Chapter 9

**Three-Level and Mixed-Level**

**Factorial and Fractional Factorial Design**

## Solutions

**9.1S.** The effects of developer strength (*A*) and developer time (*B*) on the density of photographic plate film are being studied. Three strengths and three times are used, and four replicates of a 32 factorial experiment are run. The data from this experiment follow. Analyze the data using the standard methods for factorial experiments.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Development Time (minutes) | | | | | |
| Developer Strength | 10 | | 14 | | 18 | |
| 1 | 0 | 2 | 1 | 3 | 2 | 5 |
|  | 5 | 4 | 4 | 2 | 4 | 6 |
| 2 | 4 | 6 | 6 | 8 | 9 | 10 |
|  | 7 | 5 | 7 | 7 | 8 | 5 |
| 3 | 7 | 10 | 10 | 10 | 12 | 10 |
|  | 8 | 7 | 8 | 7 | 9 | 8 |

Design Expert Output

**Response:** **Data**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 224.22 8 28.03 10.66 < 0.0001 significant

*A 198.22 2 99.11 37.69 < 0.0001*

*B 22.72 2 11.36 4.32 0.0236*

*AB 3.28 4 0.82 0.31 0.8677*

Pure Error 71.00 27 2.63

Cor Total 295.22 35

The Model F-value of 10.66 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Strength and time are significant. The quadratic and interaction effects are not significant. By treating both *A* and *B* as numerical factors, the analysis can be performed as follows:

Design Expert Output

**Response:** **Data**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 214.71 2 107.35 44.00 < 0.0001 significant

*A 192.67 1 192.67 78.97 < 0.0001*

*B 22.04 1 22.04 9.03 0.0050*

Residual 80.51 33 2.44

*Lack of Fit 9.51 6 1.59 0.60 0.7255 not significant*

*Pure Error 71.00 27 2.63*

Cor Total 295.22 35

The Model F-value of 44.00 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

**9.2.** Compute the *I* and *J* components of the two-factor interaction in Problem 9.1.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | *B* |  |
|  | 11 | 10 | 17 |
| *A* | 22 | 28 | 32 |
|  | 32 | 35 | 39 |







**9.3S.** An experiment was performed to study the effect of three different types of 32-ounce bottles (*A*) and three different shelf types (*B*) -- smooth permanent shelves, end-aisle displays with grilled shelves, and beverage coolers -- on the time it takes to stock ten 12-bottle cases on the shelves. Three workers (factor *C*) were employed in this experiment, and two replicates of a 33 factorial design were run. The observed time data are shown in the following table. Analyze the data and draw conclusions.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Replicate I | | | Replicate II | | |
| Worker | Bottle Type | Permanent | End Aisle | Cooler | Permanent | End Aisle | Cooler |
| 1 | Plastic | 3.45 | 4.14 | 5.80 | 3.36 | 4.19 | 5.23 |
|  | 28-mm glass | 4.07 | 4.38 | 5.48 | 3.52 | 4.26 | 4.85 |
|  | 38-mm glass | 4.20 | 4.26 | 5.67 | 3.68 | 4.37 | 5.58 |
| 2 | Plastic | 4.80 | 5.22 | 6.21 | 4.40 | 4.70 | 5.88 |
|  | 28-mm glass | 4.52 | 5.15 | 6.25 | 4.44 | 4.65 | 6.20 |
|  | 38-mm glass | 4.96 | 5.17 | 6.03 | 4.39 | 4.75 | 6.38 |
| 3 | Plastic | 4.08 | 3.94 | 5.14 | 3.65 | 4.08 | 4.49 |
|  | 28-mm glass | 4.30 | 4.53 | 4.99 | 4.04 | 4.08 | 4.59 |
|  | 38-mm glass | 4.17 | 4.86 | 4.85 | 3.88 | 4.48 | 4.90 |

Design Expert Output

**Response:** **Time**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 28.28 26 1.09 14.50 < 0.0001 significant

*A 0.41 2 0.21 2.74 0.0828*

*B 17.75 2 8.88 118.34 < 0.0001*

*C 7.66 2 3.83 51.09 < 0.0001*

*AB 0.12 4 0.029 0.39 0.8163*

*AC 0.11 4 0.027 0.36 0.8319*

*BC 1.68 4 0.42 5.60 0.0021*

*ABC 0.55 8 0.069 0.92 0.5145*

Pure Error 2.03 27 0.075

Cor Total 30.31 53

The Model F-value of 14.50 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case B, C, BC are significant model terms.

Factors *B* and *C*, shelf type and worker, and the *BC* interaction are significant. For the shortest time regardless of worker chose the permanent shelves. This can easily be seen in the interaction plot below.



**9.4.** An experiment is run in a chemical process using a 32 factorial design. The design factors are temperature and pressure, and the response variable is yield. The data that result from this experiment are shown below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Pressure, psig | | |
| Temperature, °C | 100 | 120 | 140 |
| 80 | 47.58, 48.77 | 64.97, 69.22 | 80.92, 72.60 |
| 90 | 51.86, 82.43 | 88.47, 84.23 | 93.95, 88.54 |
| 100 | 71.18, 92.77 | 96.57, 88.72 | 76.58, 83.04 |

(a) Analyze the data from this experiment by conducting an analysis of variance. What conclusions can you draw?

# Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 3187.13 8 398.39 4.37 0.0205 significant

*A* *1096.93* *2* *548.47* *6.02* *0.0219*

*B* *1503.56* *2* *751.78* *8.25* *0.0092*

*AB* *586.64* *4* *146.66* *1.61* *0.2536*

Pure Error 819.98 9 91.11

Cor Total 4007.10 17

The Model F-value of 4.37 implies the model is significant. There is only

a 2.05% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B are significant model terms.

Temperature and pressure are significant. Their interaction is not. An alternate analysis is performed below with *A* and *B* treated as numeric factors:

# Design Expert Output

**Response:** **Yield**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 3073.27 5 614.65 7.90 0.0017 significant

*A 850.76 1 850.76 10.93 0.0063*

*B 1297.92 1 1297.92 16.68 0.0015*

*A2 246.18 1 246.18 3.16 0.1006*

*B2 205.64 1 205.64 2.64 0.1300*

*AB 472.78 1 472.78 6.08 0.0298*

Residual 933.83 12 77.82

*Lack of Fit 113.86 3 37.95 0.42 0.7454 not significant*

*Pure Error 819.98 9 91.11*

Cor Total 4007.10 17

The Model F-value of 7.90 implies the model is significant. There is only

a 0.17% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case A, B, AB are significant model terms.

(b) Graphically analyze the residuals. Are there any concerns about underlying assumptions or model adequacy?

The following residual plots are based on the first analysis shown above with the *A* and *B* treated as categorical factors. The plot of residuals versus pressure shows a decreasing funnel shape indicating a non-constant variance.





(c) Verify that if we let the low, medium and high levels of both factors in this experiment take on the levels -1, 0, and +1, then a least squares fit to a second order model for yield is



The coefficients can be found in the following table of computer output.

# Design Expert Output

**Final Equation in Terms of Coded Factors:**

Yield =

+86.81

+8.42 \* A

+10.40 \* B

-7.84 \* A2

-7.17 \* B2

-7.69 \* A \* B

(d) Confirm that the model in part (c) can be written in terms of the natural variables temperature (*T*) and pressure (*P*) as



The coefficients can be found in the following table of computer output.

Design Expert Output

**Final Equation in Terms of Actual Factors:**

Yield =

-1335.62500

+8.58737 \* Pressure

+18.55850 \* Temperature

-0.019612 \* Pressure2

-0.071700 \* Temperature2

-0.038437 \* Pressure \* Temperature

(e) Construct a contour plot for yield as a function of pressure and temperature. Based on the examination of this plot, where would you recommend running the process?



Run the process in the oval region indicated by the yield of 90.

**9.5.**

(a) Confound a 33 design in three blocks using the *ABC*2component of the three-factor interaction. Compare your results with the design in Figure 9.7.

*L* = *x*1 + *x* 2 + 2*x* 3

|  |  |  |
| --- | --- | --- |
| Block 1 | Block 2 | Block 3 |
| 000 | 100 | 200 |
| 112 | 212 | 012 |
| 210 | 010 | 110 |
| 120 | 220 | 020 |
| 022 | 122 | 222 |
| 202 | 002 | 102 |
| 221 | 021 | 121 |
| 101 | 201 | 001 |
| 011 | 111 | 211 |

The new design is a 180° rotation around the Factor *B* axis.

(b) Confound a 33 design in three blocks using the *AB*2*C* component of the three-factor interaction. Compare your results with the design in Figure 9.7.

*L* = *x*1 + 2*x*2 + *x*3

|  |  |  |
| --- | --- | --- |
| Block 1 | Block 2 | Block 3 |
| 000 | 210 | 112 |
| 022 | 202 | 120 |
| 011 | 221 | 101 |
| 212 | 100 | 010 |
| 220 | 122 | 002 |
| 201 | 111 | 021 |
| 110 | 012 | 200 |
| 102 | 020 | 222 |
| 121 | 001 | 211 |

The new design is a 180° rotation around the Factor *C* axis.

(c) Confound a 33 design three blocks using the *ABC* component of the three-factor interaction. Compare your results with the design in Figure 9.7.

*L* = *x*1 + *x*2 + *x*3

|  |  |  |
| --- | --- | --- |
| Block 1 | Block 2 | Block 3 |
| 000 | 112 | 221 |
| 210 | 022 | 101 |
| 120 | 202 | 011 |
| 021 | 100 | 212 |
| 201 | 010 | 122 |
| 111 | 220 | 002 |
| 012 | 121 | 200 |
| 222 | 001 | 110 |
| 102 | 211 | 020 |

The new design is a 90° rotation around the Factor *C* axis along with switching layer 0 and layer 1 in the *C* axis.

(d) After looking at the designs in parts (a), (b), and (c) and Figure 9.7, what conclusions can you draw?

All four designs are relatively the same. The only differences are rotations and swapping of layers.

**9.6.** Confound a 34 design in three blocks using the *AB*2*CD* component of the four-factor interaction.

The three blocks are shown below with *L* = *x*1 + 2*x*2 + *x*3 + *x*4

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Block 1 |  |  |  |  |
| 0000 | 1100 | 0110 | 0101 | 2200 | 0220 | 0202 | 1210 | 1201 |
| 0211 | 1222 | 2212 | 2221 | 0122 | 2111 | 1121 | 1112 | 2010 |
| 2102 | 0021 | 2001 | 2120 | 1011 | 2022 | 0012 | 1002 | 1020 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Block 2 |  |  |  |  |
| 1021 | 1110 | 1202 | 0001 | 0120 | 0212 | 1012 | 1101 | 1220 |
| 0200 | 0022 | 0111 | 2002 | 2121 | 2210 | 0010 | 0102 | 0221 |
| 1000 | 1122 | 1211 | 2112 | 2201 | 2020 | 2011 | 2100 | 2222 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Block 3 |  |  |  |  |
| 2012 | 2101 | 2220 | 1022 | 1111 | 1200 | 2000 | 2121 | 2211 |
| 1221 | 1010 | 1102 | 0020 | 0112 | 0201 | 1001 | 1120 | 1212 |
| 2021 | 2110 | 2202 | 0100 | 0222 | 0011 | 0002 | 0121 | 0210 |

**9.7S.** Consider the data from the first replicate of Problem 9.3. Assuming that all 27 observations could not be run on the same day, set up a design for conducting the experiment over three days with *AB2C* confounded with blocks. Analyze the data.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Block 1 | | |  |  | Block 2 | | |  |  | Block 3 | | |  |
| 000 | | = | 3.45 | | 100 | | = | 4.07 | | 200 | | = | 4.20 | |
| 110 | | = | 4.38 | | 210 | | = | 4.26 | | 010 | | = | 4.14 | |
| 011 | | = | 5.22 | | 111 | | = | 5.15 | | 211 | | = | 5.17 | |
| 102 | | = | 4.30 | | 202 | | = | 4.17 | | 002 | | = | 4.08 | |
| 201 | | = | 4.96 | | 001 | | = | 4.80 | | 101 | | = | 4.52 | |
| 212 | | = | 4.86 | | 012 | | = | 3.94 | | 112 | | = | 4.53 | |
| 121 | | = | 6.25 | | 221 | | = | 6.03 | | 021 | | = | 6.21 | |
| 022 | | = | 5.14 | | 122 | | = | 4.99 | | 222 | | = | 4.85 | |
| 220 | | = | 5.67 | | 020 | | = | 5.80 | | 120 | | = | 5.48 | |
| Totals | | = | 44.23 | |  | |  | 43.21 | |  | |  | 43.18 | |

The analysis of variance below identifies factors *B* and *C* as significant.

Design Expert Output

**Response: Time**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Block 0.079 2 0.040

Model 13.57 18 0.75 10.49 0.0040 significant

*A 0.11 2 0.055 0.77 0.5052*

*B 8.42 2 4.21 58.63 0.0001*

*C 3.81 2 1.91 26.53 0.0010*

*AB 0.30 4 0.075 1.04 0.4573*

*AC 0.11 4 0.028 0.39 0.8105*

*BC 0.81 4 0.20 2.83 0.1234*

Residual 0.43 6 0.072

Cor Total 14.08 26

The Model F-value of 10.49 implies the model is significant. There is only

a 0.40% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.

In this case B, C are significant model terms.

**9.8.** Outline the analysis of variance table for the 34 design in nine blocks. Is this a practical design?

|  |  |
| --- | --- |
| Source | DF |
| *A* | 2 |
| *B* | 2 |
| *C* | 2 |
| *D* | 2 |
| *AB* | 4 |
| *AC* | 4 |
| *AD* | 4 |
| *BC* | 4 |
| *BD* | 4 |
| *CD* | 4 |
| *ABC (AB*2*C,ABC*2*,AB*2*C*2*)* | 6 |
| *ABD (ABD,AB*2*D,ABD*2*)* | 6 |
| *ACD (ACD,ACD*2*,AC*2*D*2*)* | 6 |
| *BCD (BCD,BC*2*D,BCD*2*)* | 6 |
| *ABCD* | 16 |
| Blocks (*ABC,AB*2*C*2*,AC*2*D,BC*2*D*2) | 8 |
| Total | 80 |

Any experiment with 81 runs is large. Instead of having three full levels of each factor, if two levels of each factor could be used, then the overall design would have 16 runs plus some center points. This two-level design could now probably be run in 2 or 4 blocks, with center points in each block. Additional curvature effects could be determined by augmenting the experiment with the axial points of a central composite design and additional enter points. The overall design would be less than 81 runs.

**9.19S.** Consider the data in Problem 9.3. If *ABC* is confounded in replicate I and *ABC2* is confounded in replicate II, perform the analysis of variance.

*L*1 = *x*1 + *x*2 + *x*3 *L*2 = *x*1 + *x*2 + 2*x*2

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Block 1 | | |  |  |  | Block 2 | | |  |  |  | Block 3 | | |  |  |  | Block 1 | | |  |  |  | Block 2 | | |  |  |  | Block 3 | | |  |
| 000 | | = | 3.45 | |  | 001 | | = | 4.80 | |  | 002 | | = | 4.08 | |  | 000 | | = | 3.36 | |  | 100 | | = | 3.52 | |  | 200 | | = | 3.68 | |
| 111 | | = | 5.15 | |  | 112 | | = | 4.53 | |  | 110 | | = | 4.38 | |  | 101 | | = | 4.44 | |  | 201 | | = | 4.39 | |  | 001 | | = | 4.40 | |
| 222 | | = | 4.85 | |  | 220 | | = | 5.67 | |  | 221 | | = | 6.03 | |  | 011 | | = | 4.70 | |  | 111 | | = | 4.65 | |  | 211 | | = | 4.75 | |
| 120 | | = | 5.48 | |  | 121 | | = | 6.25 | |  | 122 | | = | 4.99 | |  | 221 | | = | 6.38 | |  | 021 | | = | 5.88 | |  | 121 | | = | 6.20 | |
| 102 | | = | 4.30 | |  | 100 | | = | 4.07 | |  | 101 | | = | 4.52 | |  | 202 | | = | 3.88 | |  | 002 | | = | 3.65 | |  | 102 | | = | 4.04 | |
| 210 | | = | 4.26 | |  | 211 | | = | 5.17 | |  | 212 | | = | 4.86 | |  | 022 | | = | 4.49 | |  | 122 | | = | 4.59 | |  | 222 | | = | 4.90 | |
| 201 | | = | 4.96 | |  | 202 | | = | 4.17 | |  | 200 | | = | 4.20 | |  | 120 | | = | 4.85 | |  | 220 | | = | 5.58 | |  | 020 | | = | 5.23 | |
| 012 | | = | 3.94 | |  | 010 | | = | 4.14 | |  | 011 | | = | 5.22 | |  | 210 | | = | 4.37 | |  | 010 | | = | 4.19 | |  | 110 | | = | 4.26 | |
| 021 | | = | 6.21 | |  | 022 | | = | 5.14 | |  | 020 | | = | 5.80 | |  | 112 | | = | 4.08 | |  | 212 | | = | 4.48 | |  | 012 | | = | 4.08 | |

The sums of squares for *A, B, C, AB, AC*, and *BC* are calculated as usual. The only sums of squares presenting difficulties with calculations are the four components of the *ABC* interaction (*ABC, ABC2,* *AB2C*, and *AB2C2*). *ABC* is computed using replicate I and *ABC2* is computed using replicate II. *AB2C* and *AB2C2* are computed using data from both replicates.

We will show how to calculate *AB2C* and *AB2C2* from both replicates. Form a two-way table of *A* x *B* at each level of *C*. Find the *I*(*AB*) and *J*(*AB*) totals for each third of the *A* x *B* table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | *A* |  |  |  |  |
| *C* | *B* | 0 | 1 | 2 |  | I | J |
|  | 0 | 6.81 | 7.59 | 7.88 |  | 26.70 | 27.55 |
| 0 | 1 | 8.33 | 8.64 | 8.63 |  | 27.25 | 27.17 |
|  | 2 | 11.03 | 10.33 | 11.25 |  | 26.54 | 25.77 |
|  | 0 | 9.20 | 8.96 | 9.35 |  | 31.41 | 31.24 |
| 1 | 1 | 9.92 | 9.80 | 9.92 |  | 30.97 | 31.29 |
|  | 2 | 12.09 | 12.45 | 12.41 |  | 31.72 | 31.57 |
|  | 0 | 7.73 | 8.34 | 8.05 |  | 26.09 | 26.29 |
| 2 | 1 | 8.02 | 8.61 | 9.34 |  | 27.31 | 26.11 |
|  | 2 | 9.63 | 9.58 | 9.75 |  | 25.65 | 26.65 |

The *I* and J components for each third of the above table are used to form a new table of diagonal totals.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *C* |  | *I*(*AB*) |  |  | *J*(*AB*) |  |
| 0 | 26.70 | 27.25 | 26.54 | 27.55 | 27.17 | 25.77 |
| 1 | 31.41 | 30.97 | 31.72 | 31.24 | 31.29 | 31.57 |
| 2 | 26.09 | 27.31 | 25.65 | 26.29 | 26.11 | 26.65 |

*I* Totals: *I* Totals:

85.06, 85.26, 83.32 85.49, 85.03, 83.12

*J* Totals: *J* Totals:

85.73, 83.60, 84.31 83.35,8 5.06, 85.23





If it were necessary, we could find *ABC2* as *ABC2*= *I*[*C* x *J*(*AB*)] and *ABC* as *J*[*C* x *J*(*AB*)]. However, these components must be computed using the data from the appropriate replicate.

The analysis of variance table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | *SS* | *DF* | *MS* | *F*0 |
| Replicates | 1.06696 | 1 |  |  |
| Blocks within Replicates | 0.2038 | 4 |  |  |
| *A* | 0.4104 | 2 | 0.2052 | 5.02 |
| *B* | 17.7514 | 2 | 8.8757 | 217.0 |
| *C* | 7.6631 | 2 | 3.8316 | 93.68 |
| *AB* | 0.1161 | 4 | 0.0290 | <1 |
| *AC* | 0.1093 | 4 | 0.0273 | <1 |
| *BC* | 1.6790 | 4 | 0.4198 | 10.26 |
| *ABC* (rep I) | 0.0452 | 2 | 0.0226 | <1 |
| *ABC2* (rep II) | 0.1020 | 2 | 0.0510 | 1.25 |
| *AB2C* | 0.1307 | 2 | 0.0754 | 1.60 |
| *AB2C2* | 0.1265 | 2 | 0.0633 | 1.55 |
| Error | 0.8998 | 22 | 0.0409 |  |
| Total | 30.3069 | 53 |  |  |

**9.10.** Consider the data from replicate I in Problem 9.3. Suppose that only a one-third fraction of this design with *I=ABC* is run. Construct the design, determine the alias structure, and analyze the design.

The design is 000, 012, 021, 102, 201, 111, 120, 210, 222.

The alias structure is: *A = BC = AB*2*C*2

*B = AC = AB*2*C*

*C = AB = ABC*2

*AB*2 *= AC*2*= BC*2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | *C* |  |
| *A* | *B* | 0 | 1 | 2 |
|  | 0 | 3.45 |  |  |
| 0 | 1 |  |  | 5.48 |
|  | 2 |  | 4.26 |  |
|  | 0 |  |  | 6.21 |
| 1 | 1 |  | 5.15 |  |
|  | 2 | 4.96 |  |  |
|  | 0 |  | 3.94 |  |
| 2 | 1 | 4.30 |  |  |
|  | 2 |  |  | 4.85 |

|  |  |  |
| --- | --- | --- |
| Source | SS | DF |
| *A* | 2.25 | 2 |
| *B* | 0.30 | 2 |
| *C* | 2.81 | 2 |
| *AB2* | 0.30 | 2 |
| Total | 5.66 | 8 |

**9.11.** From examining Figure 9.9, what type of design would remain if after completing the first 9 runs, one of the three factors could be dropped?

The remaining design is a full 32 factorial.

**9.12S.** Construct a  design with *I=ABCD*. Write out the alias structure for this design.

The 27 runs for this design are as follows:

|  |  |  |
| --- | --- | --- |
| 0000 | 1002 | 2001 |
| 0012 | 1011 | 2010 |
| 0021 | 1020 | 2022 |
| 0102 | 1101 | 2100 |
| 0111 | 1110 | 2112 |
| 0120 | 1122 | 2121 |
| 0201 | 1200 | 2202 |
| 0210 | 1212 | 2211 |
| 0222 | 1221 | 2220 |

*A = AB*2*C*2*D*2 *= BCD B = AB*2*CD = ACD C = ABC*2*D = ABD D = ABCD*2 *= ABC*

*AB = ABC*2*D*2 *= CD AB*2 *= AC*2*D*2 *= BC*2*D*2 *AC = AB*2*CD*2 *= BD AC*2 *= AB*2*D*2 *= BC*2*D*

*BC = AB*2*C*2*D = AD BC*2 *= AB*2*D = AC*2*D BD*2 *= AB*2*C = ACD*2 *CD*2 *= ABC*2 *= ABD*2

*AD*2 *= AB*2*C*2 *= BCD*2

**9.13.** Verify that the design in Problem 9.12 is a resolution IV design.

The design in Problem 9.12 is a Resolution IV design because no main effect is aliased with a component of a two-factor interaction, but some two-factor interaction components are aliased with each other.

**9.14S.** Construct a 35-2 design with *I=ABC* and *I=CDE*. Write out the alias structure for this design. What is the resolution of this design?

The complete defining relation for this design is: *I* = *ABC = CDE = ABC*2*DE = ABD*2*E*2

This is a resolution III design with *x*4 = 2*x*1 + 2*x*2 + *x*3 and *x*5 = *x*1 + *x*2 + 2*x*4 (mod 3).

|  |  |  |
| --- | --- | --- |
| 00000 | 00112 | 00221 |
| 10022 | 10101 | 10210 |
| 20011 | 20120 | 20202 |
| 01022 | 01101 | 01210 |
| 11011 | 11120 | 11202 |
| 21000 | 21112 | 21221 |
| 02011 | 02120 | 02202 |
| 12000 | 12112 | 12221 |
| 22022 | 22101 | 22210 |

To find the alias of any effect, multiply the effect by *I* and *I*2. For example, the alias of *A* is:

*A = AB*2*C*2 *= ACDE = AB*2*CDE = AB*2*DE = BC = AC*2*D*2*E*2 *= BC*2*DE = BD*2*E*2

**9.15.** Construct a 39-6 design, and verify that it is a resolution III design.

Use the generators *I* = *AC*2*D*2, *I* = *AB*2*C*2*E*, *I* = *BC*2*F*2, *I* = *AB*2*CG*, *I* = *ABCH*2, and *I* = *ABJ*2

|  |  |  |
| --- | --- | --- |
| 000000000 | 021201102 | 102211001 |
| 022110012 | 212012020 | 001212210 |
| 011220021 | 100120211 | 211100110 |
| 221111221 | 122200220 | 020022222 |
| 210221200 | 010011111 | 222020101 |
| 202001212 | 201122002 | 200210122 |
| 112222112 | 002121120 | 121021010 |
| 101002121 | 111010202 | 110101022 |
| 120112100 | 220202011 | 012102201 |

To find the alias of any effect, multiply the effect by *I* and *I*2. For example, the alias of *C* is:

*C* = *C*(*BC*2*F*2) = *BF*2, At least one main effect is aliased with a component of a two-factor interaction.

**9.16.** Construct a 4 x 23 design confounded in two blocks of 16 observations each. Outline the analysis of variance for this design.

Design is a 4 x 23, with *ABC* at two levels, and *Z* at 4 levels. Represent *Z* with two pseudo-factors *D* and *E* as follows:

|  |  |  |
| --- | --- | --- |
| Factor | Pseudo- | Factors |
| *Z* | *D* | *E* |
| *Z*1 | 0 | 0 = (1) |
| *Z*2 | 1 | 0 = *d* |
| *Z*3 | 0 | 1 = *e* |
| *Z*4 | 1 | 1 = *de* |

The 4 x 23 is now a 25 in the factors *A, B, C, D* and *E*. Confound *ABCDE* with blocks. We have given both the letter notation and the digital notation for the treatment combinations.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Block 1 | | |  |  |  | Block 2 | | |  |
| (1) | | = | 0000 | |  | *a* | | = | 1000 | |
| *ab* | | = | 1100 | |  | *b* | | = | 0100 | |
| *ac* | | = | 1010 | |  | *c* | | = | 0010 | |
| *bc* | | = | 0110 | |  | *abc* | | = | 1110 | |
| *abcd* | | = | 1111 | |  | *bcd* | | = | 0111 | |
| *abce* | | = | 1112 | |  | *bce* | | = | 0112 | |
| *cd* | | = | 0011 | |  | *acd* | | = | 1011 | |
| *ce* | | = | 0012 | |  | *ace* | | = | 1012 | |
| *de* | | = | 0003 | |  | *ade* | | = | 1003 | |
| *abde* | | = | 1103 | |  | *bde* | | = | 0103 | |
| *bcde* | | = | 0113 | |  | *abcde* | | = | 1113 | |
| *be* | | = | 0102 | |  | *abd* | | = | 1101 | |
| *ad* | | = | 1001 | |  | *abe* | | = | 1102 | |
| *ae* | | = | 1002 | |  | *d* | | = | 0001 | |
| *acde* | | = | 1013 | |  | *e* | | = | 0002 | |
| *bd* | | = | 0101 | |  | *cde* | | = | 0013 | |

|  |  |
| --- | --- |
| Source | *DF* |
| *A* | 1 |
| *B* | 1 |
| *C* | 1 |
| *Z (D+E+DE)* | 3 |
| *AB* | 1 |
| *AC* | 1 |
| *AZ (AD+AE+ADE)* | 3 |
| *BC* | 1 |
| *BZ (BD+BE+BDE)* | 3 |
| *CZ (CD+CE+CDE)* | 3 |
| *ABC* | 1 |
| *ABZ (ABD+ABE+ABDE)* | 3 |
| *ACZ (ACD+ACE+ACDE)* | 3 |
| *BCZ (BCD+BCE+BCDE)* | 3 |
| *ABCZ (ABCD+ABCE)* | 2 |
| Blocks (or *ABCDE*) | 1 |
| Total | 31 |

**9.17.** Outline the analysis of variance table for a 2232 factorial design. Discuss how this design may be confounded in blocks.

Suppose we have *n* replicates of a 2232 factorial design. *A* and *B* are at 2 levels, and *C* and *D* are at 3 levels.

|  |  |  |
| --- | --- | --- |
| Source | *DF* | Components for Confounding |
| *A* | 1 | *A* |
| *B* | 1 | *B* |
| *C* | 2 | *C* |
| *D* | 2 | *D* |
| *AB* | 1 | *AB* |
| *AC* | 2 | *AC* |
| *AD* | 2 | *AD* |
| *BC* | 2 | *BD* |
| *BD* | 2 | *CD,CD2* |
| *CD* | 4 | *ABC* |
| *ABC* | 2 | *ABD* |
| *ABD* | 2 | *ACD,ACD2* |
| *ACD* | 4 | *BCD,BCD2* |
| *BCD* | 4 | *ABCD,ABCD2* |
| *ABCD* | 4 |  |
| Error | 36(n-1) |  |
| Total | 36n-1 |  |

Confounding in this series of designs is discussed extensively by Margolin (1967). The possibilities for a single replicate of the 2232 design are:

2 blocks of 18 observations 4 blocks of 9 observations 9 blocks of 4 observations

3 blocks of 12 observations 6 blocks of 6 observations

For example, one component of the four-factor interaction, say *ABCD2*, could be selected to confound the design in 3 blocks of 12 observations each, while to confound the design in 2 blocks of 18 observations each we would select the *AB* interaction. Cochran and Cox (1957) and Anderson and McLean (1974) discuss confounding in these designs.

**9.18.** Starting with a 16-run 24 design, show how two three-level factors can be incorporated in this experiment. How many two-level factors can be included if we want some information on two-factor interactions?

Use column *A* and *B* for one three-level factor and columns *C* and *D* for the other. Use the *AC* and *BD* columns for the two, two-level factors. The design will be of resolution V.

**9.19.** Starting with a 16-run 24 design, show how one three-level factor and three two-level factors can be accommodated and still allow the estimation of two-factor interactions.

Use columns *A* and *B* for the three-level factor, and columns *C* and *D* and *ABCD* for the three two-level factors. This design will be of resolution V.

**9.20.** In Problem 8.24, you met Harry Peterson-Nedry, a friend of the author who has a winery and vineyard in Newberg, Oregon. That problem described the application of two-level fractional factorial designs to their 1985 Pinor Noir product. In 1987, they wanted to conduct another Pinot Noir experiment. The variables for this experiment were

Variable Levels

Clone of Pinot Noir Wadenswil, Pommard

Berry Size Small, Large

Fermentation temperature 80F, 85F, 90/80F, 90F

Whole Berry None, 10%

Maceration Time 10 days, 21 days

Yeast Type Assmanhau, Champagne

Oak Type Troncais, Allier

Harry decided to use a 16-run two-level fractional factorial design, treating the four levels of fermentation temperature as two two-level variables. As in the Chapter 8 Problem, the rankings from a taste-test panel was the response variable. The design and the resulting average ranks are shown below:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | Clone | Size | Berry Temp. | Ferm. Berry | Whole Time | Macer. Type | Yeast Type | Oak Rank | Average |
| 1 | – | – | – | – | – | – | – | – | 4 |
| 2 | + | – | – | – | – | + | + | + | 10 |
| 3 | – | + | – | – | + | – | + | + | 6 |
| 4 | + | + | – | – | + | + | – | – | 9 |
| 5 | – | – | + | – | + | + | + | – | 11 |
| 6 | + | – | + | – | + | – | – | + | 1 |
| 7 | – | + | + | – | – | + | – | + | 15 |
| 8 | + | + | + | – | – | – | + | – | 5 |
| 9 | – | – | – | + | + | + | – | + | 12 |
| 10 | + | – | – | + | + | – | + | – | 2 |
| 11 | – | + | – | + | – | + | + | – | 16 |
| 12 | + | + | – | + | – | – | – | + | 3 |
| 13 | – | – | + | + | – | – | + | + | 8 |
| 14 | + | – | + | + | – | + | – | – | 14 |
| 15 | – | + | + | + | + | – | – | – | 7 |
| 16 | + | + | + | + | + | + | + | + | 13 |

(a) Describe the aliasing in this design.

The design is a resolution IV design such that the main effects are aliased with three factor interactions.

Design Expert Output

Term Aliases

Intercept ABCG ABDH ABEF ACDF ACEH ADEG AFGH BCDE BCFH BDFG BEGH CDGH CEFG DEFH

A BCG BDH BEF CDF CEH DEG FGH ABCDE

B ACG ADH AEF CDE CFH DFG EGH

C ABG ADF AEH BDE BFH DGH EFG

D ABH ACF AEG BCE BFG CGH EFH

E ABF ACH ADG BCD BGH CFG DFH

F ABE ACD AGH BCH BDG CEG DEH

G ABC ADE AFH BDF BEH CDH CEF

H ABD ACE AFG BCF BEG CDG DEF

AB CG DH EF ACDE ACFH ADFG AEGH BCDF BCEH BDEG BFGH

AC BG DF EH ABDE ABFH ADGH AEFG BCDH BCEF CDEG CFGH

AD BH CF EG ABCE ABFG ACGH AEFH BCDG BDEF CDEH DFGH

AE BF CH DG ABCD ABGH ACFG ADFH BCEG BDEH CDEF EFGH

AF BE CD GH ABCH ABDG ACEG ADEH BCFG BDFH CEFH DEFG

AG BC DE FH ABDF ABEH ACDH ACEF BDGH BEFG CDFG CEGH

AH BD CE FG ABCF ABEG ACDG ADEF BCGH BEFH CDFH DEGH

(b) Analyze the data and draw conclusions.

All of the main effects except Yeast and Oak are significant. The Maceration Time is the most significant. None of the interactions were significant.



Design Expert Output

**Response:** **Rank**

**ANOVA for Selected Factorial Model**

**Analysis of variance table [Partial sum of squares]**

**Sum of** **Mean** **F**

**Source** **Squares** **DF** **Square** **Value** **Prob > F**

Model 328.75 6 54.79 43.83 < 0.0001 significant

*A* *30.25* *1* *30.25* *24.20* *0.0008*

*B* *9.00* *1* *9.00* *7.20* *0.0251*

*C* *9.00* *1* *9.00* *7.20* *0.0251*

*D* *12.25* *1* *12.25* *9.80* *0.0121*

*E* *12.25* *1* *12.25* *9.80* *0.0121*

*F* *256.00* *1* *256.00* *204.80* *< 0.0001*

Residual 11.25 9 1.25

Cor Total 340.00 15

The Model F-value of 43.83 implies the model is significant. There is only

a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev. 1.12 R-Squared 0.9669

Mean 8.50 Adj R-Squared 0.9449

C.V. 13.15 Pred R-Squared 0.8954

PRESS 35.56 Adeq Precision 19.270

**Coefficient** **Standard** **95% CI** **95% CI**

**Factor** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 8.50 1 0.28 7.87 9.13

A-Clone -1.38 1 0.28 -2.01 -0.74 1.00

B-Berry Size 0.75 1 0.28 0.12 1.38 1.00

C-Ferm Temp 1 0.75 1 0.28 0.12 1.38 1.00

D-Ferm Temp 2 0.88 1 0.28 0.24 1.51 1.00

E-Whole Berry -0.87 1 0.28 -1.51 -0.24 1.00

F-Macer Time 4.00 1 0.28 3.37 4.63 1.00

**Final Equation in Terms of Coded Factors:**

Rank =

+8.50

-1.38 \* A

+0.75 \* B

+0.75 \* C

+0.88 \* D

-0.87 \* E

+4.00 \* F

(c) What comparisons can you make between this experiment and the 1985 Pinot Noir experiment from Problem 8.30?

The experiment from Chapter 8 indicates that Yeast, Barrel, Whole Cluster and the Clone X Yeast interactions were significant. This experiment indicates that Maceration Time, Whole Berry, Clone and Fermentation Temperature are significant.

**9.21.** Suppose there are four three-level categorical factors and a style two-level continuous factor. What is the minimum number of runs required to estimate all main effects and two-factor interactions? Construct this design.

A minimum of 42 runs are required to estimate all main effects and two factor interactions. Below is a *D-*optimal design with 42 runs.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run | Factor *A* | Factor *B* | Factor *C* | Factor *D* | Factor *E* |
| 1 | 2 | 1 | 1 | 1 | -1 |
| 2 | 1 | 1 | 2 | 1 | +1 |
| 3 | 3 | 3 | 2 | 3 | -1 |
| 4 | 2 | 2 | 1 | 3 | -1 |
| 5 | 1 | 1 | 1 | 2 | +1 |
| 6 | 3 | 2 | 3 | 3 | -1 |
| 7 | 1 | 2 | 1 | 2 | -1 |
| 8 | 1 | 1 | 3 | 2 | +1 |
| 9 | 2 | 1 | 3 | 2 | +1 |
| 10 | 1 | 2 | 2 | 2 | +1 |
| 11 | 2 | 3 | 3 | 1 | -1 |
| 12 | 3 | 2 | 2 | 3 | +1 |
| 13 | 2 | 1 | 3 | 1 | -1 |
| 14 | 3 | 1 | 1 | 2 | +1 |
| 15 | 2 | 3 | 1 | 2 | -1 |
| 16 | 3 | 1 | 3 | 3 | +1 |
| 17 | 2 | 2 | 2 | 1 | -1 |
| 18 | 3 | 3 | 1 | 2 | +1 |
| 19 | 1 | 1 | 3 | 3 | -1 |
| 20 | 3 | 3 | 3 | 2 | -1 |
| 21 | 3 | 2 | 1 | 1 | -1 |
| 22 | 1 | 3 | 3 | 3 | -1 |
| 23 | 1 | 3 | 2 | 3 | +1 |
| 24 | 2 | 1 | 2 | 2 | -1 |
| 25 | 1 | 3 | 2 | 2 | -1 |
| 26 | 3 | 2 | 3 | 1 | +1 |
| 27 | 1 | 3 | 1 | 1 | -1 |
| 28 | 2 | 2 | 3 | 3 | +1 |
| 29 | 1 | 3 | 3 | 1 | +1 |
| 30 | 3 | 1 | 1 | 3 | -1 |
| 31 | 2 | 3 | 2 | 2 | +1 |
| 32 | 1 | 2 | 3 | 1 | -1 |
| 33 | 3 | 2 | 2 | 2 | -1 |
| 34 | 2 | 3 | 1 | 1 | +1 |
| 35 | 3 | 1 | 2 | 1 | -1 |
| 36 | 3 | 3 | 2 | 1 | +1 |
| 37 | 1 | 2 | 1 | 3 | +1 |
| 38 | 2 | 1 | 2 | 3 | -1 |
| 39 | 3 | 1 | 1 | 1 | +1 |
| 40 | 2 | 3 | 3 | 3 | +1 |
| 41 | 1 | 2 | 2 | 3 | -1 |
| 42 | 2 | 2 | 1 | 2 | +1 |

**9.22S.** Reconsider the experiment in Problem 9.21. Construct a design with *N* = 48 runs and compare it to the design you constructed in Problem 9.21.

When generating the *D-*optimal design in Design Expert software, an additional six runs were included for lack of fit. The resulting design is shown below. As with the design created in Problem 9.21, the three level factors are balanced with equal runs at each individual factor level, and the two level factor is very slightly unbalanced with 25 low level runs and 23 high level runs.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run | Factor *A* | Factor *B* | Factor *C* | Factor *D* | Factor *E* |
| 1 | 1 | 1 | 3 | 3 | -1 |
| 2 | 3 | 3 | 1 | 1 | -1 |
| 3 | 2 | 3 | 3 | 3 | +1 |
| 4 | 3 | 1 | 2 | 1 | -1 |
| 5 | 1 | 2 | 3 | 2 | +1 |
| 6 | 2 | 1 | 2 | 3 | -1 |
| 7 | 2 | 2 | 3 | 3 | -1 |
| 8 | 3 | 1 | 2 | 2 | +1 |
| 9 | 2 | 2 | 1 | 1 | +1 |
| 10 | 1 | 3 | 3 | 3 | +1 |
| 11 | 2 | 2 | 1 | 2 | -1 |
| 12 | 3 | 2 | 2 | 3 | -1 |
| 13 | 1 | 3 | 1 | 2 | -1 |
| 14 | 2 | 3 | 1 | 2 | +1 |
| 15 | 3 | 1 | 3 | 1 | -1 |
| 16 | 1 | 2 | 2 | 3 | +1 |
| 17 | 3 | 3 | 3 | 3 | -1 |
| 18 | 1 | 2 | 2 | 1 | +1 |
| 19 | 1 | 1 | 1 | 2 | +1 |
| 20 | 3 | 3 | 2 | 3 | +1 |
| 21 | 2 | 2 | 3 | 1 | -1 |
| 22 | 3 | 3 | 3 | 2 | +1 |
| 23 | 2 | 1 | 1 | 1 | -1 |
| 24 | 1 | 2 | 1 | 1 | -1 |
| 25 | 3 | 1 | 3 | 3 | +1 |
| 26 | 1 | 3 | 2 | 3 | -1 |
| 27 | 3 | 1 | 1 | 1 | +1 |
| 28 | 1 | 3 | 3 | 1 | -1 |
| 29 | 2 | 1 | 3 | 2 | -1 |
| 30 | 1 | 3 | 2 | 2 | +1 |
| 31 | 3 | 2 | 3 | 2 | -1 |
| 32 | 2 | 3 | 2 | 1 | +1 |
| 33 | 1 | 1 | 2 | 2 | -1 |
| 34 | 2 | 1 | 1 | 3 | +1 |
| 35 | 2 | 2 | 2 | 2 | +1 |
| 36 | 2 | 3 | 2 | 2 | -1 |
| 37 | 3 | 1 | 1 | 3 | -1 |
| 38 | 1 | 1 | 2 | 1 | +1 |
| 39 | 3 | 2 | 1 | 2 | +1 |
| 40 | 1 | 3 | 1 | 1 | +1 |
| 41 | 2 | 3 | 1 | 3 | -1 |
| 42 | 3 | 2 | 3 | 1 | +1 |
| 43 | 2 | 1 | 3 | 1 | +1 |
| 44 | 1 | 2 | 1 | 2 | -1 |
| 45 | 3 | 2 | 2 | 3 | +1 |
| 46 | 3 | 1 | 1 | 1 | -1 |
| 47 | 1 | 2 | 2 | 2 | -1 |
| 48 | 2 | 3 | 3 | 3 | -1 |

**9.23.** Reconsider the experiment in Problem 9.21. Suppose that you are only interested in main effects. Construct a design with *N* = 12 runs for this experiment.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Run | Factor *A* | Factor *B* | Factor *C* | Factor *D* | Factor *E* |
| 1 | 1 | 3 | 2 | 3 | +1 |
| 2 | 1 | 2 | 3 | 2 | -1 |
| 3 | 2 | 2 | 1 | 3 | +1 |
| 4 | 1 | 1 | 1 | 1 | -1 |
| 5 | 3 | 2 | 2 | 1 | -1 |
| 6 | 3 | 1 | 3 | 3 | -1 |
| 7 | 3 | 1 | 1 | 2 | +1 |
| 8 | 3 | 3 | 3 | 1 | +1 |
| 9 | 2 | 1 | 3 | 1 | +1 |
| 10 | 2 | 3 | 2 | 2 | -1 |
| 11 | 2 | 2 | 1 | 2 | -1 |
| 12 | 1 | 3 | 2 | 1 | -1 |

**9.24.** An article in the *Journal of Chemical Technology and Biotechnology* (“A Study of Antifungal Antibiotic Production by *Thermomomospora* sp MTCC 3340 Using Full Factorial Design,” 2003, Vol. 78, pp. 605-610) investigated three independent variables – concentration of carbon source (glucose), concentration of nitrogen source (soybean meal), and temperature of incubation for their effects on the production of antifungal antibiotic by the isolate *Thermomonospora* sp MTCC 3340. A 33 factorial design was conducted and the results are shown in the table in Table P9.1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Run | % Carbon | % Nitrogen | Temp | Activity |
| 1 | 2 | 0.5 | 25 | 25.84 |
| 2 | 2 | 1 | 25 | 51.86 |
| 3 | 2 | 3 | 25 | 32.59 |
| 4 | 5 | 0.5 | 25 | 20.48 |
| 5 | 5 | 1 | 25 | 25.84 |
| 6 | 5 | 3 | 25 | 12.87 |
| 7 | 7.5 | 0.5 | 25 | 20.48 |
| 8 | 7.5 | 1 | 25 | 25.84 |
| 9 | 7.5 | 3 | 25 | 10.2 |
| 10 | 2 | 0.5 | 30 | 51.86 |
| 11 | 2 | 1 | 30 | 131.33 |
| 12 | 2 | 3 | 30 | 41.11 |
| 13 | 5 | 0.5 | 30 | 41.11 |
| 14 | 5 | 1 | 30 | 104.11 |
| 15 | 5 | 3 | 30 | 32.59 |
| 16 | 7.5 | 0.5 | 30 | 65.42 |
| 17 | 7.5 | 1 | 30 | 82.53 |
| 18 | 7.5 | 3 | 30 | 51.86 |
| 19 | 2 | 0.5 | 35 | 41.11 |
| 20 | 2 | 1 | 35 | 104.11 |
| 21 | 2 | 3 | 35 | 32.59 |
| 22 | 5 | 0.5 | 35 | 32.59 |
| 23 | 5 | 1 | 35 | 82.53 |
| 24 | 5 | 3 | 35 | 25.84 |
| 25 | 7.5 | 0.5 | 35 | 51.86 |
| 26 | 7.5 | 1 | 35 | 65.42 |
| 27 | 7.5 | 3 | 35 | 41.11 |

(a) Analyze the data from this experiment.

The ANOVA is shown below. Factor *B*, and the *B*2 and *C*2 quadratic terms appear to be significant.

Design Expert Output

**Response 1 Activity  
 ANOVA for Response Surface Quadratic Model  
 Analysis of variance table [Partial sum of squares - Type III]  
 Sum of Mean F p-value  
 Source Squares df Square Value Prob > F**

Model 19338.43 9 2148.71 8.63 < 0.0001 significant

*A-Carbon* *433.77* *1* *433.77* *1.74* *0.2043*  
  *B-Nitrogen* *280.10* *1* *280.10* *1.13* *0.3036*  
  *C-Temperature* *2913.48* *1* *2913.48* *11.70* *0.0033*  
  *AB* *50.78* *1* *50.78* *0.20* *0.6572*  
  *AC* *97.05* *1* *97.05* *0.39* *0.5407*  
  *BC* *215.04* *1* *215.04* *0.86* *0.3657*  
  *A2* *486.58* *1* *486.58* *1.95* *0.1801*  
  *B2* *7503.07* *1* *7503.07* *30.14* *< 0.0001*  
  *C2* *4642.23* *1* *4642.23* *18.65* *0.0005*  
 Residual 4231.91 17 248.94  
 Cor Total 23570.33 26

Std. Dev. 15.78 R-Squared 0.8205  
 Mean 48.34 Adj R-Squared 0.7254  
 C.V. % 32.64 Pred R-Squared 0.5795

PRESS 9911.08 Adeq Precision 10.086

(b) Fit a second-order model to the activity response. Construct contour plots and response surface plots to assist in interpreting the results of this experiment.

The model, contour plot, and response surface plot are shown below. Because factor *A* was not significant, the plots are shown with factors *B* and *C* only.

Design Expert Output

**Coefficient** **Standard** **95% CI** **95% CI**

**Term** **Estimate** **DF** **Error** **Low** **High** **VIF**

Intercept 106.17 1 11.02 82.91 129.43

A-Carbon -5.04 1 3.82 -13.09 3.01 1.06  
 B-Nitrogen -3.95 1 3.72 -11.80 3.90 1.12  
 C-Temperature 13.07 1 3.82 5.01 21.13 1.05  
 AB 1.94 1 4.30 -7.13 11.01 1.05  
 AC 2.84 1 4.55 -6.76 12.44 1.00  
 BC -4.00 1 4.30 -13.08 5.08 1.05  
 A2 9.09 1 6.50 -4.63 22.81 1.00  
 B2 -58.48 1 10.65 -80.95 -36.00 1.12  
 C2 -27.82 1 6.44 -41.41 -14.23 1.00

**Final Equation in Terms of Actual Factors:**

Activity =

-1046.32378

-20.43903 \* Carbon  
 +144.34481 \* Nitrogen  
 +69.50973 \* Temperature  
 +0.56468 \* Carbon \* Nitrogen  
 +0.20654 \* Carbon \* Temperature  
 -0.64000 \* Nitrogen \* Temperature  
 +1.20237 \* Carbon2  
 -37.42422 \* Nitrogen2  
 -1.11262 \* Temperature2





(c) What operating conditions would you recommend to optimize this process?

Set factor *B*, Nitrogen, at approximately 1.7, and factor *C*, Temperature, at 31, maximizes the Activity. Factor *A* is not significant, so any level can be chosen depending on the manufacturing preferences.

**9.25.** Construct a minimum run *D-*optimal resolution IV design for 10 factors. Find the alias relationships. What approach would you recommend for analyzing the data from this experiment?

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | *K* |
| – | + | + | – | – | – | + | + | – | – |
| + | + | + | + | + | – | + | + | + | + |
| – | + | – | – | + | + | – | + | + | – |
| – | + | – | + | – | – | – | + | – | + |
| + | + | – | – | – | – | – | – | + | – |
| + | + | – | – | + | + | + | – | – | + |
| – | – | – | – | + | – | + | – | + | + |
| + | – | – | + | + | – | + | + | – | – |
| – | + | + | + | + | – | – | – | – | – |
| – | + | – | + | – | + | + | – | + | – |
| + | – | – | + | + | + | – | – | + | + |
| – | – | – | – | – | + | – | – | – | – |
| + | – | + | + | – | – | + | – | – | + |
| + | – | + | – | + | + | + | – | + | – |
| – | – | + | + | + | + | + | + | – | + |
| – | – | + | + | – | – | – | + | + | – |
| + | + | + | + | – | + | – | + | – | – |
| – | + | + | – | – | + | – | – | + | + |
| + | – | – | – | – | + | + | + | + | + |
| + | – | + | – | + | – | – | + | – | + |

The alias relationships for the main effects and two factor interactions are as follows:

[Intercept] = Intercept *-* 0.333 *\* BE -* 0.333 *\* BG -* 0.333 *\* BK +* 0.333 *\* CD*

*+* 0.333 *\* CH -* 0.333 *\* CJ +* 0.333 *\* DH -* 0.333 *\* DJ +* 0.333 *\* EG +* 0.333 *\* EK*

*+* 0.333 *\* GK -* 0.333 *\* HJ*

[*A*] = *A* [*B*] = *B* [*C*] = *C* [*D*] = *D* [*E*] = *E*

[*F*] = *F* [*G*] = *G* [*H*] = *H* [*J*] = *J* [*K*] = *K*

[*AB*] = *AB +* 0.333 *\* BE +* 0.333 *\* BG +* 0.333 *\* BK +* 0.667 *\* CD +* 0.667 *\* CH*

*+* 0.333 *\* CJ +* 0.667 *\* DH +* 0.333 *\* DJ +* 0.667 *\* EG +* 0.667 *\* EK - FJ*

*+* 0.667 *\* GK +* 0.333 *\* HJ*

[*AC*] = *AC -* 0.333 *\* BC +* 0.667 *\* BD +* 0.667 *\* BH +* 0.333 *\* CE +* 0.333 *\* CG*

*-* 0.667 *\* DE +* 0.333 *\* DK +* 0.667 *\* EJ - FK -* 0.667 *\* GH +* 0.667 *\* GJ*

*+* 0.333 *\* HK -* 0.333 *\* JK*

[*AD*] = *AD +* 0.667 *\* BC -* 0.333 *\* BD +* 0.667 *\* BH -* 0.667 *\* CE +* 0.333 *\* CG*

*+* 0.333 *\* DE +* 0.333 *\* DK +* 0.667 *\* EJ - FG +* 0.333 *\* GH -* 0.333 *\* GJ*

*-* 0.667 *\* HK +* 0.667 *\* JK*

[*AE*] = *AE -* 0.333 *\* BE +* 0.667 *\* BG +* 0.667 *\* BK -* 0.667 *\* CD +* 0.333 *\* CH*

*+* 0.667 *\* CJ +* 0.333 *\* DH +* 0.667 *\* DJ +* 0.333 *\* EG +* 0.333 *\* EK - FH*

*-* 0.667 *\* GK -* 0.333 *\* HJ*

[*AF*] = *AF +* 0.333 *\* BC +* 0.333 *\* BD +* 0.333 *\* BH - BJ -* 0.333 *\* CE -* 0.333 *\* CG*

*- CK -* 0.333 *\* DE - DG -* 0.333 *\* DK - EH +* 0.333 *\* EJ -* 0.333 *\* GH +* 0.333 *\* GJ*

*-* 0.333 *\* HK +* 0.333 *\* JK*

[*AG*] = *AG +* 0.667 *\* BE -* 0.333 *\* BG +* 0.667 *\* BK +* 0.333 *\* CD -* 0.667 *\* CH*

*+* 0.667 *\* CJ - DF +* 0.333 *\* DH -* 0.333 *\* DJ +* 0.333 *\* EG -* 0.667 *\* EK*

*+* 0.333 *\* GK +* 0.667 *\* HJ*

[*AH*] = *AH +* 0.667 *\* BC +* 0.667 *\* BD -* 0.333 *\* BH +* 0.333 *\* CE -* 0.667 *\* CG*

*+* 0.333 *\* DE -* 0.667 *\* DK - EF -* 0.333 *\* EJ +* 0.333 *\* GH +* 0.667 *\* GJ*

*+* 0.333 *\* HK +* 0.667 *\* JK*

[*AJ*] = *AJ +* 0.333 *\* BC +* 0.333 *\* BD - BF +* 0.333 *\* BH +* 0.667 *\* CE +* 0.667 *\* CG*

*+* 0.667 *\* DE +* 0.667 *\* DK +* 0.333 *\* EJ +* 0.667 *\* GH +* 0.333 *\* GJ +* 0.667 *\* HK*

*+* 0.333 *\* JK*

[*AK*] = *AK +* 0.667 *\* BE +* 0.667 *\* BG -* 0.333 *\* BK +* 0.333 *\* CD - CF +* 0.333 *\* CH*

*-* 0.333 *\* CJ -* 0.667 *\* DH +* 0.667 *\* DJ -* 0.667 *\* EG +* 0.333 *\* EK +* 0.333 *\* GK*

*+* 0.667 *\* HJ*

Forward selection regression should be used for analyzing the experiment. Hierarchy should be considered.

**9.26.** Construct a minimum run *D-*optimal resolution IV design for 12 factors. Find the alias relationships. What approach would you recommend for analyzing the data from this experiment?

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* | *K* | *L* | *M* |
| – | – | + | – | – | + | + | – | – | – | – | + |
| – | – | – | – | + | + | – | – | + | + | + | + |
| – | + | – | + | + | + | + | + | – | + | – | + |
| – | + | – | – | – | – | – | + | + | – | – | + |
| + | + | – | + | – | + | + | – | + | – | + | + |
| – | + | + | – | – | + | + | + | + | + | + | – |
| – | – | – | + | – | + | – | + | – | – | + | – |
| – | – | + | + | + | – | + | + | + | – | + | + |
| + | – | – | – | + | + | + | + | + | – | – | – |
| – | – | – | + | – | – | + | – | + | + | – | – |
| + | – | – | – | – | – | + | + | – | + | + | + |
| – | + | – | – | + | – | + | – | – | – | + | – |
| + | + | – | + | + | – | – | + | + | + | + | – |
| + | + | + | + | – | – | + | + | – | – | – | – |
| + | – | + | – | – | – | – | – | + | – | + | – |
| + | – | + | + | + | + | + | – | – | + | + | – |
| – | – | + | – | + | – | – | + | – | + | – | – |
| + | – | – | + | + | – | – | – | – | – | – | + |
| + | + | + | – | + | – | + | – | + | + | – | + |
| + | + | – | – | – | + | – | – | – | + | – | – |
| – | + | + | + | – | – | – | – | – | + | + | + |
| + | + | + | – | + | + | – | + | – | – | + | + |
| – | + | + | + | + | + | – | – | + | – | – | – |
| + | – | + | + | – | + | – | + | + | + | – | + |

The alias relationships for the main effects and two factor interactions are as follows:

[Intercept] = Intercept

[*A*] = *A* [*B*] = *B* [*C*] = *C* [*D*] = *D* [*E*] = *E*

[*F*] = *F* [*G*] = *G* [*H*] = *H* [*J*] = *J* [*K*] = *K*

[*L*] = *L* [*M*] = *M*

[*AB*] = *AB -* 0.333 *\* CD +* 0.333 *\* CE -* 0.333 *\* CF +* 0.333 *\* CG +* 0.333 *\* CH*

*-* 0.333 *\* CJ -* 0.333 *\* CK -* 0.333 *\* CL +* 0.333 *\* CM -* 0.333 *\* DE -* 0.333 *\* DF*

*+* 0.333 *\* DG +* 0.333 *\* DH +* 0.333 *\* DJ -* 0.333 *\* DK +* 0.333 *\* DL -* 0.333 *\* DM*

*-* 0.333 *\* EF -* 0.333 *\* EG +* 0.333 *\* EH +* 0.333 *\* EJ +* 0.333 *\* EK +* 0.333 *\* EL*

*+* 0.333 *\* EM -* 0.333 *\* FG -* 0.333 *\* FH -* 0.333 *\* FJ -* 0.333 *\* FK +* 0.333 *\* FL*

*+* 0.333 *\* FM -* 0.333 *\* GH +* 0.333 *\* GJ -* 0.333 *\* GK -* 0.333 *\* GL +* 0.333 *\* GM*

*-* 0.333 *\* HJ -* 0.333 *\* HK +* 0.333 *\* HL -* 0.333 *\* HM +* 0.333 *\* JK +* 0.333 *\* JL*

+ 0.333 \* JM - 0.333 \* KL - 0.333 \* KM + 0.333 \* LM

[*AC*] = *AC -* 0.333 *\* BD +* 0.333 *\* BE -* 0.333 *\* BF +* 0.333 *\* BG +* 0.333 *\* BH*

*-* 0.333 *\* BJ -* 0.333 *\* BK -* 0.333 *\* BL +* 0.333 *\* BM -* 0.333 *\* DE +* 0.333 *\* DF*

*+* 0.333 *\* DG +* 0.333 *\* DH -* 0.333 *\* DJ +* 0.333 *\* DK -* 0.333 *\* DL -* 0.333 *\* DM*

*+* 0.333 *\* EF +* 0.333 *\* EG -* 0.333 *\* EH -* 0.333 *\* EJ +* 0.333 *\* EK +* 0.333 *\* EL*

*+* 0.333 *\* EM -* 0.333 *\* FG +* 0.333 *\* FH -* 0.333 *\* FJ +* 0.333 *\* FK +* 0.333 *\* FL*

*+* 0.333 *\* FM -* 0.333 *\* GH -* 0.333 *\* GJ +* 0.333 *\* GK -* 0.333 *\* GL -* 0.333 *\* GM*

*-* 0.333 *\* HJ -* 0.333 *\* HK -* 0.333 *\* HL +* 0.333 *\* HM +* 0.333 *\* JK -* 0.333 *\* JL*

*+* 0.333 *\* JM -* 0.333 *\* KL +* 0.333 *\* KM -* 0.333 *\* LM*

[*AD*] = *AD -* 0.333 *\* BC -* 0.333 *\* BE -* 0.333 *\* BF +* 0.333 *\* BG +* 0.333 *\* BH*

*+* 0.333 *\* BJ -* 0.333 *\* BK +* 0.333 *\* BL -* 0.333 *\* BM -* 0.333 *\* CE +* 0.333 *\* CF*

*+* 0.333 *\* CG +* 0.333 *\* CH -* 0.333 *\* CJ +* 0.333 *\* CK -* 0.333 *\* CL -* 0.333 *\* CM*

*-* 0.333 *\* EF -* 0.333 *\* EG -* 0.333 *\* EH -* 0.333 *\* EJ +* 0.333 *\* EK +* 0.333 *\* EL*

*-* 0.333 *\* EM +* 0.333 *\* FG -* 0.333 *\* FH +* 0.333 *\* FJ +* 0.333 *\* FK +* 0.333 *\* FL*

*+* 0.333 *\* FM -* 0.333 *\* GH -* 0.333 *\* GJ -* 0.333 *\* GK +* 0.333 *\* GL -* 0.333 *\* GM*

*+* 0.333 *\* HJ +* 0.333 *\* HK -* 0.333 *\* HL -* 0.333 *\* HM +* 0.333 *\* JK +* 0.333 *\* JL*

*+* 0.333 *\* JM +* 0.333 *\* KL -* 0.333 *\* KM -* 0.333 *\* LM*

[*AE*] = *AE +* 0.333 *\* BC -* 0.333 *\* BD -* 0.333 *\* BF -* 0.333 *\* BG +* 0.333 *\* BH*

*+* 0.333 *\* BJ +* 0.333 *\* BK +* 0.333 *\* BL +* 0.333 *\* BM -* 0.333 *\* CD +* 0.333 *\* CF*

*+* 0.333 *\* CG -* 0.333 *\* CH -* 0.333 *\* CJ +* 0.333 *\* CK +* 0.333 *\* CL +* 0.333 *\* CM*

*-* 0.333 *\* DF -* 0.333 *\* DG -* 0.333 *\* DH -* 0.333 *\* DJ +* 0.333 *\* DK +* 0.333 *\* DL*

*-* 0.333 *\* DM +* 0.333 *\* FG +* 0.333 *\* FH -* 0.333 *\* FJ -* 0.333 *\* FK +* 0.333 *\* FL*

*-* 0.333 *\* FM -* 0.333 *\* GH +* 0.333 *\* GJ +* 0.333 *\* GK -* 0.333 *\* GL -* 0.333 *\* GM*

*+* 0.333 *\* HJ -* 0.333 *\* HK +* 0.333 *\* HL -* 0.333 *\* HM +* 0.333 *\* JK -* 0.333 *\* JL*

*-* 0.333 *\* JM +* 0.333 *\* KL -* 0.333 *\* KM -* 0.333 *\* LM*

[*AF*] = *AF -* 0.333 *\* BC -* 0.333 *\* BD -* 0.333 *\* BE -* 0.333 *\* BG -* 0.333 *\* BH*

*-* 0.333 *\* BJ -* 0.333 *\* BK +* 0.333 *\* BL +* 0.333 *\* BM +* 0.333 *\* CD +* 0.333 *\* CE*

*-* 0.333 *\* CG +* 0.333 *\* CH -* 0.333 *\* CJ +* 0.333 *\* CK +* 0.333 *\* CL +* 0.333 *\* CM*

*-* 0.333 *\* DE +* 0.333 *\* DG -* 0.333 *\* DH +* 0.333 *\* DJ +* 0.333 *\* DK +* 0.333 *\* DL*

*+* 0.333 *\* DM +* 0.333 *\* EG +* 0.333 *\* EH -* 0.333 *\* EJ -* 0.333 *\* EK +* 0.333 *\* EL*

*-* 0.333 *\* EM -* 0.333 *\* GH +* 0.333 *\* GJ -* 0.333 *\* GK +* 0.333 *\* GL -* 0.333 *\* GM*

*+* 0.333 *\* HJ -* 0.333 *\* HK -* 0.333 *\* HL +* 0.333 *\* HM -* 0.333 *\* JK -* 0.333 *\* JL*

*+* 0.333 *\* JM -* 0.333 *\* KL -* 0.333 *\* KM +* 0.333 *\* LM*

[*AG*] = *AG +* 0.333 *\* BC +* 0.333 *\* BD -* 0.333 *\* BE -* 0.333 *\* BF -* 0.333 *\* BH*

*+* 0.333 *\* BJ -* 0.333 *\* BK -* 0.333 *\* BL +* 0.333 *\* BM +* 0.333 *\* CD +* 0.333 *\* CE*

*-* 0.333 *\* CF -* 0.333 *\* CH -* 0.333 *\* CJ +* 0.333 *\* CK -* 0.333 *\* CL -* 0.333 *\* CM*

*-* 0.333 *\* DE +* 0.333 *\* DF -* 0.333 *\* DH -* 0.333 *\* DJ -* 0.333 *\* DK +* 0.333 *\* DL*

*-* 0.333 *\* DM +* 0.333 *\* EF -* 0.333 *\* EH +* 0.333 *\* EJ +* 0.333 *\* EK -* 0.333 *\* EL*

*-* 0.333 *\* EM -* 0.333 *\* FH +* 0.333 *\* FJ -* 0.333 *\* FK +* 0.333 *\* FL -* 0.333 *\* FM*

*-* 0.333 *\* HJ -* 0.333 *\* HK -* 0.333 *\* HL -* 0.333 *\* HM -* 0.333 *\* JK -* 0.333 *\* JL*

*+* 0.333 *\* JM +* 0.333 *\* KL +* 0.333 *\* KM +* 0.333 *\* LM*

[*AH*] = *AH +* 0.333 *\* BC +* 0.333 *\* BD +* 0.333 *\* BE -* 0.333 *\* BF -* 0.333 *\* BG*

*-* 0.333 *\* BJ -* 0.333 *\* BK +* 0.333 *\* BL -* 0.333 *\* BM +* 0.333 *\* CD -* 0.333 *\* CE*

*+* 0.333 *\* CF -* 0.333 *\* CG -* 0.333 *\* CJ -* 0.333 *\* CK -* 0.333 *\* CL +* 0.333 *\* CM*

*-* 0.333 *\* DE -* 0.333 *\* DF -* 0.333 *\* DG +* 0.333 *\* DJ +* 0.333 *\* DK -* 0.333 *\* DL*

*-* 0.333 *\* DM +* 0.333 *\* EF -* 0.333 *\* EG +* 0.333 *\* EJ -* 0.333 *\* EK +* 0.333 *\* EL*

*-* 0.333 *\* EM -* 0.333 *\* FG +* 0.333 *\* FJ -* 0.333 *\* FK -* 0.333 *\* FL +* 0.333 *\* FM*

*-* 0.333 *\* GJ -* 0.333 *\* GK -* 0.333 *\* GL -* 0.333 *\* GM +* 0.333 *\* JK -* 0.333 *\* JL*

*-* 0.333 *\* JM +* 0.333 *\* KL +* 0.333 *\* KM +* 0.333 *\* LM*

[*AJ*] = *AJ -* 0.333 *\* BC +* 0.333 *\* BD +* 0.333 *\* BE -* 0.333 *\* BF +* 0.333 *\* BG*

*-* 0.333 *\* BH +* 0.333 *\* BK +* 0.333 *\* BL +* 0.333 *\* BM -* 0.333 *\* CD -* 0.333 *\* CE*

*-* 0.333 *\* CF -* 0.333 *\* CG -* 0.333 *\* CH +* 0.333 *\* CK -* 0.333 *\* CL +* 0.333 *\* CM*

*-* 0.333 *\* DE +* 0.333 *\* DF -* 0.333 *\* DG +* 0.333 *\* DH +* 0.333 *\* DK +* 0.333 *\* DL*

*+* 0.333 *\* DM -* 0.333 *\* EF +* 0.333 *\* EG +* 0.333 *\* EH +* 0.333 *\* EK -* 0.333 *\* EL*

*-* 0.333 *\* EM +* 0.333 *\* FG +* 0.333 *\* FH -* 0.333 *\* FK -* 0.333 *\* FL +* 0.333 *\* FM*

*-* 0.333 *\* GH -* 0.333 *\* GK -* 0.333 *\* GL +* 0.333 *\* GM +* 0.333 *\* HK -* 0.333 *\* HL*

*-* 0.333 *\* HM -* 0.333 *\* KL +* 0.333 *\* KM -* 0.333 *\* LM*

[*AK*] = *AK -* 0.333 *\* BC -* 0.333 *\* BD +* 0.333 *\* BE -* 0.333 *\* BF -* 0.333 *\* BG*

*-* 0.333 *\* BH +* 0.333 *\* BJ -* 0.333 *\* BL -* 0.333 *\* BM +* 0.333 *\* CD +* 0.333 *\* CE*

*+* 0.333 *\* CF +* 0.333 *\* CG -* 0.333 *\* CH +* 0.333 *\* CJ -* 0.333 *\* CL +* 0.333 *\* CM*

*+* 0.333 *\* DE +* 0.333 *\* DF -* 0.333 *\* DG +* 0.333 *\* DH +* 0.333 *\* DJ +* 0.333 *\* DL*

*-* 0.333 *\* DM -* 0.333 *\* EF +* 0.333 *\* EG -* 0.333 *\* EH +* 0.333 *\* EJ +* 0.333 *\* EL*

*-* 0.333 *\* EM -* 0.333 *\* FG -* 0.333 *\* FH -* 0.333 *\* FJ -* 0.333 *\* FL -* 0.333 *\* FM*

*-* 0.333 *\* GH -* 0.333 *\* GJ +* 0.333 *\* GL +* 0.333 *\* GM +* 0.333 *\* HJ +* 0.333 *\* HL*

*+* 0.333 *\* HM -* 0.333 *\* JL +* 0.333 *\* JM -* 0.333 *\* LM*

[*AL*] = *AL -* 0.333 *\* BC +* 0.333 *\* BD +* 0.333 *\* BE +* 0.333 *\* BF -* 0.333 *\* BG*

*+* 0.333 *\* BH +* 0.333 *\* BJ -* 0.333 *\* BK +* 0.333 *\* BM -* 0.333 *\* CD +* 0.333 *\* CE*

*+* 0.333 *\* CF -* 0.333 *\* CG -* 0.333 *\* CH -* 0.333 *\* CJ -* 0.333 *\* CK -* 0.333 *\* CM*

*+* 0.333 *\* DE +* 0.333 *\* DF +* 0.333 *\* DG -* 0.333 *\* DH +* 0.333 *\* DJ +* 0.333 *\* DK*

*-* 0.333 *\* DM +* 0.333 *\* EF -* 0.333 *\* EG +* 0.333 *\* EH -* 0.333 *\* EJ +* 0.333 *\* EK*

*-* 0.333 *\* EM +* 0.333 *\* FG -* 0.333 *\* FH -* 0.333 *\* FJ -* 0.333 *\* FK +* 0.333 *\* FM*

*-* 0.333 *\* GH -* 0.333 *\* GJ +* 0.333 *\* GK +* 0.333 *\* GM -* 0.333 *\* HJ +* 0.333 *\* HK*

*+* 0.333 *\* HM -* 0.333 *\* JK -* 0.333 *\* JM -* 0.333 *\* KM*

[*AM*] = *AM +* 0.333 *\* BC -* 0.333 *\* BD +* 0.333 *\* BE +* 0.333 *\* BF +* 0.333 *\* BG*

*-* 0.333 *\* BH +* 0.333 *\* BJ -* 0.333 *\* BK +* 0.333 *\* BL -* 0.333 *\* CD +* 0.333 *\* CE*

*+* 0.333 *\* CF -* 0.333 *\* CG +* 0.333 *\* CH +* 0.333 *\* CJ +* 0.333 *\* CK -* 0.333 *\* CL*

*-* 0.333 *\* DE +* 0.333 *\* DF -* 0.333 *\* DG -* 0.333 *\* DH +* 0.333 *\* DJ -* 0.333 *\* DK*

*-* 0.333 *\* DL -* 0.333 *\* EF -* 0.333 *\* EG -* 0.333 *\* EH -* 0.333 *\* EJ -* 0.333 *\* EK*

*-* 0.333 *\* EL -* 0.333 *\* FG +* 0.333 *\* FH +* 0.333 *\* FJ -* 0.333 *\* FK +* 0.333 *\* FL*

*-* 0.333 *\* GH +* 0.333 *\* GJ +* 0.333 *\* GK +* 0.333 *\* GL -* 0.333 *\* HJ +* 0.333 *\* HK*

*+* 0.333 *\* HL +* 0.333 *\* JK -* 0.333 *\* JL -* 0.333 *\* KL*

Forward selection regression should be used for analyzing the experiment. Hierarchy should be considered.

**9.27.** Suppose that you must design an experiment to investigate nine continuous factors. It is thought that running all factors at two levels is adequate but that all two-factor interactions are of interest.

(a) How many runs are required to estimate all main effects and two factor-interactions?

1 for the intercept term

9 for the main effects

 for the two-factor interactions

Total of 46 runs.

(b) Find a minimum-run *D-*optimal design that is suitable for this problem.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* |
| + | – | – | + | + | – | – | – | – |
| + | – | + | – | + | – | + | + | + |
| + | + | – | + | + | – | – | + | + |
| + | + | + | – | + | + | + | – | + |
| + | + | – | – | + | – | – | – | + |
| – | – | – | – | + | + | – | + | + |
| + | – | + | + | + | + | – | + | + |
| + | + | + | + | + | + | – | – | – |
| + | – | – | + | – | – | + | – | + |
| + | – | – | + | – | + | – | + | – |
| – | + | – | – | + | + | – | – | – |
| – | – | + | – | – | – | – | + | – |
| – | + | + | – | + | – | – | + | + |
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| + | + | – | – | – | + | + | – | – |
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| – | – | – | – | + | + | + | + | – |
| – | + | – | – | – | + | + | – | + |
| – | – | – | + | + | + | + | – | + |
| + | – | – | – | – | – | – | + | + |
| – | – | + | + | + | + | + | + | + |
| – | + | + | – | – | – | – | – | – |
| – | – | + | + | – | + | + | – | – |
| + | + | + | – | – | – | + | – | + |
| – | + | + | + | – | – | + | + | + |
| – | + | + | + | + | – | – | + | – |
| + | – | – | – | – | – | + | + | – |
| + | – | + | + | + | + | + | – | – |
| – | + | + | + | – | + | – | + | + |
| – | + | – | – | + | – | + | + | + |
| – | + | – | + | – | – | – | – | + |
| + | + | + | + | + | – | + | – | – |
| – | – | – | + | + | – | + | + | – |
| – | – | + | – | + | – | + | – | – |
| + | + | + | + | – | – | – | + | – |
| – | – | – | – | – | – | – | – | – |
| – | – | + | + | + | – | – | – | + |
| + | + | + | – | – | + | + | + | – |
| + | + | – | – | + | – | – | + | – |

**9.28.** Suppose that you must design an experiment to investigate seven continuous factors. Running all factors at two levels is thought to be appropriate but that only the two-factor interactions involving factor *A* are of interest.

(a) How many runs are required to estimate all main effects and two factor-interactions?

1 for the intercept term, 7 for the main effects, and 6 for the two-factor interactions involving A, for a total of 14 runs.

(b) Find a minimum-run *D-*optimal design that is suitable for this problem.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* | *G* |
| – | – | – | + | – | – | + |
| + | + | – | + | – | – | – |
| – | + | + | – | – | – | + |
| – | + | – | – | + | + | + |
| + | + | + | + | + | + | – |
| + | – | + | + | – | – | + |
| – | + | + | + | + | – | – |
| + | – | – | – | + | + | + |
| + | + | + | – | – | + | + |
| + | + | + | – | + | – | – |
| – | + | – | – | – | – | – |
| + | – | + | – | – | + | – |
| – | – | + | – | + | + | – |
| – | + | + | + | – | + | – |

**9.29S.** Suppose that you must design an experiment to investigate six continuous factors. It is thought that running all factors at two levels is adequate but that only the *AB*, *AC*, and *AD* two-factor interactions are of interest.

(a) How many runs are required to estimate all main effects and two factor-interactions?

1 for the intercept term, 6 for the main effects, and 3 for the two-factor interactions involving *AB*, *AC* and *AD*, for a total of 10 runs.

(b) Find a minimum-run *D-*optimal design that is suitable for this problem.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *A* | *B* | *C* | *D* | *E* | *F* |
| + | + | – | – | – | – |
| + | + | + | + | + | + |
| – | – | – | + | – | + |
| + | – | + | + | – | – |
| + | – | – | + | + | + |
| – | – | + | + | + | – |
| – | – | + | – | – | + |
| – | + | – | – | + | – |
| + | – | + | – | + | + |
| – | + | + | + | – | + |

**9.30.** Suppose that you must design an experiment with six categorical factors. Factor *A* has six levels, factor *B* has five levels, factor *C* has five levels, factor *D* has three levels, and factors *E* and *F* have two levels. You are interested in main effects and two factor interactions.

(a) How many runs are required to estimate all of the effects that are of interest?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Intercept | 1 | *AB* | 20 | *BE* | 4 |
| *A* | 5 | *AC* | 20 | *BF* | 4 |
| *B* | 4 | *AD* | 10 | *CD* | 8 |
| *C* | 4 | *AE* | 5 | *CE* | 4 |
| *D* | 2 | *AF* | 5 | *CF* | 4 |
| *E* | 1 | *BC* | 16 | *DE* | 2 |
| *F* | 1 | *BD* | 8 | *DF* | 2 |
|  |  |  |  | *EF* | 1 |
| Total | 131 |  |  |  |  |

(b) Find a *D-*optimal design suitable for this problem.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Order | *A* | *B* | *C* | *D* | *E* | *F* |  | Order | *A* | *B* | *C* | *D* | *E* | *F* |
| 1 | L2 | L5 | L4 | L1 | L1 | L1 |  | 67 | L2 | L1 | L4 | L1 | L2 | L2 |
| 2 | L5 | L3 | L5 | L2 | L1 | L2 |  | 68 | L6 | L3 | L5 | L3 | L2 | L1 |
| 3 | L3 | L2 | L4 | L3 | L2 | L1 |  | 69 | L3 | L2 | L2 | L2 | L1 | L2 |
| 4 | L5 | L4 | L2 | L1 | L1 | L2 |  | 70 | L5 | L4 | L3 | L2 | L2 | L1 |
| 5 | L1 | L2 | L3 | L2 | L2 | L2 |  | 71 | L4 | L4 | L5 | L3 | L1 | L2 |
| 6 | L4 | L1 | L3 | L1 | L1 | L1 |  | 72 | L1 | L1 | L1 | L1 | L1 | L1 |
| 7 | L1 | L3 | L2 | L2 | L1 | L2 |  | 73 | L4 | L3 | L4 | L2 | L1 | L2 |
| 8 | L6 | L5 | L1 | L1 | L2 | L1 |  | 74 | L6 | L5 | L2 | L3 | L2 | L1 |
| 9 | L5 | L4 | L5 | L1 | L2 | L2 |  | 75 | L3 | L2 | L3 | L2 | L1 | L1 |
| 10 | L4 | L2 | L5 | L2 | L2 | L1 |  | 76 | L5 | L5 | L2 | L1 | L2 | L2 |
| 11 | L2 | L2 | L1 | L3 | L1 | L1 |  | 77 | L6 | L1 | L3 | L3 | L2 | L1 |
| 12 | L5 | L1 | L1 | L2 | L2 | L2 |  | 78 | L2 | L4 | L1 | L3 | L1 | L2 |
| 13 | L2 | L3 | L5 | L2 | L2 | L2 |  | 79 | L6 | L2 | L5 | L1 | L2 | L1 |
| 14 | L4 | L5 | L3 | L2 | L2 | L2 |  | 80 | L1 | L3 | L4 | L2 | L2 | L1 |
| 15 | L3 | L4 | L2 | L3 | L2 | L2 |  | 81 | L1 | L5 | L1 | L3 | L1 | L1 |
| 16 | L6 | L5 | L3 | L1 | L1 | L2 |  | 82 | L2 | L3 | L4 | L3 | L2 | L1 |
| 17 | L5 | L2 | L1 | L2 | L2 | L2 |  | 83 | L3 | L4 | L5 | L1 | L1 | L1 |
| 18 | L4 | L1 | L4 | L3 | L1 | L1 |  | 84 | L4 | L4 | L3 | L2 | L1 | L2 |
| 19 | L5 | L3 | L2 | L3 | L2 | L2 |  | 85 | L2 | L1 | L2 | L1 | L1 | L2 |
| 20 | L1 | L4 | L5 | L2 | L1 | L1 |  | 86 | L3 | L3 | L2 | L2 | L2 | L1 |
| 21 | L6 | L4 | L4 | L1 | L1 | L1 |  | 87 | L4 | L1 | L1 | L3 | L1 | L2 |
| 22 | L4 | L2 | L3 | L1 | L2 | L2 |  | 88 | L5 | L1 | L4 | L1 | L2 | L1 |
| 23 | L2 | L3 | L1 | L2 | L2 | L1 |  | 89 | L2 | L4 | L5 | L3 | L2 | L1 |
| 24 | L3 | L5 | L5 | L3 | L1 | L2 |  | 90 | L6 | L5 | L5 | L2 | L1 | L1 |
| 25 | L2 | L1 | L3 | L2 | L1 | L2 |  | 91 | L4 | L4 | L1 | L2 | L2 | L2 |
| 26 | L2 | L1 | L5 | L1 | L2 | L1 |  | 92 | L2 | L5 | L2 | L2 | L1 | L1 |
| 27 | L6 | L2 | L2 | L2 | L2 | L2 |  | 93 | L1 | L5 | L3 | L1 | L2 | L1 |
| 28 | L1 | L4 | L1 | L1 | L2 | L2 |  | 94 | L5 | L1 | L5 | L3 | L1 | L1 |
| 29 | L3 | L5 | L4 | L2 | L2 | L1 |  | 95 | L3 | L3 | L4 | L3 | L1 | L2 |
| 30 | L4 | L3 | L3 | L3 | L2 | L2 |  | 96 | L6 | L1 | L4 | L2 | L2 | L1 |
| 31 | L4 | L1 | L5 | L2 | L1 | L2 |  | 97 | L1 | L3 | L3 | L3 | L1 | L1 |
| 32 | L2 | L3 | L2 | L1 | L1 | L1 |  | 98 | L1 | L5 | L5 | L2 | L2 | L2 |
| 33 | L6 | L2 | L4 | L3 | L1 | L2 |  | 99 | L6 | L4 | L1 | L3 | L2 | L1 |
| 34 | L5 | L5 | L1 | L3 | L2 | L1 |  | 100 | L4 | L2 | L2 | L3 | L1 | L2 |
| 35 | L1 | L5 | L2 | L1 | L1 | L1 |  | 101 | L2 | L2 | L2 | L1 | L2 | L1 |
| 36 | L3 | L2 | L5 | L3 | L2 | L2 |  | 102 | L3 | L4 | L4 | L2 | L1 | L2 |
| 37 | L4 | L4 | L2 | L2 | L2 | L1 |  | 103 | L6 | L2 | L3 | L3 | L1 | L1 |
| 38 | L2 | L5 | L3 | L3 | L2 | L2 |  | 104 | L4 | L1 | L1 | L2 | L2 | L1 |
| 39 | L1 | L2 | L4 | L1 | L1 | L2 |  | 105 | L6 | L5 | L5 | L3 | L2 | L2 |
| 40 | L6 | L2 | L1 | L2 | L1 | L2 |  | 106 | L4 | L2 | L1 | L1 | L1 | L1 |
| 41 | L3 | L2 | L1 | L1 | L1 | L2 |  | 107 | L2 | L1 | L2 | L3 | L2 | L1 |
| 42 | L5 | L5 | L3 | L3 | L1 | L1 |  | 108 | L3 | L3 | L5 | L2 | L1 | L1 |
| 43 | L1 | L4 | L4 | L3 | L2 | L2 |  | 109 | L4 | L4 | L4 | L1 | L2 | L1 |
| 44 | L5 | L1 | L2 | L2 | L1 | L1 |  | 110 | L5 | L2 | L3 | L3 | L1 | L2 |
| 45 | L3 | L3 | L3 | L1 | L2 | L1 |  | 111 | L3 | L1 | L3 | L2 | L2 | L2 |
| 46 | L6 | L4 | L5 | L1 | L1 | L2 |  | 112 | L2 | L2 | L5 | L3 | L1 | L2 |
| 47 | L2 | L2 | L3 | L1 | L1 | L2 |  | 113 | L6 | L3 | L4 | L1 | L2 | L2 |
| 48 | L4 | L1 | L2 | L1 | L2 | L2 |  | 114 | L1 | L4 | L2 | L3 | L1 | L1 |
| 49 | L5 | L3 | L1 | L1 | L1 | L2 |  | 115 | L3 | L4 | L1 | L2 | L2 | L1 |
| 50 | L2 | L4 | L4 | L2 | L1 | L1 |  | 116 | L5 | L5 | L4 | L2 | L2 | L2 |
| 51 | L4 | L5 | L4 | L3 | L1 | L2 |  | 117 | L2 | L4 | L3 | L1 | L2 | L1 |
| 52 | L6 | L3 | L3 | L2 | L1 | L1 |  | 118 | L1 | L2 | L1 | L3 | L2 | L1 |
| 53 | L3 | L1 | L1 | L3 | L1 | L1 |  | 119 | L4 | L3 | L5 | L1 | L2 | L1 |
| 54 | L1 | L2 | L5 | L1 | L2 | L1 |  | 120 | L6 | L1 | L2 | L3 | L1 | L2 |
| 55 | L2 | L4 | L2 | L2 | L2 | L2 |  | 121 | L5 | L2 | L5 | L3 | L2 | L1 |
| 56 | L5 | L2 | L2 | L1 | L1 | L1 |  | 122 | L5 | L3 | L3 | L1 | L2 | L1 |
| 57 | L4 | L3 | L1 | L3 | L1 | L1 |  | 123 | L1 | L5 | L4 | L2 | L1 | L2 |
| 58 | L6 | L4 | L3 | L2 | L2 | L2 |  | 124 | L6 | L4 | L2 | L1 | L2 | L1 |
| 59 | L1 | L1 | L5 | L3 | L2 | L2 |  | 125 | L2 | L5 | L1 | L1 | L2 | L2 |
| 60 | L3 | L5 | L4 | L1 | L2 | L2 |  | 126 | L3 | L5 | L1 | L2 | L1 | L2 |
| 61 | L4 | L5 | L5 | L1 | L1 | L1 |  | 127 | L4 | L3 | L2 | L1 | L1 | L2 |
| 62 | L3 | L3 | L1 | L3 | L2 | L2 |  | 128 | L1 | L1 | L2 | L2 | L2 | L1 |
| 63 | L2 | L2 | L4 | L2 | L1 | L2 |  | 129 | L5 | L4 | L4 | L3 | L1 | L1 |
| 64 | L3 | L4 | L3 | L3 | L1 | L1 |  | 130 | L2 | L2 | L3 | L2 | L2 | L1 |
| 65 | L6 | L1 | L1 | L1 | L2 | L2 |  | 131 | L3 | L3 | L5 | L1 | L1 | L2 |
| 66 | L5 | L4 | L1 | L2 | L1 | L1 |  |  |  |  |  |  |  |  |

(c) Suppose that the experimenter decides that this is too many runs. What strategy would you recommend?

A design for main effects could be initially run with only 18 runs which would identify the significant factors. This could be followed by a second experiment that would include the significant factors and levels of interest.

**9.31.** Suppose that you need to design an experiment for 9 factors. Three of these factors are categorical with three levels, and the remaining six are continuous. You are interested in the main effects and the two-factor interactions.

(a) How many runs are required for this experiment?

76 runs are required.

(b) Construct a *D-*optimal design with the minimum number of runs. What are the relative variances of the model parameters?

JMP Output

**Prediction Variance Profile – D-optimal**



(c) Construct an *I-*optimal design with the minimum number of runs. What are the variances of the model parameters? How does this design compare to the *D-*optimal design you constructed in part (b)?

The *I-*optimal design has smaller variances for the factors. *I-*optimal designs minimize variance. For comparison, the *D-*efficiency of the *D-*optimal design is 73.50353, the *D-*efficiency of the *I-*optimal design is 64.37547.

JMP Output

**Prediction Variance Profile – I-optimal**



**9.32S.** Suppose that you need to design an experiment for 5 factors. You are interested in the main effects for all factors, but only the *AB* and *AC* factor interactions.

(a) How many runs are required for this experiment?

8 runs are required.

(b) Construct a *D-*optimal design with the minimum number of runs. What are the relative variances of the model parameters?

JMP Output

**Prediction Variance Profile – D-optimal**



(c) Construct an *I-*optimal design with the minimum number of runs. What are the relative variances of the model parameters? How does this design compare to the *D-*optimal design you constructed in (b)? The designs are the same.

JMP Output

**Prediction Variance Profile – I-optimal (8 runs)**



(d) Construct and *I-*optimal design with 12 runs. How does this design compare to the minimum-run design from part (c)?

The variance has decreased from 0.125 to 0.09375. The *D-*efficiency has decreased from 1 to 95.67926.

JMP Output

**Prediction Variance Profile – I-Optimal (12 runs)**



**9.33.** Construct a minimum run *D-*optimal design for 8 two-level factors. Design this experiment to estimate all main effects and as many two-factor interactions as possible. Compare this design to the 16-run no-confounding design for 8 factors. Which design would you prefer? Why?

JMP Output

**Prediction Variance Profile – 9 run minimum D-Optimal**



JMP Output

**Prediction Variance Profile – 16 run no-confounding design**



The 16 run design is balanced – the variance profiles are very uniform and less than the 9 run *D-*optimal. The *D-*efficiency of the 16 run design is 1.00, the *D-*efficiency for the 9 run design is 93.19834. The nine run design is not bad if 9 runs is all you can afford.

**9.34.** Suppose that you need to conduct an experiment for 4 categorical factors. Factor *A* has 5 levels, factor *B* has 4 levels and the other two factors each have 3 levels. You are interested in all main effects and the two-factor interactions.

(a) How many runs are required?

56 runs are required.

(b) How many runs would you actually recommend?

60 runs are recommended.

(c) Find an appropriate *D-*optimal design.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Run | *A* | *B* | *C* | *D* |  | Run | *A* | *B* | *C* | *D* |  | Run | *A* | *B* | *C* | *D* |
| 1 | L3 | L1 | L1 | L3 |  | 21 | L4 | L2 | L2 | L1 |  | 41 | L3 | L1 | L2 | L1 |
| 2 | L2 | L2 | L2 | L3 |  | 22 | L3 | L4 | L3 | L1 |  | 42 | L5 | L4 | L3 | L2 |
| 3 | L1 | L4 | L2 | L2 |  | 23 | L5 | L1 | L2 | L3 |  | 43 | L3 | L1 | L3 | L2 |
| 4 | L4 | L3 | L2 | L2 |  | 24 | L1 | L4 | L1 | L2 |  | 44 | L4 | L4 | L3 | L2 |
| 5 | L5 | L1 | L3 | L1 |  | 25 | L2 | L1 | L3 | L3 |  | 45 | L3 | L4 | L1 | L2 |
| 6 | L3 | L2 | L1 | L1 |  | 26 | L2 | L3 | L1 | L1 |  | 46 | L1 | L1 | L1 | L3 |
| 7 | L2 | L3 | L2 | L3 |  | 27 | L5 | L2 | L2 | L2 |  | 47 | L5 | L2 | L3 | L1 |
| 8 | L4 | L4 | L2 | L3 |  | 28 | L4 | L4 | L1 | L1 |  | 48 | L5 | L3 | L2 | L2 |
| 9 | L1 | L4 | L1 | L1 |  | 29 | L5 | L4 | L2 | L1 |  | 49 | L1 | L1 | L2 | L2 |
| 10 | L5 | L2 | L1 | L3 |  | 30 | L1 | L3 | L1 | L3 |  | 50 | L3 | L2 | L3 | L3 |
| 11 | L4 | L3 | L3 | L1 |  | 31 | L1 | L1 | L3 | L1 |  | 51 | L3 | L3 | L2 | L1 |
| 12 | L1 | L3 | L2 | L1 |  | 32 | L4 | L2 | L1 | L2 |  | 52 | L2 | L4 | L2 | L2 |
| 13 | L1 | L3 | L3 | L2 |  | 33 | L1 | L2 | L2 | L3 |  | 53 | L4 | L1 | L1 | L1 |
| 14 | L2 | L2 | L1 | L2 |  | 34 | L2 | L1 | L2 | L1 |  | 54 | L5 | L3 | L3 | L3 |
| 15 | L3 | L2 | L2 | L2 |  | 35 | L2 | L4 | L3 | L1 |  | 55 | L3 | L3 | L3 | L3 |
| 16 | L5 | L4 | L1 | L3 |  | 36 | L4 | L3 | L1 | L3 |  | 56 | L2 | L3 | L3 | L2 |
| 17 | L4 | L1 | L2 | L2 |  | 37 | L2 | L4 | L1 | L3 |  | 57 | L5 | L3 | L1 | L1 |
| 18 | L1 | L4 | L3 | L3 |  | 38 | L1 | L2 | L3 | L2 |  | 58 | L2 | L1 | L1 | L2 |
| 19 | L3 | L3 | L1 | L2 |  | 39 | L5 | L1 | L1 | L2 |  | 59 | L1 | L2 | L1 | L1 |
| 20 | L4 | L2 | L3 | L3 |  | 40 | L3 | L4 | L2 | L3 |  | 60 | L4 | L1 | L3 | L3 |