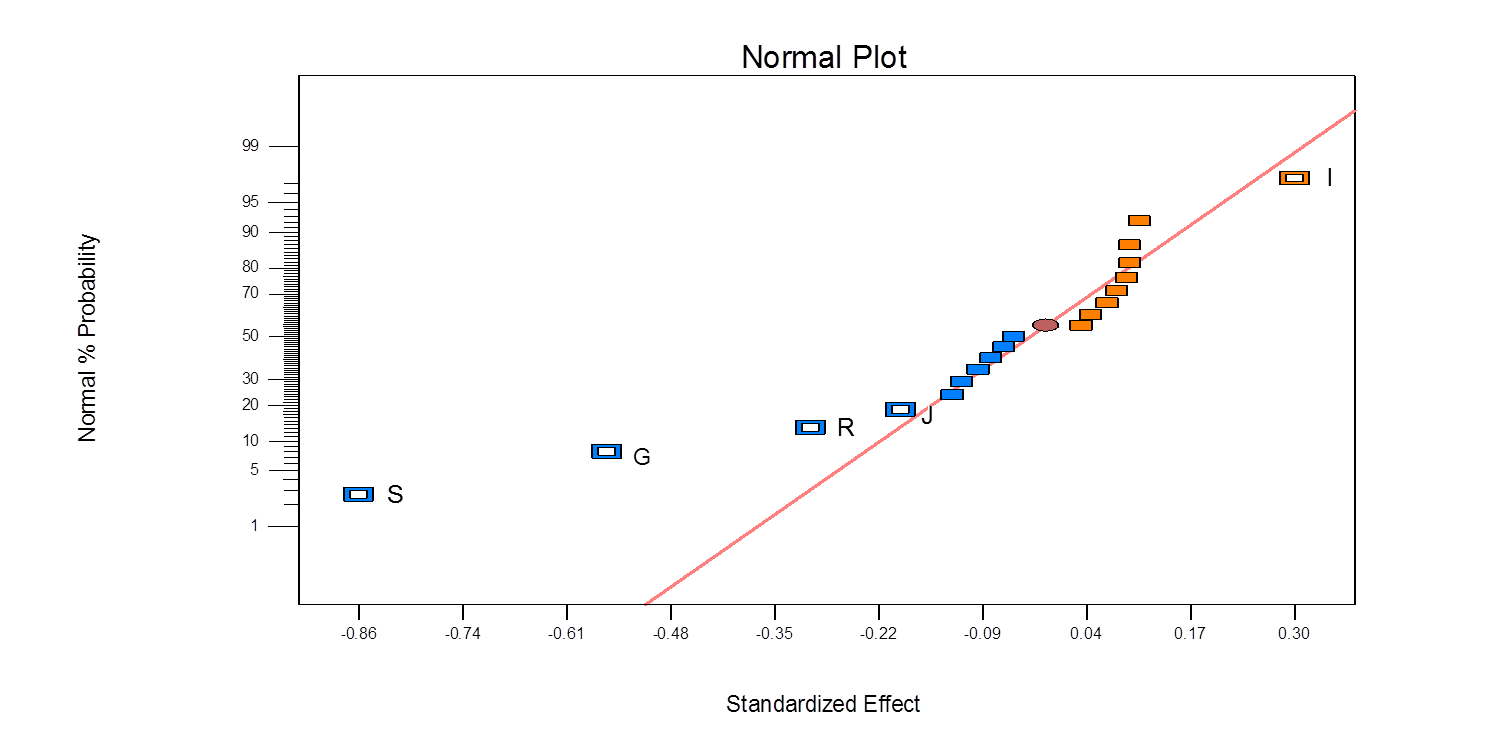
- 0.2 \* MO - 0.2 \* MP + 0.2 \* MQ + 0.2 \* MR + 0.2 \* NO - 0.6 \* NP + 0.2 \* NQ

- 0.2 \* NR + 0.2 \* OP - 0.2 \* OQ - 0.2 \* OR + 0.2 \* PQ - 0.2 \* PR - 0.2 \* QR

(b) Show that for the 20-run Plackett-Burman in Table P8.10, the weights (or correlations) that multiply the two-factor interactions in each alias chain are either -0.2, 0.2, or -0.6. Of the 153 interactions that are aliased with each main effect, 144 have weights of -0.2 or 0.2, while nine interactions have weights of -0.6

See the Design Expert output shown above.

(c) Verify that the five largest main effects are *S*, *G*, *R*, *I* and *J*.



Design Expert Output

**Response 2 Response rate  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 6.36 5 1.27 35.43 < 0.0001 significant

*G-Sticker* *1.55* *1* *1.55* *43.03* *< 0.0001*  
  *I-Copy Message* *0.44* *1* *0.44* *12.20* *0.0036*  
  *J-Letter headline* *0.18* *1* *0.18* *5.13* *0.0399*  
  *R-2nd buckslip* *0.46* *1* *0.46* *12.86* *0.0030*  
  *S-Interest rate* *3.73* *1* *3.73* *103.91* *< 0.0001*  
 Residual 0.50 14 0.036  
 Cor Total 6.87 19

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 1.30 1 0.042 1.21 1.39

G-Sticker -0.28 1 0.042 -0.37 -0.19 1.00  
 J-Copy Message 0.15 1 0.042 0.057 0.24 1.00  
 K-Letter headline -0.096 1 0.042 -0.19 -5.106E-003 1.00  
 S-2nd buckslip -0.15 1 0.042 -0.24 -0.061 1.00  
 T-Interest rate -0.43 1 0.042 -0.52 -0.34 1.00

(d) Factor *S* (interest rate) and *G* (presence of a sticker) are by far the largest main effects. The correlation between the main effect of *R* (2nd buckslip) and the *SG* interaction is -0.6. This means that a significant *SG* interaction would bias the estimate of the main effect of *R* by -0.6 times the value of the interaction. This suggests that it may not be the main effect of factor *R* that is important, but the two factor interaction between *S* and *G*.

(e) Since this design projects into a full factorial in any three factors, obtain the projection in factors *S*, *G* and *R* and verify that it is a full factorial with some runs replicated. Fit a full factorial model involving all three of these factors and the interactions (you will need to use a regression program to do this). Show that *S*, *G* and the *SG* interaction are significant.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Test Cell | *G* | *R* | *S* | *GR* | *GS* | *RS* | *GRS* | run | Orders | Resp Rate |
| 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | *gr* | 52 | 1.04% |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | *grs* | 38 | 0.76% |
| 3 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | *gs* | 42 | 0.84% |
| 4 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | (1) | 134 | 2.68% |
| 5 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | (1) | 104 | 2.08% |
| 6 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | *g* | 60 | 1.20% |
| 7 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | *gr* | 61 | 1.22% |
| 8 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | *s* | 68 | 1.36% |
| 9 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | *gr* | 57 | 1.14% |
| 10 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | *gs* | 30 | 0.60% |
| 11 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | *r* | 108 | 2.16% |
| 12 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | *rs* | 39 | 0.78% |
| 13 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | *rs* | 40 | 0.80% |
| 14 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | *rs* | 49 | 0.98% |
| 15 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | *gs* | 37 | 0.74% |
| 16 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | (1) | 99 | 1.98% |
| 17 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | *gr* | 86 | 1.72% |
| 18 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | *rs* | 43 | 0.86% |
| 19 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | *gs* | 47 | 0.94% |
| 20 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | *gr* | 104 | 2.08% |

The projected design is shown in the above table, along with the terms to estimate the interaction. Note that all 8 corners of the 23 are represented at least once. The analysis shown below shows that the significant factors are *G* and *S* (*A* and *C*). *R* (*B*) is not significant. The *GS* (*AC*) interaction has a higher *t*-value than the *R* (*C*) factor that it was confounded with in the original Plackett-Burman design.

Design Expert Output

**Response 2 Response rate  
 ANOVA for selected factorial model  
 Analysis of variance table [Partial sum of squares - Type III]**

**Sum of** **Mean** **F** **p-value**  
 **Source** **Squares** **df** **Square** **Value** **Prob > F**

Model 6.20 7 0.89 15.87 < 0.0001 significant

*A-G - Sticker* *1.31* *1* *1.31* *23.50* *0.0004*  
  *B-R - 2nd buckslip* *0.048* *1* *0.048* *0.86* *0.3718*  
  *C-S - Interest rate* *1.91* *1* *1.91* *34.23* *< 0.0001*  
  *AB* *0.074* *1* *0.074* *1.33* *0.2706*  
  *AC* *0.29* *1* *0.29* *5.25* *0.0408*  
  *BC* *0.063* *1* *0.063* *1.12* *0.3099*  
  *ABC* *0.026* *1* *0.026* *0.46* *0.5084*  
 Pure Error 0.67 12 0.056  
 Cor Total 6.87 19

**Coefficient** **Standard** **95% CI** **95% CI**  
 **Factor** **Estimate** **df** **Error** **Low** **High** **VIF**

Intercept 1.33 1 0.066 1.18 1.47

A-G - Sticker -0.32 1 0.066 -0.46 -0.18 1.56  
 B-R - 2nd buckslip -0.061 1 0.066 -0.21 0.083 1.56  
 C-S - Interest rate -0.39 1 0.066 -0.53 -0.24 1.56  
 AB 0.076 1 0.066 -0.068 0.22 1.56  
 AC 0.15 1 0.066 7.414E-003 0.30 1.56  
 BC -0.070 1 0.066 -0.21 0.074 1.56  
 ABC 0.045 1 0.066 -0.099 0.19 1.00

**8.31.** Consider the following experiment:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Run | Treatment Combination |  | *A* | *B* | *C* | *D=AB* | *E=-AC* |
| 1 | *d* |  | -1 | -1 | -1 | 1 | -1 |
| 2 | *ae* |  | 1 | -1 | -1 | -1 | 1 |
| 3 | *b* |  | -1 | 1 | -1 | -1 | -1 |
| 4 | *abde* |  | 1 | 1 | -1 | 1 | 1 |
| 5 | *cde* |  | -1 | -1 | 1 | 1 | 1 |
| 6 | *ac* |  | 1 | -1 | 1 | -1 | -1 |
| 7 | *bce* |  | -1 | 1 | 1 | -1 | 1 |
| 8 | *abcd* |  | 1 | 1 | 1 | 1 | -1 |

Answer the following questions about this experiment:

(a) How many factors did this experiment investigate?

5.

(b) How many factors are in the basic design?

3.

(c) Assume that the factors in the experiment are represented by the initial letters of the alphabet (i.e. A, B, etc.), what are the design generators for the factors beyond the basic design?

*D* = *AB* and *E* = –*AC*, therefore *I* = *ABD* – *ACE.*

(d) Is this design a principal fraction?

No, it is not a principal fraction.

(e) What is the complete defining relation?

*I* = *ABD* –*ACE* – *BDCE.*

(f) What is the resolution of this design?

It is a resolution III design.

**8.32.** Consider the following experiment:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | Treatment Combination |  | *A* | *B* | *C* | *D=ABC* |  | *y* |
| 1 | (1) |  | -1 | -1 | -1 | -1 |  | 8 |
| 2 | *ad* |  | 1 | -1 | -1 | 1 |  | 10 |
| 3 | *bd* |  | -1 | 1 | -1 | 1 |  | 12 |
| 4 | *ab* |  | 1 | 1 | -1 | -1 |  | 7 |
| 5 | *cd* |  | -1 | -1 | 1 | 1 |  | 13 |
| 6 | *ac* |  | 1 | -1 | 1 | -1 |  | 6 |
| 7 | *bc* |  | -1 | 1 | 1 | -1 |  | 5 |
| 8 | *abcd* |  | 1 | 1 | 1 | 1 |  | 11 |
|  |  | Avg (+) | 8.5 | 4.5 | 8.75 | 11.5 |  |  |
|  |  | Avg (-) | 9.5 | 9.25 | 9.25 | 26 |  |  |
|  |  | Effect | -0.5 | -2.375 | -0.25 | -7.25 |  |  |

(a) How many factors did this experiment investigate?

4.

(b) What is the resolution of this design?

IV.

(c) Calculate the estimates of the effects.

See table above.

(d) What is the complete defining relation?

*I* = *ABCD*.

**8.33.** An unreplicated 25-1 fractional factorial experiment with four center points has been run in a chemical process. The response variable is molecular weight. The experimenter has used the following factors:

|  |  |  |
| --- | --- | --- |
| Factor | Natural levels | Coded levels (x’s) |
| *A* – time | 20, 40 (minutes) | -1, 1 |
| *B* – temperature | 160, 180 (deg C) | -1, 1 |
| *C* – concentration | 30, 60 (percent) | -1, 1 |
| *D* – stirring rate | 100, 150 (rpm) | -1, 1 |
| *E* – catalyst type | 1, 2 (type) | -1, 1 |

Suppose that the prediction equation from this experiment is . What is the predicted response at *A* = 30, *B* = 165, *C* = 50, *D* = 135, and *E* = 1?

*A* = 3, is *A* = 0 in coded units, *B* = 165 is *B* = -0.5 in coded units. These are the only two factors in the prediction equation.



**8.34.** An unreplicated 24-1 fractional factorial experiment has been run. The experimenter has used the following factors:

|  |  |  |
| --- | --- | --- |
| Factor | Natural levels | Coded levels (x’s) |
| *A* | 20, 50 | -1, 1 |
| *B* | 200, 280 | -1, 1 |
| *C* | 50, 100 | -1, 1 |
| *D* | 150, 200 | -1, 1 |

(a) Suppose that this design has four center points that average 100. The average of the 8 factorial design points is 95. What is the sum of squares for pure quadratic curvature?



(b) Suppose that the prediction equation that results from this experiment is . Find the predicted response at *A* = 41, *B* = 280, *C* = 60 and *D* = 185.

*A* = 41, is *A* = 0.4 in coded units, *B* = 280 is *B* = 1 in coded units. These are the only two factors in the prediction equation.



**8.35.** A 26-2 factorial experiment with three replicates has been run in a pharmaceutical drug manufacturing process. The experimenter has used the following factors:

|  |  |  |
| --- | --- | --- |
| Factor | Natural levels | Coded levels (x’s) |
| *A* | 50, 100 | -1, 1 |
| *B* | 20, 60 | -1, 1 |
| *C* | 10, 30 | -1, 1 |
| *D* | 12, 18 | -1, 1 |
| *E* | 15, 30 | -1, 1 |
| *F* | 60, 100 | -1, 1 |

(a) If two main effects and one two-factor interaction are included in the final model, how many degrees of freedom for error will be available?

A 26-2 factorial experiment has 16 runs. We are estimating four model parameters – the intercept, two main effects and one two-factor interactions. This leaves 12 degrees of freedom for error. This assumes a non-hierarchical model.

(b) Suppose that the significant factors are *A*, *C*, *AB* and *AC*. What other effects need to be included to obtain a hierarchical model?

The hierarchical model would include *A*, *B*, *C*, *AB* and *AC*.

**8.36.** Consider the following design:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D* | *E* | *y* | Block = *ABC* |
| 1 | -1 | -1 | -1 | -1 | -1 | 63 | -1 |
| 2 | 1 | -1 | -1 | -1 | 1 | 21 | 1 |
| 3 | -1 | 1 | -1 | -1 | 1 | 36 | 1 |
| 4 | 1 | 1 | -1 | -1 | -1 | 99 | -1 |
| 5 | -1 | -1 | 1 | -1 | 1 | 24 | 1 |
| 6 | 1 | -1 | 1 | -1 | -1 | 66 | -1 |
| 7 | -1 | 1 | 1 | -1 | -1 | 71 | -1 |
| 8 | 1 | 1 | 1 | -1 | 1 | 54 | 1 |
| 9 | -1 | -1 | -1 | 1 | -1 | 23 | -1 |
| 10 | 1 | -1 | -1 | 1 | 1 | 74 | 1 |
| 11 | -1 | 1 | -1 | 1 | 1 | 80 | 1 |
| 12 | 1 | 1 | -1 | 1 | -1 | 33 | -1 |
| 13 | -1 | -1 | 1 | 1 | 1 | 63 | 1 |
| 14 | 1 | -1 | 1 | 1 | -1 | 21 | -1 |
| 15 | -1 | 1 | 1 | 1 | -1 | 44 | -1 |
| 16 | 1 | 1 | 1 | 1 | 1 | 96 | 1 |
| Avg(+) | 58 | 64.125 | 54.875 | 54.25 | 56 |  |  |
| Avg(-) | 50.5 | 44.375 | 53.625 | 54.25 | 52.5 |  |  |
| Effect | 3.75 | 9.875 | 0.625 | 0 | 1.75 |  |  |

(a) What is the generator for column *E*?

*E* = *ABC.*

(b) If *ABC* is confounded with blocks, run 1 above goes in the \_\_\_ block. Answer either + or –.

*ABC* goes in to the – block.

(c) What is the resolution of this design?

Resolution IV design.

(d) Find the estimates of the main effects and their aliases.

See table above.

**8.37.** Consider the following design:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D* | *E* | *y* | Block=*ABE* |
| 1 | -1 | -1 | -1 | -1 | -1 | 65 | -1 |
| 2 | 1 | -1 | -1 | -1 | 1 | 25 | -1 |
| 3 | -1 | 1 | -1 | -1 | 1 | 30 | -1 |
| 4 | 1 | 1 | -1 | -1 | -1 | 89 | -1 |
| 5 | -1 | -1 | 1 | -1 | 1 | 25 | 1 |
| 6 | 1 | -1 | 1 | -1 | -1 | 60 | 1 |
| 7 | -1 | 1 | 1 | -1 | -1 | 70 | 1 |
| 8 | 1 | 1 | 1 | -1 | 1 | 50 | 1 |
| 9 | -1 | -1 | -1 | 1 | 1 | 20 | 1 |
| 10 | 1 | -1 | -1 | 1 | -1 | 70 | 1 |
| 11 | -1 | 1 | -1 | 1 | -1 | 80 | 1 |
| 12 | 1 | 1 | -1 | 1 | 1 | 30 | 1 |
| 13 | -1 | -1 | 1 | 1 | -1 | 60 | -1 |
| 14 | 1 | -1 | 1 | 1 | 1 | 20 | -1 |
| 15 | -1 | 1 | 1 | 1 | 1 | 40 | -1 |
| 16 | 1 | 1 | 1 | 1 | -1 | 90 | -1 |
| Avg(+) | 54.25 | 59.875 | 51.875 | 51.25 | 30 |  |  |
| Avg(–) | 48.75 | 43.125 | 51.125 | 51.75 | 73 |  |  |
| Effect | 2.75 | 8.375 | 0.375 | -0.25 | -21.5 |  |  |

(a) What is the generator for column *E*?

*E = –ABCD.*

(b) If *ABE* is confounded with blocks, run 16 above goes in the \_\_\_ block. Answer either + or –.

Run 16 is in the – block.

(c) What is the resolution of this design?

It is a resolution V design.

(d) Find the estimates of the main effects and their aliases.

See the table above.

**8.38.** Consider the following design:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D=–ABC* | *E=–BC* | *y* | Block = *AB* |
| 1 | -1 | -1 | -1 | 1 | -1 | 50 | 1 |
| 2 | 1 | -1 | -1 | -1 | -1 | 20 | -1 |
| 3 | -1 | 1 | -1 | -1 | 1 | 40 | -1 |
| 4 | 1 | 1 | -1 | 1 | 1 | 25 | 1 |
| 5 | -1 | -1 | 1 | -1 | 1 | 45 | 1 |
| 6 | 1 | -1 | 1 | 1 | 1 | 30 | -1 |
| 7 | -1 | 1 | 1 | 1 | -1 | 40 | -1 |
| 8 | 1 | 1 | 1 | -1 | -1 | 30 | 1 |

(a) What is the generator for column *D*?

*D = –ABC*

(b) What is the generator for column *E*?

*E = –BC*

(c) If this design were run in two blocks with the *AB* interaction confounded with blocks, the run *d* would be in the block where the sign of *AB* is \_\_\_\_? Answer either + or –.

Run *d*, is the combination (-1, -1, -1, 1, -1), it is run 1 and is in the + block.

**8.39.** Consider the following design:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Run | A | B | C | D | E | y |
| 1 | -1 | -1 | -1 | 1 | 1 | 40 |
| 2 | 1 | -1 | -1 | -1 | 1 | 10 |
| 3 | -1 | 1 | -1 | -1 | -1 | 30 |
| 4 | 1 | 1 | -1 | 1 | -1 | 20 |
| 5 | -1 | -1 | 1 | -1 | -1 | 40 |
| 6 | 1 | -1 | 1 | 1 | -1 | 30 |
| 7 | -1 | 1 | 1 | 1 | 1 | 20 |
| 8 | 1 | 1 | 1 | -1 | 1 | 30 |
| 9 | 1 | 1 | 1 | -1 | -1 |  |
| 10 | -1 | 1 | 1 | 1 | -1 |  |
| 11 | 1 | -1 | 1 | 1 | 1 |  |
| 12 | -1 | -1 | 1 | -1 | 1 |  |
| 13 | 1 | 1 | -1 | 1 | 1 |  |
| 14 | -1 | 1 | -1 | -1 | 1 |  |
| 15 | 1 | -1 | -1 | -1 | -1 |  |
| 16 | -1 | -1 | -1 | 1 | -1 |  |

(a) What is the generator for column *D*?

*D = –ABC*

(b) What is the generator for column *E*?

*E = BC*

(c) If this design were folded over, what is the resolution of the combined design?

Resolution IV design.

**8.40S.** In an article in *Quality Engineering* (“An Application of Fractional Factorial Experimental Designs,” 1988, Vol. 1 pp. 19-23) M.B. Kilgo describes an experiment to determine the effect of CO2 pressure (*A*), CO2 temperature (*B*), peanut moisture (*C*), CO2 flow rate (*D*), and peanut particle size (*E*) on the total yield of oil per batch of peanuts (*y*). The levels she used for these factors are shown in Table P8.11. She conducted the 16-run fractional factorial experiment shown in Table P8.12.

**Table P8.11**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *A* | *B* | *C* | *D* | *E* |
| Coded | Pressure | Temp | Moisture | Flow | Particle Size |
| Level | (bar) | (C) | (% by weight) | (liters/min) | (mm) |
| -1 | 415 | 25 | 5 | 40 | 1.28 |
| 1 | 550 | 95 | 15 | 60 | 4.05 |

**Table P8.12**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *A* | *B* | *C* | *D* | *E* | y |
| 1 | 415 | 25 | 5 | 40 | 1.28 | 63 |
| 2 | 550 | 25 | 5 | 40 | 4.05 | 21 |
| 3 | 415 | 95 | 5 | 40 | 4.05 | 36 |
| 4 | 550 | 95 | 5 | 40 | 1.28 | 99 |
| 5 | 415 | 25 | 15 | 40 | 4.05 | 24 |
| 6 | 550 | 25 | 15 | 40 | 1.28 | 66 |
| 7 | 415 | 95 | 15 | 40 | 1.28 | 71 |
| 8 | 550 | 95 | 15 | 40 | 4.05 | 54 |
| 9 | 415 | 25 | 5 | 60 | 4.05 | 23 |
| 10 | 550 | 25 | 5 | 60 | 1.28 | 74 |
| 11 | 415 | 95 | 5 | 60 | 1.28 | 80 |
| 12 | 550 | 95 | 5 | 60 | 4.05 | 33 |
| 13 | 415 | 25 | 15 | 60 | 1.28 | 63 |
| 14 | 550 | 25 | 15 | 60 | 4.05 | 21 |
| 15 | 415 | 95 | 15 | 60 | 4.05 | 44 |
| 16 | 550 | 95 | 15 | 60 | 1.28 | 96 |

1. What type of design has been used? Identify the defining relation and the alias relationships.

A , 16-run design, with *I*=*–ABCDE.*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *(–ABCDE)=* | *–BCDE* | *A* | *=* | *–BCDE* |
| *B* | *(–ABCDE)=* | *–ACDE* | *B* | *=* | *–ACDE* |
| *C* | *(–ABCDE)=* | *–ABDE* | *C* | *=* | *–ABDE* |
| *D* | *(–ABCDE)=* | *–ABCE* | *D* | *=* | *–ABCE* |
| *E* | *(–ABCDE)=* | *–ABCD* | *E* | *=* | *–ABCD* |
| *AB* | *(–ABCDE)=* | *–CDE* | *AB* | *=* | *–CDE* |
| *AC* | *(–ABCDE)=* | *–BDE* | *AC* | *=* | *–BDE* |
| *AD* | *(–ABCDE)=* | *–BCE* | *AD* | *=* | *–BCE* |
| *AE* | *(–ABCDE)=* | *–BCD* | *AE* | *=* | *–BCD* |
| *BC* | *(–ABCDE)=* | *–ADE* | *BC* | *=* | *–ADE* |
| *BD* | *(–ABCDE)=* | *–ACE* | *BD* | *=* | *–ACE* |
| *BE* | *(–ABCDE)=* | *–ACD* | *BE* | *=* | *–ACD* |
| *CD* | *(–ABCDE)=* | *–ABE* | *CD* | *=* | *–ABE* |
| *CE* | *(–ABCDE)=* | *–ABD* | *CE* | *=* | *–ABD* |
| *DE* | *(–ABCDE)=* | *–ABC* | *DE* | *=* | *–ABC* |

1. Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Error A 7.5 225 2.17119

Model B 19.75 1560.25 15.056

Error C 1.25 6.25 0.0603107

Error D 0 0 0

Model E 44.5 7921 76.4354

Error AB 5.25 110.25 1.06388

Error AC 1.25 6.25 0.0603107

Error AD -4 64 0.617582

Error AE 7 196 1.89134

Error BC 3 36 0.34739

Error BD -1.75 12.25 0.118209

Error BE 0.25 0.25 0.00241243

Error CD 2.25 20.25 0.195407

Error CE -6.25 156.25 1.50777

Error DE 3.5 49 0.472836

Lenth's ME 11.5676

Lenth's SME 23.4839



1. Perform an appropriate statistical analysis to test the hypothesis that the factors identified in part above have a significant effect on the yield of peanut oil.

Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 9481.25 2 4740.63 69.89 < 0.0001 significant

*B* *1560.25* *1* *1560.25* *23.00* *0.0003*

*E* *7921.00* *1* *7921.00* *116.78* *< 0.0001*

Residual 881.75 13 67.83

Cor Total 10363.00 15

The Model F-value of 69.89 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 8.24 R-Squared 0.9149

Mean 54.25 Adj R-Squared 0.9018

C.V. 15.18 Pred R-Squared 0.8711

PRESS 1335.67 Adeq Precision 18.017

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 54.25 1 2.06 49.80 58.70

B-Temperature 9.88 1 2.06 5.43 14.32 1.00

E-Particle Size 22.25 1 2.06 17.80 26.70 1.00

1. Fit a model that could be used to predict peanut oil yield in terms of the factors that you have identified as important.

Design Expert Output

**Final Equation in Terms of Coded Factors:**

Yield =

+54.25

+9.88 \* B

+22.25 \* E

**Final Equation in Terms of Actual Factors:**

Yield =

-5.49175

+0.28214 \* Temperature

+16.06498 \* Particle Size

1. Analyze the residuals from this experiment and comment on model adequacy.

The residual plots are satisfactory. There is a slight tendency for the variability of the residuals to increase with the predicted value of *y*.





**8.41S.** A 16-run fractional factorial experiment in nine factors was conducted by Chrysler Motors Engineering and described in the article “Sheet Molded Compound Process Improvement,” by P.I. Hsieh and D.E. Goodwin (*Fourth* *Symposium on Taguchi Methods*, American Supplier Institute, Dearborn, MI, 1986, pp. 13-21). The purpose was to reduce the number of defects in the finish of sheet-molded grill opening panels. The design, and the resulting number of defects, *c*, observed on each run, is shown in Table P8.14. This is a resolution III fraction with generators *E=BD, F=BCD, G=AC, H=ACD*, and *J=AB.*

**Table P8.14**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | c |  | F&T’s Modification |
| 1 | – | – | – | – | + | – | + | – | + | 56 | 7.48 | 7.52 |
| 2 | + | – | – | – | + | – | – | + | – | 17 | 4.12 | 4.18 |
| 3 | – | + | – | – | – | + | + | – | – | 2 | 1.41 | 1.57 |
| 4 | + | + | – | – | – | + | – | + | + | 4 | 2.00 | 2.12 |
| 5 | – | – | + | – | + | + | – | + | + | 3 | 1.73 | 1.87 |
| 6 | + | – | + | – | + | + | + | – | – | 4 | 2.00 | 2.12 |
| 7 | – | + | + | – | – | – | – | + | – | 50 | 7.07 | 7.12 |
| 8 | + | + | + | – | – | – | + | – | + | 2 | 1.41 | 1.57 |
| 9 | – | – | – | + | – | + | + | + | + | 1 | 1.00 | 1.21 |
| 10 | + | – | – | + | – | + | – | – | – | 0 | 0.00 | 0.50 |
| 11 | – | + | – | + | + | – | + | + | – | 3 | 1.73 | 1.87 |
| 12 | + | + | – | + | + | – | – | – | + | 12 | 3.46 | 3.54 |
| 13 | – | – | + | + | – | – | – | – | + | 3 | 1.73 | 1.87 |
| 14 | + | – | + | + | – | – | + | + | – | 4 | 2.00 | 2.12 |
| 15 | – | + | + | + | + | + | – | – | – | 0 | 0.00 | 0.50 |
| 16 | + | + | + | + | + | + | + | + | + | 0 | 0.00 | 0.50 |

1. Find the defining relation and the alias relationships in this design.

*I = ABJ = ACG = BDE = CEF = DGH = FHJ = ABFH = ACDH = ADEJ = AEFG = BCDF = BCGJ = BEGH = CEHJ = DFGJ = ABCEH = ABDFG = ACDFJ = ADEFH = AEGHJ = BCDHJ = BCFGH = BEFGJ = CDEGJ = ABCDEG = ABCEFJ = ABDGHJ = ACFGHJ = BDEFHJ = CDEFGH = ABCDEFGHJ*

1. Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

The effects are shown below in the *Design Expert* output. The normal probability plot of effects identifies factors *A*, *D*, *F*, and interactions *AD*, *AF*, *BC*, *BG* as important.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A -9.375 351.562 7.75573

Model B -1.875 14.0625 0.310229

Model C -3.625 52.5625 1.15957

Model D -14.375 826.562 18.2346

Error E 3.625 52.5625 1.15957

Model F -16.625 1105.56 24.3895

Model G -2.125 18.0625 0.398472

Error H 0.375 0.5625 0.0124092

Error J 0.125 0.0625 0.0013788

Model AD 11.625 540.563 11.9252

Error AE 2.125 18.0625 0.398472

Model AF 9.875 390.063 8.60507

Error AH 1.375 7.5625 0.166834

Model BC 11.375 517.563 11.4178

Model BG -12.625 637.562 14.0651

Lenth's ME 13.9775

Lenth's SME 28.3764



1. Fit an appropriate model using the factors identified in part (b) above.

The analysis of variance and corresponding model is shown below. Factors *B*, *C*, and *G* are included for hierarchal purposes.

Design Expert Output

**Response:** **c**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 4454.13 10 445.41 28.26 0.0009 significant

*A* *351.56* *1* *351.56* *22.30* *0.0052*

*B* *14.06* *1* *14.06* *0.89* *0.3883*

*C* *52.56* *1* *52.56* *3.33* *0.1274*

*D* *826.56* *1* *826.56* *52.44* *0.0008*

*F* *1105.56* *1* *1105.56* *70.14* *0.0004*

*G* *18.06* *1* *18.06* *1.15* *0.3333*

*AD* *540.56* *1* *540.56* *34.29* *0.0021*

*AF* *390.06* *1* *390.06* *24.75* *0.0042*

*BC* *517.56* *1* *517.56* *32.84* *0.0023*

*BG* *637.56* *1* *637.56* *40.45* *0.0014*

Residual 78.81 5 15.76

Cor Total 4532.94 15

The Model F-value of 28.26 implies the model is significant. There is only

a 0.09% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 3.97 R-Squared 0.9826

Mean 10.06 Adj R-Squared 0.9478

C.V. 39.46 Pred R-Squared 0.8220

PRESS 807.04 Adeq Precision 17.771

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 10.06 1 0.99 7.51 12.61

A-A -4.69 1 0.99 -7.24 -2.14 1.00

B-B -0.94 1 0.99 -3.49 1.61 1.00

C-C -1.81 1 0.99 -4.36 0.74 1.00

D-D -7.19 1 0.99 -9.74 -4.64 1.00

F-F -8.31 1 0.99 -10.86 -5.76 1.00

G-G -1.06 1 0.99 -3.61 1.49 1.00

AD 5.81 1 0.99 3.26 8.36 1.00

AF 4.94 1 0.99 2.39 7.49 1.00

BC 5.69 1 0.99 3.14 8.24 1.00

BG -6.31 1 0.99 -8.86 -3.76 1.00

**Final Equation in Terms of Coded Factors:**

c =

+10.06

-4.69 \* A

-0.94 \* B

-1.81 \* C

-7.19 \* D

-8.31 \* F

-1.06 \* G

+5.81 \* A \* D

+4.94 \* A \* F

+5.69 \* B \* C

-6.31 \* B \* G

**Final Equation in Terms of Actual Factors:**

c =

+10.06250

-4.68750 \* A

-0.93750 \* B

-1.81250 \* C

-7.18750 \* D

-8.31250 \* F

-1.06250 \* G

+5.81250 \* A \* D

+4.93750 \* A \* F

+5.68750 \* B \* C

-6.31250 \* B \* G

1. Plot the residuals from this model versus the predicted number of defects. Also, prepare a normal probability plot of the residuals. Comment on the adequacy of these plots.



There is a significant problem with inequality of variance. This is likely caused by the response variable being a count. A transformation may be appropriate.

1. In part (d) you should have noticed an indication that the variance of the response is not constant (considering that the response is a count, you should have expected this). The previous table also shows a transformation on c, the square root, that is a widely used variance stabilizing transformation for count data (refer to the discussion of variance stabilizing transformations in Chapter 3). Repeat parts (a) through (d) using the transformed response and comment on your results. Specifically, are the residual plots improved?

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Error A -0.895 3.2041 4.2936

Model B -0.3725 0.555025 0.743752

Error C -0.6575 1.72922 2.31722

Model D -2.1625 18.7056 25.0662

Error E 0.4875 0.950625 1.27387

Model F -2.6075 27.1962 36.4439

Model G -0.385 0.5929 0.794506

Error H 0.27 0.2916 0.390754

Error J 0.06 0.0144 0.0192965

Error AD 1.145 5.2441 7.02727

Error AE 0.555 1.2321 1.65106

Error AF 0.86 2.9584 3.96436

Error AH 0.0425 0.007225 0.00968175

Error BC 0.6275 1.57502 2.11059

Model BG -1.61 10.3684 13.894

Lenth's ME 2.27978

Lenth's SME 4.62829

The analysis of the data with the square root transformation identifies only *D*, *F*, the *BG* interaction as being significant. The original analysis identified factor *A* and several two factor interactions as being significant.



Design Expert Output

**Response:** **sqrt**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 57.42 5 11.48 6.67 0.0056 significant

*B* *0.56* *1* *0.56* *0.32* *0.5826*

*D* *18.71* *1* *18.71* *10.87* *0.0081*

*F* *27.20* *1* *27.20* *15.81* *0.0026*

*G* *0.59* *1* *0.59* *0.34* *0.5702*

*BG* *10.37* *1* *10.37* *6.03* *0.0340*

Residual 17.21 10 1.72

Cor Total 74.62 15

The Model F-value of 6.67 implies the model is significant. There is only

a 0.56% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 1.31 R-Squared 0.7694

Mean 2.32 Adj R-Squared 0.6541

C.V. 56.51 Pred R-Squared 0.4097

PRESS 44.05 Adeq Precision 8.422

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 2.32 1 0.33 1.59 3.05

B-B -0.19 1 0.33 -0.92 0.54 1.00

D-D -1.08 1 0.33 -1.81 -0.35 1.00

F-F -1.30 1 0.33 -2.03 -0.57 1.00

G-G -0.19 1 0.33 -0.92 0.54 1.00

BG -0.80 1 0.33 -1.54 -0.074 1.00

**Final Equation in Terms of Coded Factors:**

sqrt =

+2.32

-0.19 \* B

-1.08 \* D

-1.30 \* F

-0.19 \* G

-0.80 \* B \* G

**Final Equation in Terms of Actual Factors:**

sqrt =

+2.32125

-0.18625 \* B

-1.08125 \* D

-1.30375 \* F

-0.19250 \* G

-0.80500 \* B \* G

The residual plots are acceptable; although, there appears to be a slight “u” shape to the residuals versus predicted plot.



1. There is a modification to the square root transformation proposed by Freeman and Tukey (“Transformations Related to the Angular and the Square Root,” *Annals of Mathematical Statistics*, Vol. 21, 1950, pp. 607-611) that improves its performance. F&T’s modification to the square root transformation is:



Rework parts (a) through (d) using this transformation and comment on the results. (For an interesting discussion and analysis of this experiment, refer to “Analysis of Factorial Experiments with Defects or Defectives as the Response,” by S. Bisgaard and H.T. Fuller, *Quality Engineering*, Vol. 7, 1994-5, pp. 429-443.)

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Error A -0.86 2.9584 4.38512

Model B -0.325 0.4225 0.626255

Error C -0.605 1.4641 2.17018

Model D -1.995 15.9201 23.5977

Error E 0.5025 1.01002 1.49712

Model F -2.425 23.5225 34.8664

Model G -0.4025 0.648025 0.960541

Error H 0.225 0.2025 0.300158

Error J 0.0275 0.003025 0.00448383

Error AD 1.1625 5.40562 8.01254

Error AE 0.505 1.0201 1.51205

Error AF 0.8825 3.11523 4.61757

Error AH 0.0725 0.021025 0.0311645

Error BC 0.7525 2.26503 3.35735

Model BG -1.54 9.4864 14.0613

Lenth's ME 2.14001

Lenth's SME 4.34453

As with the square root transformation, factors *D*, *F*, and the *BG* interaction remain significant.



Design Expert Output

**Response:** **F&T**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 50.00 5 10.00 5.73 0.0095 significant

*B* *0.42* *1* *0.42* *0.24* *0.6334*

*D* *15.92* *1* *15.92* *9.12* *0.0129*

*F* *23.52* *1* *23.52* *13.47* *0.0043*

*G* *0.65* *1* *0.65* *0.37* *0.5560*

*BG* *9.49* *1* *9.49* *5.43* *0.0420*

Residual 17.47 10 1.75

Cor Total 67.46 15

The Model F-value of 5.73 implies the model is significant. There is only

a 0.95% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 1.32 R-Squared 0.7411

Mean 2.51 Adj R-Squared 0.6117

C.V. 52.63 Pred R-Squared 0.3373

PRESS 44.71 Adeq Precision 7.862

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 2.51 1 0.33 1.78 3.25

B-B -0.16 1 0.33 -0.90 0.57 1.00

D-D -1.00 1 0.33 -1.73 -0.26 1.00

F-F -1.21 1 0.33 -1.95 -0.48 1.00

G-G -0.20 1 0.33 -0.94 0.53 1.00

BG -0.77 1 0.33 -1.51 -0.034 1.00

**Final Equation in Terms of Coded Factors:**

F&T =

+2.51

-0.16 \* B

-1.00 \* D

-1.21 \* F

-0.20 \* G

-0.77 \* B \* G

**Final Equation in Terms of Actual Factors:**

F&T =

+2.51125

-0.16250 \* B

-0.99750 \* D

-1.21250 \* F

-0.20125 \* G

-0.77000 \* B \* G

The following interaction plots appear as they did with the square root transformation; a slight “u” shape is observed in the residuals versus predicted plot.



**8.42S.** An experiment is run in a semiconductor factory to investigate the effect of six factors on transistor gain. The design selected is the  shown in Table P8.15.

**Table P8.15**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Standard | Run |  |  |  |  |  |  |  |
| Order | Order | *A* | *B* | *C* | *D* | *E* | *F* | Gain |
| 1 | 2 | – | – | – | – | – | – | 1455 |
| 2 | 8 | + | – | – | – | + | – | 1511 |
| 3 | 5 | – | + | – | – | + | + | 1487 |
| 4 | 9 | + | + | – | – | – | + | 1596 |
| 5 | 3 | – | – | + | – | + | + | 1430 |
| 6 | 14 | + | – | + | – | – | + | 1481 |
| 7 | 11 | – | + | + | – | – | – | 1458 |
| 8 | 10 | + | + | + | – | + | – | 1549 |
| 9 | 15 | – | – | – | + | – | + | 1454 |
| 10 | 13 | + | – | – | + | + | + | 1517 |
| 11 | 1 | – | + | – | + | + | – | 1487 |
| 12 | 6 | + | + | – | + | – | – | 1596 |
| 13 | 12 | – | – | + | + | + | – | 1446 |
| 14 | 4 | + | – | + | + | – | – | 1473 |
| 15 | 7 | – | + | + | + | – | + | 1461 |
| 16 | 16 | + | + | + | + | + | + | 1563 |

1. Use a normal plot of the effects to identify the significant factors.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 76 23104 55.2714

Model B 53.75 11556.2 27.6459

Model C -30.25 3660.25 8.75637

Error D 3.75 56.25 0.134566

Error E 2 16 0.0382766

Error F 1.75 12.25 0.0293055

Model AB 26.75 2862.25 6.84732

Model AC -8.25 272.25 0.6513

Error AD -0.75 2.25 0.00538265

Error AE -3.5 49 0.117222

Error AF 5.25 110.25 0.26375

Error BD 0.5 1 0.00239229

Error BF 2.5 25 0.0598072

Error ABD 3.5 49 0.117222

Error ABF -2.5 25 0.0598072

Lenth's ME 9.63968

Lenth's SME 19.57



1. Conduct appropriate statistical tests for the model identified in part (a).

Design Expert Output

**Response:** **Gain**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 41455.00 5 8291.00 239.62 < 0.0001 significant

*A* *23104.00* *1* *23104.00* *667.75* *< 0.0001*

*B* *11556.25* *1* *11556.25* *334.00* *< 0.0001*

*C* *3660.25* *1* *3660.25* *105.79* *< 0.0001*

*AB* *2862.25* *1* *2862.25* *82.72* *< 0.0001*

*AC* *272.25* *1* *272.25* *7.87* *0.0186*

Residual 346.00 10 34.60

Cor Total 41801.00 15

The Model F-value of 239.62 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 5.88 R-Squared 0.9917

Mean 1497.75 Adj R-Squared 0.9876

C.V. 0.39 Pred R-Squared 0.9788

PRESS 885.76 Adeq Precision 44.419

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 1497.75 1 1.47 1494.47 1501.03

A-A 38.00 1 1.47 34.72 41.28 1.00

B-B 26.87 1 1.47 23.60 30.15 1.00

C-C -15.13 1 1.47 -18.40 -11.85 1.00

AB 13.38 1 1.47 10.10 16.65 1.00

AC -4.12 1 1.47 -7.40 -0.85 1.00

**Final Equation in Terms of Coded Factors:**

Gain =

+1497.75

+38.00 \* A

+26.87 \* B

-15.13 \* C

+13.38 \* A \* B

-4.12 \* A \* C

**Final Equation in Terms of Actual Factors:**

Gain =

+1497.75000

+38.00000 \* A

+26.87500 \* B

-15.12500 \* C

+13.37500 \* A \* B

-4.12500 \* A \* C

1. Analyze the residuals and comment on your findings.

The residual plots are acceptable. The normality and equality of variance assumptions are verified. There does not appear to be any trends or interruptions in the residuals versus run order plot.











1. Can you find a set of operating conditions that produce gain of ?

Yes, see the graphs below.





**8.43S.** Heat treating is often used to carbonize metal parts, such as gears. The thickness of the carbonized layer is a critical output variable from this process, and it is usually measured by performing a carbon analysis on the gear pitch (top of the gear tooth). Six factors were studied on a  design: *A* = furnace temperature, *B* = cycle time, *C* = carbon concentration, *D* = duration of the carbonizing cycle, *E* = carbon concentration of the diffuse cycle, and *F* = duration of the diffuse cycle. The experiment is shown in Table P8.16.

**Table P8.16**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Standard | Run |  |  |  |  |  |  |  |
| Order | Order | *A* | *B* | *C* | *D* | *E* | *F* | Pitch |
| 1 | 5 | – | – | – | – | – | – | 74 |
| 2 | 7 | + | – | – | – | + | – | 190 |
| 3 | 8 | – | + | – | – | + | + | 133 |
| 4 | 2 | + | + | – | – | – | + | 127 |
| 5 | 10 | – | – | + | – | + | + | 115 |
| 6 | 12 | + | – | + | – | – | + | 101 |
| 7 | 16 | – | + | + | – | – | – | 54 |
| 8 | 1 | + | + | + | – | + | – | 144 |
| 9 | 6 | – | – | – | + | – | + | 121 |
| 10 | 9 | + | – | – | + | + | + | 188 |
| 11 | 14 | – | + | – | + | + | – | 135 |
| 12 | 13 | + | + | – | + | – | – | 170 |
| 13 | 11 | – | – | + | + | + | – | 126 |
| 14 | 3 | + | – | + | + | – | – | 175 |
| 15 | 15 | – | + | + | + | – | + | 126 |
| 16 | 4 | + | + | + | + | + | + | 193 |

1. Estimate the factor effects and plot them on a normal probability plot. Select a tentative model.

Design Expert Output

Term Effect SumSqr % Contribtn

Model Intercept

Model A 50.5 10201 41.8777

Error B -1 4 0.016421

Model C -13 676 2.77515

Model D 37 5476 22.4804

Model E 34.5 4761 19.5451

Error F 4.5 81 0.332526

Error AB -4 64 0.262737

Error AC -2.5 25 0.102631

Error AD 4 64 0.262737

Error AE 1 4 0.016421

Error BD 4.5 81 0.332526

Model CD 14.5 841 3.45252

Model DE -22 1936 7.94778

Error ABD 0.5 1 0.00410526

Error ABF 6 144 0.591157

Lenth's ME 15.4235

Lenth's SME 31.3119

Factors *A*, *C*, *D*, *E* and the two factor interactions *CD* and *DE* appear to be significant. The *CD* and *DE* interactions are aliased with BF and AF interactions respectively. Because factors B and F are not significant, *CD* and *DE* were included in the model. The model can be found in the Design Expert Output below.



1. Perform appropriate statistical tests on the model.

Design Expert Output

**Response:** **Pitch**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 23891.00 6 3981.83 76.57 < 0.0001 significant

*A* *10201.00* *1* *10201.00* *196.17* *< 0.0001*

*C* *676.00* *1* *676.00* *13.00* *0.0057*

*D* *5476.00* *1* *5476.00* *105.31* *< 0.0001*

*E* *4761.00* *1* *4761.00* *91.56* *< 0.0001*

*CD* *841.00* *1* *841.00* *16.17* *0.0030*

*DE* *1936.00* *1* *1936.00* *37.23* *0.0002*

Residual 468.00 9 52.00

Cor Total 24359.00 15

The Model F-value of 76.57 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 7.21 R-Squared 0.9808

Mean 135.75 Adj R-Squared 0.9680

C.V. 5.31 Pred R-Squared 0.9393

PRESS 1479.11 Adeq Precision 28.618

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 135.75 1 1.80 131.67 139.83

A-A 25.25 1 1.80 21.17 29.33 1.00

C-C -6.50 1 1.80 -10.58 -2.42 1.00

D-D 18.50 1 1.80 14.42 22.58 1.00

E-E 17.25 1 1.80 13.17 21.33 1.00

CD 7.25 1 1.80 3.17 11.33 1.00

DE -11.00 1 1.80 -15.08 -6.92 1.00

**Final Equation in Terms of Coded Factors:**

Pitch =

+135.75

+25.25 \* A

-6.50 \* C

+18.50 \* D

+17.25 \* E

+7.25 \* C \* D

-11.00 \* D \* E

**Final Equation in Terms of Actual Factors:**

Pitch =

+135.75000

+25.25000 \* A

-6.50000 \* C

+18.50000 \* D

+17.25000 \* E

+7.25000 \* C \* D

-11.00000 \* D \* E

1. Analyze the residuals and comment on model adequacy.

The residual plots are acceptable. The normality and equality of variance assumptions are verified. There does not appear to be any trends or interruptions in the residuals versus run order plot. The plots of the residuals versus factors *C* and *E* identify reduced variation at the lower level of both variables while the plot of residuals versus factor *F* identifies reduced variation at the upper level. Because *C* and *E* are significant factors in the model, this might not affect the decision on the optimum solution for the process. However, factor *F* is not included in the model and may be set at the upper level to reduce variation.











1. Interpret the results of this experiment. Assume that a layer thickness of between 140 and 160 is desirable.

The graphs below identify a region that is acceptable between 140 and 160.





**8.44.** An article by L.B. Hare (“In the Soup: A Case Study to Identify Contributors to Filling Variability”, Journal of Quality Technology, Vol. 20, pp. 36-43) describes a factorial experiment used to study the filling variability of dry soup mix packages. The factors are *A* = number of mixing ports through which the vegetable oil was added (1, 2), *B* = temperature surrounding the mixer (cooled, ambient), *C* = mixing time (60, 80 sec), *D* = batch weight (1500, 2000 lb), and *E* = number of days between mixing and packaging (1,7). Between 125 and 150 packages of soup were sampled over an eight hour period for each run in the design and the standard deviation of package weight was used as the response variable. The design and resulting data are shown in Table P8.17.

**Table P8.17**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Std Order | *A* – Mixer Ports | *B* – Temp | *C* – Time | *D* – Batch Weight | *E* – Delay | *y* – Std Dev |
| 1 | –1 | –1 | –1 | –1 | –1 | 1.13 |
| 2 | 1 | –1 | –1 | –1 | 1 | 1.25 |
| 3 | –1 | 1 | –1 | –1 | 1 | 0.97 |
| 4 | 1 | 1 | –1 | –1 | –1 | 1.70 |
| 5 | –1 | –1 | 1 | –1 | 1 | 1.47 |
| 6 | 1 | –1 | 1 | –1 | –1 | 1.28 |
| 7 | –1 | 1 | 1 | –1 | –1 | 1.18 |
| 8 | 1 | 1 | 1 | –1 | 1 | 0.98 |
| 9 | –1 | –1 | –1 | 1 | 1 | 0.78 |
| 10 | 1 | –1 | –1 | 1 | –1 | 1.36 |
| 11 | –1 | 1 | –1 | 1 | –1 | 1.85 |
| 12 | 1 | 1 | –1 | 1 | 1 | 0.62 |
| 13 | –1 | –1 | 1 | 1 | –1 | 1.09 |
| 14 | 1 | –1 | 1 | 1 | 1 | 1.10 |
| 15 | –1 | 1 | 1 | 1 | 1 | 0.76 |
| 16 | 1 | 1 | 1 | 1 | –1 | 2.10 |

(a) What is the generator for this design?

The design generator is *I = -ABCDE.*

(b) What is the resolution of this design?

This design is Resolution *V.*

(c) Estimate the factor effects. Which effects are large?

Design Expert Output

Term Effect SumSqr % Contribtn

Require Intercept

Error A 0.145 0.0841 3.48388

Model B 0.0875 0.030625 1.26865

Error C 0.0375 0.005625 0.233018

Model D -0.0375 0.005625 0.233018

Model E -0.47 0.8836 36.6035

Error AB 0.015 0.0009 0.0372829

Error AC 0.095 0.0361 1.49546

Error AD 0.03 0.0036 0.149132

Error AE -0.1525 0.093025 3.8536

Error BC -0.0675 0.018225 0.754979

Error BD 0.1625 0.105625 4.37556

Model BE -0.405 0.6561 27.1792

Error CD 0.0725 0.021025 0.87097

Error CE 0.135 0.0729 3.01992

Model DE -0.315 0.3969 16.4418

Lenth's ME 0.337389

Lenth's SME 0.684948

Factor *E* and the two factor interactions *BE* and *DE* appear to be significant. Factors *B*, and *D*, are included to satisfy model hierarchy. The analysis of variance and model can be found in the Design Expert Output below.



Design Expert Output

**Response:** **Std Dev**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 1.97 5 0.39 8.94 0.0019 significant

*B 0.031 1 0.031 0.69 0.4242*

*D 5.625E-003 1 5.625E-003 0.13 0.7284*

*E 0.88 1 0.88 20.03 0.0012*

*BE 0.66 1 0.66 14.87 0.0032*

*DE 0.40 1 0.40 9.00 0.0134*

Residual 0.44 10 0.044

Cor Total 2.41 15

The Model F-value of 8.94 implies the model is significant. There is only

a 0.19% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 0.21 R-Squared 0.8173

Mean 1.23 Adj R-Squared 0.7259

C.V. 17.13 Pred R-Squared 0.5322

PRESS 1.13 Adeq Precision 9.252

**Final Equation in Terms of Coded Factors:**

Std Dev =

+1.23

+0.044 \* B

-0.019 \* D

-0.24 \* E

-0.20 \* B \* E

-0.16 \* D \* E

**Final Equation in Terms of Actual Factors:**

Temperature Cool

Std Dev =

-0.11292

+7.65000E-004 \* Batch Weight

+0.35667 \* Delays

-2.10000E-004 \* Batch Weight \* Delays

Temperature Ambient

Std Dev =

+0.51458

+7.65000E-004 \* Batch Weight

+0.22167 \* Delays

-2.10000E-004 \* Batch Weight \* Delays

(d) Does a residual analysis indicate any problems with the underlying assumptions?

Often a transformation such as the natural log is required for the standard deviation response; however, the following residuals appear to be acceptable without the transformation.





(e) Draw conclusions about this filling process.

From the interaction plots below, the lowest standard deviation can be achieved with the Temperature at ambient, Batch Weight at 2000 lbs, and a Delay of 7 days.



**8.45.** Consider the  design.

(a) Suppose that the design had been folded over by changing the signs in the column *B* instead of column *A*. What changes would have resulted in the effects that can be estimated from the combined design?

Minitab Output – Fold Over on *A*

**Fractional Factorial Design**

Factors: 6 Base Design: 6, 16 Resolution: IV

Runs: 32 Replicates: 1 Fraction: 1/2

Blocks: 1 Center pts (total): 0

Design Generators (before folding): E = ABC, F = BCD

Folded on Factors: A

Alias Structure

I + BCDF

A + ABCDF

B + CDF

C + BDF

D + BCF

E + BCDEF

F + BCD

AB + ACDF

AC + ABDF

AD + ABCF

AE + ABCDEF

AF + ABCD

BC + DF

BD + CF

BE + CDEF

BF + CD

CE + BDEF

DE + BCEF

EF + BCDE

ABC + ADF

ABD + ACF

ABE + ACDEF

ABF + ACD

ACE + ABDEF

ADE + ABCEF

AEF + ABCDE

BCE + DEF

BDE + CEF

BEF + CDE

ABCE + ADEF

ABDE + ACEF

ABEF + ACDE

Minitab Output – Fold Over on *B*

**Fractional Factorial Design**

Factors: 6 Base Design: 6, 16 Resolution: IV

Runs: 32 Replicates: 1 Fraction: 1/2

Blocks: 1 Center pts (total): 0

Design Generators (before folding): E = ABC, F = BCD

Folded on Factors: B

Alias Structure

I + ADEF

A + DEF

B + ABDEF

C + ACDEF

D + AEF

E + ADF

F + ADE

AB + BDEF

AC + CDEF

AD + EF

AE + DF

AF + DE

BC + ABCDEF

BD + ABEF

BE + ABDF

BF + ABDE

CD + ACEF

CE + ACDF

CF + ACDE

ABC + BCDEF

ABD + BEF

ABE + BDF

ABF + BDE

ACD + CEF

ACE + CDF

ACF + CDE

BCD + ABCEF

BCE + ABCDF

BCF + ABCDE

ABCD + BCEF

ABCE + BCDF

ABCF + BCDE

Both combined designs are still resolution *IV*, there are some two-factor interactions aliased with other two-factor interactions. In the combined design folded on *A*, all two-factor interactions with *A* are now aliased with four-factor or higher interactions. The sparsity of effects principle would tell us that the higher order interactions are highly unlikely to occur. In the combined design folded on *B*, all two-factor interactions with *B* are now aliased with four-factor or higher interactions.

(b) Suppose that the design had been folded over by changing the sign in the column *E* instead of column *A*. What changes would have resulted in the effects that can be estimated from the combined design?

Minitab Output – Fold Over on *B*

**Fractional Factorial Design**

Factors: 6 Base Design: 6, 16 Resolution: IV

Runs: 32 Replicates: 1 Fraction: 1/2

Blocks: 1 Center pts (total): 0

Design Generators (before folding): E = ABC, F = BCD

Folded on Factors: E

Alias Structure

I + BCDF

A + ABCDF

B + CDF

C + BDF

D + BCF

E + BCDEF

F + BCD

AB + ACDF

AC + ABDF

AD + ABCF

AE + ABCDEF

AF + ABCD

BC + DF

BD + CF

BE + CDEF

BF + CD

CE + BDEF

DE + BCEF

EF + BCDE

ABC + ADF

ABD + ACF

ABE + ACDEF

ABF + ACD

ACE + ABDEF

ADE + ABCEF

AEF + ABCDE

BCE + DEF

BDE + CEF

BEF + CDE

ABCE + ADEF

ABDE + ACEF

ABEF + ACDE

In the combined design folded on *E*, all two-factor interactions with *E* are now aliased with four-factor or higher interactions.

**8.46S.** Consider the  design. Suppose that fold over of this design is run by changing the signs in column *A*. Determine the alias relationship in the combined design.

Minitab Output

**Fractional Factorial Design**

Factors: 7 Base Design: 7, 16 Resolution: IV

Runs: 32 Replicates: 1 Fraction: 1/4

Blocks: 1 Center pts (total): 0

Design Generators (before folding): E = ABC, F = BCD, G = ACD

Folded on Factors: A

Alias Structure

I + BCDF + BDEG + CEFG

A + ABCDF + ABDEG + ACEFG

B + CDF + DEG + BCEFG

C + BDF + EFG + BCDEG

D + BCF + BEG + CDEFG

E + BDG + CFG + BCDEF

F + BCD + CEG + BDEFG

G + BDE + CEF + BCDFG

AB + ACDF + ADEG + ABCEFG

AC + ABDF + AEFG + ABCDEG

AD + ABCF + ABEG + ACDEFG

AE + ABDG + ACFG + ABCDEF

AF + ABCD + ACEG + ABDEFG

AG + ABDE + ACEF + ABCDFG

BC + DF + BEFG + CDEG

BD + CF + EG + BCDEFG

BE + DG + BCFG + CDEF

BF + CD + BCEG + DEFG

BG + DE + BCEF + CDFG

CE + FG + BCDG + BDEF

CG + EF + BCDE + BDFG

ABC + ADF + ABEFG + ACDEG

ABD + ACF + AEG + ABCDEFG

ABE + ADG + ABCFG + ACDEF

ABF + ACD + ABCEG + ADEFG

ABG + ADE + ABCEF + ACDFG

ACE + AFG + ABCDG + ABDEF

ACG + AEF + ABCDE + ABDFG

BCE + BFG + CDG + DEF

BCG + BEF + CDE + DFG

ABCE + ABFG + ACDG + ADEF

ABCG + ABEF + ACDE + ADFG

**8.47S.** Reconsider the  design in Problem 8.46S.

(a) Suppose that a fold over of this design is run by changing the signs in column *B*. Determine the alias relationship in the combined design.

Minitab Output

**Fractional Factorial Design**

Factors: 7 Base Design: 7, 16 Resolution: IV

Runs: 32 Replicates: 1 Fraction: 1/4

Blocks: 1 Center pts (total): 0

Design Generators (before folding): E = ABC, F = BCD, G = ACD

Folded on Factors: B

Alias Structure

I + ACDG + ADEF + CEFG

A + CDG + DEF + ACEFG

B + ABCDG + ABDEF + BCEFG

C + ADG + EFG + ACDEF

D + ACG + AEF + CDEFG

E + ADF + CFG + ACDEG

F + ADE + CEG + ACDFG

G + ACD + CEF + ADEFG

AB + BCDG + BDEF + ABCEFG

AC + DG + AEFG + CDEF

AD + CG + EF + ACDEFG

AE + DF + ACFG + CDEG

AF + DE + ACEG + CDFG

AG + CD + ACEF + DEFG

BC + ABDG + BEFG + ABCDEF

BD + ABCG + ABEF + BCDEFG

BE + ABDF + BCFG + ABCDEG

BF + ABDE + BCEG + ABCDFG

BG + ABCD + BCEF + ABDEFG

CE + FG + ACDF + ADEG

CF + EG + ACDE + ADFG

ABC + BDG + ABEFG + BCDEF

ABD + BCG + BEF + ABCDEFG

ABE + BDF + ABCFG + BCDEG

ABF + BDE + ABCEG + BCDFG

ABG + BCD + ABCEF + BDEFG

ACE + AFG + CDF + DEG

ACF + AEG + CDE + DFG

BCE + BFG + ABCDF + ABDEG

BCF + BEG + ABCDE + ABDFG

ABCE + ABFG + BCDF + BDEG

ABCF + ABEG + BCDE + BDFG

(b) Compare the aliases from this combined design to those from the combined design from Problem 8.35. What differences resulted by changing the signs in a different column?

Both combined designs are still resolution IV, there are some two-factor interactions aliased with other two-factor interactions. In the combined design folded on *A*, all two-factor interactions with *A* are now aliased with four-factor or higher interactions. The sparsity of effects principle would tell us that the higher order interactions are highly unlikely to occur. In the combined design folded on *B*, all two-factor interactions with *B* are now aliased with four-factor or higher interactions.

**8.48S.** Consider the  design.

(a) Suppose that a partial fold over of this design is run using column *A* (+ signs only). Determine the alias relationship in the combined design.

By choosing a fold over design in *Design Expert*, sorting on column *A*, and deleting the rows with a minus sign for *A* in the second block, the alias structures are identified below.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept + ABCE + ABFG + ACDG + ADEF + BCDF + BDEG + CEFG

[A] = A - ABCE - ABFG - ACDG - ADEF + ABCDF + ABDEG + ACEFG

[B] = B + ACE + AFG + CDF + DEG + ABCDG + ABDEF + BCEFG

[C] = C + ABE + ADG + BDF + EFG + ABCFG + ACDEF + BCDEG

[D] = D + ACG + AEF + BCF + BEG + ABCDE + ABDFG + CDEFG

[E] = E + ABC + ADF + BDG + CFG + ABEFG + ACDEG + BCDEF

[F] = F + ABG + ADE + BCD + CEG + ABCEF + ACDFG + BDEFG

[G] = G + ABF + ACD + BDE + CEF + ABCEG + ADEFG + BCDFG

[AB] = AB - ACE - AFG + ACDF + ADEG - ABCDG - ABDEF + ABCEFG

[AC] = AC - ABE - ADG + ABDF + AEFG - ABCFG - ACDEF + ABCDEG

[AD] = AD - ACG - AEF + ABCF + ABEG - ABCDE - ABDFG + ACDEFG

[AE] = AE - ABC - ADF + ABDG + ACFG - ABEFG - ACDEG + ABCDEF

[AF] = AF - ABG - ADE + ABCD + ACEG - ABCEF - ACDFG + ABDEFG

[AG] = AG - ABF - ACD + ABDE + ACEF - ABCEG - ADEFG + ABCDFG

[BC] = BC + DF + ABC + ADF + BEFG + CDEG + ABEFG + ACDEG

[BD] = BD + CF + EG + ABCG + ABEF + ACDE + ADFG + BCDEFG

[BE] = BE + DG + ABE + ADG + BCFG + CDEF + ABCFG + ACDEF

[BF] = BF + CD + ABF + ACD + BCEG + DEFG + ABCEG + ADEFG

[BG] = BG + DE + ABG + ADE + BCEF + CDFG + ABCEF + ACDFG

[CE] = CE + FG + ACE + AFG + BCDG + BDEF + ABCDG + ABDEF

[CG] = CG + EF + ACG + AEF + BCDE + BDFG + ABCDE + ABDFG

[ABD] = ABD + ACF + AEG - ABCG - ABEF - ACDE - ADFG + ABCDEFG

[BCE] = BCE + BFG + CDG + DEF + ABCE + ABFG + ACDG + ADEF

[BCG] = BCG + BEF + CDE + DFG + ABCG + ABEF + ACDE + ADFG

**Factorial Effects Defining Contrast**

I = BCDF = BDEG = CEFG

(b) Rework part (a) using the negative signs to define the partial fold over. Does it make any difference which signs are used to define the partial fold over?

Both partial fold over designs produce the same alias relationships as shown below.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept + ABCE + ABFG + ACDG + ADEF + BCDF + BDEG + CEFG

[A] = A - ABCE - ABFG - ACDG - ADEF + ABCDF + ABDEG + ACEFG

[B] = B + ACE + AFG + CDF + DEG + ABCDG + ABDEF + BCEFG

[C] = C + ABE + ADG + BDF + EFG + ABCFG + ACDEF + BCDEG

[D] = D + ACG + AEF + BCF + BEG + ABCDE + ABDFG + CDEFG

[E] = E + ABC + ADF + BDG + CFG + ABEFG + ACDEG + BCDEF

[F] = F + ABG + ADE + BCD + CEG + ABCEF + ACDFG + BDEFG

[G] = G + ABF + ACD + BDE + CEF + ABCEG + ADEFG + BCDFG

[AB] = AB - ACE - AFG + ACDF + ADEG - ABCDG - ABDEF + ABCEFG

[AC] = AC - ABE - ADG + ABDF + AEFG - ABCFG - ACDEF + ABCDEG

[AD] = AD - ACG - AEF + ABCF + ABEG - ABCDE - ABDFG + ACDEFG

[AE] = AE - ABC - ADF + ABDG + ACFG - ABEFG - ACDEG + ABCDEF

[AF] = AF - ABG - ADE + ABCD + ACEG - ABCEF - ACDFG + ABDEFG

[AG] = AG - ABF - ACD + ABDE + ACEF - ABCEG - ADEFG + ABCDFG

[BC] = BC + DF + ABC + ADF + BEFG + CDEG + ABEFG + ACDEG

[BD] = BD + CF + EG + ABCG + ABEF + ACDE + ADFG + BCDEFG

[BE] = BE + DG + ABE + ADG + BCFG + CDEF + ABCFG + ACDEF

[BF] = BF + CD + ABF + ACD + BCEG + DEFG + ABCEG + ADEFG

[BG] = BG + DE + ABG + ADE + BCEF + CDFG + ABCEF + ACDFG

[CE] = CE + FG + ACE + AFG + BCDG + BDEF + ABCDG + ABDEF

[CG] = CG + EF + ACG + AEF + BCDE + BDFG + ABCDE + ABDFG

[ABD] = ABD + ACF + AEG - ABCG - ABEF - ACDE - ADFG + ABCDEFG

[BCE] = BCE + BFG + CDG + DEF + ABCE + ABFG + ACDG + ADEF

[BCG] = BCG + BEF + CDE + DFG + ABCG + ABEF + ACDE + ADFG

**Factorial Effects Defining Contrast**

I = BCDF = BDEG = CEFG

**8.49.** Consider a partial fold over for the  design. Suppose that the signs are reversed in column *A*, but the eight runs are retained are the runs that have positive signs in column *C*. Determine the alias relationship in the combined design.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept - ABE + BCDF - ACDEF

[Block 1] = Block 1 + ABE + ABCE + ADEF + ACDEF

[Block 2] = Block 2 - ABE - ABCE - ADEF - ACDEF

[A] = A + BCE + DEF + ABCDF

[B] = B + ACE + CDF + ABDEF

[C] = C + ABE + BDF + ACDEF

[D] = D + BCF - ABDE - ACEF

[E] = E + ABC + ADF + BCDEF

[F] = F + BCD - ABEF - ACDE

[AB] = AB + ABC + ADF + ACDF

[AC] = AC - BCE - DEF + ABDF

[AD] = AD - BDE - CEF + ABCF

[AE] = AE + ACE + ABDEF + ABCDEF

[AF] = AF - BEF - CDE + ABCD

[BC] = BC + DF - ACE - ABDEF

[BD] = BD + CF + ABEF + ACDE

[BE] = BE + BCE + DEF + CDEF

[BF] = BF + CD + ABDE + ACEF

[CE] = CE - ABC - ADF + BDEF

[DE] = DE + BEF + CDE + BCEF

[EF] = EF + BDE + CEF + BCDE

[ABD] = ABD + ACF + BEF + CDE

[ABF] = ABF + ACD + BDE + CEF

[ADE] = ADE + ABEF + ACDE + ABCEF

[AEF] = AEF + ABDE + ACEF + ABCDE

**Factorial Effects Defining Contrast**

I = BCDF

**8.50.** Consider a partial fold over for the  design. Suppose that the partial fold over of this design is constructed using column *A* (+ signs only). Determine the alias relationship for the combined design.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept - BD - CE - FG - ABCF + ABCG + ABEF - ABEG + ACDF

- ACDG - ADEF + ADEG + BCDE + BDFG + CEFG - BCDEFG

[Block 1] = Block 1 + BD + CE + FG + ABD + ACE + AFG + BCF + BEG + CDG + DEF

+ ABCF + ABEG + ACDG + ADEF + BCDEFG + ABCDEFG

[Block 2] = Block 2 - BD - CE - FG - ABD - ACE - AFG - BCF - BEG - CDG - DEF

- ABCF - ABEG - ACDG - ADEF - BCDEFG - ABCDEFG

[A] = A + BD + CE + FG + BCG + BEF + CDF + DEG + ABCF + ABEG + ACDG + ADEF

+ ABCDE + ABDFG + ACEFG + BCDEFG

[B] = B + AD + CF + EG + ACG + AEF + CDE + DFG + ABCE + ABFG + BCDG + BDEF

+ ABCDF + ABDEG + BCEFG + ACDEFG

[C] = C + AE + BF + DG + ABG + ADF + BDE + EFG + ABCD + ACFG + BCEG + CDEF

+ ABCEF + ACDEG + BCDFG + ABDEFG

[D] = D + AD + CF + EG + ACF + AEG + BCE + BFG + ABCE + ABFG + BCDG + BDEF

+ ABCDG + ABDEF + CDEFG + ACDEFG

[E] = E + AE + BF + DG + ABF + ADG + BCD + CFG + ABCD + ACFG + BCEG + CDEF

+ ABCEG + ACDEF + BDEFG + ABDEFG

[F] = F + AG + BC + DE + ABE + ACD + BDG + CEG + ABDF + ACEF + BEFG + CDFG

+ ABCFG + ADEFG + BCDEF + ABCDEG

[G] = G + AG + BC + DE + ABC + ADE + BDF + CEF + ABDF + ACEF + BEFG + CDFG

+ ABEFG + ACDFG + BCDEG + ABCDEG

[AB] = AB - AD - CF + CG + EF - EG - ABCE - ABFG + ACDE + ADFG + BCDF - BCDG

- BDEF + BDEG + ABCEFG - ACDEFG

[AC] = AC - AE - BF + BG + DF - DG - ABCD + ABDE - ACFG + AEFG + BCEF - BCEG

- CDEF + CDEG + ABCDFG - ABDEFG

[AF] = AF - AG - BC + BE + CD - DE - ABDF + ABDG - ACEF + ACEG + BCFG - BEFG

- CDFG + DEFG + ABCDEF - ABCDEG

**Factorial Effects Defining Contrast**

I = ABCG = ABEF = ACDF = ADEG = BCDE = BDFG = CEFG

**8.51.** Consider a partial fold over for the  design. Suppose that the partial fold over of this design is constructed using column *A* (+ signs only). Determine the alias relationship for the combined design.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept + ABCE + ABFG + ACDG + ADEF + BCDF + BDEG + CEFG

[Intercept] = Intercept - BD - CE + BCDE

[Block 1] = Block 1 + BD + CE + ABD + ACE

[Block 2] = Block 2 - BD - CE - ABD - ACE

[A] = A + BD + CE + ABCDE

[B] = B + AD + CDE + ABCE

[C] = C + AE + BDE + ABCD

[D] = D + AD + BCE + ABCE

[E] = E + AE + BCD + ABCD

[AB] = AB - AD - ABCE + ACDE

[AC] = AC - AE - ABCD + ABDE

[BC] = BC + DE + ABE + ACD

[BE] = BE + CD + ABE + ACD

[ABC] = ABC - ABE - ACD + ADE

**Factorial Effects Defining Contrast**

I = BCDE

**8.52.** Reconsider the 24-1 design in Example 8.1. The significant factors are *A*, *C*, *D*, *AC + BD*, and *AD* + *BC*. Find a partial fold over design that will allow the *AC*, *BD*, *AD*, and *BD* interactions to be estimated.

By constructing a partial fold over reversing the signs in column *A* and using column *A* (+ signs only), the *AC*, *BD*, *AD*, and *BD* interactions can be estimated as shown below. This could also be accomplished by reversing the signs of any one of the factors *A*, *B*, *C*, and *D*.

Design Expert Output

**Factorial Effects Aliases**

**[Est Terms] Aliased Terms**

[Intercept] = Intercept - BCD

[Block 1] = Block 1 + BCD + ABCD

[Block 2] = Block 2 - BCD - ABCD

[A] = A + BCD

[B] = B + ACD

[C] = C + ABD

[D] = D + ABC

[AB] = AB - ACD

[AC] = AC - ABD

[AD] = AD - ABC

[BC] = BC + ABC

[BD] = BD + ABD

[CD] = CD + ACD

**8.53.** Construct a supersaturated design for *k* = 8 factors in *N* = 6 runs.

We have chosen the Hadamard matrix design approach using the following Plackett-Burman design for *N* = 12 runs and *k* = 11 factors. By sorting on the 11th factor, *L*, deleting the rows with the positive levels on *L*, and removing the last 3 factors, the design is reduced to 6 runs with 8 factors. Either the positive or negative levels on *L* could have been chosen. Also, the JMP statistics software package has the capability to generate supersaturated designs.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *I* | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | *K* | *L* |
| 1 | + | + | + | – | + | + | + | – | – | – | + | – |
| 2 | + | – | + | + | – | + | + | + | – | – | – | + |
| 3 | + | + | – | + | + | – | + | + | + | – | – | – |
| 4 | + | – | + | – | + | + | – | + | + | + | – | – |
| 5 | + | – | – | + | – | + | + | – | + | + | + | – |
| 6 | + | – | – | – | + | – | + | + | – | + | + | + |
| 7 | + | + | – | – | – | + | – | + | + | – | + | + |
| 8 | + | + | + | – | – | – | + | – | + | + | – | + |
| 9 | + | + | + | + | – | – | – | + | – | + | + | – |
| 10 | + | – | + | + | + | – | – | – | + | – | + | + |
| 11 | + | + | – | + | + | + | – | – | – | + | – | + |
| 12 | + | – | – | – | – | – | – | – | – | – | – | – |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Original Run | Run | *I* | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
| 1 | 1 | 1 | 1 | 1 | –1 | 1 | 1 | 1 | –1 | –1 |
| 3 | 2 | 1 | 1 | –1 | 1 | 1 | –1 | 1 | 1 | 1 |
| 4 | 3 | 1 | –1 | 1 | –1 | 1 | 1 | –1 | 1 | 1 |
| 5 | 4 | 1 | –1 | –1 | 1 | –1 | 1 | 1 | –1 | 1 |
| 9 | 5 | 1 | 1 | 1 | 1 | –1 | –1 | –1 | 1 | –1 |
| 12 | 6 | 1 | –1 | –1 | –1 | –1 | –1 | –1 | –1 | –1 |

**8.54.** Consider the 28–3 design in problem 8.29. Suppose that the alias chain involving the *AB* interaction was large. Recommend a partial fold–over design to resolve the ambiguity about this interaction.

The full defining relationship is: *I + ABCF + ABDG + CDFG + ADEFH + ACEGH + BCDEH*; and the alias structure for the *AB* interaction is *AB + CF + DG*. The following 16 runs were created by changing the sign of *A* in the original experiment and then choosing the runs where A is at the low (–) level.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* |
| –1 | –1 | –1 | –1 | –1 | 1 | 1 | 1 |
| –1 | 1 | –1 | –1 | –1 | –1 | –1 | –1 |
| –1 | –1 | 1 | –1 | –1 | –1 | 1 | –1 |
| –1 | 1 | 1 | –1 | –1 | 1 | –1 | 1 |
| –1 | –1 | –1 | 1 | –1 | 1 | –1 | –1 |
| –1 | 1 | –1 | 1 | –1 | –1 | 1 | 1 |
| –1 | –1 | 1 | 1 | –1 | –1 | –1 | 1 |
| –1 | 1 | 1 | 1 | –1 | 1 | 1 | –1 |
| –1 | –1 | –1 | –1 | 1 | 1 | 1 | –1 |
| –1 | 1 | –1 | –1 | 1 | –1 | –1 | 1 |
| –1 | –1 | 1 | –1 | 1 | –1 | 1 | 1 |
| –1 | 1 | 1 | –1 | 1 | 1 | –1 | –1 |
| –1 | –1 | –1 | 1 | 1 | 1 | –1 | 1 |
| –1 | 1 | –1 | 1 | 1 | –1 | 1 | –1 |
| –1 | –1 | 1 | 1 | 1 | –1 | –1 | –1 |
| –1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**8.55.** Construct a supersaturated design for *k* = 12 factors in *N* = 10 runs.

We have chosen the Hadamard matrix design approach using the following Plackett–Burman design for *N* = 20 runs and *k* = 19 factors. By sorting on the 19th factor, *T*, deleting the rows with the negative levels on *T*, and removing the last 7 factors, the design is reduced to 10 runs with 12 factors. Either the positive or negative levels on *T* could have been chosen. Also, the JMP statistics software package has the capability to generate supersaturated designs.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *I* | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | *K* | *L* | *M* | *N* | *O* | *P* | *Q* | *R* | *S* | *T* |
| 1 | + | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – |
| 2 | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + |
| 3 | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + |
| 4 | + | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – |
| 5 | + | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – |
| 6 | + | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – | – |
| 7 | + | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + | – |
| 8 | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – | + |
| 9 | + | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + | – |
| 10 | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – | + |
| 11 | + | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + | – |
| 12 | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + | + |
| 13 | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + | + |
| 14 | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + | + |
| 15 | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – | + |
| 16 | + | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – | – |
| 17 | + | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + | – |
| 18 | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + | + |
| 19 | + | + | – | – | + | + | + | + | – | + | – | + | – | – | – | – | + | + | – | + |
| 20 | + | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Original Run | Run | *I* | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | *K* | *L* | *M* |
| 2 | 1 | + | – | + | + | – | – | + | + | + | + | – | + | – |
| 3 | 2 | + | + | – | + | + | – | – | + | + | + | + | – | + |
| 8 | 3 | + | – | – | – | – | + | + | – | + | + | – | – | + |
| 10 | 4 | + | – | + | – | – | – | – | + | + | – | + | + | – |
| 12 | 5 | + | – | + | – | + | – | – | – | – | + | + | – | + |
| 13 | 6 | + | + | – | + | – | + | – | – | – | – | + | + | – |
| 14 | 7 | + | + | + | – | + | – | + | – | – | – | – | + | + |
| 15 | 8 | + | + | + | + | – | + | – | + | – | – | – | – | + |
| 18 | 9 | + | – | – | + | + | + | + | – | + | – | + | – | – |
| 19 | 10 | + | + | – | – | + | + | + | + | – | + | – | + | – |

**8.56.** Consider the full 25 factorial design in Problem 6.42. Suppose that this experiment had been run as a 25-1 fractional factorial. Set up the fractional design using the principle fraction. Using the 16 runs associated with this design from the original experiment analyze the data and compare your results with those obtained from the analysis of the original full factorial.

The principle fraction for the experiment is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *y* |
| –1 | –1 | –1 | –1 | 1 | 7.93 |
| 1 | –1 | –1 | –1 | –1 | 5.56 |
| –1 | 1 | –1 | –1 | –1 | 5.77 |
| 1 | 1 | –1 | –1 | 1 | 12 |
| –1 | –1 | 1 | –1 | –1 | 9.17 |
| 1 | –1 | 1 | –1 | 1 | 3.65 |
| –1 | 1 | 1 | –1 | 1 | 6.4 |
| 1 | 1 | 1 | –1 | –1 | 5.69 |
| –1 | –1 | –1 | 1 | –1 | 8.82 |
| 1 | –1 | –1 | 1 | 1 | 17.55 |
| –1 | 1 | –1 | 1 | 1 | 8.87 |
| 1 | 1 | –1 | 1 | –1 | 8.94 |
| –1 | –1 | 1 | 1 | 1 | 13.06 |
| 1 | –1 | 1 | 1 | –1 | 11.49 |
| –1 | 1 | 1 | 1 | –1 | 6.25 |
| 1 | 1 | 1 | 1 | 1 | 26.05 |

The first analysis that includes the main effects and two factor interactions shows nothing significant. Significant factors are masked because it is a saturated design, i.e. there are not enough degrees of freedom to estimate the full model and 2–factor interactions are aliased with 3–factor interactions. By removing the two factors with the smallest effect, *AC* and *CE*, and re–running the analysis, most of the factors are now significant.



Minitab Output

**Factorial Regression: Y versus A, B, C, D, E**

Analysis of Variance

Source DF Adj SS Adj MS F–Value P–Value

Model 13 461.935 35.533 1228.47 0.001

Linear 5 238.236 47.647 1647.27 0.001

A 1 38.007 38.007 1313.99 0.001

B 1 0.469 0.469 16.22 0.056

C 1 2.496 2.496 86.31 0.011

D 1 125.776 125.776 4348.36 0.000

E 1 71.487 71.487 2471.46 0.000

2–Way Interactions 8 223.699 27.962 966.72 0.001

A\*B 1 42.641 42.641 1474.19 0.001

A\*D 1 54.023 54.023 1867.68 0.001

A\*E 1 28.409 28.409 982.16 0.001

B\*C 1 7.981 7.981 275.91 0.004

B\*D 1 1.188 1.188 41.08 0.023

B\*E 1 23.814 23.814 823.32 0.001

C\*D 1 22.610 22.610 781.68 0.001

D\*E 1 43.034 43.034 1487.76 0.001

Error 2 0.058 0.029

Total 15 461.993

Model Summary

S R–sq R–sq(adj) R–sq(pred)

0.170074 99.99% 99.91% 99.20%

This model accounts for more of the variation than the full factorial. The R2 predicted is 99.20%. The issue is that the 2–factor interactions are aliased with the 3–factor interactions and these can only be separated with additional runs. The Residual plots look fine.



**8.57.** Consider the full 25 factorial design in Problem 6.42. Suppose that this experiment had been run as a 25-1 fractional factorial. Set up the fractional design using the alternate fraction. Using the 16 runs associated with this design from the original experiment analyze the data and compare your results with those obtained from the analysis of the principal fraction in Problem 8.48.

The runs for the alternate fraction are:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *y* |
| –1 | –1 | –1 | –1 | –1 | 8.11 |
| 1 | –1 | –1 | –1 | 1 | 5 |
| –1 | 1 | –1 | –1 | 1 | 7.47 |
| 1 | 1 | –1 | –1 | –1 | 5.82 |
| –1 | –1 | 1 | –1 | 1 | 9.86 |
| 1 | –1 | 1 | –1 | –1 | 7.8 |
| –1 | 1 | 1 | –1 | –1 | 3.23 |
| 1 | 1 | 1 | –1 | 1 | 11.61 |
| –1 | –1 | –1 | 1 | 1 | 12.43 |
| 1 | –1 | –1 | 1 | –1 | 14.23 |
| –1 | 1 | –1 | 1 | –1 | 9.2 |
| 1 | 1 | –1 | 1 | 1 | 25.38 |
| –1 | –1 | 1 | 1 | –1 | 8.68 |
| 1 | –1 | 1 | 1 | 1 | 18.85 |
| –1 | 1 | 1 | 1 | 1 | 11.78 |
| 1 | 1 | 1 | 1 | –1 | 9.12 |

The first analysis that includes the main effects and two factor interactions shows nothing significant. Significant factors are masked because it is a saturated design, i.e. there are not enough degrees of freedom to estimate the full model and 2–factor interactions are aliased with 3–factor interactions. By removing the factor with the smallest effect, *AC*, and re–running the analysis, most of the factors are now significant.



This model accounts for more of the variation than the full factorial. The R2 predicted is 98.67%. The issue is that the 2–factor interactions are aliased with the 3–factor interactions and these can only be separated with additional runs. The Residual plots look unusual because of the low number of degrees of freedom for err.

Minitab Output

**Factorial Regression: y versus A, B, C, D, E**

Analysis of Variance

Source DF Adj SS Adj MS F–Value P–Value

Model 14 446.106 31.865 1370.16 0.021

Linear 5 291.616 58.323 2507.85 0.015

A 1 45.731 45.731 1966.41 0.014

B 1 0.114 0.114 4.90 0.270

C 1 2.814 2.814 121.00 0.058

D 1 161.100 161.100 6927.15 0.008

E 1 81.857 81.857 3519.80 0.011

2–Way Interactions 9 154.490 17.166 738.11 0.029

A\*B 1 11.306 11.306 486.17 0.029

A\*D 1 35.790 35.790 1538.95 0.016

A\*E 1 8.338 8.338 358.51 0.034

B\*C 1 19.250 19.250 827.74 0.022

B\*D 1 0.965 0.965 41.51 0.098

B\*E 1 29.025 29.025 1248.06 0.018

C\*D 1 22.349 22.349 961.00 0.021

C\*E 1 6.695 6.695 287.89 0.037

D\*E 1 20.771 20.771 893.13 0.021

Error 1 0.023 0.023

Total 15 446.130

Model Summary

S R–sq R–sq(adj) R–sq(pred)

0.1525 99.99% 99.92% 98.67%



**8.58.** Consider the full 25 factorial design in Problem 6.42. Suppose that this experiment had been run as a 25-2 fractional factorial. Set up the fractional design using the principal fraction. Using the 8 runs associated with this design from the original experiment analyze the data and compare your results with those obtained from the analysis of original design. Then construct the fold–over design and analyze the data from the combined design.

The resulting design is a Resolution III design with main effects aliased with 2–factor interactions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *y* | effect |
| –1 | –1 | –1 | 1 | 1 | 12.43 | *de* |
| 1 | –1 | –1 | –1 | –1 | 5.56 | *a* |
| –1 | 1 | –1 | –1 | 1 | 7.47 | *be* |
| 1 | 1 | –1 | 1 | –1 | 8.94 | *abd* |
| –1 | –1 | 1 | 1 | –1 | 8.68 | *cd* |
| 1 | –1 | 1 | –1 | 1 | 3.65 | *ace* |
| –1 | 1 | 1 | –1 | –1 | 3.23 | *bc* |
| 1 | 1 | 1 | 1 | 1 | 26.05 | *abcde* |



None of the effects are significant in this small design. If the effect of *C* is eliminated, then there is a degree of freedom for error. Even with this, no effect is significant.

By doing the full fold over, i.e. changing the signs on every run, the resulting design is:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *y* | effect |
| –1 | –1 | –1 | 1 | 1 | 12.43 | *de* |
| 1 | –1 | –1 | –1 | –1 | 5.56 | *a* |
| –1 | 1 | –1 | –1 | 1 | 7.47 | *be* |
| 1 | 1 | –1 | 1 | –1 | 8.94 | *abd* |
| –1 | –1 | 1 | 1 | –1 | 8.68 | *cd* |
| 1 | –1 | 1 | –1 | 1 | 3.65 | *ace* |
| –1 | 1 | 1 | –1 | –1 | 3.23 | *bc* |
| 1 | 1 | 1 | 1 | 1 | 26.05 | *abcde* |
| 1 | 1 | 1 | –1 | –1 | 5.69 | *abc* |
| –1 | 1 | 1 | 1 | 1 | 11.78 | *bcde* |
| 1 | –1 | 1 | 1 | –1 | 11.49 | *acd* |
| –1 | –1 | 1 | –1 | 1 | 9.86 | *ce* |
| 1 | 1 | –1 | –1 | 1 | 12 | *abe* |
| –1 | 1 | –1 | 1 | –1 | 9.2 | *bd* |
| 1 | –1 | –1 | 1 | 1 | 17.55 | *ade* |
| –1 | –1 | –1 | –1 | –1 | 8.11 | *(1)* |



From the half–normal plot, Factors *D* and *E* are significant. The defining relationship is *A=BCDE*. This is a Resolution IV design. The model simplifies into the main effects of *D* and *E*. There is one unusual residual.

Minitab Output

**Factorial Regression: y versus D, E**

Analysis of Variance

Source DF Adj SS Adj MS F–Value P–Value

Model 2 259.16 129.58 8.08 0.005

Linear 2 259.16 129.58 8.08 0.005

D 1 159.71 159.71 9.96 0.008

E 1 99.45 99.45 6.20 0.027

Error 13 208.51 16.04

Total 15 467.66

Model Summary

S R–sq R–sq(adj) R–sq(pred)

4.00487 55.42% 48.56% 32.46%

Coded Coefficients

Term Effect Coef SE Coef T–Value P–Value VIF

Constant 10.11 1.00 10.09 0.000

D 6.32 3.16 1.00 3.16 0.008 1.00

E 4.99 2.49 1.00 2.49 0.027 1.00

Regression Equation in Uncoded Units

y = 10.11 + 3.16 D + 2.49 E



**8.59.** Consider the 25-2 fractional factorial constructed in Problem 8.58. Compare this design with the two one–half fractions of the 25-1. Is the fold–over design the same as either of the one–half fractions?

The fold–over is not the same as either of the two fractions. The two fractions are resolution V, the fold–over is resolution IV. The half–fraction design has a more complete and better fitting model. But if the experimenter cannot afford to run all of the runs at once, the fold–over is a reasonable alternative.

**8.60.** The resolution of a two–level fractional factorial design is the number of words in the defining relation.

True **False**

**8.61.** For a half fraction of a two–level fractional factorial design the maximum possible resolution is equal to the number of factors.

**True** False

**8.62.** The 12 run Plackett–Burman for up to 11 factors design is a regular fraction.

True **False**

**8.63.** The design points of the 2*k-p* family are the corners of a cube in *k*–dimensional space and they project into a full factorial in any subset of the original *k* factors.

True **False**

**8.64.** It is a good practice to keep the number of factor levels low and the region of interest small in a screening experiment.

True **False**

**8.65.** Consider a 2*k-p* fractional factorial design. If the principal fraction is run first – first block – (I = ABCD) and then later augmented with the alternate fraction – second block, the four–factor interaction effect is confounded with blocks.

**True** False

**8.66.** In a 2*k-*2 design, every effect has 4 aliases.

True **False**

**8.67.** In a 2*k-*3 design, the complete defining relation has 15 words.

**True** False

**8.68.** The aberration of a fractional factorial design is related to the length of the longest word in the defining relation.

True **False**